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Physical mechanisms of generation of coherent radiation in an ultrarelativistic free-electron laser

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Contents

Abstract. A systematic comprehensive account is given of the methods and results of theoretical simulation of the dynamics of the processes of collective electromagnetic interaction in a nonequilibrium system formed by an electron beam and an undulator. The physical mechanisms of the phenomena responsible for the emission of stimulated (coherent) radiation are identified for specific nonequilibrium systems belonging to the class of free-electron lasers.

1. Introduction

Studies of the physical nature of the mechanisms responsible for the dynamics of growth of collective radiative instabilities of charged particle fluxes in material media (and in external fields) have been an inherent part of the whole history of modern theoretical microwave electronics right from its foundation (see, for example, Refs [1 – 57]†). The foundation stone of the relevant physical theory was laid down in the forties by the classical papers of Ginzburg [1 – 3] in which the main features of the theory have been formulated and which have retained their essential validity even today. They include primarily the following:

— the dominant role of the elementary effects of the emission of radiation by charges moving in material media and in external fields as primary electromagnetic field sources in the investigated collective radiative instabilities (including those used in microwave electronics);

— the feasibility of a significant increase in the peak power of the radiation emitted by moving charges when use is made of the coherence of elementary radiators;

— a method for the generation of quasimonochromatic bremsstrahlung, based on the use of a spatially periodic external field as the material medium;

— a proposal for a device for realisation of this method of bremsstrahlung generation, called later an undulator (see Refs [3] and [76]).

The possibility of existence of quantitative relationships between the characteristics of elementary effects of the emission of radiation by moving charges and the parameters of collective radiative instabilities of charged particle fluxes in the same material media has been mooted in the very first original papers on a self-consistent theory of instabilities (see, for example, Refs [4 – 13]).

Moreover, some of these and later investigations of this subject have provided a direct quantitative proof of the

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†Here and later we shall cite mainly those communications in which the elementary effects of the emission of radiation by moving charges are used to calculate and interpret collective radiative instabilities of charged particle fluxes. More detailed information on the self-consistent theory of these instabilities can be found in monographs [58 – 75].

existence of such interactions. In particular, an increment in the kinetic cyclotron instability of an electron flux, rotating out of phase in a homogeneous external magnetic field, was first deduced [6] by direct summation of the intensities of noncoherent spontaneous† cyclotron radiation of electron (for details see monographs [14, 26, 29, 32]).

A similar method has since been used to calculate the kinetic increments for a number of other instabilities of nonmonoenergetic electron fluxes, including those due to elementary effects of the Cherenkov excitation of longitudinal waves in an isotropic plasma [11] and also those due to synchrotron (undulator) radiation emitted by electrons in a spatially periodic magnetic field [16, 28].

In the last three and a half decades the same fundamental ideas of Ginzburg [1 – 3] have led to the discovery of physical mechanisms and quantitative relationships between the parameters of a number of key collective hydrodynamic radiative instabilities of high-intensity monoenergetic fluxes of point-like electrons (and of their coherent bunches), on the one hand, and the characteristics of the corresponding elementary effects of the emission of radiation, on the other. For example, a corpuscular approach [12, 17, 25, 42, 52] (see also monographs [18, 74]) has made it possible to account for the physical nature of a transverse instability in resonant linear electron accelerators.

The same method has made it possible to establish a functional relationship between the spectra of spontaneous and stimulated undulator radiation emitted by monoenergetic electron beams in an undulator free-electron laser (FEL) self-oscillator when the gain achieved in the undulator length is relatively low (see Refs [9, 20, 30, 31]). The same method is used in Ref. [23] to explain for the first time the physical nature of a hydrodynamic polarisation instability of a monoenergetic electron beam in an isotropic plasma predicted theoretically earlier [4, 77].

The search for the physical mechanisms responsible for the development of absolute instabilities of travelling free-electron fluxes in resonant electrodynamic structures has not only established the dominant role of spontaneous transition radiation in the development of these instabilities [23, 38, 42, 52, 55], but has also resulted in detection of the hitherto unknown stimulated transition interaction of a uniformly moving charge with the field of a regular (monochromatic) wave [38 – 40, 45, 78].

Finally, the work done in the last few years (see Refs [47 – 49, 53, 54, 56, 57]) has revealed physical mechanisms and quantitative relationships governing the formation of high-intensity coherent radiation in ultrarelativistic undulator FELs operating in the regime of amplification of the undulator radiation emitted by the beam electrons themselves. The need to include the effects of coherence of elementary radiators and of the reaction of the emitted field on the motion of these radiators (resulting in the formation of coherently emitting bunches) has made it necessary to use unconventional methods, first proposed in Ref. [24].

The chief aim and the main final practical results of these investigations [47 – 49, 56, 57] has been the solution of the problem of finding the minimum wavelength of the coherent radiation generated in ultrarelativistic FELs. The physical aspects of this problem are as follows.

An increase in the electron energy $E = m_0c^2\gamma$ (γ is the relativistic factor) can reduce the undulator radiation wavelength $\lambda_{\mathrm{u.r}} = D/2\gamma^2$ down to tens of picometres (when the length of the undulator period D is of the order of a few centimetres and the energy of the beam electrons is $E_{\max} \approx 50$ GeV, which corresponds to the highest Stanford electron linac energy [79]). However, in view of the absence of mirrors capable of sufficiently strong reflection of these wavelengths, the only acceptable way of generating coherent microwave radiation by a high-intensity ultrarelativistic electron beam is to create conditions enabling collective amplification, by an electron beam of the undulator radiation emitted by the beam electrons themselves (SASE mode‡).

In the SASE mode the collective undulator radiation field of the individual electrons groups these electrons into coherently emitting bunches. The resultant coherent radiation of these bunches is called stimulated (see Refs [13, 44]) because, in the linear stage of the development of the corresponding radiative instability, the intensity of the radiation emitted by the bunches is proportional to the square of the amplitude of the radiated field (as in the classical Einstein theory).

The SASE mode has been proposed [80] as a way of generating high-intensity coherent radiation of wavelengths from VUV to soft x-rays. This mode has been described in detail theoretically in the high-gain approximation [81 – 84].

However, the approach used in the formalised theory of FELs [81 – 84] does not make it possible to estimate the range of validity of this approximation, or to find how far one can reduce the wavelength of stimulated (coherent) radiation emitted by FELs in the SASE mode and what are the means for reducing the wavelength.

In fact in accordance with external qualitative criteria a nonequilibrium system formed by an electron beam and an undulator in the SASE mode does not differ in any way from a classical source of incoherent undulator radiation or (which is equivalent) a second-generation source of synchrotron radiation. Under these conditions we naturally face the following question. What parameters, in which direction, and to what extent should they be altered to make this system operate optimally as an FEL in which the actual field of undulator radiation of the beam electrons causes these electrons to form coherently emitting bunches?

In the last few years this question has been tackled by detailed theoretical investigations carried out by two mutually complementary methods (not used hitherto in the FEL theory). The first of them involves a fully scalable analytic simulation (in the dipole approximation) of the dynamics of longitudinal motion of a monoenergetic flux of point-like electrons in the collective field of undulator radiation of each of the individual electrons in the flux. This situation is considered in the laboratory reference system on the basis of the solution of a self-consistent system of the Maxwell equations for the microwave field and the Klimontovich kinetic equation for the beam particles (solved by the method of characteristics; see Refs [56, 57]).

The essence of the second method is analytic and numerical simulation of the quantitative relationships governing the Thomson scattering of electromagnetic

†The spontaneous radiation of moving charges is the name given in classical radioelectronics to those induced, by given current of these charges, solutions of the Maxwell equations which satisfy the radiation conditions at infinity.

‡SASE stands for self-amplification of spontaneous emission.

waves by bunches of charged particles with a given idealised profile. This method has revealed the physical nature of the mechanisms responsible for the generation of coherent radiation by electrons in a monoenergetic beam (in its own reference system; see Refs [53, 54, 85–87]).

The results obtained in this way and the conclusions that follow from a corpuscular theory of undulator FELs agree sufficiently well not only with the corresponding results of the classical formalised theory of strong amplification in FELs in the appropriate limiting cases (see, for example, Refs 73, 88]), but also with analogous results of theoretical quantum electronics (see Refs [89, 90]), and also with an analysis of the available data of experimental investigations, which we reported in Ref. [49].

There are many reasons why a systematic account of the methods and results of the corpuscular theory of FELs is needed, the most important of which are the following:

— as pointed out earlier, only such a theory provides the methodology for a fully scalable theoretical simulation of the processes of radiative interaction of a beam of electrons with an undulator throughout the full range of values of the external parameters of such a nonequilibrium system, ranging from the emission of stimulated coherent radiation, typical of FELs, to that of a classical source of second-generation incoherent undulator radiation;

— only this theory can reveal the contribution made to these processes by the effects of coherence of the undulator radiation emitted by the individual sources (beam electrons), which is the physically dominant contribution to an increase in the rate of radiative deceleration of a high-intensity monoenergetic electron beam in an undulator, compared with the rate of radiative deceleration of each electron of the same energy in the same undulator;

— the reviews and monographs on the theory of FELs published so far have not given sufficient attention to the physical nature and qualitative relationships governing the mechanisms by which stimulated coherent undulator radiation is generated in FELs;

— the relevant scientific data on the key aspects of these relationships have been published only in scattered communications, not specially devoted to FELs, and in monographs of a number of related branches of modern theoretical physics, such as classical electrodynamics [91], quantum electronics [89–90], plasma theory [14, 26, 29, 32, 58, 59, 61–67, 69], theory of charged-particle accelerators [18, 72, 74, 92, 93], and theoretical microwave electronics [60, 68, 94].

In view of the considerable scientific, methodological, and practical interest in the problem of minimisation of the wavelength of coherent radiation emitted by undulator FELs, both in the task of construction of efficient radiation sources and in the identification of the physical mechanisms of the formation of such radiation, we shall provide a systematic general account of all the currently known results of theoretical investigations of this subject. For the sake of maximum clarity in the description of the main physical and methodological aspects of the problem, we shall consider only the simplest model of a non-equilibrium system of this type:

(1) we shall assume that a beam of electrons is monoenergetic, unbounded, and spatially homogeneous, and that its volume density is not too high, so that it is possible to confine the analysis to the limiting case of the Thomson scattering;

(2) we shall also assume that the undulator is unbounded and that its magnetic field is perfectly periodic, circularly polarised, and not too strong, so that the dipole approximation can be used in the description of the dynamics of transverse motion and of the undulator radiation emitted by electrons;

(3) we shall study the radiative instability of a flux of electrons in an undulator only under amplification conditions.

We shall use these assumptions to describe first the classical formalised hydrodynamic theory of an FEL amplifier, based on the simulation of an electron beam by a flux of a charged liquid (Section 2). We shall then present the corpuscular (kinetic) theory of amplification of a regular signal in the same nonequilibrium system, based on simulation of an electron beam by a flux of point-like charged particles (Section 3).

The main quantitative relationships governing the coherent radiative interaction of electrons in a monoenergetic beam, considered in its own reference system, will be revealed by the methods of analytic and numerical simulation of the Thomson scattering of plane electromagnetic waves by bunches of charged particles with a given idealised profile, characterised by just one parameter with the dimensions of length (Section 4).

Finally, in the last part of this review (Section 5), we shall consider the results of the two alternative approaches from which we shall formulate appropriate conclusions.

2. Hydrodynamic theory of undulator radiation from a free-electron laser amplifier

2.1 Physical formulation and initial equations of the problem

We shall consider a monoenergetic beam of electrons of energy $E \equiv m_0 c^2 \gamma$ with a spatially homogeneous equilibrium density n_0. We shall assume that this beam is travelling along the Oz axis of a helical undulator where the magnetic field is

$$\boldsymbol{H}_{\mathrm{w}}(z) \equiv \boldsymbol{H}_0 \cdot \left[\boldsymbol{e}_x \cos(k_{\mathrm{w}} z) + \boldsymbol{e}_y \sin(k_{\mathrm{w}} z) \right] . \qquad (1)$$

Here $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ are unit vectors along the Ox and Oy axes of a Cartesian coordinate system; $k_{\mathrm{w}} \equiv 2\pi/D$ (where D is the length of an undulator period).

We shall also assume that the amplitude H_0 of the magnetic field in the undulator is not too high: the relevant condition is the strong inequality

$$p \equiv \frac{|e| H_0 D}{2\pi m_0 c^2} \ll 1 .$$

In this approximation we need to determine the explicit analytic dependence on the external parameters of the investigated nonequilibrium system, of its main collective characteristic, which is the complex gain experienced by a slow charge space wave in the beam and of the wave of the electromagnetic radiation emitted by the beam at a frequency ω close to the undulator radiation frequency of individual beam electron $\omega_{\mathrm{u.r}} \equiv 2\pi c/\lambda_{\mathrm{u.r}} = 4\pi c \gamma^2/D$.

We shall now describe the methodology used in a self-consistent linear hydrodynamic theory of an undulator-radiation FEL amplifier. We must stress here that this theory simulates an electron beam by a continuous charged liquid moving in the combined field of an undulator described by expression (*1*) and that of amplified electromagnetic radiation which has the electric and magnetic field components $\boldsymbol{E}$ and $\boldsymbol{H}$, respectively.

The initial system of equations describing the physical problem under consideration consists of the hydrodynamic equations of motion and continuity for the Euler velocity v and the Euler volume density n of the beam (liquid):

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = e\left[\boldsymbol{E} + \frac{1}{c} v \times (\boldsymbol{H}_{\mathrm{w}} + \boldsymbol{H})\right], \tag{2}$$

$$\frac{\partial n}{\partial t} + \operatorname{div}\boldsymbol{j} = 0 ,$$

$$\boldsymbol{j} \equiv env, \quad \boldsymbol{p} \equiv m_0 v\gamma, \quad \gamma \equiv \left(1 - \frac{v^2}{c^2}\right)^{-1/2}, \tag{2b}$$

and the Maxwell equations for the corresponding microwave fields $\boldsymbol{E}$ and $\boldsymbol{H}$:

$$\operatorname{curl}\boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{H}}{\partial t}, \qquad \operatorname{curl}\boldsymbol{H} = \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \frac{4\pi}{c}\boldsymbol{j} .$$

We shall consider only one-dimensional collective oscillations (independent of the transverse coordinates x and y) of the system and adopt the linear approximation. We shall represent the velocity v and the beam density n by sums of appropriate equilibrium time-independent quantities and small hf corrections, which vary harmonically with the Euler time t (frequency ω) and have amplitudes that depend only on the longitudinal coordinate z:

$$v_z(z,t) = v_0 + \tilde{v}_{\|}(z)\,\boldsymbol{\Phi}(t), \quad n(z,t) = n_0 + \tilde{n}(z)\,\boldsymbol{\Phi}(t),$$

$$\tilde{v}_{\perp}(z,t) = -\frac{pc}{\gamma}\frac{\boldsymbol{H}_{\mathrm{w}}(z)}{H_0} + \tilde{v}_{\perp}(z)\,\boldsymbol{\Phi}(t),$$

$$\boldsymbol{E}(z,t) = \left[\tilde{\boldsymbol{E}}_{\perp}(z) + \tilde{\boldsymbol{E}}_{\|}(z)\right]\boldsymbol{\Phi}(t),$$

$$\boldsymbol{H}(z,t) = \boldsymbol{H}_{\perp}(z)\,\boldsymbol{\Phi}(t), \quad \boldsymbol{\Phi}(t) \equiv \exp(-\mathrm{i}\omega t),$$

where the subscripts $\|$ and $\perp$ represent the vector components which are, respectively, parallel and perpendicular to the undulator axis.

