# Exotic mesons: the search for glueballs 

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Abstract. The current status of the search for glueballs is surveyed.

## 1. Introduction

Quantum chromodynamics (QCD) has been verified sufficiently thoroughly in the range of hard interactions (or at short distances). In this case the QCD coupling constant $\alpha_{s}\left(k^{2}\right)$ becomes small: the theory is asymptotically free at $k^{2} \gg 1 \mathrm{GeV}^{2}$. This makes it possible to carry out calculations within the framework of the perturbative approach and to compare the theoretical results with experiments. The success of QCD in the description of the hard processes leads us to expect that QCD will prove to be the correct theory for the strong interactions.

Nevertheless, we must bear in mind that quantitative comparisons of QCD with experiments cannot at present be carried out in the range of soft interactions (at large distances), where $\alpha_{s}\left(k^{2}\right) \sim 1$. Therefore, qualitative consequences, confirming the principal concepts of the theory, are particularly important. One example of these

[^0]consequences, although it comes from a different application of QCD, is the colour transparency: squeezed hadron configurations, which are selected by the hard processes, do not interact with nucleons and they pass right through nuclear matter. This reflects a fundamental feature of the theory, which is its gauge invariance. Another example is the systematics of low-lying $\mathrm{q} \overline{\mathrm{q}}$ and qqq states. The systematics of hadrons demonstrates definitely that the low-lying hadrons consist of quarks, which are fundamental objects in QCD.

Equally important is the observation of particles which contain another fundamental QCD object, that is, a gluon. These particles are glueballs, which consist solely of gluons, and hybrids, which consist of quarks and gluons. Those hadrons which do not fit the $\mathrm{q} \overline{\mathrm{q}}$ or qqq classification are called exotic: glueballs and hybrids are exotic hadrons.

The problem of whether glueballs exist has provoked opposite answers. A serious argument in support of their existence is that in all the phenomenological approaches that describe successfully the low-lying $\mathrm{q} \overline{\mathrm{q}}$ and qqq states, both glueballs and hybrids appear as natural generalisation.

The quantitative QCD methods do not work in the strong interaction range and the only possible calculation technique available at present is the development of QCDmotivated models based on experiments. This phenomenological approach is very effective and it makes it possible to tackle a very wide range of tasks. Possibly this may be not just a temporary measure, reflecting our current inability to solve problems in the soft interaction range on the basis of the fundamental QCD Lagrangian. It seems
very likely that phenomenological models will be used extensively also in future even when we learn how to 'solve' QCD. A relevant example comes from the research on condensed matter: phenomenological models and effective interactions are used also for such problems and this is done very frequently without derivation of the assumed interactions from basic principles of electrodynamics.

Model analyses are very fruitful in the spectroscopy of low lying $\mathrm{q} \overline{\mathrm{q}}$ and qqq states: not only have they made it possible to calculate numerous spectroscopic characteristics of hadrons, but they have also provided rich information on the properties of QCD at large distances. However, as we give the appropriate due respect to phenomenological approaches in dealing with low-lying hadrons, we should stress that in answering the question of the existence of exotic mesons we are not demanding too much of spectroscopic calculations. At this stage the problem is the classification of hadrons. In other words, we are seeking the answer to whether there are 'extra' mesons which do not fit the $q \bar{q}$ systematics, but would fit better the class of glueballs (this means that such extra mesons should have properties typical of the states with an enriched gluon component).

The main topic of interest to us here is that of glueball states. A detailed discussion of mesons as glueball candidates has become possible because of the recently reported precision experimental data obtained in the search for exotic mesons and gluon-rich states. However, in order to analyse in detail the current status of the problem of exotic mesons, we must turn first of all to the existing systematics of $q \bar{q}$ states and to consider the problem of extra mesons in the range $1000-2000 \mathrm{MeV}$.

## 2. Systematics of $\mathbf{q} \bar{q}$ mesons and candidates for exotic states

The systematics of low-lying hadrons, which are $S$-wave quark states, is well established and has remained unshakeable for the last thirty years: this systematics has made possible the breakthrough to the physics of quarks.

The subject under discussion here is that of the $q \bar{q}$ states with masses above 1000 MeV . The classification of the $q \bar{q}$ mesons is given in Table 1, which on the whole reflects the standard view on the situation in the range of masses $1000-1700 \mathrm{MeV}$ (see, for example, Ref. [1]). The controversy comes in the case of the scalar mesons belonging to the ${ }^{3} \mathrm{P}_{0}\left(0^{++}\right)$multiplets.

In Ref. [1], the lowest multiplet of scalar mesons looks as follows:

$$
\begin{equation*}
{ }^{3} \mathrm{P}_{0}\left(0^{++}\right): \mathrm{a}_{0}(980), \mathrm{f}_{0}(980), \mathrm{f}_{0}(1300), \mathrm{K}_{0}(1430) \tag{1}
\end{equation*}
$$

This multiplet differs from that given in Table 1 by the scalar resonance $I=0, J^{P C}=0^{++}$, located near 1000 MeV . In the $\pi \pi$ amplitude, found by the Chew-Low method with an extraction of the $t$-channel pion exchange, there is a wide bump in the region of $800-1000 \mathrm{MeV}$ and a narrow dip at the $\mathrm{K} \overline{\mathrm{K}}$ threshold $[2,3]$. When the momentum $|t|$ transferred to a two-pion system is increased, this wide bump disappears and a threshold cusp appears as a narrow resonance. An analysis of the $\pi \pi$ amplitude in the region of the $\mathrm{K} \overline{\mathrm{K}}$ threshold shows $[4,5]$ that the narrow cusp corresponds to two poles located in the complex plane of $s$ (energy squared) on the second and third sheets. Such a two-pole structure near a threshold singularity usually corresponds to one bound state. This bound state (narrow resonance) is considered in Ref. [1] as one of the members of the ${ }^{3} \mathrm{P}_{0}$ multiplet. It is at present difficult to determine the analytic structure of this wide bump. It is possible that it does not correspond to any resonance: this is in fact suggested in Ref. [1]. However, it can also be interpreted as a wide resonance, as is done in Table 1. A detailed discussion of the $f_{0}(1000)$ resonance is given in Section 8.

The interpretation of the wide bump of the $\pi \pi$ amplitude as a resonance is supported also by spectroscopic calculations [6, 7]. These calculations, carried out in widely differing approaches, give the spectra of the P -wave $\mathrm{q} \overline{\mathrm{q}}$ states. The masses of the mesons representing the members of the ${ }^{3} \mathrm{P}_{0}$ multiplet are smaller than the masses of the mesons in the ${ }^{3} \mathrm{P}_{1}$ and ${ }^{3} \mathrm{P}_{2}$ multiplets. The $\mathrm{f}_{0}$ meson with a predominant nonstrange quark composition has a mass in the region of 1000 MeV and a very large width: $\Gamma \approx 500-1000 \mathrm{MeV}$ [6] or $\Gamma>400 \mathrm{MeV}$ [7]. (The band corresponding to $\Gamma \leqslant 400 \mathrm{MeV}$ in the complex plane is considered in Ref. [7]; when the decay channels are included, a pole 'escapes' deeper into the complex plane to a region which is not monitored.)

