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Once again about the equivalence principle

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Abstract. The formulation of the equivalence principle underlying the general theory of relativity is considered. Implications of the equivalence principle are discussed with regard to a charge placed in a locally uniform gravitational field or a uniformly accelerated reference frame (the problem was addressed earlier elsewhere, for example by Ginzburg [1] in 1969). The criticism by Logunov and coworkers [2] of the canonical equivalence principle in electrodynamics as presented in the earlier review by Ginzburg [1] is shown to be irrelevant.

1. The equivalence principle (EP) and its role in physics in general, specifically in the general theory of relativity (GTR), has been extensively considered in numerous publications. Another detailed discussion of EP is beyond the scope of the present communication, but we believe it appropriate to recall once again its formulation and offer a few comments from the classical book by Pauli ([3], p. 196):

''For an infinitely small four-dimensional world-region (i.e. a world-region which is so small that the spacetime variation of gravity can be neglected in it), there always exists such a coordinate frame K_0 (X_1, X_2, X_3, X_4) in which gravitation has no influence either on the motion of a material point or any other physical process. In short, in an infinitely small world-region, any gravitational field can be destroyed by means of coordinate transformation. The local coordinate frame K_0 can be thought of as a freely soaring small box which is exposed to the effect of no other external forces but gravitation in which it falls freely.

''It is evident that the transformation of the coordinates is possible only because the gravitational field possesses the fundamental property of imparting the same acceleration to all bodies, in other words because gravitational and inertial masses always are equal. This inference is based on the most reliable experimental findings.''

We have selected this formulation of EP because the book in question [3] was first published as early as 1921. This means that the physical sense and the local (so to say) character of EP were quite obvious for many physicists within a few years after the construction of GTR was completed in 1915.

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Certain authors (see for instance Ref. [4]) distinguish between the Newtonian or weak equivalence principle (WEP) and the Einsteinian or strong equivalence principle (SEP). WEP is understood as being EP applied exclusively to mechanics, i.e. as the statement that all bodies travelling in a gravitational field are uniformly accelerated. This means that an observer enclosed in a freely falling elevator has no way to perceive accelerated motion of the bodies (in the past, such an elevator was frequently referred to as Einstein's elevator; nowadays it would rather be called a satellite). Of course, we ignore here possible tidal effects arising from inhomogeneity of the field, which tend to be vanishingly small with decreasing size of the elevator.

The transition to SEP accomplished by Einstein implies that all physical laws, including mechanical and electromagnetic, hold true in a falling elevator exactly as they do in the absence of gravity. Naturally, the definition of EP as suggested in Ref. [3] also included SEP.

Since strong, electromagnetic, and weak interactions all contribute to the body's inertial mass, experimental verification of the equality of inertial and gravitational masses may be regarded as a check of SEP just as it is a check of WEP (see Ref. [4]). The fact that the gravitational interaction contributes to the value of the inertial mass does not interfere with the universal character of the equality of inertial and gravitational masses (see Ref. [4], p. 162).

In elaborating GTR and in later works, Einstein used different SEP formulations and commented on them as he thought proper (see Refs [2] and [5], p. 32). However, his views were never in conflict with the formulation of EP by Pauli [3] (see above).

It should be emphasised that quantum events are beyond the scope of the present paper. EP would hold true if quantum effects (specifically, zero-field oscillations in vacuum) were taken into account provided an appropriate vacuum state has been chosen [6].

2. It is also worth discussing the application of EP to a charge e placed in a gravitational field or a uniformly accelerated reference frame. This problem attracted the attention of many authors and was examined in a number of papers [7, 8, 9, 10, 1, 2] for several reasons which will become clear from this review.

In examining different reference frames and different situations, we shall to a certain extent remain confined to classical mechanics (Newtonian approximation), where notions of the inertial reference frame, the external gravitational field with acceleration due to gravity $\boldsymbol{g}$, etc., are physically determined.

The first system to be discussed is the inertial reference frame K without a gravitational field. This system can be physically realised somewhere in outer space at a sufficiently large distance from all masses. Evidently, a free charge e in system K does not radiate. This is especially clear when charge velocity (with respect to system K) $v = \mathbf{0}$. In this case, it simply has 'nothing to emit', and its field is the Coulomb field. In other reference frames to be discussed below, charge velocity at a given moment is also considered to equal zero.