Substitution of the above functions in the system of equations (2), retention of terms linear in the amplitudes of the hf corrections, and exclusion of the field $\boldsymbol{H}_{\perp}(z)$ gives a system of linear homogeneous differential equations in terms of the total derivatives (with respect to the longitudinal coordinate z) and with coefficients which depend periodically on this coordinate:

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2} + k_0^2\right)\tilde{E}_{\pm} = -4\pi e\mathrm{i}k_0\left[n_0\tilde{\beta}_{\pm}(z) - \tilde{n}\frac{p}{\gamma_0}\exp(\pm\mathrm{i}k_{\mathrm{w}}z)\right],$$

$$m_{\perp}\hat{\mathrm{D}}\tilde{\beta}_{\pm} = \frac{e}{c}\left[\frac{\mathrm{i}}{\omega}\hat{\mathrm{D}}\tilde{E}_{\pm} \pm \mathrm{i}\tilde{\beta}_{\|}H_0\exp(\pm\mathrm{i}k_{\mathrm{w}}z)\right],$$

$$m_{\|}\hat{\mathrm{D}}\beta_{\|} = \frac{e}{c}\left\{\frac{\mathrm{i}}{2k_0}\frac{p}{\gamma_0}\left[\frac{\mathrm{d}\tilde{E}_{+}}{\mathrm{d}z}\exp(-\mathrm{i}k_{\mathrm{w}}z) + \frac{\mathrm{d}\tilde{E}_{-}}{\mathrm{d}z}\exp(+\mathrm{i}k_{\mathrm{w}}z)\right]\right.$$

$$\left. +\tilde{E}_{\|} + \frac{\mathrm{i}}{2}H_0\left[\tilde{\beta}_{+}\exp(-\mathrm{i}k_{\mathrm{w}}z) - \tilde{\beta}_{-}\exp(+\mathrm{i}k_{\mathrm{w}}z)\right]\right\}, \tag{3a}$$

$$\hat{\mathrm{D}}\tilde{n} + cn_0\frac{\mathrm{d}\tilde{\beta}_{\|}}{\mathrm{d}z} = 0 .$$

The following notation is used above:

$$\hat{\mathrm{D}} \equiv -\mathrm{i}\omega + v_0\frac{\mathrm{d}}{\mathrm{d}z}, \qquad \tilde{E}_{\pm} = \tilde{E}_x \pm \mathrm{i}\tilde{E}_y ,$$

$$\tilde{\beta}_{\pm} \equiv \frac{\tilde{v}_x \pm \mathrm{i}\tilde{v}_y}{c}, \quad \tilde{\beta}_{\|} \equiv \frac{\tilde{v}_{\|}}{c}, \quad k_0 \equiv \frac{\omega}{c}, \tag{3b}$$

$$m_{\perp} \equiv m_0\gamma_0, \qquad m_{\|} \equiv m_0\gamma_0^3 .$$

The periodic dependence of the coefficients of the system of equations (3a) on the longitudinal coordinate is a consequence of the periodic structure of the magnetic field in the undulator and it disappears in the limit $H_0 \to 0$. It is this inhomogeneity of the medium (of the field $\boldsymbol{H}_{\mathrm{w}}$) that ensures the interaction of normal waves of the nonequilibrium beam–undulator system that correspond to this limit. We shall now classify these waves and calculate the gain for small but finite values of the parameter p.

2.2 Spectrum of normal waves of the system and analytic asymptotes of the gain

In classification of the spectrum of normal waves of the system of equations (3) we shall consider the case $H_0 = 0$ when all the coefficients of this system become constant. The system then splits into two independent groups of equations corresponding to two types of waves. The first two equations yield the spectrum of transverse circularly polarised electromagnetic waves with the dispersion described by

$$k_{\mathrm{t}}^2 = k_0^2 - \frac{\omega_{\mathrm{b}\perp}^2}{c^2}, \qquad \omega_{\mathrm{b}\perp}^2 \equiv \frac{4\pi e^2 n_0}{m_{\perp}}, \tag{4a}$$

and the last three equations give the spectrum of longitudinal charge density waves with the dispersion

$$k_{\mathrm{l}} \equiv \frac{\omega}{v_0} \pm \frac{\omega_{\mathrm{b}\|}}{v_0}, \qquad \omega_{\mathrm{b}\|}^2 \equiv \frac{\omega_{\mathrm{b}\perp}^2}{\gamma_0^2} . \tag{4b}$$

On the right of expressions (4a) and (4b) we have retained only the leading terms of the relevant expansions in terms of the volume density of the beam, which here and later we shall regard as relatively small.

In the case of finite values of the amplitude H_0 of the undulator field, which acts as the pump wave (see Refs [62, 32]), the transverse (t) and longitudinal (l) waves in this nonequilibrium system are no longer independent and begin to interact with one another.

The interaction of the transverse and longitudinal waves becomes strongest under the condition of a resonance which corresponds to the law of conservation of the momentum of the interacting waves:

$$k_{\mathrm{l}}^{(0)} = k_{\mathrm{t}}^{(0)} + k_{\mathrm{w}} . \tag{4c}$$

In this case, if the particle flux density in the beam is relatively low, the wave number of space charge waves $k_{\mathrm{l}}^{(0)} \equiv \omega_*/v_0$ is exactly equal to the sum of the wave numbers $k_{\mathrm{t}}^{(0)} \equiv \omega_*/c$ and $k_{\mathrm{w}} \equiv 2\pi/D$ of a transverse electromag-netic wave E_- and the undulator field. The frequency ω_*, which satisfies this law, is exactly equal to the frequency $\omega_{\mathrm{u.r}}$ of the undulator radiation in the same undulator: $\omega_* = \omega_{\mathrm{u.r}} = 4\pi c\gamma_0^2/D$.

The intensity of this collective three-wave interaction can be estimated directly from the initial system of equations (3) by the substitutions

$$\tilde{E}_-(z) = \frac{m_0 c^2}{|e|D}\, a_-(z) \exp(\mathrm{i}k_{\mathrm{t}}^{(0)} z) \,,$$
$$\tilde{\beta}_-(z) = b_-(z) \exp(\mathrm{i}k_{\mathrm{t}}^{(0)} z) \,,$$
$$\tilde{\beta}_{||}(z) = g(z) \exp(\mathrm{i}k_1^{(0)} z) \,, \tag{5}$$
$$\tilde{n}(z) = n_0 h(z) \exp(\mathrm{i}k_1^{(0)} z) \,.$$

The resultant system of equations for the required dimensionless amplitudes $a_-(\xi)$, $h(\xi)$, and $g(\xi)$ becomes

$$\frac{\mathrm{d}a_-}{\mathrm{d}\xi} = -\frac{2\pi r_0 n_0 D^2 p}{\gamma_0}\, h(\xi)\,, \tag{6a}$$

$$\frac{\mathrm{d}h}{\mathrm{d}\xi} = -\mathrm{i}k_1^{(0)} D\, g(\xi) \,, \tag{6b}$$

$$\frac{\mathrm{d}g}{\mathrm{d}\xi} = -\frac{p}{2\gamma_0^4}\, a_-(\xi) \,, \tag{6c}$$

where $r_0 \equiv e^2/m_0 c^2$ is the classical electron radius and $\xi \equiv z/D$.

The system of equations (6) is equivalent to one differential equation in terms of the third-order total derivatives. We shall rewrite this system in terms of the depth of modulation $h(\xi)$ of the beam density, which represents the grouping of the particles into coherently emitting bunches (see, for example, Refs [83–88]):

$$\left(\frac{\mathrm{d}^3}{\mathrm{d}\xi^3} - \mathrm{i}\kappa^3\right) h(\xi) = 0\,, \qquad \kappa^3 \equiv \frac{4\pi^2 r_0 n_0 D^2 p^2}{\gamma_0^3} \,. \tag{7a}$$

Eqn (7a) has a solution which arises exponentially with the coordinate ξ:

$$h(\xi) = \mathrm{const}\exp\left\{\kappa\xi \exp\frac{\pi \mathrm{i}}{6}\right\} , \tag{7b}$$

where the gain is

$$\Gamma_{\max} = \frac{\sqrt{3}}{2}\,\kappa \,. \tag{7c}$$

It is this solution that describes the amplification of a slow space charge wave in the beam. The phase velocity of this wave is less than the equilibrium beam velocity v_0:

$$v_{\mathrm{ph}} \equiv \frac{\omega_{\mathrm{u.r}}}{k_1^{(0)} + \kappa/2D} = \frac{v_0}{1 + \kappa/8\pi\gamma_0^2} \,. \tag{7d}$$

It follows from Eqn (6a) that the same exponential law is obeyed not only by this space charge wave, but also by the rise of the amplitude a_- of the transverse electromagnetic wave. It is this feature that distinguishes decisively the parametric amplifier with distributed parameters discussed above, from a spatially homogeneous amplifier such as a travelling-wave tube in which the exponential rise of the ampli-tude is observed only for a slow space charge wave [94].

If the law of conservation of the momentum (4c) is not obeyed, so that the working frequency of the investigated FEL amplifier is not equal to the frequency of the undulator radiation of the beam electrons ($\omega = \omega_{\mathrm{u.r}} + \Delta$, where $|\Delta| \ll \kappa\omega_{\mathrm{u.r}}$), the gain Γ is less than the maximum value described by formula (7c) and it falls as the modulus $|\Delta|$ increases.

In fact, if $|\Delta| \gg \kappa\omega_{\mathrm{u.r}}$, the first equation of the system (3) yields not a differential but an algebraic relationship between the beam modulation depth h and the amplitude E_- of the electromagnetic field:

$$E_- = \frac{4\pi e \mathrm{i} k_0 n_0 p \tilde{h}}{\left[k_0^2 - (k_1^{(0)} - k_{\mathrm{w}})^2\right]\gamma_0} \exp\left[\mathrm{i}(k_1^{(0)} - k_{\mathrm{w}})z\right] . \tag{8a}$$

Substitution of expression (8a) on the right of the third equation in the system (3) and simple algebraic transformations give a system, analogous to the system (6), of two homogeneous second-order differential equations for the depths of modulation of the beam in respect of the velocity ($\tilde{g}$) and density ($\tilde{h}$):

$$\frac{\mathrm{d}\tilde{g}}{\mathrm{d}\xi} = \frac{2\pi e^2 \mathrm{i} n_0 p^2 k_1^{(0)} D}{\left[k_0^2 - (k_1^{(0)} - k_{\mathrm{w}})^2\right]\gamma_0}\, \tilde{h}\,, \qquad \frac{\mathrm{d}\tilde{h}}{\mathrm{d}\xi} = -\mathrm{i}k_1^{(0)} D\tilde{g} \,. \tag{8b}$$

The condition of absence of the trivial (zeroth) solution of this system yields an analytic formula for the gain in one period (see Ref. [88]):

$$\Gamma(\Delta) \equiv \left(\frac{\kappa^3}{2\pi}\,\frac{\omega_{\mathrm{u.r}}}{\Delta}\right)^{1/2} , \qquad \omega_{\mathrm{u.r}} \gg \Delta \gg \kappa\omega_{\mathrm{u.r}} \,. \tag{8c}$$

2.3 Analysis of the physical meaning of the results

Expressions (7) and (8) represent the essence of the familiar (see, for example, Refs [70, 71, 75, 88]) results of the self-consistent linear hydrodynamic theory of an undulator-radiation FEL amplifier. However, the results given above are obtained by a formalised method of weakly coupled parametrically interacting normal waves in the nonequilibrium beam–undulator system. Therefore, they do not give direct answers to a number of questions of methodological importance, which relate to the physical nature of the mechanism of collective amplification of coherent undulator radiation emitted by individual electrons. This is not surprising, because our liquid model does not deal with the contribution of each individual radiator and the final theoretical results contain only the collective parameters of these radiators, which are the volume particle density n, the charge $\rho_{\mathrm{e}} \equiv ne$, and the mass $\rho_{\mathrm{m}} \equiv nm_0$.

In the case of microwave undulator-radiation FEL amplifiers operating in the SASE regime, which are considered here, the most important are the following two questions:

(1) What are the mechanisms and the conditions in the investigated nonequilibrium system which can ensure the emission of stimulated coherent undulator radiation by elementary radiators?

(2) What is the role of the undulator radiation emitted by individual radiators in the transfer of kinetic energy of the beam to the field by the emitted electromagnetic radiation in an FEL amplifier?

The arguments used in answering these two questions relate them. Nevertheless, separate consideration of the questions makes it possible to understand better the physical content of the questions and to suggest ways of finding the appropriate quantitatively justified answers.

The first of these questions is of methodological importance because the answer to this question makes it possible to identify the conditions and limits of the validity of the formalised hydrodynamic linear self-consistent theory of an FEL amplifier or, which is equivalent, the conditions for a transition of our nonequilibrium system from a source of incoherent undulator radiation to the SASE mode (see Section 1). We can intuitively assume that, in the space of the external parameters of the system, the range of validity of the method and of the results of the hydrodynamic theory is limited from below by the volume density of the beam particles n_0 and from above by the electron energy γ_0.

This hypothesis is based mainly on the fact that simulation of a beam by a liquid can be justified physically only if the number of the beam particles (in a region with the characteristic size of the order of the undulator radiation wavelength $\lambda'_{\mathrm{u.r}}$ in the beam rest frame), $Q' \equiv n'_0 \lambda'^3_{\mathrm{u.r}} = n_0 D^3/8\gamma_0^3$, is sufficiently large compared with unity: $Q' \gg 1$. The last condition was formulated in Refs [47–48] and can be argued additionally by comparing the dependences, on the beam energy γ_0, of the total energy fluxes of the stimulated (coherent) radiation generated in the SASE mode and of the incoherent undulator radiation emitted by the beam.

It follows in fact from the nonlinear theory of an FEL amplifier (see, for example, Refs [73, 83, 84]) that the electron efficiency η_{e} is of the order of the maximum gain $\Gamma_{\max}$ of this amplifier:

$$\eta_{\mathrm{e}} \simeq \Gamma_{\max} \simeq \frac{\Gamma_0}{\gamma_0} \,. \tag{9a}$$

It follows from formula (9a) that, in the case of a beam of given volume density in an undulator with a given period D and a fixed amplitude H_0 of the magnetic field, the power of the coherent radiation emitted in the SASE mode is independent of the beam energy γ_0:

$$P_{\mathrm{coh}}(\gamma_0) \approx J_0 m_0 c^2 \Gamma_0 \,, \tag{9b}$$

where J_0 is the total flux of the beam particles crossing the undulator ($[J_0] = \mathrm{s}^{-1}$) and the calculated undulator length $L_{\mathrm{opt}} \approx D/\Gamma_{\max}$) increases proportionately to γ_0.

On the other hand, for the same optimal undulator length L_{opt}, the power P_{incoh} of the incoherent undulator radiation emitted by this beam increases proportionately to γ_0^3:

$$P_{\mathrm{incoh}} = J_0 |F_z^{\mathrm{rad}}|_1 L_{\mathrm{opt}} = J_0 \frac{4\pi}{\Gamma_0} e^2 p^2 k_{\mathrm{w}}^1 \gamma_0^3 \,, \tag{9c}$$

because the force $|F_z^{\mathrm{rad}}|_1$ of the radiative deceleration of an electron by the field of its undulator radiation is proportional to γ_0^2 [91].