Inclusion of $\mathrm{f}_{0}(980)$ in the ${ }^{3} \mathrm{P}_{0} \mathrm{q} \bar{q}$ multiplet has been criticised specifically because of its small width. In his paper, Close [8] represented the ${ }^{3} \mathrm{P}_{0}$ multiplet as

$$
\begin{equation*}
{ }^{3} \mathrm{P}_{0}\left(0^{++}\right): \mathrm{a}_{0}(1320), \mathrm{f}_{0}(1400), \mathrm{f}_{0}(1505), \mathrm{K}_{0}(1430) \tag{2}
\end{equation*}
$$

and interpreted $\mathrm{f}_{0}(980)$ and $\mathrm{a}_{0}(980)$ as the Gribov 'minions' (particles more compact than known particles) [9, 10]. It must be stressed immediately that $\mathrm{f}_{0}(1505)$ decays relatively weakly in the $\mathrm{K} \overline{\mathrm{K}}$ channel, which makes it an unlikely

Table 1. Quark - antiquark mesons (ground states and radial excitations).

|  | Ground states, 12 |  |  |  | First radial excitation, $2 L$ |  |  |  | Second radial excitation, $3 L$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{1} \mathrm{~S}_{0}\left(0^{-+}\right)$ | $\pi(140)$ | $\eta(550)$ | $\eta^{\prime}(960)$ | K (500) | $\pi(1300)$ | $\eta$ (1295) | $\eta^{\prime}(1440)$ | K (1430) | $\pi(1770)$ | - | - | K (1830) |
| ${ }^{3} \mathrm{~S}\left(1^{--}\right)$ | $\rho(760)$ | $\omega(760)$ | $\Phi$ (1020) | $\mathrm{K}^{*}(890)$ | $\rho(1450)$ | $\omega(1420)$ | $\Phi(1680)$ | $\mathrm{K}^{*}(1410)$ | - | - | - | - |
| ${ }^{1} \mathrm{P}\left(1^{+-}\right)$ | $\mathrm{b}_{1}(1235)$ | $\mathrm{h}_{1}(1170)$ | $\mathrm{h}_{1}(1380)$ | $\mathrm{K}_{1}(1270)$ | - | - | - | - | - | - | - | - |
| ${ }^{3} \mathrm{P}\left(0^{++}\right)$ | $\mathrm{a}_{0}(980)$ | $\mathrm{f}_{0}(1000)$ | $\mathrm{f}_{0}(1240)$ | $\mathrm{K}_{0}^{*}(1430)$ | $a_{0}(1440)$ | $\mathrm{f}_{0}(1370)$ | $\mathrm{f}_{0}(1590)$ | $\mathrm{K}_{0}^{*}(1430)$ | - | - | - | - |
| ${ }_{3}^{3} \mathrm{P}_{1}\left(1^{++}\right)$ | $\mathrm{a}_{1}(1260)$ | $\mathrm{f}_{1}(1285)$ | $\mathrm{f}_{1}(1510)$ | $\mathrm{K}_{1}(1400)$ |  |  |  |  | - | - | - | - |
| ${ }^{3} \mathrm{P}_{2}\left(2^{++}\right)$ | $\mathrm{a}_{2}(1320)$ | $\mathrm{f}_{2}(1270)$ | $\mathrm{f}_{2}^{\prime}(1525)$ | $\mathrm{K}_{2}^{*}(1430)$ | - | $\mathrm{f}_{2}(1650)$ | $\mathrm{f}_{2}(1810)$ | $\mathrm{K}_{2}(1980)$ | - | $\mathrm{f}_{2}(2010)$ | - | - |
| ${ }^{1} \mathrm{D}_{2}\left(2^{-+}\right)$ | $\pi_{2}(1670)$ | $\eta_{2}(1650)$ | $\eta_{2}(1870)$ | $\mathrm{K}_{2}(1770)$ | - | - | - | - | - | - | - |  |
| ${ }^{3} \mathrm{D}_{1}\left(1^{--}\right)$ | $\rho(1700)$ | $\omega(1600)$ | - | $\mathrm{K}^{*}(1680)$ | - | - | - | - | - | - | - | - |
| ${ }^{3} \mathrm{D}_{2}\left(2^{--}\right)$ | - | - | - | $\mathrm{K}_{2}(1820)$ | - | - | - | - | - | - | - | - |
| ${ }^{3} \mathrm{D}_{3}\left(3^{--}\right)$ | $\rho_{3}(1690)$ | $\omega_{3}(1670)$ | $\Phi_{3}(1850)$ | $\mathrm{K}_{3}^{*}(1780)$ | - | - | - | - | - | - | - | - |

candidate for a state with a large sse component. Bugg [11] took account of this circumstance and identified the second $\mathrm{f}_{0}$ meson with $\mathrm{f}_{0}(1590)$, discovered by the GAM S group [12].

According to Bugg, we have

$$
\begin{equation*}
{ }^{3} \mathrm{P}_{0}\left(0^{++}\right): \mathrm{a}_{0}(1415), \mathrm{f}_{0}(1335), \mathrm{f}_{0}(1590), \mathrm{K}_{0}(1430) \tag{3}
\end{equation*}
$$

Bugg proposes that the $\mathrm{f}_{0}(980)$ and $\mathrm{a}_{0}(980)$ mesons be considered as $K \bar{K}$ molecules, and $\mathrm{f}_{0}(1505)$ be regarded as a glueball.

A possible interpretation of $\mathrm{f}_{0}(980)$ and $\mathrm{a}_{0}(980)$ as molecules is considered in Ref. [13]. The experimental results $[3,14,15]$ provide a strong argument against this hypothesis. In the charge-exchange $\pi^{-} p \rightarrow \pi^{0} \pi^{0}$ n reaction at $40 \mathrm{GeV} \mathrm{s}^{-1}[3,14]$ the $f_{0}(980)$ resonance can be seen at low values of $t$ (the four-momentum squared transferred to a nucleon) in the form of a dip, which is due to destructive interference with the wide $\pi \pi$ bump, corresponding to $\mathrm{f}_{0}(1000)$. However, this bump gradually disappears with increase in $|t|$, but the $f_{0}(980)$ signal remains: the dependence of the $\mathrm{f}_{0}(980)$ production cross section on $t$ is typical of one-pion exchange. The predominance of the $\mathrm{f}_{0}(980)$ production at high transferred momenta is difficult to understand on the basis of a quasimolecular (or deuteron-like) structure of the resonance: the $\pi \rightarrow \mathrm{f}_{0}(980)$ transition vertex contains a form factor with the $t$ dependence governed by the dimensions of the created composite system ('molecule'). This form factor suppresses creation of loosely bound composite systems in the range of high transferred momenta. A detailed discussion of the structure of $\mathrm{f}_{0}(980)$ is given in Section 8.

The $\mathrm{a}_{0}(980)$ resonance can also be seen in the pp collisions at 450 GeV as a narrow peak against the background of a low-level continuous spectrum [15], which is again an argument against the hypothesis of a loosely bound molecule-like structure. A discussion of the structure of $\mathrm{a}_{0}(980)$ can be found in Section 9.

The $f_{0}(1505)$ resonance was discovered in a re-analysis of the Crystal Barrel data on the $\mathrm{p} \overline{\mathrm{p}}$ annihilation at rest: $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{0} \pi^{0} \pi^{0}[16,17]$. An irregularity in the region of 1500 MeV has been interpreted earlier as the D-wave resonance $\mathrm{AX}_{2}$ (1515) [18]. A combined analysis of the data on the formation of $\pi^{0} \pi^{0} \pi^{0}, \pi^{0} \pi^{0} \eta$, and $\pi^{0} \eta \eta$ has also shown that these reactions result in strong production also of other scalar resonances: $\mathrm{a}_{0}(1440), \mathrm{a}_{0}(980), \mathrm{f}_{0}(1370)$, and $\mathrm{f}_{0}(980)[17,19,20]$. The $\mathrm{a}_{0}(1440)$ and $\mathrm{f}_{0}(1370)$ resonances are listed in Table 1 as the first radial excitation of the ${ }^{3} \mathrm{P}_{0}\left(0^{++}\right)$multiplet together with the $\mathrm{f}_{0}(1590)$ resonance. One must point out immediately that the status of $f_{0}(1590)$ requires a special discussion: we shall return to this resonance in connection with the problem of mixing of quark and gluon states.

