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Photon noise: observation, squeezing, interpretation

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Contents

Abstract. A review of modern concepts of photon noise (PN), which is observed even in the case of an ideal laser, is presented. Methods of transformation, squeezing, and nondemolition observation of PN, are described. The optical nondemolition methods seem to be very important for the interpretation of PN. The experiments with PN suppression by negative electron feedback are analysed in detail within two alternative approaches, which could be called the a priori and a posteriori concepts. According to the first approach, PN exists in the laser beam from the beginning, while according to the second it appears only in the detectors. The theory based on the a priori concept predicts the squeezing of the in-loop field — in contrast to the a posteriori one. Several possible crucial experiments using the nondemolition methods are discussed.

1. Introduction

An ideal laser must emit a ‘pure sinusoid’ $E_0 \sin(\omega_0 t)$ with constant amplitude and frequency. However, when the laser light is detected, one observes the photocurrent $i(t)$ to fluctuate: in addition to the constant component I_0, it contains some noise with a spectral density proportional to I_0 and a wide frequency band $\Delta\Omega$ limited only by the inertia of detector and electronics. This noise is named *photon noise*, or *quantum, vacuum, shot* noise.

The history of PN studies goes back to the well-known works of Einstein, in which he revived Newton’s corpuscular theory of light at a new level. However, despite the long period of time that has elapsed since the discovery of PN, up to now its interpretation has met certain difficulties. These difficulties are partially connected with the general problem of describing the measurement process in quantum mechanics (see, for example, Ref. [1]). Recently, the problem of the PN interpretation has acquired a quite unexpected sharpness and even a certain practical significance in connection with the experiments of Yamamoto et al. (see below). In this connection, it appears worth discussing the state of the problem in the light of experience gained in quantum optics in recent years.

Getting rid of the PN, which limits the informational capacity of optical connection systems and the accuracy of optical measurements, remains an unsolved problem in contemporary quantum optics. Here large hopes are being placed on the *squeezed light* [2 – 8], in which PN decreases at some frequencies (Fig. 1).

Technically, the simplest and clearest method for decreasing (squeezing) the observed PN is the use of a negative electronic feedback (FB) stabilising the light intensity and photocurrent [9 – 21]. Then the alternating component of the detector’s current after amplification modulates the intensity of the incident laser light in the opposite phase (Fig. 2). Such a scheme successively ‘suppresses’ the real modulation (determined or noise-induced) of the incident light, as well as the detector’s shot noise and intrinsic noise of the amplifier.

The first experiments of this type were carried out by Yamamoto et al. in 1986 [9] and by Fofanov in 1988 [10, 11].

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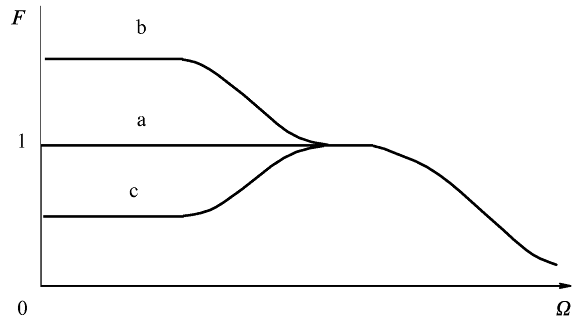

Figure 1. Photocurrent noise intensity (per unit frequency band), F, as a function of frequency, Ω, in the case of (a) ideal laser coherent light (*Poissonian* noise), (b) additional noise modulation of the laser producing *super-Poissonian* ('excess') noise, and (c) amplitude-squeezed laser light (*sub-Poissonian* noise). The maximum frequency modulation is assumed to be less than the limiting frequency of the electronics.

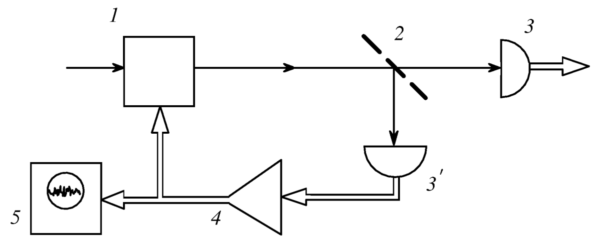

Figure 2. The scheme of the detector's PN squeezing by a negative feedback (FB). *1* — modulator, *2* — beam splitter, *3* and *3′* — detectors, *4* — amplifier, *5* — spectrum analyser. The noise of the external detector *3* increases when the FB is switched-on.

The maximum suppression of the photocurrent shot noise by this means has recently been observed by Masalov et al. [12] (by 18 times in spectral density at frequencies near $\Omega/2\pi \approx 20$ MHz). One uses two ways for controlling the light intensity — through the excitation current of the laser [9 – 11, 13] and with the help of an electro-optical modulator [12].

However, a principal question arises as to whether the quantum fluctuations of the light incident on the laser will be suppressed here as well. There exist two opposite opinions on this subject. According to the 'optimistic' point of view, the noise of both the current and the light are being squeezed [6, 9, 12, 15 – 17], whereas according to the 'pessimistic' one, only the current noise is being squeezed [19 – 21]. Pessimists believe that the light remains unsqueezed (classical) and even gains some extra fluctuations due to the modulation. These fluctuations are anticorrelated with the detector's shot noise, which causes the observed suppression of the latter. (In some works, only current noise is considered and this dilemma is not discussed at all.)

One can put forward two apparently convincing arguments in favour of the pessimistic point of view, on the basis of the well-known properties of optical modulators and beam splitters. Note in advance that they both failed in the framework of the theory presented in this paper, which mainly uses the approach of Shapiro et al. [15].

The first argument is based upon the fact that a classical source of the field (a linear light modulator in the FB loop would provide an example) always adds only a coherent, classical component to the initial field. Hence by introducing the FB, the original classical light must remain classical, unsqueezed.

Further, it was discovered experimentally that the 'external' light beam extracted from the FB loop by a beam splitter not only reveals no squeezing, but even has excessive fluctuations. This fact would appear also to prove the correctness of the pessimists, as the beam splitter is thought always to keep the type of the light statistics — its squeezing or unsqueezing — unchanged, so that the 'in-loop' light incident on the beam splitter should also be super-Poissonian, unsqueezed.

It is essential that even if the 'in-loop' light is actually squeezed, the problem of its extraction for further applications remains. For this one should use, instead of conventional beam splitters, rather complicated methods of optical 'nondemolition' measurements of intensity (QND methods) [1, 22 – 29]. The practical realisation of such methods of measurements [27 – 29], as well as observation of the squeezing of PN in laser light (see Refs [2 – 8]), has great meaning in the history of PN studies. Optical QND methods make it possible to solve experimentally the question about the in-loop field squeezing in the presence of FB, so it ceases to be an academic problem.

Notice here that the schemes with FB can find another application as well. Thus, on the basis of these schemes, a new method for the 'electronic' amplification and generation of light without the use of population inversion [30] has recently been suggested and studied experimentally. In this method, the amplification of the original laser light modulation in the output beam is essentially due to the FB.

The theory of optical systems with FB has been developed in Refs [6, 9, 12 – 21]. The models used can be divided in two main groups which we refer to as *a priori* and *a posteriori*. The first group predicts the squeezing of the in-loop field under some conditions; the second one denies this possibility or ignores it.

In *a priori* models [6, 9, 12, 13, 15 – 17], the PN is present in the light field from the beginning; it formally arises because of the use of nonnormally-ordered quantum field correlation functions (or because of introducing Poissonian sources of noise into the classical kinetic field equations [17]).

In *a posteriori* models [19 – 21], the PN arises only in the photodetector (according to Refs [19, 21], as a result of the *quantum measurement* process), so that owing to the FB the field can acquire only excessive classical fluctuations.

In the present work we shall try to show that in order to describe experiments with FB and with a nondemolition measuring instrument, the *a priori* concept must be used. We stress that the proposed classification makes sense for describing only the specified group of experiments. But for the majority of optical effects, both approaches yield the same predictions.

Some papers have used a semiclassical description in which the field is not quantised and only its intensity is considered [14, 15, 17]. The photoelectron statistics have been computed in Refs [14, 15, 21] by the theory of stochastic 'point' processes with delayed self-action. This approach is based essentially on the *a posteriori* concept of PN and cannot in principle predict the in-loop field squeezing due to FB and describe the case when the original light has some preliminary squeezing.

In Ref. [19], based on quantum measurement theory and a model of a laser with power controlled by the amplified photocurrent, the conclusion is drawn that the field emitted by the laser with FB is classical.

In Ref. [20], classical character of the in-loop field is justified on the basis of very general arguments about the correspondence between classical and quantum electrodynamics when describing the interaction between macroscopic objects.

Ref. [21] stresses the connection of the problem of photodetection with FB with the theory of continuous quantum measurements. The paper disputes the applicability in this case of the conventional theory of photodetection [31], which expresses the observed photocurrent statistics through correlation functions of the incident light, since ''...Glauber's approach is valid only for the fields whose sources are not correlated with the detector's atomic states''. Note, however, that the characteristic atomic timescales for photoionisation processes are certainly many orders of magnitude less than the real time delay in the FB loop, so that the correlations of the incident light with the detector's atomic states are hardly of importance in the case considered. In fact, it is just Glauber's correlations that have been used in Ref. [21] to determine the initial absolute probabilities for the point process, from which conditional probabilities were derived afterwards.

A concrete calculation in Ref. [21] is made under the assumption that the in-loop field is in a coherent state. Such an approach excludes immediately the possibility of considering squeezed states for the initial (before the modulator) or in-loop (between the modulator and the detector) field and makes the subsequent theory essentially classical.

However, some models [6, 9, 12, 15 – 17], which are based upon the *a priori* concept for PN, lead to the squeezed in-loop field. The degree of suppression of the in-loop field PN predicted in Ref. [21] and Ref. [17] is the same.

In Ref. [12], the degree of the in-loop field squeezing is calculated and its unusual properties are emphasised (see Section 3.5).

In Ref. [15], Heisenberg operators of the in-loop field depend on the previous photocurrent values, which leads to a self-consistent change of these operators and accordingly of their commutators. This paper also uses the semiclassical model as well; it is found that in the case of coherent initial light, the two models yield the same predictions for the current fluctuations observed with the use of two conventional detectors — an in-loop detector and an external detector.

In Ref. [17], the laser and the FB loop are described by kinetic equations with Langevin forces. Here an *a priori* 'corpuscular' concept is actually used: the light is represented by a photon flux with originally Poissonian fluctuations which are suppressed by the FB under some conditions. The in-loop light passing through the beam splitter may consist of a sub-Poissonian photon flux, and the fraction of the beam reflected by the beam splitter is always super-Poissonian. The reason for such strange behaviour of the beam splitter is not discussed.

Let us enumerate some unexpected, paradoxical features of the quantum *a priori* theory of the optical systems with FB. Unfortunately, they have almost never been discussed in the literature, which stimulated to a large extent the writing of the present review.

1. The relative value of the detector's current fluctuations F_i at some frequencies can be made arbitrarily small, whereas according to the conventional theory of the squeezed light photodetection, F_i reaches a minimum value $1-\eta$, where η is the detector's quantum efficiency. This limit was significantly surpassed in the experiments [12].

2. Further, F_i turns out to be less than the calculated relative PN of the incident light F_N [12] — again contrary to the generally accepted concept. Moreover, when the in-loop light passes through the beam splitter or an absorber, its relative noise decrease and the light can even turn from classical to nonclassical (this effect may be called 'dissipative squeezing').

3. The amplified, clearly classical signal $w(t)$ controlling the modulator is taken to be proportional to the Heisenberg field operator [15]. It thus 'realises' the quantum stochastic process that appears due to quantum fluctuations of the field.

4. Operators of the in-loop field propagating from the modulator to the detector do not satisfy the standard commutation relations: $[a, a^+]$ is not now 1. As a result, there is a break down of the uncertainty relation for the variance of two field quadratures q, p describing respectively the amplitude and phase noise modulation: one of them is squeezed without the corresponding stretching of the other [15] [$a = (q + \mathrm{i}p)/\sqrt{2}$ is the photon annihilation operator]. Light with such unusual properties may be called 'supersqueezed'; another name — *anticorrelated light* — was suggested in Ref. [12].

The above points mean that the optical experiments with FB and their analysis appears to be of a certain methodical interest not only for quantum optics, but also in a broader sense, for example, for deeper understanding of the essence of quantum fluctuations in general, for the quantum theory of measurements, etc.

The purpose of the present work is to provide a general concept of PN and of the methods for its calculation, transformation, and squeezing. We use a conceptually simple and natural description that permits parallel consideration of both essentially quantum and pure classical features of the phenomena under discussion.

Different types of squeezed light and methods for its preparation are described in detail in the review literature [2 – 8], so that the present review focuses on the description of only one type — *amplitude-squeezed* laser light (under the conditions considered it coincides with *quadrature-squeezed* light) and on its preparation by a parametric down-conversion (Section 2).

A lot of attention will be given to PN suppression in the scheme with FB (Section 3). This is connected with the fact that here a purely methodical question about the essence of PN relates directly to an important practical problem — the strong possibility of light squeezing by FB. As we have already pointed out, there are two opposite opinions on this subject in the literature. In the present work an attempt is made to justify the possibility of the in-loop field squeezing on the grounds of the existence of nondemolition methods for PN registration. For comparison, an alternative '*a posteriori*' quantum model which does not produce the field squeezing is also considered. The ultimate choice between the different approaches can obviously be made only after the corresponding crucial experiments have been performed.

Much attention will have to be paid to the semiclassical calculations and their comparison with more consecutive quantum computations. As the semiclassical approach is clearly inadequate to describe the observed effects in modern quantum optics, we shall not dwell on it here. Instead, a 'classical analogue' will be considered — a fully classical visual model that follows from the quantum model at the limiting values of some parameters.

The presentation begins with an elementary discussion of the existing interpretations of PN, its general properties, and new methods for its experimental detection. Several 'crucial' experiments demonstrating the necessity of the *a priori* description of PN in some cases are also suggested (Section 2.1). In Section 2.2 the photodetection theory is briefly presented with the accent on the methodical moments. In Section 2.3 the notions of weakly modulated coherent light and quadrature signals q and p describing, respectively, the amplitude (AM) and phase (PM) modulation of the coherent light are introduced. Section 2.4 provides description for the transformation of the PN and modulated coherent fields by beam splitters and homodyne detectors. Section 2.5 is devoted to parametric down-conversions and an important concept in modern quantum optics—quadrature-squeezed light and its close classical analogue, classical squeezed light. Here a simple example illustrates the idea of quantum nondemolition measurements.

In Section 3.1, the dynamics of the optical system with FB as in Fig. 2 (in spectral representation) is computed in the linear approximation. As in other linear problems of quantum optics, it is essentially the same in the quantum and the classical description. In Section 3.2 the classical Green function for the system and the associated commutation relations which appear during the transition to quantum theory are considered. Fluctuations at different points of the system are calculated in Section 3.3. The possibility of watching the in-loop field by nondemolition methods is justified in Section 3.4. In Section 3.5 the formalism used and the paradoxes listed above are discussed. An alternative theory considered in Section 3.6 comes from the *a posteriori* concept of the PN and hence does not give squeezing of the in-loop field. Finally, in Section 3.7 a simple corpuscular model of the system with FB that permits one to understand some of its features is analysed. The uncertainty relations and Cauchy–Schwartz inequalities for the field spectral densities are derived in the Appendix.

2. General description of PN

2.1 Properties and different interpretations of PN

Several interpretations are known for the observed shot noise of a photodetector illuminated by an ideal laser beam.

1. In the framework of the *semiclassical theory*, the electromagnetic field is considered as classical and the matter is assumed to obey quantum laws. The photoelectrons are then believed to emerge randomly in the detector with a probability proportional to the field intensity. An ideal laser field has a constant intensity proportional to E_0^2, so that all moments of time are equivalent (after averaging over the optical period $2\pi/\omega_0 \sim 10^{-14}$ s has been performed). As a result, the instants of the photoelectron appearances makes up a *Poisson random process*, which leads to the observed photocurrent fluctuations. Of the same character are the fluctuations of the current that passes through a vacuum diode in the saturation regime and which are described by the Schottky formula. (We remind the reader that the conventional sources of current, for example the inductive electromotive force, produce no Poisson fluctuations.) Therefore the semiclassical model can be called *a posteriori*, as here the PN appears only as a result of the detection process.

The PN is often associated with the fact of the electric charge discreteness. Notice that a strictly periodic regular sequence of pulses contains no noise, its spectrum consists of the harmonics of the pulse timing frequency N_0. Therefore, in order to explain the PN, in addition to the charge discreteness one should also assume a random character of free electrons birth moments under the action of light of constant intensity.

It is important that the light can, in principle, be detected by a pure analogue device, for example a microcalorimeter. Then according to the postulates of quantum theory (see below), the calorimeter's energy must increase only by a discrete multiple of $\hbar\omega_0$, with the moment of the subsequent energy transfer being random every time. The analogue photodetectors must thus also reveal the PN, which contradicts the assumption about its association with the charge discreteness.

2. According to another widespread concept, lasers as well as all other light emitters radiate a random sequence of energy packets — photons — and an ideal detector simply converts them into the observed pulses of current with the original chaotic character of the photon time distribution. In this visual '*a priori*' picture, the PN is originally present in the radiation field due to its *corpuscular* structure, so that discrete and analogue detectors get equal rights. Here, however, well-known difficulties emerge connected with the explanation of wave phenomena depending on the phase of the field — interference and, diffraction.

3. In the framework of quantum theory, the electromagnetic field is a quantum object, whereas the photodetector together with electronic amplifiers is considered as a classical device. The mostly widespread — *Copenhagen* — interpretation usually assumes that PN is the appearance of the quantum fluctuations arising *during the observation* of the field energy by macroscopic detectors. What has taken place *before the observation* is usually not well known: even in the best case one may only find *the state-vector* for the field $|\psi\rangle$.

One of the most important achievements in quantum optics is the conclusion that the field of an ideal laser is described to a good approximation by a *coherent state* $|\psi\rangle = |E_0\rangle$. Then according to the quantum formalism, the measured field energy, and correspondingly the number of photons (i.e. energy divided by $\hbar\omega_0$), has no definite values, but fluctuate in accordance with the Poisson distribution. In the case of other field states, the character of the energy fluctuations may be non-Poissonian.

The observed quantum fluctuations of the energy (number of photons) can be absent only in the case that $|\psi\rangle$ is an eigenvector of the energy operator. The practical preparation of such states with a given photon number represents one of unsolved problems in quantum optics. The light in this state can be visualised by a regular flux of equally distant particles (an effect of the ideal *photon antibunching*).

Which of the three basic pictures described above is the 'most correct'? What 'actually' is PN? In the framework of the Copenhagen interpretation of the quantum formalism, this question makes no sense as quantum theory predicts only the statistics of counts in macroscopic devices — photodetectors in the given case — and the statistics cannot be used for recovering all properties of the incident light. Thus in the framework of the modern quantum theory of light, the statement that 'light consists of photons' can be neither confirmed nor discarded (the photon notion in modern quantum optics is discussed in Ref. [32] in more detail). As a result, different interpretations (unlike different concepts) should be evaluated not from the point of view of their 'correctness' but only by consideration of their convenience for the qualitative description of some class of experiments.

For example, although according to postulates of quantum theory of measurement PN appears only during the process of detection, it is convenient, nevertheless, to assume that PN has an *a priori* character, i.e. that the quantum fluctuations are present in the free radiation field before the detector. These are referred to as 'quantum', 'zero-point', or 'vacuum' fluctuations of the field which are added to a laser or some other 'real' field. In what follows we shall try to show that such an interpretation describes more adequately the PN in the presence of a feedback, i.e. in that case it acquires the status of a concept.

The *a priori* PN formally appears when one uses *nonordered* products of field operators for describing the statistics of the observed macroscopic effects (see Sections 2.2 and 2.3). Then the noncommutativity of these operators is significant: $aa^+ - a^+a \equiv [a, a^+] = 1$. Here a and a^+ are the operators of photon annihilation and creation in one mode of the field — a plane monochromatic wave with a certain polarisation. Operators like a^+a or a^+a^+aa, in which the annihilation operators act first on state-vectors $|\psi\rangle$ to the right of them, so that vacuum averages of such operators vanish, are called ordered.

Let us consider how PN appears directly in experiments.

Let the intensity of the incident stationary light on the detector be sufficiently small, then at the detector's output one will observe separate pulses of current (Fig. 3). We shall measure the number of such pulses appeared in a fixed time interval T much longer than the coherence time of the light (this condition makes the consecutive observations statistically independent). In the repeated tests, the number of photons observed in such a way n_T will fluctuate. Modern photon counters have a high efficiency ($\eta \approx 1$) and allow

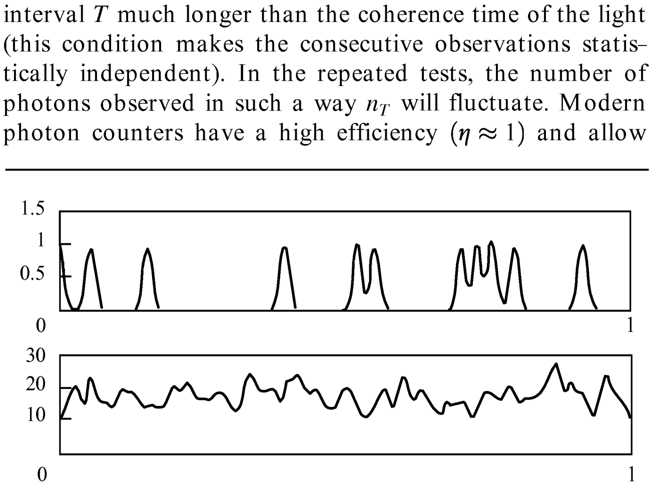

Figure 3. Discrete and analogue detection. The superposition I_0 of pulses with the form $f_i(t) = \exp[-(t - t_i)^2/\tau^2]$, where $\tau = 0.01$ s. Moments of time t_i are randomly distributed in the shown time interval, the duration of which is taken to be 1 s. In the upper plot $I_0 = 10$ s^{-1}, i.e. $I_0\tau = 0.1$; in the lower, $I_0 = 1000$ s^{-1}, i.e. $I_0\tau = 10$.

measurement of the statistics of the random number n_T reliably enough to find the distribution $p(n_T)$ and moments of the distribution $\langle n_T^k \rangle$. According to quantum theory, these statistics fully determine the field state.

As was already noted, the field of an ideal laser is well represented by a coherent state, $|\psi\rangle = |E_0\rangle$. In such a state the number of pulses n_T obeys the Poisson distribution and correspondingly has a variance equal to the mean number of pulses $\langle n_T \rangle$: $\langle \Delta n_T^2 \rangle = \langle n_T^2 \rangle - \langle n_T \rangle^2 = \langle n_T \rangle$. The 'Poissonian' character of the photocurrent in the case of laser light and, in particular, the formula $\langle \Delta n_T^2 \rangle = \langle n_T \rangle$, is confirmed with a high degree of accuracy in the experiments.

Notice that in theoretical formulas the angle brackets mean the operation of quantum averaging over the field state $|\psi\rangle$: $\langle \ldots \rangle = \langle \psi | \ldots | \psi \rangle$; these quantum averages are assumed to be coincident with the results of stationary experiments averaged over time.

Nonlaser beams usually display additional, super-Poissonian fluctuations of the number n_T, allowing one to speak of *photon bunching*. In particular, in the case of thermal single-mode light, Einstein's formula $\langle \Delta n_T^2 \rangle = \langle n_T \rangle + \langle n_T \rangle^2$ is valid and the Brown – Twiss effect connected with it occurs. In the case of thermal sources, the excess noise $\langle n_T \rangle^2$ can be visually explained by the interference of wave packets — photons emitted by individual atoms of the source. Since the phases of the waves in these packets are randomly changed, the amplitude of the resultant field strongly fluctuates.