The two main conclusions follow from the dependence plotted in Fig. 1:

(1) neglect of the contribution of the incoherent undulator radiation to the dynamics of the processes of motion and emission of radiation by the beam particles in an undulator is permissible only in the range of beam energies, limited from above, where the strong inequality $P_{\mathrm{incoh}}(\gamma_0) \ll P_{\mathrm{coh}}$ is obeyed;

(2) a satisfactory fully scalable theoretical simulation of these processes requires dropping the main initial assumption in the hydrodynamic theory of an FEL amplifier, which is that an electron beam can be simulated by a flux of a continuous liquid and, instead, this beam should be considered as a flux of point-like radiators (see Refs [47, 48, 56, 57]).

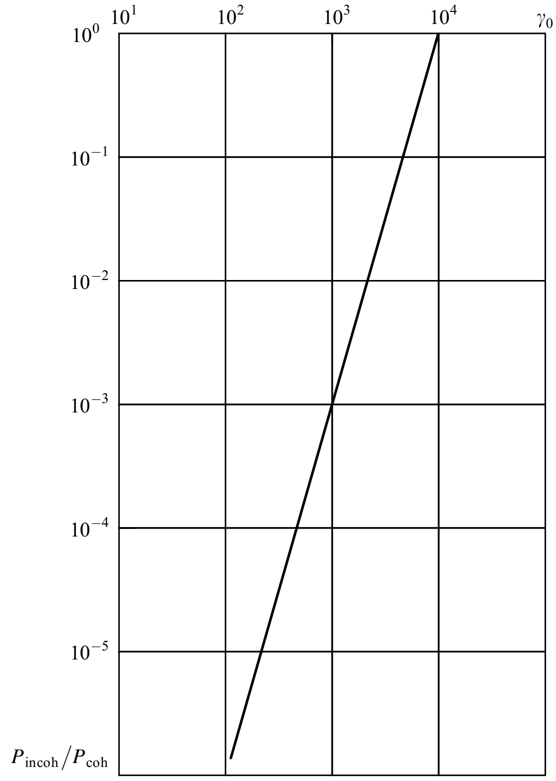

Figure 1. Logarithmic dependence of the ratio of the coherent and incoherent losses $P_{\mathrm{incoh}}/P_{\mathrm{coh}}$ in an FEL on the beam energy γ_0 [$\Gamma_0 = 1$, $r_0 = 10^{-13} D$, $\log(8\pi^2 p^2/3) = 1$].

The last conclusion follows also from the attempts to find a reasoned answer to the second of the questions formulated above. In fact, the dominant role of the undulator radiation of individual electrons in the process of collective amplification of coherent electromagnetic radiation by these electrons is not in doubt, because in the system under consideration the conditions for alternative mechanisms of the emission of radiation by moving charges are not satisfied. The reasons why these conditions are not satisfied are as follows: there is no medium that would ensure a reduction in the phase velocity of electromagnetic waves, which is essential for the emission of spontaneous Vavilov–Cherenkov radiation (and/or the appearance of the anomalous Doppler effect); there are no electrodynamic inhomogeneities of the medium needed for the emission of spontaneous and stimulated transition radiation by uniformly moving beam electrons [19, 23, 38, 39].

On the other hand, the important role of the undulator radiation emitted by individual beam electrons in the mechanism of the radiative instability of the beam under amplification conditions which is considered here, is supported also by the existence of a maximum of the gain Γ at the frequency $\omega_* = \omega_{\mathrm{u.r}}$ [see expressions (7) and (8)]. Nevertheless, the hydrodynamic theory of an FEL amplifier does not provide a direct proof, based on quantitative results, to the answer just stated.

We shall conclude this section by mentioning that the problem in hand is also of interest because of the familiar vanishing at the frequency $\omega_* = \omega_{\mathrm{u.r}}$ of the intensity of stimulated radiation generated in a relatively short undulator-radiation FEL oscillator excited by a monoenergetic initially unmodulated electron flux.

The last result has been obtained earlier by several methods [20, 30, 31], including summation of the undulator radiation losses of individual electrons [30]. However, the existence of a difference between the positions of the maxima of the intensity of stimulated radiation emitted by an FEL amplifier and by an FEL oscillator on the frequency axis has not been discussed. This difference will be explained later (Section 3.4) on the basis of an analysis of the specific features of the mechanisms of the stimulated interaction between the beam electrons and the undulator radiation fields emitted by them.

We can summarise the above discussion by concluding that complete explanation of the conditions and quantitative relationships for realisation of the mechanisms of stimulated coherent radiative interaction of an electron beam with an undulator operating in the SASE mode, and explanation of the physical nature of these mechanisms, requires a change from the simulation of a beam of electrons by a flux of a continuous liquid to be simulation of an electron beam by a flux of point-like electrons. The methodological approach and the analytic results of a self-consistent corpuscular (kinetic) theory of an undulator-radiation FEL amplifier given below represents generalisation of the published work on this subject.

3. Corpuscular theory of an undulator free-electron laser amplifier

3.1 Physical model and general scheme of the analysis method

The physical model of a nonequilibrium beam – undulator system considered below is completely identical with the model adopted above. We are still assuming that the electron beam is monoenergetic and spatially homogeneous and that the magnetic field in the undulator is described by expression (1). The final aim is still the same: calculation of the functional dependence of the gain on the external parameters of the system. The only difference is the method used to achieve this aim. Dropping the simulation of a beam by a continuous liquid, we shall represent it by a flux of point-like electrons moving in the combination of the magnetic field in the undulator described by expression (1) and of the resulting undulator radiation field of all the individual beam electrons.

We shall follow Refs [56, 57] and use first the dipole approximation for the explicit configuration of the field emitted by a single electron (see Section 3.2), and we shall then find analytic asymptotes of the longitudinal force exerted on the beam by the total undulator radiation of all the electrons. We shall then obtain and solve analytically the equations of motion of the beam electrons experiencing this force (Section 3.3), and we shall calculate the gain.

3.2 Pattern of the field of a relativistic electron in a helical undulator

Let us assume that an electron labelled by the number s enters an undulator at $z = 0$ at an initial moment t_s, and travels in the positive direction of the Oz axis in the field described by expression (1). The trajectory of this electron is

$$\boldsymbol{R}_s(\boldsymbol{r}, t; q_s) = \boldsymbol{r}_s + a_{\mathrm{w}}\left[\boldsymbol{e}_y \cos(k_{\mathrm{w}} z) - \boldsymbol{e}_x \sin(k_{\mathrm{w}} z)\right] + \boldsymbol{e}_z z_s(\tau, q_s) \; . \qquad (10)$$

The following notation is used here for an electron: $\boldsymbol{r}_s \equiv \boldsymbol{e}_x x_s + \boldsymbol{e}_y x_s$ is the radius vector in the $z = 0$ plane (at $t = t_s$); $\boldsymbol{q}_s \equiv (\boldsymbol{r}_s, t_s)$ is the set of the input parameters; $\boldsymbol{R}_s \equiv \boldsymbol{e}_x X_s + \boldsymbol{e}_y Y_s + \boldsymbol{e}_z Z_s$ is the Lagrangian trajectory [where $Z_s(\tau, q_s) \equiv v_0\tau + \Delta(\tau, q_s)$, where $\tau \equiv t - t_s$ is the Lagrangian time, $\Delta(\tau, q_s)$ is the longitudinal displacement of the electron relative to its equilibrium trajectory $v_0\tau$, $\Delta(0, q_s) = 0$]; $a_{\mathrm{w}} \equiv \omega_{\mathrm{w}}/k_{\mathrm{w}}^2 v_0 \gamma_0$ (where $\omega_{\mathrm{w}} \equiv |e|H_0/m_0 c$, $\gamma_0 \equiv (1 - v_0^2/c^2)^{-1/2}$, $|e|$ and m_0 are the absolute charge and rest mass of the electron, respectively).

We now have to find explicit analytic forms of the functional dependences of the pattern of the electron field on the external parameters of the system, which are $\boldsymbol{r}_s$, γ_0, $p \equiv |e|H_0 D/2\pi m_0 c^2$.

3.2.1 Configuration of the total field of a relativistic electron

We shall consider the explicit form of the field created by an electron as it moves along the trajectory described by formula (10). The initial expression for the field of the retarded potential of a point charge, which moves at a velocity $v(t)$ and experiences an acceleration $\dot{v}$ (see Ref. [91]):

$$\boldsymbol{E}(\boldsymbol{r}, t) = \frac{e}{(R' - \boldsymbol{\beta}'\cdot\boldsymbol{R}')^3}\left\{(1 - \beta'^2)(\boldsymbol{R} - \boldsymbol{\beta}' R') + \frac{1}{c^2}\left[\boldsymbol{R}' \times \left((\boldsymbol{R}' - \boldsymbol{\beta}' R') \times \dot{v}'\right)\right]\right\}, \qquad (11a)$$

$$\boldsymbol{H}(\boldsymbol{r}, t) = \frac{\boldsymbol{R}' \times \boldsymbol{E}}{R'}\,, \qquad (11b)$$

$$t' + \frac{R'}{c} = t\,, \quad \boldsymbol{R}' \equiv \boldsymbol{r} - \boldsymbol{R}_s(t', q_s)\,, \qquad (11c)$$

where the primes identify the functions of the retarded time t'.

The first term in Eqn (11a) in the limit $R' \to \infty$ decreases inversely proportionately to R^2 and remains finite in the limit $c \to \infty$. Hence, this term describes the quasistatic field of a uniformly moving charge corresponding to the Lienard – Wiechert potential (see Ref. [91]).

The second term in Eqn (11a), which decreases at infinity in proportion to the amplitude of a spherically diverging wave and is also proportional to the charge acceleration, describes the bremsstrahlung field of the charge generated by its accelerated motion [91].

In general, the formulas in the system (11) are cumbersome and not very suitable for practical application because of the complexity of the functional dependences of the retarded time t' and of the radius vector $\boldsymbol{R}'$ on the four-dimensional Euler coordinate of the point of observation $(\boldsymbol{r}, t)$. We shall therefore use a number of additional assumptions which make it possible to simplify greatly these dependences.

First of all, we shall limit our discussion to ultrarelativistic energies of a charge ($\gamma_0^2 \gg 1$). Moreover, we shall adopt the dipole approximation (when the strong inequality $p \equiv |e|H_0 D/2\pi m_0 c^2 \ll 1$ is satisfied) in the investigation of the configuration of the field described by the system (11) and we shall seek explicit dependences of the right-hand sides on the coordinates $\boldsymbol{r}$ and $\boldsymbol{r}_s$, and on time t by the method of iteration in terms of the small parameter p.

Under these conditions the main contribution to the electromagnetic field, created by a charge in the undulator, contributes terms which are linear in the parameter p. In view of the relative smallness of this parameter, we shall represent the vector $\boldsymbol{R}'$ in formulas (11) by two terms. The first of these terms will be calculated in the zeroth approximation with respect to p, which corresponds to uniform rectilinear motion of a charge along the undulator axis. The second (linear in p) term is due to the accelerated motion of a charge in the undulator.

The initial equation now becomes

$$\boldsymbol{R}' - \boldsymbol{\beta}_0 R' = \boldsymbol{R}_s'(t,\, q_s)\,. \tag{12a}$$

Its solution for the parameter $R' \equiv |\boldsymbol{r} - \boldsymbol{R}_s(t',\, q_s)|$, accurate to terms linear in p, is

$$R' = \gamma_0^2\left[(\boldsymbol{\beta}_0\cdot\boldsymbol{R}_0) + R_*\right] - \frac{\rho a_{\mathrm{w}}}{R_*}\sin\chi\,. \tag{12b}$$

The following notation is used above:

$$R_* \equiv \left((\delta z)_s^2 + \frac{\rho^2}{\gamma_0^2}\right)^{1/2}, \qquad (\delta z)_s \equiv z - Z_s(\tau,\, q_s)\,,$$

$$\chi \equiv \varphi - \vartheta\,, \qquad \vartheta \equiv k_{\mathrm{w}} z_0 = k_{\mathrm{w}}\left\{z - \gamma_0^2\left[(\delta z)_s + \beta_0 R_*\right]\right\},$$

$$\rho \equiv \left[(x - x_s)^2 + (y - y_s)^2\right]^{1/2},$$

$$\varphi = \arctan\frac{y - y_s}{x - x_s}\,,$$

$$\boldsymbol{R}_0(\tau) = \boldsymbol{e}_x(x - x_s) + \boldsymbol{e}_y(y - y_s) + \boldsymbol{e}_z(\delta z)_s\,.$$

Substitution of expressions (4a) and (12b), and of the expression for the velocity on the right-hand side of formula (11a), followed by an expansion as a series in terms of the small parameter p, gives the explicit form of the total field of a moving electron in the undulator, which consists of two terms. One term describes the radiation field of the charge (r) and the second represents its quasistatic field (q):

$$\begin{aligned}\boldsymbol{E}^{(\mathrm{r})}(\boldsymbol{r},\, t;\, q_s) = \frac{|e|\beta_\perp}{R_*}\Bigg\{&\frac{\rho\boldsymbol{R}_0}{R_*^2\gamma_0^2}\left[\frac{3}{R_*}\cos\chi + k_0\left(1 - \frac{3}{(k_0R_*)^2}\right)\sin\chi\right]\\ &- \mathrm{Re}\left[(\boldsymbol{e}_x - \mathrm{i}\boldsymbol{e}_y)\exp(\mathrm{i}\vartheta)\right]\\ &\times\left[\frac{1}{R_*}\left(1 + \beta_0\frac{(\delta z)_s}{R_*}\right) + \mathrm{i}k_0\left(1 + \beta_0\frac{(\delta z)_s}{R_*} - \frac{1}{(k_0R_*)^2}\right)\right]\Bigg\}\,,\end{aligned} \tag{13a}$$

$$\boldsymbol{E}^{(\mathrm{q})}(\boldsymbol{r},\, t;\, q_s) = -\frac{|e|\boldsymbol{R}_0}{R_*^3\gamma_0^2}\,, \tag{13b}$$

where $\beta_\perp \equiv v_\perp/c = p/\gamma_0$, $k_0 \equiv k_{\mathrm{w}}\beta_0\gamma_0^2$.

In the case of a charge with the initial coordinates $t = t_s$, $x = x_s$, $y = y_s$, $z = 0$, injected into a region $z > 0$, the asymptotes of the fields described by expressions (13) are valid only in the range of influence of the field of the retarded radiation emitted by the charge and governed by the inequality $c(t - t_s) > (z^2 + \rho^2)^{1/2}$.

Therefore, the structure of the field described by expressions (13) takes account of the finite propagation time of the front of the field of the charge itself, which differs from zero only at the points of observation located within the region of influence of the field with the boundary defined by† $(z^2 + \rho^2)^{1/2} = c(t - t_s)$.

