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Giant dipole resonance and evolution of concepts of nuclear dynamics (on the 50th anniversary of A B Migdal's paper "Quadrupole and dipole γ emission from nuclei")

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Abstract. Fifty years ago, in his paper on "Quadrupole and dipole γ emission from nuclei", A B Migdal introduced implicitly the concept of a dynamic collective model in nuclear physics and used this concept to predict a giant dipole resonance. Evolution of the theory of this resonance has had an enormous influence on the formation of modern concepts relating to the dynamics of nuclei. A brief historical introduction is followed in this paper by an account of the conceptual aspects of the subsequent evolution of the ideas on the nature of the giant dipole resonance. This evolution has followed a complex path from the initial identification of a nucleus with a liquid drop to its representation as a system of independent nucleons. Recent investigations have made it possible to understand the interrelationships between these apparently diametrically opposed concepts, to bring them closer together, and to demonstrate the equivalence of the description of the giant dipole resonance with either concept.

1. Introduction

1.1 This year fifty years will have passed from the publication of A B Migdal's "Quadrupole and dipole γ emission from nuclei" [1]. The title and the specific result reported in this paper, an indication of a strong upward energy shift of intense dipole transitions in nuclei, at first sight seem to be of purely 'local' importance. In reality, however, the work of Arkadii Beinusovich occupies a special place in the history of nuclear physics.

First, this paper predicted one of the most important phenomena in nuclear physics, which is the giant dipole resonance (GDR). The GDR is a wide (5 – 10 MeV) maximum of the curve representing the absorption of γ photons by a nucleus, which in the case of heavy nuclei is located at energies 14 – 16 MeV. In the case of light nuclei it is shifted towards somewhat higher energies (20 – 25 MeV). Without any exaggeration we can say that the formation of the GDR and also of giant resonances of different origin, together with the properties and role of these resonances in various nuclear processes, have been the central point around which the main discussions in nuclear physics have taken place in the last 30 – 35 years.

Second, A B Migdal was the first to understand that the main GDR parameter, which is the energy of its maximum, is governed, on the one hand, by the symmetry energy (more exactly, the coefficient β) in the Bethe – Weizsacker formula for the binding energy of a nucleus and, on the other, by the average kinetic energy of nucleons, which he found by applying the sum rule.

Third, and perhaps as important as the prediction of the GDR, is the fact that Migdal was the first to introduce the concept of quantum collective excitation modes in nuclear physics.

1.2 We shall consider the conceptual aspects of A B Migdal's paper and the current views on the GDR in later sections. We shall begin with a small detour to the history of nuclear physics.

The GDR predicted by Migdal was discovered several years later by Baldwin and Klaiber [2]. Soon after, Goldhaber and Teller [3] interpreted the GDR as a manifestation of collective, i.e. associated with synchronous motion of a

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large number of nucleons, dipole proton–neutron oscillations. One of the oscillations models considered by Goldhaber and Teller, the hydrodynamic model (model III), was subsequently developed by Steinwedel and Jensen [4] and Danos [5]. The dependences of the GDR properties on the shape of a nucleus were investigated by Okamoto [6] and Danos [7]. At approximately the same time, Rainwater [8] introduced surface oscillation modes in nuclear physics and these were investigated subsequently by A Bohr and Mottelson [9, 10], and by many other authors. A complete dynamic model of coupled dipole and surface oscillations was formulated in the early sixties by Danos and Greiner [11], and also by Semenko [12]. Some aspects of the interaction of photons with nuclei were considered by Baldin [13] within the framework of more general concepts. An important role was played by the work of McDaniel, Walker and Stearns [14], who investigated elastic scattering of 17.6 MeV photons by a number of nuclei. Their results were used by Danos to support the collective model [15].

In addition to the evolution and extensive use of the concept of collective types of motion in a nucleus, the late forties and early fifties saw a literal breakthrough in nuclear physics of the idea of independent motion of nucleons and the special role of single-particle degrees of freedom [16]. The final stage of extensive investigations of the effects of these degrees of freedom in nuclei has been a general acceptance that a nucleus represents, in the first approximation, a system of independently moving nucleons, which is known as the independent-particle model (IPM). This has brought to the fore the problem of interpretation of the GDR in terms of single-particle degrees of freedom and the question of the general relationship between the collective and single-particle forms of motion.

Wilkinson [17] was the first to investigate the general properties of dipole transitions in the IPM. It was found that such dipole transitions are grouped in the range of energies equal to the average energy separation between the shells. This means that the phenomenon of the GDR appears also in the IPM, but the energy of this resonance for heavy nuclei is approximately half that found experimentally. The 'collective nature' of the GDR, in the sense defined above, seems to be absent.

In the early fifties, Reifman, working under Heisenberg, in the now completely forgotten note [18] demonstrated that — using the modern language — the IPM can accommodate collective motion if the residual interaction is taken into account. Soon after, Brink [19] showed that the IPM with the oscillator potential can be used also to predict configurations in which centres of masses of protons and neutrons are excited separately. In other words, we can say that the IPM can be regarded as including potentially the collective model of dipole oscillations.

The role of the residual interaction in the GDR was also investigated by a group at Moscow University [20]. Finally, Elliott and Flowers [21] and to an even greater extent Brown and Bosterli [22] demonstrated explicitly the particle–hole mechanism of formation of collective modes of motion of nucleons in the IPM. The mathematical 'apparatus' used to describe this mechanism is nowadays called the random phase approximation (RPA). This has been followed by the development of a more or less consistent picture of the formation of the GDR (and also of other giant resonances) in nuclei, the main features of which agree with the experimental results. This led to the growth of an entire particle–hole 'industry' occupied with the research on the GDR and other giant resonances. The most important stages in the development of this industry were as follows:

— a detailed investigation of the form of the phenomenological particle–hole interaction [23];
— identification of the mechanisms of dissipation of the collective forms of motion [24–27];
— inclusion of nonmagic nuclei in the RPA scheme [28–31];
— development by Migdal and his colleagues of a consistent theory of the final Fermi systems [32–34];
— development of methods for including in the RPA scheme a continuous spectrum of single-particle excitations [35–37];
— establishment of a relationship between realistic nucleon–nucleon forces and the effective particle–hole interaction [38];
— demonstration that the RPA approach is equivalent to the macroscopic approach of the Vlasov equation in the case of a collisionless plasma; this has made it possible to approach from a new angle the relationship between the collective and single-particle motions in nuclei [39–41].