One of the most important achievements in quantum optics is the introduction of the concept of light with *photon anti-bunching*, which yields during detection a variance $\langle \Delta n_T^2 \rangle$ less than $\langle n_T \rangle$, and the development of the principles of its generation. Such light cannot be described by semi-classical theory, in the framework of which there is obviously no field that produces during the detection noise less than the wave of constant amplitude E_0 (the in-loop field in a system with negative FB is an exception discussed below).

If the intensity is sufficiently high, individual pulses of photocurrent overlap (see Fig. 3). Then it is convenient to go over from the discrete observable value n_T to a continuous analogue random value $i(t) = n_T/T$, i.e. to the photocurrent (divided by the electron charge e; for simplicity we put $e = 1$). For stationary sources the value $\langle i \rangle = \langle n_T \rangle / T = I_0$ does not depend on time; it has the meaning of light intensity in units of photons s^{-1} (mean photon flux) multiplied by the detector's quantum efficiency η. The condition of strong pulse overlap obviously has the form $I_0\tau \gg 1$, where $\tau \sim 2\pi/\Delta\Omega$ is the pulse duration and $\Delta\Omega$ is the frequency band of the detector and electronics. Then the statistics of photocurrent fluctuations will approach Gaussian form. An important characteristic of the current fluctuations is their spectral density $\langle i^2(\Omega) \rangle$ at different frequencies.

Thus, depending on the type of detector used — discrete or analogue — there are two main types of PN observational appearance: fluctuations $\langle \Delta n_T^2 \rangle$ of the discrete number of photocounts n_T over a sampling time T, and fluctuations of the current $i(t)$ at some frequency Ω with spectral density $\langle i^2(\Omega) \rangle$. Correspondingly, two types of nonclassical light are distinguished: with photon antibunching and with sub-Poissonian (subshot) noise. Below we shall discuss only the latter case.

Let us present some qualitative relations. In the case of an ideal laser beam, the spectral density of the photocurrent is constant within the bandwidth of the detector and electronics $\Delta\Omega$; as will be shown in the next section, it is determined by the well-known Schottky formula:

$$\left\langle i^2(\Omega)\right\rangle = I_0 \, . \tag{2.1.1}$$

Here $\Omega \neq 0$ and the integration over both negative and positive circular frequencies is assumed. If one goes over to positively determined conventional frequencies $f = \Omega/2\pi > 0$, a factor of $2/2\pi$ is included ($\langle i^2(f)\rangle = I_0/\pi$). This level of fluctuations is called the standard quantum limit (SQL). We shall relate the term PN just to that Poissonian part of the noise.

Let us estimate the value of PN. The current fluctuations variance is $\langle \Delta i^2\rangle = I_0\Delta\Omega/\pi$, where $\Delta\Omega$ is the effective electronics bandwidth. The standard deviation ('uncertainty' of the current) is equal to the square root of the variance, $\Delta i = (I_0\Delta\Omega/\pi)^{1/2}$. The relative value of the PN is characterised by the relation

$$\delta = \frac{\Delta i}{I_0} = \left(\frac{\Delta\Omega}{\pi I_0}\right)^{1/2} \, . \tag{2.1.2}$$

Let the power of the laser with wavelength 0.5 μm be 1 mW. With $\eta = 1$ the mean fluxes of electrons and photons will be the same: $I_0 = N_0 = 2.5 \times 10^{15}$ s^{-1} (in conventional units, the current is $eN_0 \sim 0.4$ mA). Hence at $\Delta\Omega/2\pi = 10^9$ Hz we get $\delta = 10^{-3}$, i.e. the uncertainty in intensity and photocurrent is about $\pm 0.1\%$.

Notice that the condition of the strong overlapping of the photocurrent pulses $I_0 \gg 1/\tau = \Delta\Omega/2\pi$, which allows the transition to the analogue description, automatically ensures that the relative fluctuations are small as well, $\delta^2 \ll 1$. Let $\Omega < \Delta\Omega$ be the frequency at which the spectral density of the photocurrent is measured, then $I_0\tau \gg 1$ implies also $I_0 T_\Omega \gg 1$, where $T_\Omega = 2\pi/\Omega$ is the period of oscillations with frequency Ω. Therefore, the analogue description of the photocurrent in terms of spectral density suggests a lot of pulses of the current to occur over the oscillation period T_Ω.

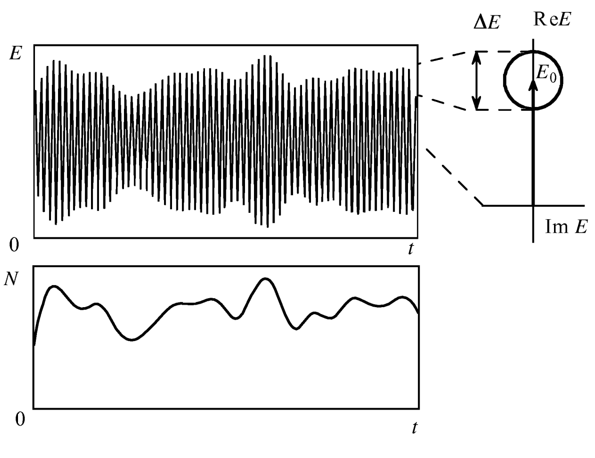

Figure 4. A visual representation of the PN of a coherent field with an amplitude E_0 as a stochastic amplitude modulation with a standard deviation $\Delta E = \sqrt{\Delta\Omega/4\pi}$. In contrast to the 'real' stochastic modulation (excess noise), the 'vacuum' modulation spectrum is unlimited, so the recording equipment bandwidth $\Delta\Omega$ should be taken into account in advance. The field is normalised so that $|E_0|^2$ is equal to the mean power of the light beam divided by $\hbar\omega_0$, i.e. to the mean photon flux. The upper figure shows the dependence of the field strength on time and the corresponding polar diagram (to the right). The 'envelope' of the field is shown in the lower figure. In the case of squeezed light, an excess noise modulation is added which is anticorrelated with the vacuum modulation, therefore ΔE decreases.

The current fluctuations $\Delta i \equiv \left(\langle \Delta i^2\rangle\right)^{1/2}$, in accordance with the *a priori* concept, can be visualised as a result of slow fluctuations of the beam power $\hbar\omega_0 N$ and correspondingly, of the field amplitude ΔE (Fig. 4). We draw attention to the difference of the picture shown in Fig. 4 from the naive picture of a random sequence of photon-beams.

We stress that the obvious *a priori* representation of the PN in Fig. 4 as a noise AM-modulation of the coherent field is not universal. For example, it is of little use for describing experiments with discrete detection when $\delta \gg 1$ and the photocurrent statistics n_T is measured over a fixed sample interval T. As we already noted, the numbers n_T are distributed according to a Poissonian law which is hard to represent by the stochastic amplitude modulation of a monochromatic field. This is an example, characteristic for quantum models, of the dependence of visual *a priori* concepts on type of the measurement device used. Such a dependence follows from the Copenhagen treatment of quantum mechanics. Other examples of the inadequacy of Fig. 4 are given in the discussion to Fig. 5.

Let us find ΔE from the Schottky formula for power fluctuations $\Delta N = (N_0\Delta\Omega/\pi)^{1/2}$. Let $E(t) = (E_0 + \Delta E)\times\cos(\omega_0 t)$; then, because of the equal contribution of the magnetic field, the beam power divided by $\hbar\omega_0$ takes the form $N = cA(E_0^2 + 2E_0\Delta E)/8\pi\hbar\omega_0$, where A is the beam cross-section. After renormalising the field strength $E \to E(8\pi\hbar\omega_0/cA)^{1/2}$, we get $N = E_0^2 + 2E_0\Delta E$. Now E has the dimension s$^{-1/2}$ and the field amplitude uncertainty has the form

$$\Delta E = \frac{\Delta N}{2E_0} = \left(\frac{\Delta\Omega}{4\pi}\right)^{1/2} \, . \tag{2.1.3}$$

The relative fluctuations of the field amplitude (the modulation depth) is half the relative power amplitude fluctuations δ

$$\frac{\Delta E}{E_0} = \frac{\delta}{2} = \left(\frac{\Delta\Omega}{4\pi I_0}\right)^{1/2} \, . \tag{2.1.4}$$

Thus, the depth of the vacuum modulation depends on the bandwidth of the measurement equipment and on the light intensity, i.e. it must increase as part of the light flux is absorbed.

As is clear from the polar diagram in Fig. 4, in addition to the amplitude fluctuations, vacuum phase fluctuations are also present (in Fig. 4, fluctuations of the period length $\Delta T = \Delta\phi/\omega_0$ should correspond to them, which is hard to represent on the chosen scales)

$$\Delta\phi = \frac{\Delta E}{E_0} = \left(\frac{\Delta\Omega}{4\pi N_0}\right)^{1/2} = \frac{\delta}{2} \, . \tag{2.1.5}$$

For the parameters chosen above, we have $\Delta\phi = \pm 5 \times 10^{-4}$ rad.

Hence, the product of uncertainties of the photon flux and the phase is equal to the effective bandwidth of the electronics expressed in Hertz:

$$\Delta N \Delta\phi = \frac{\Delta\Omega}{2\pi} \, . \tag{2.1.6}$$

The ratio of N to the bandwidth in Hertz is just the number of photons in one longitudinal mode n (so far only one transverse mode has been assumed, i.e. a beam narrower than the coherence radius). As the result, Eqn (2.1.6) takes the usual form $\Delta n \Delta \phi = 1$.

In the case of the amplitude-squeezed light, the 'circle of uncertainty' in Fig. 4 is transformed into an ellipse and ΔN decreases with respect to the standard quantum limit $(N_0 \Delta\Omega/\pi)^{1/2}$ as $\Delta\phi$ increases correspondingly.

Usually, a relative quantity is measured — the Fano factor for the spectral density of the photocurrent $F(\Omega) = \langle i^2(\Omega)\rangle / I_0$, which must be unity for an ideal laser, $F(\Omega) = 1$ (at $\Omega < \Delta\Omega$). Noncoherent (or noise-modulated coherent) sources of light produce, apart from PN, excess noise as well, with $F(\Omega) > 1$ (see Fig. 1) and the photocurrent (and light) is called super-Poissonian. Sub-Poissonian, or squeezed, light yields $F(\Omega) < 1$ in some frequency band during the detection; it is then called nonclassical.

Consider now several possible experimental schemes illustrating the properties of PN and possibilities for its suppression by FB (Fig. 5).

The scheme in Fig. 5a reveals the independence of the relation $\langle i^2(\Omega)\rangle = I_0$ on the light intensity which can be decreased by absorption. This means that (in the absence of FB) the depth of the hypothetical 'vacuum' noise modulation $\Delta E/E_0 \cong 1/\sqrt{I_0}$ [see Eqn (2.1.4) and Fig. 4], unlike the usual modulation, increases as a result of the absorption.

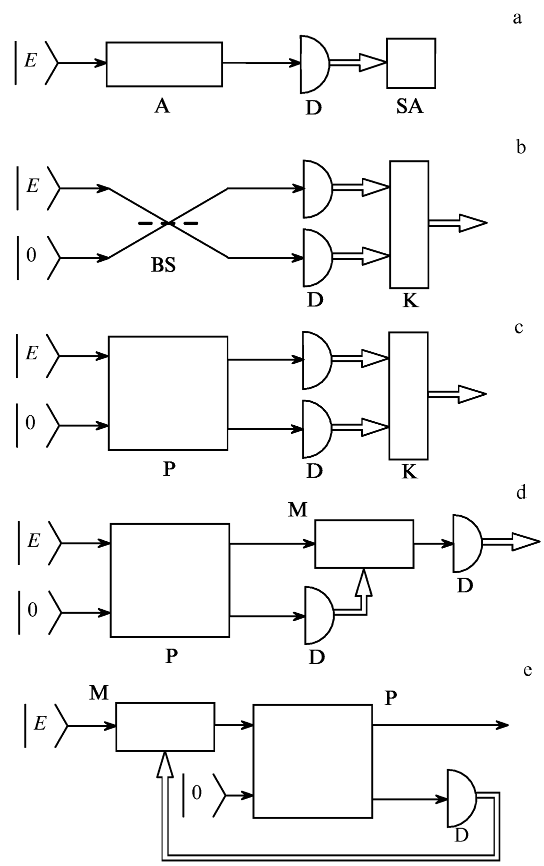

Figure 5. Schematic diagram of experiments for investigating PN suppression. $|E\rangle$ is a coherent state of the incident field (at the first input of the system), $|0\rangle$ is a vacuum state (at the second input), A is the absorber, D are detectors, SA is the spectrum analyser, BS is the beam splitter, K is the correlometer, P is the nondemolition parametric transformer, M is the modulator.

In Fig. 5b the initial light beam is split into two parts by a partially transparent mirror and intensity correlations in the two output beams are observed by means of two detectors. This device is called a Brown – Twiss correlometer. Such correlometers detect only excess, classical modulations, so the coherent state produces no correlation — again in contradiction to the obvious '*a priori*' picture in Fig. 4. In photon language, the absence of the Brown – Twiss effect in the case of laser light is clearly explained by the chaotic distribution of the original Poissonian photon flux at the beam splitter. The two output beams then show independent Poissonian fluctuations.

However, if one uses, instead of the partially transparent mirror, a parametric 'nondemolition beam splitter' (Fig. 5c) described below in Section 2.5, then the correlation will be observed even in the case of laser light. This statement follows from the experiments described in Refs [27, 29] and is confirmed by calculations [see Eqn (2.5.14)]. It is of great importance for the PN theory and evidence that the currents in both detectors from Fig. 5c preserve information about the PN in the original light beam — contrary to the *a posteriori* concept! One may say that the 'nondemolition beam splitter' enables one to control individual occurrences of PN, to prepare its (enhanced) light and electronic replicas.

It is natural now to assume that if one feeds an amplified signal from one of the detectors not into the correlator, but into the modulator mounted before the second detector (Fig. 5d), one can suppress the PN of the latter, i.e. prepare the squeezed light.

But on the other hand, it can be shown that the modulator controlled by a classical macroscopic signal, cannot transform classical (unsqueezed) light into nonclassical light. The paradox is solved by assuming that the controlling signal of the modulator cannot be considered as classical in this case (Section 3.5). The experimental confirmation of such effects would clearly be important.

A natural modification of the scheme (Fig. 5d) is the interchanging of the position of the modulator and the 'nondemolition beam splitter' (Fig. 5e). This scheme will be examined in detail in Section 3.4. Notice that the scheme in Fig. 5e differs from that in Fig. 2 only in replacing a conventional beam splitter by a 'nondemolition' one.

2.2 Light intensity and photocurrent fluctuations

For a quantitative description of fluctuations in time, one needs to take into account a set of longitudinal modes (frequency components). A light beam with one transverse mode is described in some cross-section by a *positive-frequency* function of the following form:

$$A(t) = (2\pi)^{-1/2} \int_0^\infty d\omega \exp(-i\omega t) a(\omega) \,. \tag{2.2.1}$$

Here $a(\omega)$ is the photon annihilation operator in a longitudinal mode with frequency ω. The total field has the form $E(t) = A(t) + A^+(t)$. We are interested in a quasimonochromatic optical field with a central frequency ω_0 of order 10^{14} Hz and a bandwidth $\Delta\Omega$ limited by the response of the electronics to values not exceeding 10^9 Hz, and thus in Eqn (2.2.1) we omitted the integrand factor

$\sqrt{\omega}$. With the normalisation chosen, the operator $N(t) = A^+(t)A(t)$ is the operator of energy flux (power or intensity) divided by $\hbar\omega_0$ (i.e. the flux measured in units of photon s^{-1}). The field in an arbitrary cross-section of the beam is found from Eqn (2.2.1) simply by considering the delay: $t \to t - \tau$.

To describe the field dynamics, we shall use the Heisenberg representation, so that our formulas conserve the form when going over to the classical description (here the symbol for Hermitian conjugation '+' should be read as that of complex conjugation '*'). Quantum specifics appear only in the noncommutativity of the operators A, A^+ and, at the last stages of calculations, in the averaging procedure. In the quantum version, the latter is being made over the initial state of the field $|\psi\rangle$ at the optical system input. In classical optics, the function $A(t)$ is called the *analytical signal* and the averaging is performed with the help of some distribution function describing the incident field statistics.

In order to determine the main features of the photodetection theory, we neglect the difference of the detector's quantum efficiency η from 1 and its response ($\Delta\Omega = \infty$). We shall assume that the detector's surface is less than the field coherency area. Let us fix some time interval $[0, T)$ and assume initially that a certain number of pulses $n = n_T$ arose in this interval at some instants t_i, $0 \leqslant t_i < T$, $i = 1, \ldots, n$ (we consider n and $\{t_i\}$ as independent random values). Under this condition, the photocurrent observed at some instant t is represented by the sum of n pulses:

$$i_n(t) = \sum_{i=1}^{n} \delta(t - t_i) \ . \tag{2.2.2}$$

As usually in physics, expressions with generalised functions like $\delta(t)$ make sense only when integrated with some weight function — in the given case, with the electronics transmission function $k(t)$.

Formula (2.2.2) determines the observed quantity (c-number) through the set $\{t_i\}$. The quantum model for the detector's atom photoionisation can be used to express the statistics for this set through a set of normally-ordered correlation functions of the free field [31]:

$$\begin{aligned} &\langle A^+(t)A(t)\rangle \equiv \langle N(t)\rangle \equiv G_1(t) \ , \\ &\langle A^+(t)A^+(t')A(t)A(t')\rangle \equiv \langle :N(t)N(t'):\rangle \equiv G_2(t, t') \ , \end{aligned} \tag{2.2.3}$$

and so on. Here $A(t)$ are Heisenberg operators of the field at the detector's surface (q-numbers) which are connected with the field operators at the input of the optical system by classical (phenomenological) Green functions [33]; the colon denotes the operation of the normal ordering. The averaging is taken over some initial state of the field defined by the properties of the light source. Thus, some products of the operators turn out to be *observable*. As a result, a link $q \to c$ between q- and c-numbers is established — a necessary element in any quantum model describing experiment. Then one can use the well-developed theory of random point systems.

Formally similar relations between a stochastic field and the photocurrent are postulated in the semiclassical theory of photodetection as well.

An important point here is the assumption used in Eqn (2.2.2) of the discrete character of the information transfer $q \to c$ which causes the shot (Poissonian) noise of the photocurrent in the case of a coherent state of the field. This is visualised by a fully random photon distribution in time. There are no photons in the semiclassical theory, and this is the 'discretisation', justified by the charge discreteness that yields the shot noise, and the field intensity fluctuations can only increase current fluctuations and produce an 'excess' noise (in the absence of FB). At the same time, in quantum theory field states are possible in which the excess noise provides a negative contribution and compensates Poissonian fluctuations. Visually, this corresponds to a temporally regular photon distribution (*antibunching of photons*).

Notice that our description may be applied to other types of photodetectors as well; for example, to those based on the thermal action of light. Then Eqn (2.2.2) determines (in units of $\hbar\omega_0$) the power and energy transmitted. Hence it is clear that in quantum theory, charge discreteness has no significant bearing on the appearance of shot noise; the noise formally arises as a result of postulating the discreteness of the relation $q \to c$.

From Eqn (2.2.2) we find the current correlation function

$$\begin{aligned} \overline{i_n(t)i_n(t+\tau)} &= \sum_{i=1}^{n}\sum_{j=1}^{n} \overline{\delta(t - t_i)\,\delta(t - t_j + \tau)} \\ &= \delta(\tau)\sum_{i=1}^{n} \overline{\delta(t - t_i)} + \sum_{i \neq j}^{n} \overline{\delta(t - t_i)\,\delta(t - t_j + \tau)} \ . \end{aligned} \tag{2.2.4}$$

The diagonal part of the double sum separated in the latter equality and depending on the current pulse autoconvolution yields the 'white' noise caused by the process discreteness. The nondiagonal part describes changes connected with a possible regularity in the instants $\{t_i\}$ ('bunching' or 'anti-bunching' of points t_i).

The horizontal line in Eqn (2.2.4) means *classical* averaging over the time distribution $\{t_i\}$

$$\begin{aligned} \overline{f(t_i)} &= \int_0^T \mathrm{d}t_i\, f(t_i) w_1(t_i) \ , \\ \overline{f(t_i, t_j)} &= \int_0^T\int_0^T \mathrm{d}t_i\,\mathrm{d}t_j\, f(t_i, t_j) w_2(t_i, t_j) \ . \end{aligned} \tag{2.2.5}$$

Here $w_k(t_1, \ldots, t_k)$ are the distribution densities determining the probabilities for the k points to be inside small intervals near the moments of time $t_1, \ldots, t_k$. Here the appearance of other points in the interval considered is not excluded, i.e. the condition $k = n$ is not required. Obviously, $w_k(t_1, \ldots, t_k) = 0$ at $k > n$. In a more general case, conditional distributions $w_k(t_1, \ldots, t_k|n)$ at $k = n$ are introduced (see Ref. [21]).

According to Ref. [31], w_k are proportional to the normally ordered correlation functions G_k defined in Eqn (2.2.3) (the field is considered to be stationary):

$$w_1(t) = \frac{G_1}{C_1} \ , \quad w_2(t, t') = \frac{G_2(t - t')}{C_2} \ , \ \ldots \tag{2.2.6}$$

Here C_k are normalisation coefficients. Assuming in Eqn (2.2.5) that $f = 1$ at $T \gg \tau_{\mathrm{coh}}$, $T \gg 1/G_1$ (here τ_{coh} is the timescale on which $G_2(\tau)$ differs from $G_2(\infty) = G_1^2$ significantly), we find

$$\begin{aligned} C_1 &= \int_0^T \mathrm{d}t\, G_1 = TG_1 = \langle n\rangle \ , \\ C_2 &= \int_0^T\int_0^T \mathrm{d}t\,\mathrm{d}t'\, G_2(t - t') \approx (TG_1)^2 = \langle n\rangle^2 \ . \end{aligned} \tag{2.2.7}$$

Thus, when $i \neq j$,

$$\overline{\delta(t-t_i)\delta(t-t_j+\tau)}$$
$$= \langle n\rangle^2 \int_0^T \int_0^T \mathrm{d}t_i\,\mathrm{d}t_j\, G_2(t_i-t_j)\delta(t-t_i)\delta(t-t_j+\tau) = \frac{G_2(\tau)}{\langle n\rangle^2}\,, \tag{2.2.8}$$

so that Eqn (2.2.4) takes the form

$$\overline{i_n(t)i_n(t+\tau)} = \frac{\delta(\tau)G_1 n}{\langle n\rangle} + \frac{G_2(\tau)n(n-1)}{\langle n\rangle^2}\,. \tag{2.2.9}$$

Now we average Eqn (2.2.9) over n with $T \to \infty$. Assuming $\langle n^2\rangle \approx \langle n\rangle^2 \gg \langle n\rangle$, we get

$$\overline{i(t)i(t+\tau)} = G_1\delta(\tau) + G_2(\tau)$$
$$= \langle N\rangle\delta(\tau) + \langle :N(t)N(t+\tau):\rangle\,. \tag{2.2.10}$$

Thus, the classical correlation function for the current and quantum normally ordered intensity correlation function for the field differ from each other only by the term $G_1\delta(\tau)$ describing the 'white' shot noise.

Making use of the permutation relation $[A(t), A^+(t')] = \delta(t-t')$ [which follows from Eqn (2.2.1)], one can represent Eqn (2.2.10) in the form

$$\overline{i(t)i(t+\tau)} = G_2'(\tau) \equiv \langle A^+(t)A(t)A^+(t+\tau)A(t+\tau)\rangle$$
$$\equiv \langle N(t)N(t+\tau)\rangle\,. \tag{2.2.11}$$

In the case of an ideal laser beam described by a coherent state $|E_0\rangle$ there are no excess fluctuations ($G_2 = \langle N\rangle^2 = \bar{i}^2$), so that the PN is found from

$$\overline{i(t)i(t+\tau)} - \bar{i}^2 = \langle N\rangle\delta(\tau) = \langle N\rangle\left[A(t), A^+(t+\tau)\right]\,. \tag{2.2.11a}$$

Here the relation between the PN and noncommutativity of the field operators is explicitly seen.

