Excitation of an electric instability by heating

E D Eidel'man

Contents

1. Introduction	1231
2. Excitation of cellular motion	1233
3. Surface waves	1236
4. Advective flows	1237
5. Film thickness	1238
6. Thin film of a liquid semiconductor (semimetal) with arbitrary heating from above	1239
7. Excitation of an electric instability by heating in other liquid media	1240
7.1 Electrolytes; 7.2 Liquid crystals; 7.3 Insulators	
8. Comparison with experiments	1242
8.1 Benard's experiments; 8.2 Laser heating experiments; 8.3 Mechanism of formation of a molten zone	
9. Concluding comments: synergetic aspects	1244
References	1244

Abstract. A theory is proposed of the mechanism of the excitation of free thermal convection, which is activated by an electric force and creates an electric instability. This thermoelectric mechanism can account for the excitation of cellular motion when a medium is heated from above by, for example, laser radiation.

1. Introduction

The problem of the excitation of thermal convection by the buoyancy force has been investigated thoroughly. These investigations were made by Benard, Rayleigh, Jeffreys, Chandrasekhar, Ostroumov, Gershuni, Zhukhovitskii, and many others (see Refs [1-4] and the literature cited there). Methods have been developed for the study of this excitation mechanism. In the field of the force due to gravity ρg (ρ is the density of a liquid or a gas and g is the free-fall acceleration), a liquid is in mechanical equilibrium if the force due to gravity is balanced out by an excess pressure $\rho g = \nabla p$ (p is the pressure). There may be no macroscopic motion in a medium of this kind even if the temperature T varies along the liquid. It is, however, clear that such equilibrium may become unstable as a result of an increase in its temperature $A = |\nabla T|$ by heating. Equilibrium may be unstable only when a certain (excitation) condition is satisfied. Then disordered fluctu-

E D Eidel'man St Petersburg Chemical-Pharmaceutical Institute, ul. prof. Popova 14, 197376 St Petersburg Tel. (7-812)2341523. E-mail: gromov@relat.spb.su

Received 23 December 1994; revision received 20 March 1995 Uspekhi Fizicheskikh Nauk **165** (11) 1249–1294 (1995) Translated by A Tybulewicz ation flows appear and grow in a liquid, leading to mixing and establishment of a different temperature distribution. This motion is called convection.

Let us imagine that an element of a liquid accidentally has a fluctuation velocity v. Then the motion of this element creates a heat flux $\rho C_p vA$ (C_p is the specific heat of the liquid at constant pressure). This heat flux creates a fluctuation T_1 , which is a small deviation of temperature from its value at the same place in an immobile medium. This flux should be balanced out by a flux associated with heat conduction: $\rho C_p \varkappa \Delta T_1$ (\varkappa is the thermal diffusivity and $\rho C_p \varkappa$ is the thermal conductivity). Therefore, if the characteristic dimension of such fluctuation motion is h, a small temperature deviation $T_1 \approx vA h^2/\varkappa$ appears.

On the other hand, the heating of a liquid results in its thermal expansion described by the coefficient $\beta = -(1/\rho) \partial \rho / \partial T$. If a heated liquid element is displaced, it reaches a region with a different temperature and, consequently, an excess Archimedean (buoyancy) force $\rho\beta T_1 g$ appears; it represents the excess of the Archimedean force, acting on a heated (by T_1) element of the liquid, over and above the force due to gravity. Naturally, since $\beta > 0$, such an excess is in principle possible only if the cold layers are above the hotter layers, i.e. if the heating is from below. In fact, if a liquid is heated from above, the buoyancy force returns a more strongly heated element back to the heated region, i.e. equilibrium should always be stable. Therefore, the excitation of convection by the buoyancy force is possible only as a result of heating from below. The buoyancy force is frequently also called the lifting force.

Equilibrium is disturbed when the force (or its density) $\rho\beta T_1 g$ exceeds the viscosity force (or its density) $\rho v\Delta v$, where v is the kinematic viscosity. A comparison of these forces demonstrates that we need a heating effect A to ensure that the condition $\rho\beta gAh^4 > \rho v\varkappa$ is obeyed. This is

the necessary condition for free convection under the action of the buoyancy force (Boussinesq condition). In reality, the buoyancy force should exceed the dissipative forces by a large factor.

Conditions sufficient for the appearance of instabilities (the growth of small perturbations) can be found most simply by searching for nontrivial solutions of systems of homogeneous differential equations linearised in terms of small perturbations and subject to uniform boundary conditions. The existence of a range of the values of the parameters for which such solutions exist implies that a spontaneous transition of a medium to a new state is possible. If these values are physically permissible, this means that an instability can appear. This approach has been used very successfully in analyses of thermal convection and also in finding the conditions for the existence of nontrivial solutions of a linearised system of the equations of motion, continuity, and heat transfer in an incompressible liquid heated in the gravity field:

$$\frac{\partial v}{\partial t} - v\Delta v + \frac{1}{\rho_0} \nabla p + \beta T_1 g = 0, \qquad (1)$$

$$\operatorname{div} v = 0, \qquad (2)$$

$$\frac{\partial T_1}{\partial t} - \varkappa \, \Delta T_1 + (\nu \cdot \nabla) \, T_0 = 0. \tag{3}$$

As before, v is the velocity of a liquid which is at rest under equilibrium conditions; $p_1 = p - p_0$, $T_1 = T - T_0$, $\rho_1 = \rho - \rho_0$ are the deviations of the pressure, temperature, and density from their equilibrium values; v, \varkappa , g, β are all defined above.

The problem of convection is solved for a specific region. It is usual to employ a model of a layer or film which is infinite along two directions (x and y) and of thickness h along the third direction. Uniform boundary conditions are imposed on the boundaries of this layer.

In the usual Boussinesq approximation, the system of equations (1)-(3) has constant coefficients and can be transformed into a system of algebraic equations if the solution is sought in the form of a Fourier expansion with its harmonics proportional to

$$\exp(i\omega t) \exp(ik_x x + ik_y y + ik_z z).$$
(4)

The solution gives the dependences of the external parameters — such as, for example, heating governed by the temperatures of the 'hot' $T_{\rm h}$ and 'cold' $T_{\rm c}$ boundary surfaces of the layer: $A = (T_{\rm h} - T_{\rm c})/h$ —on the wave vector $k^2 = k_{\perp}^2 + k_z^2$, $k_{\perp}^2 = k_x^2 + k_y^2$ on condition that the imaginary part of the frequency $\omega = \omega + i\omega''$ vanishes.

The result is the following characteristic equation:

$$-k^{2}(k^{2} - \mathrm{i}\,\omega)(k^{2} - \mathrm{i}\,\omega\,\mathrm{Pr}) \mp \mathrm{Ra}\,k_{\perp}^{2} = 0.$$
(5)

All the quantities in the above equation are dimensionless. The units of length and time are *h* (layer thickness) and h^2/ν . The minus sign in the above equation corresponds to heating from above and the plus sign represents heating from below. The parameter $Pr = \nu/\varkappa$ is the Prandtl number.

The heating term includes the Rayleigh number

$$Ra = \frac{\rho \beta g A h^4}{\rho \nu \varkappa},\tag{6}$$

which governs the range of existence of nontrivial solutions with a growth increment greater than zero. The Rayleigh number defines the ratio of the lifting force (the force creating the instability) to the dissipative forces.

If we satisfy the conditions of uniformity of the boundary conditions, we obtain the dispersion equation. Let us say that there are boundary conditions (at 'free and isothermal' boundaries) such that their adoption makes it possible to find the value $k_z = \pi$ convenient for a qualitative analysis of the resultant effects. The double eigenvalue problem is then solved. We shall complete this account of methodological procedures by pointing out that the symmetry of the resultant cell (the translation symmetry along the layer) is determined by minimisation of the Rayleigh number Ra as a function of $w = k_z^2/k_\perp^2$.

In the case of the usual convection, the excitation condition is

$$\operatorname{Ra} > \operatorname{Ra}^* = \frac{27\pi^4}{4} \approx 660 \text{ for } w = \frac{1}{2}.$$
 (7)

An instability appears only if a liquid is heated from below $(\beta > 0)$. The growth of this instability is aperiodic.

A numerical solution is needed for other types of boundary conditions. The threshold value of the Rayleigh number Ra^{*} then increases, but the excitation is not forbidden. The qualitative conditions for the appearance of an instability are retained.

The excitation of cellular motion by the thermocapillary effect was tackled later (see Refs [5-7] and the review in Ref. [2]). This effect originates from the temperature dependence of the surface tension. This mechanism is characterised by the Marangoni number

$$M = \frac{\sigma A h^2}{\rho v \varkappa} \,. \tag{8}$$

Here σ is the temperature coefficient of the surface tension. The physical meaning of M is exactly the same as that of the Rayleigh number. It gives the factor by which the force creating an instability, i.e. the surface tension force, exceeds the dissipative force.

Let us imagine that an element of a liquid floats up to its surface. It is acted upon by an excess (thermocapillary) force (or its density) $\sigma\Delta T_1$. Naturally, if the surface is at a higher temperature than the temperature of this element, then this force tends to restore its position (since $\sigma > 0$) and equilibrium is always stable. If the surface of the liquid is colder, this force results in an instability if it can overcome the viscosity force.

A comparison of $\sigma \Delta T_1$ with $\rho v \Delta v$ shows that we need a heating effect A to ensure that M > 1. This necessary condition is an analogue of the Boussinesq condition for the excitation by the buoyancy force. It should be stressed that the mechanism of the appearance of an instability under the action of the thermocapillary force does not work when a free surface is heated (this force can only act on a free surface).

Let us now formulate the sufficient conditions for such excitation.

The thermocapillary force acts on a free (but not isothermal) surface. Therefore, the relevant boundaryvalue problem has to be solved numerically. The excitation threshold is given by the criterion

$$M > M^* \approx 80 \quad \text{for} \quad w \approx 0.5, \tag{9}$$

if the boundary z = h is free and thermally insulated and the lower boundary (at z = 0) is an interface with a solid. The excitation is again possible only on heating (the solid) from below ($\sigma > 0$) and the excitation grows aperiodically.

A comparison of the mechanisms of excitation by the buoyancy and thermocapillary forces shows that the thermocapillary mechanism predominates in thin layers of thickness

$$h < h_{\mathrm{Ra,M}} = \left(\frac{\sigma}{\rho g \beta}\right)^{1/2}.$$
 (10)

2. Excitation of cellular motion

2.1 The main subject of this review is the thermoelectric excitation mechanism which is governed by the dimensionless number [8]

$$\mathbf{E} = \mathbf{I}^2 = \frac{\varepsilon \gamma^2 A^2 h^2}{\rho v \varkappa} \tag{11}$$

Here, ε is the permittivity and γ is the Seebeck coefficient. The physical meaning of this number resembles the meaning of the Rayleigh Ra and Marangoni M numbers defined by expressions (6) and (8), respectively. The number E is the ratio of the electric force induced by heating to the dissipative force. Like the buoyancy and thermocapillary mechanisms discussed above, the thermoelectric mechanism can result in an instability of a liquid and can induce cellular motion. The excitation is possible also as a result of heating from above and the appearance of cells is accompanied by the formation of electric field structures.