It follows from expression (13a) that the phase of the field ϑ, radiated by a charge in the direction of its helical path in the undulator $(z > Z_s,\ \rho = 0,\ \Delta = 0)$, is a linear function of the Euler coordinate z of the point of observation and of the Euler time t:

$$\vartheta(z > Z_s,\ \rho = 0) = \omega_+\left(t - t_s - \frac{z}{c}\right). \tag{14}$$

This means that the radiation field travelling forward along this axis is a plane wave of frequency

$$\omega_+ \equiv \frac{k_{\mathrm{w}}v_0}{1 - \beta_0}\,, \tag{14a}$$

whose wavelength is

$$\lambda_+ \equiv D\,\frac{1 - \beta}{\beta_0} \approx \lambda_{\mathrm{u.r}} = \frac{D}{2\gamma_0^2}$$

and the phase velocity is equal to the velocity of light c:

$$v_{\mathrm{ph}}^{(+)} = \frac{\omega_+}{k_+} = c\,, \qquad k_+ \equiv \frac{\beta_0}{1 - \beta_0}\,k_{\mathrm{w}}\,. \tag{14b}$$

Expressions (13a) and (13b) also give the phase, frequency, and phase velocity of the radiation field emitted in the backward direction $(z < Z_s,\ \rho = 0)$:

$$\vartheta(z < Z_s,\ \rho = 0) = \omega_-\left(t - t_s + \frac{z}{c}\right), \tag{15}$$

$$\omega_- \equiv \frac{k_{\mathrm{w}}v_0}{1 + \beta_0} \approx \frac{\omega_+}{4\gamma_0^2}\,, \tag{15}$$

$$v_{\mathrm{ph}}^{(-)} = -\frac{\omega_-}{k_-} = -c\,, \qquad k_- \equiv \frac{\beta_0}{1 + \beta_0}\,k_{\mathrm{w}}\,. \tag{15b}$$

Moreover, it is evident from expressions (13) that an increase in the distance ρ from the charge path axis reduces the fields generated by the charge at a rate γ_0 times slower than along the axis. Hence, it follows in particular that the characteristic size of the region of influence of the field (where a charge can emit coherently with its neighbours) in the transverse direction is γ_0 times greater than in the longitudinal direction.

3.2.2 Configuration of the radiation field

It follows from expression (10) that the trajectory of an electron in an undulator is a helix with the radius a_{w} and the period D. Therefore, in an analysis of the topography of the radiation field of an electron it is convenient to use a cylindrical system of coordinates in which the Oz axis coincides with the axis of the helical orbit: $x = x_s + \rho\cos\varphi$, $y = y_s + \rho\sin\vartheta$. According to expression (10), the transverse coordinates of an electron in this system become

$$\rho_s = a_{\mathrm{w}}\,, \qquad \varphi_s(\tau,\, q_s) = k_{\mathrm{w}}Z_s(\tau,\, q_s) + \frac{\pi}{2}\,, \qquad v_{\varphi s} = v_\perp\,. \tag{16}$$

Transformation of the right-hand sides of expression (13a) and of the corresponding expression for $\boldsymbol{H}$

†Strictly speaking, this is true only when an electron–positron pair is created at the entry to the undulator and the electron which then appears begins to move along the undulator axis. The electromagnetic radiation field of the charge and its quasistatic field then have a shared boundary of the region of their influence, which is defined by $c(t - t_s) > (z^2 + \rho^2)^{1/2}$.

[see formula (11b)] to the same system of coordinates gives expressions for the field components which govern the radiative energy losses experienced by the charge:

$$H_\rho^{(r)} = -\frac{|e|\beta_\perp}{R_*}\left[\frac{1}{R_*}\left(\beta_0 + \frac{(\delta z)_s}{R_*}\right)\sin\chi - k_0\left(\beta_0 + \frac{(\delta z)_s}{R_*} - \frac{\beta_0}{(k_0R_*)^2}\right)\cos\chi\right], \quad (17a)$$

$$E_\varphi^{(r)} = \frac{|e|\beta_\perp}{R_*}\left[\frac{1}{R_*}\left(1 + \beta_0\frac{(\delta z)_s}{R_*}\right)\sin\chi - k_0\left(1 + \beta_0\frac{(\delta z)_s}{R_*} - \frac{1}{(k_0R_*)^2}\right)\cos\chi\right], \quad (17b)$$

$$E_z^{(r)} = \frac{|e|\beta_\perp\rho}{R_*^3\gamma_0^2}(\delta z)_s\left[\frac{3}{R_*}\cos\chi + k_0\left(1 - \frac{3}{(k_0R_*)^2}\right)\sin\chi\right]. \quad (17c)$$

It should be stressed that expressions (17), like the initial formulas (13), are obtained in a linear approximation with respect to the parameter $p \ll 1$, and are valid equally in the near-field and far-field zones of the radiation. In particular, it follows from these formulas that the longitudinal component of the electric field $E_z^{(r)}$ of the radiation vanishes at the point $z = Z_s(\tau, q_s)$, where the charge is located $[(\delta z)_s = 0]$.

The transverse radiation fields can be determined at the point $z = Z_s(\tau, q_s)$ by calculation of the phase $\chi(z = Z_s) = \pi/2 + \gamma_0^2 k_w R_*$ with the aid of expression (12c). Subsequent expansion of the fields $E_\varphi^{(r)}$ and $H_\rho^{(r)}$ containing this phase in expressions (17) as series in R_* for $\gamma_0^2 k_w R_* \ll 1$ gives, in the dipole approximation, explicit expressions for the amplitudes of the transverse radiation fields at the point where the charge is located:

$$\begin{aligned} E_\varphi^{(r)}(z = Z_s, \rho = a_w) &= \frac{2}{3}|e|\beta_\perp k_w^2\beta_0^2\gamma_0^4 = \frac{2}{3}|e|\beta_\perp k_0^2, \\ H_\rho^{(r)}(z = Z_s, \rho = a_w) &= -\beta_0 E_\varphi^{(r)}(z = Z_s, \rho = a_w). \end{aligned} \quad (18)$$

It follows from expressions (18) that the rate of change of the charge energy, due to its radiative losses,

$$\left(\frac{dE^{\text{rad}}}{dt}\right)_1 \equiv -|e|(\boldsymbol{v}\cdot\boldsymbol{E}) = -|e|v_\varphi E_\varphi(z = Z_s, \rho = a_s) = -\frac{2}{3}c(r_0 H_w \beta_0\gamma_0)^2 \quad (18a)$$

and the corresponding effective amplitude of the radiation friction force (longitudinal force of deceleration by radiation) acting on the charge

$$(F_z^{\text{rad}})_1 \equiv \frac{v_0}{c^2}\left(\frac{dE^{\text{rad}}}{dt}\right)_1 = -\frac{2}{3}e^2\beta_\perp^2 k_w^2\beta_0^3\gamma_0^4 \quad (18b)$$

are identical with the corresponding classical results (see, for example, Refs [91, 95, 98]).

3.3 Amplification of the field of undulator radiation of a monoenergetic electron beam

When the structure of the undulator radiation field of an individual electron is known, it is possible to find the total field due to a flux of electrons, which acts on every radiator. In general, the total field depends on the parameters of the beam itself and on the selection of a specific FEL regime.

Subject to certain constraints on the beam parameters, the total undulator radiation field of a flux of electrons and the longitudinal bunching force governed by this field can be calculated in an explicit analytic form. In view of the complexity of the structure of the longitudinal force (discussed below), it is useful to limit our analysis to a rigorously monoenergetic nondiverging beam and also to investigate separately the limiting cases of high- and low-intensity beams.

The relevant dimensionless beam intensity parameter is defined above as the number of the beam particles in a cube with its side equal to the wavelength of the emitted wave in the beam rest frame: $Q' \equiv n_0'\lambda_{\text{u.r}}'^3$, where n_0' is the volume density of the beam particles in this system. Since Q' is a scalar, it is independent of the selected reference system and can be expressed as follows in terms of the beam and radiation parameters in the laboratory reference system:

$$Q' = Q = n_0\gamma_0^2\lambda_{\text{u.r}}^3 = n_0\frac{D^3}{8\gamma_0^4}.$$

3.3.1 Longitudinal force of the total field of undulator radiation emitted by beam electrons

The total Lorentz force at the point $\boldsymbol{r}$ at a moment t is equal to the sum of the force exerted by the fields of the individual radiators:

$$\boldsymbol{F}^{(\text{tot})}(\boldsymbol{r}, t) \equiv \sum_s \boldsymbol{F}^{(b)}(\boldsymbol{r}, t; q_s). \quad (19)$$

The summation over s on the right extends only to those electrons whose region of influence contains the investigated point $\boldsymbol{r}$.

We shall be interested only in the longitudinal component of the total Lorentz force, which is responsible for the grouping of electrons into coherently emitting bunches (when $Q \gg 1$). In terms of the Euler variables, the longitudinal force exerted by a radiator with the serial number s on a moving electron, which has the coordinate r at the moment t, is given by

$$F_z^{(b)}(\boldsymbol{r}, t; q_s) = -|e|\Big\{E_z(\boldsymbol{r}, t; q_s) - \beta_\perp \operatorname{Re}\left[H_y(\boldsymbol{r}, t; q_s) + iH_x(\boldsymbol{r}, t; q_s)\right]\exp(ik_w z)\Big\}. \quad (19a)$$

The longitudinal electric field E_z in expression (19a) is given by formula (13a) and the expression for the transverse magnetic field $H_y + iH_x$ can be obtained by utilising formulas (13a) and (11b):

$$\boldsymbol{H} = \boldsymbol{H}^{(r)} + \boldsymbol{H}^{(q)},$$

$$\begin{aligned} H_y^{(r)}(\boldsymbol{r}, t; q) + iH_x^{(r)}(\boldsymbol{r}, t; q) = -|e|\beta_\perp\left[\exp(-i\vartheta)\right] \times \Bigg\{&\frac{1}{R_*^2}\left(\beta_0 + \frac{(\delta z)_s}{R_*}\right) - \frac{ik_0}{R_*}\left(\beta_0 + \frac{(\delta z)_s}{R_*} - \frac{\beta_0}{(k_0R_*)^2}\right) \\ &+ \frac{i\beta_0\rho^2 k_0}{2R_*^3\gamma_0^2}\left[1 - \frac{3}{(k_0R_*)^2} + \frac{3i}{k_0R_*} - \left(1 - \frac{3}{(k_0R_*)^2} - \frac{3i}{k_0R_*}\right)\exp(-2i\chi)\right]\Bigg\}, \end{aligned} \quad (20a)$$

$$H_y^{(q)}(\boldsymbol{r}, t; q_s) + iH_x^{(c)}(\boldsymbol{r}, t; q_s) = -\frac{|e|\beta_0\rho}{R_*^3\gamma_0^2}\exp(-i\varphi), \quad (20b)$$

where the notation is the same as in Section 3.2.

It is evident from the above formulas that in general the configuration of the longitudinal bunching force exerted by a flux of monoenergetic radiators is fairly complex. This is why the result of summation on the right-hand side of expression (19) depends strongly not only on the average volume density of the electron radiators and on the details of their spatial distribution (in the case of relatively low values of the volume density), but also on the specific selection of the FEL operating regime. We shall now consider the limiting cases of high- and low-intensity beams.

(a) High-intensity electron beam ($Q \gg 1$). In this limiting case the average distance between the beam particles is considerably less than the wavelength of the undulator radiation field and we can change over from summation to integration in expression (19):

$$F_z^{(\mathrm{tot})}(\boldsymbol{r}, t) = \int \mathrm{d}t_s \int_{-\infty}^{\infty} \mathrm{d}x_s \int_{-\infty}^{\infty} \mathrm{d}y_s\, j_0(\boldsymbol{r}_s, t_s) \times F_z^{(\mathrm{b})}(\boldsymbol{r}, t; q_s)\, \Theta\big[z - Z_s(\tau, q_s)\big] . \quad (21)$$

If, moreover, the density j_0 of the radiator flux in the space of the input parameters of this flux is independent of the transverse coordinates x_s and y_s (the radiators are distributed in an axially symmetric manner relative to the point of observation of the radiation), it follows that integration with respect to the Cartesian coordinates in Eqn (21) reduces to integration with respect to the cylindrical coordinates ρ and φ introduced above:

$$\int_{-\infty}^{\infty} \mathrm{d}x_s \int_{-\infty}^{\infty} \mathrm{d}y_s = \int_0^{\infty} \rho\, \mathrm{d}\rho \int_0^{2\pi} \mathrm{d}\varphi \ldots .$$

If this substitution is made in Eqn (21) for the total bunching force, subject to formula (19a), and if the nature of the fields described by formulas (17c), (20a), and (20b) is taken into account, we can show that the total longitudinal electric field $E_z^{(\mathrm{tot})}$ vanishes, whereas its quasistatic part (q) is antisymmetric relative to the longitudinal coordinate of the source,† and the radiative part (r) additionally vanishes as a result of averaging over the angle φ [see expression (17c) for $E_z^{(\mathrm{r})}$].

For the same reason, there is no contribution to Eqn (21) by the quasistatic part of the transverse component of the magnetic field. Hence it follows that the grouping of the beam particles into coherent bunches is solely due to the magnetic component of the total undulator radiation field of the beam electrons.

It should be noted that the magnetic component of the Lorentz force plays the dominant role in grouping the beam particles in an undulator into coherent bunches and this is well known in the case when the configuration of the grouping radiation field is assumed to be given and has the form of a plane transverse electromagnetic wave (see, for example, Refs [30, 41, 71]). We shall not make this assumption. We shall consider instead the grouping of the beam particles by the field of their own undulator radiation and create an opportunity for investigating that range of the beam parameters in which the electromagnetic radiation field is emitted by the beam electrons.

†Here, we are assuming that the beam intensity is limited from above to such an extent that we can ignore the longitudinal polarisation fields of the beam oscillations. In terms of the scattering theory (see, for example, Refs [44, 70]), we are ignoring here the case of Raman scattering.

In this limiting case of injection of a high-intensity beam distributed homogeneously over its cross section, the magnetic component of the total undulator radiation field of the charges creates the following longitudinal grouping force

$$F_z^{(\mathrm{tot})}(\boldsymbol{r}, t) = |e|\beta_\perp \operatorname{Re} \int \mathrm{d}t_s \int_0^{\infty} \rho\, \mathrm{d}\rho \int_0^{2\pi} \mathrm{d}\varphi\, j_0(t_s) \times \Big[H_y^{(\mathrm{r})}(\boldsymbol{r}, t; q_s) + \mathrm{i}H_x^{(\mathrm{r})}(\boldsymbol{r}, t; q_s)\Big] \exp(\mathrm{i}k_\mathrm{w} z) \times \Theta\Big[z - Z_s(\tau, q_s)\Big]\, \Theta\Big[c\tau - (z^2 + \rho^2)^{1/2}\Big] . \quad (21\mathrm{a})$$

The first and second step functions Θ in expression (21a) determine respectively the lower ($t_s^{(\min)}$) and upper ($t_s^{(\max)}$) boundaries of the integration domain in terms of the moments at which the beam particles enter the undulator when the coordinates of the point of observation z and t are given:

$$t_s^{(\min)}(z, t) = t - \frac{z}{v_0} , \qquad t_s^{(\max)}(z, t) = t - \frac{z}{c} .$$

The upper limit of integration with respect to ρ depends on the longitudinal coordinate of the point of observation (z) and on the Lagrangian time $\tau \equiv t - t_s$:

$$\rho_{\max}(t, z) = (c^2\tau^2 - z^2)^{1/2} .$$

Substituting in Eqn (21) the magnetic field of the radiation from an elementary source, given by expressions (20), and integrating with respect to ρ and φ (taking account of the finite size of the region of influence of the field of the source), we obtain the following analytic expression for the longitudinal components of the Lorentz force exerted by the coherent radiation of the beam particles:

$$\begin{aligned} F_z^{(\mathrm{tot})}(z, t) &= -4\pi e^2 \beta_\perp^2 \gamma_0^2 \int_{t - z/v_0}^{t - z/c} \mathrm{d}t_s\, j_0(t_s) \\ &\times \cos\Big[k_\mathrm{m}(z - v_0\tau - \Delta)\Big] , \\ k_\mathrm{m} &\equiv k_+ + k_\mathrm{w} = \frac{\omega_\mathrm{m}}{v_0} \approx 2\gamma_0^2 k_\mathrm{w} . \end{aligned} \quad (21\mathrm{b})$$

The right-hand side of expression (21b) can be simplified further if we specify the nature of the function describing the distribution of the time t_s at which the flux particles enter the undulator. We shall now consider the grouping force, described by expression (21b) in the regime of amplification of a regular external signal, known as the amplification mode.