Let us now examine reference frame K_g, i.e. an inertial reference frame in which the gravitational field has acceleration $\boldsymbol{g}$. In the restricted space-region of interest, field $\boldsymbol{g}$ may be assumed to be uniform and constant in time. The reference frame K_g can be realised, with a certain degree of error, in a relatively small domain nearby any mass, e.g. close to the Earth's surface. In such a system, a free charge is likely to propagate with constant acceleration $\boldsymbol{g}$.

The problem of radiation in this or in a more general uniformly accelerated motion is a matter of long-standing discussion (see Refs [1, 3, 7, 8, 9, 10, 11] and publications cited in the bibliography of each report). It is now quite clear that a uniformly accelerated charge radiates in the sense that the flow of the Poynting vector through the surface surrounding the charge is nonzero:

$$P = \frac{2e^2}{3c^3} w^2 , \quad (1)$$

where $w^2/c^4 = -w^i w_i$, w^i is the four-vector of charge acceleration, and c is the velocity of light (see Ref. [1] and chapter 3 in Ref. [11] for details). If charge velocity v at the instant of emission is low compared with c, then $w^2 = |\dot{v}|^2$; in this case, the flow of the Poynting vector in system K_g is

$$P = \frac{2e^2}{3c^3} g^2 . \quad (2)$$

Finally, there is a reference frame K_a, that is a field-free system with acceleration $\boldsymbol{a} = -\boldsymbol{g}$ relative to reference frame K. In agreement with EP, a free charge placed in reference frame K_a (in an elevator) should be expected to behave precisely the way it does in system K_g; in particular, it must emit energy (2).

Moreover, Ginzburg [1] mentions reference frame K_{ga} which falls freely in system K_g, i.e. a reference frame with acceleration $\boldsymbol{g}$ with respect to K_g. Therefore, system K_{ga} is a locally inertial reference frame in which a charge does not radiate. In other words, system K_{ga} is in fact the reference frame K_0 mentioned in the above formulation of EP [3].

The presence of radiation in system K_a looks at first sight like a paradox because charge e is not accelerated with respect to reference frame K and cannot radiate. Indeed, this paradox has become a matter of debate in the literature (see for example Ref. [1]).

The thing is that expression (1) is Lorentz covariant, i.e. it is the same in any inertial reference frame. However, this expression need not be preserved, as it is actually not, upon the transition to noninertial systems like a uniformly accelerated reference frame K_a.

This situation is similar to that which is well-known to take place in the case of electromagnetic fields in different inertial reference frames. In a reference frame K where a charge is at rest, only the Coulomb field of the charge is present, whereas both the transverse electric field and the magnetic field are inherent in inertial reference frames K′ which are in uniform motion with respect to frame K. Similarly, transition from reference frame K to system K_a results in a change of the charge field: energy $P = 0$ in K while $P \neq 0$ in K_a.

One more paradox is the possibility 'to create radiation', i.e. to make energy P nonzero by means of transition from one reference frame to another. Nevertheless, the transition from reference frame K to K_a fails to give rise to new particles, e.g. electrons, androns, photons, etc.

The solution of this apparent paradox is to be found in the fact that $P \neq 0$ is not equivalent to the presence of photons, that is free solutions of the electromagnetic field equations. This problem has been discussed at greater length in [11].

A group of authors (A A Logunov, M A Mestvirishvili, and Yu V Chugreev [2]) turn down the propositions of Ginzburg [1]. These authors argue: ''...V L Ginzburg's statement that a charge in frame K_a radiates is utterly wrong, ...just as wrong as his statement that a charge in system K_{ga} does not radiate. If either statement were true, it would mean the possibility to transform radiation away by the choice of the reference frame, which is physically unattainable.'' In fact, Logunov et al [2] question the validity of EP or, to be more precise, Einstein's EP, which they thus set off against weak EP. Here is a quotation from Ref. [2]: ''...the equivalence principle as formulated above holds true for mechanical processes but is not applicable to electrodynamic ones. This suggests the possibility to ascertain, by making measurements inside the system, whether a reference frame is inertial or falls freely in a uniform gravitational field.'' Here is another extract from this paper [2], practically to the same effect: ''...an inertial reference frame with a static homogeneous gravitational field is not equivalent, in physical terms, to a uniformly accelerated reference frame free of a gravitational field. Therefore, the equivalence principle is not fulfilled for electromagnetic phenomena.''