According to Table 1 , the $\mathrm{f}_{0}(1240)$ resonance is a scalar $s \bar{s}$ partner of the wide $f_{0}(1000)$ resonance. The $f_{0}(1240)$ resonance has been seen in the $\mathrm{K} \overline{\mathrm{K}}$ channel [21], but undoubtedly this requires confirmation. In the compilation given in [1] this resonance is dropped into the common 'basket' of scalar resonances in the $1300-1400 \mathrm{MeV}$ range; we shall return to this resonance in Section 9.

It is also assumed in Table 1 that the $\mathrm{K}_{0}(1430)$ scalar resonance has actually a two-pole structure: in fact, the $\mathrm{K}_{0}(1430)$ resonance is very wide with $\Gamma \approx 300 \mathrm{MeV}$; this width is approximately twice the widths of other strange P wave resonances: $\Gamma\left[\mathrm{K}_{1}(1270)\right] \approx 90 \mathrm{MeV}, \Gamma\left[\mathrm{K}_{1}(1400)\right] \approx$ $170 \mathrm{MeV}, \quad \Gamma\left[\mathrm{K}_{2}(1430)\right] \approx 100 \mathrm{MeV}$. This allows us to
assume that there are actually two resonances in this region: their masses are about 1300 and 1500 MeV , corresponding to $\mathrm{K}_{0}^{(1)}(1300)$ and $\mathrm{K}_{0}^{(2)}(1500)$, and their widths are of the order of 150 MeV .

Table 1 includes two new isoscalar $2^{-+}$resonances: $\eta_{2}(1650)$ and $\eta_{2}(1870)$, observed by the Crystal Barrel group in the $p \bar{p} \rightarrow \eta \pi^{0} \pi^{0} \pi^{0}$ reaction with the $a_{2}^{0}(1320) \pi^{0}$ and $f_{2}(1270) \eta$ decay channels, respectively [22]. These resonances fill in a natural manner the $D$-wave $\mathrm{q} \overline{\mathrm{q}}$ multiplet ${ }^{1} \mathrm{D}_{2}\left(2^{-+}\right)$.

A comparison of the data presented in Table 1 with the experimental values reveals directly that there is a group of isoscalar resonances with $J^{P C}=0^{++}$and $2^{++}$, which is superfluous to the $q \bar{q}$ classification. These are primarily $\mathrm{f}_{0}(980)$ and $\mathrm{f}_{0}(1505)$. In the $1600-2100 \mathrm{MeV}$ range there are also scalar resonances: a resonance with the 1740 MeV mass $\left(J^{P C}=0^{++}\right.$or $\left.2^{++}\right)$[23]; we shall label it $\mathrm{f}_{J}(1740)$ with $J=0$ or 2. Re-analysis of the Mark-III data on $\mathrm{J} / \psi \rightarrow \gamma+\pi^{+} \pi^{-} \pi^{+} \pi^{-}[24]$ reveals two strong resonances in a system of four pions: $f_{2}(1780)$ and $f_{0}(2100)$. The $f_{2}(1780)$ resonance is close to $f_{2}(1810)$, which can be seen in the $4 \pi^{0}$ mode [25, 26]; it may be in fact the latter resonance or the sum of the contributions $\mathrm{f}_{2}(1710)+\mathrm{f}_{2}(1810)$.

The $J / \psi \rightarrow \gamma+K \bar{K}$ decay reveals formation of a $\mathrm{f}_{2}(1710) \rightarrow \mathrm{K} \overline{\mathrm{K}}$ resonance [27]: it also does not fit the systematics in Table 1.

Let us now summarise: the group of isoscalar resonances $\mathrm{O}^{++}$and $2^{++}$, including

$$
\begin{align*}
& f_{0}(980), f_{0}(1505), f_{0}(2100), f_{J}(1740) \quad(J=0,2), \\
& f_{2}(1710), f_{2}(1780) \tag{4}
\end{align*}
$$

does not fit the classification in Table 1. It is necessary to consider whether any of these resonances are indeed extra from the point of view of the $q \bar{q}$ systematics.

One other mysterious resonance can be added to the list of candidates for the class of exotics: it was observed by the GAMS group [28] in the $\eta \eta^{\prime}$ channel as a narrow peak with the 1910 MeV mass. This resonance does not decay by the $\eta \eta, \pi^{0} \pi^{0}$, and $K_{s}^{0} K_{s}^{0}$ channels, which provides grounds for proposing that the orbital momentum of the $\eta \eta^{\prime}$ system is odd: $L=1,3, \ldots$. We are then dealing with a resonance characterised by exotic quantum numbers, which are impossible in the $\mathrm{q} \overline{\mathrm{q}}$ system. The minimum variant, $L=1$, gives $J^{P C}=1^{-+}$:

$$
\begin{equation*}
1^{-+}(1910) \tag{5}
\end{equation*}
$$

An alternative, although with dynamics that cannot be understood at all, would be the assumption that $L$ is even; then, for $L=0$ or 2 we would have to include this resonance in the group described by the set of expressions (4) above: $\mathrm{f}_{J}(1910)(J=0$ or 2$)$.

The excess of the isoscalar $\mathrm{O}^{++}$and $2^{++}$resonances, described by the set of expressions (4) suggests the existence of glueball exotics: the lowest two-glueball states should have these precise quantum numbers, as estimated by a procedure considered below. However, before we discuss these glueballs, let us consider whether we can 'save' the $q \bar{q}$ systematics by including these resonances in the next multiplets of the radial excitations ${ }^{3} \mathrm{P}_{0}$ and ${ }^{3} \mathrm{P}_{2}$.

Figs 1 a and 1 b show the trajectories of the mesons that belong to the ${ }^{1} \mathrm{~S}_{0}$ and ${ }^{3} \mathrm{~S}_{1}$ multiplets [1,29]: they fit well the trajectories which are linear functions of $M^{2}$. We can use the same method to estimate the masses of the $f_{0}$ and $f_{2}$


Figure 1. Trajectories of mesons, treated as radial excitations of Swave $q \bar{q}$ multiplets, fitting well dependences linear in $M^{2}(a, b)$ and estimates of the masses of mesons belonging to the ${ }^{3} \mathrm{P}_{2} \mathrm{q} \overline{\mathrm{q}}$ and ${ }^{3} \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ multiplets found by means of trajectories linear in $M^{2}$ (c, d). The masses of the $3^{3} \mathrm{P}_{2} \mathrm{q} \bar{q}$ and $3^{3} \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ states are listed in expressions (6).
mesons belonging to the $3^{3} \mathrm{P}_{0}$ and $3^{3} \mathrm{P}_{2}$ multiplets (Figs 1c and 1 d ). This gives the following values of the masses:

$$
\begin{align*}
& 3^{3} \mathrm{P}_{0}: \mathrm{f}_{0}(1720 \pm 40), \mathrm{f}_{0}(1880 \pm 40) \\
& 3^{3} \mathrm{P}_{2}: \mathrm{f}_{2}(1950 \pm 50), \mathrm{f}_{2}(2070 \pm 50) \tag{6}
\end{align*}
$$

The masses of the mesons in the $1^{3} \mathrm{~F}_{2}$ multiplet can be estimated similarly:

$$
\begin{equation*}
1^{3} \mathrm{~F}_{2}: \mathrm{f}_{2}(2000 \pm 80), \mathrm{f}_{2}(2100 \pm 80) \tag{7}
\end{equation*}
$$