Let us assume that a temperature fluctuation T_1 appears in an inhomogeneously heated liquid. Then, if the thermoelectric effect can appear, an electric field $\gamma \nabla T_1$ and an electric charge $\varepsilon \gamma \Delta T_1$ are induced. The thermoelectric field, due to the heating nonuniformity γA , acts on this fluctuation-induced charge and creates a force $\varepsilon \gamma^2 A \Delta T_1$. This bulk force can set a liquid in motion if it exceeds the dissipative force $\rho \nu \varkappa \Delta T_1/A$. The ratio of the thermoelectric and dissipative forces is the number E.

The thermoelectric and buoyancy forces are of bulk nature. Qualitative ideas and an exact solution of the problem of excitation in a liquid layer with free isothermal boundaries are given for liquid semiconductors in Ref. [9] with buoyancy, Rayleigh, and thermoelectric excitation mechanisms all taken into account.

When the approximations usual in dealing with electroconvection are adopted, the system of equations (1)-(3)should be supplemented by the equations of continuity for the current and by the electrostatic equation:

div
$$(E_1 - \gamma \nabla T_1) = 0$$
; div $E_1 = \frac{en_1}{\varepsilon}$; curl $E_1 = 0$. (12)

Here, E is the electric field intensity and n is the density of carriers whose charge is e. An electric force $en_1\gamma\nabla T_0/\rho_0$ now appears in Eqn (1).

It has been shown that convection occurs aperiodically and the solution of the double eigenvalue problem leads to the excitation conditions:

$$-k^{6} \mp \operatorname{Ra} k_{\perp}^{2} + \operatorname{E} k^{2} k_{\perp}^{2} = 0.$$
 (13)

In the case of the purely thermoelectric excitation mechanism, an instability appears if

$$E > E^* = 4\pi^2 \approx 40$$
, $I > I^* = 2\pi \approx 6.3$ for $w = 1$. (14)

The excitation is possible for any direction of heating, includ-ing heating from above. This can be understood

qualitatively in the sense that a temperature fluctuation T_1 induces a fluctuation of the thermoelectric field $E_1 = \gamma \nabla T_1$, which leads to the appearance of a space charge en_1 . In an 'external' thermoelectric field γA such a charge experiences a force $en_1\gamma A$. This force can set the liquid in motion if it exceeds the dissipative force by the necessary factor E^{*}.

The conditions for the excitation by the lifting (Rayleigh) and thermoelectric mechanisms are compared in Refs [8, 9] on the basis of relation (13). The thermoelectric mechanism predominates in thin layers of thickness

$$h < h_{\mathrm{Ra,I}} = \left(\frac{\varkappa v \gamma^2 \varepsilon}{\rho \beta^2 g^2}\right)^{1/6}.$$
 (15)

The influence of the Rayleigh and thermoelectric mechanisms on one another is also considered in Refs [8, 9]. When liquid is heated from below, the two mechanisms reinforce one another, which leads to the possibility of pre-Rayleigh convection in liquid semiconductors. When heating is from above, the Rayleigh mechanism balances out the excitation by the thermoelectric mechanism, but the excitation is still possible in layers of thickness less than 10 μ m.

The thermoelectric convection can be described by a theory similar to that developed for the Rayleigh convection mechanism (see, for example, Ref. [1]).

Problems with the boundary conditions close to those encountered in experiments, but in the absence of the thermocapillary effect, are solved numerically in Ref. [10]. It is shown there that conditions lead to some increase of the value of E^* (to about 43) without altering the symmetry of the excited cell.

Amplitudes of the resultant convection and structures are calculated in Ref. [11] for a small excess of the actual value of E above the threshold E^* . It is shown there that the amplitudes obey $V \propto (I - I^*)^{1/2}$. When the layer thickness is reduced, the excitation mechanism changes and this results in abrupt changes in w and V. These abrupt changes represent a second-order phase transition under 'soft' excitation conditions.

The possibility of excitation of an instability in the presence of the Rayleigh and thermoelectric mechanisms, of rotation, and possibly also of an external magnetic field, is considered in Ref. [12]. The relationships are found here between four dimensionless numbers: Ra, E, the Taylor number τ , representing the action of the Coriolis force, and the Hartmann number H, representing the action of the Ampere force, as in the case of the usual Rayleigh convection. Under these conditions, branches with oscillatory growth may appear. The action of rotation and/or of a magnetic field increases the necessary threshold values of Ra^{*} and E^{*}, but it cannot 'forbid' such excitation.

Calculations are reported in Refs [13, 14] of the influence of the thermocapillary excitation mechanism on the thermoelectric mechanism, which predominates in layers of thick-ness

$$h < h_{\rm M,I} = h_{\rm c} = \left(\frac{\rho \varkappa v \varepsilon \gamma^2}{\sigma^2}\right)^{1/2},\tag{16}$$

where h_c is the thickness of the layer in which the effect of the thermocapillary mechanism is equivalent to that of the thermoelectric mechanism.

The influence, on the excitation, of the conditons of heat transfer from a free surface, which are characterised by the Biot number Bi, are also investigated in Refs [13, 14].

Numerical calculations are carried out by assuming the most favourable conditions for the thermocapillary excitation mechanism, i.e. when the free surface is thermally insulated (Bi = 0). The results obtained show that heating from below can result in pre-Marangoni convection and heating from above ensures that the thermocapillary mechanism balances out the excitation by the thermoelectric mechanism, but cannot suppress the latter when a layer is up to 100 μ m thick. This result is similar to the effect of the interaction between the Rayleigh and thermoelectric mechanisms, i.e. a second-order phase transition takes place.

2.2 The results of the calculations relating to the excitation of cellular motion in the presence of the thermoelectric force are given in Figs 1-8.



Figure 1. Liquid film with thermoelectric properties heated from below (a) and from above (b): (1) lifting (buoyancy) force, representing the difference between the Archimedean force and that due to gravity; (2) surface tension (thermocapillary) force; (3) thermoelectric (electrostatic) force. It is obvious that heating from above can induce an instability only because of the thermoelectric force.

Fig. 1 shows a liquid film acted upon by all three forces discussed above. We can see that when this film is heated from above, an instability induced by the buoyancy force cannot appear. This force can set the film in motion only when it is heated from below. We can also see that heating of a free surface cannot induce an instability under the action of the surface tension forces. An instability can be created by the thermoelectric forces when the liquid is heated from below and from above, from the free-surface side and from the side of the solid substrate on which the film is located.

Fig. 2 gives the dependences on the reciprocal of the film thickness 1/h. We can see that heating from the substrate side (from below) results in mutual enhancement of all the three forces under consideration. They all tend to create an instability so that convection appears for smaller temperature differences $T_{\rm h} - T_{\rm c}$, compared with the differences necessary for the excitation by one of these forces or even by two of them.

Fig. 3 shows that the only force capable of exciting an instability is thermoelectric when heating is from the freesurface side (from above). The surface tension and buoyancy forces suppress the action of the electric force. Excitation is possible only in the thinnest layers, since the thermoelectric force is independent of the liquid layer thickness, whereas the buoyancy and thermocapillary forces decrease with reduction in the layer thickness.



Figure 2. Illustration of the action of thermoelectric forces (heating from below), which facilitate excitation of cellular motion by the thermo-capillary forces (Marangoni cells): (1) dependence of the Marangoni number M_1^* needed for the excitation and representing the combined effect of the forces when the thermocapillary effect predominates; (2) changes in the limiting cell size l_1^* that appear as a result of such excitation. (Similar dependences are obtained also on excitation of the Rayleigh convection.)



Figure 3. Competition between the excitation of the thermoelectric convection and its suppression by the surface tension forces (heating from above): (1) dependence of the number I_M^* , representing the combined effect of the two forces with predominance of the thermoelectric effect; (2) changes in the longitudinal dimension l_M^* of a cell. (Similar dependences apply also to the combined effect of the buoyancy and thermoelectric forces.)

The motion excited by the thermoelectric force in a convection cell is demonstrated in Fig. 4. Cells of this kind are excited also by the other forces considered here. However, the thermoelectric force gives rise also to electric field structures. Naturally, electric fields are not excited by the buoyancy or thermocapillary forces.

Neutral stability curves are plotted in Figs 5-7. The neutral stability curve established under the action of the buoyancy and thermocapillary forces is well known (see, for example, Ref. [2]), but our figures give the stability curves for all possible combinations of these forces.

The most important feature of curves on the neutral stability surfaces is the presence of branches corresponding to heating from above (Fig. 5) and to heating of the free surface (Fig. 6).

In Fig. 6 the quantity Bi is the Biot number representing heat transfer from the free boundary at the top. If Bi = 0, this boundary is thermally insulated. The lower boundary is



Figure 4. Convection excitation cell when the thermoelectric force predominates: (a) lines of flow of the current; (b) lines of the electric field intensity (l and h are the cell dimensions). The film boundaries are free and isothermal.



Figure 5. Neutral stability curves for the case when the buoyancy and thermoelectric forces act on a liquid film heated from below (1) and from above (2). The film boundaries are free and isothermal. The nature of these dependences is also qualitatively similar for other types of the boundary conditions.

the interface with solid and is isothermal. The dashed line in Fig. 6 shows how we can find the values of M and I for a specific liquid and a given layer thickness h, characterised by a slope angle φ , such that $\tan \varphi = h_c/h$, where h_c is defined by expression (16).

In Fig. 7 the lower boundary is solid and isothermal; the upper boundary is free and thermally insulated. Under other conditions of heat transfer from the upper boundary of a film or layer, the shape of the neutral stability surface remains basically the same, but it passes now below the surface shown in Fig. 7.

The increase in the numbers M and I necessary for such excitation is evidence of the influence of suppression by the nonelectric forces. The same effect is demonstrated also in Fig. 7 under the action of all three forces.



Figure 6. Neutral stability curves under conditions close to those encountered in the experiments involving heating from the solid (1) and from the free surface (2). A film is acted upon by the surface tension (thermocapillary forces) and thermoelectric forces.



Figure 7. Neutral stability surfaces expected when the buoyancy, surface tension, and thermoelectric forces act simultaneously on a film. One surface (1) near the origin of the coordinate system corresponds to heating from below and the other surface (2), rising upwards, corresponds to heating from above. The surfaces are plotted for the conditions closest to those found in experiments.