The main milestones in the early evolution of the physics of photonuclear research are discussed in detail in conference proceedings [42]. The subsequent theoretical developments and new experimental data can be found in a recently published review volume [43].

2. Giant dipole resonance as excitation of a collective mode of a nucleus

2.1 In modern terms, Migdal's estimate of the average energy $\bar{\omega}$ of the absorbed dipole phonons is obtained from the formula

$$\bar{\omega} = \left(\frac{\sigma_{\text{integ}}}{\sigma_{-2}}\right)^{1/2}, \tag{1}$$

where

$$\sigma_{\text{int}} = \int \sigma(\omega)\,\mathrm{d}\omega \tag{2}$$

is the total cross section of the absorption of a photon by a nucleus $\sigma(\omega)$, integrated with respect to energy, and

$$\sigma_{-2} = \int \frac{\sigma(\omega)}{\omega^2}\,\mathrm{d}\omega\,. \tag{3}$$

If, for the sake of simplicity, we assume that the potential energy commutes with the nucleon coordinates (in reality this is not quite true because of the exchange and spin–orbit forces), we find that σ_{integ} is a model-independent quantity:

$$\begin{aligned}\sigma_{\text{integ}} &= 4\pi^2 \sum_n \omega_{n0}\,|\hat{d}_{n0}|^2 = 4\pi^2 \langle 0\,|[\hat{d}\,[\hat{H}\,\hat{d}]]|0\rangle \\ &= 4\pi^2 \langle 0|[\hat{d}\,[\hat{T}\,\hat{d}]]|0\rangle = \frac{2\pi^2 e^2}{M}\,\frac{NZ}{A},\end{aligned} \tag{4}$$

where $|0\rangle$ is the vector of the ground state of the nucleus, $\hat{T}$ is the kinetic energy operator, $\hat{d}$ is the dipole transition operator

$$\hat{d} = e\left\{\sum_{i=1}^{Z} \hat{z}_i\,\frac{N}{A} - \sum_{i=Z+1}^{N+Z} \hat{z}_i\,\frac{Z}{A}\right\}, \tag{5}$$

and $\hat{z}_i$ is the operator representing the z projection of the coordinates of the ith nucleon. The rest of the notation is as follows: M is the mass of a nucleon; A, N, and Z are, respectively, the mass number, and the numbers of neutrons and protons in a nucleus; $[\hat{H}\hat{d}]$ and $[\hat{T}\hat{d}]$ are the commutators of the complete Hamiltonian of the kinetic energy of the nuclear system with the dipole transition operator.

If we take into account the exchange nature of the nuclear forces, we have to introduce a small correlation into formula (4): this correction represents the ratio of the effective mass M^* of a nucleon in a nucleus to the mass of a free nucleon. However, this correction alters only slightly the quantitative result obtained by Migdal [1].

On the other hand, the quantity σ_{-2} is model-dependent and Migdal estimated it with the aid of collective coordinates. We can easily see that σ_{-2} is identical, apart from a factor, with the static polarisability α of a nucleus:

$$\sigma_{-2} = 2\pi^2\alpha. \tag{6}$$

The polarisability α can be found from the dipole moment induced by an external electric field. In its turn the induced dipole moment of the nucleus is governed by minimisation of the energy of a nucleus in an external field. The local variant of the Bethe–Weizsacker formula for the binding energy of a nucleus, in which for example the symmetry energy

$$E_{\mathrm{s}} = \frac{\beta(N-Z)^2}{A} \tag{7}$$

becomes

$$E_{\mathrm{s}} = \beta \int \mathrm{d}^3x \, \frac{\left[\rho_{\mathrm{p}}(\boldsymbol{x}) - \rho_{\mathrm{n}}(\boldsymbol{x})\right]^2}{\rho(\boldsymbol{x})}, \tag{8}$$

where $\rho_{\mathrm{p,n}}(\boldsymbol{x})$ are respectively the densities of protons (p) and neutrons (n), together with the assumption that $\rho = \rho_{\mathrm{p}} + \rho_{\mathrm{n}} = \mathrm{const}$, enabled Migdal to show that the polarisability α of a nucleus is given by

$$\alpha = \frac{e^2R^2A}{40\beta}, \tag{9}$$

where R is the radius of the nucleus. Migdal then substituted formula (6) into formula (1) and found that for $a = 200$, $Z = 80$, and $R = 5$ fm, this average energy is

$$\bar{\omega} \simeq 16 \ \mathrm{MeV}. \tag{10}$$

2.2 We shall now consider the conceptual aspects of the work of A B Migdal. We must stress once again that he used the collective model. This is obvious from the fact that the dynamic variable in his analysis is the density $\rho_{\mathrm{p}}(\boldsymbol{x})$ in the distribution of protons.

The collective model in the liquid drop form had been used in nuclear physics long before Migdal. For example, it was used by Bohr and Wheeler [44] in an analysis of the binding energy of nuclei and also of the process of nuclear fission. However, only the effects of the potential energy of collective motion have been used in the previous treatments. We can therefore say that the concept of a quantum collective model (with the essential element of including not only the potential but also the kinetic energy of collective motion) had not been stated explicitly. In fact, it is in Migdal's paper that the kinetic and potential terms were included simultaneously for the first time. We can easily see from the preceding subsection that the polarisability α is governed entirely by the potential energy of the collective variable ρ_{p}, i.e. by the symmetry energy E_{s}; on the other hand, the total cross section σ_{integ} is governed, as can be seen from formula (4), by the kinetic energy of the Hamiltonian. One should mention also some nontrivial features of the variable $\rho_{\mathrm{p}}(\boldsymbol{x})$. In Migdal's analysis it is model-independent in the sense that it does not require the hydrodynamic interpretation. Actually, this variable is of quantum nature and is one of the components of a quantum object which is the density matrix $\rho_{\mathrm{mn}}(\boldsymbol{x})$:

$$\rho_{mn}(\boldsymbol{x}) = \left\langle n \middle| \sum \delta\left(\boldsymbol{x} - \boldsymbol{x}_i\right) \middle| m \right\rangle, \tag{11}$$

where m, and n are the indices of the state vectors, $\boldsymbol{x}_i$ are the coordinates of the nucleons, and $\delta(\boldsymbol{x})$ is the usual δ function. Formula (11) represents one of the rules for transition from microscopic (x_i) to collective variables.

2.3 The collective model is frequently associated with the hydrodynamic model. However, we can see from an analysis of Migdal's paper that the concept of the collective model is wider than that of its hydrodynamic variant.