In this mode the density of a beam entering the undulator is modulated harmonically at a modulation frequency ω_m assumed to be equal to the frequency ω_+ of the radiation emitted forward by an electron in the undulator [see Eqn (14a)]:

$$\begin{aligned} j_0(t_s) &= j_0\Big\{1 + \operatorname{Re}\big[h(0)\exp(-\mathrm{i}\omega_\mathrm{m} t_s)\big]\Big\} , \\ \omega_\mathrm{m} &\equiv k_\mathrm{m} v_0 = \omega_+ . \end{aligned} \quad (22\mathrm{a})$$

Here, $j_0 \equiv n_0 v_0$ and $h(0)$ is the dimensionless depth of modulation of the beam density at the entry to the undulator in the amplification mode when this modulation is imposed by an external source $[(|h(0)| \ll 1)]$.

If the particles are initially grouped harmonically into coherent bunches, the field of their undulator radiation modulates the longitudinal displacements of electron radiators at the same frequency $\omega_{\rm m}$:

$$\Delta(\tau,\, t_s) = {\rm Re}\left[a(z)\exp(-{\rm i}\omega_{\rm m}t_s)\right] \, . \tag{22b}$$

Substitution of expressions (22a) and (22b) into expression (21b), and neglect of the rapidly oscillating (at the frequency $\omega_{\rm m}$) terms in the integral used in expression (21b) gives the total grouping force considered in the linear approximation ($k_{\rm m}a \ll 1$):

$$F_z^{(\rm tot)}(z,\, t) = -\pi e^2\beta_\perp^2 n_0\, {\rm Re}\left[\exp({\rm i}\varphi_1)\int_0^z {\rm d}z' f(z')\right] ,$$
$$\varphi_1 \equiv k_{\rm m}z - \omega_{\rm m}t \, . \tag{23}$$

The notation $f(z) \equiv h(0) - {\rm i}k_{\rm m}a$ is used in expressions (23). We can see from the definition of the function $f(z)$ that it is equal to the sum of the depths of modulation of the beam at the entry to the undulator [$h(0)$] and in the interior of the latter where it is affected by the radiation field [$({\rm i}k_{\rm m}a(z),\, a(0)=0)$]. Hence, it follows that the function $f(z)$ is equal to the total depth of modulation of the beam density: $f(z) = h(z)$.

It follows that, in the investigated limiting case of a beam with a sufficiently high intensity when the rms distance between its particles is much less than the wavelength of the emitted wave, the total grouping force of the coherent undulator radiation emitted by the beam particles is a plane wave travelling at the phase velocity equal to the unperturbed velocity of the beam (compare with Refs [47, 48]):

$$F_z^{(\rm tot)}(x,\, t) = A(z)\cos\varphi_1(z,\, t) \, , \tag{23a}$$

$$A(z) \equiv \frac{3}{\pi}\left(F_z^{(\rm rad)}\right)_1 \int_0^{z/D} {\rm d}\zeta\, h(\zeta)\, Q \, . \tag{23b}$$

For simplicity of a physical analysis of the results, we shall express the amplitude of the grouping force $A(z)$ in terms of the force of the radiative friction exerted by the dipole undulator radiation emitted by a single electron $(F_z^{(\rm rad)})_1$ [see expression (18b)]. We shall also adopt a dimensionless variable $\zeta = z/D$ in the integral.

The results make it obvious that it is coherence of the undulator radiation of electrons in a high intensity ($Q \gg 1$) beam that ensures an increase in the rate of their grouping into coherently emitting bunches and therefore reduces the radiative relaxation length of the beam in the undulator.

The increase in the amplitude of the grouping force because of coherent addition of the radiation fields of the individual electrons within each coherent bunch takes into account the factor $h(\zeta)Q$ on the right-hand side of expression (23b). In turn, the increase in the right-hand side of this expression, described by an integral with respect to the longitudinal coordinate, takes account of the coherent addition of the radiation fields of a sequence of bunches following the one under investigation [47, 48].

The phase φ_1 of the force described by expressions (23) corresponds to a wave which travels along the undulator axis at the velocity $v_{\rm ph} \equiv \omega_{\rm m}/k_{\rm m}$, equal to the longitudinal beam velocity v_0. Therefore, it is this force that ensures grouping of the beam particles into coherent bunches by the Veksler – McMillan phase stability mechanism [74, 92, 93], justifying the 'grouping force' name.

(b) Low-intensity beam ($Q \ll 1$). In this limiting case the replacement of the summation on the right-hand side of expression (19) by integration is not permissible and, moreover, we cannot assume that the function $j_0(\boldsymbol{r}_s,\, t_s)$ is independent of the transverse coordinates $\boldsymbol{r}_s$. Then, on the right-hand side of expression (20a), we need to retain only the terms which decrease at the slowest rate with increase in the distance between the beam particles (i.e. inversely proportionately to the distance).

In this limiting case the asymptotic expression for the bunching force is

$$F_z^{(\rm b)}(\boldsymbol{r},\, t;\, q_s) = -e^2\beta_\perp^2\,\frac{k_0}{R_*}\left\{\left(\beta_0 + \frac{(\delta z)_s}{R_*}\right)\sin\psi_s - 2\,\frac{\beta_0\rho_s^2}{R_*^2\gamma_0^2}\left[\sin\psi_s + \sin(\psi_s + 2\varphi - 2k_{\rm w}z)\right]\right\} ,$$
$$\psi_s \equiv k_{\rm w}z - \vartheta = k_{\rm w}\gamma_0^2\left[(\delta z)_s + \beta_0 R_*\right] . \tag{24a}$$

It follows from the general nature of the dependences of the function R_* on ρ that, for all the electrons with a serial number s moving in a shared coherence tube† together with the electrons under consideration, we can assume that the ratio ρ/γ_0 is small compared with the characteristic average distance $d_\parallel$ between the longitudinal coordinates of these electrons:

$$\frac{\left\langle \rho_{ps}^2\right\rangle^{1/2}}{\gamma_0} \ll d_\parallel \equiv \left\langle (Z_p - Z_s)^2\right\rangle^{1/2} .$$

The angular brackets denote here the operation of statistical averaging over all possible values of the indices s and p; we have introduced also the function $\rho_{ps} = [(x_p - x_s)^2 + (y_p - y_s)^2]^{1/2}$.

Under these conditions, the right-hand side of expression (24a) becomes simplest for the force which an electron with a serial number s exerts on an electron with a serial number p:

$$F_z^{(\rm b)}(\boldsymbol{R}_p,\, \boldsymbol{R}_s) = -\frac{e^2\beta_\perp^2 k_0}{|\delta z_{ps}|}\left[\beta_0 + {\rm sgn}\,(\delta z)_{ps}\right]\sin\psi_s \, ,$$
$$(\delta z)_{ps} \equiv Z_p - Z_s \, . \tag{24b}$$

In the subsequent analysis the factor of decisive influence is the estimated ratio of the modulus of the right-hand side of expression (24b) to the amplitude of the radiative friction force $|F_z^{(\rm rad)}|_1$ acting on each electron [see expression (18b)]:

$$\left\langle \left| F_z^{(\rm b)}(\boldsymbol{R}_p,\, \boldsymbol{R}_s)\right|\right\rangle \approx \frac{\lambda_0}{d_\parallel}\left|F_z^{(\rm rad)}\right|_1 . \tag{24}$$

It follows from the above definition of the number of particles Q in a coherent bunch that $\lambda_0/d_\parallel \approx Q$. Substituting this estimate on the right of expressions (24), we readily find that in the limiting case of a low-intensity beam, when the strong inequality $Q \ll 1$ is obeyed, the amplitude of the radiative interaction force in each electron emitter pair is considerably less than the intrinsic force of radiative deceleration exerted on each of them because of a reduction in the field in inverse proportion to the distance

†A coherence tube (i.e. a region of coherence in the transverse direction) with a given electron is a cylinder of radius of the order of $\lambda_0\gamma_0 \approx D/4\pi\gamma_0$ and of height equal to the longitudinal dimension of the region of influence of the electron when the cylinder axis is oriented along the equilibrium trajectory of this electron.

between the radiators in the far-field zone of each of them (for details, see Section 4).

It must be stressed particularly that in the limiting case when $Q \ll 1$ the last conclusion remains valid also if we take account of the influence of the coherent neighbours of the investigated radiator. In fact, the amplitude of the bunching force, described by expression (24b), decreases with increase in the serial number s of a neighbour, whereas the corresponding phase ψ_s varies in a random manner.

It therefore follows that each electron radiator experiences the radiation fields of only a limited number of neighbours with appropriately small amplitudes, so that its motion is governed primarily by the force exerted by its own radiative friction $(F_z^{(\mathrm{rad})})_1$, as expected from the physical nature of the investigated effect (Section 1).

3.3.2 Dynamics of the longitudinal motion of beam particles

We shall now describe a collective radiative instability of a monoenergetic electron flux in an undulator.

(a) High values of the volume beam density ($Q \gg 1$)*; amplification mode.* The equation describing the longitudinal motion of the beam particles, corresponding to the force described by expression (23a), has the following form in terms of the Euler variables:

$$\hat{D}\tilde{p}_z = F_z^{(\mathrm{tot})}(z,\ t)\ , \tag{25}$$

where the notation is as follows:

$$\hat{D} \equiv \frac{\partial}{\partial t} + v_0 \frac{\partial}{\partial z}\ , \qquad \tilde{p}_z \equiv m_{||}\tilde{v}_z = m_0 \tilde{v}_z \gamma_0^3\ .$$

We can find the explicit form of the equation for the amplitude by introducing, in expression (25), the Euler density $n_{\mathrm{E}}(z, t)$ and the Euler velocity $v_{\mathrm{E}}(z, t)$ of the beam particles:

$$n_{\mathrm{E}}(z,\ t) = n_0 + \tilde{n}(z,\ t) = n_0\left\{1 + \mathrm{Re}\left[h(z)\exp\left(\mathrm{i}\varphi_1(z,\ t)\right)\right]\right\}\ , \tag{25a}$$

$$v_{\mathrm{E}}(z,\ t) = v_0 + \tilde{v}_z(z,\ t) = v_0\left\{1 + \mathrm{Re}\left[g(z)\exp\left(\mathrm{i}\varphi_1(z,\ t)\right)\right]\right\}\ . \tag{25b}$$

Substitution of expressions (25a) and (25b) in expression (25), and neglect of the rapidly oscillating (at the frequency ω_{m}) terms yields the following integrodifferential equation which relates the functions $g(z)$ and $h(z)$ (see Refs [45, 47, 48]):

$$m_{||}v_0^2\,\frac{\mathrm{d}g}{\mathrm{d}z} = (F^{(\mathrm{rad})})_{\mathrm{tot}} = \frac{3}{\pi}\,Q(F_z^{(\mathrm{rad})})_1 \int_0^{\zeta} \mathrm{d}\zeta'\,h(\zeta')\ . \tag{26a}$$

The relationship between the functions $g(z)$ and $h(z)$, which follows from the equation of continuity

$$\frac{\mathrm{d}h}{\mathrm{d}z} + \mathrm{i}k_{\mathrm{m}}\,g = 0\ , \tag{26b}$$

leads to the following equation for $h(\zeta)$ in terms of total third-order derivatives:

$$\left(\frac{\mathrm{d}^3}{\mathrm{d}\zeta^3} - \mathrm{i}\kappa^3\right)h(\zeta) = 0\,, \qquad \kappa \equiv \frac{(4\pi^2 n_0 r_0 D^2 p^2)^{1/3}}{\gamma_0}\ . \tag{26}$$

The system of equations (26) describes the dynamics of the growth of the amplitude of a space charge wave due to grouping of the beam particles into bunches by the field of their stimulated (coherent) radiation in a undulator. We can easily see that it agrees fully with the result given by formula (7a), obtained in the same approximation ($p^2 \ll 1$, $\gamma_0 \gg 1$) by the methods of formalised hydrodynamic theory of FELs.

(b) Finite values of the parameter Q ($Q \gtrsim 1$)*; SASE mode.* In the SASE mode the main sources of the initial (amplified) field are fluctuations of the density of the beam particles which form coherent bunches [84, 97, 98]. Then the radiation field of these bunches aggregates into coherent bunches other beam particles which enter the region of influence of the field and this finally results in the emission of intense stimulated (coherent) radiation.

The specific feature of the SASE mode is that fluctuations of the beam particle density appear at random and are uncorrelated with one another not only between one realisation of an electric current pulse to another, but also within the limits of each pulse between one coherence tube and another.

It follows from the above that fully scalable quantitative simulation of the dynamics of motion of beam particles in the SASE mode requires substitution of the radiation field described by formula (20a) into the bunching force given by expression (19). In particular, only this method can be used to describe the SASE mode near the stimulated emission threshold (i.e. near the lower wavelength limit of coherent undulator radiation), where the rms number of particles Q in a coherent bunch is not too large compared with unity.

In view of the statistical nature of the expected results, it is necessary to carry out calculations for a sufficiently large number of different spatial distributions of the beam particles entering an undulator. Such calculations can be carried out only numerically and they require selection of specific values of the parameters in the simulated experiment. We shall therefore give here only the necessary methodological details of the calculations, representing the Lagrangian dynamics of the beam particles in the field of the bunching force described by formulas (19) and (20), and an estimate (given above) of the order of magnitude of the ratio of the modulus of the bunching force to the modulus of the radiative friction force of an individual electron when $Q \ll 1$ [see expression (24)].

3.4 Analysis of the physical meaning of the results

We have thus used a fully scalable analytic simulation of the dynamics of self-consistent motion of a monoenergetic flux of point-like electrons in the total undulator radiation field of each of the electrons, to demonstrate the following results.

(1) The aggregation of the beam electrons by the total undulator radiation field into coherently emitting bunches occurs only when the strong inequality $Q \gg 1$ is obeyed, because this ensures operation of a nonequilibrium system as a source of stimulated coherent electromagnetic radiation, i.e. its operation as an FEL.

(2) The method of description of a beam by a flux of a continuous liquid, used in the formalised classical self-consistent theory of FELs, is valid only if the condition $Q \gg 1$ is obeyed. This is indicated in particular by the agreement between the gain per one undulator period, given by formula (7), and the corresponding result of the corpuscular theory, given by expression (26): not only the functional dependences on the external parameters of

the investigated nonequilibrium system, but also the numerical value of the gain are in agreement.

