

METHODOLOGICAL NOTES

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The theory of the classical gravitational field

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Abstract. Equations for the massive gravitational field have been derived in the framework of the special theory of relativity on the basis of the geometrisation principle. Graviton mass has been shown to be crucial for the elaboration of the relativistic theory of gravitation. According to this theory, a homogeneous and isotropic universe develops in a series of alternating cycles, from high to low density, and cannot be anything other than flat. The theory predicts the presence of a large amount of latent mass in the universe and prohibits the existence of 'black holes'. Also, the theory explains all observable events so far known to occur in the solar system.

1. Introduction

Einstein's general theory of relativity (GTR), for which the principal equations were proposed by Gilbert and Einstein in 1915, opened a new stage in the study of gravitational phenomena. However, from the very beginning, this theory encountered, for all its progress and advances, serious difficulties with the evaluation of the physical characteristics of the gravitational field and the formulation of the energy – momentum conservation laws†.

†The Editorial Board would remind readers that problems dealt with in the present paper were more than once discussed in *Physics – Uspekhi* [see A A Logunov et al. **155** (3) 1988, Ya B Zel'dovich **155** (3) 1988, L P Grishchuk and A A Logunov et al. **160** (8) 1990].

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Einstein was perfectly aware of the fundamental importance of the energy – momentum conservation laws. Moreover, he believed the total tensor of matter and of the gravitational field, taken together, to be the source of the latter. In 1913, he wrote that ''the tensor of gravitational field $\vartheta_{\mu\nu}$ is a true source of the field as well as the tensor of material systems $\Theta_{\mu\nu}$. An exceptional position of the energy of the gravitational field compared with that of all other forms of energy would have inadmissible consequences.'' In the same work, Einstein came to the conclusion that ''in the general case the gravitational field is characterised by ten spacetime functions'', components of the metric tensor $g_{\mu\nu}$ of the Riemannian space. However, elaboration of the theory along this line did not enable Einstein to use the tensor of matter and the gravitational field as a field source since a pseudotensor in the Riemannian space arose in GTR, instead of the gravitational field tensor.

In 1918, SchroSdinger demonstrated that all the components of the energy – momentum pseudotensor of the gravitational field outside a spherically symmetric source can be made to vanish provided an appropriate system of coordinates is chosen. With regard to this, Einstein noted: ''As for the ideas of SchroSdinger, their persuasiveness lies in the analogy with electrodynamics where the stresses and energy densities of any field are nonzero. However, I cannot find any reason why the situation should be the same for gravitational fields. Gravitational fields can be specified without introducing stresses and energy densities.''

Thus, Einstein abandoned the concept of the gravitational field being a classical field of the Faraday – Maxwell type possessing energy – momentum density, even though he made an important step by linking the gravitational field with a tensor value. The value taken by Einstein was the metric tensor of Riemannian space, $g_{\mu\nu}$. Einstein seems to have considered this line of reasoning to be quite natural since his views on the gravitational field developed under the influence of the principle of equivalence between forces of inertia and gravitational forces, which he introduced as follows: "...for any infinitely small world-region, the coordinates can always be chosen in such a way that the gravitational field in that world-region vanishes."

Einstein was very persistent in emphasising this idea. In 1923, he wrote: "For any infinitely small vicinity of a point in an arbitrary gravitational field, the local system of coordinates can be specified in such a state of motion that there will be no gravitational field with respect to this coordinate system (the local inertial system)." This assertion gave rise to the belief that the gravitational field cannot be localised. According to Einstein, the presence of the energy–momentum pseudotensor is in good conformity with the equivalence principle†.

The above statement of Einstein is in fact not fulfilled in GTR because, for a physical characterisation of the field in this theory, it is necessary to consider the Riemann curvature tensor. That we have a clear understanding of this fact we owe to Synge. According to this author: "If we accept the idea that spacetime is a Riemann four-dimensional space (and if we are relativists we must), then surely our first task is to get the feel of it just as early navigators had to get the feel of a spherical ocean. And the first thing we have to get the feel of is the Riemann tensor, for it is the gravitational field: if it vanishes, and only then, there is no field at all. Yet, strangely enough, this most important fact has been pushed into the background." Synge proceeded as follows: "In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does not or does vanish. This is an absolute property, it has nothing to do with any observer's world line."

Thus, in compliance with GTR, matter (all matter fields with the exception of the gravitational field) is characterised by the energy–momentum tensor, whereas the physical characteristic of the gravitational field is the Riemann curvature tensor. The former tensor is of second rank while the latter is a fourth-rank tensor, which implies a fundamental difference between the properties of matter and the gravitational field in GTR.

The introduction of the energy–momentum pseudotensor of the gravitational field in GTR turned out to be of no great help to Einstein in his attempt to make provision for the energy–momentum conservation laws in his theory. Gilbert seems to have understood this fairly well when he wrote in 1917 as follows: "...I declare that for the general theory of relativity, that is in the case of general invariance of the Hamiltonian function, there are no energy equations that... correspond to energy equations in orthogonal-invariant theories; I could even note this fact as being a characteristic feature of the theory."

Unlike any other physical theory, GTR is fundamentally incapable of the introduction of the energy–momentum and angular momentum conservation laws because of the absence of the ten-parameter group of spacetime transformation. The laws of energy–momentum and angular momentum conservation are fundamental laws of nature. These laws impose universal physical properties on all forms of matter and allow their interconversion to be quantitatively analysed. Not surprisingly, it is tempting to have a gravitational theory that includes all energy–momentum and angular momentum conservation laws, with the gravitational field possessing energy–momentum density, similar to the case of the Faraday–Maxwell electromagnetic field.

In GTR, the scalar Lagrangian density of the gravitational field contains second-order derivatives of the field, at variance with all other physical theories.

About 50 years ago, Nathan Rosen showed that introduction of the metric $\gamma_{\mu\nu}$ of Minkowski space, along with the Riemannian metric $g_{\mu\nu}$, makes it possible to obtain a scalar Lagrangian density of the gravitational field with respect to arbitrary coordinate transformations, containing derivatives of not higher than the first order [1]. Specifically, Rosen devised such a Lagrangian density, which led to the Gilbert–Einstein equations. In this way, the bimetric formalism arose. However, this approach made elaboration of the gravitational theory even more complicated since using tensors $\gamma_{\mu\nu}$ and $g_{\mu\nu}$ allows rather a large number of scalar densities with respect to arbitrary coordinate transformation to be written and it is utterly unclear which scalar density to choose as the Lagrangian density for the development of the theory of gravitation.

Following this approach, Rosen used different scalar densities as the Lagrangian density to develop a variety of gravitational theories which, generally speaking, predict different gravitational effects. It will be shown below that it is possible to combine Poincare's idea of the gravitational field [2] as a Faraday–Maxwell physical field with Einstein's idea of a Riemannian spacetime geometry in the framework of the special theory of relativity (STR), which describes events both in inertial and in noninertial reference frames. For this purpose, the geometrisation principle that reflects the universal character of the gravitational interaction between the field and matter will be employed and the mass of the graviton introduced. It is the geometrisation principle that may prove helpful in the search for the infinite-dimensional non-commutative gauge group necessary to construct the Lagrangian density of the gravitational field proper. This line of reasoning has led to the relativistic theory of gravitation (RTG) [3], which contains all conservation laws, as is the case with all other physical theories.

This theory assumes the conserved total tensor of matter and gravitational field to be the source of the field, in conformity with Einstein's idea of the gravitational theory. It will be shown that general physical requirements lead to unambiguous construction of the complete system of equations for the massive gravitational field. Eqns (66) and (67) in this theory are fundamentally different from the Gilbert–Einstein equations since they retain the notion of an inertial coordinate system, and forces due to gravity are fundamentally different from those due to inertia, as the former are generated by a physical field. It should be specially emphasised that the rest mass being a property of gravitational field is a matter of principle (see below). This paper presents, once again, the basic principles and

†Question: Misner, Thorne, and Wheeler explain why the gravitational field is impossible to localise (see their book *Gravitation*, chapters 19 and especially 20, paragraph 20.4). I do not share their opinion. Hence, the question arises: what is wrong with the arguments of Misner and co-workers?

According to GTR, this is exactly the case with the gravitational field energy as was long ago stated by V A Fock. In the relativistic theory of gravitation (RTG), the Faraday–Maxwell physical gravitational field in Minkowski space is introduced which allows the notion of an energy–momentum tensor of the gravitational field to be used. The difference of opinion ensues from different starting points in GTR and RTG. An experiment is needed to verify the validity of either approach although general theoretical considerations are also of great value.

equations of the theory, with some amendments and modifications.

The relativistic theory of gravitation with graviton mass is a field theory in the same sense as classical electrodynamics; therefore, it may be called classical gravidynamics.

2. Basic propositions of RTG

In developing the theory of the gravitational field, we shall proceed from the following basic propositions.

Proposition I

RTG is based on the special theory of relativity, which means that Minkowski space (a pseudo-Euclidean spacetime geometry) is the fundamental space for all physical fields, including the gravitational field. This proposition is necessary and sufficient for the laws of conservation of energy–momentum and angular momentum to be valid for matter and gravitational field taken together. In other words, the Minkowski space reflects the dynamic properties shared by all forms of matter. This ensures their having common physical characteristics, which in turn allows interconversion of these forms to be quantitatively described.

The Minkowski space is not to be regarded as a priori existent; it reflects properties of matter and is therefore inseparable from it. Nevertheless, its independence of the form of matter is sometimes considered a formal reason to examine it regardless of matter.

Minkowski space admits description both in inertial (e.g. Galilean coordinates) and in noninertial (accelerated) coordinate systems. This appears evident from the mathematical point of view because Minkowski space allows for the introduction of a broad class of possible coordinate systems, including curvilinear ones. Nevertheless, this rather simple consideration has long remained unperceived even by prominent physicists. This can be accounted for by the fact that many authors considered Minkowski space to be a formal geometric interpretation of the special theory of relativity. This view tends to a narrowing of the limits of STR. The most general thesis of STR was assumed as the basis in the development of RTG that runs as follows: all physical processes, including gravitational ones, occur in a four-dimensional world, that is in space and time with pseudo-Euclidean geometry. In such a representation of STR, the process of clock synchronisation and the principle of the constant velocity of light, become immaterial because they are considered to be of little and limited importance, and only the interval is supposed to have physical sense.

At the beginning of this century, H Poincare wrote in his book *Science and Hypothesis* that although ''...experience necessarily played a role in the origin of geometry, it would be a mistake to conclude that geometry is, at least partially, an experimental science. If it were an experimental science, it would have only transient and approximate—highly approximate—significance.'' He also noted: ''Geometry studies but a specific 'group' of displacements whereas the general notion of the group preexists in the human mind, at least as a possibility.

''Experience directs us in this choice but does not make it obligatory; it prompts which geometry is more convenient rather than which is correct.'' This statement indicates the necessity of using a pseudo-Euclidean spacetime geometry if one proceeds from such fundamental physical principles as the laws of energy–momentum and angular momentum conservation. But this choice is not only convenient, it is actually the only one possible as long as the conservation laws are considered valid. In 1921, A Einstein wrote in his paper on ''Geometry and experiment'': ''the question whether this continuum has a Euclidean, Riemannian, or any other structure is a question of physics proper which must be answered with the aid of experiment, rather than a matter of agreement of choice on the grounds of mere expediency.''