This means that we are not expecting additional $\mathrm{q} \overline{\mathrm{q}}$ states with $I=0$ and $J^{P C}=0^{++}$below 1650 MeV or with $J^{P C}=2^{++}$below 1900 MeV : the $\mathrm{f}_{0}(1505)$ and $\mathrm{f}_{2}(1710)$ resonances are definitely extra from the point of view of the $\mathrm{q} \overline{\mathrm{q}}$ systematics. The $\mathrm{f}_{0}(980)$ resonance is, as mentioned above, a good candidate for an exotic state, but its nature is still controversial (see Section 8). There may be also controversy about the nature of other resonances belonging to the groups described by the sets of expressions (4) and (5), but two resonances

$$
\begin{equation*}
\mathrm{f}_{0}(1505), \mathrm{f}_{2}(1710) \tag{8}
\end{equation*}
$$

definitely fall out from the $\mathrm{q} \overline{\mathrm{q}}$ classification: we are obviously dealing here with exotics. The suspicion falls first on glueballs. In the following sections we shall discuss the reactions in which we can expect enhanced glueball formation and also the information that can be obtained from the reactions with enhanced production of the gluon states.

## 3. Resonance in the $\eta \eta^{\prime}$ spectrum at 1910 MeV , tentatively due to $J^{P C}=1^{-+}$

This resonance was discovered by the GAMS group in the $\pi^{-} \mathrm{p} \rightarrow \mathrm{nX}(1910) \rightarrow \mathrm{n} \eta \eta^{\prime}$ reaction [28]. The spectrum of the $\eta \eta^{\prime}$ masses is shown in Fig. 2a. The X(1910) resonance is not observed in the spectra of $\eta \eta, \mathrm{K}_{\mathrm{S}}^{0} \mathrm{~K}_{\mathrm{S}}^{0}$, and $\pi^{0} \pi^{0}$ at the next level of accuracy (see Ref. [28] and the literature cited there), as indicated by the following inequalities applicable to the branching ratio ( BR ) values:
$\frac{\operatorname{BR}(\eta \eta)}{\operatorname{BR}\left(\eta \eta^{\prime}\right)}<\frac{1}{20}, \quad \frac{\operatorname{BR}\left(\mathrm{~K}_{\mathrm{S}}^{0} \mathrm{~K}_{\mathrm{S}}^{0}\right)}{\operatorname{BR}\left(\eta \eta^{\prime}\right)}<\frac{1}{15}, \quad \frac{\operatorname{BR}\left(\pi^{0} \pi^{0}\right)}{\operatorname{BR}\left(\eta \eta^{\prime}\right)}<\frac{1}{15}$.


Figure 2. Spectrum of the $\eta^{\prime} \eta$ system in the $\pi^{-} p \rightarrow n \eta \eta^{\prime}$ reaction, plotted for $0.35<|t|<0.6 \mathrm{GeV}^{2} c^{-2}$. (a) The shaded histogram is the spectrum after subtraction of the background under the $\eta^{\prime}$ peak. (b, c) Distributions in terms of the polar (b) and azimuthal (c) angles for the decay of the $\mathrm{X}(1910)$ meson in the Gottfried-Jackson (GJ) system, plotted for $0.35<|t|<0.6 \mathrm{GeV}^{2} c^{-2}$. (d) Differential $t$ distribution of the $\eta \eta^{\prime}$ events in the region of the $\mathrm{X}(1910)$ peak.

The spin of the $\mathrm{X}(1910)$ resonance has not been determined and we shall consider the variants $J=0,1$, 2 , and 3. If the spin of this resonance is even, $J^{P C}=0^{++}$or, $2^{++}$, then we cannot understand at all why partial decay in the $\eta \eta, K_{S}^{0} K_{S}^{0}$, and $\pi^{0} \pi^{0}$ channels is suppressed. The $\eta \eta$ system is similar to $\eta \eta^{\prime}$ channels.

If the spin is odd, the decay in the $\eta \eta, \mathrm{K}_{\mathrm{S}}^{0} \mathrm{~K}_{\mathrm{S}}^{0}$, and $\pi^{0} \pi^{0}$ channels is forbidden and we have a natural explanation of why there is no resonance signal of these states. The quantum numbers $1^{-+}$and $3^{-+}$are exotic: such numbers are impossible in the $q \bar{q}$ system. It is reasonable to assume that in this case we observe explicit exotics and we are dealing with the lower exotic state, i.e. $J^{P C}=1^{-1}$.

The exotic quantum numbers $1^{-+}$can apply to a twogluon glueball GG if its component (or effective) gluons are massive, or they may apply to the hybrid Gqq. The mechanism of creation of the $1^{-+}$glueball in the $\pi^{-} \mathrm{p} \rightarrow \mathrm{n}+\mathrm{GG}\left(1^{-+}\right)$reaction is illustrated in Fig. 3: we are dealing either with the $t$-channel exchange involving pion-like states of the $\pi(1300)$ or $\pi(1770)$ type, for which the $\mathrm{GG}\left(1^{-+}\right) \rightarrow \pi \pi$ decay is forbidden (Fig. 3a), or with the exchange of the $\mathrm{a}_{1}$ meson ( Fig .3 b ). The pionlike exchange diagram may be valid for low momenta transferred. For high momenta transferred the diagram with the $a_{1}$ exchange predominates.

The diagram (Fig. 3b) corresponds to the $t$-channel exchange involving the $a_{1}$ meson and it has a transition vertex of the $\mathrm{X}\left(1^{-+}\right)$resonance with the following structure:

$$
\begin{equation*}
\left(e_{a_{1}} \varepsilon_{\mathrm{X}}\right) \tag{10}
\end{equation*}
$$

where $\boldsymbol{e}_{\mathrm{a}_{1}}$ is the polarisation vector of the $1^{++}$meson. Therefore, the diagram in Fig. 3b does not lead to the angular dependence of the decay products in the Gottfried-Jackson (GJ) frame.


Figure 3. Quark diagrams representing creation of the $X(1910)$ meson on the assumption that it is of glueball nature.

The transition vertex of the $\mathrm{X}\left(1^{-+}\right)$resonance for the exchange of the $t$-channel state with the quantum numbers of the pion (3a) has the structure

$$
\begin{equation*}
\left(\boldsymbol{k} \boldsymbol{\varepsilon}_{\mathrm{X}}\right) \tag{11}
\end{equation*}
$$

where $\boldsymbol{k}$ is the relative momentum of the incident pion and of the $t$-channel state in the GJ system, and $\boldsymbol{\varepsilon}$ is the polarisation vector of the resonance. Therefore, in this system the angular distribution of the particles, which are the decay products, behaves as $\cos ^{2} \theta_{\mathrm{GJ}}$ in the region where this diagram predominates.

The experimental data on the angular dependence of $\eta \eta^{\prime}$ in the GJ system (Figs 2b and 3c) are in agreement with the $\mathrm{X}\left(1^{-+}\right)$creation mechanism illustrated in Fig. 3b. It is at high values of $|t|$ that we have

$$
\begin{equation*}
W\left(\cos \theta_{\mathrm{GJ}}\right)=\text { const. } \tag{12}
\end{equation*}
$$

It follows that for $|t|>0.35 \mathrm{GeV}^{2} \mathrm{~s}^{-2}$ we have the diagrams in Fig. 3b.

In the range of large momenta transferred the $|t|$ dependence of the cross section is relatively weak: $\mathrm{d} \sigma / \mathrm{d} t$ $\left[\pi^{-} \mathrm{p} \rightarrow \mathrm{X}(1910)\right] \propto \exp [B t]$ and $B \approx 2 \mathrm{GeV}^{-2} c^{2}$ (Fig. 2d).