Thus, in the case of an ideal detector the classical current correlation function (measured when averaging over time) repeats, according to Eqn (2.2.11), the quantum intensity correlation function of the field G_2'. The computations made above enabled us to reveal connections between the noncommutativity of the field operators, the discrete character of the information transfer $q \to c$, and the observed shot (photon) noise.

We emphasise an important difference in the physical meaning of two terms in Eqn (2.2.10) describing the PN and excess noise: while $G_1\delta(\tau)$ has zero correlation time, the characteristic scale τ_{coh} of the change $G_2(\tau)$ (time of correlation or of the second-order coherence) is finite and depends on the properties of the light source. In the case of sources of squeezed light, the contribution of the second term to the spectral density can be negative so that the PN is compensated, but only in a limited frequency interval of order $1/\tau_{\mathrm{coh}}$. Here one can assume that the shot and excess stochastic signals are anticorrelated (have opposite signs). This obvious picture of the PN squeezing 'mechanism' applies when a negative FB is used (Section 3).

Quantum efficiencies different from unity of the detector without FB are taken into account simply by the substitution $A \to A\sqrt{\eta}$ into the normally ordered expressions [33]:

$$\overline{i(t)i(t+\tau)} = \eta G_1\delta(\tau) + \eta^2 G_2(\tau) = I_0\delta(\tau) + \eta^2 G_2(\tau)\,, \tag{2.2.12}$$

(here $I_0 = \bar{i}$). Fourier transformation of this expression with account taken of the transmission function of the electronics $k(\Omega)$ yields the spectral density of the current fluctuations at frequency Ω:

$$\overline{i^2(\Omega)} = |k(\Omega)|^2 \int \mathrm{d}\tau \exp(\mathrm{i}\Omega\tau)\langle i(t)i(t+\tau)\rangle$$
$$= |k(\Omega)|^2\left[I_0 + \eta^2 G_2(\Omega)\right]\,. \tag{2.2.13}$$

Here $G_2(\Omega)$ is the spectrum of light intensity fluctuations.

From here at $k = 1$ we get the relation between the Fano factors for the current and light:

$$F_i(\Omega) - 1 = \eta\left[F_N(\Omega) - 1\right]\,. \tag{2.2.14}$$

On the basis of Eqns (2.2.13) and Eqn (2.2.14), the measured photocurrent fluctuations are usually used for obtaining information about intensity fluctuations of the light and its super- or sub-Poissonian character.

The FB violates the relations obtained. In 'semiclassical' theory this is explained by the dependence of the probability of the appearance of a successive point t_n on the preceding events $\{t_i\}$, $t_i < t_n - \tau$, where τ is the delay in the FB loop [14, 15, 21]. An account of FB in quantum theory will be given in Section 3.

In the following sections classical (experimental) and quantum averaging will be denoted by the unique symbol $\langle\ldots\rangle$.

2.3 Coherent light with weak modulation

Let one mode with a frequency ω_0 be in a coherent state with a large amplitude E: $|\psi\rangle_0 = |E\rangle_0$, and other modes be in arbitrary states. Then the positive-frequency part of the field can be represented as

$$E(t) = E\exp(-\mathrm{i}\omega_0 t) + A(t)\,. \tag{2.3.1}$$

Now, the operator $A(t)$ and its Fourier image $A(\omega)$ describe all other modes which are assumed to be weakly excited in comparison with the central mode (symbolically, $A(t) \ll E$). Here it is convenient to consider the 'weak' field $A(t)$ as *modulating* and the component $E\exp(-\mathrm{i}\omega_0 t)$ as a *carrier* (of course, such a point of view makes sense only at $\Delta\Omega \ll \omega_0$). The fixed complex amplitude E can be considered as a c-number.

In order to distinguish the signals of the amplitude (AM) and phase (PM) modulation, we introduce 'slow' (radiofrequency) Hermitian operators $Q(t)$ and $P(t)$ (they are called the first and second field *quadrature*):

$$Q(t,\phi) \equiv \frac{A(t)\exp(\mathrm{i}\omega_0 t - \mathrm{i}\phi) + A^+(t)\exp(-\mathrm{i}\omega_0 t + \mathrm{i}\phi)}{\sqrt{2}}\,,$$
$$P(t,\phi) \equiv -\frac{A(t)\exp(\mathrm{i}\omega_0 t - \mathrm{i}\phi) - A^+(t)\exp(-\mathrm{i}\omega_0 t + \mathrm{i}\phi)}{\mathrm{i}\sqrt{2}}$$
$$= Q\left(t, \phi + \frac{\pi}{2}\right)\,. \tag{2.3.2}$$

Here ϕ is an arbitrary phase in general, but when describing the coherent field modulation with an amplitude E, one should assume $\phi = \arg(E)$, then $Q(t,\phi)$ and $P(t,\phi)$ are proportional to the AM and PN signals,

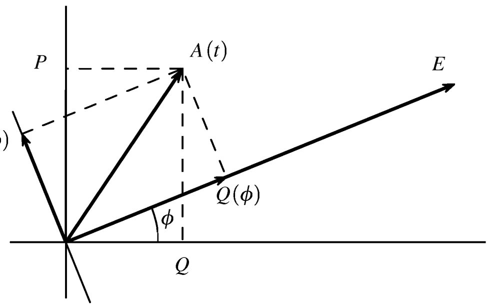

Figure 6. A monochromatic field with amplitude E and a weak field $A(t)$ can be represented as a modulated field. The projections $Q(\phi)$ and $P(\phi)$ describe the AM and PM, respectively (ϕ is the monochromatic field phase).

respectively (see Fig. 6). Inverse transformations have the form

$$A(t) = \frac{[Q(t) + \mathrm{i}P(t)]\exp(-\mathrm{i}\omega_0 t + \mathrm{i}\phi)}{\sqrt{2}} ,$$

$$A^+(t) = \frac{[Q(t) - \mathrm{i}P(t)]\exp(\mathrm{i}\omega_0 t - \mathrm{i}\phi)}{\sqrt{2}} . \qquad (2.3.3)$$

Thus, in classical theory $Q(t)$ and $P(t)$ are the real and imaginary parts of a 'slow' complex amplitude $A(t)\exp(\mathrm{i}\omega_0 t)$ (which is 'almost' stationary in a coordinate frame rotating with the frequency ω_0); in quantum theory, the attributes *real* and *imaginary* should be replaced by *Hermitian* and *anti-Hermitian*, respectively. The total field has the form

$$E(t) + E^+(t) = \left[|E| + \sqrt{2}\,Q(t) + \mathrm{i}\sqrt{2}\,P(t)\right]\exp(-\mathrm{i}\omega_0 t + \mathrm{i}\phi) + \left[|E| + \sqrt{2}\,Q(t) - \mathrm{i}\sqrt{2}\,P(t)\right]\exp(\mathrm{i}\omega_0 t - \mathrm{i}\phi) . \qquad (2.3.3a)$$

It is also convenient to determine projections of the signal $Q(t, \phi)$ (which is a vector on the complex plane) on some basic axes (see Fig.6): $Q(t, 0) \equiv Q(t)$, $P(t, 0) \equiv P(t)$. Then

$$\begin{aligned} Q(t,\phi) &= Q(t)\cos\phi + P(t)\sin\phi , \\ P(t,\phi) &= -Q(t)\sin\phi + P(t)\cos\phi . \end{aligned} \qquad (2.3.4)$$

Nonzero commutators of the field operators are written as

$$\begin{aligned} &\left[A(t), A^+(t)\right] = \delta(t - t') , \qquad \left[Q(t), P(t')\right] = \mathrm{i}\delta(t - t') , \\ &\left[Q(t,\phi), Q(t',\phi')\right] = \mathrm{i}\sin(\phi' - \phi)\delta(t - t') . \end{aligned} \qquad (2.3.5)$$

Now we go over to the spectral representation. We denote

$$\begin{aligned} a(\Omega) &\equiv a(\omega_0 + \Omega) \equiv (2\pi)^{-1/2}\int \mathrm{d}t \exp\left[\mathrm{i}(\omega_0 + \Omega)\right]A(t) , \\ q(\Omega, \phi) &\equiv (2\pi)^{-1/2}\int \mathrm{d}t \exp(\mathrm{i}\Omega t) Q(t, \phi) , \end{aligned} \qquad (2.3.6)$$

(here $|\Omega| \ll \omega_0$ and the case $\Omega = 0$ is excluded), then

$$\begin{aligned} q(\Omega, \phi) &= q^+(-\Omega, \phi) \\ &= \frac{a(\Omega)\exp(-\mathrm{i}\phi) + a^+(-\Omega)\exp(\mathrm{i}\phi)}{\sqrt{2}} \\ &= q(\Omega)\cos\phi + p(\Omega)\sin\phi , \end{aligned}$$

$$\begin{aligned} &q(\Omega) = \frac{a(\Omega) + a^+(-\Omega)}{\sqrt{2}} , \quad p(\Omega) = -\frac{a(\Omega) - a^+(-\Omega)}{\mathrm{i}\sqrt{2}} , \\ &a(\Omega) = \frac{[q(\Omega,\phi) + \mathrm{i}p(\Omega,\phi)]\exp(\mathrm{i}\phi)}{\sqrt{2}} , \\ &a^+(\Omega) = \frac{[q(-\Omega,\phi) - \mathrm{i}p(-\Omega,\phi)]\exp(-\mathrm{i}\phi)}{\sqrt{2}} . \end{aligned} \qquad (2.3.7)$$

Expression (2.3.5) then yields

$$\begin{aligned} &\left[q(\Omega,\phi), q(\Omega',\phi')\right] = \mathrm{i}\sin(\phi' - \phi)\,\delta(\Omega + \Omega') , \\ &\left[q(\Omega), q(\Omega')\right] = \left[p(\Omega), p(\Omega')\right] = 0 , \\ &\left[q(\Omega), p(\Omega')\right] = \mathrm{i}\delta(\Omega + \Omega') , \\ &\left[q(\Omega), p^+(\Omega')\right] = \mathrm{i}\delta(\Omega - \Omega') , \\ &\left[a(\Omega), a^+(\Omega')\right] = \delta(\Omega - \Omega') . \end{aligned} \qquad (2.3.8)$$

Below the frequency argument will often be omitted: $q \equiv q(\Omega)$, $p \equiv p(\Omega)$, $a \equiv a(\Omega)$, $q^+ = q(-\Omega)$, $p^+ = p(-\Omega)$.

In classical theory one can assume some or all components of the field to be zero. For example, for a harmonic AM with some frequency Ω_1, only one quadrature $q(\Omega_1, \phi)$ is nonzero. However, in quantum theory such an assumption is prohibited, as it violates the commutation relations (2.3.8). To avoid this, each transformation of the operator $q \to \gamma q$ must be followed by a consistent transformation $p \to p/\gamma$ (the condition of *unitary* transformation). In other words, the PN always yields some AM and PM minimum noise, with a constant product of the modulation coefficients: one can decrease (increase) the amplitude PN only at the expense of the corresponding increase (decrease) of the phase PN (an exception to this rule will be considered in Section 3.2). If the mean field is in the vacuum state, then AM and PM are equal to each other, but in the case of quadrature-squeezed state they are not.

In addition to the unavoidable quantum noise modulation, 'coherent' modulation by a specific signal can, of course, occur; for example, for the harmonic AM $\langle q(\Omega_1, \phi)\rangle \neq 0$. *Beating* of two coherent components, a combination of AM and PM, is also possible.

Let us find the beam intensity operator $E^+(t)E(t)$. By omitting the constant part $|E|^2$ and neglecting the weak contribution $A^+(t)A(t)$, we get the alternating part of the intensity in the form

$$\begin{aligned} N(t) &\equiv E^* A(t)\exp(\mathrm{i}\omega_0 t) + EA^+(t)\exp(-\mathrm{i}\omega_0 t) \\ &= \sqrt{2}\,|E| Q(t,\phi) , \end{aligned} \qquad (2.3.9)$$

where $\phi = \arg(E)$. Thus, the operator $Q(t, \phi)$ multiplied by $\sqrt{2}\,|E|$ is the operator of the *envelope* or AM-*signal* (this is valid only in the linear-in-Q/E approximation). Therefore, the alternating part of photocurrent in an analogue photodetector must be determined by the operator $Q(t, \phi)$.

A similar signal describing the phase modulation has the form

$$N'(t) = \sqrt{2}\,|E| Q\left(t, \phi + \frac{\pi}{2}\right) = \sqrt{2}\,|E| P(t,\phi) . \qquad (2.3.9a)$$

In the spectral representation we have

$$\begin{aligned} N(\Omega) &= E^* a(\Omega) + Ea^+(-\Omega) = \sqrt{2}\,|E| q(\Omega,\phi) , \\ N'(\Omega) &= -\mathrm{i}\left[E^* a(\Omega) - Ea^+(-\Omega)\right] = \sqrt{2}\,|E| p(\Omega,\phi) . \end{aligned} \qquad (2.3.9b)$$

Formulas (2.3.9) describe the signal at the output of an ideal ($\eta = 1$) homodyne photodetector with a coherent field amplitude E. Thus, we represented the superposition of a strong field in a coherent state and a weak arbitrary field (including 'vacuum' noise) as a *modulated* field (see Fig. 4).

Consider now the weak field fluctuations. A stationary field is described by spectral density $n(\Omega)$ which is determined by the relation $\langle a^{+}(\Omega)a(\Omega')\rangle = n(\Omega)\delta(\Omega-\Omega')$. Thus, $n(\Omega)$ is a dimensionless coefficient of the δ-function. In addition, in the case of quadrature-squeezed light, the anomalous correlator $m(\Omega)=m(-\Omega)$, determined from $\langle a(\Omega)a(\Omega')\rangle = m(\Omega)\delta(\Omega+\Omega')$, is also nonzero. The field is then periodically nonstationary: its properties change with period $(2\pi/\omega_0)/2$. Let us introduce the following notation for the quadrature spectral densities:

$$\begin{aligned}
\left\langle q^2(\Omega)\right\rangle &\equiv \left\langle q^{+}(\Omega)q(\Omega)\right\rangle \equiv \frac{\left\langle q(\Omega)q(\Omega')\right\rangle}{\delta(\Omega+\Omega')}\,,\\
\left\langle p^2(\Omega)\right\rangle &\equiv \left\langle p^{+}(\Omega)p(\Omega)\right\rangle \equiv \frac{\left\langle p(\Omega)p(\Omega')\right\rangle}{\delta(\Omega+\Omega')}\,,\\
\left\langle q(\Omega)\,p^{+}(\Omega)\right\rangle &\equiv \left\langle p(\Omega)q^{+}(\Omega)\right\rangle^{*} \equiv \frac{\left\langle q(\Omega)p(\Omega')\right\rangle}{\delta(\Omega+\Omega')}\,,\\
\left\langle p(\Omega)\,q^{+}(\Omega)\right\rangle &\equiv \left\langle q(\Omega)p^{+}(\Omega)\right\rangle^{*} \equiv \frac{\left\langle p(\Omega)q(\Omega')\right\rangle}{\delta(\Omega+\Omega')}\,.
\end{aligned}$$

It is shown in the Appendix that the following uncertainty relation for the quadrature spectral densities applies (where we put $\langle q\rangle = \langle p\rangle = 0$):

$$\left\langle q^2(\Omega)\right\rangle\left\langle p^2(\Omega)\right\rangle \geqslant \left|\left\langle q(\Omega)\,p^{+}(\Omega)\right\rangle\right|^2 \geqslant \frac{1}{4}\,. \qquad (2.3.10)$$

The field states in which the equality holds are called the *states with minimum uncertainty*.

With the help of Eqn (2.3.7) we find the second moments of the quadratures

$$\begin{aligned}
2\left\langle q^2(\Omega)\right\rangle &= n(\Omega)+n(-\Omega)+1+2\,\mathrm{Re}\left[m(\Omega)\right]\,,\\
2\left\langle p^2(\Omega)\right\rangle &= n(\Omega)+n(-\Omega)+1-2\,\mathrm{Re}\left[m(\Omega)\right]\,,\\
2\left\langle q(\Omega)\,p^{+}(\Omega)\right\rangle &= 2\,\mathrm{Im}\left[m(\Omega)\right]+\mathrm{i}\left[n(\Omega)-n(-\Omega)+1\right]\,,\\
2\left\langle p^{+}(\Omega)\,q(\Omega)\right\rangle &= 2\,\mathrm{Im}\left[m(\Omega)\right]+\mathrm{i}\left[n(\Omega)-n(-\Omega)-1\right]\,.
\end{aligned} \qquad (2.3.11)$$

It follows from here that $\langle q^2(\Omega)\rangle$ and $\langle p^2(\Omega)\rangle$ are even real functions Ω. Inverse transformations have the form

$$\begin{aligned}
n(\Omega)+1 &= \left\langle a(\Omega)a^{+}(\Omega)\right\rangle\\
&= \frac{1}{2}\left[\left\langle q^2(\Omega)\right\rangle + \left\langle p^2(\Omega)\right\rangle + 2\,\mathrm{Im}\left\langle q(\Omega)\,p^{+}(\Omega)\right\rangle\right]\,,\\
n(-\Omega) &= \left\langle a(-\Omega)^{+}a(-\Omega)\right\rangle\\
&= \frac{1}{2}\left[\left\langle q^2(\Omega)\right\rangle + \left\langle p^2(\Omega)\right\rangle - 2\,\mathrm{Im}\left\langle q(\Omega)\,p^{+}(\Omega)\right\rangle\right]\,,\\
m(\Omega) &= \left\langle a(\Omega)a(-\Omega)\right\rangle\\
&= \frac{1}{2}\left[\left\langle q^2(\Omega)\right\rangle - \left\langle p^2(\Omega)\right\rangle + 2\mathrm{i}\,\mathrm{Re}\left\langle q(\Omega)\,p^{+}(\Omega)\right\rangle\right]\,.
\end{aligned} \qquad (2.3.11\mathrm{a})$$

The parameter m describes the *quadrature squeezing*: for example, if $m=m^{*}>0$ the depth of the noise AM exceeds the PM and if $m<0$, the PM is greater than the AM (see Fig. 7g and 7h). In the case of the vacuum weak field, $n=m=0$, $\langle q^2\rangle = \langle p^2\rangle = -\mathrm{i}\langle qp^{+}\rangle = -\mathrm{i}\langle pq^{+}\rangle = 1/2$.

The terms $\frac{1}{2}$ ('zero-point fluctuations') in Eqn (2.3.11) emerged because of the noncommutativity of the field operators. In classical theory as well as in quantum theory when the averaging of normally ordered operators of type $a^{+}a$ or $:q^2:\equiv q^2-1/2$ is performed, these terms are absent. It is these operators that describe the observed PN, i.e. quantum fluctuations during energy flux measurements that arise in detectors even for the vacuum weak field when $n=m=0$. Formally, the PN can be defined through differences between the symmetrised and normally ordered products of the operators A and A^{+}.

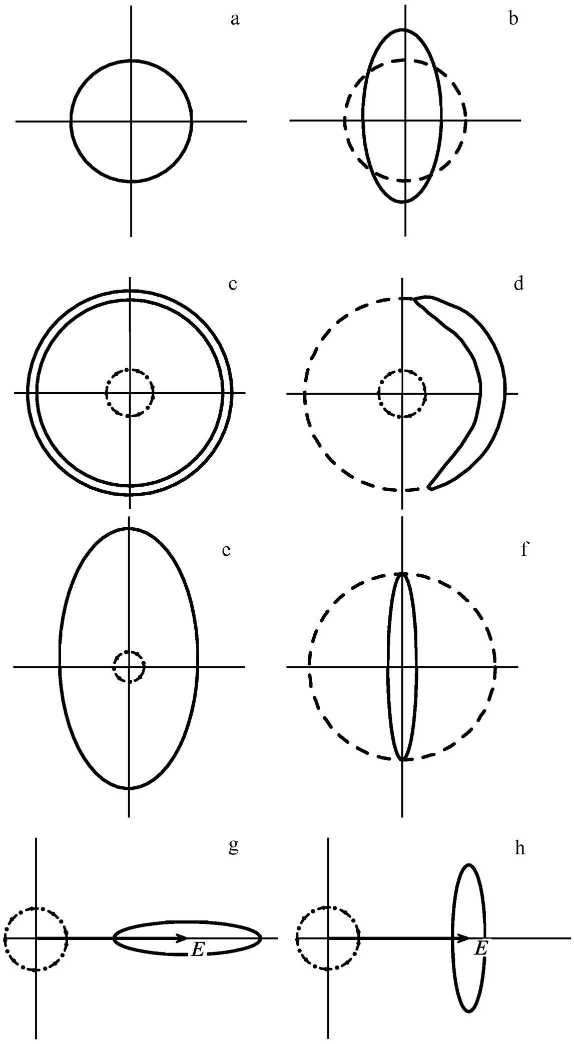

Figure 7. Different types of squeezed light: (a) vacuum, (b) squeezed vacuum, (c) energy-squeezed, (d) amplitude-squeezed, (e) classically squeezed, (f) 'supersqueezed', (g) and (h) quadrature-squeezed vacuum.

$$q^2 - :q^2: = \frac{1}{2}\left[a(\Omega), a^{+}(\Omega)\right] = \frac{1}{2}\,. \qquad (2.3.12)$$

The full noise without PN is called *excess noise*. It is proportional to $2:q^2(\Omega): = n(\Omega)+n(-\Omega)+2\,\mathrm{Re}\left[m(\Omega)\right]$. In the case of squeezed light this quantity may be negative.

From Eqn (2.3.9) it follows that in the considered approximation of linearity in A/E the intensity fluctuation N (4th moment of the total field) is proportional to the quadrature fluctuations q (2d moment of the weak field), which simplifies the calculations substantially:

$$\left\langle N^2(\Omega)\right\rangle = 2|E|^2\left\langle q^2(\Omega,\phi)\right\rangle\,. \qquad (2.3.13)$$

A convenient measure of the relative fluctuations is the Fano factor

$$F(\Omega,\phi) \equiv \frac{\langle N^2(\Omega)\rangle}{|E|^2} = 1 + n(\Omega) + n(-\Omega) + 2\,\mathrm{Re}\big[m(\Omega)\exp(\mathrm{i}2\phi)\big] \;. \quad (2.3.14)$$

The Fourier-image of the function F is the AM-signal correlation function:

$$\big\langle Q(t)Q(t')\big\rangle = \frac{1}{2\pi}\int \mathrm{d}\Omega \exp\big[-\mathrm{i}\Omega(t-t')\big]\big\langle q^2(\Omega)\big\rangle \;. \quad (2.3.15)$$

For $F=1$, the Poissonian spectral density is obtained: $\langle N^2(\Omega)\rangle = |E|^2$; for $F<1$ it is sub-Poissonian and for $F>1$ it is the super-Poissonian (see Fig. 1). The excess noise is described by the function $F(\Omega)-1$.

During the absorption, splitting, or detection of the light beam $F-1$ is multiplied by the transmission coefficient T or η. In other words, the absorption (energy dissipation) has an effect only on the excess noise, whereas the PN remains unchanged. As a result, during the absorption or amplification the 'non-Poissonian' character of the fluctuations described by the quantity $|F-1|$ must, it would appear, decrease. An exception to this rule will be discussed in Section 3.3.

Let $2\phi + \arg(m) = \pi$ and $n(\Omega) = n(-\Omega)$, then Eqn (2.3.14) takes the form

$$F(\Omega) = 1 + 2\big[n(\Omega) - m(\Omega)\big] \;. \quad (2.3.16)$$

Therefore, during the 'vacuum' modulation when $n=m=0$, we have $F=1$, and a weak stationary field with intensity $n(\Omega) = n(-\Omega) \equiv n$ and $m=0$ yields the super-Poissonian noise $F = 1+2n$. To get the sub-Poissonian noise, $m>n$ is needed. In classical theory this condition is impossible to satisfy, as it contradicts the Cauchy–Schwartz inequality (see the Appendix). There is no such a limitation, however, in quantum theory. It will be shown in Section 2.5 that, by means of a parametric down-conversion, the light beam can be made sub-Poissonian.