Meanwhile, the validity of SEP has been proved with a very high degree of accuracy [4]. Specifically, it has been shown that the equality of inertial and gravitational masses holds true up to 10^{-12} [4, 12]. Apart from this argument, the reasoning of Logunov et al [2] is beneath criticism in itself. That the identification of energy P [see (1)] and the energy of free electromagnetic radiation (photon flow) is unsound has already been mentioned in the foregoing discussion. Therefore, the possibility to create or frustrate 'emission' of P by the choice of a reference frame does not contradict current views. It will be shown below by means of direct calculation that a charge in reference frame K_a emits energy provided the transition from reference frame K to frame K_a is correctly accomplished; moreover, the emitted energy is exactly that given by Eqn (2).

It should be emphasised that we introduce the notion of radiation and variable P in different reference frames because it is this variable that is commonly referred to in the literature reviewed in the present paper. As far as the application of EP is concerned, it would be possible and even more rational to be confined to fields themselves. It follows from EP that the electromagnetic fields of a charge in reference frames K_g and K_a are similar, which immediately implies equality of the quadratic-in-field values P in these reference frames.

3. Any physical theory has to deal with the problem of measurement, that is a methodological problem of the evaluation of pertinent physical parameters, the choice of instruments, and the ways to interpret the results. This problem is considered to be of primary importance in quantum theory where its discussion remains a matter of great concern in the current literature.

However, the problem extends far beyond the limits of quantum theory. Specifically, the features of rulers and clocks need to be more accurately specified so far as measurement of spatial distances and time intervals between different events are concerned. For example, in comparing lengths and time intervals in two inertial reference frames K and K$'$ which are in motion relative to each other at a constant speed, any conclusion about the contraction of travelling rulers (rods) and the deceleration of running clocks is made with the rulers and the clocks in each system being regarded as similar. It is common to speak about rigid or standard rulers and standard or ideal clocks. Generally speaking, it must be possible to transfer these rulers and clocks between systems to ensure the identity of the rulers and clocks in reference frames K and K$'$. However, this is impossible to do without accelerating both the rulers and the clocks; they must be insensitive to acceleration.

Furthermore, the length of the rulers and the running of the clocks must be independent of their location in space and time. If this requirement cannot be met (e.g. clocks showing different time in reference frames K and K$'$ are used), the Lorentz transformations and their known corollaries will be incorrect.

Of course, this line of reasoning holds equally true in nonrelativistic physics where clocks in reference frames K and K$'$ are also considered similar whenever the Galilean transformations need to be defined. That is probably why this problem is usually assumed to be generally known and the said requirements of rulers and clocks are just implied rather than explicitly specified in various textbooks (see for instance Ref. [13]). There is certainly some reason for this assumption, but we believe it opportune to highlight the problem of measurement when dealing with the special theory of relativity in order to avoid misinterpretation. An example of a detailed discussion of the requirements to rulers and especially to clocks in the relativistic theory is provided in Ref. [14], dedicated to 'the twins paradox'.

In special relativity (inertial reference frame K or Minkowski space), the Cartesian coordinate system $x^i = (ct, x, y, z)$ is most frequently used in which the square of the interval (borrowing notation from Ref. [13]) is

$$ds^2 = c^2\,dt^2 - dx^2 - dy^2 - dz^2 \,. \tag{3}$$

In such a system, the above agreement as regards rulers and clocks actually looks almost evident.

Generally speaking, GTR uses curvilinear coordinates $x^i = (x^0, x^1, x^2, x^3)$ so that

$$ds^2 = g_{ik}(x^i)\,dx^i\,dx^k \,. \tag{4}$$

But the relationship between coordinates x^i and 'real' distance and time measured with standard rulers and clocks is not so obvious. This problem was discussed by Landau and Lifshitz ([13], paragraph 84), who showed that intervals of real time or proper time $d\tau$ at a given space point are related to dx^0 as

$$d\tau = \frac{1}{c}\sqrt{g_{00}}\,dx^0 \,. \tag{5}$$

For the distance dl between two close points, one has

$$dl^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha}g_{0\beta}}{g_{00}}\right) dx^\alpha\,dx^\beta \,. \tag{6}$$

We shall use Cartesian coordinates x^i with interval (3) in the inertial reference frame K. For the transition to the uniformly accelerated reference frame K_a, corresponding coordinates x'^i must be chosen rather than arbitrary ones to allow us to compare the results of the measurement with those in system K. Doubtless, Einstein was perfectly aware of this when he wrote the following in one of his papers ([15], p. 189) published in 1912, i.e. before the construction of GTR was completed:

"Suppose that system K (coordinates x, y, z)† is in uniformly accelerated motion towards axis X. This acceleration is uniform in Born's sense, which means that acceleration of the coordinate origin in this system is constant with respect to such a non-accelerated system relative to which points of system K are at rest (that is, they have infinitely low speed, to put it more precisely). According to the equivalence principle, such a system K is exactly equivalent to a certain system at rest where a mass-free gravitational field of a given form acts (masses that generate this field can be imagined as infinitely remote). Spatial measurements in system K are performed by means of scales which have identical lengths if compared with each other at rest in a selected place of the system. All geometric properties as well as the relations between coordinates x, y, z and other lengths must be examined with the use of such scales. These rules must not be taken for granted; instead, they contain certain physical assumptions which may sometimes happen to be incorrect. ...It is feasible to imagine scales, as well as axes of coordinates, in the form of absolutely rigid rods despite the fact that absolutely rigid bodies cannot exist in the relativistic theory. In fact, absolutely rigid measuring rods may be imagined as being composed of a large number of bodies which are not absolutely rigid; they are connected in such a way that they do not transfer pressure between them when one or another rod stops. We shall measure time t in the reference frame K with a clock positioned in spatial points of the system so that the time interval measured in this way and necessary for a light beam to travel between two points A and B of the frame is independent of the instant when the light beam was emitted from A... ."

Furthermore, in the same paper, Einstein examined system K_g with a gravitational field, but we feel it would be inappropriate to dwell on this subject here.

4. Thus, our objective is to construct coordinates in the accelerated reference frame K_a that would satisfy the above requirements. It will be shown below that such coordinates include the coordinates $x'^i = (c\eta, \xi, \chi, \rho)$, described for instance in Ref. [16] (paragraph 18.6) and usually referred to as Moller coordinates.

The relationship between Moller coordinates and coordinates $x^i = (ct, x, y, z)$, which were previously used

†The system denoted here by K is evidently our system K_a.

in the inertial reference frame K may be presented in the following form:

$$ct = \rho \sinh \frac{a\eta}{c}\,, \tag{7}$$

$$x = \xi\,, \tag{8}$$

$$y = \chi\,, \tag{9}$$

$$z = \rho \cosh \frac{a\eta}{c}\,, \tag{10}$$

where $\rho \geqslant 0$. In Ref. [16], this transformation was introduced by considering system K_a as a continuous transition between instantaneously co-moving systems.

It is easy to see that

$$ds^2 = g'_{ik}\, dx'^i\, dx'^k = \frac{a^2\rho^2}{c^2}\, d\eta^2 - d\rho^2 - d\xi^2 - d\chi^2\,. \tag{11}$$

According to Eqns (7)–(10) and (6), in Moller coordinates,

$$dl^2 = d\rho^2 + d\xi^2 + d\chi^2\,, \tag{12}$$

i.e. the physical length element coincides with the coordinate one. Element dl is independent of the coordinates, and measuring rulers may be regarded as rigid (see Ref. [6], paragraph 2.2, for details).

Furthermore, in compliance with Eqns (5) and (11), where $x^0 = c\eta$,

$$d\tau = \frac{a\rho}{c^2}\, d\eta\,, \tag{13}$$

with variable η (coordinate time) being the proper time at the point $\rho = c^2/a$.

It follows from the definition of the Moller coordinates (7)–(10) that they cover only a part of the Minkowski space in which $z > c|t|$. Logunov and co-workers [2] consider this to be a defect since, in their opinion: "...from the physical point of view, transition to a new reference frame should always ensure transformations of all points of the spacetime in the initial reference frame". We consider this statement to be completely wrong because in GTR reference frames of 'global' nature are far from being always considered.

In the case in question, the discussion concerns small domains of spacetime in keeping with the spirit of EP. It is in these domains that the equivalence of reference frames K_a and K_g is to be demonstrated. Moreover, it is physically meaningless to consider uniformly accelerated motion at all times, i.e. in the interval $-\infty < t < \infty$, as is emphasised by many authors (see for instance Refs [1, 11]). To summarise, the transition to Moller coordinates (7)–(10) for the description of the uniformly accelerated frame K_a meets all necessary requirements†.