It is known [2-4] that the ratio of the longitudinal and transverse dimensions of a cell is the most reliable indicator or characteristic of that excitation mechanism which sets the investigated medium in motion. Changes in the boundary conditions have practically no effect on this ratio (see Ref. [1] for the buoyancy forces, Ref. [2] for



Figure 8. Abrupt change in the longitudinal dimension of a convection cell near the points where the excitation mechanism changes.

the thermocapillary forces, and Ref. [10] for the thermoelectric forces). A change in the main excitation mechanism alters greatly this ratio. Such a change occurs when the film or layer thickness is varied and it represents a second-order phase transition. The relevant results are plotted in Fig. 8.

3. Surface waves

In addition to the excitation of cellular motion and advective flow, the mechanisms based on the action of the lifting, thermocapillary, and electric forces lead also to the excitation of waves on the surface of a liquid layer. Gravity and capillary waves are discussed in detail in Refs [15, 16] and the influence of the thermocapillary effect is considered in Ref. [17]. Much work has appeared recently (for a review see Ref. [18]) on the formation of waves and other structures on the surfaces subjected to laser radiation.

It is under the conditions of laser irradiation that it is important to allow also for the thermoelectric effect when considering the formation of surface structures of size in excess of 10^{-3} cm [19]. In addition to the Rayleigh and thermocapillary excitation mechanisms, the thermoelectric mechanism can also act then. Therefore, in addition to gravity and capillary surface waves, there may be surface waves excited by the thermoelectric mechanism. Calculations show that an analytic form of the dispersion equation can be derived in the absence of dissipation.

The main distinction of the excitation of surface waves is the replacement of the condition representing the absence of outflow, $v_z = 0$, by the condition of the absence of a pressure jump. This condition, together with all the other factors, can be written as follows:

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\alpha}{\rho} \,\Delta_\perp v_z - g v_z - 2v \,\frac{\partial^2 v_z}{\partial t \,\partial z} \mp \frac{\varepsilon \gamma^2 A}{\rho} \,\frac{\partial^2 T}{\partial t \,\partial z} \,. \tag{17}$$

Here, ψ is the scalar velocity potential, α is the surface tension, and $\Delta_{\perp} = \partial^2/\partial x^2 + \partial^2/\partial y^2$. However, in general, the velocity is expressed in terms not only of the scalar, but also of the vector potential. This makes it possible to take nonpotential flows into account. The rule of signs and the rest of the notation are the same as those used above.



Figure 9. Typical dispersion relationship for surface waves excited by heating from above. It takes account of the main contribution from the thermoelectric effect.

Other boundary conditions retain their previous meaning. The condition describing the influence of the capillary effect,

$$\rho v \frac{\partial}{\partial t} \left(\frac{\partial^2}{\partial z^2} - \Delta_{\perp} \right) v v_z = -\sigma \left(\frac{\partial}{\partial z} \Delta_{\perp} T_1 \pm A \Delta v_z \right), \quad (18)$$

can be simplified by ignoring the thermoelectric mechanism (this is demonstrated by an estimate given in Ref. [20]).

We can now solve the problem formulated in Section 2 on the assumption that the upper surface of a liquid is displaced by ζ and that $v_z = \partial \zeta / \partial t$. The results of a numerical solution are shown in Fig. 9; these results are obtained from Eqns (1)-(3) and (12), which include the nonlinear terms.

Qualitative results, which account fully for the profile of the curve in Fig. 9, can be obtained simply by considering the conditions for the excitation of waves in the absence of the dissipative forces. We can then assume that $v_z = \partial \psi / \partial z$, i.e. we can assume that the wave motion is of the potential type. Moreover, $\zeta \ll h$, i.e. the investigated liquid can be regarded as 'deep'. Then the conditions at the bottom do not affect the motion on the surface. Finally, the thermoelectric effect can be isolated by considering the waves formed on an isothermal surface, because the thermocapillary effect does not appear on such a surface.

This model can be used to find the solution in the form given by expression (4) where ω and k_x are real and $k_z = -ik$ is purely imaginary. As in all cases of surface waves [15, 16], it is found that $k_x = k$. We can now use relationship (17) to find that

$$\omega^2 = \frac{\alpha}{\rho} k^3 + gk - k^2 \frac{\varepsilon \gamma^2 A^2}{\rho}$$
(19)

is valid irrespective of the direction of heating. This dispersion equation contains not only the familiar capillary (first term) and gravity (second term) terms, but also the new thermoelectric term which is a quadratic function of the wave vector.

The thermoelectric effect has to be taken into account also when internal waves in a liquid are considered [1, 2]. In fact, the instability considered in Section 2 applies to internal waves which appear inside the liquid, but decay before reaching the threshold heating value. The method used for finding the conditions inducing growth of internal waves can be used to obtain qualitative results and qualitative answers as to whether an instability is possible or impossible under given conditions.

Since the competition between the excitation mechanisms is important in thin films or layers, it is necessary to compare the conditions for the excitation of surface and internal waves. There may be situations in which the thermal properties of a film are determined specifically by the propagation of surface waves [17]. However, we can see that this is important only for the surface structures with dimensions of the order of nanometres [18]. A comparison of the possibility of excitation of convective motion and surface waves under conditions close to those encountered in experiments will be made later (see Section 8.2).

4. Advective flows

The same mechanisms which excite cellular motion can also excite advective flows. As pointed out above (Section 2) cellular motion appears as a result of heating across a layer when the temperature rise exceeds a certain critical value. For example, advective flows appear in the middle part of a wide rectangular cell with a flat horizontal bottom. If one vertical wall of the cell is hot and the opposite one is cold, equilibrium cannot be established in the liquid contained by the cell and motion occurs no matter how small the temperature difference. Such flows have been studied by Levich, Davies, Birikh, and others (see, for example, Refs [2, 21-24] and the literature cited there).

In calculations of advective flows the temperature gradient at the boundaries along a layer can be regarded as constant and the motion of the liquid is parallel to the bottom of the cell. The system of equations is the same as in Section 2, but naturally they are now nonlinear.

The projections of the equation of motion along x and z are used to exclude the pressure from them and the field is assumed to be purely thermoelectric. The result is the equation

$$\frac{\partial^3 v_x}{\partial z^3} - \frac{\beta g}{\nu} \frac{\partial T}{\partial x} = -\frac{\varepsilon \gamma^2}{\rho \nu \varkappa} \frac{\partial v_x}{\partial z} \left(\frac{\partial T}{\partial x}\right)^2 \tag{20}$$

which has to be solved simultaneously with the heat transport equation

$$v_x \frac{\partial T}{\partial x} - \varkappa \left(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial x^2} \right) = 0.$$
(21)

The continuity equation gives the relationship between v_x and v_z . The dependence on the second longitudinal coordinate is omitted in order to simplify the problem, which can then be solved exactly [25].

Naturally, for completeness, it is necessary to supplement the system of equations (20) and (21) with the boundary conditions. At the bottom of the cell the boundary is always assumed to be solid. The other boundary can be solid or free. The thermocapillary effects appear on a free boundary. Moreover, in solving the problem of a liquid film it is convenient to use the condition that the flow is closed.

We shall retain the symbol A for the temperature gradient at the boundaries along the liquid, so that T = Ax, and we shall retain also the forms of expressions (6), (8), and (11) for the dimensionless numbers Ra, M, and E.

The exact solution of the nonlinear problem which takes account of all three possible mechanisms shows that, in a region where the motion across the layer can be ignored, flow is possible only in the presence of the Archimedean or thermocapillary forces. Thus, in the simplest case we have

$$v_{x} = \frac{\varkappa}{h} \frac{\text{Ra}}{2l^{2}} \left\{ 2\xi - 1 + \frac{\cos\left(l\xi\right) - \cos\left[l(1-\xi)\right]}{1 - \cos l} \right\}.$$
 (22)

Here, $\xi = z/h$ is the dimensionless coordinate across the layer. It is evident that the thermoelectric mechanism alters the flow in such a way that it grows abruptly in a 'resonant' manner under the same conditions (represented by $I_* = 2\pi$) as those necessary for the excitation of motion when heating is across the layer. This remains true for all types of boundary conditions. The solutions are cumbersome. The results of numerical investigations are presented in Figs 10-12.

The curves in Fig. 10 represent the velocity v_x in units of \varkappa/h (curve a), the transverse thermoelectric field E_z in units of γA (curve b), and the component F_z of the force in the bulk of the liquid acting across the layer, in units of $\varepsilon \gamma^2 A^2/h$ (curve c). All these curves are plotted for the point close to the loss of stability I < I_{*}. The coefficients of proportionality are $I_*^2(I_* - I)/Ra$, $I_*^3(I_* - I)/Ra$, and $I_*^5(I_* - I)^2/Ra$, respectively.

The corresponding coefficients of proportionality in Fig. 11 are $I_*(I_* - I)/M$, $I_*^3(I_* - I)/M$, $I_*^5(I_* - I)^2/M^2$, respectively ($I_* \approx 4.5$).



Figure 10. Advective flow characteristics when both boundaries are solid ($I_* \approx 6.3$).



Figure 11. Advective flow characteristics when the $\xi = 1$ boundary is free and $\xi = 0$ is solid. The film thickness is such that the lifting (buoyancy) force can be ignored compared with the thermocapillary force.



Figure 12. Same as in Figure 10, but for a thick film so that the thermocapillary force can be neglected compared with the lifting (buoyancy) force.

The coefficients of proportionality in Fig. 12 are $I_*^3(I_* - I)/R$, $I_*^5(I_* - I)/Ra$, $I_*^8(I_* - I)^2/Ra^2$ respectively.

It is known [4] that advective flows are stable in the absence of electric forces. The thermoelectric mechanism creates a bulk force $F_z = enE_z$, which results in the loss of stability at I \approx I_{*}. The condition I \approx I_{*} evidently means that the force F_z can overcome the dissipative forces. Under the action of the force F_z the motion consists not only of purely longitudinal ($v = v_x$) flow, but it also has components perpendicular to the layer surface.

The thermoelectric effect alters somewhat also the conditions for heat exchange in advective flows, but qualitative changes in heat exchange are related specifically to the loss of stability [13]. Since $E_z = \gamma \partial T / \partial z$, and the flow of heat across a layer is $q_z = \rho C_p \varkappa \partial T / \partial z$, the nature of the heat flux q_z can be deduced from the behaviour of E_z in Figs 10-12 (the units of heat flux are $\rho C_p \varkappa A$). When heating ensures that the condition $I \approx I_*$ is satisfied, the heat flux also increases resonantly. Across the layer it rises as $(I - I_*)^{-1}$ and along the layer it even grows as $(I - I_*)^{-2}$. The influence of the thermoelectric field is usually ignored in heat exchange calculations [26, 27]. However, an external transverse electric field does not destabilise advective flows and, therefore, it does not produce abrupt changes in the heat exchange conditions (see Refs [28-30] and the literature cited there).