Notice that the classical limitation $F>1$ relates only to the *field's* Fano factor, which is expressed through the field amplitude correlators according to Eqn (2.3.16). No such limitation exists for the Fano factor of the *current*, so that the smoothing of the current fluctuations by an RC-chain below the Poissonian level $F=1$ is not, of course, a quantum effect.

2.4 Transformation of PN by beam splitters

Let us consider a transformation (mixing) of two light beams that differ in direction (or polarisation) by a partially transparent mirror or a polarising prism. In the absence of dissipation and dispersion, the transformation is described by real transmission and reflection coefficients $u \equiv \sqrt{T}$, $v \equiv \sqrt{R}$, with $T+R=1$. The output fields have the form

$$E_1'(t) = uE_1(t) + vE_2(t)\,, \qquad E_2'(t) = -vE_1(t) + uE_2(t)\;. \quad (2.4.1)$$

The transformed components are indicated by primes. This transformation conserves the energy flux: $N_1' + N_2' = N_1 + N_2$ (here $N = E^+(t)E(t)$ is the total intensity). Similar linear relations hold for the coherent field amplitudes E_k and quadratures $Q_k(t,\phi)$:

$$E_1' = uE_1 + vE_2\,, \quad Q_1'(t,\phi) = uQ_1(t,\phi) + vQ_2(t,\phi)\,,$$
$$E_2' = -vE_1 + uE_2\,, \quad Q_2'(t,\phi) = -vQ_1(t,\phi) + uQ_2(t,\phi)\;. \quad (2.4.2)$$

According to Eqn (2.3.9), the output signals are equal to

$$N_1'(t) = \sqrt{2}\,|E_1'|Q_1'(t,\phi_1')\,,$$
$$N_2'(t) = \sqrt{2}\,|E_2'|Q_2'(t,\phi_2')\,, \quad (2.4.3)$$

In the case of homodyne detection, the light enters, for example, input 1 ($E_1 = 0$) and the coherent field E_2 input 2 (Fig. 8). Let $E_2 = E_2^*$, then

$$N_1' = \sqrt{2}\,E_2\big(RQ_2 + \sqrt{TR}\,Q_1\big)\,,$$
$$N_2' = \sqrt{2}\,E_2\big(TQ_2 - \sqrt{TR}\,Q_1\big)\,, \quad (2.4.4)$$

i.e. due to the negative sign in Eqn (2.4.1), the signal Q_1 modulates the output coherent fields in the 'opposite phase', while the heterodyne noise Q_2 does so in 'phase'. This yields

$$N_1'(t) + N_2'(t) = \sqrt{2}\,E_2 Q_2(t) = N_2(t)\,,$$
$$TN_1'(t) - RN_2'(t) = \sqrt{2TR}\,E_2 Q_1(t)\;. \quad (2.4.5)$$

Thus, the sum of the output signals does not depend on the input signal Q_1 and their weighted difference — on the noise Q_2 modulating the coherent (homodyne) field. The latter fact enables one to decrease the effect of parasitic modulation of the laser field E_2 in homodyne receivers [34, 35], which is very important, in particular, in quadrature-squeezed light studies.

Now let coherent components with the same frequency be present in the both input channels, with the phase-modulated field, $\langle P_1(t)\rangle \neq 0$ in channel 1 while the field in the channel 2 has only the coherent component E_2 that plays the role of a homodyne field. Clearly, by selecting the field phase E_2, one can transform the PM into AM, i.e. the device can serve as a phase detector with the output photocurrent $\langle N_1'\rangle$ proportional to $\langle P_1(t)\rangle$. For another homodyne phase, we get an amplitude detector measuring $\langle Q_1(t)\rangle$. In the case of squeezed light, by measuring the amplitude and phase of the homodyne field E_2, one can move and rotate the uncertainty ellipse for the output field (see Fig. 8).

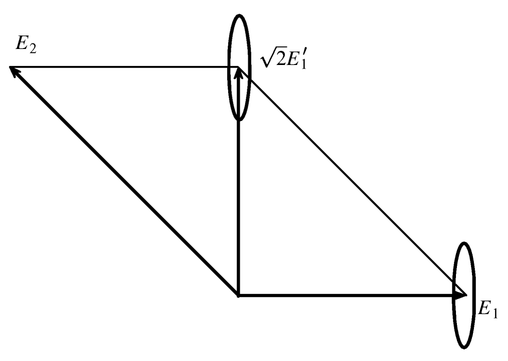

Figure 8. The transformation of the phase modulation of field E_1 into the amplitude modulation of the output field E_1' by a beam splitter and a homodyne field E_2 with the appropriate phase and amplitude.

Thus, one can measure by choice one of the quadratures, but it is impossible to measure both quadratures simultaneously by one detector. In quantum theory this conclusion, which is associated with the noncommutativity of the quadrature operators, plays a central role. In Section 2.5 we shall consider a realistic way of getting information about a quadrature Q_1 of some mode without perturbing this quadrature ('nondemolition', or QND-measurements).

Let us further consider transformation of the quadrature variance by a beam splitter. According to Eqn (2.4.2), for arbitrary frequency Ω and angle ϕ

$$\begin{aligned}
\langle (q_1')^2 \rangle &= T\langle q_1^2\rangle + R\langle q_2^2\rangle + 2uv\,\mathrm{Re}\langle q_1 q_2^+\rangle\,,\\
\langle (q_2')^2 \rangle &= R\langle q_1^2\rangle + T\langle q_2^2\rangle - 2uv\,\mathrm{Re}\langle q_1 q_2^+\rangle\,,\\
\langle q_1'(q_2')^+ \rangle &= T\langle q_1 q_2^+\rangle - R\langle q_2 q_1^+\rangle + uv\big(\langle q_2^2\rangle - \langle q_1^2\rangle\big)\,.
\end{aligned} \qquad (2.4.6)$$

Here $q \equiv q(\Omega, \phi)$, $q^+ \equiv q(-\Omega, \phi)$, $\langle q^2\rangle \equiv \langle qq^+\rangle = \langle q^+ q\rangle$. By summing the first two equalities, we find that the sum of the quadrature variances is conserved (independently of a possible correlation between q_1 and q_2):

$$\langle (q_1')^2\rangle + \langle (q_2')^2\rangle = \langle q_1^2\rangle + \langle q_2^2\rangle\,. \qquad (2.4.6a)$$

We also find the variance for the sum and difference of the output signals at $T = R = \frac{1}{2}$:

$$q_\pm \equiv q_1 \pm q_2\,,\quad \langle q_+^2\rangle = 2\langle q_1^2\rangle\,,\quad \langle q_-^2\rangle = 2\langle q_2^2\rangle\,. \qquad (2.4.6b)$$

Therefore if, for example, the vacuum is at input 2, the fluctuations of the signal difference will be Poissonian independently of the field statistics at the other input (this fact is used for the apparature calibration).

Similar relations are valid for the normally ordered operators $:q^2: \equiv q^2 - \frac{1}{2}$ that describe only the excess noise. Let, for instance, $\langle :q_2^2: \rangle = 0$, then

$$\begin{aligned}
\langle :(q_1')^2: \rangle &= T\langle :q_1^2: \rangle + 2uv\,\mathrm{Re}\,\langle q_1 q_2^+\rangle\,,\\
\langle :(q_2')^2: \rangle &= R\langle :q_1^2: \rangle - 2uv\,\mathrm{Re}\,\langle q_1 q_2^+\rangle\,,\\
\langle :(q_1')^2: \rangle &+ \langle :(q_2')^2: \rangle = \langle :q_1^2: \rangle\,.
\end{aligned} \qquad (2.4.6c)$$

These relations show that the partitioning of the excess noise from channel 1 into two output channels depends significantly on the presence of an initial correlation $\langle q_1 q_2^+\rangle$, in particular on its sign. This fact explains, as mentioned in the introduction the 'anomalous' effect of the output beam splitter in the FB loop (in the framework of *a priori* models).

At $\langle q_1 q_2^+\rangle$ the beam splitter, according to Eqn (2.4.6), performs the transformation

$$F_1' = TF_1 + RF_2\,,\quad F_2' = RF_1 + TF_2\,. \qquad (2.4.6d)$$

At input 2 let there be the coherent component plus vacuum, i.e. $F_2 = 1$, then

$$F_1' - 1 = T(F_1 - 1)\,, \qquad (2.4.7)$$

or $\langle :(q_1')^2: \rangle = T\langle :q_1^2: \rangle$. Thus, the beam splitter decreases the absolute value of the excess noise, i.e. diminishes the 'non-Poissonian' character of the statistics.

This conclusion can be generalised [33]: the losses act on the normally ordered operators trivially, in the same way as on the field components in classical models. This concerns the field transformation by a detector too: T is substituted by the detector's quantum efficiency η [see Eqn (2.2.14)]. Let PN in the light incident on the detector be fully suppressed, $F = 0$; then, according to Eqn (2.2.14),

$$F'_{\min} = 1 - \eta\,. \qquad (2.4.8)$$

Thus, the ultimate observed suppression of the PN is restricted, it would appear, by the detector's efficiency. However, this limit was significantly surpassed in the experiments of Ref. [12] (see Fig. 14). The point is that in Eqns (2.4.7), (2.4.8) no correlation is assumed between the input fields in channels 1 and 2: $\langle q_1 q_2^+\rangle = 0$. However, the FB just establishes such a correlation.

Let us consider now a balance homodyne detection [34, 35]. Let the intensities of two beams be measured by two identical detectors with an efficiency η, with the registered sum and difference of the current components being $i_\pm = i_1 \pm i_2$. Constant components of the currents are assumed to be the same. In analogy with Eqn (2.4.7), we go over from currents to light fluxes: $F_\pm \equiv \langle i_\pm^2\rangle / 2I_0 = \eta(\langle N_\pm^2\rangle/2I_0 - 1) + 1$, where $N_\pm = N_1' \pm N_2'$. Here normalisation by the total detectors currents $2I_0$ is used, since if the beams are independent, the noises are added together: $\langle i_\pm^2(\Omega)\rangle = 2I_0$. In the case of $T = R = \frac{1}{2}$, we find with the help of Eqn (2.4.5) that

$$F_+ - 1 = \eta\big(2\langle q_2^2\rangle - 1\big)\,,\quad F_- - 1 = \eta\big(2\langle q_1^2\rangle - 1\big)\,. \qquad (2.4.9)$$

Thus the noise of the current difference does not depend on the homodyne field noise q_2.

2.5 Squeezing and nondemolition measurements of PN

Let a light beam be passed through a wide-band parametric transformer (PT) of the travelling wave type with one transverse mode — a transparent nonlinear crystal excited by a double-frequency pump $2\omega_0$. The pump's field then must be coherent with a 'carrier' field E, i.e. both beams must be generated by the same drive laser.

According to simple models, either quantum or classical (see Ref. [36]), the field at the PT output has the following form

$$a' = ga - fa^+\,,\quad a'^+ = ga^+ - fa\,. \qquad (2.5.1)$$

Here $a \equiv a(\Omega)$, $a^+ \equiv a^+(-\Omega)$, $g = \cosh\Gamma$, $f = \sinh\Gamma$, Γ is the amplification degree proportional to the pump amplitude, the pump phase is taken to be equal $\pi/2$, and the PT bandwidth is assumed to be much broader than the frequency span under consideration $\Delta\Omega$. The transformed components are indicated by primes. Thus, the transformer mixes the spectral components with frequencies $\omega_0 \pm \Omega$.

It follows from Eqn (2.5.1) that

$$\begin{aligned}
q' &= \exp(-\Gamma)q\,,\quad p' = \exp(\Gamma)p\,,\\
q'(\phi) &= \exp(-\Gamma)\cos(\phi q) + \exp(\Gamma)\sin(\phi q)\,.
\end{aligned} \qquad (2.5.1a)$$

Therefore, the PT is a phase-sensitive device: it amplifies p- and weakens q-quadratures (at the pump phase chosen here). This relates both to 'real' signals (determined or noisy) and to the PN as well. The latter surprising fact was demonstrated in a number of experiments (see Refs [2–8]).

When a coherent component with phase $\phi = 0$ is present at the PT input, it is transformed as a q-quadrature, i.e. is weakened too (however, it can be recovered afterwards with the help of a beam splitter and an additional homodyne field). Thus, the PT amplifies PM and weakens AM of the input field, including quantum modulation.

Notice that at an arbitrary pump phase, one should assume $f = \exp(\mathrm{i}\phi_0)$ in Eqn (2.5.1), where ϕ_0 is the pump phase plus $\pi/2$, so that we get instead of Eqn (2.5.1a)

$$q'(\phi) = [\cosh\Gamma + \sinh\Gamma\cos(\phi_0 - 2\phi)]q(\phi) + [\cosh\Gamma + \sinh\Gamma\sin(\phi_0 - 2\phi)]\,p(\phi)\ . \quad (2.5.1\text{b})$$

At the PT input let a stationary noise be present with intensity $n(\Omega) = n(-\Omega) \equiv n$; then according to Eqns (2.5.1), (2.5.1a) we have at the output,

$$\langle (q')^2\rangle = \left(\frac{1}{2}+n\right)\exp(-2\Gamma)\,, \quad \langle (p')^2\rangle = \left(\frac{1}{2}+n\right)\exp(2\Gamma),$$

$$\langle (q')^2(\phi)\rangle = \left(\frac{1}{2}+n\right)[\exp(-2\Gamma)\cos^2\phi + \exp(2\Gamma)\sin^2\phi]\ ,$$

$$n' = \left(\frac{1}{2}+n\right)\cosh(2\Gamma) - \frac{1}{2}\,,$$

$$m' = -\left(\frac{1}{2}+n\right)\sinh(2\Gamma) = -\left[\left(\frac{1}{2}+n'\right)^2 - \left(\frac{1}{2}+n\right)^2\right]^{1/2}\,,$$

$$n' + m' = \left(\frac{1}{2}+n\right)\exp(-2\Gamma) - \frac{1}{2}\,. \quad (2.5.2)$$

Notice that according to Eqn (2.5.1a), the cross correlators like $\langle q^+ p\rangle$ are not changed during the transformation, i.e. their vacuum values $\pm i/2$ are conserved.

The function $\langle [q'(\phi)]^2\rangle$ in polar coordinates forms uncertainty ellipses with axes proportional to $\exp(2\Gamma)$ and $\exp(-2\Gamma)$ (see Fig. 7). These numbers represent the coefficients of stretching and squeezing for quadrature variance. The product of the uncertainties is $[\langle (q')^2\rangle\langle (p')^2\rangle]^{1/2} = \frac{1}{2}+n$, i.e. a lower limit in Eqn (2.3.10) is reached at $n = 0$.

From Eqn (2.5.2), we find the Fano factor after the transformation

$$F' = F\exp(-2\Gamma) = 2\langle (q')^2\rangle = (1+2n)\exp(-2\Gamma)\ . \quad (2.5.3)$$

Thus, with a sufficiently strong pumping, both the PN and the input excess noise $2n$ are suppressed (the latter effect is obviously purely classical). We recall that, according to the approximate relation (2.3.9), the contribution of a^+a to the PN is not taken into account here, which can notably increase F' at a large amplification Γ or a small amplitude E (a method exists for compensating this contribution too [37, 38]). The suppression or amplification of the PN during the parametric transformation occurs, of course, only within a limited frequency range determined by the PT bandwidth.

In the case of vacuum at the input ($n = E = 0$), the PT radiates only its own spontaneous noise, and the output field state then is called *squeezed vacuum*. For the squeezed vacuum

$$n' = \sinh^2\Gamma\,, \quad m' = -\cosh\Gamma\sinh\Gamma\,, \quad \frac{m'}{n'} = -\coth\Gamma\ . \quad (2.5.4)$$

These relations violate the Cauchy–Schwartz inequality $\{|m'|/n'\}_{\text{clas}} \leqslant 1$ (see the Appendix) that applies in classical theory, in which connection the own PT emission is called nonclassical. Notice that the smaller the value of Γ, the stronger the deviation from the classical limit is.

In the opposite limiting case, at $n \gg \frac{1}{2}$, one can neglect quantum fluctuations and our model then describes the classical parametric transformation of the usual stationary noise (with identical fluctuations of both quadratures: $\langle q^2\rangle = \langle p^2\rangle = n$) into a periodically nonstationary one with different quadrature variances [39]. One can say that the PT transforms the input Gaussian chaotic light into a classical squeezed light [40–44]. Fig. 7 explains different types of squeezed light and the accepted terminology.

In real experiments, the PT usually emits a squeezed vacuum and in order to get quadrature-squeezed light, a coherent (homodyne) component from the initial laser should be added to it. A beam splitter can be used for this purpose. According to Eqn (2.4.6) and Eqn (2.5.2), at the beam splitter output $F_1'' = T[\exp(-2\Gamma) - 1] + 1$, so that at $\Gamma \gg 1$, the Fano factor F_1'' tends to $1 - T = R$ — the beam splitter reflection coefficient that can be made sufficiently small at the expense of decreasing the light intensity at the output.

In practice, in order to observe the effects of squeezing, a balance homodyning is used (see Section 2.4). Making use of Eqn (2.5.3) and Eqn (2.4.9), we find the Fano factor for the difference of the currents:

$$F_- = \eta[\exp(-2\Gamma) - 1] + 1\ . \quad (2.5.3\text{a})$$

From this at $\Gamma \gg 1$ we get $F_- = 1 - \eta$ [cf. Eqn (2.4.8)].

Consider then the transformation of two light beams by a wide bandwidth PT of the travelling wave type with two transverse modes that differ by polarisation or direction. Now, instead of Eqn (2.5.1), the following transformation occurs:

$$a_1' = ga_1 + \exp(i\phi_0)fa_2^+\,, \quad a_2'^+ = ga_2^+ + \exp(-i\phi_0)fa_1\ . \quad (2.5.5)$$

Here $a_k \equiv a_k(\Omega)$, $a_k^+ \equiv a_k^+(-\Omega)$, $g = \cosh\Gamma$, $f = \sinh\Gamma$, ϕ_0 is the pump phase plus $\pi/2$. From this, we find the following connections between the modes quadratures:

$$\begin{aligned} q_1' &= gq_1 + fq_2\cos\phi_0 + fp_2\sin\phi_0\ , \\ q_2' &= gq_2 + fq_1\cos\phi_0 + fp_1\sin\phi_0\ , \\ p_1' &= gp_1 + fq_2\sin\phi_0 - fp_2\cos\phi_0\ , \\ p_2' &= gp_2 + fq_1\sin\phi_0 - fp_1\cos\phi_0 \end{aligned} \quad (2.5.6)$$

[here $q_1 \equiv q_1(\Omega)$ etc.] In particular, at $\phi_0 = 0$,

$$\begin{aligned} q_1' &= gq_1 + fq_2\,, \quad p_1' = gp_1 - fp_2\ , \\ q_2' &= gq_2 + fq_1\,, \quad p_2' = gp_2 - fp_1\ . \end{aligned} \quad (2.5.7)$$

For practical purposes, the following property coming from Eqn (2.5.7) is of interest: $q_1' - q_2' = \exp(-\Gamma)(q_1 - q_2)$. At $\Gamma \gg 1$, the difference between two output signals tends to zero, i.e. it is squeezed'. Let the field incident on the transformer be in a vacuum state, then

$$\langle (q_1' - q_2')^2\rangle = \exp(-2\Gamma)\langle q_1^2 + q_2^2\rangle = \exp(-2\Gamma)\ , \quad (2.5.8)$$

i.e. $F_- = \exp(-2\Gamma)$ [compare Eqn (2.5.3) and Eqn (2.5.3a)]. Thus, at $\eta = 1$, $\Gamma \gg 1$, the same amplitudes and appropriate phases of the coherent components, the difference between currents in two homodyne detectors at the two-mode PT output contains no PN [38]. The fluctuations of the difference of the weak field intensities $a_1^+a_1 - a_2^+a_2$, which do not depend on the coherent components, are then compensated as well, independently of Γ [37, 38]. Visually, the latter effect is explained by the simultaneous photon creation in the signal and idle beam.

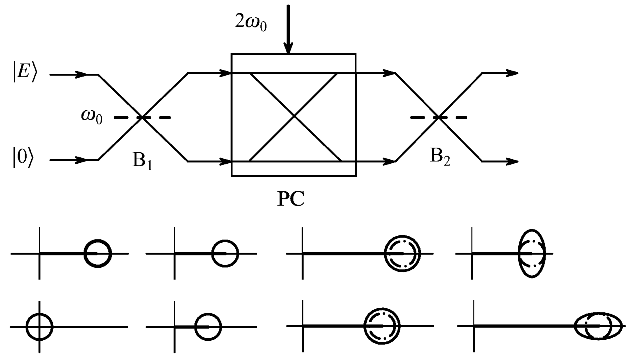

Figure 9. A nondemolition parametric 'beam splitter'. B_1 and B_2 are beam splitters, PC is a parametric amplifier transformer. The lower part of the figure contains polar diagrams illustrating the consecutive transformations of the coherent and noise components at $f = 1$, $g = 1.4$, $T_1 = T_2 = 0.85$.

An interesting possibility is provided by a combination of three consecutive transformations (2.4.1), (2.5.5), and again (2.4.1) shown in Fig. 9 [29]. By multiplying the matrices of these transformations at $\phi_0 = 0$, we obtain ($u = \cos\theta$, $v = \sin\theta$)

$$a_1'(\Omega) = g\left[\cos(2\theta)a_1(\Omega) + \sin(2\theta)a_2(\Omega)\right] + fa_2^+(-\Omega) ,$$
$$a_2'(\Omega) = g\left[-\sin(2\theta)a_1(\Omega) + \cos(2\theta)a_2(\Omega)\right] + fa_1^+(-\Omega) . \tag{2.5.9}$$

Now let $f \equiv \sinh\Gamma = -\tan(2\theta)$, then $\sin(2\theta) = -\tanh\Gamma$, $g\cos(2\theta) = 1$, $g\sin(2\theta) = -f$, $T = (1 + 1/g)/2$, $R = (1 - 1/g)/2$. As a result, we find the following links between the output and input quadratures:

$$q_1' = q_1 , \quad p_1' = p_1 - 2fp_2 ,$$
$$q_2' = q_2 + 2fq_1 , \quad p_2' = p_2 . \tag{2.5.10}$$

Coherent components are transformed as q-quadratures:

$$E_1' = E_1 , \quad E_2' = E_2 + 2fE_1 = 2fE_1 . \tag{2.5.10a}$$

For example, let $f = \frac{1}{2}$, then $g = 1.12$, $\Gamma = 0.48$, $T = 0.95$, and

$$q_1' = q_1 , \quad p_1' = p_1 - p_2 ,$$
$$q_2' = q_2 + q_1 , \quad p_2' = p_2 , \tag{2.5.11}$$
$$E_1' = E_2' = E_1 .$$

We recall that the same linear relations apply in classical theory as well. In quantum theory, they are interpreted as proof for the possibility of the *quantum nondemolition measurement* of one quadrature in the transverse mode (beam) of the field [26]. Let the 'signal' q_1 be measured in mode 1, then mode 2 is used for probing or measuring. The beam splitter permits interaction between the signal and measuring instrument. By watching q_2', one can get information about the original signal q_1 from the term $2fq_1$ in Eqn (2.5.10). It is essential that q_1 is not perturbed ($q_1' = q_1$) here, whereas the second quadrature p_1 receives some addition (the term $-2fp_2$), which is interpreted as a *back-action* of the measuring device (the second beam in the given case) on the observed quantum object (the first beam). By taking back-action into account, we can ensure that the uncertainty relation (2.3.10) is satisfied during the interaction. Here this invariance is a consequence of the unitarity of the transformation matrices for mode amplitudes used.