Of course, the statement is basically correct, but the question arises as to which experimental facts are necessary unambiguously to characterise geometry. I believe such facts to be the fundamental laws of energy–momentum and angular momentum conservation because these laws reflect general dynamic properties of matter. It is precisely these facts that lead to pseudo-Euclidean spacetime geometry as the simplest one.

This means that in establishing the structure of the spacetime geometry, it is reasonable to proceed not from selected experimental facts (e.g. light propagation and test body motion) but from the basic physical principles deduced by generalising numerous experimental findings which characterise different forms of matter.

The Minkowski space has deep physical sense because it determines universal properties of matter such as energy, momentum, and angular momentum.

The gravitational field is described by a symmetric second-rank tensor $\phi^{\mu\nu}$ and is a real physical field possessing energy–momentum density, rest mass m, and polarisation states corresponding to spin 2 and 0. Elimination of representations corresponding to spin 1 and 0 from field states $\Phi^{\mu\nu}$ is accomplished by the constraining of the components $\phi^{\mu\nu}$ by the field equation

$$D_\mu \phi^{\mu\nu} = 0 , \tag{1}$$

where D_μ is the covariant derivative in the Minkowski space.

Eqn (1) not only excludes nonphysical field states but also introduces the metric $\gamma_{\mu\nu}$ of the Minkowski space into the theory, which makes it possible to separate forces of inertia from the action of the gravitational field. The choice of the Galilean metric $\gamma_{\mu\nu}$ makes it possible to eliminate completely the action of inertial forces while the Minkowski metric allows the notions of standard length and time interval to be introduced in the absence of a gravitational field. It will be shown below that the interaction between tensor gravitational field and matter can be introduced as though it deformed the Minkowski space, changing metric properties without affecting causality.

Proposition II

The gravitational field being described by the symmetric second-rank tensor $\phi^{\mu\nu}$, and its interaction with other fields being considered universal, there is a unique opportunity to 'join' this field in the Lagrangian density of matter directly to tensor $\gamma^{\mu\nu}$, according to the following rule:

$$L_{\mathrm{M}}(\tilde{\gamma}^{\mu\nu}, \phi_A) \to L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A) , \tag{2}$$

where

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu} , \quad \tilde{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu} ,$$

$$\tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\, \gamma^{\mu\nu} , \quad \tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\, \phi^{\mu\nu} ; \tag{3}$$

where ϕ_A represents matter fields; $g = \det g_{\mu\nu}$; $\gamma = \det \gamma_{\mu\nu}$; $\tilde{g}^{\mu\nu}\tilde{g}_{\nu\sigma} = \delta^{\mu}_{\sigma}$. Tensor $\gamma^{\mu\nu}$ is determined from the last equality. Tensor $\gamma_{\mu\nu}$ is used to raise and lower indices in $\gamma^{\mu\nu}$, and the metric tensor of the Riemannian space performs the same operations with tensor $g^{\mu\nu}$. By 'matter' we mean all its forms excepting the gravitational field.

Such a mode of interaction between the gravitational field and matter introduces the notion of the effective Riemannian space in which motion of matter occurs. This mode is referred to as the geometrisation principle. According to the geometrisation principle, the motion of matter under the action of the gravitational field $\phi^{\mu\nu}$ in the Minkowski space with metric $\gamma_{\mu\nu}$ is identical to its motion in the effective Riemannian space with metric $g_{\mu\nu}$. The effective Riemannian space is literally of field origin because of the presence of the gravitational field $\phi^{\mu\nu}$.

Metric properties are determined by the effective Riemann space tensor in the presence of a gravitational field and by the Minkowski space tensor $\gamma_{\mu\nu}$ in the absence of the field. For this reason, the theory can explain how the size of a body and the rate at which a clock runs change under the influence of a gravitational field. A theory that does not contain tensor $\gamma_{\mu\nu}$ in its field equations is unable, in principle, to answer such questions. GTR characterises the field by the metric tensor $g_{\mu\nu}$, whereas in RTG it is determined by the tensor value $\phi^{\mu\nu}$, and the effective Riemannian space is constructed with the help of the field $\phi^{\mu\nu}$ and the Minkowski metric tensor $\gamma^{\mu\nu}$ to fix the choice of the coordinate system.

Our theory includes the Galilean (inertial) system of coordinates, and acceleration has the absolute meaning. The movement of a test body in the effective Riemannian space occurs along its geodesic line, but it is not free motion since it is induced by the gravitational field. Had the test body been charged, it would have emitted electromagnetic waves because its motion in the field would have been an accelerated motion.

Because the effective Riemannian space is produced by the gravitational field $\phi^{\mu\nu}$ in the Minkowski space, it is essential that the former can always be defined in the same coordinate system. This means that the theory has to do only with Riemannian spaces which are covered by a single map. From the standpoint of our theory, Riemannian spaces with complicated topology are totally excluded as not being of field-like nature. It is worthwhile to note that the equations of motion for matter do not include the metric tensor $\gamma_{\mu\nu}$ of the Minkowski space since matter travels in the effective Riemannian space. The Minkowski space is supposed to interfere with the motion of matter only via the Riemannian space metric tensor $g_{\mu\nu}$ derived from equations which contain metric tensor $\gamma_{\mu\nu}$.

In conclusion, although the geometrisation principle allows for the description of motion in the effective Riemannian space, the metric of the initial Minkowski space is not eliminated; rather, it is retained in the gravitational field equations to preserve the notion of an inertial system in which the forces of inertia are identically zero.

3. Gauge transformation group

Since the Lagrangian density of matter has the form

$$L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A) , \tag{4}$$

it is easy to find the gauge group of transformations at which this density is changed only by a divergence. To this end, the invariance of the action

$$S_{\mathrm{M}} = \int L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A)\, \mathrm{d}^4 x \tag{5}$$

may be used under an arbitrary infinitesimal change of coordinates

$$x'^{\alpha} = x^{\alpha} + \xi^{\alpha}(x) , \tag{6}$$

where ξ^{α} is the infinitesimal four-vector of the coordinate shift. Under these coordinate transformations, the field functions $\tilde{g}^{\mu\nu}$, ϕ_A are changed in the following manner:

$$\begin{aligned} \tilde{g}'^{\mu\nu}(x') &= \tilde{g}^{\mu\nu}(x) + \delta_{\xi}\tilde{g}^{\mu\nu}(x) + \xi^{\alpha}(x)\mathrm{D}_{\alpha}\tilde{g}^{\mu\nu}(x) , \\ \phi'_A(x') &= \phi_A(x) + \delta_{\xi}\phi_A(x) + \xi^{\alpha}(x)\mathrm{D}_{\alpha}\phi_A(x) , \end{aligned} \tag{7}$$

where the expressions

$$\begin{aligned} \delta_{\xi}\tilde{g}^{\mu\nu}(x) &= \tilde{g}^{\mu\alpha}\mathrm{D}_{\alpha}\xi^{\nu}(x) + \tilde{g}^{\nu\alpha}\mathrm{D}_{\alpha}\xi^{\mu}(x) - \mathrm{D}_{\alpha}(\xi^{\alpha}\tilde{g}^{\mu\nu}) , \\ \delta_{\xi}\phi_A(x) &= -\xi^{\alpha}(x)\mathrm{D}_{\alpha}\phi_A(x) + F^{B;\alpha}_{A;\beta}\,\phi_B(x)\mathrm{D}_{\alpha}\xi^{\beta}(x) \end{aligned} \tag{8}$$

are Lie variations.

Operators δ_{ξ} fulfil the conditions of a Lie algebra, that is the commutation relation

$$[\delta_{\xi_1}, \delta_{\xi_2}](\cdot) = \delta_{\xi_3}(\cdot) \tag{9}$$

and the Jacobi identity

$$[\delta_{\xi_1}, [\delta_{\xi_2}, \delta_{\xi_3}]] + [\delta_{\xi_3}, [\delta_{\xi_1}, \delta_{\xi_2}]] + [\delta_{\xi_2}, [\delta_{\xi_3}, \delta_{\xi_1}]] = 0 ,$$

where

$$\xi^{\nu}_3 = \xi^{\mu}_1\mathrm{D}_{\mu}\xi^{\nu}_2 - \xi^{\mu}_2\mathrm{D}_{\mu}\xi^{\nu}_1 = \xi^{\mu}_1\partial_{\mu}\xi^{\nu}_2 - \xi^{\mu}_2\partial_{\mu}\xi^{\nu}_1 . \tag{10}$$

In order to have (9), it is necessary to fulfil the following conditions

$$F^{B;\mu}_{A;\nu}\,F^{C;\alpha}_{B;\beta} - F^{B;\alpha}_{A;\beta}\,F^{C;\mu}_{B;\nu} = f^{\mu\alpha;\tau}_{\nu\beta;\sigma}\,F^{C;\sigma}_{A;\tau} , \tag{11}$$

where the structure constants f are given by

$$f^{\mu\alpha;\tau}_{\nu\beta;\sigma} = \delta^{\mu}_{\beta}\delta^{\alpha}_{\sigma}\delta^{\tau}_{\nu} - \delta^{\alpha}_{\nu}\delta^{\mu}_{\sigma}\delta^{\tau}_{\beta} . \tag{12}$$

It is easy to see that they satisfy the Jacobi identity

$$f^{\alpha\nu;\sigma}_{\beta\mu;\tau}\,f^{\tau\rho;\omega}_{\sigma\varepsilon;\delta} + f^{\nu\rho;\sigma}_{\mu\varepsilon;\tau}\,f^{\tau\alpha;\omega}_{\sigma\beta;\delta} + f^{\rho\alpha;\sigma}_{\varepsilon\beta;\tau}\,f^{\tau\nu;\omega}_{\sigma\mu;\delta} = 0 , \tag{13}$$

and possess properties of antisymmetry,

$$f^{\alpha\nu;\rho}_{\beta\mu;\sigma} = -f^{\nu\alpha;\rho}_{\mu\beta;\sigma} .$$

Under coordinate transformation (6), variation of the action is zero:

$$\delta_{\mathrm{c}}S_{\mathrm{M}} = \int_{\Omega'} L'_{\mathrm{M}}(x')\, \mathrm{d}^4 x' - \int_{\Omega} L_{\mathrm{M}}(x)\, \mathrm{d}^4 x = 0 . \tag{14}$$

The first integral in Eqn (14) can be written as

$$\int_{\Omega'} L'_{\mathrm{M}}(x')\, \mathrm{d}^4 x' = \int_{\Omega} J L'_{\mathrm{M}}(x')\, \mathrm{d}^4 x ,$$

where

$$J = \det\left(\frac{\partial x'^{\alpha}}{\partial x^{\beta}}\right).$$

To the first order in ξ^{α}, determinant J is

$$J = 1 + \partial_{\alpha}\xi^{\alpha}(x) . \tag{15}$$

Taking into account the expansion

$$L'_{\mathrm{M}}(x') = L'_{\mathrm{M}}(x) + \xi^{\alpha}(x)\frac{\partial L_{\mathrm{M}}}{\partial x^{\alpha}} ,$$

and Eqn (15), one may represent the expression for the variation in the form

$$\delta_{\mathrm{c}} S_{\mathrm{M}} = \int_{\Omega} \left\{ \delta L_{\mathrm{M}}(x) + \partial_{\alpha}[\xi^{\alpha} L_{\mathrm{M}}(x)] \right\} \mathrm{d}^4 x = 0 .$$