We can therefore reach the conclusion that, first, the thermoelectric effect does not by itself induce advective flows when heating is from the side and, second, the action of electric forces leads—in a certain range of parameters—to a loss of the stability of advective flows excited by the buoyancy and thermocapillary forces. This loss of stability occurs under the conditions close to those which are necessary for the excitation of cellular motion by the thermo-electric forces (Section 1), which is one further consequence of the fact that the electric forces are independent of the direction of heating.

It is evident from Figs 11 and 12 that in the presence of a free surface the necessary velocity of motion will be found specifically on that surface. Intense motion near the surface naturally helps in heat transfer. This has to be taken into account in discussing the phenomena which appear as a result of heating from above [13, 14].

5. Film thickness

The thickness of a liquid film depends on the thermal conditions on its surface [15]. The linear equations in the preceding section can be used to calculate the film thickness if they are solved subject to the boundary conditions stating the absence of a pressure jump given in Section 3 [31]. Integration gives

$$\rho v v_z = \left(\frac{z^2}{2} - zh\right) \left(g \frac{\mathrm{d}(\rho h)}{\mathrm{d}x} - \varepsilon \gamma^2 A \frac{\mathrm{d}A}{\mathrm{d}x}\right) -\frac{1}{2} \left(\frac{z^3}{3} - zh^2\right) \frac{\mathrm{d}\rho}{\mathrm{d}x} + z \frac{\mathrm{d}\alpha}{\mathrm{d}x} \,.$$
(23)

This relationship, together with the condition that the flow of a liquid in a film is closed, allows us to solve the problem provided we know the dependences of the required quantities along the layer. The closed-flow condition enables us to derive the equation which determines the film thickness

$$\frac{1}{3}\rho d(h^2) + \frac{1}{4}h^2 d\rho = \frac{1}{g}\left(d\alpha + \frac{2}{3}\epsilon\gamma^2 Ah dA\right).$$
 (24)

It is always assumed [1, 2] that the initial properties of an instability are retained later when the process is well advanced. Therefore, we can use the expressions for the distributions of the temperature $\beta T_1 = \Theta \cos(2\pi x/l)$ and density $\rho = \rho_0(1 - \beta T_1)$ along a layer, as usually adopted in the theory of convection in a liquid. These expressions apply to the cases of the Rayleigh [1] and thermoelectric [9] convection. They can readily be found by continuing calculations described in Section 1. We can also apply the usual law for the amplitude, i.e. we can assume the amplitude Θ in the expression for the temperature, like the velocity amplitude V (Section 2), is proportional to the square root of the 'supercriticality' $(T_h - T_h^*)^{1/2}$. The amplitude in the expression for the temperature can be taken from Ref. [11]. We then have

$$\Theta = \beta A h \left(\frac{E - E^*}{E^*}\right)^{1/2} \frac{2\sqrt{2}}{\pi} \left(1 + \frac{h^2}{l^2}\right)^{-1/2}.$$
 (25)

If E/Ra ≤ 1 in a region defined by $x/h_0 \geq 1$ (h_0 is the constant thickness of an undisturbed layer), we find that

ļ

k

$$\frac{h - h_0}{h_0} = \frac{5}{16} \Theta \left[\frac{2E\pi^2}{R} \frac{x}{l} - \frac{6}{5} \cos\left(\frac{2\pi x}{l}\right) \right].$$
 (26)

Hence, we can see that the layer thickness increases during motion along the layer from the hot to the cold regions. The surface slope $\Theta E/R$ is small. The greatest difference between the layer thicknesses is about $3\Theta/8$.

If a specific form of the dependence of the temperature on the coordinate along the layer is not assumed, then

$$\frac{a-h_0}{h_0} = \frac{3}{8} \beta T + \frac{2M}{Ra} - \frac{11}{b^2} \frac{E \operatorname{Bi}}{Ra^2} \beta T$$

$$-\frac{E h_0}{Ra} \int_0^x (1+\beta T) \frac{\partial^2(\beta T)}{\partial x^2} \, \mathrm{d}x \,, \qquad (27)$$

where $b = l/h_0$. The results of numerical calculations of the layer thickness are presented in Fig. 13 and in many respects they are identical with the results obtained from formula (26). In Fig. 13 the ratio of the dimensions along



Figure 13. Model of a liquid film showing the shape of its surface which it assumes under the action of the thermoelectric effect.

the layer l and perpendicular to the layer (for a constant thickness h_0) is about 45, which agrees with the values observed experimentally (Section 8). It is clear from this figure that the shape of a system which forms under the influence of the thermoelectric force is the same (matter is removed from the hot to the cold regions) as usual [18]. Naturally, in real experiments it is practically impossible to avoid evaporation of some of the material and the shape of the crater is governed by the vapour pressure.

6. Thin film of a liquid semiconductor (semimetal) with arbitrary heating from above

We have assumed so far that a temperature gradient acts either across a layer, A_{z} (Sections 2 and 3), or along a layer, A_x (Sections 4 and 5). In reality it is possible that $A_z \approx A_x$ (Section 8). Then, first, if heating is from above, the thermoelectric force can induce motion, whereas the buoyancy and thermocapillary forces tend to stabilise a liquid. Second, a comparison of the dimensionless numbers shows that the thermoelectric force predominates in thin liquid layers which are formed initially as a result of melting by radiation. Laser radiation used in such melting has not only a vertical component A_z (with the z axis perpendicular to the surface of the material), but also a longitudinal component A_x . Under the action of A_x some motion appears as a result of heating, no matter how weak (Section 4), and the velocity of such motion $v_x \gg v_z$ is directed parallel to the x axis along the layer and is proportional to the numbers Ra and M. These numbers are small for thin layers and, therefore, the velocities are low.

On the other hand, the thermoelectric instability, which essentially represents electroconvection, is independent of the direction of heating and is excited when $I > I_*$. In the vertical heating case, this is a demonstration of the threshold nature of the excitation process, whereas in the case of heating from one side an instability appears in the form of a 'resonant' rise near I_* . This can be seen from solutions of the type described by expression (22), which represents purely longitudinal motion. The actual value of the critical number I_* is calculated in Ref. [25] and it is governed by the boundary conditions.

In a comparison with the experiments it is natural to consider the excitation in extremely thin (but 'macroscopic') initially molten layers in the presence of the gradients A_z and A_x , i.e. in the case of heating both from above (opposite to the force of gravity g) and from one side. The influence of other forces can then be ignored and we can assume that initially the liquid is at rest.

Linearised equations of motion (1) (two projections), of heat transport (3), and of electrostatics (12)—considered in the film approximation $l \ge h$ —relate (Section 1) the

$$\frac{\partial p_1}{\partial z} = e n_1 \gamma A_z, \quad \frac{\partial p_1}{\partial x} = \rho v \frac{\partial^2 v}{\partial z^2},$$

$$e n_1 = \varepsilon \gamma \frac{\partial^2 T_1}{\partial z^2}, \quad \varkappa \frac{\partial^2 T_1}{\partial z^2} = v A_x.$$
(28)

The continuity equation makes it possible to find the vertical velocity component which does not occur in the equations derived in the adopted approximation. The field is assumed to be 'purely' thermoelectric: $E = \gamma \nabla T$.

Elimination of all the variables with the exception of v gives

$$\frac{\partial^3 v}{\partial z^3} = \frac{\varepsilon \gamma^2 A_z A_x}{\rho \varkappa v} \frac{\partial v}{\partial x} . \tag{29}$$

If we introduce a natural unit of length, which is the layer thickness h, it becomes obvious that the onset of an instability can be described by the following dimensionless 'film' number:

$$E^{f} = \frac{\varepsilon \gamma^{2} A_{x} A_{z} h^{2}}{\rho \nu \varkappa}, \qquad (30)$$

which is fully analogous to the number E representing the thermoelectric effect when A_x and A_z act independently. Eqn (29) is solved by separation of variables. Let us assume that

$$v = v(z) \exp\left(-\frac{\mathrm{i}\,k_x x}{h}\right)\,,\tag{31}$$

where the 'separation constant' is $k_x > 0$ since heating at the 'centre' (at x = 0) is the strongest. In fact, k_x determines the size l of the region of motion in the longitudinal direction in accordance with the law $k_x = 2\pi h/l$ in a cell of rectangular shape.

The system of equations (28) is supplemented by uniform boundary conditions. The lower boundary z = 0can always be regarded as solid (v = 0). This is the boundary with a bulk material. The upper boundary is with air (z = h) and it should be regarded as free so that $\partial v/\partial z = 0$. Moreover, the flow in the liquid should be closed. The condition for the excitation of a finite velocity of flow is independent of the thermal conditions at the boundaries.

Calculations show that motion appears when the 'film' dimensionless number reaches the value $E_*^{f} \approx 3.4/k_x$.

On the other hand, it follows from the results given in Section 4, that the excitation of an instability occurs for the values of E close to 20. Hence it readily follows that $l = bh \approx 37h$ [32].

We shall now calculate the layer thickness and the characteristics of convective motion under the simultaneous action of all the factors mentioned above. We shall confine ourselves to the two-dimensional problem and steady-state flows. We shall seek the following quantities:

$$v_x, \quad h = h_0 + h_1(x,z), \quad en, \quad E_x = \gamma \frac{\partial T}{\partial x},$$

$$E_z = \gamma \frac{\partial T}{\partial z}, \quad T = T_0 + A_x x + A_z z + T_1(x,z).$$
(32)

The problem is self-consistent since the film thickness and the characteristics of convection depend on one another. The possibility of appearance of surface waves is ignored. The equations describing the problem and combining all the equations used earlier are:

$$\rho \nu \Delta_2 \nu_x - \frac{\partial p}{\partial x} = -\epsilon \gamma^2 \frac{\partial T}{\partial x} \Delta_2 T + \nu_x \frac{\partial \nu_x}{\partial x} + \nu_z \frac{\partial \nu_x}{\partial z},$$

$$\rho \nu \Delta_2 \nu_z - \frac{\partial p}{\partial z} + \rho \beta_g T_1$$

$$= -\epsilon \gamma^2 \frac{\partial T}{\partial z} \Delta_2 T + \nu_x \frac{\partial \nu_z}{\partial x} + \nu_z \frac{\partial \nu_z}{\partial z},$$

$$\varkappa \Delta_2 T = \nu_x \frac{\partial T}{\partial x} + \nu_z \frac{\partial T}{\partial z}, \quad \Delta_2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$
(33)

As usual, these equations are supplemented by the equations of continuity (2) and electrostatics (12). These equations should be solved subject to the conditions $v_x = v_z = T_1 = 0$ at z = 0, i.e. the boundary with the bulk is assumed, as usual, to be solid and isothermal.