(3) If the strong inequality $Q \gg 1$ is not obeyed, there is no mutually coherent amplification of the undulator radiation of the individual beam electrons and this nonequilibrium system operates as a traditional source of incoherent undulator radiation.

Before we discuss the methodological aspects of the results given above, we must identify those results that provide argued answers to the questions posed in the concluding part of Section 2.3.

First of all, we note that the dominant role of the coherence factor in the operation of FELs was first postulated on the basis of a semiphenomenological corpuscular theory [47] (see also Ref. [48]). It is the requirement of a quantitative inclusion of the effects of coherence of the undulator radiation of individual electrons (both within the bunches formed by the undulator radiation field and between these bunches) that guided our selection of the initial equations (23) for the description of the dynamics of the longitudinal motion, accurate to within a numerical coefficient of the order of unity ($C = 3\pi$).

Since the system of equations (23) was derived by us by a more general method of direct summation of analytic asymptotes of the undulator radiation fields of the individual electrons for the case when $Q \gg 1$, it follows that in this limiting case the above physical aspects are not only valid qualitatively, but are also included correctly — in the quantitative sense — in Refs [47, 48].

It thus follows that the coherence of elementary radiators plays the key role in ensuring a significant rise of the amplitude of the bunching force, described by expression (23b), compared with the force of radiative deceleration of individual electrons in the same undulator, which is necessary for the aggregation of electron radiators to form coherently emitting bunches in the relatively short length of the undulator and for the operation of the nonequilibrium system as an FEL.

Since there is no bunching of electron radiators if $Q \ll 1$, the equality formulated first in Ref. [47]

$$Q' \equiv n_0' \lambda_{\mathrm{u.r}}'^3 = \frac{n_0 D^3}{8\gamma_0^4} = 1$$

does indeed divide the ranges of the external parameters of the electron – undulator system in which the system either emits stimulated coherent undulator radiation ($Q' \gg 1$, FEL mode) or incoherent undulator radiation ($Q' \ll 1$, second-generation source of cyclotron and undulator radiation).

The following comments should be made about the role of the undulator radiation of individual electrons in the mechanism of collective amplification of electromagnetic waves in the FEL mode.

The dominant continuation of the undulator radiation to the formation of coherently emitting bunches can be seen directly from an analysis given above, where there are no other sources of the longitudinal bunching force [described by expression (23)] apart from the undulator radiation of the individual electrons when $Q' \gg 1$. Therefore, the position of the gain maximum on the frequency axis [compare expressions (7) and (8), and also (26)] corresponds exactly to the frequency of the undulator radiation of an individual electron.

The physical reasons for vanishing, at a certain frequency of the intensity maximum of stimulated coherent radiation from an FEL oscillator, mentioned in Section 2.3, are due to the specific realisation in this oscillator of the mechanism of stimulated interaction between a beam of electrons with an electromagnetic field emitted by this beam by a stimulated process. We shall discuss in detail this point because, to the best of our knowledge, it has been ignored in the literature.

Oscillators loaded by monoenergetic transit electron fluxes of relatively low intensity have been investigated thoroughly in theoretical microwave electronics (including calculations of the threshold currents and electron efficiencies) for all the elementary effects resulting in the emission of radiation by moving charges, including the Vavilov – Cherenkov radiation [60], synchrotron radiation [20, 30, 46, 68], and transition radiation [19, 23, 38, 39].

Common distinguishing features of oscillators, loaded with monoenergetic initially unmodulated electron fluxes, are:

— the presence of a cavity where the energy of the stimulated (coherent) radiation field of the beam particles accumulates at one of the normal frequencies of the resonator ($\Omega_0' \equiv \mathrm{Re}\,\Omega_0$);

— the linear dependence of the growth increment of the amplitude of the field of normal cavity oscillations ($\delta'' \equiv \mathrm{Im}\,\Omega_0$) on the total beam current passing through the cavity;

— the nonmonotonic dependence of this increment in the case of Cherenkov, transition, and FEL oscillators on the angle θ of the phase slip of the beam particles relative to the field of a normal cavity oscillation excited by these particles:

$$\delta'' = \frac{\Omega_0'}{\pi s} G^3 \frac{\mathrm{d}}{\mathrm{d}\theta} \left\{ \left[\frac{\sin(\theta/2)}{\theta/2} \right]^2 \right\} C \,. \tag{27}$$

The following notation is introduced in expression (27): $\theta \equiv \Omega_0' L / v_0 - \pi s$; L is the length of the interaction region; v_0 is the velocity of the beam particles; s is the serial number of a longitudinal harmonic of the oscillations; G is a well-known parameter, equal to the gain at the selected wavelength (in the case of systems with local emission of radiation by moving charges in the interaction region, which can be, for example, the Cherenkov or undulator radiation). In our case of an undulator-radiation FEL oscillator, the parameter G is equal to the product of the number of the undulator periods $N_0 \ll N_{\max}$ and the maximum gain $\Gamma_{\max}$: $G \equiv N_0 \Gamma_{\max}$.

The factor G on the right of expression (27) represents a dimensionless normalised constant, which depends on the radial profile of the beam particle density and on the actual nature of the elementary effect resulting in the emission of radiation by the beam electrons in the interior of a cavity.

In accordance with the initial physical assumptions adopted in the theory of oscillators of this type, which leads to expressions such as (27), under the conditions considered here the parameter G is small compared with unity: $G \ll 1$. Two conclusions follow from this inequality and they relate to the principal physical properties of the process of stimulated collective radiative interaction between the beam electrons and the field of their electromagnetic radiation in the investigated nonequilibrium system (electron beam + cavity + medium ensuring slowing down of the concurrent wave representing normal oscillations in the resonator:

(1) It follows directly from expression (27) that in the limiting case $G \ll 1$ the increment δ'' is much less than the

characteristic spacing $\Delta\Omega \equiv \pi|v_g|/L$ between the normal frequencies of the cavity, corresponding to the nearest adjacent values of the serial number s of the longitudinal harmonic (v_g is the group velocity of the concurrent wave at the frequency Ω_0, which is $v_g \approx v_0$):

$$\frac{\delta''}{\Delta\Omega} = \frac{\Omega_0 L}{\pi s|v_g|}\, G^3 \ll 1 \;. \tag{27a}$$

It follows from inequality (27a) that it should be possible to generate quasimonochromatic electromagnetic oscillations in devices of this type. (For this reason such devices are called monotrons in the literature on theoretical microwave electronics [19, 60].)

(2) It also follows from expression (27) that in this system the characteristic transit time of the beam electrons $\tau_p \equiv L/v_0$ across the interaction region is considerably less than the rise time $\tau_r \equiv 1/\delta''$ of the amplitude of the field representing the normal cavity oscillations:

$$\frac{\tau_p}{\tau_r} \approx G^3 \ll 1 \;. \tag{27b}$$

It follows from this inequality that the main part of the working duration of a pulse of the beam electrons in oscillators of this type interacts in a stimulated manner not with the field of the coherent radiation emitted by these electrons (as in the case of the FEL amplifier discussed above), but with the radiation field of the preceding pulses stored in the cavity. Under these conditions the cavity locks the phases of the stimulated radiation of the portions of the previously unmodulated beam entering the cavity and thus ensures the radiation coherence even in the case of those beam portions which are separated from one another by a distance considerably greater than the length L of the interaction region. However, such phase locking occurs only under special conditions.

This is because the Veksler–McMillan phase stability mechanism, ensuring the grouping of beam particles to form coherently emitting bunches by the field of the concurrent wave, results in the formation of bunches at zero phase of this wave when the beam particles are synchronised exactly with the radiation they emit ($v_{ph}^{(s)} \equiv Q_0 L/\pi s = v_0$). It should be noted that the square of the frequency of the phase oscillations is positive [45, 74, 92, 93]. At zero phase the particles cannot exchange energy with the field. This is the reason why, in particular, the increment described by expressions (27) vanishes at the point $\theta = 0$.

The bunches formed in this way can transfer their kinetic energy to the radiation field only if they overtake slightly the wave representing the bunching force of their undulator radiation [see expressions (23)], which is accumulated inside the cavity, and the bunches should be displaced from zero phase of the wave field to the decelerating phase [45]. These bunches can emit in a stimulated manner (and in coherence with their predecessors) only in the decelerating phase and this ensures an increase in the amplitude of the field of the concurrent electromagnetic wave necessary for the development of the instability. Such an increase in turn enhances the depth of modulation of the subsequent portions of the beam and thus provides a positive feedback in the investigated nonequilibrium system.

In the undulator-radiation FEL amplifier discussed above (as in the case of the Cherenkov free-electron microwave amplifiers of the travelling-wave tube type; see Ref. [94]) the reduction in the phase velocity of the amplified slow space charge wave, needed for the above-mentioned phase slip of the electron bunches, is ensured automatically by the corresponding positive beam shift of the wave number of the space charge wave: $\Delta k \equiv \kappa/2D$ [see formula (7d)].

In the case of FEL oscillators with a relatively low current, of the kind described by expressions (27), the main source of the phase slip of the bunches, necessary for a shift to the decelerating phase of the field of the concurrent wave of normal cavity oscillations, is breakdown of the law of conservation of the momentum of the interacting waves, described by expression (4c), and the deviation of the appropriate frequency Ω_0' from $\omega_{u.r}$.

If the velocity of the beam particles v_0 exceeds the phase velocity $v_{ph}^{(s)}$ of the beats of the concurrent electromagnetic wave (t) and the pump wave (w)

$$v_0 > v_{ph}^{(s)} \equiv \frac{\Omega_0}{k_t^{(s)}} + k_w \;, \tag{28a}$$

the corresponding phase shift θ during the time taken by an electron to pass through the undulator is positive: $0 < \theta < 2\pi$. In this range of the values of θ, near the entry to the undulator, coherent electron bunches form at zero phase of the field of the bunching force, described by expression (23). When these bunches approach the undulator exit, they shift to the range of the decelerating phases of this force.

Naturally, very small and very large displacements of the bunches make small contributions to the resultant increment given by formula (27). For this reason, the stimulated undulator radiation of the beam electrons reaches its highest effectiveness as a result of bunch displacement when $\theta = \theta_* \approx 2.65$ and the right-hand side of formula (27) is maximal [30, 71, 74].

In this case the optimal frequency Ω_+ of an FEL oscillator exceeds the frequency $\omega_{u.r}$ by an amount which increases on reduction in the undulator length:

$$\Omega_+ = \omega_{u.r}\left(1 + \frac{\theta_* D}{2\pi L}\right) , \tag{28b}$$

and in the distance equal to the length of the interaction region a bunch can overtake the travelling bunching-force wave by a distance which is almost half the wavelength of this wave.

It follows that the difference between the optimal frequencies of an FEL amplifier and an oscillator is not in any way in conflict with the dominant role of the undulator radiation of individual electrons in the amplifier and this is physically justified by the specific nature of the mechanism of generation of stimulated (coherent) undulator radiation by a long-pulse beam ($\tau_r \gg \tau_p$) in the oscillator cavity.

4. Nature of coherent amplification of undulator radiation

The results of the preceding section indicate a considerable weakening of the undulator radiation field of the beam electrons when the conditions are changed from those corresponding to $Q \gg 1$ to $Q \ll 1$. However, it is not clear from these results how the undulator radiation field of the individual emitters is amplified when $Q \gg 1$ and what is the law governing the fall of the amplification efficiency with reduction in the parameter Q.

The explanation of the reason for the fall of the amplification efficiency, given in Section 3.4, is based on reduc-tion in the degree of coherence of elementary radiators. In fact, this explanation is supported only by the phenomenological theory [47, 48]. We shall describe the physical nature and the quantitative relationships governing the mechanism of the coherent radiative interaction of the beam electrons with one another both within the bunches formed by the force described by Eqn (23), and between the bunches.

4.1 Physical model and method of analysis of the problem

The purpose of this section is to justify the selection of the physical model representing the process under considera-tion and the method used in the theoretical description of this process.

We shall adopt a reference system in which a mono-energetic electron flux is at rest. The field of a plane undulator in the ultrarelativistic case ($\gamma_0 \gg 1$) considered here is close to the field of a plane electromagnetic wave. Therefore, radiative interaction of the beam bunches with the undulator field reduces in this system to the Thomson scattering of a plane electromagnetic wave by a periodic sequence of immobile charged particle bunches.

The strength of this interaction can be described in a natural manner by the degree of coherence of the Thomson scattering, which is defined by the ratio of the total intensity of the radiation scattered by a system of charge scatterers to the total intensity of the scattering of the same wave considered independently for each individual charge form-ing the investigated system.

The final aim of this analysis is to determine the functional dependences of the degree of coherence both on the total number of scatterers in a beam, on the distances between them within the individual bunches, and on the repetition period of such bunches.

We shall consider a bunch consisting of N identical point-like charged particles, each of which has a charge q and a mass m. The equilibrium position of this particle in space is represented by the vector $\boldsymbol{r}_s \equiv \boldsymbol{e}_x x_s + \boldsymbol{e}_y y_s + \boldsymbol{e}_z z_s$, where s is the particle number ($1 \leqslant s \leqslant N$). We shall assume that a plane linearly polarised electromagnetic wave of frequency ω_0 is propagating along the Oz axis and that the electric field of this wave is parallel to the Ox axis:

$$E_x(z, t) = E_0 \cos(\omega_0 t - k_0 z)\,, \quad k_0 = \omega_0/c\,.$$

We shall find the functional dependences of the total intensity of the radiation scattered by a bunch on the number of scatterers N and on the geometric dimensions of a bunch, normalised to the scattered wavelength $\lambda_0 \equiv c/\omega_0$.

4.2 Coherence factor for the total radiation intensity

We shall define the coherence factor for the total radiation intensity as the ratio of the total intensity of the bremsstrahlung of a bunch $I_{\text{tot}}^{(N)}$ at a frequency $\omega = \omega_0$ to the sum of the intensities of the incoherent bremsstrah-lung $I_{\text{incoh}}^{(N)}$ emitted by the charged particles forming this bunch:

$$K_N \equiv \frac{I_{\text{tot}}^{(N)}}{I_{\text{incoh}}^{(N)}}\,, \tag{29}$$

where the following notation is used:

$$I_{\text{incoh}}^{N} = N I_{\text{incoh}}^{(1)} = N\,\frac{q^2\omega_0^4\alpha^2}{3c^3}\,, \quad \alpha \equiv \frac{qE_0}{m\omega_0^2}\,.$$

The total bremsstrahlung power $I_{\text{tot}}^{(N)}$ for a bunch will be defined as the sum of the radiation losses experienced by each of the scattering charges in a bunch in the combined field decelerating radiatively all its neighbours:

$$I_{\text{tot}}^{(N)} = \sum_{s=1}^{N}\sum_{p=1}^{N} I_s^{(p)}\,. \tag{29a}$$

In formula (29a) the scatterer numbers s and p assume the values $1, 2, \ldots, N$, and the symbol $I_s^{(p)}$ denotes the intensity of the radiation emitted by a charge with the serial number s in a field created, at the point of equilibrium $\boldsymbol{r}_s$ of this charge, by its neighbour labelled by the number p:

$$I_s^{(p)} \equiv -q\left\langle \boldsymbol{V}_s(t)\cdot\boldsymbol{E}^{(p)}(\boldsymbol{r}_s, t)\right\rangle\,. \tag{29b}$$

The angular brackets in expression (29b) denote aver-aging over one period of the field of the scattered wave $T_0 \equiv 2\pi/\omega_0$, $\boldsymbol{V}_s(t)$ is the oscillatory velocity of a charge with the number s in the field of the electromagnetic wave scattered by this charge:

$$V_s(t) = \omega_0\alpha\sin(\omega_0 t - k_0 z)\,, \tag{29c}$$

and $\boldsymbol{E}^{(p)}(\boldsymbol{r}, t)$ is the field created by a scatterer with the number p at the point of observation $\boldsymbol{r}$ at the moment t.