We may therefore conclude that the available experimental results support the hypothesis that the $\eta \eta^{\prime}(1910)$ resonance is exotic and it is characterised by $J^{P C}=1^{-+}$. We shall discuss later the hypothesis of the glueball nature of the $1^{-+}(1910)$ resonance (according to this hypothesis the soft gluons should have an effective mass). We shall also consider, on the basis of the rules of the $1 / N$ expansion [30, 31], why glueballs decay preferentially into $\eta$ and $\eta^{\prime}$.

## 4. Resonance $f_{2}(1710)$ and mechanism of radiative decay of $J / \Psi$

This resonance can be seen clearly in the $K \overline{\mathrm{~K}}$ spectra of the $J / \psi \rightarrow \gamma+K \bar{K}$ radiative decay [27]. The $\mathrm{f}_{2}$ resonance decays also into the $\pi \pi$ channel [32] and possibly also into the $\sigma \sigma \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-}$[24] channel.

The resonant decays of the $\mathrm{J} / \psi$ meson are reactions with an enriched creation of gluon states in the hadron component. Possible types of the processes are shown in Fig. 4. A photon may be emitted both by charmed quarks (Figs $4 a$ and $4 b$ ) and by light quarks (Figs $4 c$ and $4 d$ ). The processes illustrated in Figs 4 a and 4 b determine the formation of glueballs of mesons with an admixture of the glueball component. The $q \bar{q}$ and $q \bar{q} G$ mesons are produced in the processes of the kind shown in Fig. 4c. One of the successes of the glueball physics is a suppression of processes shown in Figs 4c and 4d (as well as that given in Fig. 4b). The radiative decay of $J / \psi$ is dominated by the two-gluon transition and the three-gluon transition is suppressed. The following estimate of partial widths is given in Ref. [52]:

$$
\begin{equation*}
\frac{\Gamma(\mathrm{J} / \psi \rightarrow \gamma \mathrm{gg})}{\Gamma(\mathrm{J} / \psi \rightarrow \mathrm{ggg})} \simeq(19 \pm 6) \times 10^{-2} \tag{13}
\end{equation*}
$$

This means that $\Gamma(\mathrm{J} / \psi \rightarrow \gamma \mathrm{ggg})$ is not more than $3 \%-4 \%$ of $\Gamma(\mathrm{J} / \psi \rightarrow \gamma \mathrm{gg})$. Therefore, projections of the two-gluon state onto the hadron states are observed in the radiative decay of $J / \psi$.

The discovery of the $f_{2}(1710)$ resonance was fraught with difficulties: in the initial analyses of the experimental results the value $J^{P C}=2^{++}$was much preferred to other


Figure 4. Diagrams representing the main contribution to the creation of glueballs in the radiative decays of $J / \psi$ (a) and the diagrams making small corrections to the main contribution (b, c, d).
quantum numbers, but a subsequent incorrect analysis predicted zero spin [33]. Only a recent re-analysis of the results restored the initial prediction: the 1710 resonance is of the tensor type [34].

## 5. Problem of extracting information from reactions of production of three particles: discovery of $f_{\mathbf{0}}(\mathbf{1 5 0 5})$

The results obtained by the Crystal Barrel collaboration in a study of the formation of three mesons by the $p \bar{p}$ annihilation at rest are exceptionally rich in the number of observed events. They undoubtedly open a new page in the study of mesons with masses in the range $1000-1600 \mathrm{MeV}$. However, such large statistics and high accuracy of the measurements mean that interference phenomena and interaction effects in the final state should be taken into account at the same time and in a satisfactory manner. In this section we shall present the key aspects of a method for the analysis of such reactions by considering the following processes as an example:

$$
\mathrm{p} \overline{\mathrm{p}} \rightarrow\left\{\begin{array}{ccc}
\pi^{0} & \pi^{0} & \pi^{0}  \tag{14}\\
\pi^{0} & \pi^{0} & \eta \\
\pi^{0} & \eta & \eta
\end{array}\right.
$$

The Crystal Barrel investigation led to the discovery of $\mathrm{f}_{0}(1505)$.

Let us start by discussing the structure of the amplitude of three-pion production. This production may involve the following hydrogen-like $\mathrm{p} \overline{\mathrm{p}}$ levels:

$$
\begin{gather*}
{ }^{1} \mathrm{~S}_{0}: J^{P}=0^{-}  \tag{15}\\
{ }^{3} \mathrm{P}_{1}: \\
{ }^{3} \mathrm{P}_{2}:
\end{gather*} 1^{+},
$$

It should be stressed that annihilation from different levels may differ from process to process, for example, it may differ for $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi \pi \pi$ and $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi \pi$ : the probabilities of these processes depend on the annihilation radii, while the radii themselves vary with the annihilation channels. Therefore, the problem of the level from which the process begins has to be solved separately for each reaction. We shall assume that the annihilation into three pions originates from the ${ }^{1} \mathrm{~S}_{0}$ level. In this case the amplitude has the following structure:

$$
\begin{equation*}
\delta_{a b} \delta_{c z} A(1,2 ; 3)+\delta_{b c} \delta_{a z} A(2,3 ; 1)+\delta_{c a} \delta_{b z} A(3,1 ; 2) \tag{16}
\end{equation*}
$$

Here the indices 1,2 , and 3 apply to the pion momenta, whereas $a, b$, and $c$ determine their charges; $z$ is the isospin of the initial state: $z=0$ for $\mathrm{p} \overline{\mathrm{p}}$ and $z=1$ for $\mathrm{p} \bar{n}$. The amplitude satisfies the symmetry condition $A(1,2 ; 3)=A(2,1 ; 3)$. The amplitudes of the transitions involving different channels are.

$$
\begin{aligned}
& \mathrm{p} \overline{\mathrm{p}} \rightarrow\left\{\begin{array}{cc}
\pi^{0} \pi^{0} \pi^{0} & A(1,2 ; 3)+A(2,3 ; 1)+A(3,1 ; 2) \\
\pi^{+} \pi^{-} \pi^{0} & A(1,2 ; 3)
\end{array}\right. \\
& \overline{\mathrm{p} n} \rightarrow\left\{\begin{array}{cc}
\pi^{-} \pi^{-} \pi^{+} & A(2,3 ; 1)+A(3,1 ; 2) \\
\pi^{0} \pi^{0} \pi^{-} & A(1,2 ; 3)
\end{array}\right.
\end{aligned}
$$

In a rough approximation the amplitude $A(1,2,3)$ can be represented as the sum of the resonance production amplitude and a relatively smooth background function.

In this case we have

$$
\begin{equation*}
A(1,2 ; 3)=\alpha+\sum_{R} \frac{a_{R} X_{R}}{s_{12}-M_{R}^{2}+\mathrm{i} \Gamma_{R} M_{R}} \tag{18}
\end{equation*}
$$

where $\alpha$ and $a_{R}$ are smooth functions of the squares of the pair energies $s_{i k}$, and $X_{R}$ is a centrifugal factor, which is 1 for the resonances with $J=0$ and equal to

$$
\begin{equation*}
X_{2}=\left(k_{a} k_{b}-\frac{1}{3} k^{2} \delta_{a b}\right)\left(k_{3 a} k_{3 b}-\frac{1}{3} k_{3}^{2} \delta_{a b}\right) \tag{19}
\end{equation*}
$$

for $J=2$; here, $\boldsymbol{k}$ and $\boldsymbol{k}_{3}$ are, respectively, the relative momentum of particles 1 and 2 , in the system of these two pions at rest, and the momentum of the third pion in the centre-of-mass system of the three mesons; $a$ and $b$ are spatial indices.