Since the integration volume Ω is arbitrary, one has the identity

$$\delta L_{\mathrm{M}}(x) = -\partial_{\alpha}[\xi^{\alpha}(x) L_{\mathrm{M}}(x)] , \tag{16}$$

where the Lie variation δL_{M} is given by

$$\delta L_{\mathrm{M}}(x) = \frac{\partial L_{\mathrm{M}}}{\partial \tilde{g}^{\mu\nu}} \delta\tilde{g}^{\mu\nu} + \frac{\partial L_{\mathrm{M}}}{\partial(\partial_{\alpha}\tilde{g}^{\mu\nu})} \delta(\partial_{\alpha}\tilde{g}^{\mu\nu}) + \frac{\partial L_{\mathrm{M}}}{\partial \phi_A} \delta\phi_A + \frac{\partial L_{\mathrm{M}}}{\partial(\partial_{\alpha}\phi_A)} \delta(\partial_{\alpha}\phi_A) . \tag{17}$$

Hence, if the scalar density depends only on $\partial_{\alpha}\tilde{g}^{\mu\nu}$ and its derivatives, transformation (8) will result in its change by a divergence:

$$\delta L[\tilde{g}^{\mu\nu}(x)] = -\partial_{\alpha}\left\{\xi^{\alpha}(x) L[\tilde{g}^{\mu\nu}(x)]\right\} , \tag{16a}$$

where the Lie variation is

$$\delta L[\tilde{g}^{\mu\nu}(x)] = \frac{\partial L}{\partial \tilde{g}^{\mu\nu}} \delta\tilde{g}^{\mu\nu} + \frac{\partial L}{\partial(\partial_{\alpha}\tilde{g}^{\mu\nu})} \delta(\partial_{\alpha}\tilde{g}^{\mu\nu}) + \frac{\partial L}{\partial(\partial_{\alpha}\partial_{\beta}\tilde{g}^{\mu\nu})} \delta(\partial_{\alpha}\partial_{\beta}\tilde{g}^{\mu\nu}) . \tag{17a}$$

Lie variations (8) were obtained in the context of coordinate transformations (6). However, it is possible to approach this problem from the other side and examine transformations (8) as gauge transformations. In this case, the arbitrary infinitesimal four-vector $\xi^{\alpha}(x)$ would be a gauge vector rather than a coordinate shift vector. In the following discussion, the designation $\varepsilon^{\alpha}(x)$ is used to emphasise the difference between the gauge group and the coordinate transformation group, whereas the transformations of the field functions

$$\tilde{g}^{\mu\nu}(x) \to \tilde{g}^{\mu\nu}(x) + \delta\tilde{g}^{\mu\nu}(x) ,$$
$$\phi_A(x) \to \phi_A(x) + \delta\phi_A(x) \tag{18}$$

with the additions

$$\delta_{\varepsilon}\tilde{g}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha}\mathrm{D}_{\alpha}\varepsilon^{\nu}(x) + \tilde{g}^{\nu\alpha}\mathrm{D}_{\alpha}\varepsilon^{\mu}(x) - \mathrm{D}_{\alpha}(\varepsilon^{\alpha}\tilde{g}^{\mu\nu}) ,$$
$$\delta_{\varepsilon}\phi_A(x) = -\varepsilon^{\alpha}(x)\mathrm{D}_{\alpha}\phi_A(x) + F^{B;\alpha}_{A;\beta}\,\phi_B(x)\,\mathrm{D}_{\alpha}\,\varepsilon^{\beta}(x) , \tag{19}$$

are referred to as gauge transformations.

In perfect conformity with formulas (9) and (10), these operators conform to the same Lie algebra, i.e. the commutation relation

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}](\cdot) = \delta_{\varepsilon_3}(\cdot) \tag{20}$$

and the Jacobi identity

$$[\delta_{\varepsilon_1}, [\delta_{\varepsilon_2}, \delta_{\varepsilon_3}]] + [\delta_{\varepsilon_3}, [\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]] + [\delta_{\varepsilon_2}, [\delta_{\varepsilon_3}, \delta_{\varepsilon_1}]] = 0 . \tag{21}$$

Similarly to Eqn (10), one has

$$\varepsilon_3^{\nu} = \varepsilon_1^{\mu}\mathrm{D}_{\mu}\varepsilon_2^{\nu} - \varepsilon_2^{\mu}\mathrm{D}_{\mu}\varepsilon_1^{\nu} = \varepsilon_1^{\mu}\partial_{\mu}\varepsilon_2^{\nu} - \varepsilon_2^{\mu}\partial_{\mu}\varepsilon_1^{\nu} .$$

The gauge group arose from the geometrised structure of the scalar Lagrangian density for matter, $L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A)$, which changes by no more than a divergence under gauge transformations (19), on account of identity (16). Therefore, the geometrisation principle that determines the universal character of the interaction between matter and the gravitational field enabled us to formulate a non-commutative infinite-dimensional gauge group (19).

A significant difference between gauge and coordinate transformations will be apparent in the critical point of the theory dealing with the construction of a scalar Lagrangian density for the gravitational field proper. The difference is due to the fact that the metric tensor $\gamma_{\mu\nu}$ does not change under gauge transformations, which leads, because of Eqns (3), to

$$\delta_{\varepsilon}\tilde{g}^{\mu\nu}(x) = \delta_{\varepsilon}\tilde{\phi}^{\mu\nu}(x) .$$

Based on Eqn (19), one has the following transformation for the field:

$$\delta_{\varepsilon}\tilde{\phi}^{\mu\nu}(x) = \tilde{g}^{\mu\alpha}\,\mathrm{D}_{\alpha}\,\varepsilon^{\nu}(x) + \tilde{g}^{\nu\alpha}\,\mathrm{D}_{\alpha}\,\varepsilon^{\mu}(x) - \mathrm{D}_{\alpha}(\varepsilon^{\alpha}\,\tilde{g}^{\mu\nu}) .$$

However, this transformation for the field is somewhat different from that based on the coordinate shift:

$$\delta_{\xi}\,\tilde{\phi}^{\mu\nu}(x) = \tilde{\phi}^{\mu\alpha}\,\mathrm{D}_{\alpha}\xi^{\nu}(x) + \tilde{\phi}^{\nu\alpha}\,\mathrm{D}_{\alpha}\xi^{\mu}(x) - \mathrm{D}_{\alpha}(\xi^{\alpha}\,\tilde{\phi}^{\mu\nu}) .$$

Gauge transformations (19) do not result in altered equations of motion for matter since any transformation of this kind leads to a change in the Lagrangian density of matter of no more than a divergence.

4. Lagrangian density and equations of motion for the gravitational field proper

It is well known that the use of the tensor $g_{\mu\nu}$ alone does not allow one to construct a scalar Lagrangian density of the gravitational field proper with respect to the rather arbitrary coordinate transformations in the form of quadratic derivatives of not higher than first order. This accounts for the Lagrangian density inevitably comprising the metric $\gamma_{\mu\nu}$ along with the metric $g_{\mu\nu}$. But the former metric does not change under gauge transformation (19). Therefore, strong limitations on the Lagrangian density of the gravitational field proper are needed to ensure that this transformation causes no more than a divergence change in the density. It is here that the basic difference arises between gauge and coordinate transformations.

While coordinate transformations impose virtually no limits on the structure of the scalar Lagrangian density of the gravitational field proper, gauge transformations allow the Lagrangian density to be found. A direct general method for constructing the Lagrangian has been described in monograph [3].

Here, a simpler method of constructing the Lagrangian is employed. Based on Eqn (16), it may be concluded that the simplest scalar densities $\sqrt{-g}$ and $\tilde{R} = \sqrt{-g}R$, (where R is the scalar curvature of the effective Riemannian space) undergo the following changes under gauge transformation (19):

$$\sqrt{-g} \to \sqrt{-g} - \mathrm{D}_{\nu}(\varepsilon^{\nu}\sqrt{-g}) , \tag{22}$$

$$\tilde{R} \to \tilde{R} - \mathrm{D}_{\nu}(\varepsilon^{\nu}\tilde{R}) . \tag{23}$$

The scalar density $\tilde{R}$ is expressed via the Christoffel symbols

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{24}$$

in the form

$$\tilde{R} = -\tilde{g}^{\mu\nu}(\Gamma^{\lambda}_{\mu\nu}\Gamma^{\sigma}_{\lambda\sigma} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\lambda}) - \partial_\nu(\tilde{g}^{\mu\nu}\Gamma^{\sigma}_{\mu\sigma} - \tilde{g}^{\mu\sigma}\Gamma^{\nu}_{\mu\sigma}) . \tag{25}$$

Since the Christoffel symbols are not tensor values, none of the items in Eqn (25) is a scalar density. However, introduction of the tensor values $G^{\lambda}_{\mu\nu}$, where

$$G^{\lambda}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\mathrm{D}_\mu g_{\sigma\nu} + \mathrm{D}_\nu g_{\sigma\mu} - \mathrm{D}_\sigma g_{\mu\nu}) , \tag{26}$$

allows scalar density to be written in the form

$$\tilde{R} = -\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}) - \mathrm{D}_\nu(\tilde{g}^{\mu\nu}G^{\sigma}_{\mu\sigma} - \tilde{g}^{\mu\sigma}G^{\nu}_{\mu\sigma}) . \tag{27}$$

It should be noted that each group of terms in Eqn (27) taken separately behaves as scalar density under arbitrary coordinate transformations. Because of Eqns (22) and (23), the expression

$$\lambda_1(\tilde{R} + \mathrm{D}_\nu Q^\nu) + \lambda_2\sqrt{-g} \tag{28}$$

is invariant up to a divergence under arbitrary gauge transformations. By choosing the vector density

$$Q^\nu = \tilde{g}^{\mu\nu}G^{\sigma}_{\mu\sigma} - \tilde{g}^{\mu\sigma}G^{\nu}_{\mu\sigma} ,$$

it is possible to remove from the previous expression terms with derivatives of higher than first order and obtain the Lagrangian density

$$-\lambda_1\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}) + \lambda_2\sqrt{-g} . \tag{29}$$

It is therefore evident that the requirement that the Lagrangian density of the gravitational field proper should change by a divergence under gauge transformation (19) unambiguously defines the structure of Lagrangian density (29). However, if we restrict ourselves to this density, the gravitational field equations will be gauge invariant and the Minkowski space metric $\gamma_{\mu\nu}$ will not appear in the system of equations determined by the Lagrangian density (29). Since this approach results in the disappearance of the Minkowski space metric, the gravitational field cannot be represented as a physical field of the Faraday–Maxwell type in the Minkowski space.