At the other boundary, which is on the surface of the liquid, the condition stating the absence of a pressure jump [compare with Eqn (17)] and the condition of the influence of the thermocapillarity are satisfied. If no waves can form, these conditions can be written as follows:

$$p = p_0 + \rho(h-z) g - \frac{1}{2} \varepsilon \gamma^2 \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial z} \right)^2 \right] - \frac{1}{2} \rho(v_x^2 + v_z^2),$$
(34)

$$\rho v \left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z} \right) = -\sigma \frac{\partial T}{\partial x}$$
(35)

on the surface z = h. On this surface the heating parameters A_x and A_z are given functions of x. Naturally, the closed-flow condition

$$\int_{0}^{m} v_x \, \mathrm{d}z = 0 \tag{36}$$

should also be satisfied.

In the first approximation the quantities ρ , A_x , and A_z are regarded as constant and the layer is very thin. Then the thickness *h* is described by the results obtained for this case in Sections 5 and the results for convection are the same as those given in Sections 1 and 6.

In the second approximation, we have $\rho = \rho_0 [1 - \beta(T - T_0)]$, and A_x and A_z are linear functions of x on the surface z = h. The layer thickness is assumed to be intermediate between $h_{M,I} = h_c$ and $h_{Ra,M}$ [see expressions (16) and (10)]. Therefore, in the second approximation the problem is solved on the assumption that the main mechanism for the excitation or suppression of an instability is related to the thermocapillary force. The results of such calculations are given in Fig. 14 and they are in many respects identical with those reported by Sanochkin [33]. However, in contrast to Sanochkin's results, we find that $l \approx 50 h_0$, and the shape of the surface has the form described by expression (26). This automatically yields temperatures close to $T_1 = \Theta \cos(2\pi x/l) \cos(2\pi z/h)$.

A more detailed analysis of the results given in this section can be found below in a discussion of the mechanism of the formation of a zone which is melted by laser radiation (Section 8.3).



Figure 14. Surface shape and convection characteristics found for a liquid film by solving the self-consistent problem: (a) isotherms; (b) lines of flow of the current. The straight line $\xi = 1$ ($\xi = z/h$) represents the film thickness before heating. The straight line $\xi = 0.68$ crosses the lines of flow of the current at the points where the liquid moves across the film. The rest of the notation is the same as in Figure 13.

7. Excitation of an electric instability by heating in other liquid media

We have considered so far the example of liquid semiconductors or semimetals [34-38]. However, similar pheno-mena can occur also in other liquid media. We shall now consider briefly the possibility of excitation of an electric instability by a heat flux (heating) in electrolytes, in thermotropic liquid crystals of different types, and in insulators.

We shall also discuss briefly the possibility of excitation when there is a flux of mass in a liquid. There is a very close analogy between the possibility of excitation as a result of heat flow and mass flow (see Refs [39-41] and the literature cited there). This analogy is based on the analogy of the equations describing heat flow and diffusion:

$$\frac{\partial c_1}{\partial t} - D\Delta c_1 + (\mathbf{v} \cdot \nabla) c_0 = 0$$
(37)

where the concentration c of the transported component is an analogue of the temperature T and the diffusion coefficient D is an analogue of the thermal diffusivity \varkappa .

7.1 Electrolytes

The values of the Seebeck coefficient of electrolytes are high [42]. Therefore, all the phenomena described above can occur in electrolytes [43]. However, an electrolyte is a conductor with ambipolar conduction, i.e. it contains carriers of charges of both signs and a current can flow in them. The relevant system of equations should be supplemented by the equations of continuity for the currents which include terms due to conduction, diffusion, thermoelectricity, and convection [44].

If there is no external electric field, the results are fully identical with those obtained for liquid semiconductors. In a weak external electric field, i.e. in a field inducing a current but not altering the thermal balance, the excitation condition described by expression (13) has an additional term

$$\frac{\varepsilon\gamma\left(\boldsymbol{E}_{0}\cdot\boldsymbol{\nabla}\boldsymbol{T}_{0}\right)h^{2}}{\rho\nu\varkappa}\frac{k^{6}}{k_{\perp}^{2}},$$
(38)

which can be explained exactly as in the case of the thermoelectric term in Section 1. In fact, if temperature

deviates by T_1 as a result of fluctuations, an electric field $E_1 = \gamma \nabla T_1$ appears and in an electrolyte a space charge $e(n_1^+ - n_1^-)$, associated with this field, is formed (the superscripts identify the sign of the carriers to which a given quantity applies). The charge may be due to the thermoelectric effect $(\epsilon\gamma\Delta T_1)$ or due to the mobility of ion carriers during diffusion. The latter mechanism of charge separation is particularly important, if in addition to the electric field γA there is also an external field E_0 . A fluctuation of the space charge can be estimated as $(n^{\pm}e^{2}\gamma/k_{\rm B})(T_{1}/T_{0})$ (k_B is the Boltzmann constant). The presence of the space charge in the field γA or in an external field leads to the appearance of a force of electric nature and of density $(n^{\pm}e^{2}\gamma E_{0}/k_{\rm B})(T_{1}/T_{0})$. It is important to stress that this force does not appear in the absence of the thermoelectric effect, even in an external field. It is this force that excites motion in competition with the buoyancy and thermocapillary forces. A steady-state inhomogeneity of the carrier density n^{\pm} under ambipolar conduction conditions does not in fact appear if a current cannot flow in a medium. However, if a current can flow, a temperature inhomogeneity gives rise to a redistribution of ions and, conversely, a deviation from homogeneity of the carrier distribution leads to nonuniform heating of the liquid. Therefore, it is unimportant what external agency is acting-heating or flow; they always induce one another. In experiments it is more convenient to heat a layer. We shall consider this case by the same methods as before. However, if an electric field is strong, we cannot assume that the condition of quasineutrality is satisfied under equilibrium conditions. This is why an equilibrium state of an electrolyte is analysed in Refs [44, 45] and it is shown there that the above results are valid as long as the Debye radius $R_{\rm D}$ is much less than the characteristic size h, which is always true.

The influence of the surface tension forces on the excitation of a thermal instability in an electrolyte is considered in Ref. [46]. The surface tension in an electrolyte may depend not only on temperature, but also on the carrier density and even on the electric field. The influence of these effects on the thermoelectricity and their interaction with one another are discussed in Ref. [46].

The excitation is mainly due to the influence of the concentration of a solute on the surface tension forces. This is due to the fact that in reality we have $\varkappa/D \ge 10^2$ for electrolytes. In general, heating and a change in the solute concentration tend to cancel each other out. It has been shown that the influence of the dependence of the surface tension on the electric potential is always weak.

Phenomena very similar to those investigated in electrolytes can occur also in an electron-hole gas of charge carriers in solids [44]. The usual convection in such solids is discussed in Refs [47, 48].

7.2 Liquid crystals

Liquid crystals have the properties of semiconductors [49, 50]. Therefore, all the phenomena considered in Sections 1-6 are also possible in liquid-crystal materials. However, liquid crystals are distinguished by a great variety of hydrodynamic properties and they represent a natural class of materials which can be used as an example in the study of the interaction of thermal, hydrodynamic, and electric properties.

The interaction between hydrodynamic and electric phenomena in the excitation of instabilities in liquid crytals

of different types is considered in Ref. [51]. The system of equations (1)-(3) and the equations of electrodynamics are supplemented by the equation for the director n, which sets the orientation of molecules. Only thermotropic liquid crystals are dealt with in Refs [52-55].

The influence of anisotropy on the excitation conditions is discussed in Ref. [56].

In the case of nematics it has been shown that one needs to allow for the anisotropy of the dissipative characteristics such as the thermal diffusivity

$$\boldsymbol{\varkappa}_{ik} = \boldsymbol{\varkappa} \boldsymbol{\delta}_{ik} + \boldsymbol{\varkappa}_{a} n_{i} n_{k} , \qquad (39)$$

where \varkappa_{ik} is the thermal diffusivity determined by the isotropic (\varkappa) and anisotropic (\varkappa_a) diffusivities of media which transmit not only hydrostatic pressures, but also torsion and bending (the elastic constants are assumed to be the same and are denoted by *G*). As a result, the critical temperature gradient at the point of excitation decreases by a factor

$$1 + \frac{\varkappa_a \nu_a}{G} \tag{40}$$

compared with that reported in Refs [57, 58]. This makes pre-Rayleigh convection possible. The numbers Ra and M decrease by the same factor and the number E decreases quadratically.

It follows that thermoelectric convection in liquid crystals may be excited earlier in thicker layers than in conventional liquid semiconductors.

The appearance of the factor referred to above can be understood on the basis of the same qualitative considerations which have been employed repeatedly earlier (Sections 1 and 7.2).

The heat flux $\varkappa_a \Delta T_1$, which appears in an anisotropic liquid because of temperature fluctuations is dissipated in the work involved in altering the orientation of molecules (i.e. altering the orientation of the director $\partial n_i/\partial x_k$). This work is $\varkappa_a A |\partial n/\partial x|$. The change in the orientation of the director n_{1i} is the result of motion in a medium with an anisotropic viscosity. We therefore have $\nu_a v \approx G \partial n/\partial x$.

It follows that the heat flux is expended both in convective heat transfer represented by vA (Section 1) and in rotation of the director $vA\varkappa_a\nu_a/G$. The addition of these two quantities gives the result represented by expression (40).

The influence of the anisotropy of the dissipative coefficients applies to all classes of liquid crystals. However, specific effects occur in certain classes.

It is shown in Ref. [59] that excitation accompanied by oscillatory growth is possible in smectics with equidistant layers and a director perpendicular to the layer surfaces. This happens although in a system of forces acting in a smectic there is no pseudovector. The frequency of growth oscillations is governed by the elastic constant of the layers $B = s^2$, where s is the velocity of special acoustic waves (second sound) associated with this elastic constant [60].

One further effect associated with second sound is considered in Ref. [61]: this is the excitation of surface second-sound waves. A dispersion equation is derived and a study is made of the influence of an electric field on the excitation process. The waves represent a new type of surface waves [62, 63].

The influence of the elastic properties of a medium not only on the buoyancy but also on the thermoelectric excitation mechanism is considered in Ref. [51]. Heating from above is considered in Ref [64] as a possible way of making excitation easier.