In fitting the theoretical and experimental results it is very important to note that $\alpha$ and $a_{R}$ are complex quantities: their complex nature is due to the interaction of the particles in the initial and final states, which is not taken into account explicitly in the rough approximation adopted here.

Representation of the amplitude in the form described by formula (18) can be regarded as separation of the leading amplitude singularities, more specifically, the pole singularities. However, if the accuracy of experimental results is sufficiently high, similar to that obtained by the Crystal Barrel collaboration for the reactions described by expression (14), the precision of representation of the amplitude in the form given by formula (18) is insufficient: it is necessary to separate the singularities which follow the leading ones. Such singularities are associated with the re-scattering of the mesons which have just been formed. The terms of the amplitude described by formula (18) are shown diagrammatically in Figs 5a and 5b: the diagram in Fig. 5a describes a smooth background amplitude $\alpha$, and that in Fig. 5 b represents the process of creation of resonances. The diagrams with the re-scattering of mesons in the final state are given in Figs $5 \mathrm{c}-5 \mathrm{f}$. If the re-scattering is taken into account, threshold singularities appear also in the resonance width:

$$
\begin{equation*}
\Gamma_{R} M_{R} \rightarrow \sum_{n} \gamma_{n} \frac{k_{12}^{2 L+1}}{\sqrt{s_{12}}} \tag{20}
\end{equation*}
$$

Here $k_{12}^{2 L+1}$ is the relative momentum of two particles, into which a resonance may decay (they are pions if $k_{12}=k$ ) and $L$ is their orbital momentum.

The threshold singularities are particularly important when the threshold is close to a resonance. The $K \bar{K}$ channel provides a suitable example: $\mathrm{f}_{0}(980), \mathrm{f}_{0}(1000)$, and



Figure 5. Diagrams describing different processes involving creation and re-scattering of pions in the $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi \pi \pi$ reaction.
$a_{0}(980)$ resonances have large partial widths of the decay to the $\mathrm{K} \overline{\mathrm{K}}$ state. Re-scattering leads also to threshold singularities in the vertices $a_{R}$ (Fig. 5d).

Another problem encountered in the representation of the production amplitude in the form given by formula (18) is related to the overlap of the resonances $f_{0}(980)$ and $\mathrm{f}_{0}(1000)$. This can be taken into account together with the strong coupling of these two resonances to the channels $\pi \pi$ and $K \bar{K}$ if the S-wave amplitude is considered in the twochannel approximation. This can be done conveniently on the basis of the $N / D$ method [35]. The amplitude $A \sigma^{\sim}$ is represented by the ratio $\mathrm{No} / \mathrm{Do}$ :

$$
\begin{equation*}
\hat{A}=\binom{A(\pi \pi \rightarrow \pi \pi) A(\pi \pi \rightarrow \mathrm{~K} \overline{\mathrm{~K}})}{A(\mathrm{~K} \overline{\mathrm{~K}} \rightarrow \pi \pi) A(\mathrm{~K} \overline{\mathrm{~K}} \rightarrow \mathrm{~K} \overline{\mathrm{~K}})}=\frac{\hat{N}}{\hat{D}} \tag{21}
\end{equation*}
$$

The matrix $\mathrm{No}^{\sim}$ describes the interaction forces and contains what are known as the left-hand singularities, whereas $D \sigma^{\sim}$ describes the right-hand amplitude singularities which govern re-scattering of the particles (threshold singularities at $s=4 m_{\pi}^{2}$ and $4 m_{\mathrm{K}}^{2}$ ). The amplitude of the $\pi \pi$ scattering can be written in the form

$$
\begin{equation*}
A(\pi \pi \rightarrow \pi \pi)=\frac{a(\pi \pi \rightarrow \pi \pi)}{\operatorname{det}|\hat{D}|} \tag{22}
\end{equation*}
$$

Zeros of $\operatorname{det}|\hat{D}|$ correspond to amplitude resonances. The factor $(\operatorname{det}|\hat{D}|)^{-1}$ is common for all the amplitudes described by expression (21). Moreover, the same factor appears also in the amplitude of three-pion creation, $A(1,2 ; 3)$, which is very important for the subsequent analysis. This is related to the unitary condition which applies to the production amplitude in the two-pion channel. Therefore, the contribution of two Breit - Wigner poles to formula (18), $R(980)+R(1000)$, should be modified in accordance with expression (22):

$$
\begin{equation*}
R(980)+R(1000) \rightarrow \frac{a_{R}}{\operatorname{det}|\hat{D}|} \tag{23}
\end{equation*}
$$

where $\operatorname{det}|\hat{D}|$ should be found by fitting the data for the $\pi \pi$ amplitude when $\sqrt{s_{12}} \leqslant 1.2 \mathrm{GeV}$. The sufficiently simple


Figure 6. S-wave phase shifts of the amplitude of the $\pi \pi$ scattering with $I=0$. The continuous curve shows the results of fitting of the data to expression (24).
and yet realistic parameterisation of the $\pi \pi$ amplitudes is

$$
\begin{equation*}
\operatorname{det}|\hat{D}|=\left(s-s_{1}\right)\left(s-s_{2}\right)-\mathrm{i} \rho_{\pi \pi} \gamma_{1}-\mathrm{i} \rho_{\mathrm{KK}} \gamma_{2}-\rho_{\pi \pi} \rho_{\mathrm{KK}} \gamma_{3} \tag{24}
\end{equation*}
$$

Here, $\quad \gamma_{i}=\alpha_{i}+\beta_{i} s, \quad \rho_{\pi \pi}=\left[\left(s-4 m_{\pi}^{2}\right) / s\right]^{1 / 2}, \quad \rho_{\mathrm{KK}}=$ $\left[\left(s-4 m_{\mathrm{K}}^{2}\right) / s\right]^{1 / 2}, \quad s_{1}=0.305, \quad s_{2}=1.335, \quad \alpha_{1}=5.810$, $\alpha_{2}=2.882, \quad \alpha_{3}=-0.589, \quad \beta_{1}=-6.111, \quad \beta_{2}=-3.503$, $\beta_{3}=1.007$ (all the values are in giga-electron-volts). Fig. 6 shows how well the results for the $\pi \pi \rightarrow \pi \pi$ amplitude can be fitted by means of expression (24).

The most remarkable feature of the $p \bar{p} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ reaction is a strong interference of the resonances in the range $1300-1500 \mathrm{MeV}$ with the low-energy resonances $f_{0}(980)$ and $f_{0}(1000)$ in the crossing channels. This implies a

$$
\begin{aligned}
& s_{23} / \mathrm{GeV}^{2} \\
& \text { a } \\
& \cos \theta_{13}
\end{aligned}
$$