In the Lagrangian density (29), the introduction of the metric $\gamma_{\mu\nu}$ with the help of Eqn (1) does not seem appreciably to redeem the situation, because the physical values (the interval, the tensor of Riemannian space curvature , and the tensor $t^{\mu\nu}_g$ of the gravitational field) are dependent on the choice of gauge, which is physically irrelevant. In order to retain the field concept in the Minkowski space and avoid such an ambiguity, it is necessary to add to the Lagrangian density of the gravitational field a term which breaks the gauge group. At first sight, this appears to clear the way for arbitrariness as regards the choice of Lagrangian density, since the gauge group can be broken in more than one fashion. However, this turns out not to be the case because the physical limitation imposed by Eqns (1) on the polarisation properties of the gravitational field as a field with spin 2 and 0 requires that the term breaking group (19) be chosen in such a way as to ensure that Eqns (1) are corollaries of the system of equations for the gravitational field and matter fields; only then is it possible to avoid the emergence of an over-determined system of differential equations. This can be achieved by introducing into the scalar Lagrangian density of the gravitational field a term of the form

$$\gamma_{\mu\nu}\tilde{g}^{\mu\nu} , \tag{30}$$

which [given conditions (1) and transformations (19)] also changes by a divergence, but only with respect to the class of vectors meeting the condition

$$g^{\mu\nu}\mathrm{D}_\mu\mathrm{D}_\nu\varepsilon^\sigma(x) = 0 . \tag{31}$$

There is a very similar situation in electrodynamics with the nonzero photon rest mass. Using terms (28)–(30), one can present the total scalar Lagrangian density:†

$$L_{\mathrm{g}} = -\lambda_1\tilde{g}^{\mu\nu}(G^{\lambda}_{\mu\nu}G^{\sigma}_{\lambda\sigma} - G^{\lambda}_{\mu\sigma}G^{\sigma}_{\nu\lambda}) + \lambda_2\sqrt{-g} + \lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu} + \lambda_4\sqrt{-\gamma} . \tag{32}$$

The last term in Eqn (32) is introduced to make the Lagrangian density vanish in the absence of a gravitational field. The contraction of the class of gauge vectors following introduction of term (30) automatically makes Eqns (1) a corollary of the gravitational field equations. This will be clear from the forthcoming discussion.

In agreement with the principle of least action, the equations for the gravitational field proper have the form

$$\frac{\delta L_g}{\delta\tilde{g}^{\mu\nu}} = \lambda_1 R_{\mu\nu} + \frac{1}{2}\lambda_2 g_{\mu\nu} + \lambda_3\gamma_{\mu\nu} = 0 . \tag{33}$$

Here,

$$\frac{\delta L_{\mathrm{g}}}{\delta\tilde{g}^{\mu\nu}} = \frac{\partial L_{\mathrm{g}}}{\partial\tilde{g}^{\mu\nu}} - \partial_\sigma\left[\frac{\partial L}{\partial(\partial_\sigma\tilde{g}^{\mu\nu})}\right] ,$$

where tensor $R_{\mu\nu}$ is the Ricci tensor written in the form

$$R_{\mu\nu} = \mathrm{D}_\lambda G^{\lambda}_{\mu\nu} - \mathrm{D}_\mu G^{\lambda}_{\nu\lambda} + G^{\sigma}_{\mu\nu}G^{\lambda}_{\sigma\lambda} - G^{\sigma}_{\mu\lambda}G^{\lambda}_{\nu\sigma} . \tag{34}$$

Eqns (33) are expected to be identically satisfied in the absence of a gravitational field. Hence,

$$\lambda_2 = -2\lambda_3 . \tag{35}$$

The energy–momentum tensor density of the gravitational field in the Minkowski space is found to be

$$t^{\mu\nu}_{\mathrm{g}} = -2\frac{\delta L_{\mathrm{g}}}{\delta\gamma_{\mu\nu}} = 2\sqrt{-\gamma}\left(\gamma^{\mu\alpha}\gamma^{\nu\beta} - \frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta}\right)\frac{\delta L_{\mathrm{g}}}{\delta\tilde{g}^{\alpha\beta}} + \lambda_1 J^{\mu\nu} - 2\lambda_3\tilde{g}^{\mu\nu} - \lambda_4\tilde{\gamma}^{\mu\nu} , \tag{36}$$

where

$$J^{\mu\nu} = \mathrm{D}_\alpha\mathrm{D}_\beta(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu} + \gamma^{\alpha\nu}\tilde{g}^{\beta\mu} - \gamma^{\alpha\beta}\tilde{g}^{\mu\nu} - \gamma^{\mu\nu}\tilde{g}^{\alpha\beta}) . \tag{37}$$

†Question: it is stated on p. 180 that in GTR the scalar Lagrangian density of the gravitational field contains second derivatives of $g_{\mu\nu}$. However, in GTR they are combined into the total derivative term, which does not vary. This is precisely what the author does in paragraph 3 of the paper. What are the differences and the advantages?

In GTR, the elimination of terms with second derivatives forming a divergence from the Lagrangian density of the gravitational field results in a certain expression which contains only first-order derivatives [see the first term in formula (25)]. But this expression is not a scalar density with respect to arbitrary coordinate transformations. RTG, similar to all other physical theories, deals with a scalar density which contains derivatives of not higher than the first order [see formula (32)]. This allows the notion of energy–momentum tensor of the gravitational field to be introduced. And this makes the difference.

Provided expression (36) takes into account the dynamic equations (33), the following equation is available for the gravitational field proper:†

$$\lambda_1 J^{\mu\nu} - 2\lambda_3 \tilde{g}^{\mu\nu} - \lambda_4 \tilde{\gamma}^{\mu\nu} = t_{\mathrm{g}}^{\mu\nu} . \quad (38)$$

In order to have this equation satisfied in the absence of a gravitational field, it has to be assumed that

$$\lambda_4 = -2\lambda_3 . \quad (39)$$

Since the equality

$$\mathrm{D}_\mu t_{\mathrm{g}}^{\mu\nu} = 0 , \quad (40)$$

always holds for the gravitational field proper, it follows from Eqn (38) that

$$\mathrm{D}_\mu \tilde{g}^{\mu\nu} = 0 . \quad (41)$$

Therefore, Eqn (1) — which defines the polarisation states of the field — directly ensues from Eqn (38). Taking into consideration Eqns (41), one can write the field equations (38) in the form

$$\gamma^{\alpha\beta} \mathrm{D}_\alpha \mathrm{D}_\beta \tilde{\phi}^{\mu\nu} - \frac{\lambda_4}{\lambda_1} \tilde{\phi}^{\mu\nu} = -\frac{1}{\lambda_1} t_{\mathrm{g}}^{\mu\nu} . \quad (42)$$

In Galilean coordinates, this equation has the simple form

$$\Box \tilde{\phi}^{\mu\nu} - \frac{\lambda_4}{\lambda_1} \tilde{\phi}^{\mu\nu} = -\frac{1}{\lambda_1} t_{\mathrm{g}}^{\mu\nu} . \quad (43)$$

It is natural to interpret the numerical factor $-\lambda_4/\lambda_1 = m^2$ as the squared graviton mass, while the value $-1/\lambda_1$ must be taken to be equal to 16π. This allows all the unknown constants included in the Lagrangian density to be defined:

$$\lambda_1 = -\frac{1}{16\pi} , \qquad \lambda_2 = \lambda_4 = -2\lambda_3 = \frac{m^2}{16\pi} . \quad (44)$$

The scalar Lagrangian density of the gravitational field proper has the following form:

$$L_{\mathrm{g}} = \frac{1}{16\pi} \tilde{g}^{\mu\nu} (G^\lambda_{\mu\nu} G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma} G^\sigma_{\nu\lambda})$$
$$- \frac{m^2}{16\pi} \left(\frac{1}{2} \gamma_{\mu\nu} \tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma} \right) . \quad (45)$$

The corresponding dynamic equations for the gravitational field proper may be written as

$$J^{\mu\nu} - m^2 \tilde{\phi}^{\mu\nu} = -16\pi t_{\mathrm{g}}^{\mu\nu} , \quad (46)$$

or

$$R^{\mu\nu} - \frac{m^2}{2} (g^{\mu\nu} - g^{\mu\alpha} g^{\nu\beta} \gamma_{\alpha\beta}) = 0 . \quad (47)$$

These equations significantly restrict the class of gauge transformations, leaving only the trivial ones that meet the Killing constraints. Such transformations follow from Lorentz invariance and occur in any theory.

The Lagrangian density obtained as above leads to Eqns (47), which imply that Eqns (41) must follow from them. For this reason, one has ten equations for ten unknown field functions outside matter. By means of Eqns (41), unknown field functions $\phi^{0\alpha}$ are readily expressed through the field functions ϕ^{ik}, where the labels i and k take the values 1, 2, 3. Therefore, the structure of the mass term which breaks the gauge group in the Lagrangian density of the gravitational field proper is unambiguously determined by the field polarisation properties‡.

5. Equation of motion for the gravitational field and matter

The total Lagrangian density of matter and gravitational field is

$$L = L_{\mathrm{g}} + L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A), \quad (48)$$

where L_{g} is defined by expression (45).

Based on Eqn (48) and the principle of least action, the complete system of equations for matter and gravitational field is

$$\frac{\delta L}{\delta \tilde{g}^{\mu\nu}} = 0 , \quad (49)$$

$$\frac{\delta L_{\mathrm{M}}}{\delta \phi_A} = 0 . \quad (50)$$

Variation of the action $\delta_{\mathrm{c}} S_{\mathrm{M}}$ by an arbitrary infinitesimal change of the coordinates is zero. Therefore,

$$\delta_{\mathrm{c}} S_{\mathrm{M}} = \delta_{\mathrm{c}} \int L_{\mathrm{M}}(\tilde{g}^{\mu\nu}, \phi_A) \, \mathrm{d}^4 x = 0 .$$

Hence, the following identity can be obtained [3]:

$$g_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = -\mathrm{D}_\nu \left[\frac{\delta L_{\mathrm{M}}}{\delta \phi_A} F^{B;\nu}_{A;\mu} \phi_B(x) \right] - \frac{\delta L}{\delta \phi_A} D_\mu \phi_A(x) . \quad (51)$$

Here, $T^{\lambda\nu} = -2(\delta L_{\mathrm{M}}/\delta g_{\lambda\nu})$ is density of matter tensor in the Riemannian space, and ∇_λ is the covariant derivative in this space with metric $g_{\lambda\nu}$. Identity (51) yields the equation

$$\nabla_\lambda T^{\lambda\nu} = 0 , \quad (52)$$

provided the equations of motion for matter (50) are fulfilled.

In the case when the number of equations (50) for matter equals four, the equivalent equations (52) may be used instead of them. Since we are going to deal only with

† Question: Eqn (33) is the equation for the gravitational field, and Eqn (36) defines $t_{\mathrm{g}}^{\mu\nu}$. Then, Eqn (43) also defines $t_{\mathrm{g}}^{\mu\nu}$. At the same time, Eqn (43) looks like a wave equation for $\phi^{\mu\nu}$ with source $t_{\mathrm{g}}^{\mu\nu}$ on the right-hand side which needs a separate expression to be given through first-order derivatives of $\phi^{\mu\nu}$, if the gravitational field is to be described as an ordinary physical field. Where is such an expression for $t_{\mathrm{g}}^{\mu\nu}$?

Relation (36) is an identity (or a definition). But this identity also turns into equation as soon as field equation (33) is applied. An expression for $t_{\mathrm{g}}^{\mu\nu}$ can be obtained by substituting Eqns (44) and (55) into (36). $t_{\mathrm{g}}^{\mu\nu}$ inevitably contains terms with second derivatives since the gravitational field gives rise to the effective Riemannian space, due to geometrisation. It is this part of the gravitational field tensor that contributes to the formation of Riemannian space. The gravitational field is a special field because its interaction involves second-order derivatives in the field equations. This distinguishes it from any other field.