The convection due to a strong temperature dependence of the pitch of the molecular helix in cholesterics is considered in Refs [65-67]. The temperature dependence of the pitch q alters the permittivity of the medium and it therefore results in characteristic electric effects. It is found that an instability is possible as a result of heating from above and that it grows in an oscillatory manner. The frequency of the growth oscillations depends very strongly (as h^{-15}) on the thickness of the cholesteric liquid-crystal layer. This is a new instability effect [68, 69]. The solution is methodologically complicated because the acting electrophoretic force (a more detailed discussion of this force and also of other properties of insulators, which apply also to liquid crystals, is provided in Section 7.3) is a quantity which oscillates rapidly over distances which are short compared with h and which are of the same order as the size of the cholesteric molecules. Macroscopic characteristics can be obtained by a special type of averaging. The results show that the number which governs the possibility of excitation is

$$\frac{A^2h^2}{q^2} \left(\frac{\partial q}{\partial T}\right)^2 \mathcal{E} \quad . \tag{41}$$

If a cholesteric represents a conducting medium, then in an external electric field transverse to the cholesteric axis we can expect spatially periodic distributions (structures) in the pitch of the cholesteric helix. A theory developed in Ref. [70] for conducting media with steady-state flow is used in Ref. [67] to calculate structures in cholesteric liquid crystals. The calculations reported in Ref. [67] are carried out for an external field considerably lower than that at which the pitch of the molecular helix begins to increase with the field. It is shown in Refs [63, 64] how the interaction with new, typical only of cholesterics, mechanisms affects the usual Rayleigh and thermoelectric mechanisms.

7.3 Insulators

We shall finish the review of published material on the excitation of electroconvection by heating by considering the situation when an instability is not induced by heating but by the mass transport, i.e. when ∇T is replaced with ∇c . It is found that the hydrodynamic and electric phenomena interact in an external electric field in a manner similar to the interaction of the same phenomena in liquid semiconductors during heating. The analogue of the thermoelectric force is the electrostriction [28, 71–75].

An analysis of a liquid insulator, through which another insulator flows, can be found in Refs [76-78]. Then ∇c leads to $\nabla \varepsilon$ and the analogue of the thermoelectric force is the electrophoretic force $(E_1 \cdot E_0) \nabla \varepsilon$ [79]. In fact, in this case we can develop a theory which is fully analogous to that given above for liquid semiconductors (Sections 1-6). The number

$$E_{c} = \frac{\varepsilon E_0^2 A^2 h^2}{\rho v D}, \qquad (42)$$

representing the ratio of the electrophoretic $(A = |\nabla c|)$ and dissipative forces is used in Ref. [77] and the excitation condition is found to be

$$-k^8 + \mathcal{E}_c \, k_\perp^2 k_z^2 = 0 \,\,, \tag{43}$$

which ignores the buoyancy forces. It is has been found that instability is possible for

$$E_c > E_c^* \approx 924 \text{ for } w = \frac{1}{3},$$
 (44)

with any direction of the current. Moreover, the amplitude expected for a small excess of E_c over E_c^* is calculated in Ref. [78].

The interaction of the electrophoretic force with the temperature-dependent permittivity and with the buoyancy force during heating of a layer from one side is tackled in Ref. [80]. It is shown that, as in the cases discussed in Section 4, a strong rise is observed when a certain condition is obeyed by a number analogous in meaning to the number E_c , although the motion itself is possible only in the presence of the buoyancy force [the velocity is proportional to the number Ra, exactly as in expression (22)].

We shall conclude this section by noting that when the interaction of specific (elastic, electric, etc.) properties of a liquid is taken into account, the conditions for the excitation of thermal convection change and even qualitatively new types of instabilities can appear. Obviously, similar results can also be expected for other media.

8. Comparison with experiments

The available experimental data, which can be interpreted only if we use the results of calculations on electroconvection as a result of heating, can be divided into two groups. The first group comprises the experiments in which convection is observed in thin films of spermaceti, carried out by Benard [81], which are described and analysed in Refs [5, 6]. The second group consists mainly of the experiments in which the frequently mentioned heating from above is provided by laser radiation.

8.1 Bénard's experiments

Benard carried out his experiments on a film of spermaceti (sperm whale oil). The data on spermaceti can be found in Refs [82-84]. A spermaceti film up to 1 mm thick was placed on a flat metal substrate. Its upper surface was in contact with air. This arrangement reproduced the model of an infinite flat film heated from below. The lower boundary was solid and isothermal, and the upper was free and thermally insulated.

Estimates given in Refs [8, 9] show that the inequalities $h_c = h_{M,I} < h < h_{Ra,M}$ should be satisfied in these experiments. Expression (16) yields $h_{M,I} \approx 10 \ \mu\text{m}$ and expression (10) gives $h_{Ra,M} \approx 2 \ \text{mm}$. The value of γ is a parameter which is most difficult to estimate (see Ref. [8]). It follows that the main mechanism resulting in the formation of the Benard cells is the thermocapillary effect (see Refs [5-7] and the review in Ref. [2]).

The thermoelectric mechanism in the thinnest films used in the experiments ($h \approx 10 h_c \approx 0.1 \text{ mm}$) can be taken into account by utilising the solutions given in Ref. [20]; this gives $M \approx M^* (1 - 0.4 \text{ E})$. Under experimental conditions it was found that $E \approx 0.1$. The change in M is only a few percent. Moreover, the change in the ratio l/h of the dimensions of the cell, which appears at the moment of excitation, is also small.

However, it should be pointed out that the thermoelectric mechanism can account for ageing of liquid-crystal displays [85]. All other numerous experiments on heating from below (such as those described in Refs. [1] and [28]) have been carried out on films of such thicknesses that the influence of the thermocapillarity and thermoelectricity has not been felt.

8.2 Laser heating experiments

The first investigation of the thermoelectric convection [86] was purely theoretical, but beginning from Ref. [87] it has been recognised that it is necessary to apply a theory of electroconvection that appears as a result of heating, in order to explain the laser melting results. Following the work of Mirkin [88, 89], it has been generally recognised [18, 90-92] that convective cellular motion is induced in the melt. However, nonelectric mechanisms cannot excite such motion.

It should be stressed that convective motion appears specifically during heating. This can be judged on the basis of penetration of alloying materials [93, 94]. However, cellular motion can appear under the influence of nonelectric mechanisms as a result of cooling from above [95, 96].

An explanation of the excitation of cellular motion during melting or heating of a liquid (without evaporation) by laser radiation is of major importance in technological applications [97-101].

The influence of the thermoelectric effect on convection is observed in experiments involving heating of liquid steel carrying a film of tungsten carbide on its surface [93, 94] or for motion of liquid films of silicon-tellurium alloys [92]. This influence can be estimated from the difference between the hot $(T_{\rm h})$ and cold $(T_{\rm c})$ surfaces: $T_{\rm h} - T_{\rm c} \approx Ah \approx 10^3 \text{ K} - 10^4 \text{ K}$ and from the parameters of a liquid: $\rho \approx 1$ to 10 g cm⁻³, $\beta \approx (9$ to $6) \times 10^{-4} \text{ K}^{-1}$, $\nu \approx \varkappa \approx 5 \times 10^{-2}$ to $1 \text{ mm}^2 \text{ s}^{-1}$, $\sigma = (1 \text{ to} 3) \times 10^{-5} \text{ N m}^{-1} \text{ K}^{-1}$ [102-103].

It is moreover necessary to know the value of the Seebeck coefficient γ . However, the values of this coefficient at the melting point or slightly above are not given in the cited papers. Nevertheless, it is known that the electric conductivity and the Seebeck coefficient do not change greatly at the melting point. The Seebeck coefficient of solid semiconductors at temperatures above the Debye value T_D is governed by the rise in temperature and can be written in the form $\gamma = a(k_B/e)(T_D/T)$. The constant a in this expression is such that γ is of the order of 100 μ V K⁻¹ [104–106].

The thermoelectric effect can set a liquid in motion when neither the buoyancy force ($\beta > 0$) nor the temperature dependence of the surface tension ($\sigma > 0$) can suppress an instability, provided

$$A > A_{\rm I} = \left(\frac{\rho \varkappa \nu}{\varepsilon}\right)^{1/2} \frac{{\rm I}^*}{\gamma h} > \frac{\rho \beta g h^2 {\rm E}^*}{\varepsilon \gamma^2 {\rm R}^*} > \frac{\sigma {\rm E}^*}{\varepsilon \gamma^2 {\rm M}^*}, \tag{45}$$

where Ra^{*} > 800, M^{*} > 80, E^{*} > 45. The thermoelectric effect excites an instability if the heating exceeds the value A_1 . When the heating is from above, surface waves may be excited [17, 18]. However, first, such waves cannot set the interior of a molten layer in motion to the observed depths (up to 150 µm, according to the data in Refs [93, 94]) and, therefore, cannot ensure penetration of the alloying element (tungsten). Second, it is necessary to compare the excitation conditions. It is shown in Ref. [17] that the excitation of such waves occurs when $A > A_w = \rho gh/\sigma$. A comparison of the conditions for the excitation of the thermoelectric convection and surface waves demonstrates that this convection is excited first [87] if the liquid layer thickness is less than

$$h < h_* \approx \left(\frac{\varkappa \nu}{\rho \varepsilon}\right)^{1/2} \frac{I_* \sigma}{\gamma g}$$
 (46)

Estimates show that $h_* < 1$ mm. Under experimental conditions the thickness of the alloyed layer proved to be about 0.1-0.03 mm, i.e. the thermoelectric instability was indeed excited first.

It is assumed [1, 2] that the properties of the initial instability may be observed also after the motion becomes advanced.

The theoretical values of b = 54 and b = 37 for rectangular and hexagonal cells, calculated on the assumption that two three-dimensional cells are excited immediately (see Section 6 and Ref. [33]), are in good agreement with the result $l/h \approx 47$, obtained from an analysis of the data given in a table of the dimensions of the alloyed zone [94]. These results were taken from Refs [8, 9, 32].

8.3 Mechanism of formation of a molten zone [13, 14]

Let us assume that the laser radiation energy just exceeds the heat of fusion and that the duration of the laser pulses is of the order of the characteristic time $h^2/v = 10^{-2}$ s = 10 ms required to establish an equilibrium regime.

During the first stage the layer melted by laser radiation is very thin $(h \leq h_c)$. Under these conditions the motion is along the layer and this motion begins to change to cellular only when the heating effect reaches a value corresponding to $I_* \approx 4.5$ (Section 6). It is obvious that the value $I_*^2 = E_* \approx 20$ can be used under the conditions of arbitrary heating and it is permissible to assume that the heating is approximately the same along perpendicular directions: $A_x \approx A_z \approx A$. We can then determine the ratio of the dimensions of the molten zone, which is approximately 40. The properties that appear first will be maintained for a long time in the molten zone. In any case, this is true when convection is excited [1, 2].

Longitudinal motion which then appears in a thin layer transports a hot liquid along its boundary (Fig. 14) and it also transports heat. Therefore, the molten zone spreads rapidly until its size along the sample becomes approximately 40 times greater than in the direction of depth.