Figure 7. (a) Dalitz plot of the $\mathrm{p} \overline{\mathrm{p}}$ (at rest) $\rightarrow \pi^{0} \pi^{0} \pi^{0}$ reaction (sections $I, I I$, and $I I I$ are identified by arrows). (b) $z$-Distributions for fixed values of $s_{12}\left(z=\cos \theta_{13}\right.$, where $\theta_{13}$ is the angle between the momenta of the pions 1 and 3 in the centre-of-mass reference system of the particles 1 and 2).
strong interference between those amplitudes $A(i, j ; k)$ which occur in the set of expressions (17) Since information on the low-energy resonances can be obtained from other reactions (for example, from the $\pi \mathrm{N} \rightarrow \pi \pi \mathrm{N}$ data in the range of small momenta transferred to the nucleon), it is found that this interference can indeed be used to find the characteristics of large-mass resonances (Fig. 7). Let us consider the range of the Dalitz plots with large values of $s_{12}$ (corresponding to the top right-hand corner of the Dalitz plot in Fig. 7a). The $z$ distributions $\left(z=\cos \theta_{13}\right)$ given in Fig. 7b correspond to fixed values of $s_{12}$, which represents cuts along the $s_{23}$ axis (Fig. 7a). The cut at $\sqrt{s_{23}}=1536 \mathrm{MeV}$ gives the $z$ distribution, which resembles the distribution with a large admixture of the $D$ wave to the $\pi \pi$ system: the D-wave distribution is proportional to $\left(z^{2}-1 / 3\right)^{2}$. However, the cuts corresponding to different values of $\sqrt{s_{12}}$ are in conflict with this idea: the bump in the region of $z=0$ becomes narrower when $s_{12}$ is reduced and becomes stronger with increase in $s_{12}$. This behaviour can be explained in a natural manner by interference with the resonances in the 13 and 23 channels: suppression of the formation probability in the region of $z= \pm 1$ is the result of destructive interference with $\mathrm{f}_{0}(980)$. The curves in Fig. 7b describe the $z$ distributions obtained by fitting the data to the curves: the central peak is due to the interference of $f_{0}(1505)$ with $f_{0}(980)$ and $f_{0}(1000)$.

Simultaneous fit of the three reactions described by expression (14) is made in Refs [19, 20] within the approach described above: the meson production amplitudes are represented in the form of formula (18), but account is taken also of the threshold singularities and of the effects of the overlap of the resonances described by expression (23). Moreover, the fitting procedure is also applied to the data obtained by a phase analysis of the S-wave $\pi \pi$ amplitude with $I=0$.

The results of this procedure can be represented conveniently in the form of an Argand diagram. The amplitude $A(1,2 ; 3)$ is expanded in terms of partial waves in the channel 12 :

$$
\begin{equation*}
A(1,2 ; 3)=\sum_{l}(2 l+1) A_{l}(1,2 ; 3) P_{l}(z) \tag{25}
\end{equation*}
$$

Fig. 8 gives the values of $A_{0}(1,2 ; 3)$ and $A_{2}(1,2 ; 3)$ plotted as a function of $M=\sqrt{s_{12}}$. The circles formed by the amplitude in the Argand diagram correspond to resonances, just as in the case of the elastic scattering amplitude: the amplitude 'moves' with increase in $M$ along a circle in the anticlockwise direction.

The Argand diagram in Fig. 8a shows the $f_{0}(980)$ and $\mathrm{f}_{0}(980)$ resonances: a poorly formed circle corresponding to $f_{0}(1505)$, and a clear strong signal due to $f_{0}(1505)$ can be seen. The parameters of the $f_{0}(1505)$ resonance are

$$
\begin{equation*}
M=1505 \pm 20 \mathrm{MeV}, \quad \Gamma=150 \pm 20 \mathrm{MeV} \tag{26}
\end{equation*}
$$

A simultaneous analysis of the reactions described by expression (14) reveals also creation of one more scalar resonance characterised by $I=0, \mathrm{f}_{0}(1360)$, and two scalar resonances with $I=1: a_{0}(980)$ and $a_{0}(1450)$. The parameters of the new resonances, $\mathrm{f}_{0}(1360)$ and $a_{0}(1460)$, are as follows:

$$
\begin{array}{lll}
\mathrm{f}_{0}(1360): & M=1360 \mathrm{MeV}, & \Gamma=265 \mathrm{MeV} \\
\mathrm{f}_{0}(1450): & M=1450 \mathrm{MeV}, & \Gamma=270 \mathrm{MeV} \tag{27}
\end{array}
$$



Figure 8. Argand diagrams of the $\pi \pi \mathrm{S}$-wave amplitude obtained by fitting the data for the $p \bar{p} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ reaction (a) and for the $\pi \pi D$ wave amplitude (b).

These resonances are interpreted in Table 1 as the first resonant $q \bar{q}$ excitations of the $3 \mathrm{P}_{0}$ multiplet.

An analysis of the reactions described by expression (14) was made in Refs [19, 20] taking account, as mentioned above, of the pole singularities of the resonance production amplitude and of the threshold singularities due to the re-scattering of mesons in the final state. More distant singularities which affect the meson formation amplitude are those in triangular diagrams (Figs 5e and 5f) associated with the process of production of a resonance, its subsequent two-particle decay, and re-scattering of the decay product by the third particle. Singularities of the triangular diagrams were taken into account [20] in the $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ reaction: the S -wave rescattering of pions in the $I=0$ state was taken into account. The amplitude $A_{0}(1,2 ; 3)$, described by expression (25) was split into two: the amplitude describing diagrams of the $A_{\mathrm{B}+\mathrm{R}}\left(s_{12}\right)$ type ( F igs $5 \mathrm{a}-5 \mathrm{~d}$ ) and the amplitude of the triangular diagrams $A_{\mathrm{t}}\left(s_{12}\right)$, which is described by

$$
\begin{equation*}
A_{0}(1,2 ; 3)=A_{0}\left(s_{12}\right)=A_{\mathrm{B}+\mathrm{R}}\left(s_{12}\right)+A_{\mathrm{t}}\left(s_{12}\right) \tag{28}
\end{equation*}
$$

The fit of the experimental data is demonstrated in Figs 9b and 9c: the total amplitude $A_{\mathrm{B}+\mathrm{R}}+A_{\mathrm{t}}$ (Fig. 9b) differs little from the amplitude $A_{0}$ derived ignoring the triangular


Figure 9. Argand diagrams for the $\pi \pi$ S-wave amplitude $A_{0}$, found ignoring the triangular diagram (a), for the amplitude $A_{0}$ described by expression (28), i.e. taking account of the triangular diagram (b), for the amplitude $A_{\mathrm{B}+\mathrm{R}}[$ see expression (28)] (c), and for the amplitude of the triangular diagram $A_{\mathrm{t}}(\mathrm{d})$.
graph (Fig. 9a). However, the amplitude $A_{\mathrm{B}+\mathrm{R}}$, describing the direct production of mesons and the production of resonances (Fig. 9c), differs very greatly from the solution shown in Fig. 9a: this figure demonstrates clearly the production of the $f_{0}(980)$ resonance and the corresponding circle is evidently different. The triangular diagrams have little influence on the amplitude of the $\mathrm{f}_{0}(1505)$ resonance. This is not surprising: the singularity of a triangular graph is located at relatively low energies of the $\pi \pi$ system.

We can draw the following conclusion which is based on the results of an analysis of the reactions described by expression (14):
(1) the experimental results obtained by the Crystal Barrel collaboration definitely indicate the existence of the scalar resonance $f_{0}(1505)$, which decays into $\pi \pi$ and $\eta \eta$;
(2) this conclusion is not affected by the re-scattering of the mesons formed in this way; the characteristic features of a triangular diagram are important in determination of the characteristics of low-lying resonances such as $f_{0}(980)$.

## 6. Glueballs and quantum chromodynamics

As pointed out above, it is not at present possible to calculate the masses of bound states on the basis of the first principles of QCD. The quark - gluon interactions in the
soft range (at distances of the order of 1 fm ) are described within the framework of a QCD-motivated phenomenology. Glueballs, as composite systems of gluons, were predicted a very long time ago on the basis of this approach [36-40]. In this section we shall discuss briefly the predictions of the QCD-motivated phenomenology applicable to the glueball case.