‡Question: the reason for introducing graviton mass is unclear. Does it not coincide with the reason for the introduction of the Λ-term? If not, why is the massive gravitational field not regarded as a material one?

In RTG, a Minkowski space is introduced which allows the notion of the inertial coordinate system to be retained. Acceleration has absolute meaning. The gravitational field is considered as a physical tensor field of the Faraday–Maxwell type with polarisation properties corresponding to representations with spin 2 and 0. The source of the gravitational field in such an approach is the energy–momentum tensor of matter. The gravitational field creates the effective Riemannian space because of the geometrisation principle.

such equations below, we shall always use equations for matter in the form of Eqn (52). Thus, the complete system of equations for matter and gravitational field will look like

$$\frac{\delta L}{\delta \tilde{g}^{\mu\nu}} = 0 \,, \tag{53}$$

$$\nabla_\lambda T^{\lambda\nu} = 0 \,. \tag{54}$$

The behaviour of matter is described by velocity v, density ρ, and pressure p. The gravitational field is defined by the ten components of the tensor $\phi^{\mu\nu}$.

All in all, there are 15 unknowns. To determine them, an equation of state for matter needs to be added to the previous 14 equations (53)–(54). If one takes into consideration the relations

$$\frac{\delta L_{\rm g}}{\delta \tilde{g}^{\mu\nu}} = -\frac{1}{16\pi} R_{\mu\nu} + \frac{m^2}{32\pi}\,(g_{\mu\nu} - \gamma_{\mu\nu}) \,, \tag{55}$$

$$\frac{\delta L_{\rm M}}{\delta \tilde{g}^{\mu\nu}} = \frac{1}{2\sqrt{-g}}\left(T_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} T\right) , \tag{56}$$

the system of Eqns (53)–(54) may be presented as

$$\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\right) + \frac{m^2}{2}\left[g^{\mu\nu} + \left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2}\, g^{\mu\nu} g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = \frac{8\pi}{\sqrt{-g}}\, T^{\mu\nu} \,, \tag{57}$$

$$\nabla_\lambda T^{\lambda\nu} = 0 \,. \tag{58}$$

It follows from the Bianchi identity

$$\nabla_\mu \left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\right) = 0 \,,$$

and Eqns (57) that,

$$m^2\sqrt{-g}\left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2}\, g^{\mu\nu} g^{\alpha\beta}\right)\nabla_\mu \gamma_{\alpha\beta} = 16\pi \nabla_\mu T^{\mu\nu} \,. \tag{59}$$

Taking into account the equation

$$\nabla_\mu \gamma_{\alpha\beta} = -G^\sigma_{\mu\alpha}\gamma_{\sigma\beta} - G^\sigma_{\mu\beta}\gamma_{\sigma\alpha} \,, \tag{60}$$

where $G^\sigma_{\mu\alpha}$ is defined by formula (26), one finds that

$$\left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2}\, g^{\mu\nu} g^{\alpha\beta}\right)\nabla_\mu \gamma_{\alpha\beta} = \gamma_{\mu\lambda} g^{\mu\nu}(\mathrm{D}_\sigma g^{\sigma\lambda} + G^\beta_{\alpha\beta} g^{\alpha\lambda}) \,. \tag{61}$$

However,

$$\sqrt{-g}(\mathrm{D}_\sigma g^{\sigma\lambda} + G^\beta_{\alpha\beta} g^{\alpha\lambda}) = \mathrm{D}_\sigma \tilde{g}^{\lambda\sigma} \,, \tag{62}$$

which accounts for expression (61) taking the form

$$\sqrt{-g}\left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2}\, g^{\mu\nu} g^{\alpha\beta}\right)\nabla_\mu \gamma_{\alpha\beta} = \gamma_{\mu\lambda} g^{\mu\nu}\,\mathrm{D}_\sigma \tilde{g}^{\lambda\sigma} \,. \tag{63}$$

With the aid of Eqn (63), expression (59) may be presented as

$$m^2 \gamma_{\mu\lambda} g^{\mu\nu} \mathrm{D}_\sigma \tilde{g}^{\lambda\sigma} = 16\pi \nabla_\mu T^{\mu\nu} \,.$$

and rewritten in the form of

$$m^2 \mathrm{D}_\sigma \tilde{g}^{\lambda\sigma} = 16\pi \gamma^{\lambda\nu} \nabla_\mu T^\mu_\nu \,, \tag{64}$$

The latter relation allows substitution of Eqn (58) by the equation

$$\mathrm{D}_\sigma \tilde{g}^{\nu\sigma} = 0 \,. \tag{65}$$

Finally, the system of equations (57) and (58) can be reduced to a system of gravitational equations of the form

$$\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} R\right) + \frac{m^2}{2}\left[g^{\mu\nu} + \left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2}\, g^{\mu\nu} g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = \frac{8\pi}{\sqrt{-g}}\, T^{\mu\nu} \,, \tag{66}$$

$$\mathrm{D}_\mu \tilde{g}^{\mu\nu} = 0 \,. \tag{67}$$

These equations are form-invariant with respect to Lorentz transformations. In other words, events are described by the same equations in any inertial (Galilean) coordinate system.

A specific inertial (Galilean) system of coordinates is defined by the nature of the physical problem (i.e. initial and boundary conditions). The description of a given physical problem appears to be different in different inertial (Galilean) coordinate systems, but this is not in conflict with the relativistic principle. Introduction of the tensor

$$N^{\mu\nu} = R^{\mu\nu} - \frac{m^2}{2}\,(g^{\mu\nu} - g^{\mu\alpha} g^{\nu\beta}\gamma_{\alpha\beta}) \,, \quad N = N^{\mu\nu} g_{\mu\nu} \,,$$

allows the system of equations (66) and (67) to be written in the form

$$N^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} N = \frac{8\pi}{\sqrt{-g}}\, T^{\mu\nu} \,, \tag{66a}$$

$$\mathrm{D}_\mu \tilde{g}^{\mu\nu} = 0 \,. \tag{67a}$$

Alternatively, it can be presented as

$$N^{\mu\nu} = \frac{8\pi}{\sqrt{-g}}\left(T^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu} T\right) , \tag{68}$$

$$\mathrm{D}_\mu \tilde{g}^{\mu\nu} = 0 \,, \tag{69}$$

or

$$N_{\mu\nu} = \frac{8\pi}{\sqrt{-g}}\left(T_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} T\right) , \tag{68a}$$

$$\mathrm{D}_\mu \tilde{g}^{\mu\nu} = 0 \,. \tag{69a}$$

It should be specially emphasised that both systems of equations (68) and (69) include the metric tensor of the Minkowski space.

Transformations of coordinates in which the Minkowski space metric remains form-invariant establish links between physically equivalent reference frames. Inertial systems constitute their simplest variety. For this reason, possible gauge transformations satisfying the Killing constraints,

$$\mathrm{D}_\mu \varepsilon_\nu + \mathrm{D}_\nu \varepsilon_\mu = 0 \,,$$

leave us within the class of physically equivalent reference frames.

Provided that changes in the characteristics of the Riemannian space and the motion of matter can be measured with infinite accuracy, Eqns (68a) and (69a) may be used to determine the Minkowski space metric and find Galilean (inertial) coordinate systems. This implies that the Minkowski space is, in principle, observable.

The existence of the Minkowski space is reflected in the conservation laws. Therefore, their verification in physical phenomena means a concurrent check of the spacetime structure.

The system of gravitational equations can assume another equivalent form:†

$$\gamma^{\alpha\beta} \mathrm{D}_\alpha \mathrm{D}_\beta \tilde{\phi}^{\mu\nu} + m^2 \tilde{\phi}^{\mu\nu} = 16\pi t^{\mu\nu} , \quad (70)$$

$$\mathrm{D}_\mu \tilde{\phi}^{\mu\nu} = 0 , \quad (71)$$

where $t^{\mu\nu} = -2(\delta L / \delta\gamma_{\mu\nu})$ is the conserved density of the energy–momentum tensor of matter and gravitational field in the Minkowski space. Such a form resembles the equations of electrodynamics with the photon mass μ in the absence of gravity:

$$\gamma^{\alpha\beta} \mathrm{D}_\alpha \mathrm{D}_\beta A^\nu + \mu^2 A^\nu = 4\pi j^\nu , \quad (72)$$

$$\mathrm{D}_\nu A^\nu = 0 . \quad (73)$$

The vector field source A^ν in electrodynamics is the conserved electromagnetic current j^ν generated by charged bodies. In RTG, the source of the tensor field is the conserved total tensor of energy–momentum of matter and gravitational field. This accounts for the nonlinearity of the gravitational equations even for the gravitational field proper.

It should be particularly noted that Eqns (66) contain not only the known cosmological term but also the term with metric $\gamma_{\mu\nu}$ of the Minkowski space, both terms entering the equations with the common constant that coincides with the graviton mass which is therefore very small. The second mass term in Eqns (66) containing metric $\gamma_{\mu\nu}$ accounts for the appearance of repulsive forces which are especially potent in strong gravitational fields. This influences both the collapse and the development of the universe‡.

It has been demonstrated in the foregoing discussion that graviton rest mass is crucial for the elaboration of the gravitational field theory. It is due to the presence of graviton mass that the theory predicts that the homogeneous and isotropic universe cannot be other than flat.

To conclude, it is worthwhile to note that the theory of the tensor gravitational field in the Minkowski space which introduces the effective Riemannian spacetime is valid only on the condition that the gravitational field possesses rest mass.

6. Causality principle in RTG

Similar to other physical field theories, RTG has been elaborated in the framework of STR. The latter theory maintains that any motion of a point test body occurs within the causality light cone in Minkowski space. This implies that noninertial reference frames associated with test bodies are also located inside the causality light cone of the pseudo-Euclidean spacetime. In this way, the total class of possible noninertial systems is defined. The local equality of three-dimensional forces of inertia and gravity in their action on a material point may be expected to apply if the causality light cone in the effective Riemannian space does not spread beyond that of the Minkowski space. Only in this case, the three-dimensional force of the gravitational field that affects the test body can be locally compensated by turning to the admissible noninertial reference system associated with this body.

Had the light cone of the effective Riemannian space spread beyond the causality light cone of the Minkowski space this would have meant that there is no admissible noninertial reference frame for such a 'gravitational field' in which this 'field of force' is liable to compensation while it acts on a material point. In other words, local compensation of the three-vector of gravitational force by the force of

†Equation (70) provides the simplest derivation of the interval for the Riemannian space in the first approximation in the gravitational constant. It is this interval that explains all gravitational effects in the solar system with the exception of the shift of Mercury's perihelion, which requires the next approximation in the gravitational constant.