If a particle (a charge) in a reference frame K_a is at rest, its coordinates ρ_A, ξ_A, and χ_A are constant, i.e. $d\rho_A = d\xi_A = d\chi_A = 0$. In system K, the particle is in hyperbolic motion, that is $z_A^2 - (ct_A)^2 = \rho_A^2$ (henceforth, index A will be omitted). The velocity v of the particle, its acceleration, and proper time τ are

$$v = \frac{dz}{dt} = \frac{(a\rho/c)\sinh(a\eta/c)\, d\eta}{(1/c)\,(a\rho/c)\cosh(a\eta/c)\, d\eta}$$

$$= \frac{c \sinh(a\eta/c)}{\cosh(a\eta/c)} = \frac{c(ct/\eta)}{\sqrt{1 + (ct/\rho)^2}} = \frac{c^2 t}{z}\,,$$

$$\frac{dv}{dt} = \frac{c^2\rho^2}{(\rho^2 + c^2t^2)^{3/2}} = \frac{c^2\rho^2}{z^3}\,, \tag{14}$$

$$\tau = \int_0^t \sqrt{1 - \frac{v^2}{c^2}}\, dt = \frac{a\rho}{c^2}\,\eta\,.$$

The four-vectors of velocity u^i and acceleration w^i, together with the vector dw^i/ds, for this particle, if expressed via the coordinates ct, ξ, χ and ρ, have the form

$$u^i = \frac{dx^i}{ds} = \left(\cosh\frac{a\eta}{c}\,,\ 0,\ 0,\ \sinh\frac{a\eta}{c}\right),$$

$$w^i = \frac{du^i}{ds} = \left(\frac{1}{\rho}\sinh\frac{a\eta}{c}\,,\ 0,\ 0,\ \frac{1}{\rho}\cosh\frac{a\eta}{c}\right), \tag{15}$$

$$\frac{dw^i}{ds} = \left(\frac{1}{\rho^2}\cosh\frac{a\eta}{c}\,,\ 0,\ 0,\ \frac{1}{\rho^2}\sinh\frac{a\eta}{c}\right),$$

since, for instance,

$$u^0 = \frac{dx^0}{ds} = \frac{d(ct)}{(a\rho/c)\, d\eta}, \qquad w^0 = \frac{d[\cosh(a\eta/c)]}{(a\rho/c)\, d\eta}\,.$$

It is known (see for example Ref. [11]) that for uniformly accelerated motion

$$\frac{dw^i}{ds} + w^k w_k u^i = 0\,,$$

which follows from Eqn (15) because $w^k w_k = -1/\rho^2$. Therefore a particle at rest in reference frame K_a is in uniformly accelerated motion in system K. Conversely, a particle at rest in reference frame K will be uniformly accelerated in reference frame K_a provided it travels at low speed [8].

5. It is now time to prove that a charge which does not radiate in the inertial system K, when at rest, does radiate if placed in reference frame K_a. If the charge is at rest at the point of reference frame K with coordinates $x = y = 0$, $z = c^2/a$, it is supposed to have a purely Coulomb field in this system, that is the tensor of the electromagnetic field will have the following form (see Ref. [13], paragraph 24, for notation):‡

$$F^{ik} = \begin{pmatrix} 0 & -\dfrac{e}{r_0^3}\,x & -\dfrac{e}{r_0^3}\,y & -\dfrac{e}{r_0^3}\left(z - \dfrac{c^2}{a}\right) \\ \dfrac{e}{r_0^3}\,x & 0 & 0 & 0 \\ \dfrac{e}{r_0^3}\,y & 0 & 0 & 0 \\ \dfrac{e}{r_0^3}\left(z - \dfrac{c^2}{a}\right) & 0 & 0 & 0 \end{pmatrix}, \tag{16}$$

†It should be noted that Ref. [10] suggests coordinates which generalise the Moller coordinates and cover the entire space.