The intensity of the bremsstrahlung emitted by a single charge in its own field of radiative deceleration ($s = p$), described by formulas (29), represents the intensity of the dipole bremsstrahlung under the Thomson scattering conditions, which is well-known in classical electrody-namics (see, for example, Ref. [91]):

$$I_s^{(s)} = I_1^{(1)} \equiv \frac{q^2\alpha^2\omega_0^4}{3c^3}\,.$$

It follows from expression (11a) for the field of an accelerating charge that under the investigated conditions the right-hand side of expression (29) is determined uniquely by the total number of scatterers N and by their relative positions in a bunch:

$$K_{\text{tot}}^{(N)} \equiv \sum_{s=1}^{N}\sum_{p=1}^{N}\frac{G_{sp}}{N} = 1 + \sum_{s=1}^{N}\sum_{p=1}^{N}\frac{G_{sp}(s\neq p)}{N}\,. \tag{30}$$

The following notation is used above::

$$G_{sp} \equiv \frac{I_s^{(p)} + I_s^{(p)}}{2I_s^{(s)}}\,p = 3\left[\left(1 - 3\frac{v_{sp}^{(x)2}}{v_{sp}^2}\right)v_{sp}\cos v_{sp} + \left(v_{sp}^2 - v_{sp}^{(x)2} - 1 + 3\frac{v_{sp}^{(x)2}}{v_{sp}^2}\right)\sin v_{sp}\right]\frac{\cos v_{sp}^{(z)}}{2v_{sp}^3}\,, \tag{30a}$$

$$\boldsymbol{v}_{sp} = k_0(\boldsymbol{r}_s - \boldsymbol{r}_p) = \boldsymbol{e}_x v_{sp}^{(x)} + \boldsymbol{e}_y v_{sp}^{(y)} + \boldsymbol{e}_z v_{sp}^{(z)}\,, \quad v_{sp}^2 \equiv (\boldsymbol{v}_{sp}, \boldsymbol{v}_{sp})\,. \tag{30b}$$

4.3 Analytic asymptotes and numerical calculations of the coherence factor

In general, formula (30) is too complex not only for analytic investigation but even for numerical calculations (because of the cumbersome nature of the procedure for numbering the scatterers, particularly when their number is large: $N \gg 1$). Therefore, it is of interest to consider a bunch which has the shape of a parallelepiped with a rigorously periodic array of scatterers along the directions of each of the parallelepiped edges parallel to the Cartesian coordinate axes.

We shall denote the total number of scatterers on an edge $i = x$, y, and z by the symbol n_i and the period of these scatterers along a chain parallel to a given edge by the symbol $d_2^{(i)}$. The total length of the edge in question is then $d_{\text{tot}}^{(i)} = (n_i - 1)d_2^{(i)}$ and the total number of scatterers N in a bunch is equal to the product of the numbers n_i: $N \equiv n_x n_y n_z$.

This shape and structure of a bunch simplifies greatly not only the general form of the right-hand side of expression (30), but also has a number of advantages in analytic and numerical studies of the dependences of this side on the number of scatterers in a bunch and on the distances between them.

In fact, when a bunch is selected in this form, the procedure for numbering the scatterers in numerical calculations is greatly simplified. Moreover, the use of the periodicity of a sequence of scatterers in each chain parallel to the edge of a bunch makes it possible to replace the summation of the contributions of the individual pairs (differing in the values of the number s and p) on the right-hand side of expression (30) with the summation of the contributions of pairs differing in respect of the distances between them.

The distance between two scatterers occupying arbitrary positions in one of the chains is $d_{li} = (l_i - 1)d_2^{(i)}$, where $0 \leqslant l_i \leqslant n_i$. We can easily show that the total number M_i of pairs in a chain is

$$M_i(n_i, l_i) = (n_i - l_i)(2 - \delta_{l_i 0}) . \tag{31a}$$

The number of pairs differing in respect of the fixed distance between them is governed by the values of the scalars l_x, l_y, l_z, and it is equal to the product of the numbers M_i:

$$M(\boldsymbol{n}, \boldsymbol{l}) = \prod_i M_i(n_i, l_i) . \tag{31b}$$

The total number of such pairs

$$M_{\text{tot}}(\boldsymbol{n}) \equiv \sum_{l=0}^{l=n} M(\boldsymbol{n}, \boldsymbol{l}) = N\big[N - (n_x n_y + n_y n_z + n_z n_x) - 1 + n_x + n_y + n_z\big] \tag{31c}$$

is then found to be less than the number of terms on the right-hand side of expression (30a), which is equal to $N(N-1)$.

The contribution of each of the pairs is determined by the corresponding tensor $G_{s,q}$. Therefore, the final formula for the coherence factor of the adopted model bunch is

$$K_{\text{tot}}^{(N)}(\boldsymbol{n}, \boldsymbol{d}_2) = \sum_{l_x=0}^{n_x} \sum_{l_y=0}^{n_y} \sum_{l_z=0}^{n_z} M_x(n_x, l_x) M_y(n_y, l_y) M_z(n_z, l_z)\, G(\boldsymbol{l}, \boldsymbol{d}_2) , \tag{31}$$

where

$$G(\boldsymbol{l}, \boldsymbol{d}_2) \equiv 3\left[\left(1 - 3\frac{v_x^2}{v_t^2}\right) v_t \cos v_t + \left(v_t^2 - v_x^2 - 1 + 3\frac{v_x^2}{v_t^2}\right) \sin v_t\right] \frac{\cos v_z}{2 v_t^3} ,$$

$$\boldsymbol{v}_t \equiv \boldsymbol{e}_x v_x + \boldsymbol{e}_y v_y + \boldsymbol{e}_z v_z , \quad v_i \equiv l_i k_0 d_2^i , \quad v_t^2 \equiv (\boldsymbol{v}_t, \boldsymbol{v}_t) .$$

Finally, one of the important advantages of the selected bunch structure is that it includes, as the special limiting cases, those bunches which can be described by just one length parameter. This makes it much easier to identify and analyse the functional dependence of the right-hand side of expression (31c) on this parameter.

Among these bunches we are primarily interested (when dealing with an undulator-radiation FEL amplifier) in a cubic array of scatterers ($n_x = n_y = n_z = n = M^{1/3}$, $d_2^{(x)} = d_2^{(y)} = d_2^{(z)} = d$) and also in a linear chain of scatterers ($n_x = n_y = 1$, $n_z = n = N$, $d_2^{(z)} = d$). The former can be used to investigate the effect of coherence within one bunch and the latter can be used in a study of the effect of coherence between the bunches.

We shall now consider these two models of a cubic array and a linear chain, and we shall find explicit expressions for the coherence factors and their analytic asymptotes. We shall also give the results of numerical calculations of the dependences of these factors on the parameter d, carried out for different values of the number of scatterers.

4.3.1 Analytic representations and their asymptotes

(a) Cubic array ($N = n^3$, $d_2^{(x)} = d_2^{(y)} = d_2^{(z)} = d$). In the case of this array the functional dependence of the right-hand side of expression (31a) on the minimum distance within a pair of adjacent scatterers $\theta \equiv k_0 d$ and on the total number N becomes

$$K_{\text{tot}}^{(V)}(N, \theta) = 3\sum_{l=0}^{n} M(\boldsymbol{n}, \boldsymbol{l})\left\{\left(1 - 3\frac{v_x^2}{v_t^2}\right) v_t \cos v_t + \left(v_x^2 + v_y^2 - 1 + 3\frac{v_x^2}{v_t^2}\right) \sin v_t\right\} \frac{\cos v_z}{2(n v_t)^3} . \tag{32}$$

Formula (32) leads to simple analytic representations of its left-hand side in the case of small and large dimensions of a bunch:

$$K_{\text{tot}}^{(V)}(N, \theta) = \begin{cases} N\left[1 - (N^{2/3} - 1)\dfrac{\theta^2}{6}\right], & (n-1)\theta \ll 1 , \quad (32\text{a}) \\ 1 + \dfrac{12}{N^{1/3}\theta} \sum_l \prod_i (1 - \mu_i) \times \sin(N^{1/3}\theta\mu_t)\cos(N^{1/3}\theta\mu_z) \times \left(1 - \dfrac{\mu_x^2}{\mu_t^2}\right)\mu_t , & \theta \gg 1. \quad (32\text{b}) \end{cases}$$

The first terms on the right-hand sides of formulas (32a) and (32b) represent well-known asymptotic results of the classical Thomson scattering theory, corresponding to infinitesimally small [see formula (32a)] and infinitely large [see formula (32b)] distances between the individual scatterers [91]. When the right-hand side of formula (32a) is equal to the number of scatterers N, this corresponds physically to the maximum degree of coherence and when the right-hand side of formula (32b) is equal to unity, this means physically complete absence of the coherence.

The second terms on the right-hand sides of formulas (32a) and (32b) then include corrections due to the finite values of the parameter θ (equal to the shortest distance between two adjacent scatterers, normalised to the scattered wavelength).

(b) *Linear chain.* If on the right-hand side of expression (31) we substitute $n_x = n_y = 0$, $n_t = n_z = N$,

$\theta = k_0 d_2^z = k_0 d$, we obtain an explicit analytic representation of the coherence factor for a linear chain of scatterers with its axis parallel to the direction of propagation of the scattered electromagnetic wave:

$$K_{\text{tot}}^{(L)}(n,\theta) = 1 + 3\sum_{l=1}^{n}\Big[2(n-l)\Big]\Big\{v_l \cos v_l + (v_l^2 - 1)\sin v_l\Big\}\frac{\cos v_l}{2nv_l^3}\,, \qquad v_l \equiv l\theta\,. \tag{33}$$

Hence we can obtain analytic asymptotes of this factor for relatively short and long chains:

$$K_{\text{tot}}^{(L)}(n,\theta) = \begin{cases} n\left[1 - 7(n^2-1)\dfrac{\theta^2}{60}\right], & (n-1)\theta \ll 1\,, \quad \text{(33a)} \\ 1 + 3\displaystyle\sum_{l=1}^{n}\left(1 - \frac{l}{n}\right)\frac{\sin(2l\theta)}{2l\theta}\,, & \theta \gg 1\,. \quad \text{(33b)} \end{cases}$$

It is evident from formulas (33a) and (33b) that in this case again the correction due to the finite size of a bunch is negative and proportional to the square of the size [compare formulas (32a) and (33a)]. This is also true for a large distance between two adjacent scatterers when the coherence factor differs from unity only by a small correction inversely proportional to this distance. The only difference between formulas (33b) and (32b) is that this correction for a cubic array is inversely proportional to the number of scatterers on an edge of a cube, whereas for a linear chain it depends weakly on the number of scatterers.

4.3.2 Numerical calculations

(a) Cubic array. Numerical calculations of the functional dependences of the right-hand side of formula (32) on the shortest distance between two adjacent scatterers in the array ($\theta = k_0 d$) and on the total number of the scatterers on an edge of the cube ($n = N^{1/3}$) were carried out for the range $n\theta \leqslant \pi$ and the values of n in the interval $2 \leqslant n \leqslant 10$. The results of these calculations are represented by the graphs in Figs 2 and 3.

It is evident from these graphs that, in the selected range of values of θ, the coherence factor is a monotonically decreasing function of the argument $(n-1)\theta/\pi$ and a monotonically rising function of the number of scatterers $N = n^3$ (Fig. 2).

If $n \geqslant 3$, the reduced coherence factor $K_{\text{tot}}^{(V)}(N_i, \theta)/N$ [representing the ratio of the coherence factor $K_{\text{tot}}^{(V)}(N, \theta)$ to its maximum value $K_{\text{tot}}^{(V)}(N_i, 0) = N$] is a universal function of $n\theta/\pi$, which is independent of the total number of scatterers (Fig. 3). In particular, at the point $n\theta = \pi$ this function is approximately one-third its maximum value, equal to unity. Hence, it follows that a significant (approximately threefold) reduction in the degree of coherence of the scatterers in a bunch occurs when the dimensions of the bunch are increased to half the scattered wavelength:

$$K_{\text{tot}}^{(V)}(n,\theta) \approx \frac{1}{3}K_{\text{tot}}^{(V)}(n,0)\,, \qquad n \geqslant 3\,.$$

(b) Linear chain. Numerical calculations of the factor $K_{\text{tot}}^{(L)}(n,\theta)$ for a linear chain were carried out on the basis of formula (33) in a wider range of n ($3 \leqslant n \leqslant 120$) for finite values of the chain length defined by the inequalities $0 < (n-1)\theta \leqslant \pi$. The results of some of these calculations are presented in Figs 4 and 5 and demonstrate that:

— in the investigated range of the bunch lengths the

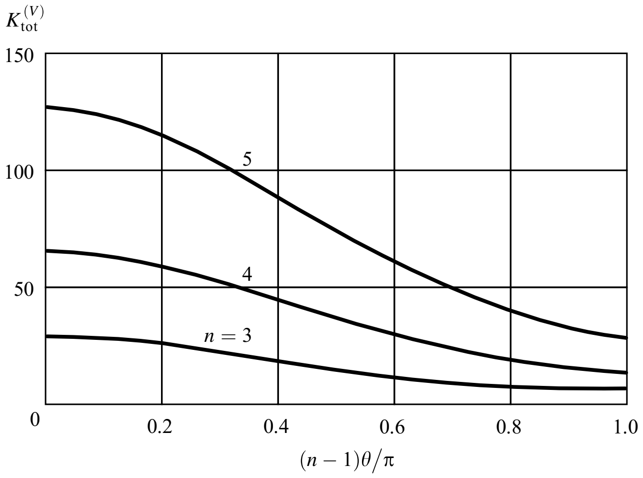

Figure 2. Dependence of the coherence factor $K_{\text{tot}}^{(V)}$ on the length $(n-1)\theta/\pi$ of an edge of a cube, plotted for three values of n.

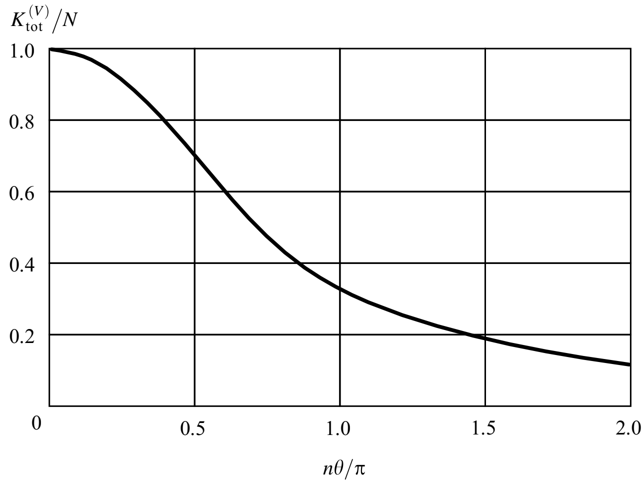

Figure 3. Dependence of the reduced coherence factor $K_{\text{tot}}^{(V)}/N$ on the length $n\theta/\pi$ of an edge of a cube.