For a static spherically symmetric body in the given approximation in the Galilean coordinates of an inertial system, Eqn (70) assumes the form of

$$\nabla \tilde{\phi}^{00} = -16\pi t^{00}, \quad \nabla \tilde{\phi}^{0i} = 0 ,$$
$$\nabla \tilde{\phi}^{ik} = 0 , \quad i,k = 1,2,3 .$$

Hence,

$$\tilde{\phi}^{0i} = 0 , \quad \tilde{\phi}^{ik} = 0 ,$$

For $\tilde{\phi}^{00}$ far from the source, the expression $\tilde{\phi}^{00} = 4M/r$ holds, and $M = \int t^{00}\, \mathrm{d}^3 x$ is the inertial mass of the source. In compliance with the geometrisation principle (3)

$$\tilde{g}^{00} = 1 + \frac{4M}{r} , \quad \tilde{g}^{0i} = 0 , \quad \tilde{g}^{11} = \tilde{g}^{22} = \tilde{g}^{33} = -1 , \quad \tilde{g}^{ik} = 0 .$$

Hence,

$$g_{00} = 1 - \frac{2M}{r} , \quad g_{11} = g_{22} = g_{33} = -\left(1 + \frac{2M}{r}\right) .$$

It follows from these expressions that the inertial mass takes the place of the active gravitational mass of the body. This can be accounted for by the fact that the energy–momentum tensor of matter serves as the source of the gravitational field rather than by the local identity of gravitational and inertial forces. The gravitational mass being small, its influence on the values obtained in examining effects in the solar system may be neglected.

‡ Question: graviton mass is supposed to have the meaning specified by Hubble's constant

$$\left(\frac{mc}{\hbar} \sim H \sim 10^{-28}\,\mathrm{cm}^{-1} \right) .$$

From the physical point of view, it is unclear how such a small mass would be able to stop stellar collapse. Evidently, the term with graviton mass becomes important only when the radius of the star differs from its gravitational radius by not more than

$$\Delta r \sim r_\mathrm{g} \left(\frac{mc}{\hbar} r_\mathrm{g} \right)^2 .$$

For stars having a mass comparable with that of the Sun, this value is around 10^{-40} cm, i.e. many orders of magnitude smaller than the Planck length. What has the classical theory of gravitation to do with all this?

Graviton mass is actually of significance in the zone adjoining the Schwarzschild sphere. It is here that the singularity of metric coefficient V is apparent. At variance with GTR, this singularity does not arise in the gravitational radius, but rather appears at a different but close point Z_g. Metric coefficient U at this point differs from zero and is always higher than zero. This makes point Z_g a turning point for a falling test body. The difference between Z_g and $2M$ is very small indeed, but it has no physical sense. Therefore, the question is irrelevant.

inertia is possible only when the gravitational field, as a physical field, acts on the particles but does not let their world-lines go beyond the causality cone in the pseudo-Euclidean spacetime. This condition is to be regarded as the causality principle which allows solutions of the system of equations (66) and (67) to be selected which have physical sense and correspond to gravitational fields.

The causality principle is not automatically satisfied because the gravitational interaction enters the coefficients as second derivatives in infield equations, that is it alters the initial spacetime geometry. This specific feature is inherent only in gravitational fields. For all other known physical fields, the interaction does not normally affect the second derivatives of field equations and is therefore unable to alter the initial pseudo-Euclidean spacetime geometry.

Now it is time to suggest an analytic formulation of the causality principle in RTG. In this theory, the motion of matter caused by the gravitational field in the pseudo-Euclidean spacetime is equivalent to that in the corresponding effective Riemannian spacetime. Therefore, on the one hand, the condition

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \geqslant 0\,, \tag{74}$$

must hold for causally related events (world-lines of particles and light). On the other hand, such events require the inequality

$$\mathrm{d}\sigma^2 = \gamma_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \geqslant 0\,. \tag{75}$$

to be satisfied. For the given reference frame associated with physical bodies, the condition

$$\gamma_{00} > 0\,. \tag{76}$$

is satisfied. In expression (75), the timelike and spacelike parts can be separated:

$$\mathrm{d}\sigma^2 = \left(\sqrt{\gamma_{00}}\,\mathrm{d}t + \frac{\gamma_{0i}\,\mathrm{d}x^i}{\sqrt{\gamma_{00}}}\right)^2 - s_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k\,. \tag{77}$$

Here, Latin indices i and k take the values 1, 2, 3;

$$s_{ik} = -\gamma_{ik} + \frac{\gamma_{0i}\gamma_{0k}}{\gamma_{00}}\,, \tag{78}$$

s_{ik} is the metric tensor of the three-dimensional space in the four-dimensional pseudo-Euclidean spacetime. The square of the spatial distance is

$$\mathrm{d}l^2 = s_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k\,. \tag{79}$$

If the velocity $v^i = \mathrm{d}x^i/\mathrm{d}t$ is presented in the form $v^i = ve^i$ (where v is the modulus of the velocity and e^i is an arbitrary unit vector in the three-dimensional space), then

$$s_{ik}e^i e^k = 1\,. \tag{80}$$

In the absence of a gravitational field, the velocity of light in the given coordinate system is easy to determine from expression (77), assuming it to equal zero:

$$\left(\sqrt{\gamma_{00}}\,dt + \frac{\gamma_{0i}\,\mathrm{d}x^i}{\sqrt{\gamma_{00}}}\right)^2 = s_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k\,.$$

Hence,

$$v = \sqrt{\gamma_{00}}\bigg/\left(1 - \frac{\gamma_{0i}\,e^i}{\sqrt{\gamma_{00}}}\right)\,. \tag{81}$$

Therefore, the arbitrary four-dimensional isotropic vector in the Minkowski space is

$$u^\nu = (1, v\,e^i)\,. \tag{82}$$

For the conditions (74) and (75) to be simultaneously fulfilled, it is necessary and sufficient that for any isotropic vector

$$\gamma_{\mu\nu}\,u^\mu u^\nu = 0\,, \tag{83}$$

the causality condition

$$g_{\mu\nu}\,u^\mu u^\nu \leqslant 0\,, \tag{84}$$

be satisfied. This would indicate that the light cone in the effective Riemannian space does not go beyond the causality light cone of the pseudo-Euclidean spacetime. The causality conditions can be written in the following form:

$$g_{\mu\nu}\,v^\mu v^\nu = 0\,, \tag{83a}$$

$$\gamma_{\mu\nu}v^\mu v^\nu \geqslant 0\,. \tag{84a}$$

In GTR, solutions of the Gilbert–Einstein equations have physical sense if they satisfy, at every point of the spacetime, the following inequality:

$$g < 0\,,$$

and also the requirement known as the energy-dominance condition and formulated as described below. For any timelike vector K_ν, the inequality

$$T^{\mu\nu}K_\mu K_\nu \geqslant 0\,,$$

must be satisfied, and the vector $T^{\mu\nu}K_\nu$ for given K_ν should not be spacelike.

In our theory, physical meaning is possessed by such solutions of Eqns (68a) and (69a) which fulfil both the above requirements and causality conditions (83a) and (84a). The last condition can be written, on the basis of Eqn (68a), in the form

$$R_{\mu\nu}K^\mu K^\nu \leqslant \frac{8\pi}{\sqrt{-g}}\left(T_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}T\right)K^\mu K^\nu + \frac{m^2}{2}\,g_{\mu\nu}K^\mu K^\nu\,. \tag{85}$$

Taking the energy–momentum density of matter to be of the form

$$T_{\mu\nu} = \sqrt{-g}\,\left[(\rho + p)U_\mu U_\nu - pg_{\mu\nu}\right]\,,$$

and using Eqn (68a), one can establish the following relation between the interval $\mathrm{d}\sigma$ of the Minkowski space and the interval $\mathrm{d}s$ of the effective Riemannian space:

$$\frac{m^2}{2}\,\mathrm{d}\sigma^2 = \mathrm{d}s^2\left[4\pi(\rho + 3p) + \frac{m^2}{2} - R_{\mu\nu}U^\mu U^\nu\right]\,,$$

where

$$U^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}s}\,.$$

Owing to the causality principle, one has the inequality

$$R_{\mu\nu}U^\mu U^\nu < 4\pi(\rho + 3p) + \frac{m^2}{2}\,,$$

which is a particular case of inequality (85), or

$$\sqrt{-g}R_{\mu\nu}\,v^\mu v^\nu \leqslant 8\pi T_{\mu\nu}v^\mu v^\nu\,. \tag{85a}$$

In 1918, A Einstein proposed the following formulation of the equivalence principle: ''Inertia and gravity are identical; hence and from the results of the special theory of relativity, it inevitably follows that the symmetric 'fundamental tensor' $g_{\mu\nu}$ defines the metric properties of space, inertial motion of bodies in this space, and also effects of gravitation.'' In GTR, identification of gravitational field and metric tensor $g_{\mu\nu}$ of the Riemannian space allows all components of the Christoffel symbol to be made zero at all points along an arbitrary line by choosing an appropriate coordinate system. But the choice of coordinate system in GTR does not eliminate the gravitational field, the motion of two close material points not being free because of the presence of the curvature tensor, which can never be nullified by the choice of coordinate system, because of its tensor properties†.

In RTG, the gravitational field is a physical field of the Faraday–Maxwell type, which accounts for the description of the gravitational force by a four-vector. For this reason, the choice of the coordinate system may be helpful for balancing the three-dimensional part of the gravitational force with forces of inertia only if conditions (83) and (84) are satisfied. The content of the equivalence principle in RTG is strikingly changed, being reduced to conditions (83) and (84) which allows a coordinate system to be chosen in which the gravitational force can be balanced by the force of inertia‡. The motion of a material point in a gravitational field, independently of the coordinate system, can never be free. This lack of freedom is especially evident if the geodesic equation is written in the form [1]

$$\frac{DU^\nu}{d\sigma} = -G^\rho_{\alpha\beta} U^\alpha U^\beta (\delta^\nu_\rho - U^\nu U_\rho) \, .$$

†Question: does the author agree with the GTR statement that gravitation is apparent because the relative acceleration of two close test particles cannot be nullified even though the acceleration of one test particle can, by the choice of coordinate system?

This is precisely what follows from GTR, but things are quite different with RTG. In the latter theory, the gravitational field is described not only by the curvature tensor but also by the four-vector of force. This means that the field described by the four-vector of force cannot be made zero. Therefore, transition to the coordinate system associated with the test body in the gravitational field is in fact the transition to the accelerated coordinate system with respect to the inertial coordinate system of the Minkowski space rather than to the locally inertial coordi-nate system as is the case with GTR. This accounts for the test body movement along the geodesic of the Riemannian space not to be a free movement under the body's own momentum independent of the action force.

‡Question: the author's statement that the equivalence principle in RTG is fundamentally different from that in GTR remains obscure. The author just calls it the 'geometrisation principle'. For the meaning in GTR see for instance *Teoriya Polya* (Field Theory) by Landau and Lifshitz, para-graph 87.

The equivalence principle in GTR and the geometrisation principle in RTG have nothing in common. According to many authors (e.g. Landau and Lifshitz in the *Teoriya Polya*, paragraph 87), the equivalence principle in GTR implies that the gravitational field can be locally eliminated by the transition to a locally inertial coordinate system. The gravitational field in RTG is a physical field and cannot be even locally excluded by the choice of a coordinate system. Forces of inertia may be used to locally balance only the three-dimensional force of gravity. In RTG, the notion of an inertial coordinate system is retained which explains why a system considered to be locally inertial in GTR is regarded as accelerated in our theory. Field equations in RTG contain the metric tensor of the Minkowski space. For this reason, forces of inertia are separated from gravitational forces as being defined by the Christoffel symbols in the Minkowski space, in compliance with the geometrisation principle. There can be no such separation in GTR.