‡We have presented here all the details of the calculation because this is exactly what the authors of the paper subjected to criticism [2] did although they published it in a journal intended for original reports. Such details are even more pertinent in the present communication, submitted to a journal devoted to reviews of current problems.

where

$$r_0^2 = \rho_0^2 + \left(z - \frac{c^2}{a}\right)^2, \qquad \rho_0^2 = x^2 + y^2 .$$

The tensor of energy–momentum of the field (see Ref. [13], paragraph 33) is

$$T^{ik} = \frac{1}{4\pi}\left(-F^{il}F^{k}{}_{l} + \frac{1}{4}g^{ik}F_{lm}F^{lm}\right) ,$$

Hence

$$4\pi T^{00} = \frac{1}{2}\frac{e^2}{r_0^4}, \qquad T^{01} = T^{02} = T^{03} = 0 ,$$

$$4\pi T^{12} = -\frac{e^2}{r_0^6}xy ,$$

$$4\pi T^{13} = -\frac{e^2}{r_0^6}x\left(z - \frac{c^2}{a}\right) ,$$

$$4\pi T^{23} = -\frac{e^2}{r_0^6}y\left(z - \frac{c^2}{a}\right) , \tag{17}$$

$$4\pi T^{11} = -\frac{e^2}{r_0^4}\left(\frac{1}{r_0^2}x^2 - \frac{1}{2}\right) ,$$

$$4\pi T^{22} = -\frac{e^2}{r_0^4}\left(\frac{1}{r_0^2}y^2 - \frac{1}{2}\right) ,$$

$$4\pi T^{33} = -\frac{e^2}{r_0^2}\left[\frac{1}{r_0^2}\left(z - \frac{c^2}{a}\right)^2 - \frac{1}{2}\right] .$$

Tensor T^{ik} is symmetric.

A shift to system K_a with coordinates (7)–(10) yields a charge with coordinates $\xi = \chi = 0$, $\rho = c^2/a$ at time $\eta = 0$. The velocity of the charge is

$$\frac{d\rho}{d\eta} = -\frac{az\sinh(a\eta/c)}{c\cosh^2(a\eta/c)} = -\frac{a\rho t}{z} = 0 ,$$

and its acceleration is

$$\frac{d^2\rho}{d\eta^2} = -\frac{a^2\rho}{c^2} = -a .$$

(It should be borne in mind that in system K the charge is at rest, with $z = c^2/a$. For this reason, in reference frame K_a the charge has acceleration $-a$ at the instant $\eta = t = 0$. This is exactly what occurs in an elevator which falls with acceleration $a = -g$, g being the free-fall acceleration in system K_g.)

Coefficients $\partial x'^i/\partial x^l$ of the tensor transformation law

$$T'^{ik} = \frac{\partial x'^i}{\partial x^l}\frac{\partial x'^k}{\partial x^m}T^{lm} \tag{18}$$

have the form

$$\frac{\partial(c\eta)}{\partial(ct)} = \frac{c^2}{a\rho}\cosh\frac{a\eta}{c} ,$$

$$\frac{\partial(c\eta)}{\partial z} = -\frac{c^2}{a\rho}\sinh\frac{a\eta}{c} ,$$

$$\frac{d\xi}{dx} = \frac{d\chi}{dy} = 1 , \tag{19}$$

$$\frac{\partial\rho}{\partial(ct)} = -\sinh\frac{a\eta}{c} ,$$

$$\frac{\partial\rho}{\partial z} = \cosh\frac{a\eta}{c} .$$

Other coefficients vanish. The coordinates $(c\eta, \xi, \chi, \rho)$ must certainly be expressed via the coordinates (ct, x, y, z) by means of relations (7)–(10), from which it follows that $z^2 - c^2t^2 = \rho^2$.

With the aid of Eqns (17)–(19), the necessary components of the energy–momentum tensor in the reference frame K_a can be found:

$$T'^{01} = \frac{c^2}{\rho a}\frac{e^2}{4\pi r_0^6}x\left(z - \frac{c^2}{a}\right)\sinh\frac{a\eta}{c} ,$$

$$T'^{02} = \frac{c^2}{\rho a}\frac{e^2}{4\pi r_0^6}y\left(z - \frac{c^2}{a}\right)\sinh\frac{a\eta}{c} , \tag{20}$$

$$T'^{03} = \frac{c^2}{\rho a}\frac{e^2}{4\pi r_0^4}\left[\frac{1}{r_0^2}\left(z - \frac{c^2}{a}\right)^2 - 1\right]\cosh\frac{a\eta}{c}\sinh\frac{a\eta}{c} .$$