Similarly, with the use of a phase detector (see Section 2.4) in output channel 1 one can measure the phase modulation of the field at input 2 'without demolition'.

The device considered mixes two light beams, but in contrast to a conventional beam splitter it copies ('clones') the input amplitudes q_1 and p_2. It can be considered as a phase-sensitive 'nondemolition beam splitter'. Notice that the device by itself does not squeeze but, instead, stretches the falling noise (see Fig. 9); for example, in the case of coherent input fields, one gets $F_1 = 1$, $F_2 = 1 + 4f^2$ at the output according to Eqn (2.5.11). Transformation (2.5.10) will be used further in Section 3.4 to prove the 'observability' of the in-loop field in the FB chain.

Let us consider once more the experiments discussed in Section 2.1 which use unusual properties of the parametric 'nondemolition beam splitter' (see Fig. 5).

In Fig. 5c, the correlation between the quantum noise in two detectors is measured at two outputs of the device considered, whose one input (subscript 1 in the given formulas and the upper input in Fig. 5c) is excited by a laser beam. According to Eqn (2.5.10), the relative signals of the detectors [i.e. normalised on $(2I_{0n})^{1/2}$, $n = 1, 2$] have the form

$$q_1'' = u_1 q_1' + v_1 q_{10} = u_1 q_1 + v_1 q_{10} ,$$
$$q_2'' = u_2 q_2' + v_2 q_{20} = u_2(q_2 + 2fq_1) + v_2 q_{20} . \tag{2.5.12}$$

Here $u_n^2 = \eta_n$, $v_n^2 = 1 - \eta_n$. The operators q_{n0} are introduced so as to ensure the unitarity of the transformation at $\eta_n \neq 1$. Thus, detector 2 ('probing'; the lower part of Fig. 5c) has an excess noise with a relative amplitude $2fu_2q_1$ correlated with the vacuum AM input signal q_1. As a result, the observed noises of the two detectors are correlated with each other:

$$\left\langle (q_1'')^2 \right\rangle = \eta_1 \langle q_1^2 \rangle + (1 - \eta_1)\langle q_{10}^2 \rangle = \frac{1}{2} ,$$
$$\left\langle (q_2'')^2 \right\rangle = \eta_2\left(\langle q_1^2 \rangle + 4f^2 \langle q_2^2 \rangle\right) + (1 - \eta_2)\langle q_{20}^2 \rangle$$
$$= \frac{1}{2}(1 + 4f^2\eta_2) ,$$
$$\left\langle q_1'' q_2'' \right\rangle = 2fu_1u_2\langle q_1^2 \rangle = \sqrt{\eta_1\eta_2}\, f . \tag{2.5.13}$$

We recall that the homodyne detection is considered here in the approximation $\Delta i \ll I_0$, which provides linearisation of the detection process and allows one (under some restrictions on the value f) not to take into account the intrinsic noise of the PT. As a result, the observed spectral densities of the noise and their correlation depend linearly on the power of the incident laser beam.

The correlation coefficient of the output signals normalised to their variances $\langle q_n^2 \rangle$ has the form

$$K = \frac{\langle q_1'' q_2'' \rangle}{\sqrt{\left\langle (q_1'')^2 \right\rangle \left\langle (q_2'')^2 \right\rangle}} = \sqrt{\frac{\eta_1\eta_2 f^2}{1/4 + \eta_2 f^2}} . \tag{2.5.14}$$

For example, at $f = \eta_1 = \eta_2 = \frac{1}{2}$ we have $K = 0.41$, and at $f^2\eta_2 \gg \frac{1}{4}$ the correlation coefficient is equal to $\sqrt{\eta_1}$. Note that here the correlation was computed for the optimal phase of the coherent fields E_1', E_2' and by neglecting the weak field contribution which is independent of them.

The scheme from Fig. 5c can be treated as a modified Brown – Twiss correlometer in which a conventional beam splitter is substituted by a 'nondemolition' one. As a result, even the light in a coherent state produces the correlation effect (which is absent in the case of a conventional beam splitter [see Section (2.4.6) at $\langle q_1 q_2^+ \rangle = 0$]. This effect was observed in Refs [27, 29].

The experiment considered enables us to make two important conclusions.

1. It provides evidence in favour of a visual *a priori* description of the PN in the given case: in fact, in accordance with the *a posteriori* point of view, if the noise were to appear in the detectors during measurement, it would be statistically independent. Obviously, the correlation can be introduced only by the signal of 'vacuum modulation' q_1 of the original field E. Notice that the detectors may be separated by a large distance to exclude their possible interaction.

Notice also that if one uses a simple PT (without additional beam splitters), there will also be correlation between currents, but it will depend on signals q_1 and q_2 at *both* inputs, which makes the conclusion on the *a priori* character of the coherent field PN q_1 less convincing.

2. The macroscopic current, $i(t)$, of both the conventional and the nondemolition homodyne detector 'realises' (makes observable) the modulating stochastic signal $Q(t)$ of the field $E(t)$ incident on the detector — even if it has 'vacuum' origin. This point, already mentioned in the introduction, underlies the quantum theory of FB in paper [15], as well as in Sections 3.1 – 3.5 of the present review.

Similarly, when making the choice of the corresponding phase of the homodyne field (Section 2.4), one can realise the vacuum phase modulation signal $P_1(t)$.

One might argue that the PT performs act of measurement during its interaction with the original beam, thus introducing the general noise of measurement into both output beams, which leads to correlation between the currents. However, the quantum measurement concept implies the transition from q-numbers to c-numbers, while here the parametric interaction is described by operator relations, and the input signals of the detectors are entirely c-numbers.

One can describe the action of the PT and beam splitters by using not the Heisenberg representation accepted here but that of Schrödinger (see Ref. [32] for comparison of these two mathematical methods). Then the state vector at the optical system input $|\psi\rangle = |E_1\rangle_1 |0\rangle_2$ is transformed into some output vector $|\psi'\rangle$ which relates to both output beams and defines the joint statistics of both detectors. Individual state vectors cannot be ascribed to the output beams: taken separately, they are in mixed and not pure states. Under such a description, the PN does not arise in detectors independently but as a result of some property of the state $|\psi'\rangle$. In essence, this is an *a priori* description as well, when information about the PN is transferred by the state vector (and not by Heisenberg operators q, p).

Let us consider one more possible experiment, shown in Fig. 5d. Here the signal of the nondemolition detector carrying, as we have been convinced, information about the PN of the original beam (term $2u_2 f q_1$), after having been amplified controls the modulator mounted across the beam path between the PT and a conventional detector (such schemes are designated by the term *feedforward*). Considering the correlation discovered above, it seems obvious that, by selecting the parameters of the scheme, one can achieve PN suppression at the modulator's output, i.e. to obtain amplitude squeezing. The calculation confirming that assumption will be given in Section 3.4 for a more efficient modification of this scheme (Fig. 5e) where the modulator is placed before the PT, i.e. with the use of a negative *feedback*.

3. PN and electronic feedback

3.1 System dynamics

The scheme of the experiment [12] and notations accepted below are presented in Fig. 10. There are four input fields $E_k(t)$, $k = 1, 2, 3, 4$, three in-loop fields $E'(t)$, $E''(t)$, $E_i(t)$, and two output fields $E_r(t)$, $E_0(t)$. Strong coherent signals are assumed to be fed only to inputs 1 and 3, i.e. $E_2 = E_4 = 0$. The additional field E_3 enables one to measure, independently of E_1, the phase of the coherent total field E'' on the detector, which plays the role of a homodyne field. The fields $E_i(t)$ and $E_4(t)$ are fictitious, they are introduced to describe the detection process.

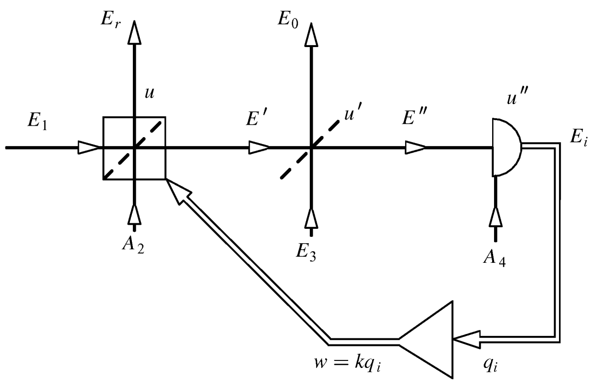

Figure 10. The experimental scheme for studying optical systems with feedback.

It is convenient to model the modulator by a beam splitter with a variable amplitude transmission $U(t) = u[1 - \varepsilon w(t)]$ and reflection $V(t) = [1 - U^2(t)]^{1/2}$ [15]. Here $w(t)$ is a real modulator signal (that has no DC component by assumption), and $u = \sqrt{T}$ and $v = \sqrt{R}$ are real transmission and reflection coefficients at $w = 0$ (no losses are assumed, so that $T + VR = 1$). The parameter ε plays the role of a switch for describing amplitude ($\varepsilon = 1$) or phase ($\varepsilon = \mathrm{i}$) modulation. If the modulation coefficient ($|w| \ll 1$) is small, we have $V(t) \approx v[1 + \chi^2 \varepsilon w(t)]$, where $\chi \equiv u/v$.

If $w(t)$ is a classical function of time (determined or stochastic), the modulator performs unitary transformation of the incident fields. Here, unlike in a usual beam splitter, modes of different frequencies interact, so that the spectral composition of the input beams is redistributed. At the same time, there is no mixing here between positive- and negative-frequency components, hence no squeezing occurs (without feedback).

However, in the model considered below, once the FB chain is looped, the function $w(t)$ becomes an operator, unitarity of transformation breaks down and the fields undergoes squeezing by amplitude or phase in the case of amplitude ($\varepsilon = 1$) or phase ($\varepsilon = \mathrm{i}$) modulator, respectively.

In the approximation of linearity on w and A the modulator performs the following transformation:

$$
\begin{aligned}
&E'(t) = UE_1(t) + VE_2(t) \approx u\left[1 - \varepsilon w(t)\right]E_1(t) + vA_2(t)\ , \\
&E_r(t) = -V^*E_1(t) + U^*E_2(t) \\
&\qquad \approx -v\left[1 + \chi^2\varepsilon^* w(t)\right]E_1(t) + uA_2(t)\ . \qquad (3.1.1)
\end{aligned}
$$

Recall that in each beam $E(t)$ is a positive-frequency field consisting of a coherent monochromatic part $E\exp(-\mathrm{i}\omega_0 t)$ with a large amplitude E and of a weak field $A(t)$. By neglecting terms of order wA, we obtain

$$
\begin{aligned}
&A'(t) = uA_1(t) + vA_2(t) - E'\exp(-\mathrm{i}\omega_0 t)\varepsilon w(t)\ , \\
&E' = uE_1\ , \\
&A_r(t) = -vA_1(t) + uA_2(t) + \chi^2 E_r \exp(-\mathrm{i}\omega_0 t)\varepsilon^* w(t)\ , \\
&E_r = -vE_1\ . \qquad (3.1.1\mathrm{a})
\end{aligned}
$$

In the spectral representation,

$$
\begin{aligned}
&a'(\Omega) = \tilde{a}'(\Omega) - E'\varepsilon w(\Omega)\ , \qquad \tilde{a}' \equiv ua_1 + va_2\ , \\
&a_r(\Omega) = \tilde{a}_r(\Omega) - \chi E'\varepsilon^* w(\Omega)\ , \quad \tilde{a}_r \equiv -va_1 + ua_2\ .
\end{aligned} \qquad (3.1.1\mathrm{b})
$$

Operators with the symbol '$\sim$' are defined through the input amplitudes a_1, a_2 with open FB loop ($a' = \tilde{a}'$, $a_r = \tilde{a}_r$). When calculating the PN, they play the role of given Langevin forces.

The next transformation of the modulated in-loop field is performed by a beam splitter leading the beam outwards with parameters u', v',

$$
\begin{aligned}
&A'' = u'A' + v'A_3\ , \qquad E'' = u'E' + v'E_3\ , \\
&A_0 = -v'A' + u'A_3\ , \quad E_0 = -v'E' + u'E_3\ .
\end{aligned} \qquad (3.1.2)
$$

Here $u'^2 + v'^2 = T' + R' = 1$. This transformation is unitary.

It is convenient to describe the detector's action by a transformation with $u''^2 = 1 - v''^2 \equiv \eta$ (the second output channel is thus ignored):

$$
A_i = u''A'' + v''A_4\ , \quad E_i = u''E''\ . \qquad (3.1.3)
$$

The fictitious field A_4 ensuring the unitarity of the transformation is in a vacuum state. The detector's current is equal, by definition, to the fictitious field intensity $E_i(t)$, so that the DC- and alternating components of the current are equal, in accordance with Eqn (2.3.9), to

$$
\begin{aligned}
&I_0 = |E_i|^2 = \eta|E''|^2\ , \\
&i(t) = \sqrt{2}\,|E_i|Q_i(t,\phi_i) = (2I_0)^{1/2}Q_i(t,\phi_i)\ .
\end{aligned} \qquad (3.1.4)
$$

Here we neglect the contribution of the weak 'modulating' field to the direct current; $\phi_i = \phi''$ is the phase of the homodyne field on the detector E''.

The link between the quantum formalism and experiment is made by postulating that the operator $i(t)$ *corresponds* to the observed photocurrent. Then the observed current fluctuations are calculated from a quantum correlation function, which in the case of a vacuum weak field, according to Eqn (2.2.12), has the form $\langle i(t)i(t')\rangle = 2I_0\langle Q(t)Q(t')\rangle = I_0\delta(t-t')$ (with no account taken of the detector's response). Notice that here a correlation function different from the normally ordered one is used (otherwise quantum noise does not arise). As is usually accepted in the quantum optical description of stationary experiments, we identify the averages over the quantum ensemble calculated in theory with the averages over time observed by experiment.

On the other hand, the PN (also called the shot noise) in the framework of the semiclassical theory of photodetection is frequently connected with charge discreteness. Our formalism also describes pure analogue detection made, for example, by a microcalorimeter or thermocouple. This approach demonstrates independence of the observed PN on the fact of discreteness of the photoelectron charge (see Section 2.2). Formally, the PN appears here as a result of the field operators' noncommutativity and the use of nonordered operators for determination of the quadrature variance.

We shall describe the action of the electronic scheme in the spectral representation by the relation

$$
\begin{aligned}
w(\Omega) &= k(\Omega)\exp(\mathrm{i}\Omega\tau)i(\Omega) \\
&= \sqrt{2}\,k(\Omega)|E_i|\exp(\mathrm{i}\Omega\tau)q_i(\Omega,\phi_i)\ . \qquad (3.1.5)
\end{aligned}
$$

Here the parameter τ makes allowance for the total delay in the closed FB loop both in the optical tract and in the connecting cables as well, and the amplification coefficient $k(\Omega)$ describes the dispersion properties of the 'electronics': detector, amplifier, and modulator. The electronic scheme is assumed not to let the direct current pass, so that the FB has no effect on the coherent parts of the fields.

Relation (3.1.5) hides the most important concept of the model (in its quantum version): it implies that the amplified macroscopic electron signal $w(t)$ controlling the modulator is, like $Q_i(t)$, an operator [15]. (The arguments in favour of such an approach have already been given in Section 2.5). Then transformation (3.1.1) ceases to be unitary.

Transformations (3.1.1)–(3.1.3) in spectral representations take the following form:

$$
\begin{aligned}
&a' = \tilde{a}' - E'\varepsilon w\ , \quad \tilde{a}' = ua_1 + va_2\ , \\
&a_r = \tilde{a}_r - \chi E'\varepsilon^* w\ , \quad \tilde{a}_r = -va_1 + ua_2\ , \\
&a'' = \tilde{a}'' - u'E'\varepsilon w\ , \quad \tilde{a}'' = u'\tilde{a}' + v'a_3\ , \qquad (3.1.6) \\
&a_0 = \tilde{a}_0 + v'E'\varepsilon w\ , \quad \tilde{a}_0 = -v'\tilde{a}' + u'a_3\ , \\
&a_i = \tilde{a}_i - u'u''E'\varepsilon w\ , \quad \tilde{a}_i = u''\tilde{a}'' + v''a_4\ .
\end{aligned}
$$

Here, for example, $\tilde{a}' \equiv \tilde{a}'(\Omega)$ is the field with excluded central mode ω_0 at the modulator's output generated by given external fields a_1 and a_2 with open FB [when $w \equiv w(\Omega) = 0$], and $a' \equiv a'(\Omega)$ is the same field with closed FB, i.e. it is a self-consistent field.

Now by using Eqn (2.3.7) we pass to quadratures. Then the following combinations arise:

$$
\begin{aligned}
&\frac{E'\left[\varepsilon w(\Omega) \pm \varepsilon^* w^+(-\Omega)\right]}{\sqrt{2}} \\
&\qquad = (\varepsilon \pm \varepsilon^*)k(\Omega)E'|E_i|\exp(\mathrm{i}\Omega\tau)q_i(\Omega,\,\phi_i) \\
&\qquad \equiv \frac{1}{2}(\varepsilon \pm \varepsilon^*)\alpha q_i(\Omega,\phi_i)\ . \qquad (3.1.7)
\end{aligned}
$$

Here the definition $\alpha \equiv 2k(\Omega)E'|E_i|\exp(\mathrm{i}\Omega\tau)$ is introduced and the property $w(\Omega) = w^+(-\Omega)$, following from the Hermitian character of $w(t)$, is used. The coherent fields amplitudes E_1, E', E_r are considered to be real at $E_3 = 0$, $\alpha = 2u'u''k(\Omega)E'^2 \times \exp(\mathrm{i}\Omega\tau)$.

By use of the amplitude modulator, one has that $\varepsilon + \varepsilon^* = 2$, $\varepsilon - \varepsilon^* = 0$, so that the modulator has no effect on P-quadratures; and in the case of the phase modulator, $\varepsilon + \varepsilon^* = 0$, $\varepsilon - \varepsilon^* = 2$ and the modulator does not affect Q-quadratures. In the latter case one needs to use an additional coherent field E_3 at input 3 which converts PM into AM; the homodyne field phase at the detector ϕ_i should be equal to $\pi/2$ (see Section 2.3). In both cases the modulator changes only one quadrature. Thus, the AM and PM cases differ from each other only by the replacing of Q- and P-quadratures, and therefore we will consider only the AM case ($\varepsilon = 1$) below. Then $p' = \tilde{p}' = up_1 + vp_2$, etc.

From Eqns (3.1.6) and (3.1.7) we find

$$\begin{aligned} &q' = \tilde{q}' - \alpha q_i(\phi_i), \quad \tilde{q}' = uq_1 + vq_2 , \\ &q_r = \tilde{q}_r - \chi\alpha q_i(\phi_i), \quad \tilde{q}_r = -vq_1 + uq_2 , \\ &q'' = \tilde{q}'' - u'\alpha q_i(\phi_i), \quad \tilde{q}'' = u'\tilde{q}' + v'q_3 , \\ &q_0 = \tilde{q}_0 + v'\alpha q_i(\phi_i), \quad \tilde{q}_0 = -v'\tilde{q}' + u'q_3 , \\ &q_i = \tilde{q}_i - u'u''\alpha q_i(\phi_i), \quad \tilde{q}_i = u''\tilde{q}'' + v''q_4 . \end{aligned} \tag{3.1.8}$$

Let us compare the intensities of two waves at the modulator's output. According to Eqn (2.3.9) and Eqn (3.1.8),

$$\begin{aligned} N'(\Omega) &= \sqrt{2}E'q' = \sqrt{2}uE_1\left[uq_1 + vq_2 - \alpha q_i(\phi_i)\right] , \\ N_r(\Omega) &= \sqrt{2}E_r q_r = -\sqrt{2}vE_1\left[-vq_1 + uq_2 - \chi\alpha q_i(\phi_i)\right] . \end{aligned} \tag{3.1.9}$$

Taking into account $\chi = u/v$, we find $N' + N_r = \sqrt{2}E_1 q_1 = N_1$. Thus, the modulator does not affect the sum of the output intensities (similar to the AM signal q_2 from channel 2), it only redistributes the fluxes.

To obtain self-consistent solutions, one needs to replace $q_i(\phi_i)$ in Eqn (3.1.8) by one of the following equivalent expressions:

$$\begin{aligned} q_i(\phi_i) &= q_i\cos\phi_i + p_i\sin\phi_i \\ &= (u''q'' + v''q_4)\cos\phi_i + p_i\sin\phi_i \\ &= (u''u'q' + u''v'q_3 + v''q_4)\cos\phi_i + p_i\sin\phi_i . \end{aligned} \tag{3.1.10}$$

Thus, from Eqn (3.1.8) and Eqn (3.1.10) we find

$$q_i = \tilde{q}_i - u'u''\alpha(q_i\cos\phi_i + p_i\sin\phi_i) .$$

Let us determine the feedback coefficient β (i.e. the transmission coefficient for the entire opto-electronic circuit) and the amplitude coefficient of the squeezing (or stretching) γ:

$$\begin{aligned} &\beta \equiv u''u'\alpha\cos\phi_i \equiv 2u''u'\cos\phi_i\, k(\Omega)E'|E_i|\exp(\mathrm{i}\Omega\tau) , \\ &\gamma \equiv \frac{1}{1+\beta} . \end{aligned} \tag{3.1.11}$$

Here $\beta = u''u'\alpha = (\eta_{\mathrm{eff}}T')^{1/2}\alpha$, where $\eta_{\mathrm{eff}} \equiv \eta\cos\phi_i$ is the effective quantum efficiency of the detector with the homodyne field phase taken into account, and $T' = u'^2$ is the beam splitter transmission. At $E_3 = 0$ we have $\phi_i = 0$ and

$$\beta = 2k(\Omega)I_0\exp(\mathrm{i}\Omega\tau) = 2k(\Omega)\eta T'T|E_1|^2\exp(\mathrm{i}\Omega\tau) . \tag{3.1.11a}$$

In this notation,

$$q_i = \gamma(\tilde{q}_i - u'u''\alpha p_i\sin\phi_i) = \gamma(\tilde{q}_i - \beta p_i\tan\phi_i) . \tag{3.1.12}$$

From here we find the self-consistent solutions for the signal at the detector's output depending on the input signals q_k, p_k, $k = 1, \ldots, 4$ and the coherent field phase ϕ_i on the detector

$$q_i(\phi_i) = \gamma\tilde{q}_i(\phi_i) , \tag{3.1.13}$$

where

$$\begin{aligned} \tilde{q}_i(\phi_i) &= \tilde{q}_i\cos\phi_i + p_i\sin\phi_i \\ &= (u''u'uq_1 + u''u'vq_2 + u''v'q_3 + v''q_4)\cos\phi_i \\ &\quad + (u''u'up_1 + u''u'vp_2 + u''v'p_3 + v''p_4)\sin\phi_i . \end{aligned} \tag{3.1.13a}$$

Thus, any external action $\{q_k, p_k\}$, such as a deterministic signal, classical or quantum noise, is multiplied by the parameter $\gamma(\Omega) = 1/[1 + \beta(\Omega)]$. Therefore, $\gamma(\Omega)$ plays the role of a spectral Green function for the photocurrent. At $|\gamma(\Omega)| \ll 1$ the scheme is a photocurrent stabiliser that accordingly does not perceive external signals. As a result, its noiseless cannot be used.

Notice that the coefficient $\beta \propto \eta_{\mathrm{eff}} = \eta\cos\phi_i$ is maximum at $\phi_i = 0$, in particular in the absence of the additional homodyne field ($E_3 = 0$). Meanwhile, this field is necessary when using a phase modulator, because in that case $\cos\phi_i$ in the β definition is replaced by $\sin\phi_i$.