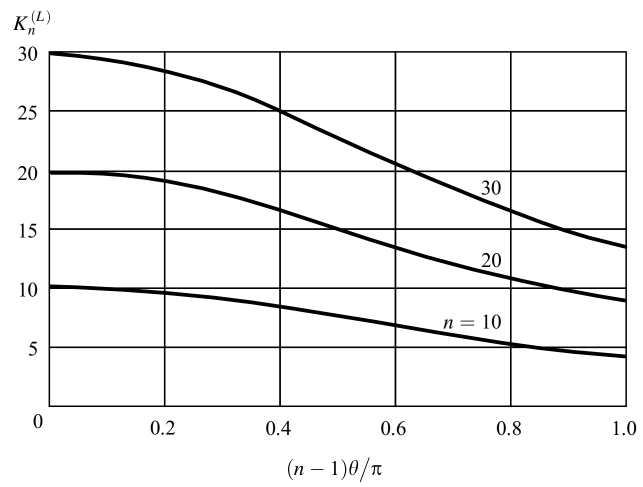

Figure 4. Dependence of the coherence factor $K_n^{(L)}$ on the length $(n-1)\theta/\pi$ of a chain, plotted for three values of n.

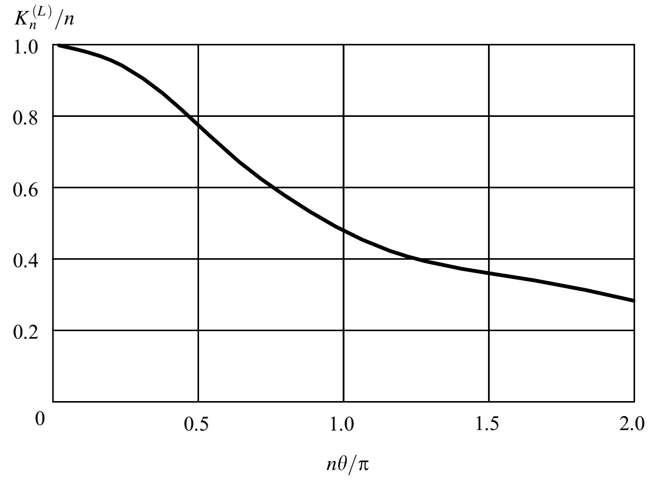

Figure 5. Dependence of the reduced coherence factor $K_n^{(L)}/n$ on the length $n\theta/\pi$ of a chain.

coherence factor decreases monotonically on increase in this length $(n-1)\theta/\pi$ and it rises monotonically with increase in the number of scatterers n (Fig. 4);
— in the range $n \geqslant 4$ the reduced coherence factor, i.e., the ratio $K_n^{(L)}/n$, is a monotonically decreasing function of the argument $n\theta/\pi$, which is independent of the number n (compare Figs 3 and 5);
— when the length of the chain is equal to half the scattered wavelength, the ratio $K_n^{(L)}/n$ is approximately $1/2$, i.e. the reduced coherence factor is larger than in the case of a cubic array (compare Figs 3 and 5).

4.3.3 Coherence between bunches

Calculations of the functional dependences of the right-hand sides of formulas (32) and (33) on the distance θ between the adjacent scatterers were carried out in the vicinity of $\theta \approx 2$. The results of those calculations which yield the efficiency or strength of the coherent interaction between the bunches are plotted in Fig. 6.

It follows from the graphs in Fig. 6 that for a linear chain of bunches the relative enhancement of the intensity of the scattered radiation, which is due to the coherent interaction between the bunches, depends weakly on the total number of bunches in a chain and even for $n = 30$ it does not exceed 40%:

$$\delta_n^{(L)}(\theta) \equiv K_{\mathrm{tot}}^{(L)}(n, \theta) - 1 \leqslant 0,4\,, \quad n \leqslant 30\,, \quad \theta \approx 2\pi\,.$$

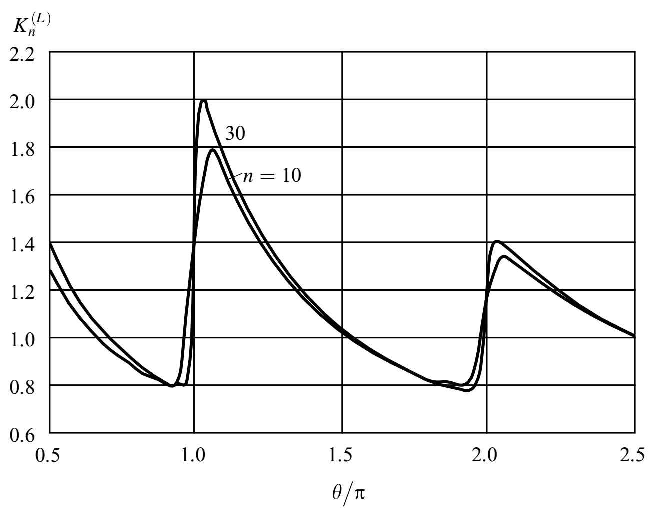

Figure 6. Dependence of the coherence factor $K_n^{(L)}$ on the distance θ/π between segments of a linear chain, plotted for two values of n.

4.4 Discussion of the physical meaning of the results

Before we discuss the results mentioned above, we must consider first the details of the physical nature of the mechanism responsible for the generation of coherent bremsstrah-lung by scattering charges.

It follows from our procedure for the calculation of the coherence factor, given by expression (30), that the radiative interaction of each pair of scatterers represents an element of the matrix G_{sp}. Physically, this element takes account of the mutual influence of the bremsstrahlung fields of charges with the numbers s and p. It is the specific spatial structure of the field that makes it possible to explain the reasons for the increase in the coherence factor when the distance between the scatterers in a bunch is reduced, as well as the rapid fall of this factor when the distance in question is increased.

In fact, in the case of a small bunch when all the scatterers are located in the near-field zones of the dipole bremsstrahlung of their neighbours, the field amplitudes are equal. In our case the phases of these fields are identical because the phases of oscillations of each of the scatterers are imposed by the shared field of the electromagnetic wave scattered by them. It is therefore clear that in the limiting case when $\theta \to 0$ the coherent addition of the radiative deceleration fields is most effective [91] for all the individual charges in the bunch: $G_{sp}(\theta \to 0) \to 1$. It is this addition that gives rise to the maximum of the coherence factor [compare formulas (32a) and (33a)].

In the other limiting case of a relatively large bunch ($\theta \gg 1$) each scatterer is located in the far-field zones of the dipole bremsstrahlung of all its neighbours. The field amplitudes then decrease in inverse proportion to the shortest distance within each pair of the nearest neighbours. This is supported in particular by the law describing the fall of the off-diagonal elements of the matrix G_{sp}:

$$G_{sp}(v_{sp} \gg 1) \approx 3\left[(v_{sp}^{(y)})^2 + (v_{sp}^{(z)})^2\right] \frac{\sin v_{sp} \cos v_{sp}^{(z)}}{2v_{sp}^3} \approx \mathrm{O}\left(\frac{1}{v_{sp}}\right)\,.$$

In this limiting case the scattering of the incident electromagnetic wave by a bunch is almost completely incoherent because of the fall of the strength of the exchange radiative interaction of scatterers when the distance between them increases:

$$\delta_N^{(V)}(\theta) \equiv K_{\mathrm{tot}}^{(V)}(N, \theta) - 1 = \mathrm{O}\left(\frac{1}{N^{1/3}\theta}\right)\,.$$

We must stress particularly the fairly rapid fall of the coherence factor with increase in θ when the dimensions of a bunch are comparable with half the scattered wavelength (see Figs 3 and 5), which is mentioned above. In the range $\theta \geqslant 2\pi$ this fall is so strong that it excludes almost completely the possibility of coherent amplification of the intensity of the bremsstrahlung of the neighbours (Fig. 6) even if the number of such neighbours is fairly large.

We shall conclude this section with the following comments. We have described above the relationships governing the coherence in a total energy flux of the scattered radiation, but we have ignored the angular distribution of the flux.

It is shown in Ref. [53] that in general the process of coherent amplification of the energy flux of the bremsstrahlung field can occur along certain directions even when the distances between the individual scatterers are relatively large ($\theta \gg 1$). This is particularly true of bunches with a periodic spatial structure. However, the coherence ensuring only local maxima in the angular spectrum of the radiation does not increase the total radiation intensity, but is simply evidence of a spatial redistribution of the energy flux of the incoherent bremsstrahlung of charged particles forming the investigated bunch.

For this reason the local maxima of the energy flux of the bremsstrahlung are of no interest in the case of undulator-radiation FELs, in which the considerable reduction in the length of the collective radiative beam deceleration by the field of its stimulated (coherent) undulator radiation, which is needed to reduce the undulator length, can be realised only by an increase in the total radiation intensity.

5. Conclusions

We shall summarise the results and conclusions of the analysis presented above by formulating the key stages of the process of development of a collective radiative instability of a monoenergetic relativistic beam of electrons in an undulator (an FEL amplifier operating in the SASE mode).

The primary source of an electromagnetic field in the investigated nonequilibrium system is the spontaneous undulator radiation emitted by individual beam electrons.

Moreover, if the volume density of the beam particles n_0 is sufficiently high and the energy of these particles γ_0 is not too high, so that the strong inequality $Q \equiv n_0 D^3/8\gamma_0^4 \gg 1$ is obeyed (thus ensuring a sufficiently large number of electrons in a cube whose edge is of the order of the undulator radiation wavelength in the beam rest frame), the individual radiators emit coherent undulator radiation. At least the electrons inside the bunches with dimensions not exceeding the undulator radiation wavelength (in a reference system in which the beam is at rest), when the bunches are formed by perturbations of the beam particle density, are coherent. The resultant field of the coherent undulator radiation of such a bunch is a wave travelling forward from the bunch at the velocity of light.

Finally, beats between the field of the coherent undulator radiation emitted by a bunch and a wave of periodic modulation of the transverse velocity of the beam particles in the undulator field create a longitudinal component of the Lorentz force, the phase velocity of which is equal to the unperturbed velocity of the beam. It is this force that causes aggregation of the beam particles which reach the region of influence of the force, producing coherent bunches as a result of the Veksler – McMillan phase stability mechanism.

It therefore follows that in our nonequilibrium system there is a positive feedback, which is necessary for the development of a collective radiation instability: an increase in the amplitude of the field of the coherent undulator radiation strengthens the modulation of the beam in respect of its density, which thus increases the degree of coherence of the undulator radiation emitted by individual electrons. The nonequilibrium system composed of a relativistic electron beam and an undulator then functions as an undulator-radiation FEL operating in the SASE mode.

However, if the beam intensity is low, but its energy is sufficiently high, the average distance between elementary radiators in the beam rest frame $[\langle\langle(\Delta\boldsymbol{r})^2\rangle^{1/2} \approx n_0^{-1/3}\rangle]$ proves to be considerably greater than the wavelength of their undulator radiation $\lambda'_{\rm r.r}$ in this system. This is why the coherence of the undulator radiation of the beam electrons cannot be achieved in this limiting case (this is due to the relatively rapid fall of the amplitudes of the undulator radiation fields of the individual electrons as the distance increases in their far-field zones, where the majority of the radiating neighbours is located). The nonequilibrium system then functions as the source of incoherent undulator radiation.

It therefore follows that a necessary condition for a qualitative change of the system from a source of incoherent undulator radiation to a source of stimulated (coherent) radiation is an increase in the number Q of elementary radiators in the interior of the undulator. The characteristic linear size is now of the order of the undulator radiation wavelength (in the beam rest frame) and Q increases up to values much greater than unity ($Q \gg 1$).

The corresponding minimum wavelength of the stimulated (coherent) undulator radiation emitted by a monoenergetic beam with a given density n_0 and with the maximum beam energy γ_* is given by the formulas

$$\min \lambda_{\rm u.r}^{\rm (coh)} = \left(\frac{2}{n_0 D}\right)^{1/2}, \tag{34}$$

$$\gamma_* \equiv \max \gamma_0 = \left(\frac{n_0 D^3}{8}\right)^{1/4}. \tag{35}$$

It should be stressed particularly that the theoretical value of the minimum wavelength of the coherent undulator radiation of an FEL amplifier is of purely methodological interest, because the limit is most probably unattainable in experiments. This is because this limit corresponds physically to the minimum degree of coherence of the undulator radiation (it corresponds to the boundary at which an FEL amplifier begins to act as a source of incoherent undulator radiation).

Under actual experimental conditions when considerable losses occur (for details see, for example, Refs [79, 99 – 103]), the absence or a low level of the coherence of the undulator radiation of the individual beam electrons does not result in a reduction of the collective deceleration length of the beam by the field of its undulator radiation, and, consequently, there is no reduction in the undulator length, which is always limited from above by finite values of the thermal velocity and the divergence of a beam.

For these reasons, the best parameters for experimental simulation are those which ensure the optimal compromise between the greatest possible reduction in the radiation wavelength, on the one hand, and the corresponding loss of the degree of its coherence (i.e. a reduction of the electron efficiency of an FEL and an increase in the undulator length and the degree to which a beam is monoenergetic), on the other.

In other words, the optimally shortest wavelength of the coherent radiation emitted by an electron beam in an FEL must necessarily be greater than the absolute minimum described by formula (34) and the difference should be the same as the extent to which this is permissible by the loss of the radiation coherence (including that due to an increase in the contributions of a departure from the monoenergetic state of an electron beam and its divergence).

We shall end this review by noting the aspects relating to the degree of reliability and generality of our results and conclusions.

First of all, an analysis of a set of experimental results reported in Ref. [49] confirms the conclusion of the occurrence of a correlation between the degree of coherence of the undulator radiation and the parameter Q. In fact, the maximum values of the coherence factor $K_{\rm tot}$ are obtained with the long-wavelength undulator radiation. The measured factor $K_{\rm tot}$ is, as expected, less than the value predicted by the theory of an idealised model of an FEL amplifier, because the real experimental conditions (finite width of the energy spectrum of the beam, nonzero divergence, and presence of radial beam density gradients) do not agree with the initial assumptions of the theory.

A reduction in the parameter Q reduces the measured values of the factor $K_{\rm tot}$. However, when the gain is small ($G \ll 1$) within the limits of the undulator length, where an FEL oscillator is to be realised, these values are found to be larger than those calculated theoretically for an FEL amplifier. The reason is the accumulation of the energy of the stimulated undulator radiation in the cavity, responsible for a considerable increase in the degree of coherence of the beam electrons (Section 3.4).

It then follows from Refs [89, 90] that the mechanisms of coherent amplification of the radiation emitted by elementary radiators, similar to those described in Section 4, apply also to quantum electronics.

Finally, in the case of the Cherenkov instability of a monoenergetic flux of charged discs in a waveguide (for example, in the case of formal application of the results of the hydrodynamic theory in the range $Q \ll 1$, where Q is the number of discs in one wavelength), we reach a paradoxical conclusion that it is possible to realise such an instability within the length of an amplifier $l_R \approx Q^{-1/3}$, which is considerably less than the average distance $\langle(\Delta z)^2\rangle^{1/2} \approx Q^{-1}$ between the discs.

It follows from the above that, in particular, in the range of finite values of the parameter Q (which do not satisfy the strong inequality $Q \gg 1$) the methods of the corpuscular theory of radiative instabilities are preferable, but this is true not only in the case of theoretical simulation of the SASE mode of undulator-radiation FEL amplifiers.

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