When heating is such that $I_* > 4.5$, motion develops also in the perpendicular direction. Transport of the hot liquid to the bottom of the melt rapidly increases the depth of the liquid layer. When the depth increases by a factor of 40, the surface of the melt spreads to a size 1600 times greater. However, the increase in the depth and surface area stimulates the stabilising action of the thermocapillary effect. Simultaneously, surface waves appear.

When the depth of the layer is $h \leq 10 \ \mu m$ and the temperature difference is $T_{\rm h} - T_{\rm c} \approx 10^4 \ {\rm K}$, thermoelectric convection may be excited: it establishes cellular motion with the ratio of the longitudinal and transverse dimensions in excess of 2. It is obvious that if this effect appears at all, then this happens only at the very first moments of heating in the thinnest layers. In reality the effect of such an instability in the growing melt is difficult to determine. The depth of the melt, observed actually in the experiments

 $(30-100 \ \mu\text{m})$ obviously only initially $(h < 10 \ \mu\text{m})$, is governed by the thermoelectric convection and then (for $h < 100 \ \mu\text{m}$) depends on the excitation of motion by heating from the side and on its 'resonant' growth. Finally, the greatest depth of the melt is reached by conventional heat transport.

9. Concluding comments: synergetic aspects

The problem of order and disorder has been attracting attention for a long time [107-113]. This problem is important also in the formation of melts by laser radiation which is a very modern technological method. The task is to study and describe the collective phenomena which are observed on transition from disordered to ordered systems and vice versa. A general feature of these phenomena is that as a system becomes more complex (as it becomes heated), it acquires certain phenomenological features which are difficult to detect by investigations of simpler subsystems [110, 111]. In our analysis, the Rayleigh and thermocapillary mechanisms (or subsystems) are complicated by the superposition of the thermoelectric subsystem, leading to a new technological phenomenon, which is the transport of heat by cellular motion when heating is from above.

The interaction of the subsystems determines the properties of the objects, in this case the molten zone. Synergetic properties of the object and its space-time periodicity are governed in particular by the thermoelectric instability and by the thermocapillary stabilisation. General considerations indicate that the appearance of cellular motion as a result of heating from above should also be regarded as a natural phenomenon [112], but the actual properties of an object (a molten semiconductor film) can be predicted and explained only by solving the specific problem and comparing the solution with the experimental results.

It follows that the thermoelectric convection is an example of the interaction of hydrodynamic, electric, and thermal subsystems in a disordered system. When these subsystems interact in an open thermodynamic system, such as a liquid film, self-organisation takes place and macroscopic space-time structures appear. In accordance with general ideas [107], the transition occurs in the form of growth of fluctuations. Stochastic perturbations increase and become deterministic cellular motion. This transition occurs only in a certain range of the values of the control parameters (heating A and layer thickness h) [108].

It follows from synergetic considerations [110, 111] that the resultant cells (deterministic motion) also exist in a certain range of values of the same control parameters. For example, when the layer thickness is higher than the critical value $h_{\rm MI}$ or $h_{\rm RI}$, motion is suppressed in cells heated from above.

The Benard cells are a classic example of a synergetic phenomenon. A new self-organisation mechanism in the form of the thermoelectric instability gives us a new insight into this phenomenon. The interaction of the thermoelectric mechanism with the excitation mechanisms investigated earlier (buoyancy, thermocapillary forces) provides an opportunity for revealing characteristics of the technology of melting of matter by laser radiation, which may be of importance in practice.

The new mechanism explains how heat can be transferred by convection when a free surface is heated. The above results of an investigation of the conditions for the excitation of the electric convection are applicable to a wide spectrum of media with a variety of mechanical and electric properties. Only some of them have been investigated so far: liquid semiconductors and semimetals, certain types of liquid crystals, ambipolar media, etc. Other media, such as porous materials, gas-liquid mixtures, and many others, have to be studied by the abovedescribed method for investigating the interaction of hydrodynamic and electric properties.

Acknowledgements. I am grateful to N V Kalinin and B I Lembrikov for their help in this work, and also to O V Konstantinov, R A Suris, M I D'yakonov, Yu K Stishkov, I N Toptygin and many other colleagues for discussions. I am also greatly indebted to my teachers, the late L E Gurevich and to I V Ioffe.

References

- 1. Chandrasekhar S Hydrodynamic and Hydromagnetic Instability (Oxford: Clarendon Press, 1961)
- Gershuni G Z, Zhukhovitskii E M Konvektivnaya Ustoichivost' Neszhimaemoi Zhidkosti (Convective Stability of an Incompressible Liquid) (Moscow: Nauka, 1972)
- 3. Drazin P G, Reid W H *Hydrodynamic Stability* (Cambridge: Cambridge University Press, 1981)
- 4. Gershuni G Z, Zhukhovitskii E M, Nepomnyashchii A A *Ustoichivost' Konvekti vnyk h Techenii* (Stability of Convective Flows) (Moscow: Nauka, 1989)
- 5. Pearson J K A J. Fluid Mech. 4 (5) 489 (1958)
- 6. Nield D A J. Fluid Mech. 19 341 (1964)
- 7. Berg J C, Acrivos A Chem. Eng. Sci. 20 737 (1965)
- 8. Eidel'man E D Zh. Eksp. Teor. Fiz. 103 1633 (1993) [J. Exp. Theor. Phys. 76 802 (1993)]
- 9. Eidel'man E D Zh. Tekh. Fiz. 64 (4) 29 (1994) [Tech. Phys. 39 364 (1994)]
- Eidel'man E D Zh. Tek h. Fiz. 57 1145 (1987) [Sov. Phys. Tech. Phys. 32 667 (1987)]
- 11. Eidel'man E D Zh. Tekh. Fiz. 63 (10) 193 (1993) [Tech. Phys. 38 928 (1993)]
- 12. Eidel'man E D Zh. Tekh. Fiz. 63 (10) 196 (1993) [Tech. Phys. 38 930 (1993)]
- Eidel'man E D Teplofiz. Vys. Temp. 32 418 (1994) [High Temp. 32 393 (1994)]
- 14. Eidel'man E D Fiz. Tek h. Poluprovodn. 28 1535 (1994) [Semiconductors 28 858 (1994)]
- 15. Landau L D, Lifshitz E M *Fluid Mechanics* 2nd edition (Oxford: Pergamon Press, 1987)
- Landau L D, Lifshitz, Pitaevskii L P *Electrodynamics of* Continuous Media 2nd edition (Oxford: Pergamon Press, 1984)
- 17. Levchenko E B, Chernyakov A L Zh. Eksp. Teor. Fiz. 81 202 (1981) [Sov. Phys. JETP 54 102 (1981)]
- Arutyunyan R V, Baranov V Yu, Bol'shov L A, et al. Vozdeistvie Lazernogo Izlucheniya na Materialy (Action of Laser Radiation on Materials) (Moscow: Nauka, 1989)
 Eirer Radiation on Materials) (Moscow: Nauka, 1989)
- 19. Eidel'man E D Poverkhnost' (3) 29 (1995)
- Eidel'man E D Pis'ma Zh. Tekh. Fiz. 19 (17) 90 (1993) [Tech. Phys. Lett. 19 566 (1993)]
- 21. Levich V G Fiziko-Khimicheskaya Gidrodinamika (Physicochemical Hydrodynamics) (Moscow: Fizmatgiz, 1959)
- 22. Davies J T, Rideal E K *Interfacial Phenomena* 2nd edition (New York: Academic Press, 1963)
- 23. Birikh R V Zh. Prikl. Mekh. Tek h. Fiz. (3) 69 (1966)
- 24. Levich V G, Krylov V S Annu. Rev. Fluid Mech. **1** 293 (1969)
- 25. Eidel'man E D Zh. Eksp. Teor. Fiz. **104** 3058 (1993) [J. Exp. Theor. Phys. **77** 428 (1993)]
- Sokovishin Yu A, Martynenko O G Vvedenie v Teoriyu Svobodnokonvekti vnogo Teploobmena (Introduction to the Theory of Free-Convection Heat Exchange) (Leningrad: Leningrad State University, 1982)

- Cebeci T, Bradshaw P Physical and Computational Aspects of 27. Convective Heat Transfer (Berlin: Springer, 1984)
- 28 Bologa M K, Grosu F P, Kozhukhar' I A Elektrokonvektsiya i Teploobmen (Electroconvection and Heat Exchange) (Kishinev Shtiintsa, 1977)
- 29. Bologa M K, Berkov A B Elektrokonvektivnyi Teploobmen Dispersionnykh Sistem (Electroconvective Heat Exchange in Disperse Systems) (Kishinev Shtiintsa, 1989)
- 30. Bologa M K, Smirnov G F, Didkovskii I M, Klimov S M Teploobmen pri Kipenii i Kondensatsii v Elektricheskom Pole (Heat Exchange in Boiling and Condensation in an Electric Field) (Kishinev Shtiintsa, 1987)
- Eidel'man E D Zh. Tekh. Fiz. 64 (5) (1) (1994) [Tech. Phys. 39 31. 455 (1994)]
- Eidel'man E D Zh. Eksp. Teor. Fiz. 105 (2) 295 (1994) [J. 32. Exp. Theor. Phys. 78 157 (1994)]
- Sanochkin Yu V Zh. Prikl. Mekh. Tek h. Fiz. (6) 134 (1983) 33
- Cutler M Liquid Semiconductors (New York: Academic Press, 34. 1977
- Regel' A R, Glazov V M Fizicheskie Svoistva Elektricheskikh 35. Rasplavov (Physical Properties of Electric Melts) (Moscow: Metallurgiya, 1980)
- Glazov V M, Wobst M, Timoshenko V Ya Metody Issledova-36. niya Svoistv Zhidkikh Metallov i Poluprovodnikov (Methods for Investigation of the Properties of Liquid Metals and Semiconductors) (Moscow: Metallurgiya, 1989)
- Novoselov A V, Glazov V M, Smirnova N A Termodinamika i 37. Mat erialovedenie Poluprovodnikov (Thermodynamics and Materials Science of Semiconductors) (Moscow: Metallurgiya, 1992)
- 38. Alekseev A A, Andreev A A, Prokhorenko V Ya Usp. Fiz.
- Nauk 106 393 (1972) [Sov. Phys. Usp. 15 139 (1972)] 39 Shaposhnikov I G Prikl. Mat. Mekh. 17 604 (1952)
- 40.
- Hart J E J. Fluid Mech. 49 (2) 279 (1979) 41. Turner J S Annu. Rev. Fluid Mech. 17 11 (1985)
- Izmailov N A Elektrokhimiya Rastvorov (Electrochemistry of 42.
- Solutions) (Moscow: Nauka, 1976)
- 43. Eidel'man E D Zh. Tekh. Fiz. 59 (4) 145 (1989) [Sov. Phys. Tech. Phys. 34 469 (1989)]
- Eidel'man E D Teplofiz. Vys. Temp. 33 509 (1995) 44
- 45. Ioffe I V, Eidel'man E D Pis'ma Zh. Tekh. Fiz. 4 193 (1978) [Sov. Tech. Phys. Lett. 4 77 (1978)]
- Eidel'man E D Zh. Tekh. Fiz. 65 (1) 19 (1995) [Tech. Phys. 40 46. 9 (1995)]
- Bonch-Bruevich V L Zh. Eksp. Teor. Fiz. 67 2204 (1974) 47 [Sov. Phys. JETP 40 1093 (1975)]
- Bonch-Bruevich V L Zh. Eksp. Teor. Fiz. 71 1583 (1976) 48. [Sov. Phys. JETP 44 829 (1976)]
- de Jeu W H Physical Properties of Liquid Crystalline Materials 49. (New York: Gordon and Breach, 1980)
- 50. Gevorkyan E V, Bazarov I P Statisticheskaya Fizika Zhidkikh Kristallov (Statistical Physics of Liquid Crystals) (Moscow: Moscow State University, 1992)
- Eidel'man E D Fiz. Tverd. Tela (Leningrad) 37 (1) 162 (1995) 51 [Phys. Solid State 37 86 (1995)]
- 52. Chandrasekhar S (Ed.) Liquid Crystals (Cambridge: Cambridge University Press, 1977)
- 53. de Gennes P G The Physics of Liquid Crystals (Oxford: Clarendon Press, 1974)
- 54. Landau L D, Lifshitz E M Theory of Elasticity 3rd edition (Oxford: Pergamon Press, 1986)
- 55. Kats E I, Lebedev V V Dinamika Zhidkikh Kristallov (Dynamics of Liquid Crystals) (Moscow: Nauka, 1988)
- Ioffe I V, Lembrikov B I, Eidel'man E D Pis'ma Zh. Tekh. Fiz. 56. 2 921 (1976) [Sov. Tech. Phys. Lett. 2 361 (1976)]
- 57. Dubois-Violette E C. R. Acad. Sci. 273 923 (1971)
- Dubois-Violette E, Guyon E Mol. Cryst. Liq. Cryst. 26 193 58. (1974)
- 59. Ioffe I V, Eidel'man E D Zh. Tekh. Fiz. 49 2291 (1979) [Sov. Phys. Tech. Phys. 24 1273 (1979)]
- Kaputin A P, Kaputina O A Akustika Zhidkikh Kristallov 60. (Acoustics of Liquid Crystals) (Moscow: Nauka, 1986)
- Eidel'man E D Poverkhnost' (3) 8, 26 (1995) 61.
- Rapini A Can. J. Phys. 53 968 (1975) 62.