Analogously, from Eqns (3.1.8) and (3.1.10) we obtain

$$\begin{aligned} q' &= \gamma\left\{\tilde{q}' - \alpha\left[(u''v'q_3 + v''q_4)\cos\phi_i + p_i\sin\phi_i\right]\right\} , \\ q'' &= \gamma\left[\tilde{q}'' - u'\alpha(v''q_4\cos\phi_i + p_i\sin\phi_i)\right] , \\ q_0 &= \tilde{q}_0 + v'\alpha\gamma(\tilde{q}_i\cos\phi_i + p_i\sin\phi_i) , \\ q_r &= \tilde{q}_r - \chi\alpha\gamma(\tilde{q}_i\cos\phi_i + p_i\sin\phi_i) . \end{aligned} \tag{3.1.14}$$

The output AM signals depend on the phases of the coherent components:

$$\begin{aligned} q_0(\phi_0) &= \tilde{q}_0\cos\phi_0 + p\sin\phi_0 \\ &\quad + v'\alpha\gamma\cos\phi_0(\tilde{q}_i\cos\phi_i + p_i\sin\phi_i) , \\ q_r(\phi_r) &= -q_r , \end{aligned} \tag{3.1.15}$$

(we have taken into account that the phase of the coherent wave 'reflected' by the modulator is π).

Now we turn our attention to the difference in responses to external perturbations $\{q, p\}$ at different points of the system. In formula (3.1.13) the entire external perturbation is multiplied by the factor γ. At $|\gamma| \ll 1$ this leads to a strong suppression of the external modulation and in particular to a strong squeezing of the PN in the observed photocurrent. In the first two formulas of Eqns (3.1.14) describing in-loop fields, an additional perturbation, multiplied by the factor $\alpha\gamma$ with absolute value not exceeding 1, acts. This yields an additional noise and, therefore, less squeezing than that of the current (this is one of the paradoxes mentioned in the Introduction). Finally, on the r.h.s. of the last two formulas of Eqns (3.1.14) describing output signals, a part of the external forces is not affected by the FB at all, which leads to the absence of the squeezing. Moreover, the last terms in these formulas yield excess AM noise at the output (which, in fact, can be avoided by transforming it into FM).

Let, for example, only $\langle q_1\rangle$ be nonzero (i.e. there is a harmonic modulation at the first input), then we have at the outputs,

$$\langle q_0\rangle = -uv'\gamma\langle q_1\rangle , \quad \langle q_r\rangle = -v\gamma\left[1 + (1-\chi^2)\beta\right]\langle q_1\rangle , \tag{3.1.16}$$

i.e. when $|\gamma| \ll 1$, the original modulation is suppressed. This effect can be used to 'clear' laser beams of unwanted modulation.

On the other hand, modulation $\langle q_3 \rangle$ from channel 3 influences the output signal q_0 in two ways: directly and through the FB loop. Only the latter contribution can be strongly suppressed by the FB. It is this circumstance defined by the system's dynamical properties that underlies the fact that one has no possibility to extract the squeezed light out from the FB loop using a conventional beam splitter. Meantime, in the 'nondemolition beam splitter' q_3 acts only via the FB loop (Section 3.4).

Notice that when $|\gamma| > 1$, the system under consideration can be used to amplify a weak input field, to increase the modulation depth, and to generate new components of the weak field [30]. The system may operate in the self-excitation regime when a 'self-modulation' of the original coherent field occurs. The generation frequencies Ω_n are determined by the poles of the spectral Green function for the system, i.e. by the equation $\beta(\Omega_n) = -1$. These frequencies are approximately defined by maxima of the function $|\gamma(\Omega)|$, which is plotted in Fig. 11. One may avoid the self-excitation by introducing an additional damping at these frequencies [12].

We have solved the dynamical part of the problem — we expressed internal and output AM and FM signals in terms of the input signals. This stage is classical in essence.

3.2 Green function and commutators

Let us consider now the system's response to a short (with respect to $1/\Delta\Omega$) perturbation. Let at instant $t = 0$ the amplitude of a coherent field E_1 incident on input 1 increase sharply and then drop back. Here one may assume $\langle Q_1(t) \rangle \propto \delta(t)$, so that $\langle \tilde{q}_i(\Omega) \rangle \propto \langle q_1(\Omega) \rangle = \text{const}$ and the photocurrent pulse arises according to Eqn (3.1.13):

$$\langle Q_i(t) \rangle \propto (2\pi)^{-1} \int d\Omega \exp(-i\Omega t)\gamma(\Omega) \equiv G(t) \,. \qquad (3.2.1)$$

Here the real function $G(t)$, the Fourier-image of the function $\gamma(\Omega) = \gamma^*(-\Omega)$, is interpreted as the photocurrent variable component. This is the response of the current to a short δ-like perturbation of the input coherent field. Since the model under consideration is linear, the same function $G(t)$ is used in classical and quantum versions.

By representing γ in the form $1 - \beta/(1+\beta)$, we select the original signal $G_0 = \delta(t)$ from the response function

$$G(t) = \delta(t) - \frac{1}{2\pi} \int \frac{d\Omega \exp(-i\Omega t)\beta}{1+\beta} \,. \qquad (3.2.2)$$

According to the causality principle, the second term here must vanish at $t < \tau$.

Let $\beta_0(\Omega)$ describe electronics' dispersion with no account for optical delay τ. By substituting

$$\gamma = \left[1 + \beta_0(\Omega)\exp(i\Omega\tau)\right]^{-1} = \sum_{n=0}^{\infty} \left[-\beta_0(\Omega)\right]^n \exp(in\Omega\tau) \,,$$

we find the series expansion of the Green function in powers of β

$$G(t) = \sum_{n=0}^{\infty} G_n(t - n\tau) \,,$$
$$G_n(t) \equiv \frac{1}{2\pi} \int d\Omega \exp(-i\Omega t)\left[-\beta_0(\Omega)\right]^n \,. \qquad (3.2.3)$$

Here $G_0(t) = \delta(t)$ describes the initial perturbation, $G_1(t - \tau)$ is delayed by the τ response of the amplifier (this is the Green function of the electronics), and the subsequent terms of the series $G_n(t - n\tau)$ describe the repetitive passing of the signal through the circuit.

If one neglects the electronics' dispersion, the Green function is a periodic sequence of δ-functions:

$$G(t) = \sum_{n=0}^{\infty} (-\beta_0)^n \delta(t - n\tau) \,. \qquad (3.2.4)$$

A characteristic feature here is the changing of signs (at $\beta_0 > 0$) of the system's response pulses: the first pulse (observed with a delay τ) is negative due to the modulator's transmission decrease, but later on this pulse causes the modulator's transmission increase and, correspondingly, a positive second pulse of the current.

Let the electronics act as a one-pole low-frequency filter

$$\beta_0(\Omega) = \frac{\beta_0}{1 - i\Omega\tau_a} \,, \qquad (3.2.5)$$

(here $\tau_a \approx 1/\Delta\Omega$ is an effective time constant determined by the bandwidth $\Delta\Omega$ of the detector, amplifier, or modulator), then partial pulses in Eqn (3.2.2) at $n = 1, 2, \ldots$ have the form

$$G_n(t) = \frac{(-\beta_0)^n}{2\pi} \int d\Omega \frac{\exp(-i\Omega t)}{(1 - i\Omega\tau_a)^n} = \frac{(-\beta_0)^n (t/\tau_a)^{n-1} \theta(t)}{(n-1)!\,\tau_a \exp(t/\tau_a)} \,, \qquad (3.2.6)$$

where $\theta(t)$ is a step function.

Fig. 11 shows the plot of the function $G(t) - \delta(t)$ according to Eqn (3.2.3) and Eqn (3.2.6), and of the function $|\gamma(\Omega)|^2$ in the case of $\tau = 10\tau_a$ and $\beta_0 = 0.6$.

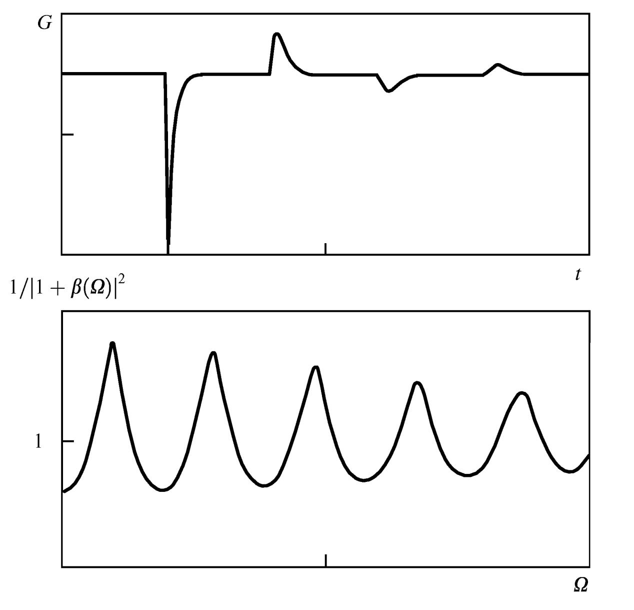

Figure 11. Temporal Green function $G(t)$ (upper figure) and frequency characteristics $1/|1 + \beta(\Omega)|^2$ (lower figure) of the opto-electronic circuit with a feedback loop [according to Eqn (3.2.5) and Eqn (3.2.6)] at $\tau = 10\tau_a$ and $\beta_0 = 0.6$. The distance between the extrema is approximately $2\pi/\tau$ by frequency and τ by time, where τ is the total delay in the circuit (with no account for electronics dispersion).

Of similar shape (but with opposite sign) are the output signals

$$\langle Q_0(t)\rangle = -uv'\langle Q_i(t)\rangle\,, \quad \langle Q_r(t)\rangle = -v\langle Q_i(t)\rangle\,, \tag{3.2.7}$$

(in the latter expression $T = R = \frac{1}{2}$ is assumed).

With $\Omega \ll \tau_a^{-1}$ and $\tau_a \ll \tau$, one can use the approximation (3.2.4); then $\gamma(\Omega) \approx 1/[1+\beta_0 \exp(\mathrm{i}\Omega\tau)]$. This function is reminiscent of the transmission coefficient for a laser Fabry–Perot resonator, or rather for a ring resonator of the total length c/τ with proper frequencies $\Omega_n = 2\pi n/\tau$ and excitation condition $\beta_0 = -1$.

Let us now find the field commutators in the presence of FB. From Eqn (2.3.8) and Eqn (3.1.14) we obtain

$$\begin{aligned} &\left[q'(\Omega), p'(-\Omega)\right] = \mathrm{i}\gamma(\Omega)\,, \\ &\left[q'(\Omega,\phi_1), q'(-\Omega,\phi_2)\right] = \mathrm{i}\gamma(\Omega)\sin(\phi_2-\phi_1)\,. \end{aligned} \tag{3.2.8}$$

Similar expressions hold for operators q'', p''. Thus, having a strong negative FB, the in-loop field operators commute with each other, i.e. they acquire classical character. This is in agreement with the fact of the current PN decrease observed in the experiments.

From Eqn (3.2.8) it follows that

$$\left[a'(\Omega), a'^{+}(\Omega')\right] = \delta(\Omega-\Omega')\,\mathrm{Re}\,\gamma(\Omega)\,, \tag{3.2.9}$$

so that

$$\begin{aligned} &\left[A'(t), A'^{+}(t')\right] \\ &\quad = \exp\left[\mathrm{i}\omega_0(t'-t)\right]\int \frac{\mathrm{d}\Omega \exp\left[\mathrm{i}\Omega(t'-t)\right]}{2\pi}\,\mathrm{Re}\,\gamma(\Omega) \\ &\quad = \delta(t-t') - \exp\left[\mathrm{i}\omega_0(t'-t)\right] \\ &\qquad \times \int \frac{\mathrm{d}\Omega \exp\left[\mathrm{i}\Omega(t'-t)\right]}{2\pi}\,\mathrm{Re}\,\frac{\beta(\Omega)}{1+\beta(\Omega)}\,. \end{aligned} \tag{3.2.10}$$

The second term in the latter expression is nonzero only at $|t-t'| > \tau$. Making use of the definition (3.2.1) for the Green function $G(t)$, we get the relation

$$\begin{aligned} &\left[A'(t), A'^{+}(t')\right] \\ &\quad = \frac{1}{2}\left[G(t-t') + G(t'-t)\right]\exp\left[\mathrm{i}\omega_0(t'-t)\right]\,. \end{aligned} \tag{3.2.11}$$

With the help of Eqn (3.1.14), it is easy to verify that the commutation relations are conserved for the output signals:

$$\left[q_0(\Omega), p_0(-\Omega)\right] = \left[q_r(\Omega), p_r(-\Omega)\right] = \mathrm{i}\,. \tag{3.2.12}$$

Finally, we take into account the FB action in the original transformation of the field by the modulator (3.1.1). Let $u' = u'' = 1$, $u = v = 1/\sqrt{2}$ for simplicity; then Eqn (3.1.8) and Eqn (3.1.14) take the form

$$\begin{aligned} &q' = \frac{\gamma(q_1+q_2)}{\sqrt{2}}\,, \qquad p' = \frac{p_1+p_2}{\sqrt{2}}\,, \\ &q_r = \frac{(\gamma-2)q_1 + \gamma q_2}{\sqrt{2}}\,, \quad p_r = \frac{-p_1+p_2}{\sqrt{2}}\,. \end{aligned} \tag{3.2.13}$$

From here, using Eqn (2.3.7) we find the action of the modulator on the operators $\{a, a^+\}$:

$$\begin{aligned} a'(\Omega) &= 2^{-3/2}\left\{\left[\gamma(\Omega)+1\right]\left[a_1(\Omega)+a_2(\Omega)\right]\right. \\ &\quad \left. + \left[\gamma(\Omega)-1\right]\left[a_1^+(-\Omega)+a_2^+(-\Omega)\right]\right\}\,, \\ a_r(\Omega) &= 2^{-3/2}\left\{\left[\gamma(\Omega)-3\right]a_1(\Omega) + \left[\gamma(\Omega)+1\right]a_2(\Omega)\right. \\ &\quad \left. + \left[\gamma(\Omega)-1\right]\left[a_1^+(-\Omega)+a_2^+(-\Omega)\right]\right\}\,. \end{aligned} \tag{3.2.14}$$

This again leads to commutators (3.2.9) and (3.2.12). This linear transformation (which is unitary only if $\gamma = 1$) differs significantly from conventional quantum-mechanical transformations. Notice the mixing of the positive and negative frequency components typical of squeezing .

3.3 PN in a system with negative feedback

Dynamical relations (3.1.13)–(3.1.15) allow the determination of the statistics of the internal and output signals through that of the input signals $\{q_k, p_k\}$ $(k = 1, \ldots, 4)$. The input correlators may be found from Eqn (2.3.11); for example, in the case of vacuum we have at all inputs,

$$\begin{aligned} \langle q_k(\Omega)q_k(-\Omega)\rangle &= \langle p_k(\Omega)p_k(-\Omega)\rangle = -\mathrm{i}\langle q_k(\Omega)p_k(-\Omega)\rangle \\ &= \mathrm{i}\langle p_k(\Omega)q_k(-\Omega)\rangle = \frac{1}{2}\,. \end{aligned} \tag{3.3.1}$$

Now at all four inputs let there be the same independent fields with a spectral density $n(\Omega) = n(-\Omega)$ and a real squeezing parameter $m(\Omega)$; then, according to Eqn (2.3.11), 'zero fluctuations' $\frac{1}{2}$ are replaced by $n + m + \frac{1}{2}$. [In fact, the fictitious field intensity a_4 cannot be nonzero, so that the example given makes sense only in the approximation $(\eta = 1)$.] Therefore, the experiments considered can in principle be repeated in the classical regime by feeding intense (so that $|n+m| \gg \frac{1}{2}$) noise radiation to all inputs. Since the system is linear, nothing should be qualitatively changed. Thus the effect of the noise suppression by FB has a close, fundamentally classical analogue. This conclusion, based on the identity of classical and quantum Green functions, may be extrapolated on all linear optical systems [32, 44], for example on parametric transformers (see Section 2.5).

In what follows we shall also need the case when fields at all but the first input are in the vacuum state, and an excess noise (positive or negative) is present at input 1, i.e. $\langle : q_1^2 : \rangle \equiv \equiv \langle q_1^2 \rangle - \frac{1}{2} \neq 0$. Normally ordered Langevin correlators at different points of the system are defined, according to Eqn (2.4.6a) and Eqn (3.1.8), by the formulas

$$\begin{aligned} &\langle : \tilde{q}_i^2 : \rangle = \eta\langle : \tilde{q}''^2 : \rangle = \eta T'\langle : \tilde{q}'^2 : \rangle = \eta T'T\langle : q_1^2 : \rangle\,, \\ &\langle : \tilde{q}_r^2 : \rangle = R\langle : q_1^2 : \rangle\,, \quad \langle : \tilde{q}_0^2 : \rangle = TR'\langle : q_1^2 : \rangle\,. \end{aligned} \tag{3.3.1a}$$

To begin with, we find fluctuations at different points of the scheme in case (3.3.1), i.e. for a vacuum at all inputs (Fig. 12). All the transformations used above are orthogonal, so that Langevin correlators of the form $\langle \tilde{q}_i(\Omega)\tilde{q}_i(-\Omega)\rangle$, $\langle p_i(\Omega)p_i(-\Omega)\rangle$, $\langle \tilde{q}_i(\Omega)p_i(-\Omega)\rangle$ take vacuum values (3.3.1). Then correlators like $\langle \tilde{q}_i(\Omega)p_i(-\Omega)\rangle = \mathrm{i}/2$ provide no contribution as they are compensated by the average $\langle p_i(\Omega)\tilde{q}_i(-\Omega)\rangle = \langle \tilde{q}_i(\Omega)p_i(-\Omega)\rangle^* = -\mathrm{i}/2$; having the opposite signs. Cross correlators like $\langle \tilde{q}_0(\Omega)\tilde{q}_i(-\Omega)\rangle$ make no contribution as well.

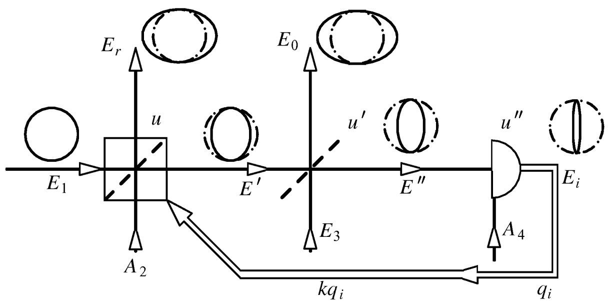

Figure 12. Quantum fluctuations of amplitude and phase at different points of the system. The amplitude fluctuations of the in-loop fields between the modulator and detector are suppressed (without the corresponding increase in phase fluctuations), but to a smaller degree than the current fluctuations. The output fields contain some excessive modulator-induced amplitude fluctuations.

As a result, from Eqn (3.1.13) we easily derive the Fano factor for the current:

$$F_i(\phi_i) = 2\langle q_i^2(\phi_i)\rangle = 2|\gamma|^2\langle \tilde{q}_i^2(\phi_i)\rangle = |\gamma|^2 \,. \qquad (3.3.2)$$

The spectral density of the current fluctuations is then equal to $\langle i^2(\Omega)\rangle = 2I_0\langle q_i^2(\Omega,\phi_i)\rangle = I_0|\gamma|^2$. Thus, at frequencies where $|\gamma(\Omega)| < 1$, the shot noise should also be less than the noise determined by the Schottky formula, which was observed, for example, in the experiments [12] (Fig. 13). The functional relation $F(I_0) = (1 + cI_0)^{-2}$, where c is a proportionality constant, was also confirmed experimentally in Ref. [12] (Fig. 14).

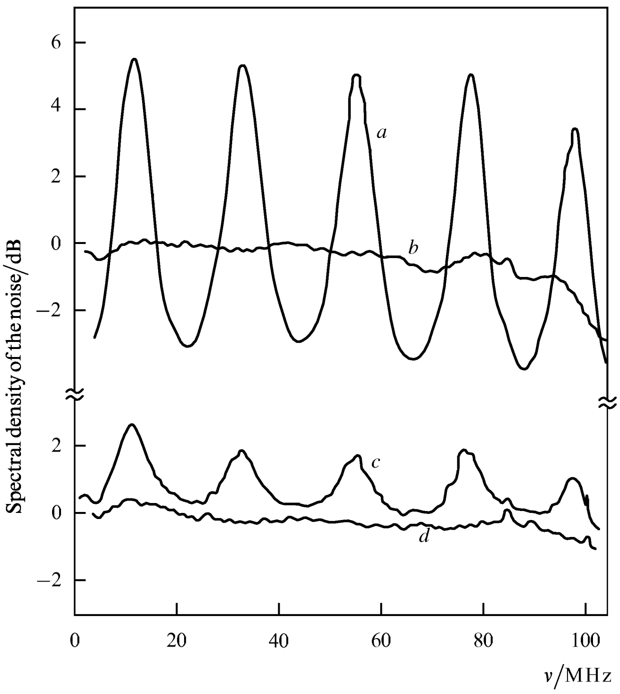

Figure 13. Dependences of the spectral density of the photocurrent fluctuations on frequency observed in Ref. [12] for internal (a) and external (c) detectors. Dependences (b) and (d) correspond to the shot level of noise. The periodicity in the feedback sign change (22 MHz) corresponds to a time delay in the feedback loop of $\tau = 45$ ns.

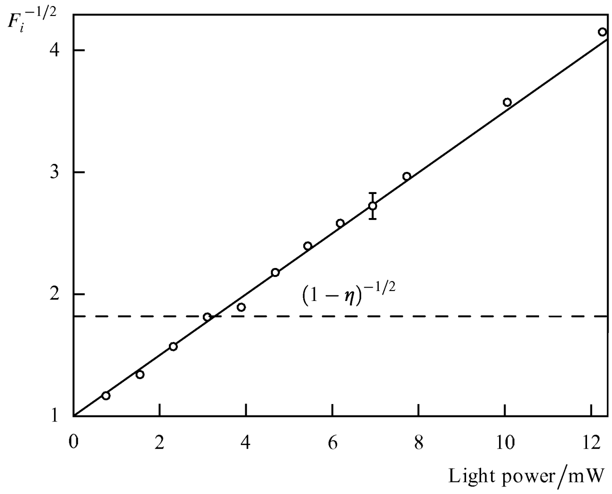

Figure 14. The relation between $F_i^{-1/2}$ and the power of light on the detector [12]. Here F_i is the relative spectral density of the inner detector's photocurrent noise at frequency 18 MHz. The dashed line is the limiting level following from Eqn (2.4.8) at $\eta = 0.68$.

Notice the periodic dependence of the PN squeezing on the homodyne field phase due to factor $\cos\phi_i$ in the coefficient β [see Eqn (3.1.11)], in full analogy with the case of the 'conventional' squeezed light [see Eqn (2.3.14); the field E_3 was absent in the experiments [9–13], so that $\phi_i = 0$].

The correlation function of the photocurrent is defined by the Fourier image of the function $F(\Omega,\phi_i)$

$$\langle Q_i(t,\phi_i)Q_i(t',\phi_i)\rangle = \frac{1}{4\pi}\int \mathrm{d}\Omega \exp\left[-\mathrm{i}\Omega(t-t')\right]|\gamma(\Omega)|^2 \,. \qquad (3.3.3)$$

For the output field according to Eqn (3.1.14) and Eqn (3.3.1)

$$F_0(\phi = 0) = 1 + |\nu'\alpha\gamma|^2 = 1 + \frac{R'}{\eta_{\mathrm{eff}}T'}\left|\frac{\beta}{1+\beta}\right|^2 ,$$

$$F_0(\phi) = F_0(0)\cos^2\phi + \sin^2\phi = 1 + \frac{R'\cos^2\phi}{\eta_{\mathrm{eff}}T'}\left|\frac{\beta}{1+\beta}\right|^2 . \qquad (3.3.4)$$

Thus, the noise is super-Poissonian at the optical output (see Fig. 13), i.e. the excess noise of the modulator provides a positive contribution to the total variance of the output field, in contrast to the in-loop field [see Eqn (3.3.6)]. At the same time, the PN of the output field remains at the vacuum level. Thus, the output field like the squeezed light has different amplitude and phase modulation, which can be studied by a homodyne detector. By selecting phase and amplitude of the homodyne field so that $\phi = \pi/2$ (i.e. by transforming the excess AM and PM), one can get rid of the excess amplitude noise at the output.