It is in general evident that a 'nucleon' nonrelativistic nucleus can be described not only in terms of the nucleon coordinates, but also by means of any complete set of dynamic variables, such as those defined by relationship (11). Therefore, there are no 'more fundamental' and 'less fundamental' degrees of freedom: the use of each of such sets of degrees of freedom should give, in the final analysis, identical results. However, the number of variables which can be used to describe a specific physical phenomenon depends on the selected set of coordinates. Therefore, instead of the concept of 'fundamentality', the pragmatic consideration of 'convenience' plays the chief role. For example, in describing a dipole resonance in a holmium nucleus we need three collective variables, whereas working with single-particle degrees of freedom, we require about 50.

The collective model begins with selection of a limited set of collective variables, i.e. variables which describe matched motion of a large number of nucleons. This set depends on the nature of the phenomenon to be analysed. For example, in a description of surface vibrations and rotations of a nucleus the collective coordinates may be the deformation parameters β and γ, and the Euler angles which govern the orientation of a nucleus in space.

The next step is the derivation of the Hamiltonian expressed in terms of selected variables and their canonical momenta. The constants and functions which determine the Hamiltonian are either regarded as parameters, to be found by comparison with experimental results, or are calculated by, for example, the Nilsson–Strutinsky method [45].

2.4 Some comments should now be made to provide more detail in the description of the GDR in the dynamic collective model. It is natural to include in this model all the collective coordinates necessary for the description of the GDR phenomenon.

The 'internal' multipole oscillations, among which the most important are dipole oscillations, are described by the harmonic Hamiltonian H_{intern}:

$$H_{\mathrm{intern}} \propto \left\{B_J^{-1} \left(\pi_J \pi_J\right)_0 + C_J \left(D_J D_J\right)_0\right\}, \tag{12}$$

where D_J and π_J are the multipole collective coordinates and the associated momenta; C_J and B_J are, respectively, the 'rigidity' coefficient and the mass coefficient; J is the angular momentum of collective oscillations. The index 0 indicates that the momenta are included in the total zero momentum.

Vibrations of the surface and rotation of a nucleus can be described by the approach developed by Bohr and Mottelson. In this approach, selection of the radius $R(\theta, \phi)$ of a nucleus

$$R(\theta, \phi) = R_0 \left(1 + \sum_{L \geqslant 2} A_L Y_L(\theta, \phi) \right), \tag{13}$$

where A_L are the internal dynamic variables, determines the rotation of the nucleus and vibrations of its surface.

The interaction of the multipole vibrations with the surface vibrations and with the rotation of a nucleus is determined by the dependence of the energy of the multipole vibrations on the static deformation of a nucleus (adiabatic approximation). In an approximation which is linear in terms of the parameter A_J, this interaction should be described by

$$H_{\text{interact}} \propto A_J D_J D_J \, . \tag{14}$$

The most important aspect of the GDR problem is the interaction between the multipole and surface vibrations. Specific effects of this interaction depend decisively on the strength of the interaction. In the case of nuclei which are rigid in relation to surface deformations, such as the ^{208}Pb nucleus considered earlier, these effects are slight. On the other hand, the interaction between the dipole and surface vibrations in 'soft' nuclei causes a very strong splitting or broadening of the GDR (Fig. 1).

On the whole, we can say that the dynamic collective model under discussion describes, in spite of the small number of initial parameters, a surprisingly wide range of experimental data on the properties of low-lying states of nuclei as well as the gross structure of the GDR. Moreover, its predictive and interpretative capabilities have not yet been fully utilised, although the paradigm has now changed and the chief place in research is occupied by the microscopic approaches.

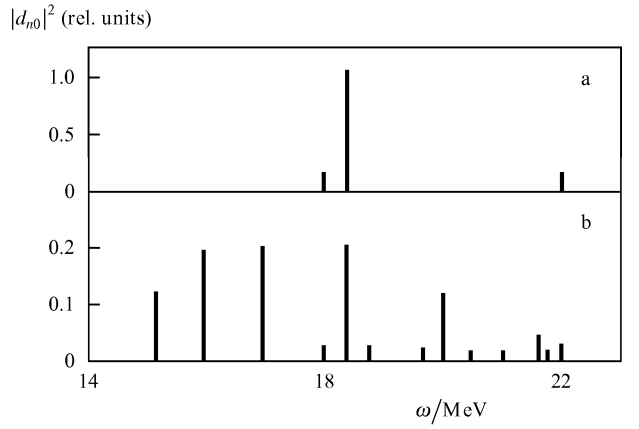

Figure 1. Giant dipole resonance in a spherical nucleus of intermediate weight, considered within the collective model framework: (a) ignoring the coupling of the dipole and surface vibrations; (b) taking account of this coupling [11]. The vertical axis gives, in relative units, the squares of the matrix elements of the dipole transition operator $|d_{n0}|^2$ [see expression (4)].

3. Giant dipole resonance as excitation of single-particle degrees of freedom of a nucleus

3.1 Single-particle degrees of freedom appear in low-energy nuclear physics as the degrees of freedom of quasiparticles (quasinucleons). Quasinucleons are generally collective quantum objects which have the quantum numbers of nucleons. They are complex superpositions of real ('hole') nucleons and of many-particle excitations such as, for example, those that create local polarisation in a nuclear medium. For simplicity, we shall use the term 'nucleon' when speaking of single-particle degrees of freedom, but we shall understand it to be a quasinucleon.

In terms of single-particle degrees of freedom a nucleus can be regarded, as in the first approximation, as a Fermi gas of nucleons placed at zero temperature in a self-consistent nuclear field.

A remarkable property of the distribution of nucleon levels in the average field is the shell structure, i.e. the grouping of levels in shells separated from one another by an energy interval considerably greater than the width of the levels (for example, in the case of ^{208}Pb, the upper shell is about 1.5 MeV wide and the separation between the shells is 7–8 MeV).

The magic nuclei, i.e. those with filled neutron and proton shells, have the simplest structure in the IPM model. Therefore, in our discussion we shall use these nuclei as an example or, more exactly, we shall consider ^{208}Pb as the 'ideal nucleus'. This heavy nucleus can be regarded as ideal in the sense that it is not affected by several physical factors (for example, pairing, a strong influence of the surface, etc.) which complicate the giant resonance pattern described in terms of the single-nucleon degrees of freedom.

3.2 The simplest excitations of magic nuclei appear as a result of motion of one nucleon from a filled to an empty (free) shell. They are described by the quantum numbers of a 'particle', which is a nucleon in an empty shell, and a 'hole', which is a vacancy in one of the filled shells. They are usually called particle–hole (ph) excitations.