- Matveenko V N, Kirsanov E A Poverkhnostnye Yavleniya v 63. Zhidkikh Kristallakh (Surface Phenomena in Liquid Crystals) (Moscow: Moscow State University, 1991)
- 64. Pieranski P, Dubois-Violette E, Guyon E Phys. Rev. Lett. 30 736 (1973)
- 65. Ioffe I V, Eidel'man E D Zh. Tekh. Fiz. 51 1702 (1981) [Sov. Phys. Tech. Phys. 26 979 (1981)]
- Eidel'man E D Zh. Tekh. Fiz. 60 (1) 214 (1990) [Sov. Phys. 66. Tech. Phys. 35 131 (1990)]
- Ioffe I V, Eidel'man E D Zh. Tekh. Fiz. 60 (1) 217 (1990) [Sov. 67 Phys. Tech. Phys. 35 133 (1990)]
- 68. Helfrich W J. Chem. Phys. 55 839 (1971)
- Dubois-Violette E J. Phys. (Paris) 34 107 (1973) 69.
- Gurevich L E, Ioffe I V Zh. Eksp. Teor. Fiz. 61 1133 (1971) 70. [Sov. Phys. JETP 34 605 (1972)]
- 71 Ostroumov G A Vzaimodeistvie Gidrodinamicheskikh i Elektricheskikh Polei (Interaction of Hydrodynamic and Electric Fields) (Moscow: Nauka, 1979)
- 72. Stishkov Yu K, Ostapenko A A Elektrodinamicheskie Techeniya v Zhidkikh Dielektrikakh (Electrodynamic Flows in Liquid Insulators) (Leningrad: Leningrad State University, 1989)
- 73. Gelmont B L, Ioffe I V Phys. Lett. A 26 253 (1968)
- Roberts P H Quart. J. Mech. Appl. Math. 22 211 (1969) 74.
- 75. Turnbull R J Phys. Fluids 11 2588 (1968)
- 76 Eidel'man E D Zh. Eksp. Teor. Fiz. 107 774 (1995) [J. Exp. Theor. Phys. 80 439 (1995)]
- Ioffe I V, Eidel'man E D Pis'ma Zh. Tekh. Fiz. 2 90 (1976) 77. [Sov. Tech. Phys. Lett. 2 34 (1976)]
- 78. Eidel'man E D Pis'ma Zh. Tekh. Fiz. 20 (22) 74 (1994) [Tech. Phys. Lett. 20 923 (1994)]
- Dukhin S S, Deryagin B V Elektroforez (Electrophoresis) 79. (Moscow: Nauka, 1976)
- 80 Eidel'man E D Pis'ma Zh. Tekh. Fiz. 20 (22) 10 (1994) [Tech. Phys. Lett. 20 893 (1994)]
- Benard H Ann. Chim. Phys. 23 62 (1901) 81
- Warth A H The Chemistry and Technology of Waxes 2nd 82. editon (New York, 1956)
- 83. Vysokomolekulyarnye Nepredel'nye Spirty iz Kashalotovogo Zhira (High-Molecular Unsaturated Alcohols from Sperm Whale Oil) (Moscow: VINITI, 1966)
- 84. Musaev T, Author's Abstract of Thesis for the Degree of Candidate of Chemical Sciences (Grozny, 1982)
- 85. Vinokur K D, Sikharulidze D G, Chilaya G S, Elashvili Z M Zhidkie Kristally so Spiral'noi Strukturoi i Ikh Ispol'zovanie dlya Sistem Otobrazheniya Informatsii (Liquid Crystals with Helical Structure and Their Use in Information Display Systems) (Tbilisi: Metsniereba, 1988)
- Ioffe I V, Kalinin N V, Eidel'man E D Pis'ma Zh. Tekh. Fiz. 2 86. 395 (1976) [Sov. Tech. Phys. Lett. 2 153 (1976)]
- 87. Ioffe I V, Eidel'man E D Pis'ma Zh. Tekh. Fiz. 15 (2) 9 (1989) [Sov. Tech. Phys. Lett. 15 41 (1989)]
- Mirkin L I Dokl. Akad. Nauk SS SR 186 305 (1969) [Sov. Phys. 88. Dok1 14 494 (1969)]
- 89. Mirkin L I Fizicheskie Osnovy Obrabotki Materiala Luchami Lazera (Physical Basis of Processing of Materials by Laser Radiation) (Moscow: Moscow State University, 1975)
- 90. Bunkin F V, Tribel'skii M I Usp. Fiz. Nauk 130 193 (1980) [Sov. Phys. Usp. 23 105 (1980)]
- Karpov S Yu, Koval'chuk Yu V, Pogorel'skii Yu V Fiz. Tekh. 91. Poluprovodn. 20 1945 (1986) [Sov. Phys. Semicond. 20 1221 (1986)1
- 92 Aleksandrov L N Kinetika Kristallizatsii i Perekristallizatsii Poluprovodnikovykh Plenok (Kinetics of Crystallisation and Recrystallisation of Semiconductor Films) (Novosibirsk: Nauka, 1985)
- 93. Betanelli A I, Danilenko L P, Loladze T N, et al. Fiz. Khim. Obrab. Mat er. (6) 22 (1972)
- 94. Rykalin N N, Uglov A A, Kokora A N Fiz. Khim. Obrab. Mat er. (6) 14 (1972)
- Astapchik S A, Tsarev G L, Bereza M A, Chibot'ko A A 95. Vestsi Akad. Navuk BSS R Ser. Fiz.-Tekh. Navuk. (2) 13 (1987)
- 96. Astapchik S A, Bereza M A Vestsi Akad. Navuk BSS R Ser. Fiz.-Tek h. Navuk (1) 29 (1989)

- 97. Rykalin N N, Uglov A A, Kokora A N *Lazernaya Elektronnoluchevaya Obrabotka Materialov* (Laser and Electron Beam Processing of Materials) (Moscow: Mashinostroenie, 1988)
- Veiko V P, Libenson M N Lazernaya Obrabotka (Laser Processing) (Leningrad: Lenizdat, 1983)
- 99. Duley W W Laser Processing and Analysis of Materials (New York: Plenum Press, 1983)
- 100. Vvedenov A A, Gladush G G Fizicheskie Protsessy pri Lazernoi Obrabotke Materialov (Physical Phenomena in Laser Processing of Materials) (Moscow: Energoatomizdat, 1985)
- 101. von Allmen M Laser-Beam Interaction with Materials (Berlin: Springer, 1987)
- Fizicheskie Postoyannye: Spavochnik (Physical Constants: Handbook) (Moscow: Nauka, 1990)
- 103. Nezhelko V I, Floka L I Poverkhnostnoe Natyazhenie Zhidkikh Metallov i Splavov: Spravochnik (Surface Tension of Liquid Metals and Alloys: Handbook) (Moscow: Mashinostroenie, 1991)
- 104. Ansel'm A I Introduction to Semiconductor Theory (Moscow: Mir; Englewood Cliffs, NJ: Prentice-Hall, 1981)
- 105. Conwell E M *High Field Transport in Semiconductors* (Suppl. 9 to Solid State Phys.) (New York: Academic Press, 1967)
- 106. Tsidil'kovskii I M Termomagnitnye i Termoelektricheskie Yavleniya v Poluprovodnikakh (Thermomagnetic and Thermoelectric Effects in Semiconductors) (Moscow: Nauka, 1973)
- 107. Glansdorff P, Prigogine I Thermodynamic Theory of Structure, Stability, and Fluctuations (New York: Wiley-Interscience, 1971)
- 108. Graham R Fluctuations, Instabilities and Phase Transitions (New York: Plenum Press, 1975)
- 109. Haken H Rev. Mod. Phys. 47 67 (1975)
- 110. Haken H Introduction to Synergetics: Nonequilibrium Phase Transitions and Self-Organization in Physics, Chemistry, and Biology (Berlin: Springer, 1977)
- 111. Haken H Advanced Synergetics: Instability Hierarchies of Self-Organizing Systems and Devices (Berlin: Springer, 1983)
- 112. Stanley H E Introduction to Phase Transition and Critical Phenomena (Oxford: Clarendon Press, 1971)
- 113. Careri G Ordine e Disordine nella Materia Latenza (1982)