It is now possible to find the energy flow of the electromagnetic field through the spherical surface

$$\xi^2 + \chi^2 + \left(\rho - \frac{c^2}{a}\right)^2 = (c\eta)^2 \tag{21}$$

at time η. Calculations are to be made up to terms of order $(a\eta/c)^2$, neglecting terms of a higher order: $O(a\eta/c)^3$, etc. Expansion of all the values in a series yields

$$z - \frac{c^2}{a} = \rho\cosh\frac{a\eta}{c} - \frac{c^2}{a}$$

$$= \rho - \frac{c^2}{a} + \frac{\rho}{2}\left(\frac{a\eta}{c}\right)^2, \qquad \sinh\frac{a\eta}{c} = \frac{a\eta}{c} .$$

The following expressions are obtained with the same accuracy at the surface of sphere (21):

$$\frac{1}{\rho} = \left(\rho + \frac{c^2}{a} - \frac{c^2}{a}\right)^{-1}$$

$$= \frac{a}{c^2}\left[1 + \frac{a}{c^2}\left(\rho - \frac{c^2}{a}\right)\right]^{-1}$$

$$= \frac{a}{c^2}\left(1 - \frac{a}{c^2}c\eta\cos\theta\right) ,$$

$$r_0^2 = \xi^2 + \chi^2 + \left(z - \frac{c^2}{a}\right)^2$$

$$= c^2\eta^2 + \frac{\rho}{2}\left(\frac{a\eta}{c}\right)^2 c\eta\cos\theta ,$$

where coordinates θ and φ are introduced such that

$$\xi = c\eta\sin\theta\cos\varphi ,$$

$$\chi = c\eta\sin\theta\sin\varphi ,$$

$$\rho - \frac{c^2}{a} = c\eta\cos\theta .$$

The flow of the Poynting vector $S^\alpha = cT^{0\alpha}$ across sphere (21) is

$$P = c\int_{-1}^{1}\int_0^{2\pi} d\varphi\, d(\cos\theta)c^2\eta^2\left\{T'^{01}\sin\theta\cos\varphi + T'^{02}\sin\theta\sin\varphi + T'^{03}\cos\theta\right\} , \tag{22}$$

where the integrand is evaluated on the surface of sphere (21) at time η; values of $T'^{0\alpha}$ are given in Eqn (20).

It is worthy of note that the surface of sphere (21) does not coincide with a constant-phase surface of the electromagnetic field. This can be accounted for by the difference between coordinate velocities of light dx^α/dx^0 at points $\rho < c^2/a$ and $\rho > c^2/a$ in reference frame K_a. Indeed, at

$$ds^2 = \frac{a^2\rho^2}{c^2}\,d\eta^2 - d\rho^2 - d\xi^2 - d\chi^2 = 0$$

the velocity

$$\frac{\sqrt{d\rho^2 + d\xi^2 + d\eta^2}}{d\eta} = \frac{a\rho}{c}\,.$$

This explains why radiation emitted at the instant $\eta = 0$ cannot simultaneously arrive at different points of the sphere. However, it is possible to show that taking this fact into account affects only higher-order terms.

Further calculation of P from Eqn (21) is simple but tedious. We went through it in the smallest detail, but it would be inappropriate to present here a minute description of the procedure because the final result is in agreement with EP as expected, that is

$$P = \frac{2e^2}{3c^3}\,a^2\,, \tag{23}$$

which coincides with (2), provided $a^2 = g^2$.

Calculation neglecting terms of $O(a\eta/c)^3$ or higher is not a defect because what we are interested in are small spacetime domains, in compliance with the spirit of EP, which was more than once emphasised in the foregoing discussion.

Calculations similar to those described in Ref. [10] unambiguously show that in system K_{ga} falling freely in reference frame K_g, the charge does not radiate even though it is certain to radiate in system K_g.

The equivalence principle constitutes the basis of GTR. Therefore, any calculation in the framework of GTR would not contradict EP. Meanwhile, the great heuristic potency of EP is apparent from the above discussion of charge radiation in reference frame K_a. The result (23) that immediately follows from EP is obtainable with the use of the machinery of GTR only after simple but tedious calculations.

6. There is one more example of the possibility of obtaining a result directly from EP when it is not at all evident without the use of EP. Let us consider, in the relativistic approximation, a certain object (a solid body or an atom) which is in motion with constant acceleration with respect to an inertial reference frame K. We wish to describe the object in the framework of nonrelativistic quantum mechanics. Specifically, we would like to describe the propagation of a conduction electron in a uniformly accelerated metal [17, 18] and find level shifts of the uniformly accelerated atom.