More complex states are built up by displacements of two, three, etc. nucleons from filled to vacant shells. They are called the 2p2h, 3p3h, etc. states. In the region of a giant resonance in heavy nuclei ($\omega \sim 14 - 16$ MeV) the number of more complex states is much greater than the number of the ph states.

We shall now determine to what extent the investigated set of single-particle degrees of freedom i.e. the IPM, can account for the GDR. In the range of the GDR energies the interaction of γ photons with a nucleus is characterised by the following features:

— the interaction is of single-particle nature and, therefore, the γ photons excite directly only the ph states;

— the γ-photon wavelength is still considerably greater than the size of a nucleus and, therefore, the main singularities of the absorption curve should be governed by the absorption of the dipole γ photons;

— in the absorption of dipole γ photons the strongest transitions are those which occur between adjacent shells.

It therefore follows that in the IPM the dipole transitions are grouped mainly in the range of energies equal to the average separation between the adjacent shells. In the ^{208}Pb nucleus the separation is 7–8 MeV. This was indeed explained by Wilkinson [17]. Therefore, the GDR phenomenon does occur in the IPM. Since the separation between the shells varies smoothly with the mass number A (as $A^{-1/3}$), the GDR predicted by this model has the external criterion of its characteristic collective nature, i.e. the resonance position depends weakly on the mass number.

However, the GDR predicted by the IPM has an energy which for heavy nuclei is about twice the experimental energy. Therefore, this very serious discrepancy in nuclear physics has to be removed by finding a mechanism which does not negate the IPM, but shifts the GDR to much higher energies.

3.3 The initial steps towards the solution of this problem were suggested by a research group working in Moscow University [20] and the solution was obtained by Elliott and Flowers [21] and Brown and Bolsterli [22]. This solution can be summarised as follows. The now familiar hole interaction between nucleons cannot be reduced entirely to the average nuclear field. Some contribution to the nucleon–nucleon forces, traditionally called the residual contribution, comes from the 'scattering' of nucleons, i.e. it alters the state of their motion in the average field or, which is equivalent, mixes the various configurations identified by indices which in the IPM are good quantum numbers.

In general, identification of the ground and excited states subject to the residual interaction is equivalent to the exact solution of the many-body problem and is therefore practically unattainable. However, since in a rough approxima-tion the IPM provides a correct description of the main properties of a nucleus, it is possible to adopt a special method of successive inclusion of the residual interaction effects.

The first and most important link in this chain of approximations is the RPA mentioned earlier. This approximation appears in a more or less natural manner in the Green function methods [32], in approximate second quantisation [28], and in the time-dependent Hartree–Fock method [46]. The physical aspect of the RPA is that a particle and a hole are regarded as one complex, which is a particle–hole degree of freedom. Therefore, the only permissible processes are conversion of one particle–hole pair into another and simultaneous creation and absorption of two such complexes (Fig. 2). The subsequent refinements representing the quasiparticle RPA [26, 29], the second RPA (SRPA) [47], the extended second RPA (ESRPA) [48], invoke other more complex degrees of freedom. In particular, account is taken of the processes of conversion of a particle–hole complex into two particle–hole pairs, etc.

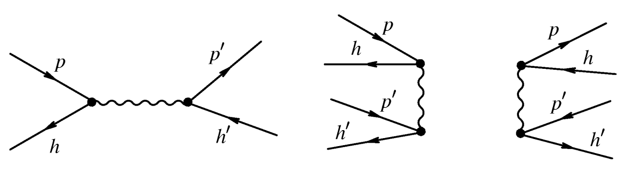

Figure 2. Particle–hole interactions included in the random phase approximation. The wavy line represents the pair interaction. A particle and a hole are identified by a line with an arrow.

Since in the IPM it is assumed that γ photons excite directly only the ph configurations, we can expect the RPA to describe correctly the gross structure of the absorption curve. The subsequent refinements therefore determine primarily the dissipation properties of the GDR, which will be discussed in Section 5.

3.4 The role of the residual interaction is demonstrated most clearly in a schematic model which makes it possible to obtain a purely analytic solution of the RPA equations [49]. In this model it is assumed that

(a) the energies of all the ph configurations taken into account are degenerate;

(b) the matrix of the particle–hole interactions i.e. the $\langle \mathrm{p'h'}|\hat{V}|\mathrm{ph}\rangle$ matrix, where $\hat{V}$ is the residual force operator, becomes factorised:

$$\langle \mathrm{p'h'}|\hat{V}|\mathrm{ph}\rangle \sim d_{\mathrm{p'h'}}\, d_{\mathrm{ph}}\,. \tag{15}$$

Here, d_{ph} are the amplitudes of the dipole particle–hole transitions. The solution of the RPA equations then gives the following results:

— all the levels, apart from one nontrivial, have the energies of the initial ph configurations and are not excited by the dipole operator;

— the nontrivial level, usually called the dipole level, is involved in all the dipole transitions and is displaced strongly in the upward direction along the energy scale. The dipole state is a coherent superposition of a large number of the ph configurations with approximately the same amplitudes. Therefore, its properties depend weakly on the details of the nuclear structure considered within the framework of the IPM.

It thus follows that the residual interaction considered within the framework of this schematic model gives rise to an excited state of the nucleus which corresponds to synchronous motion of a large number of nucleons. In other words, the residual interaction has the effect that the collective coordinate

$$\boldsymbol{R} = \frac{1}{Z}\sum_{i=1}^{Z}\boldsymbol{r}_i - \frac{1}{N}\sum_{i=Z+1}^{A}\boldsymbol{r}_i\,,$$

which represents the difference between the centres of masses of the protons and neutrons, becomes the normal coordinate of the nucleus. Since the dipole state is a superposition of a large number of the ph configurations, it takes away only a small part of the spectroscopic strength of each configuration. Therefore, the appearance of synchronous motion of nucleons does not destroy the shell structure.