This is also valid for the second output field E_r 'reflected' from the modulator

$$F_r(\phi) = 1 + |\chi\alpha\gamma\cos\phi|^2 = 1 + \frac{\cos^2(\phi)T}{\eta_{\mathrm{eff}}T'R}\left|\frac{\beta}{1+\beta}\right|^2 . \qquad (3.3.5)$$

Now we find the PN of the in-loop fields. Making use of Eqn (3.1.14) and Eqn (3.3.1) we get

$$F'(\phi=0)=\frac{1+|\beta|^2(1-\eta_{\text{eff}}T')/\eta_{\text{eff}}T'}{|1+\beta|^2}\,,$$
$$F''(\phi=0)=\frac{1+|\beta|^2(1-\eta_{\text{eff}})/\eta_{\text{eff}}}{|1+\beta|^2}\,, \qquad (3.3.6)$$

which coincides essentially with the results obtained in Refs [12, 17]. It follows from here that $F'-F''\geqslant 0$, $F''-F_i\geqslant 0$, i.e. the relative fluctuations of the field on the detector E'' are also suppressed but less than those of the current [12], and the relative noise of the in-loop field E' decreases during its passage through the beam splitter or, in general, an absorber — contrary to the rule (2.4.7). As was already said, this is a result of the correlation, which is not taken into account in Eqn (2.4.7), of the quantum and excess noise incident on the beam splitter due to FB. In the case of an absorber or detector, the correlation is meant between the incident field and phenomenological Langevin sources introduced into the theory to recover the transformation unitarity when dissipation is present.

Let $\beta^*=\beta$, $\eta_{\text{eff}}=\eta$ for simplicity, then $F''<1$ at $\beta<2\eta/(1-2\eta)$. Analogously, $F'<1$ at $\beta<2\eta T'/(1-2\eta T')$. Thus at $\eta T'<0.5$ a paradoxical situation is possible: the field before the beam splitter (or any absorber in general) is not squeezed, 'classical', but acquires a nonclassical character as the result of the absorption. Note, however, that it is more consistent to define the nonclassical character of the in-loop field in accordance with Eqn (3.2.9) from the condition $F-\text{Re}(\gamma)<0$.

According to Eqn (3.3.6), at a given detector efficiency η there is an optimal FB coefficient $\beta_{\text{opt}}=\eta/(1-\eta)$ producing a minimum field noise $F'_{\min}=1-\eta$ [12]. Thus a rather unexpected coincidence takes place: the minimum value of the Fano factor for the in-loop *field* incident on the detector coincides with the minimum value of the Fano factor for the *current* in the absence of FB and for the full PN suppression in the light falling onto the detector [see Eqn (2.4.8)].

The formulas obtained above agree with general rules of noise transformation by a beam splitter (2.4.6). Thus from Eqn (3.3.4) and Eqn (3.3.6) at $\phi_i=0$ we derive the conservation law for the sum of variances (2.4.6a): $F_0+F''=F'+F_3$ (here $F_3=1$). Further, expression (2.4.6c) after the replacement Q_1, Q_1', Q_2, $Q_2'\Rightarrow q'$, q'', q_3, q_0 takes the form $R'\langle :q'^2:\rangle-\langle :q_0^2:\rangle=2u'v'\,\text{Re}\langle q'q_3\rangle$. With the help of Eqns (3.1.4), (3.1.7), and (3.1.14), we make sure that the left-hand and right-hand sides of this equality are indeed the same.

Notice that the photocurrent and signal at the beam splitter input are correlated with each other, which can also be studied experimentally. According to Eqn (3.1.14), at $\phi_i=0$ we get

$$\langle q_0 q_i^+\rangle=\gamma^*\langle(\tilde q_0+v'\alpha\gamma\tilde q_i)\tilde q_i\rangle=\frac{1}{2}\,v'\alpha|\gamma|^2=\frac{\beta|\gamma|^2v'}{2u'u''}\,. \qquad (3.3.7)$$

Let $T'=R'=1/\sqrt{2}$ and $E_3=0$. From Eqn (2.4.5) it follows that the difference between the signal envelopes at two outputs of the beam splitter is proportional to the AM signal at input 3: $N''(t)-N_0(t)=E'Q_3(t)$. Hence, if there is vacuum at input 3, this difference, which can be observed by subtracting the currents of two detectors, will possess Poissonian fluctuations [see Eqn (2.4.6b)], which can be used to calibrate the measuring device [13]. At the same time, the signals N'' and N_0 taken separately have sub- and super-Poissonian fluctuations, respectively.

Now let us find the spectral density of the side components of the coherent field resulting from the modulation. For the output field at $\phi_i=0$ we find from Eqn (3.1.14) and Eqn (2.3.11a) that

$$n_0(\Omega)=m(\Omega)=\frac{F_0(\Omega)-1}{4}=\frac{R'}{4\eta T'}\left|\frac{\beta(\Omega)}{1+\beta(\Omega)}\right|^2\,. \qquad (3.3.8)$$

The in-loop field spectrum and parameter $m=\langle a(\Omega)a(-\Omega)\rangle$ can be determined for simplicity from the condition $T'=\eta=1$. Then $q'=q''=q_i\equiv q=\gamma\tilde q$, so that

$$\langle q^2\rangle=\frac{1}{2}\left|\gamma(\Omega)\right|^2,\qquad \langle p^2\rangle=\frac{1}{2}\,,$$
$$\langle q(\Omega)p(-\Omega)\rangle=\langle p(\Omega)q(-\Omega)\rangle^*=\frac{1}{2}\,\mathrm{i}\gamma(\Omega)\,. \qquad (3.3.9)$$

Using Eqn (2.3.11a), we get from here

$$n(\Omega)=n(-\Omega)=\frac{1}{4}\left|\frac{\beta(\Omega)}{1+\beta(\Omega)}\right|^2\,,$$
$$m=\frac{1}{4}\left(\frac{1+2\mathrm{i}\,\text{Im}\,\beta(\Omega)}{|1+\beta(\Omega)|^2}-1\right)\,. \qquad (3.3.10)$$

Let $\beta=\beta^*\neq -1$. Then according to Eqn (3.3.10)

$$\frac{m}{n}=-\left(1+\frac{2}{\beta}\right)\,. \qquad (3.3.10a)$$

From here at $\beta>-1$ we have $|m|/n>1$, whereas in classical theory this inequality should have the opposite sign: $|m|/n\leqslant 1$ (see the Appendix). Thus, the in-loop field in the presence of an arbitrarily weak FB both positive or negative should be considered nonclassical. Notice that here, as in the case of usual squeezed light [see Eqn (2.5.4)], the nonclassical parameter $|m|/n$ tends to infinity as the degree of squeezing decreases.

Let us express $F=2\langle q^2\rangle$ through m and n by means of Eqn (2.3.11). Unity in Eqn (2.3.11) arose from the commutator $[a,a^+]=1$. Replacing it by $\text{Re}(\gamma)$ in accordance with Eqn (3.2.9) yields

$$F=2\langle q^2\rangle=2n+2\,\text{Re}(m+\gamma)\,. \qquad (3.3.11)$$

By substituting Eqn (3.3.10) into this equation, we obtain again Eqn (3.3.2): $F=|\gamma|^2$. From Eqn (3.3.11) and the classical Cauchy – Schwartz inequality $n\geqslant|m|$ it follows that $F\geqslant\text{Re}(\gamma)$. From here, one more condition of nonclassical character for the in-loop field can be derived: $F<\text{Re}(\gamma)$ (instead of the usual $F<1$).

Finally, let us consider the case when strongly squeezed light is fed into input 1, so that $F_1=2\langle q_1^2\rangle=0$ and $\langle :q_1^2:\rangle=-\frac{1}{2}$ (such an initial condition makes no sense in semiclassical theory where $F_1>1$). Now according to (3.3.1a) $\langle\tilde q_i^2\rangle=(1-\eta T'T)/2$ and $\langle\tilde q_0^2\rangle=(1-TR')/2$. Let $E_3=0$, then from Eqn (3.1.13), instead of $F_i=|\gamma|^2$, we obtain

$$F_i=2|\gamma|^2\langle\tilde q_i^2\rangle=\left|\gamma(\Omega)\right|^2(1-\eta T'T)\,. \qquad (3.3.12)$$

Thus the current fluctuations reveal both the initial squeezing attenuated by losses [factor $1-\eta T'T$ (see Eqn (2.4.7)] and the squeezing due to FB. Analogously, for the output field fluctuations from Eqn (3.1.14), instead of Eqn (3.3.4), we get [cf. Eqn (3.1.4)]

$$F_0 = 2\langle\tilde{q}_0^2\rangle + \frac{2R'}{\eta T'}\left|\frac{\beta}{1+\beta}\right|^2 ,$$

$$\langle\tilde{q}_i^2\rangle = 1 - TR' + \frac{R'(1-\eta T'T)}{\eta T'}\left|\frac{\beta}{1+\beta}\right|^2 . \qquad (3.3.13)$$

The term $1 - TR'$ describes the initial squeezing attenuated by losses and independent of FB, and the last term is the FB-produced excess noise.

3.4 Observation of the in-loop field PN

According to Eqns (3.3.4) and (3.3.5) the scheme with FB considered in the previous section does not permit one to extract the squeezed in-loop light for practical use. Furthermore, for current fluctuations actually observed in that scheme, some models predict the same result as in quantum theory, but the in-loop field in the FB circuit is not squeezed, (classical) (see Section 3.6 and Refs [19–21]). A question arises as to whether one can in principle discover experimentally the in-loop field squeezing effect or whether it is a fundamentally unobservable 'thing in itself' and the disagreements are academic in nature.

Thus, there are two interconnected problems: the possibility of experimental study of the in-loop field 'to be squeezed' and the possibility of its extraction without the squeezing being lost. Both these problems can be solved with the help of nondemolition quantum measurements [22–29].

We start with a qualitative description of a possible method for observations of the in-loop field fluctuations which uses the optical Kerr effect (see Refs [15, 16, 24]), and then we dwell in more detail on a scheme with a parametric 'nondemolition beam splitter' (Section 2.5) that solves both these problems.

The in-loop beam splitter in Fig. 10 that extracts the light 'out from' the FB-loop, together with an additional external detector, performs a 'perturbing' or 'demolition' measurement of the energy flux. Such measurement, owing to the vacuum noise contribution Q_3 from input channel 3, recovers the usual PN of the original laser light and additionally reveals some excess noise introduced by the modulator [see Eqn (3.3.4)]. To discover experimentally the assumed in-loop squeezing, it is necessary to use nonlinear optical devices for information extraction.

In the in-loop beam we put, instead of a conventional beam splitter, a transparent material with a large cubic nonlinearity, for example a Kerr cell with nitrobenzol. Under the action of the light field, the optical Kerr effect occurs: the material's refractive index n varies proportionally to the in-loop field intensity $N'(t)$, i.e. the light amplitude modulation signal is carried over the refractive index $\Delta n(t) \propto N(t) \propto Q'(t)$, where Q' is the quadrature (modulating) in-loop field signal.

The modulation of the refractive index of the material $\Delta n(t)$ can be transformed into PM of an additional probing light beam (the latter thus introduces some PM back into the in-loop field which has no effect, however, on the detector's current at $\phi_i = 0$). Then PM of the probing beam is transformed into AM and is detected. The photocurrent fluctuation will have an excess component proportional to the AM noise of the in-loop field $\langle q'^2(\Omega,\phi)\rangle$.

Unfortunately, the practical realisation of efficient Kerr nondemolition detectors meets difficulties because of the weak cubic nonlinearity of the material.

Let us therefore consider a method of QND-transformation owing to a quadratic nonlinearity [26–29]. Let us replace the beam splitter in Fig. 10 by a PT with two beam splitters as described at the end of Section 2.5. After the notation in Eqn (2.5.10) has been rearranged in accordance with Fig. 10, instead of Eqn (3.1.2) we get

$$q_0 = q', \quad p_0 = p' - 2fp_3 ,$$
$$q'' = q_3 + 2fq', \quad p'' = p_3 , \qquad (3.4.1)$$

(here $f = \sinh\Gamma$ is a parametric coupling coefficient). Thus, now Q-quadrature of the output field duplicates $(q_0 = q')$ Q-quadrature of the in-loop field at the modulator's output (without signal q_3 from input vacuum channel 3 being added!), and the 'nondemolition measurement' signal $(q'' = q_3 + 2fq')$ is used to get the error signal depending on q' and q_3. One can neglect the contribution of the intrinsic PT-noise proportional to f^2 for a sufficiently coherent component. We emphasise that the PT by itself produces no squeezing at all: according to Eqn (3.4.1), $F_0 = 1$ and $F'' = 1 + 4f^2$ for open FB.

With the substitution $u' \to 2f$, $v' \to 1$, the expression for q'' in Eqn (3.4.1) coincides with the expression $q'' = u'q' + v'q_3$ used earlier for a conventional beam splitter [see Eqn (3.1.2)]. Therefore, in Eqn (3.1.11) and Eqn (3.1.14), it is sufficient to make the substitutions (we assume $E_3 = \phi_0 = \phi_i = 0$)

$$\beta = 2(uu'u''E_1)^2 k(\Omega)\exp(\mathrm{i}\Omega\tau)$$
$$\to \beta = 2(2fuu''E_1)^2 k(\Omega)\exp(\mathrm{i}\Omega\tau) ,$$
$$q' = \gamma\left[\tilde{q}' - \frac{\beta}{u'u''}(u''v'q_3 + v''q_4)\right]$$
$$\to q' = q_0 = \gamma\left[\tilde{q}' - \frac{\beta}{2fu''}(u''q_3 + v''q_4)\right] . \qquad (3.4.2)$$

From here [cf. Eqns (3.3.4) and (3.3.6)] we derive the Fano factor at the external output of the PT which coincides with that for the in-loop field

$$F_0 = F' = \frac{1 + |\beta|^2/4f^2\eta}{|1+\beta|^2} = \frac{1 + |4uE_1k(\Omega)|^2}{|1+\beta|^2} . \qquad (3.4.3)$$

Formula (3.3.6) for F'' remains unchanged.

Let $\beta = \beta^*$, then at $\beta = 4f^2\eta$ the squeezing is maximum: $F_0 = 1/(1 + 4f^2\eta)$. At $\Gamma \gg 1$ we get $F_0 = \exp(-2\Gamma)/\eta$, which differs only by a factor $1/\eta$ from the result of squeezing with the use of a conventional PT with one transverse mode [see Eqn (2.5.3)].

Notice that according to Eqns (3.4.1) and (3.4.3), the uncertainty relation (2.3.10) for the output field is satisfied:

$$4\langle q_0^2\rangle\langle p_0^2\rangle \geqslant \frac{1 + 4f^2}{1 + 4f^2\eta} \geqslant 1 . \qquad (3.4.4)$$

If one introduces an additional light absorption η_1 in front of the in-loop detector by keeping coefficients β and f constant, then η in the numerator of Eqn (3.4.3) is replaced by $\eta\eta_1$. As a result, F_0 increases. The relative difference $(F_0 - F_i)/F_i$, proportional to $1/\eta_1$ according to Eqn (3.4.3), characterises the effect of 'dissipative squeezing' — the relative field fluctuations decrease during absorption [see the discussion after formula (3.3.6)]. Thus, this effect can be experimentally studied by comparing the noise of two detectors — the in-loop and nondemolition external detectors (or of two nondemolition external detectors).

3.5 Discussion

The formal computations made in Sections 3.1 – 3.4, in spite of the simplicity of the linear algebraic relations between spectral components of the fields and current used, yield some difficulties for interpretation, as already mentioned in the Introduction.

The paradox of the violation of the rule 'absorption decreases the non-Poissonian character', can be formally resolved without special difficulty. This rule assumes no correlation of the original field and quantum noise added during the absorption, whereas the FB just establishes such a correlation. More unusual is the fact of such a quantum correlation established through the macroscopic signal controlling the modulator. We repeat in a simplified schematic way the logic of the FB-effect calculation.

Let $T' = \eta = 1$ (i.e. the beam splitter is absent and the detector's efficiency is unity), the delay τ be zero, and the additional homodyne field E_3 be equal to zero as well (then $\phi_i = 0$), so that all three in-loop fields coincide ($E' = E'' = E_i = E$; $Q' = Q'' = Q_i = Q$). We also neglect the electronics' dispersion $[k(\Omega) = k]$. Then the main relations (3.1.8) take the form

$$Q = \widetilde{Q} - W\,, \quad P = \widetilde{P}\,. \tag{3.5.1}$$

Here Q and P are the self-consistent values of the operators of the in-loop field between the modulator and detector (with no account of a strong coherent component E), $\widetilde{Q}$, $\widetilde{P}$ are the same field operators with the open FB-loop, i.e. in the absence of the modulation signal $(-W)$. (Minus sign gives negative FB.) According to Eqn (3.5.1), the in-loop field is made by a superposition of two principally different components — by the external field $\widetilde{Q}$, $\widetilde{P}$ producing PN and an additional field $(-W)$ emitted by a classical source (in fact, the modulator generates no additional fields and only redistibutes incident fluxes, but the in-loop field 'does not know' that).

As is known, a classical source of field conserves the coherent character of the original field state [31]. In the Heisenberg representation, its effect is reduced to adding to the field operators a nonoperator part ($-W$ in the given case) that coincides with the classical field of the given source [31]. As a result, the transformation (3.5.1) is unitary and the commutator remains unchanged:

$$[Q, P] = \left[\widetilde{Q} - W, \widetilde{P}\right] = \left[\widetilde{Q}, \widetilde{P}\right] = \mathrm{i}\,. \tag{3.5.2}$$

A classical signal $W(t)$, noisy or determined, modulates the amplitude E of the original coherent field. In the case of stochastic modulation, from Eqn (3.5.1) we find the field variance in the form:

$$\langle \overline{Q^2} \rangle = \left\langle \overline{(\widetilde{Q} - W)^2} \right\rangle = \langle \overline{Q^2} \rangle + \overline{W^2}\,. \tag{3.5.3}$$

Here the bar means classical averaging (over an ensemble of identical experimental devices or, by assuming ergodicity, over time) and $\langle \widetilde{Q} \overline{W} \rangle = 0$. As one might expect, during the superposition of independent random values their variances are summed (independently of the sign of W).

But in the model considered, the stochasticity of the modulating signal W is connected with quantum fluctuations of the detector's current, i.e. one should consider the operator W to correspond to the classical observable Q

$$W = \beta Q \tag{3.5.4a}$$

(here β is the transmission coefficient for the entire loop). Introducing such a relation between c- and q-numbers, i.e. making some operator an *observable* quantity, appears to be an unavoidable stage in any quantum model. It determines the external boundary of the quantum object and constructs the bridge between quantum formalism and experiment. The choice of appropriate operator is usually made by informal, intuitive considerations. The next step, according to accepted postulates of quantum theory, is equating the observed time means to the quantum means:

$$\overline{W} = \beta \langle Q \rangle\,, \quad \overline{W^2} = \beta^2 \langle Q^2 \rangle, \ldots\,. \tag{3.5.4b}$$

After substituting the latter equation into Eqn (3.5.3), we find

$$\begin{aligned} \langle \overline{Q^2} \rangle &= \langle \widetilde{Q}^2 \rangle + \beta^2 \langle \overline{Q^2} \rangle = \frac{1}{1 - \beta^2} \langle \widetilde{Q}^2 \rangle\,, \\ \overline{W^2} &= \frac{\beta^2}{1 - \beta^2} \langle \widetilde{Q}^2 \rangle\,. \end{aligned} \tag{3.5.5}$$

Therefore, regardless of the sign of β, the FB must increase current field fluctuations.

The experiments, however, demonstrate a decrease in current fluctuations (at some frequencies); they suggest the functional relation $\langle \overline{Q^2} \rangle = \langle \widetilde{Q}^2 \rangle / (1 + \beta)^2$ (see Fig. 14). The obvious error of our conclusion is in using in Eqn (3.5.3) the assumption that there is no correlation between the classical controlling signal and the external operator field. To take this into account, we substitute Eqn (3.5.4a) into Eqn (3.5.1) without averaging:

$$Q = \widetilde{Q} - \beta Q = \frac{\widetilde{Q}}{1 + \beta}\,. \tag{3.5.6}$$

When $\beta \gg 1$, we have $Q \approx 0$, i.e. according to Eqn (3.5.1) $W \approx \widetilde{Q}$ — the modulating signal $(-W)$ is fully anticorrelated with the original quantum noise. With the help of Eqn (3.5.4a) we excluded the classical signal so that here there is no classical randomness. Then, performing quantum averaging, we get a result consistent with experiment and dependent on the FB sign:

$$\begin{aligned} \langle Q^2 \rangle &= \frac{\langle \widetilde{Q}^2 \rangle}{(1 + \beta)^2} = \frac{1}{2(1 + \beta)^2}\,, \\ \langle W^2 \rangle &= \frac{\beta^2}{2(1 + \beta)^2}\,. \end{aligned} \tag{3.5.7}$$

Experiments [12] are in agreement with the result obtained from linear coupling (3.5.4a) between the macroscopic signal W and the field operator Q, i.e. with the fact that the real macroscopic voltage on the modulator is taken to be proportional (without quantum averaging) to the Heisenberg quantum mechanical operator. The relation $W = \beta Q$ between unaveraged c- and q-numbers describes the procedure of quantum measurement conserving quantum stochasticity [see also the discussion after formula (2.5.14)]

In the model considered above $\eta = 1$, so that the envelope signal of the field falling onto the detector — operator Q'' — coincided with the classical signal of the detector Q_i. Here the difference between quantum and classical signals is masked. The 'electronic' amplification $Q_i \to kQ_i$ or branching (cloning) of the electrical signal at the detector's output is not accompanied by the appearance of an additional quantum noise (in contrast to the light beams), which demonstrates its classical character. On the other hand, during branching of the electron flux in vacuum, branching noise is known to arise (in the absence of space charge). It appears that from here one may draw the conclusion that the signal Q_i can be cloned or amplified without being 'damaged' only after electrons have penetrated from vacuum into the metal, or, in more general terms, after collective degrees of freedom of the macroscopic objects have been excited. The latter condition appears to determine the moment of status changing when the q-number becomes a c-number (dequantises). But on the other hand in the model considered, the current 'realises' quantum measurement [15], thus reproducing the quantum stochasticity.

Thus, after the FB has been looped, the modulating signal $W(t)$ in the initial relation (3.5.1) plays the role of a Heisenberg operator. The relation (3.5.4a), as we have already noted above, modifies the standard commutation relations for in-loop fields $E'(t)$ and $E''(t)$ [at $|t-t'|>\tau$, see Eqn (3.2.10)]. Formally this results from the in-loop field not being free [15], i.e. there is an external, operator source of the field. The role of the source in Eqn (3.5.1) is played by the term $W(t)$. In the case when a classical signal is fed into the modulator, this term does not change the commutation relations, but by Eqn (3.5.4a) this invariance is lost. This is a general rule for interacting quantum systems: the interaction changes the dynamics of the subsystem, which is equivalent to Green functions modification and commutators connected with them [see Eqn (3.2.11)]. A peculiarity of the given case is that the system interacts (with a delay τ) with itself, with its own past.

The unusual features of the in-loop field seem to justify the introduction of a special term for it, for example, 'supersqueezed light'. This name reflects the main feature of the in-loop field — violation of the uncertainty relation for quadratures (2.3.10) at some frequencies. Another possible term — 'anticorrelated light' [12] — reflects the squeezing mechanism: the modulation of coherent amplitude by a stochastic signal varying in counter-phase with quantum noise arising in the detector.