Because the object is at rest, its behaviour can be described by means of the Schrodinger equation

$$i\hbar\,\frac{\partial\Psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\,\nabla^2 + U(x,y,z)\right]\Psi\,, \tag{24}$$

where $U(x,y,z)$ is the potential energy of a conduction electron, an electron in the atom, etc. The Schrodinger equation (24) is written in the inertial reference frame, which is not normally specified. Whenever a body or an atom is in accelerated motion (e.g. along the z axis with constant acceleration a), it is necessary to substitute $U(x,y,z)$ in Eqn (24) by

$$U\left(x,y,z-\frac{a}{2}\,t^2\right)\,. \tag{25}$$

The transition to the uniformly accelerated reference frame associated with the body, atomic nucleus, etc., is effected by the transformation

$$t' = t\,,\quad x' = x\,,\quad y' = y\,,\quad z' = z - \frac{a}{2}\,t^2\,. \tag{26}$$

After a new wave function is introduced,

$$\Psi = \Psi'\exp\left[\frac{i}{\hbar}\left(mat'z' + \frac{1}{6}\,ma^2t'^3\right)\right]\,, \tag{27}$$

it follows from Eqns (24) and (25) that

$$i\hbar\,\frac{\partial\Psi'}{\partial t'} = \left[-\frac{\hbar^2}{2m}\,\nabla'^2 + U(x',y',z') + maz'\right]\Psi'\,. \tag{28}$$

This results in precisely the same equation for Ψ' as that in the inertial reference frame, with the exception that potential energy maz' is added to the former expression. This is what immediately follows from EP [18] because in a homogeneous gravitational field with acceleration $g = -a$, the potential energy is exactly maz'. A potential of a similar type occurs in a uniform electric field. That is why elucidation of the effect of acceleration on the running of an atomic clock is possible on the basis of the known results relating to the Stark effect in atoms.

7. There is one more point to clarify: why Logunov et al. [2] have obtained a result which is in conflict with EP. The authors examined a charge at rest in the inertial reference frame K and calculated its radiation in a certain noninertial reference frame K_N, assumed to be uniformly accelerated. Furthermore, following the line of reasoning described above, they proved that the charge does not radiate in the latter system, which is true. However, the crux of the matter is that reference frame K_N is radicaly different from system K_a, which is associated with uniformly accelerated motion and discussed in connection with EP.

In fact, Logunov et al. [2] use the reference frame K_N with coordinates

$$\eta = t\,,\quad \xi = x\,,\quad \chi = y\,,\quad \rho = z - c\sqrt{\frac{c^2}{a^2} + t^2}\,. \tag{29}$$

To the second order in at/c, coordinates (29) correspond to the Galilean transformation

$$\eta = t\,,\ \xi = x\,,\ \chi = y\,,\ \rho = z - \frac{c^2}{a} - \frac{at^2}{2} + O\left(\frac{at}{c}\right)^3\,. \tag{30}$$

It is clear that such a transformation eliminates relativistic effects of order $(v/c)^2$, i.e. $(at/c)^2$. At the same time, the total radiated power (23) is of order $(a/c)^2$. This accounts for the necessity of relativistic calculations in reference frame K_a notwithstanding arbitrarily low velocity of the charge. Incidentally, the relativistic transformation rules have to be taken into account at arbitrarily low velocities and specifically in examining the Thomas precession [19].

The reference frame K_N is not rigid because the following expression holds for the physical length element dl [see Eqn (6)]:

$$dl^2 = d\xi^2 + d\chi^2 + d\rho^2 + g^2t^2\,d\eta\,d\rho\,, \tag{31}$$

i.e. dl is time dependent.

This fact immediately ensues from Eqn (30). Suppose, for instance, that the ends of a rod which is at rest in reference frame K have coordinates ρ_1 and ρ_2. Then, at each instant of time t, the length of the rod is $\rho_2 - \rho_1 = z_2 - z_1$, where z_2 and z_1 are the coordinates of its ends in reference frame K. This means that in reference frame K there is no Lorentz contraction of the rod length when the rod is at rest in system K_N. Therefore, the length of the rod in reference frame K_N is inconstant, being time dependent and increasing with time at $t > 0$. Thus, reference frame K_N is not to be compared with system K_g in the framework of EP.

To summarise, the criticism [2] of propositions suggested by one of the authors of the present communication in Ref. [1] is unsubstantiated. We failed to find mistakes in Ref. [1]. It may be inferred that the statement of Logunov and co-workers [2] on the equivalence principle as applied to electrodynamics is invalid.

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