In a real nucleus the particle–hole configurations are nondegenerate and the ph interaction is not factorisable. Nevertheless, the main effect of inclusion of the residual interaction, which is the shift of the GDR towards higher energies and setting up of collective motion, is retained. Fig. 3a [50] gives the results of a calculation of the intensities of the dipole transitions in the ^{208}Pb nucleus. It is evident from this figure that the dipole state predicted by the schematic model does not appear in its pure form: instead of one state, there are now several and they

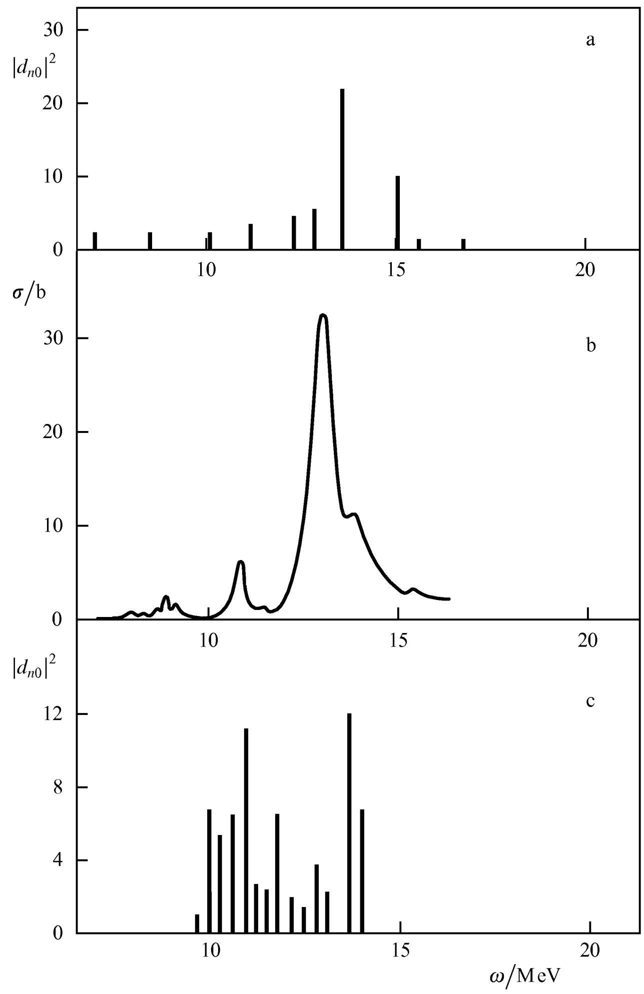

Figure 3. Absorption of γ photons in the ^{208}Pb nucleus. (a) Distribution of the squares of the matrix elements (in units of e^2 fm^2) of the dipole transition operator $|d_{n0}|^2$ [see expression (4)], considered in the random phase approximation [27]. (b) Curve representing the photoabsorption cross section (in barns) obtained in the particle–hole approximation with a continuum [37]. (c) Distribution of the squares of the matrix elements (in units of e^2 fm^2) of the dipole transition operator $|d_{n0}|^2$ obtained including the 2p2h configurations [50].

correspond to dipole transitions with nonzero intensities. In the collective model this result can be regarded as an indication that only the collective degrees of freedom have not been taken into account to a sufficient degree and that there is consequently a need to include single-particle degrees of freedom or to widen the set of collective variables.

However, a different point of view is also possible: the ph splitting of the GDR is the result of an approach in which one of the GDR splitting mechanisms (single-particle) due to the structure of the average field is included automatically in the RPA. In the relevant literature this is known, without sufficient justification, as the Landau damping [51]. Since there are other equally important splitting mechanisms, inclusion of just the Landau damping in the RPA results effectively in an excess over the precision of the approximation.

4. Explicit relationship between single-particle and collective degrees of freedom. Vlasov equations

4.1 The use of different complete sets of coordinates should, in the final analysis, give the same physical results. However, complications arise with each set of variables.

When a limited set of collective variables is used, it is necessary to 'guess', first, which of these variables are normal and, second, which of them are relevant to the physical phenomenon under discussion. In the case of the dipole resonance this was done by Migdal and soon after him by Goldhaber, Teller, and others.

It is in general quite difficult to work with single-particle degrees of freedom. However, in this case there is no need to decide anything in advance: diagonalisation of the energy matrix reveals all possible (in the adopted approximation) normal coordinates. However, when there is, as demonstrated above, a definite correspondence between the RPA and the collective theories of the GDR, a direct comparison of these two approaches is not a trivial matter. In fact, the collective model is usually understood to be its hydrodynamic variant, which is definitely in conflict with the main properties of the nucleus: the mean free path of nucleons in nuclear matter is considerably greater than the size of the nucleus.

A decisive breakthrough has been made recently in tackling this problem. It has been found that:
— the RPA equations considered in the semiclassical approximation can be written in the form of the macroscopic Vlasov equations for the function $n_{\mathrm{p}}(\boldsymbol{r}, t)$ representing the distribution of nucleons in a nucleus;
— if the distribution function $n_{\mathrm{p}}(\boldsymbol{r}, t)$ is known, there is no special difficulty in going over from single-particle degrees of freedom of the nucleus to the collective description;
— the equations which are then obtained for the collective variables are very close to the hydrodynamic equa-tions, but considered in the collisionless case.

This going over from single-particle degrees of freedom to the collective approach practically within the framework of the same physical framework is particularly valuable in an analysis of phenomena more complex than the GDR in nuclei at zero temperature. We have in mind here the processes which occur in collisions of heavy ions, the GDR in heated nuclei, and excitations in metallic clusters.

4.2 The transition to collective variables in the RPA approximation is simplest to carry out within the framework of the time-dependent Hartree–Fock method [46]. This method can be formulated in terms of the single-particle density matrix $\rho(\boldsymbol{r}, \boldsymbol{r}'; t)$, which satisfies the equation of motion

$$\mathrm{i}\,\frac{\partial\rho}{\partial t} = [h, \rho]\,. \tag{16}$$

Here, h is the single-particle Hartree–Fock Hamiltonian and

$$[h, \rho] = h\rho - \rho h. \tag{17}$$

The transition to the collective variables requires replacement of the density matrix ρ with its Wigner transform $n_p(\boldsymbol{R}; t)$ [52]:

$$n_p(\boldsymbol{R}; t) = \int \exp\left[-\mathrm{i}\frac{\boldsymbol{p}(\boldsymbol{r}-\boldsymbol{r}')}{\hbar}\right] \rho(\boldsymbol{r}, \boldsymbol{r}'; t)\;\mathrm{d}(\boldsymbol{r}-\boldsymbol{r}')\,, \tag{18}$$

where $\boldsymbol{R} = (\boldsymbol{r}+\boldsymbol{r}')/2$. The function $n_p(\boldsymbol{r};t)$ is interpreted as representing the momentum distribution of nucleons [40, 42]. For this distribution function, Eqn (16) becomes

$$\frac{\partial n_p(\boldsymbol{r};t)}{\partial t} = \frac{2}{\hbar}\sin\left(\frac{\hbar}{2}\right)\left[\vec{\nabla}_r^{(1)}\vec{\nabla}_p^{(2)} - \vec{\nabla}_p^{(1)}\vec{\nabla}_r^{(2)}\right] \times \varepsilon_p(\boldsymbol{r};t)\, n_p(\boldsymbol{r};t)\,. \tag{19}$$