Here,

$$d\sigma^2 = \gamma_{\mu\nu}\, dx^\mu\, dx^\nu , \qquad U^\nu = \frac{dx^\nu}{d\sigma} \, .$$

Free motion in the Minkowski space is described by

$$\frac{DU^\nu}{d\sigma} = \frac{dU^\nu}{d\sigma} + \gamma^\nu_{\mu\lambda} U^\mu U^\lambda = 0 \, ,$$

where $\gamma^\nu_{\mu\lambda}$ denotes Christoffel symbols in the Minkowski space. Clearly, motion along a geodesic of the Riemannian space is in fact that of a test body under the force

$$F^\nu = -G^\rho_{\alpha\beta} U^\alpha U^\beta (\delta^\nu_\rho - U^\nu U_\rho) \, ,$$

this force being a four-vector. This situation is precisely the same as in the case of other known physical forces.

In STR, forces of inertia and physical forces (electromagnetic, nuclear, etc.) are essentially different. Forces of inertia can always be nullified by a simple choice of reference frame, whereas physical forces cannot, in principle, be made zero (whatever the choice of the reference frame) since they are of vector nature in the Minkowski space. In GTR, gravitational forces are locally identical to forces of inertia, which makes them essentially different from all other physical forces§. Unlike GTR, RTG is concerned with gravitational forces that, similar to all other physical forces, have the same (vector) nature in the four-dimensional space.

Einstein believed the local identity of inertia and gravitation to be the principal cause for the equality of inertial and gravitational masses. In contrast, my opinion is that the actual cause of this equality lies in the fact that the conserved total density of the tensor of matter and gravitational field serves as the source of the latter. It is for this reason that the equality of inertial and gravitational masses does not require local identification of inertial and gravitational forces. Still, geometrisation in the sense of the geometrisation principle proves to be indispensable.

7. Some physical implications of RTG

The RTG system of equations (66) and (67) leads to qualitatively new physical conclusions which are utterly different from those of GTR. For example, the notion of collapse is altogether altered. Specifically, the process of compression in the region adjoining the Schwarzschild sphere, which is associated with the collapse of a spherically symmetric body with arbitrary mass, is terminated and replaced by dilation. This suggests the occurrence of both contracting and dilating objects in nature. Therefore, RTG rules out entirely the presence of

§Question: The author argues that ''in GTR, gravitational forces are identical to forces of inertia''. This is not true (see paragraphs 81 and 82 in *Teoriya Polya* by Landau and Lifshitz).

Einstein and many other authors (see *Teoriya Polya* by Landau and Lifshitz, paragraphs 81 and 82) maintain that in GTR forces of gravity and inertia are dissimilar in a finite region or in the entire space. As regards the infinitesimal region, Einstein (followed by Landau and Lifshitz in *Teoriya Polya* paragraph 87) argued that it is possible locally to exclude the gravitational field by the choice of coordinate system. This assertion is in a sense correct since gravitational forces enter the equations of motion for the point body without spin through the Christoffel symbols of the Riemannian space, i.e. through inertial forces. This is how my words must be understood. In this paper, I added the word 'locally', to be more precise.

'black holes', i.e. objects having no material bounds and cut off from the outside world†.

Another important conclusion pertains to the development of the homogeneous and isotropic universe. It follows from Eqns (66) and (67), and from causality conditions (83) and (84), that the homogeneous and isotropic universe has been in existence for an infinitely long time and its three-dimensional geometry is Euclidean. The universe develops in a series of alternating cycles, from maximum finite density towards the minimum density and vice versa (if there is no dissipation). The theory predicts a large amount of 'latent' mass to be present in the universe because Eqns (66) and (67) suggest that the total density of matter currently equals

$$\rho = \rho_c + \frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2 . \tag{86}$$

It is therefore clear that the density of matter for a sufficiently small graviton mass is close to the critical density ρ_c defined by Hubble's constant H:

$$\rho_c = \frac{3H^2}{8\pi G} . \tag{87}$$

RTG explains all known gravitational experiments in the solar system and allows the notion of the energy–momentum tensor to be introduced for the gravitational field (as above) as well as for other physical fields. The conserved total density of the tensor of matter and gravitational field being the source of the latter (in conformity with the geometrisation principle), it immediately follows from Eqn (70) that the inertial mass of a static body is exactly equal to its active gravitational mass. This equality does not suggest local identity of gravitation and inertia‡.

On the other hand, the motion of a neutral test body in a given gravitational field is independent of the body's mass because it occurs along a geodesic curve in the effective Riemannian space. This brings about the conclusion that the passive gravitational mass of the test body is also equal to its inert mass. Hence, the passive mass of the test body is equal to its active gravitational mass. The density of energy–momentum tensor $-2(\delta L_g/\delta g_{\mu\nu})$ in the gravitational field of the Riemannian space outside matter vanishes in accordance with Eqn (66). However, this does not imply the absence of gravitational radiation since gravitational waves carrying energy travel against an effective gravitational background.

The problem of gravitational radiation by massive gravitons has been discussed in Ref. [4] where it was shown that early calculations were based on an incorrect general expression for the intensity. Derivation of this expression was performed without due regard to the important fact that gravitons actually propagate in the effective Riemannian space rather than in the Minkowski space. This consideration brought me to the conclusion that the intensity of gravitational radiation by massive gravitons is a positive-definite value. The expression for it is presented in Ref. [4]. The system of gravitational equations (66) and (67) offers a new possibility for further studies both of basic problems and of selected gravitational phenomena.

Finally, there are several important points worthy of discussion. Is it possible to assume that the graviton mass is zero? Since in our theory the graviton mass serves to lift gauge degeneracy, it would be incorrect to omit it in Eqns (66) and (67). In other words, the graviton mass must remain nonvanishing in our theory. The system of gravitational equations (66) and (67) is hyperbolic, and the causality principle ensures the presence of a spacelike surface in the entire space which every nonspacelike curve in the Riemannian space crosses only once. There is a global Cauchy surface on which the initial physical conditions for problems may be specified

Penrose and Hawking [5] have proved singularity theorems in GTR for certain general conditions. The inequality

$$R_{\mu\nu} v^\mu v^\nu \leqslant 0 , \tag{88}$$

has been shown to hold true for isotropic vectors in the Riemannian space outside matter as follows from Eqn (68a) and as a consequence of causality conditions (85a). Therefore, the conditions of the singularity theorem are not fulfilled in RTG and its assertions are not applicable to this theory. In this theorem, spacelike events in the absence of a gravitational field will never become timelike under the effect of a gravitational field. The causality principle dictates that the effective Riemannian spacetime in RTG should be possessed of isotropic and timelike geodesic completeness.

This line of reasoning brings the following general conclusion. If the source of the gravitational field in the Minkowski space is assumed to be the conserved tensor of the energy–momentum of matter and the massive gravitational field, because of the universal character of gravitation, then this field will be apparent as a tensor field of the second rank. By analogy with electrodynamics, it is natural to write down field equations in the form

$$\Box \phi^{\mu\nu} + m^2 \phi^{\mu\nu} = \lambda t^{\mu\nu} , \quad \partial_\mu \phi^{\mu\nu} = 0 .$$

But such a system of equations follows from the Lagrange formalism only if the interaction between matter and the gravitational field occurs in compliance with the

†Question: assuming that the theory prohibits black holes, is it not possible to demonstrate this by an explicit solution of the RTG equations analogous to the Schwarzschild solution in GTR?

Yu M Loskutov (*TMF* **82** 304 1990) has demonstrated that the metric coefficients of the Riemannian space for a spherically symmetric static body in the domain near the Schwarzschild sphere have the following form:

$$ds^2 = U(Z)\,dt^2 - V(Z)\,dZ^2 - Z^2\,d\Omega^2 ,$$

$$V(Z) = \frac{Z}{Z - Z_g} , \quad U(Z) = (1 + 2mM)\,\frac{Z - Z_g}{Z} + qm^2M^2 , \quad q > 0 .$$

Singularity V arose at the point Z_g equal to

$$Z_g = 2M + (6 - c^2)m^2M^3 \ln\frac{1}{mM} , \quad c^2 < 4 ,$$

$$-g = UVZ^4 \sin^2\Theta .$$

The sphere of radius Z_g is singular, and this singularity cannot be eliminated by the choice of a coordinate system. It is therefore evident that black holes do not exist in nature, in accordance with RTG. Unfortunately, an exact solution of the spherically symmetric static problem remains to be found.

‡Question: What are active and passive gravitational masses?

A mass which generates a gravitational field is referred to as active. The action of this field on another body is determined by the passive gravitational mass. In Newtonian mechanics, the active mass of an object is exactly equal to its passive mass, by virtue of Newton's third law.

geometrisation principle, which reduces the action of this field to the effective spacetime geometry.

Therefore, the assumption of a conserved tensor of energy – momentum of matter as a universal source of the gravitational field inevitably leads to the effective Riemannian geometry.

Gravitational field theory requires the introduction of the graviton mass and in structural terms is reminiscent of electrodynamics. It is therefore very likely that the photon rest mass is also nonvanishing.

8. Mach's principle

In formulating laws of mechanics, Newton introduced the notion of absolute space which always remains unaltered and motionless. It is with respect to this space that Newton described the acceleration of a body. The acceleration was supposed to be absolute. Introduction of such abstraction proved to be very fruitful. Specifically, it gave rise to the notion of inertial reference frames in the entire space and the relativity principle for mechanical processes. It also contributed to the idea of physically distinguished states of motion. In 1923, Einstein wrote: ''Systems of coordinates in such states of motion are peculiar for the simplest form assumed by the laws of nature formulated in these coordinates.'' He further noted: ''...according to classical mechanics, there is relativity of velocity but not of acceleration''. This is how the notion of inertial reference frames in which material points unaffected by forces do not experience acceleration and remain at rest or in uniform and rectilinear motion found its way into the theory. However, Newton's absolute space and reference frames were actually introduced a priori without due regard to matter distribution in the universe.

Mach took the liberty of seriously criticising certain basic principles of Newtonian mechanics. He wrote later that he had met with great difficulties in publishing his ideas. Although Mach failed to suggest a theory devoid of the drawbacks he criticised, his ideas had great influence on the development of physical theory. Suffice it to say that Mach drew the attention of scientists to the analysis of basic physical notions.

It is appropriate to cite a few extracts from Mach's works [6] which collectively constitute the so-called Mach principle. Mach stated: ''Nobody can tell anything about absolute space and absolute motion which are conceivable entities beyond the grasp of experience.'' Then, he went on: ''Instead of referring a travelling body to the space (i.e. to a certain coordinate system), let us consider its relation to world bodies which appear to be the only tools available to determine the system of coordinates. ...even in the simplest case, when we seem to examine the interaction between two masses, we can not be altogether abstracted from the remaining world. ...If a body rotates relative to the sky of motionless stars, there appear centrifugal forces; if the body rotates relative to another body rather than the sky of motionless stars, there are no centrifugal forces. I have nothing against calling the former mode of rotation absolute provided it is kept in mind that this implies nothing but rotation relative to the sky of motionless stars.''