Frequently in quantum optics, the nonunitarity of phenomenological transformations is reconstructed by adding appropriate Langevin sources into dynamical equations (for example, the detection process (3.3.3) description at $\eta \neq 1$). This is impossible when describing the modulator, as the modulator can change only one field quadrature while both quadratures should be changed (in the opposite directions) during unitary squeezing so as to not violate the uncertainty relation (2.3.10).

3.6 *A posteriori* approach

We shall start from the following postulate: 'quantum' fluctuations observed in experiments are the result of the *quantum measurement* procedure made by the measuring equipment. It is natural to accept that the observed shot noise of the photocurrent during illumination by light in a coherent state is the result of quantum measurement too. Hence it makes no sense to speak of *a priori* existence of the PN because it is generated only 'inside' the photodetector. Then it can be adequately taken into account, as is usually done, by a procedure of 'discretisation': the transition from continuous q-numbers (field operators) to a set of discrete random classical events $\{t_i\}$ (see Section 2.2). As follows from Eqn (2.2.12), this procedure is equivalent to (in the absence of FB) a substitution of the normally ordered correlation function of the field intensity G_2 by a nonordered one G_2'. As a result, one can carry over at will the current noise back into the light flux by considering nonordered correlation functions. Then the PN is explained by noncommutativity of the field operators.

Until recently, it has appeared that these two points of view on the PN — *a priori* and *a posteriori* — give the same observational consequences and thus the choice between them is a matter of taste. Here we shall show that this conclusion is not valid when describing the in-loop field in the FB-loop.

Notice that the discretisation procedure is the only source of the observed shot noise of the current in semiclassical theories as well (which differ from '*a posteriori*' quantum theories only in replacing quantum correlation functions by classical ones). Thus, the '*a posteriori*' quantum description given below can be considered semiclassical to the same extent. For simplicity, the original state of the field will be considered as coherent and the additional homodyne field E_3 to be absent; as above, modulation and demodulation are considered in the linear approximation.

If one is interested in the statistics not of the field amplitude and its quadratures, but only of its intensity fluctuations, then one can use the very simple description given by a stochastic phenomenological equation for the current with Langevin force $\tilde{q}_i$. According to Eqn (3.1.8), the relative noise amplitude at the detector's output has the form

$$q_i = \tilde{q}_i - \beta\,, \qquad q_i = \frac{\tilde{q}_i}{1+\beta}\,, \tag{3.6.1}$$

where $q_i = i(\Omega)/\sqrt{2I_0}$, $i(\Omega)$ is the alternating component of the current at a frequency $\Omega \neq 0$ at the detector's output (which is now a c-number), $I_0 = i(0)$ its DC-component, β is the FB coefficient, and $\tilde{q}_i^2 = \frac{1}{2}$, according to Eqn (2.2.13). From here, we derive the known expression for the Fano factor of the current (which is the same in *a priori* and *a posteriori* models)

$$F_i = \frac{2\langle q_i^2\rangle}{I_0} = \frac{1}{|1+\beta|^2}\,. \tag{3.6.2}$$

According to Eqn (3.1.8), the relative signal at the modulator's output with no account taken of the quantum noise has the form $q' = -(\beta/\sqrt{\eta T'})q_i$, so that

$$\left\langle (q')^2 \right\rangle = \frac{1}{2\eta T'}\left|\frac{\beta}{1+\beta}\right|^2\,. \tag{3.6.3}$$

This is the excess noise induced by the modulation. When detecting the in-loop field by a conventional detector, PN is added which can be taken into account in advance by having added $(\tilde{q}')^2 = \frac{1}{2}$. As a result, the Fano factor for the in-loop field between the modulator and the beam splitter takes the form

$$F' = 2\left[(q')^2 + (\tilde{q}')^2\right] = 1 + \frac{1}{\eta T'}\left|\frac{\beta}{1+\beta}\right|^2\,. \tag{3.6.4}$$

From here, using the ordinary rule (2.4.7) we get at the two outputs of the beam splitter with parameters T', R',

$$F'' = 1 + \frac{1}{\eta}\left|\frac{\beta}{1+\beta}\right|^2\,, \qquad F_0 = 1 + \frac{R'}{\eta T'}\left|\frac{\beta}{1+\beta}\right|^2\,. \tag{3.6.5}$$

The latter expression coincides with Eqn (3.3.4) as well as with the results calculated by other authors (with the substitution $R' \to \eta_0 R'$ that takes into account the external detector's efficiency). Thus, calculations by all possible quantum and semiclassical methods yield the same results for the parameters F_i and F_0 observed by conventional detectors. At the same time, expressions for F' and F'' in Eqn (3.6.4) and Eqn (3.6.5) differ significantly from those obtained above in Eqn (3.3.6). Now they predict super-Poissonian fluctuations of the in-loop fields, i.e. the absence of squeezing. (The in-loop field parameters F' and F'' are usually ignored in semiclassical calculations as unobservable.)

3.7 Corpuscular model

Let vacuum be present at all additional inputs in Fig. 10. If one is interested only in small fluctuations $\Delta N \equiv n$ of the light intensity N (by ignoring different AM and PM modulations at the output, see Fig. 12), then one may use a simple visual description in terms of light beams with Poissonian initial intensity fluctuations $\tilde{n}$ changed under the action of the FB. The Poissonian fluctuations are associated with chaotic particle fluxes, therefore this model can be called *corpuscular*.

Let us change the field indexes. Let index 0 relate now to the original Poissonian flow, 1 to the flux after the modulator, 2 and $2'$ to the fluxes at the beam splitter's output, 3 to the electron flux at the detector's output, 4 to the amplified electrical signal controlling the modulator. Let N_j and $n_j(\Omega)$ be constant components and small fluctuation additions, respectively, at some frequency at which one can neglect the FB-loop dispersion. For Poissonian 'seed' parts of the fluctuations, which we designate by $\tilde{n}_j$, we have the spectral densities

$$\langle \tilde{n}_j^2 \rangle = N_j \,. \tag{3.7.1}$$

Let us denote the transmission coefficient of the modulator (with no signal), beam splitter, detector and amplifier by T_0, T_1, T_2, and T_3, respectively, and the noise introduced by these devices by f_j; then f_0 and f_1 describe the noise of the 'flux distribution' by beam splitting, f_2 the detector's noise (connected with difference T_2 from 1), and f_3 the amplifier's noise. The modulator in the linear approximation produces a noise component $-2N_1 n_4$ to the flux N_1, where $N_1 = T_0 N_0$. As a result, the fluxes are transformed as follows ($R_j = 1 - T_j$):

$$\begin{aligned} &N_{j+1} = T_j N_j \,, \quad n_{j+1} = T_j n_j + f_j - 2N_1 n_4 \delta_{j0} \,, \\ &N_2' = R_1 N_1 \,, \quad n_2' = R_1 n_1 + f_1 \,. \end{aligned} \tag{3.7.2}$$

Here the input flux N_0 and hence its fluctuations $\tilde{n}_0$, as well as the proper noise of the electronics f_3, are considered to be specified.

The spectral density of the noise sources f_j at $j = 0, 1, 2$ can be derived from the condition of 'Poissonianity' conservation (3.7.1):

$$\langle \tilde{n}_{j+1}^2 \rangle = T_j^2 \langle \tilde{n}_j^2 \rangle + \langle f_j^2 \rangle = T_j^2 N_j + \langle f_j^2 \rangle = N_{j+1} = T_j N_j \,. \tag{3.7.3}$$

From here,

$$\langle f_j^2 \rangle = T_j R_j N_j \,. \tag{3.7.4}$$

Thus, in the present model, instead of vacuum amplitude noise at the system's additional inputs [see Eqns (2.4.6), (3.1.2), (3.1.3)], phenomenological Langevin forces f_j ($j = 0, 1, 2$) are used, and the electronic amplifier's noise f_3 is added for generality.

Considering Eqns (3.7.2) and (3.7.4) we find

$$\begin{aligned} &n_1 = \tilde{n}_1 - 2N_1 n_4 \,, \quad n_2 = \tilde{n}_2 - 2T_1 N_1 n_4 \,, \\ &n_3 = \tilde{n}_3 - 2T_1 T_2 N_1 n_4 \,, \\ &n_4 = T_3 \tilde{n}_3 + f_3 - 2T_1 T_2 T_3 N_1 n_4 \,. \end{aligned} \tag{3.7.5}$$

In the first three equalities n_4 should be substituted by one of the following equivalent expressions

$$\begin{aligned} n_4 &= f_3 + T_3 n_3 = f_3 + T_3 f_2 + T_3 T_2 n_2 \\ &= f_3 + T_3 f_2 + T_3 T_2 f_1 + T_3 T_2 T_1 n_1 \,. \end{aligned} \tag{3.7.6}$$

Let us denote $\beta = 2T_1 T_2 T_3 N_1$, $\gamma = 1/(1+\beta)$, then from Eqns (3.7.5) and (3.7.6) it follows that

$$\begin{aligned} &n_1 = \gamma \left[\tilde{n}_1 - \beta \left(\frac{f_1}{T_1} + \frac{f_2}{T_1 T_2} + \frac{f_3}{T_1 T_2 T_3} \right) \right] , \\ &n_2 = \gamma \left[\tilde{n}_2 - \beta \left(\frac{f_2}{T_1 T_3} + \frac{f_3}{T_2 T_3} \right) \right] , \\ &n_3 = \gamma \left(\tilde{n}_3 - \beta \frac{f_3}{T_3} \right) , \quad n_4 = \gamma (T_3 \tilde{n}_3 + f_3) \,, \\ &n_2' = \tilde{n}_2' - \gamma \beta R_1 \left(\frac{\tilde{n}_3}{T_1 T_2} + \frac{f_3}{T_1 T_2 T_3} \right) . \end{aligned} \tag{3.7.7}$$

This is the 'corpuscular' equivalent of the dynamical equations for the system obtained in Section 3.1 [cf. formulas (3.1.13) and (3.1.14)]. Obviously, considering the amplifier's noise f_3, only fluctuations n_4 at its output are infinitely suppressed (at $\beta \to \infty$).

Taking into account Eqns (3.7.1) and (3.7.4), from here we find

$$\begin{aligned} &\langle n_1^2 \rangle = \gamma^2 \left\{ N_1 \left[1 + \frac{\beta^2 (1 - T_1 T_2)}{T_1 T_2} \right] + \left\langle \left(\frac{\beta f_3}{T_1 T_2 T_3} \right)^2 \right\rangle \right\} , \\ &\langle n_2^2 \rangle = \gamma^2 \left\{ N_2 \left(1 + \frac{\beta^2 R_2}{T_2} \right) + \left\langle \left(\frac{\beta f_3}{T_2 T_3} \right)^2 \right\rangle \right\} , \\ &\langle n_3^2 \rangle = \gamma^2 \left\{ N_3 + \left\langle \left(\frac{\beta f_3}{T_3} \right)^2 \right\rangle \right\} , \\ &\langle n_4^2 \rangle = \gamma^2 \left\{ T_3^2 N_3 + \langle f_3^2 \rangle \right\} , \\ &\langle (n_2')^2 \rangle = N_2' + \gamma^2 \beta^2 \left[\frac{N_2' R_1}{T_1 T_2} + \left\langle \left(\frac{f_3 R_1}{T_1 T_2 T_3} \right)^2 \right\rangle \right] . \end{aligned} \tag{3.7.8}$$

Let now $f_3 = 0$. Let us determine the Fano factors $F_j = \tilde{n}_j^2 / N_j$. According to Eqn (3.7.8),

$$\begin{aligned} &F_1 = \gamma^2 \left[1 + \frac{\beta^2 (1 - T_1 T_2)}{T_1 T_2} \right] , \quad F_2 = \gamma^2 \left(1 + \frac{\beta^2 R_2}{T_2} \right) , \\ &F_3 = \gamma^2 \,, \quad F_2' = 1 + \frac{\gamma^2 \beta^2 R_1}{T_1 T_2} \,. \end{aligned} \tag{3.7.9}$$

These expressions coincide with those obtained in Section 3.3.

Let us determine, also with the help of Eqn (3.7.7), the correlation

$$\langle n_3 n_2' \rangle = -\gamma \left\langle \tilde{n}_3 \gamma \beta R_1 \frac{\tilde{n}_3}{T_1 T_2} \right\rangle = -\frac{\gamma^2 \beta N_3 R_1}{T_1 T_2} \,. \tag{3.7.10}$$

After having divided this expression by $2(N_2' N_3) = 2N_3(R_1/T_1T_2)^{1/2}$, we obtain $\frac{1}{2}\beta\gamma^2(R_1/T_1T_2)^{1/2}$, which coincides with Eqn (3.3.7) up to a sign.

Thus, for the simplest experimental scheme, the corpuscular model yields the same results as the theory considered in Sections 3.1 – 3.5 above. However, with this model it is hard to describe unequal AM and PM modulations of the output field, the versions with a phase modulator and an additional homodyne field, and with the QND detection as well. The explanation of the anomalous action of beam splitters and absorbers placed into the FB loop in general remains out of the scope of this model.

Now let us apply the *a posteriori* approach. Then the original photon noise f_0 and noise of photon distribution f_1, f_2 are absent. Let the amplifier also be noiseless, then the detector is the sole source of noise. Substituting $\tilde{n}_1 = \tilde{n}_2 = f_3 = 0$ in Eqn (3.7.5), we find

$$n_1 = -2N_1 n_4 = -\frac{\beta\gamma\tilde{n}_3}{T_1T_2}\,, \qquad n_2 = -2T_1 N_1 n_4 = -\frac{\beta\gamma\tilde{n}_3}{T_2}\,,$$
$$n_3 = \tilde{n}_3 - 2T_1T_2N_1n_4 = \gamma\tilde{n}_3\,, \tag{3.7.11}$$
$$n_4 = T_3 n_3\,, \qquad n_2' = R_1 n_1\,.$$

Using $\langle \tilde{n}_3^2 \rangle = N_3 = T_2 N_2 = T_2 T_1 N_1$ we obtain

$$\langle n_1^2 \rangle = \frac{(\beta\gamma)^2 N_1}{T_1T_2}\,, \qquad \langle n_2^2 \rangle = \frac{(\beta\gamma)^2 N_2}{T_2}\,,$$
$$\langle n_3^2 \rangle = \gamma^2 N_3\,, \qquad \left\langle (n_2')^2 \right\rangle = \frac{(\beta\gamma)^2 N_2' R_1}{T_1T_2}\,. \tag{3.7.12}$$

Now all light fluxes n_1, n_2 and n_2' contain only excess noise introduced by the modulator. The same modulation noise enters formulas (3.6.4) and (3.6.5). The correlation (3.7.10) also follows from Eqn (3.7.11).

Thus, in the framework of the corpuscular phenomenological model both concepts of the PN also lead to the same results when describing experiments which use conventional AM detectors.

4. Conclusions

We have tried to present modern ideas about the nature of PN observed by means of analogue detectors, and about methods of its description, transformation, and suppression. In our discussion a large role is played by two comparatively new points — the idea of nondemolition measurement of the light field amplitude (and thus of the PN intensity) and the idea of the PN suppression by an electronic FB.

There is a significant difference between the quantum (Q), semiclassical (C), and pure classical (C*) descriptions of the PN and its suppression by FB.

In the framework of the C*-description, original fields already contain a 'real' noise component with equal or unequal quadrature variances which are changed by the action of FB. This description does not pretend to be true but only provides useful visual analogues with quantum models because of linearity of the system considered.

In C-models, one needs to introduce the stochasticity artificially by using a Langevin source of noise or by postulating photoelectron creation as a stochastic point process. In the presence of an FB this is a process with a delayed self-action which is performed through a classical field with intensity controlled by the photocurrent. In C-theories, the possible squeezing of the field — either of external or internal origin — is ignored, and the idea of nondemolition measurement is ignored as well.

In the Q-description, there are two variants which give significantly different predictions for experiments with electronic FB and nondemolition detection.

In the first, *a posteriori* variant Q_1, the PN is a quantum noise of measurement arising during the detection. As a result, the Q_1-model predictions coincide with those of C-theory: the in-loop field is not squeezed under the action of the modulator, but instead receives excess noise.

In the second, *a priori* variant of calculation Q_2, one considers potential stochasticity to be originally present in the field and quantum noise to appear 'automatically' through the noncommutativity of its operators. Here one postulates the observed current fluctuations to be determined by nonordered field operators. The macroscopic amplified photocurrent $i(t)$ is taken to be proportional to the field intensity operator (without quantum averaging). The FB modifies the phenomenological Green function of the system, thus decreasing (or increasing) the in-loop field operators' noncommutativity and squeezing (stretching) one quadrature variance without reference to the orthogonal quadrature. The Q_2-model considered here based upon Ref. [15] and using Heisenberg representation predicts three new effects (see Fig. 12): the fact of the variance squeezing for one of the in-loop field quadratures [15], the lack of the corresponding stretching for the second quadrature [15] ('supersqueezing') and, finally, the squeezing increase during the in-loop field dissipation on the way toward the detector or during the detection [12] (effect of the 'dissipative squeezing'). It seems that these effects can be observed with the help of nondemolition detectors. Their calculation in the Schrödinger representation will probably meet significant difficulties.

Notice that in the experiments, one can go beyond the linear regimes of modulation, detection, and amplification (the amplification linearity was certainly broken down in Ref. [12] near the auto-oscillations threshold). One can also use the original light in a noncoherent state. The theoretical description of such experiments must be an interesting nonlinear quantum problem.

To conclude, it is worthwhile to emphasise the difference between the terms *interpretation* and *concept*. The interpretation is usually understood as a convenient verbal and visual (i.e. customary) description of the results of mathematical computations in the framework of an already approved model. On the other hand, concepts serve as a base for choosing one or other particular model describing a given experiment or a group of experiments. Further, in the framework of the model chosen, the mathematical computations of the measured quantities and comparison with observational data are being made. In modern quantum optics a rare situation has emerged when there are two groups of models for optical systems with FB based on two alternative concepts. For an ultimate choice between them to be made, crucial experiments are obviously needed. However, given the sequence of experimental schemes as in Fig. 5c, 5d, and 5e, which use a 'nondemolition' beam splitter, the *a priori* models, despite their paradoxical features (see the introduction), appear to be preferable.

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Appendix. Uncertainty relation and Cauchy–Schwartz inequalities for spectral components

Let A and B be arbitrary operators, $|\psi\rangle$ be an arbitrary vector, and $z = x\exp(\mathrm{i}\phi)$ be an arbitrary scalar. We also define $C = A + zB$ and $|\psi'\rangle = C|\psi\rangle$. The condition of nonnegativity of the norm of vector $|\psi'\rangle$ has the form:

$$\begin{aligned}\langle\psi'|\psi'\rangle &= \langle C^+C\rangle = \big\langle (A^+ + z^*B^+)(A+zB)\big\rangle \\ &= \langle A^+A\rangle + 2x\,\mathrm{Re}\big[\exp(\mathrm{i}\phi)\langle A^+B\rangle\big] + x^2\langle B^+B\rangle \\ &\equiv c + bx + ax^2 \geqslant 0\,. \end{aligned} \tag{A.1}$$

Here the angle brackets denote averaging with the help of $|\psi\rangle$. The quadratic equation with respect to x must have no more than one real root, i.e. the condition $4ac \geqslant b^2$ must be satisfied. From here a Cauchy–Schwartz-like inequality follows:

$$\langle A^+A\rangle\langle B^+B\rangle \geqslant \big\{\mathrm{Re}\big[\exp(\mathrm{i}\phi)\langle A^+B\rangle\big]\big\}^2\,. \tag{A.2}$$

The right-hand side is maximum at $\phi = -\arg\langle A^+B\rangle$. Then a maximum lower limit for the left-hand side is obtained in the form

$$\langle A^+A\rangle\langle B^+B\rangle \geqslant \big|\langle A^+B\rangle\big|^2 = \big|\langle B^+A\rangle\big|^2\,. \tag{A.3}$$

When $\phi = 0$ or $\phi = \pi$ we get the generally more mild restrictions

$$\begin{aligned}\langle A^+A\rangle\langle B^+B\rangle &\geqslant \big(\mathrm{Re}\langle A^+B\rangle\big)^2\,,\\ \langle A^+A\rangle\langle B^+B\rangle &\geqslant \big(\mathrm{Im}\langle A^+B\rangle\big)^2\,.\end{aligned} \tag{A.4}$$

If $A = A^+$ and $B = B^+$, then $\mathrm{Im}\langle AB\rangle = -\mathrm{i}(\langle AB\rangle - \langle AB\rangle^*) = -\mathrm{i}\langle[A,B]\rangle$.

Let us substitute A and B by $\Delta A = A - \langle A\rangle$ and $\Delta B = B - \langle B\rangle$; then from the latter inequality in Eqn (A.4) we obtain the usual uncertainty relation

$$\langle\Delta A^2\rangle\langle\Delta B^2\rangle \geqslant \big(\mathrm{i}\langle[A,B]\rangle\big)^2\,. \tag{A.5}$$

Let us consider two cases.

1. Let us accept $A \equiv a(\omega_0 + \Omega) \equiv a(\Omega)$ and $B^+ \equiv a(\omega_0 - \Omega) \equiv a(-\Omega)$, where a and b are photon creation operators in modes with frequency $\omega_0 \pm \Omega$, then Eqn (A.3), in the notation of Section 2.3, takes the form

$$n(\Omega)\big[n(\Omega)+1\big] \geqslant \big|m(\Omega)\big|^2\,. \tag{A.6}$$

It is accepted here for simplicity that $n(\Omega)$ is an even function and $[a,a^+] = 1$ (i.e. discrete Fourier decomposition is used).

In classical theory, a similar inequality holds for classical averages but the 'unity' on the left-hand side that is the commutator $[a,a^+] = 1$ is absent, so that Eqn (A.6) takes the form

$$\left(\frac{|m(\Omega)|}{n(\Omega)}\right)_{\mathrm{clas}} \leqslant 1\,. \tag{A.6a}$$

The values of correlators m and n found from quantum models or by experiments satisfy, of course, Eqn (A.6), but may not satisfy Eqn (A.6a) when $n(\Omega) < 1$ [see Eqn (2.5.4) and Eqn (3.3.10a)]. The corresponding radiation is called nonclassical.

2. Let now in Eqn (A.3) $A \equiv q \equiv q(\Omega) = q^{+}(-\Omega)$ and $B \equiv p \equiv p(\Omega) = p^{+}(-\Omega)$ be quadrature operators; then we get the following restriction for spectral densities:

$$\langle q^{+}q\rangle\langle p^{+}p\rangle \geqslant \left|\langle q^{+}p\rangle\right|^{2} = \left|\langle p^{+}q\rangle\right|^{2} . \qquad \text{(A.7)}$$

Since operators q and q^{+}, as well as p and p^{+}, commutate with each other, the similar inequality in classical theory has the same form. In a similar way, by setting $A \equiv q^{+}$, $B \equiv p^{+}$, we get

$$\langle q^{+}q\rangle\langle p^{+}p\rangle \geqslant \left|\langle qp^{+}\rangle\right|^{2} = \left|\langle pq^{+}\rangle\right|^{2} . \qquad \text{(A.7a)}$$

Considering Eqn (2.3.11) we obtain from Eqn (A.7),

$$\left\langle q^{+}(\Omega)q(\Omega)\right\rangle\left\langle p^{+}(\Omega)p(\Omega)\right\rangle \geqslant \frac{1}{4}\left\{\mathrm{Im}\left[m(\Omega)\right]\right\}^{2} + \left[n(\Omega) - n(-\Omega) + 1\right]^{2} . \qquad \text{(A.8)}$$

Let $m = m^{*}$ and $n(\Omega) = n(-\Omega)$, then finally we find,

$$\left\langle q^{+}(\Omega)q(\Omega)\right\rangle\left\langle p^{+}(\Omega)p(\Omega)\right\rangle \geqslant \frac{1}{4} . \qquad \text{(A.9)}$$