Here, the indices (1) and (2) indicate that the derivatives operate on ε_p and n_p, respectively. The equation includes the Wigner transform of the single-particle Hamiltonian

$$\varepsilon_p(\boldsymbol{R};t) = \int \exp\left[-\mathrm{i}\frac{\boldsymbol{p}(\boldsymbol{r}-\boldsymbol{r}')}{\hbar}\right] h(\boldsymbol{r},\boldsymbol{r}';t)\,\mathrm{d}(\boldsymbol{r}-\boldsymbol{r}')\,, \tag{20}$$

which represents the local energy of a nucleon.

The approximation in which only the first term of the series expansion of the sine is retained, subject to the condition of locality of h, can be called the semiclassical Hartree–Fock approximation. In this approximation, Eqn (19) becomes

$$\left[\frac{\partial}{\partial t} + \vec{\nabla}_p\,\varepsilon_p(\boldsymbol{r};t)\,\vec{\nabla}_r - \vec{\nabla}_r\,\varepsilon_p(\boldsymbol{r};t)\,\vec{\nabla}_p\right] n_p(\boldsymbol{r};t) = 0\,. \tag{21}$$

The terms of higher order in $\hbar$ and the effects of nonlocality of h give rise to quantum corrections to the function $n_p(\boldsymbol{r};t)$, which is defined by Eqn (21).

Therefore, the quantum equations of the (16) type used in the RPA method and considered in the semiclassical approximation can be reduced to an equation which is identical in form with the classical Vlasov equation [53] for the distribution function of particles in a collisionless plasma.

The formulation of the RPA method in terms of the distribution function $n_p(\boldsymbol{r};t)$ allows us to go over easily to the collective variables. This is done by introduction of the moments of this function [40]. The simplest moment is the local density $\rho(\boldsymbol{r};t)$ of matter:

$$\rho(\boldsymbol{r};t) = M\int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\, n_p(\boldsymbol{r},t)\,, \tag{22}$$

where M is the mass of a nucleon.

The next in complexity are the moments which are identical with the local velocity $\boldsymbol{u}(\boldsymbol{r};t)$ and with the pressure tensor $P_{ij}(\boldsymbol{r},t)$:

$$\boldsymbol{u}(\boldsymbol{r};t) = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\,\frac{\boldsymbol{p}\, n_p(\boldsymbol{r};t)}{\rho(\boldsymbol{r};t)}\,, \tag{23}$$

$$P_{ij}(\boldsymbol{r};t) = \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi\hbar)^3}\,(P_i - Mu_i)(P_j - Mu_j)\, n_p(\boldsymbol{r};t)\,. \tag{24}$$

Eqn (21) can be rewritten in terms of these quantities in the form of the classical hydrodynamic equations:

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\rho\boldsymbol{u} = 0\,, \tag{25}$$

$$\frac{\partial u_i}{\partial t} + \frac{1}{M}\,\nabla w\, u_i = -\frac{1}{M\rho}\,\nabla_j P_{ij} \tag{26}$$

(w is the local average field) for a collisionless fluid.

If the tensor P_{ij} could have been expressed in terms of just ρ and $\boldsymbol{u}$, the result would have been a closed system of equations expressed directly in terms of collective variables. In reality, the equations for the tensor P_{ij} include higher moments of the distribution $n_p(\boldsymbol{r};t)$ and truncation of the resultant chain of equations occurs only under certain physical assumptions.

The result is a remarkable and instructive situation when the physics underlying the RPA approximation can be described equally successfully in terms of single-particle and collective variables. Details of the calculations of the characteristics of various excitations of nuclei, carried out starting from Eqn (17), can be found in the reviews mentioned earlier [27, 40–42].

5. Decay properties of the giant dipole resonance

5.1 In the preceding sections we have analysed the nature of the GDR from two alternative points of view. We shall consider the same points of view in dealing with the decay characteristics of the GDR. They include the nature of the absorption curve in the region of the GDR with all the details (fine and intermediate structure of the GDR) of the energy dependence of the cross section $\sigma_\gamma(\omega)$ and the energy spectra of the emitted nucleons.

The absorption curve is determined by the processes of spreading, dissipation, and escape to a continuous spectrum of the initial particle–hole and more complex states. The spreading and the dissipation are related concepts, but they represent different phenomena. In the course of dissipation an excited nucleus leaves the initial ph configuration and does not return to it. On the other hand, in the case of spreading the nucleus may return many times to its initial state before decay.

5.2 In the phenomenological collective model the spreading of the dipole excitation occurs mainly because of the coupling of a dipole degree of freedom to surface vibrations; dissipation is included as an additional parameter and determines the intrinsic width of the collective dipole level. However, such a natural approach to an analysis of the GDR has not found great favour (possibly because the lack of clarity in the relationship between the hydrodynamic and shell models) and the main effort has for a long time been concentrated on the approaches based on single-particle degrees of freedom.

5.3 In the RPA approach, the absorption curve of γ photons in the GDR region (and, consequently, the GDR width) is determined by the spreading of a dipole excitation mode between the ph configurations in the GDR region and by the width of the decay in various nucleon channels. The RPA with a continuum makes it possible to calculate the widths of nucleon decay of particle–hole states with the same conceptual degree of reliability as that involved in calculation of the GDR energy. Numerous calculations of this type have shown that the RPA photoabsorption curve differs greatly from that observed in the GDR region. For example, the decay width of a dipole level of ^{208}Pb is approximately an order of magnitude less than that found experimentally (Fig. 3b). This has led to the conclusion of the need to include the coupling between the ph configurations and more complex (particularly 2p2h) configurations.