Hence, the following assertion by Mach: ''...it is not necessarily to associate the law of inertia with any specific absolute space. The matter-of-course approach of a true naturalist would be to first consider the law of inertia as the approximate one, correlate it spatially with the sky of motionless stars, ...then corrections or further development of our knowledge should be anticipated based on the accumulated experience. Lange has recently published a critical paper in which he tells how it would be possible, following his principles, to introduce a new coordinate system in case the usual rough correlation with the motionless starry sky turned out to be no longer suitable in view of more accurate astronomical observations. There is no discrepancy between Lange's opinion and mine concerning the theoretical formal value of his inferences, namely that the motionless starry sky is currently the only practically suitable reference frame; nor do we disagree as regards the method for determining a new reference frame by means of gradual adjustment.'' Moreover, Mach cited C Neumann as stating: ''Since any motion must be considered in the alpha system (inertial system), it apparently serves to indirectly connect all processes that take place in the universe and, hence, contains a universal law as enigmatic as it is complicated.'' Mach concludes the quotation with the following remark: ''I think everybody would agree with this.''

This utterances of Mach raise the question of inertial reference frames and their relation to the distribution of matter, since Mach is evidently concerned here with the law of inertia, which was formulated by Newton as follows: ''Any body taken alone as a self-contained entity tends to maintain its state, be it rest or uniform rectilinear motion...'' Mach and his contemporaries appear to have had the clear understanding that such a relationship must occur in nature. The term 'Mach principle' will be further used precisely in this sense.

Mach also wrote: ''Although I believe that astronomical observations will first require only minor amendments, I do admit that the law of inertia in the simple form suggested by Newton is of limited and transient significance for us humans.'' It will be shown below that Mach was in the wrong. He failed to propose a mathematical formulation of his ideas which accounts for different authors interpreting the Mach principle in their own way. We are trying henceforth to adhere to the essence of the principle as stated by Mach.

Poincare, followed by Einstein, extended the principle of relativity to all physical phenomena. Poincare [2] formulated it in the following way: ''...the relativity principle implies that the laws of physical phenomena must be the same for a motionless observer and an observer in a state of uniform translational motion, which does not permit us to have any way to decide whether we are in the same state of motion or not.'' Application of this principle to electromagnetic phenomena led Poincare and then Minkowski to the discovery of the pseudo-Euclidean geometry of spacetime and provided further support for the hypothesis of inertial reference frames in the entire space. Such reference frames are physically distinguished, which accounts for the absolute meaning of acceleration with respect to these frames.

There are no global inertial reference frames in GTR. In 1929, Einstein noted: ''The starting point of the theory is the assertion that there is no physically distinguished state of motion, which means that neither velocity nor acceleration has absolute sense.''

The Mach principle as formulated in the framework of GTR was not required. However, it is worth mentioning that the notion of global inertial reference frames is fairly well supported by experimental findings. Specifically, the transition from the terrestrial coordinate system to that of the Sun and then of the metagalaxy leads, with increasing accuracy, to an inertial reference frame. Therefore, there is no serious reason to reject such an important notion as the inertial reference frame. On the other hand, the fundamental laws of energy–momentum and angular momentum conservation necessarily provide for the existence of global inertial reference frames. Pseudo-Euclidean spacetime geometry reflects general dynamic properties of matter and at the same time introduces inertial reference frames. Despite the fact that the pseudo-Euclidean geometry of spacetime arose as a result of the study of matter and is therefore inseparable from it, there is formally every reason to examine Minkowski space in the absence of matter. However, neither Newtonian mechanics nor special relativity explains how inertial reference frames are associated with the distribution of matter throughout the universe.

The discovery of the pseudo-Euclidean geometry of spacetime allowed a common approach to be used for examining both inertial and accelerated reference frames. Forces of inertia and forces generated by physical fields have been found to be totally different. Forces of inertia can always be nullified by the choice of an appropriate reference frame, whereas forces generated by physical fields cannot in principle be made zero by the proper choice of a reference frame since they have vector nature in the four-dimensional spacetime. Because the gravitational field in RTG is a physical field of the Faraday–Maxwell type, forces that it gives rise to cannot be made zero by the choice of a reference frame.

The situation is quite different in general relativity. Gravitational forces in this theory do not exhibit vector nature in the four-dimensional spacetime and are therefore subject to be locally made zero by the choice of a reference frame. The principal RTG equations (66) and (67) contain both the Riemann metric and the metric tensor of the Minkowski space because of the presence of the rest mass of the gravitational field. This implies the possibility, in principle, of expressing the metric of this space through geometric characteristics of the effective Riemann space as well as through those values that characterise the distribution of matter in the universe. This is easy to achieve if contravariant values in Eqns (66) are replaced by covariant ones. This results in

$$\frac{m^2}{2}\,\gamma_{\mu\nu}(x) = \frac{8\pi}{\sqrt{-g}}\left(T_{\mu\nu} - \frac{1}{2}\,g_{\mu\nu}T\right) - R_{\mu\nu} + \frac{m^2}{2}\,g_{\mu\nu}\,. \quad (89)$$

This equation contains on the right-hand side only geometric characteristics of the effective Riemannian space and values describing the distribution of matter in this field.

In principle, experimental studies on the motion of particles and light propagation in the Riemannian space may be helpful in finding the metric tensor of the Minkowski space and hence in constructing an inertial reference frame. This means that RTG elaborated in the framework of special relativity allows for the mathematical formulation of the Mach principle. Thus, the special principle of relativity has universal implications regardless of the form of matter.

Its requirements for the gravitational field are specified in the condition of form-invariance of Eqns (66) and (67) with respect to the Lorentz group. Lorentz form-invariance of physical equations remains the most crucial physical principle in the construction of the theory and allows universal characteristics for all forms of matter to be introduced.

In 1950, Einstein asked "...should we not eventually try to retain the notion of the inertial frame giving up all attempts to explain a fundamental feature of gravitational phenomena which is apparent in the Newtonian system as the equivalence of inertial and gravitating masses?" It has been shown in Section 6 that the equality of inertial and gravitational masses is an immediate corollary of Eqns (70), in which the total density of the tensor of energy–momentum of matter and gravitational field serves as the source of the field, in conformity with the geometrisation principle. This equality by no means excludes the notion of the inertial frame. In RTG, this notion is preserved totally unaltered and reflects general dynamic properties of matter, i.e. the laws of conservation of energy–momentum and angular momentum. Therefore, the equivalence of inertial and gravitating masses does not necessarily require one to abandon the notion of the inertial frame. Einstein answered his own question as follows: "Anyone who believes in the cognosciblity of nature would say — no, we should not", which is in conflict with our conclusion.

Mach's ideas greatly influenced Einstein's views of gravitation in constructing his general theory of relativity. Einstein made the following remark in one of his works: "The Mach principle: G-field is totally determined by body masses." However, even this proposition fails to be fulfilled in GTR, for solutions are possible in the absence of matter. The attempt to obviate this fact by introducing the λ-term was in vain because equations with this term in the absence of matter proved also to have nonzero solutions. It is easy to see that Einstein put quite a different meaning into the notion of 'Mach principle'. But even so, the Mach principle failed to find its way into GTR.

Is the Mach principle as formulated by Einstein included in RTG? Unlike GTR, this theory contains (in agreement with the causality principle) spacelike surfaces filling the entire space (Cauchy global surfaces). If matter is absent on one such surface, it will always be absent, in conformity with the requirement of energy dominance imposed upon the tensor of matter [5]. Since matter does exist in nature, the system of equations homogeneous over the entire space has no solutions realisable in nature. In other words, all solutions of this system are physically meaningless in a given scenario of the development of the universe. That it is possible to discard solutions of the system of homogeneous gravitational equations is due not only to the equations themselves but also to the intrinsic properties of the real universe.

In principle, the equations of our theory do not reject universes constructed from a gravitational field without matter. But such universes have been rejected by the proper development of matter. The theory is currently unable to explain why our universe contains matter. Solutions of only inhomogeneous systems of gravitational equations have physical meaning when matter is present in all or a part of space. This means that neither the gravitational field nor the effective Riemannian space of the real universe could arise in the absence of parent matter. It may be inferred that the Mach principle as formulated by Einstein also occurs in RTG.

For all that, there is an important difference in the understanding of the G-field in our theory and in GTR. The G-field was understood by Einstein to be the Riemannian metric whereas we regard it as a physical field. Such a field enters the Riemannian metric together with the flat metric which ensures that the metric does not disappear in the absence of matter and gravitational fields; instead, it is retained as the Minkowski space metric. Other formulations of the Mach principle have been suggested which differ from those given by Mach himself and Einstein, but their examination is beyond the scope of the present communication as I consider them to be obscure. Because the forces of gravity in RTG are due to a physical field of the Faraday–Maxwell type, their identity with forces of inertia is in the main out of the question.

The gist of the Mach principle is sometimes viewed as the dependence of inertial forces on interaction with matter contained in the universe. From the field standpoint, such a principle is irrelevant and may not occur in nature. The thing is that although inertial reference frames are associated with matter distribution in the universe (see above), forces of inertia do not actually result from the interaction with matter in the universe because any effect of matter is possible only via physical fields. But this means that forces generated by these fields cannot be nullified by the choice of a reference frame. Therefore, forces of inertia are immediately related not to the physical fields, but to a strictly determined geometry structure and the choice of a reference frame.

On the one hand, the pseudo-Euclidean geometry of the spacetime which reflects dynamic properties common to all forms of matter confirmed the hypothesis of inertial reference frames; on the other hand, it demonstrated that forces of inertia arising from the appropriate choice of a reference frame are expressed through the Christoffel symbols of the Minkowski space. Therefore, they are independent of the body's nature. All this has become clear after the special theory of relativity was shown to be applicable to noninertial (accelerated) reference frames just as well as to inertial ones. This allowed for a more general definition of the relativity principle [7]: "Whatever a physical reference frame is chosen (inertial or noninertial), it is always possible to indicate an infinitely great number of other reference frames in which all physical phenomena occur in the same fashion as they do in the initial reference frame; thus, we do not and can not have any experimental tool to decide in which particular system we are located within this infinite set of them."

In RTG, the forces of inertia and gravity are essentially different† in that the gravitational field becomes progressively weaker as distances from the bodies increase whereas forces of inertia may be arbitrarily strong depending on the choice of reference frame. It is only in an inertial reference frame that these forces are zero. It would therefore be improper to think that forces of inertia are inseparable from gravitational forces. The possibility of distinguishing between them is all but clear in everyday life.

Construction of RTG allowed a relationship between the inertial reference frame and the distribution of matter in the universe to be established and provided a deeper insight into the nature of forces of inertia as utterly distinct from material forces. Our theory assigns to the forces of inertia just the same role which they are known to play in any other field theory.

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†Question: we stated that "in RTG, forces of inertia and gravitational forces are essentially different..." and the final impression is that there is no such difference in GTR. However, it is precisely this difference that GTR claims (see *Teoriya Polya* by Landau and Lifshitz, paragraphs 81 and 82).

There is no connection whatsoever between forces of inertia and gravity in RTG. The three-dimensional gravitational force can be balanced by the force of inertia by the choice of an appropriate coordinate frame alone. In GTR, it is possible to speak in a narrow context (following Einstein) about the local identity of inertial and gravitational forces. See the last footnote on p. 189

However, things are not so simple as regards GTR. According to Synge, the gravitational field in GTR is characterised by the curvature tensor alone; hence, the possibility of even a local difference between gravitation and inertia. This accounts for the free fall acceleration (980 cm s^{-2}) being unrelated to the gravitational field as was demonstrated by Synge. In RTG, the gravitational field is characterised by both the curvature tensor and the four-vector of force, which makes acceleration of a free-falling body field-dependent.