A realistic calculation of the absorption curve of nonmagic nuclei is a very difficult task. Therefore, we shall limit ourselves to a brief discussion of the situation in our ideal ^{308}Pb nucleus. The usual method for the calculation of the absorption curve of such nuclei is based on the hypothesis of 'grey' 2p2h configurations for which the intensity of the interaction with the ph configurations is approximately the same. The situation is as follows. Let $G_S^{(0)}(\omega)$ be the exact particle–hole propagator. It is naturally diagonal in respect of the index S representing the exact particle–hole levels. If we include the ph $\to$ 2p2h coupling, the propagator $G_S^{(0)}(\omega)$ becomes modified to $G_{SS'}(\omega)$, which satisfies the Dyson equation whose symbolic form is as follows:

$$G(\omega) = G_0(\omega) + G_0(\omega)\Sigma(\omega)\,G(\omega) \tag{27a}$$

or

$$G(\omega) = \frac{1}{G_0^{-1}(\omega) - \Sigma(\omega)}, \tag{27b}$$

where $\Sigma(\omega)$ is the self-energy of the particle–hole states

$$\Sigma(\omega) = VG_2V\,; \tag{28}$$

V is the interaction responsible for the ph $\to$ 2p2h transition and G_2 is the exact propagator of the 2p2h states. The enormous number of the 2p2h states in the GDR region makes it impossible to take the interaction 2p2h space exactly into account. Therefore, G_2 is as a rule replaced with the 'zeroth' approximation, i.e. it is assumed that

$$G_2(\omega) = \frac{1}{\omega - \varepsilon_{2\text{p}2\text{h}} + \mathrm{i}\delta}, \tag{29}$$

where $\varepsilon_{2\text{p}2\text{h}}$ is the energy of the 2p2h states considered in the approximation of either two diagonalised ph states or of noninteracting particles and holes [26]. It is usual to assume that the phases of the $\langle 2\text{p}2\text{h}|V|\text{ph}\rangle$ amplitudes are chaotic so that the matrix $G_{SS'}(\omega)$ becomes diagonal. The cross section $\sigma_S(\omega)\gamma$, representing the absorption of a γ photon by the ph level S, averaged over the energy interval $\Delta\omega = I$, has the following energy dependence:

$$\sigma_S(\omega) \propto \operatorname{Im}\overline{G_S(\omega)} = \operatorname{Im} G\left(\omega + \mathrm{i}\,\frac{I}{2}\right)$$
$$= \frac{\Gamma_S}{(\omega - \varepsilon_S)^2 + (\Gamma_S/2)^2}\,, \tag{30}$$

where

$$\Gamma_S = 2\pi \sum_{2\text{p}2\text{h}} \frac{|\langle S|V|2\text{p}2\text{h}\rangle|^2 (I/2)}{(\omega - \varepsilon_{2\text{p}2\text{h}})^2 + (I/2)^2} \tag{31}$$

determines the width of the particle–hole level S, dependent on the energy ω. Ideas of this kind have led to the growth, in the last two decades, of a '2p2h industry' of calculations of the curve representing the absorption of γ photons by nuclei. By way of example, Fig. 3c shows the results of one of the recent calculations of this type carried out for the ^{208}Pb nucleus [50].

The overall agreement between the calculated and observed total absorption curves of γ photons shows that the mechanisms governing the decay properties of the GDR are on the whole now understood. In particular, the role of the 2p2h configurations has become clear. It must be stressed however that formula (30) is approximate because the propagator in the space of the 2p2h states is described by an approximate expression (29). The resultant error in the analysis of the GDR is at present very difficult to estimate. We shall simply note that the excitation spectrum in the GDR region, considered in this approximation, is discrete. This means that only the spreading and not the dissipation processes are taken into account.

5.4 For a more detailed understanding of the absorption curve, for example its fine structure, we must go beyond the approximation discussed above. From the fundamental point of view, it is clear which additional factors have to be taken into account: the continuum, as well as the 3p3h and more complex particle–hole states. However, if the volume of calculations is widened to take these factors into account, one meets not only technical but also various serious physical problems. They include above all the identification of the interactions of the configurations of the various subspaces, 2p2h, 3p3h, etc., as well as the problems of internal consistency of the calculations. An important breakthrough has been made recently: a self-consistent 'ph + phonon + continuum' method has been developed for inclusion of the additional 'ph + phonon' configurations in the RPA with a continuum [54, 55]. The term 'phonon' means here that one of the ph states in the subspace of the 2p2h configurations becomes collective. The results of a calculation of this type carried out for the ^{208}Pb nucleus are presented in Fig. 4. These results represent the current level of calculations of the absorption curves.

5.5 The recently established explicit relationship (see Section 3) between the IPM and the collective model may resuscitate the collective approach to an analysis of the GDR absorption curve described in Section 5.2. The dissipation parameter of the collective states needed in this approach can be found automatically by supplementing the collisionless Vlasov equation with a collision integral. This integral can be expressed directly [27] in terms of the amplitudes of transitions of the p $\to$ 2ph type, i.e. for

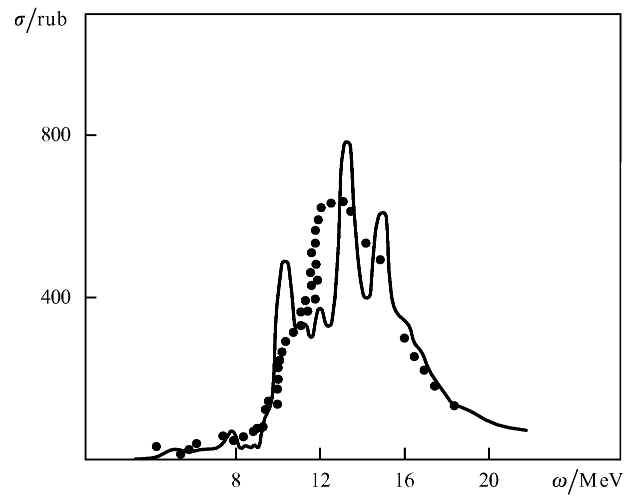

Figure 4. Curve representing the photoabsorption cross section (in millibarns) of the ^{208}Pb nucleus. The continuous curve gives the results of a calculation carried out in the 'RPA + phonon + continuum' approximation [55] and the dots give the experimental results.

example in terms of the amplitude of the particle–hole interaction in the Fermi liquid theory. It would be interesting and instructive to put this into practice and to compare the results obtained in this way with, for example, those plotted in Fig. 4.

5.6 An even more important decay characteristic of the GDR is the nature of the energy spectra of the escaping nucleons. The phenomenological collective model leads us to expect (this was pointed out long ago by Goldhaber and Teller [3]) that the energy spectra of nucleons should be statistical. The experimental results indicate that even in the case of heavy nuclei the number of high-energy nucleons is considerably greater than the number predicted by the statistical model. This effect can be understood again, as in the case of the width, if we turn back to the single-particle degrees of freedom. The question of nonstatistical nucleons was considered already by Wilkinson [17], who essentially predicted this effect by introducing the concept of 'direct resonant escape' of nucleons. From the modern point of view, this direct resonant escape is the first stage of precompound decay of the GDR [56] which occurs because γ photons excite directly primarily the particle–hole configurations that play the role of the 'incoming states' [57]. In view of the relatively weak interaction with the 2p2h configurations, a particle–hole GDR may emit 'superstatistical' nucleons up to complete dissipation. The number of such nucleons in heavy nuclei does not exceed 5% – 10%. Nucleon decay at each subsequent stage of thermalisation of the GDR approaches more and more closely the statistical predictions [58].

Identification of the nucleon decay channels can be used as a decisive test of the configurational nature of the GDR. The small number of nonstatistical nucleons of this type in heavy nuclei makes it difficult to carry out such a test. In the case of light and intermediate nuclei we find the opposite situation: the Wilkinson direct resonant decay of simple configurations, excited directly by γ photons, is in many cases the dominant mechanism. This has been used in a direct experimental confirmation of the existence of the configurational splitting of the GDR in the case of light nuclei [59].

6. Conclusions

6.1 We have considered the conceptual aspects of the evolution of our ideas on the nature of the GDR following the pioneering paper of A B Migdal and the experimental discovery of this resonance. This evolution was one of the central (or even the central) points in the establishment of the current views on the dynamics of nuclei. Initially, the theory of the GDR considered as an excitation of vibrations of the proton liquid, relative to the neutron liquid, has been based implicitly on the liquid drop model of the nucleus. However, an incontrovertible proof was soon obtained that, in the first approximation, a nucleus is a system of independently moving nucleons (independent-particle model). This has given rise to two extremely important problems:

— interpretation of the GDR on the basis of the independent-nucleon model. The main difficulty has been that the GDR energy has been strongly underestimated;

— compatibility of independent motion of nucleons with the liquid properties of nuclei, which at first sight would require that the mean free path of a nucleon in a nuclear medium should be small compared with the dimensions of the nucleus itself.

The first problem was solved without rejecting the hypothesis of independent motion of the bulk of nucleons. This was done by including the residual interaction between nucleons. It has been found that the interaction can induce synchronous small-amplitude vibrations of a large number of nucleons. There were two results of this approach: first, the experimental GDR energy was reproduced and, second, the collective coordinate of the difference between the centres of masses of protons and neutrons became a normal coordinate.

The second problem was resolved by recognising that the particle–hole equations in the RPA can be reduced, to a good approximation, to the Vlasov equations for a quantum analogue of the classical distribution function $n_{\mathrm{p}}(\boldsymbol{r}, t)$. The Vlasov equation for $n_{\mathrm{p}}(\boldsymbol{r}, t)$ can then be used to go over to equations for the collective variables such as the local density, velocity, pressure tensor, etc. The resultant equations describe a collisionless liquid and they are identical in form with the equations of classical hydrodynamics.

6.2 It is now clear that the collective description of a nucleus can be fully equivalent to the description obtained in the particle–hole RPA approach. However, a further refinement of the description of the GDR properties (its width, fine structure, energy spectra of nucleons, etc.) is possible only if the single-particle degrees of freedom are taken into account. Here, we have some inequivalence of the collective and shell models.

6.3 The nature and the limited space in this review have prevented us from discussing many other extremely important and interesting aspects of the dynamics of nuclei closely related to the GDR. We can only list some of them here.

A major role is played in the GDR by the symmetry effects in a nucleus. The most striking among these effects is probably the splitting of the GDR into two maxima observed for nonspherical but axially symmetric nuclei. In the collective model, this splitting is attributed to the difference between the radii along the two symmetry axes of the nucleus and in the microscopic approach the two maxima rise because of the considerable difference between the energies of the particle–hole configurations corresponding to longitudinal and transverse (relative to the symmetry axis of the average field) excitations and because these excitations are not miscible by the residual interaction. It should also be mentioned that we have ignored completely the single-particle aspects of the dynamics of the GDR in nonspherical nuclei.

A second effect of the symmetry of a nucleus is the isospin splitting of the GDR in nuclei with $N \neq Z$. This splitting is related to the conservation of isospin in a nucleus and to the fact that a γ photon may excite states of the isospin $T_< = (N - Z)/2$ of the ground state as well as the states with $T_> = (N - Z)/2 + 1$, which lie considerably higher on the energy scale.

Finally, the third effect of the symmetry and the characteristic features of the structure of a nucleus is the configurational splitting of the GDR in light (up to ^{40}Ca) nuclei. It arises from the approximate conservation of the 'Young scheme' quantum number and a strong dependence of the energy of the hole levels (and, consequently, of the potential) on the nucleon configuration.

6.4 The states of the nuclei responsible for the GDR may be excited not only by photons. Extensive studies are being conducted at present in which a nucleus is probed with virtual photons, pions, nucleons, etc. [60]. The whole enormous set of data on the probing of the GDR by a variety of beams leaves no doubt that the main features of the nature of the GDR are now understood correctly and this applies also to the dissipation and decay (escape) processes.

This circumstance however leads almost automatically to the prediction (and the existence!) of a large number of other giant resonances (monopole, electric quadrupole and octupole, magnetic dipole, Gamow–Teller) and also the GDR of excited states [61].

6.5 The collective model, which goes back to A B Migdal, has become very popular not only in dealing with giant resonances, but also with fission processes in nuclei. A classical description of this process goes back to the early work of Bohr and Wheeler. The next important step was made by going over to collective coordinates both on the basis of the Nilsson–Strutinsky scheme, put into practice by a group of physicists at Los Alamos (Nix et al. [62]), and in Frankfurt (Greiner et al. [63]), as well as on the basis of the time-dependent Hartree–Fock method (Cogny [64]). In particular, the last group of authors have made considerable progress and revealed fine details of the nuclear structure such as the existence of a second minimum of the potential energy.

6.6 A new branch of physics in which collective dynamics should give important results is growing at present. We have in mind here metallic clusters, condensed from a supercold metal vapour and containing several hundreds of atoms. Qutie recently, a study has been made of the absorption of photons by such systems and a giant resonance has been discovered.

6.7 Naturally, A B Migdal could not foresee 50 years ago the natural evolution of the ideas set out in his first paper on the GDR. However, since he made the necessary statements, this has opened up new and extremely fruitful directions for evolution of nuclear physics and several related fields, which is undoubtedly the historical contribution made by Arkadii Beinusovich Migdal.

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