### 6.1 Bag model

The MIT bag model predicts the spectrum and the masses of the lowest glueball states [40, 41]. The model deals with the gluon field in a static cavity, which in the simplest case is a spherical bag. The gluon field tensor satisfies the confinement condition: the gluon flux does not escape across the cavity surface:

$$
\begin{equation*}
n_{\mu} G_{\mu \nu}=0 \tag{29}
\end{equation*}
$$

Here, $n_{\mu}$ is the normal to the surface of the bag containing gluons. The boundary condition given by formula (29) makes it possible to find the eigenmodes of the field. There are two families of the solutions obtained in this way:
-a transverse electric field TE with the $(-1)^{l+1}$ parity;
-a transverse magnetic field TM with the $(-1)^{l}$ parity.
There are no solutions with $l=0$. the parameters of the bag found in the description of the $\mathrm{q} \overline{\mathrm{q}}$ and qqq states can

Table 2. Low-lying glueballs in the bag model [40, 41].

| States | $J^{P C}$ | $\begin{aligned} & \text { Mass/ } \mathrm{GeV} \\ & \left(R^{-1}=175 \mathrm{MeV}\right) \end{aligned}$ | $\begin{aligned} & \text { Mass } / \mathrm{GeV} \\ & \left(R^{-1}=274 \mathrm{MeV}\right) \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| Two gluons, ground states | $0^{++}, 2^{++}$ | 0.96 | 1.5 |
| Two gluons, first excited states | $0^{-+}, 2^{-+}$ | 1.3 | 2.0 |
| Three gluons, ground states | $0^{++}, 1^{+-}, 3^{+-}$ | 1.45 | 2.3 |
| Three gluons, first excited states | $\begin{aligned} & 3^{-+}, 2 \times 2^{-+} \\ & 1^{-+}, 0^{-+}, 3^{--} \\ & 2^{--}, 2 \times 1^{--} \end{aligned}$ | 1.8 | 2.7 |

be used to calculate the energies of these eigenmodes: the lowest state is the TE mode with $l=1$ and its energy is $E\left(1^{+}\right)=2.74 / R$ ( $R$ is the bad radius). This is followed by the (TE, $l=2$ ) mode with $E\left(2^{-}\right)=3.96 / R$ and (TE, $l=1$ ) mode with $E\left(1^{-}\right)=4.49 / R[41]$.

A singlet (in respect of the colour) state may be formed by two or three gluons. The lowest glueballs are then formed from two gluons in the TE, $l=1$ ) mode:

$$
\begin{equation*}
J^{P C}=0^{++}, 2^{++} . \tag{30}
\end{equation*}
$$

The next excited two-gluon states are formed by the $\mathrm{TE}, l=1+\mathrm{TE}, l=2$ ) and (TE, $l=1+\mathrm{TM}, l=1$ ) modes.

Their quantum numbers are

$$
\begin{equation*}
J^{P C}=2^{-+}, 0^{-+} \tag{31}
\end{equation*}
$$

A series of states such as $1^{-+}, 2^{-+}$, and $3^{-+}$does not form because the mass of a gluon is zero (the relevant method for the calculation of the gluon states can be found in Refs [42, 43]). Estimates of the masses of two-gluon and three-gluon glueballs are listed in Table 2.

The value $R=175 \mathrm{MeV}^{-1}$ (or 1.14 fm ), discussed in Ref. [41], gives the masses of the lowest glueballs $\left(\mathrm{O}^{++}\right.$and $2^{++}$) near 1000 MeV and predicts a large number of glueballs up to 1500 MeV . Extensive experimental material is now available on mesons for this range of masses and it seems a more realistic situation in which the glueball spectrum is shifted to the range $1500-2000 \mathrm{MeV}$. Table 2 gives the calculated values of the glueball masses for the case when the ground glueball states are near 1500 MeV ( $R \approx 0.7 \mathrm{fm}$ ). Characteristically, a glueball with exotic quantum numbers $1^{-+}$(three-gluon state) is then located very high at 2700 MeV . This is a direct consequence of the zero mass of a gluon in the bag cavity.

### 6.2 Lattice calculations

A lattice gauge theory was formulated by Wilson [44]. A calculation technique based on the Monte Carlo method was developed subsequently [45-47]. The years following the publication of Wilson's paper have seen many attempts to develop and improve this approach.

In QCD lattice calculations the theory is formulated in the form of a path integral:

$$
\begin{equation*}
Z=\int D \psi D U \exp \left(-S_{Q}-S_{G}\right) \tag{32}
\end{equation*}
$$

Integration is carried out with respect to all the fermion fields $\psi$ and $\bar{\psi}$, with respect to $U$, and with respect to $\mathrm{SU}(3)$ matrices representing gluon fields which are defined for each lattice node. The action, used in almost all the calculations, is described by [45]

$$
\begin{align*}
& S_{G}=\frac{\beta}{6} \sum_{x, \mu, v} P_{\mu v}(x), \\
& S_{Q}=\kappa \sum_{x, \mu} \bar{\psi}_{x}\left[\left(1-\gamma_{\mu}\right) U_{x, \mu} \psi_{x+\hat{\mu}}\right. \\
& \left.+\left(1+\gamma_{\mu}\right) U_{x-\hat{\mu}, \mu}^{+} \psi_{x-\hat{\mu}}\right]+\sum_{x} \bar{\psi}_{x} \psi_{x} . \tag{33}
\end{align*}
$$

The 'plaquette' $P_{\mu \nu}(x)$ is the trace of the product of the $U$ matrices around an elementary little square which lies in the $\mu-v$ plane and is specified by a point $x$.

In the lattice calculations of glueballs carried out so far only the gluon part of the action defined by the system of equations (33) is used. The quark degrees of freedom in the glueball states are ignored. The parameter of the gluon part of the lattice action is $\beta=6 / g_{0}^{2}$ and the parameter $\kappa$ is defined by the mass of a bare quark $\kappa=1 /\left(8+2 m_{0} a\right)$.

In the limit of an infinitesimally short lattice length, $a \rightarrow 0$, the action reduces to the standard QCD equations:

$$
\begin{align*}
& S_{G}=\frac{1}{4 g_{0}^{2}} \int \mathrm{~d}^{4} x\left(F_{\mu \nu}^{a}\right)^{2}+O(a) \\
& S_{Q}=\int \mathrm{d}^{4} x \bar{\psi}(x)\left(\partial_{\mu} \gamma_{\mu}+m_{0}\right) \psi(x)+O(a) \tag{34}
\end{align*}
$$

Here, $O(a)$ describes terms which vanish as $a$ goes to zero. The lattice length $a$ is selected so that in the limit, when $\beta \rightarrow \infty$ and $a \rightarrow 0$, the physical quantities remain constant. Calculations are carried out in a Euclidean space for imaginary time $t$.

In spite of the progress made recently in developing calculation algorithms, the glueball masses are difficult to find because of the small signal/noise ratio. The results of a calculation reported in Ref. [48] are given in Table 3 in units of the string tension $\sqrt{\sigma}$ for $\sigma=(440 \mathrm{MeV})^{2}$ (this value of $\sigma$ describes well the masses of light hadrons and is in agreement with the model of the Regge trajectories). We must stress once again that the results presented in Table 3 are based on the assumption that the glueball masses are not very sensitive to light-quark admixtures. Further progress in the calculation of the spectrum of low-lying glueballs will require determination of the role of an admixture of glueball states

Table 3. Results of lattice calculations of the glueball spectrum in gluodynamics [48]. Minimum lattice length $a=0.055 \mathrm{fm} \quad(\beta=6.4)$. The symbol 'ex' is used for the states with exotic quantum numbers.

| $J^{P C}$ | $m / \sqrt{\sigma}$ | $m / \mathrm{MeV}(\sqrt{\sigma}=440 \mathrm{MeV})$ |
| :--- | :---: | :--- |
| $0^{++}$ | $3.52 \pm 0.12$ | $1549 \pm 53$ |
| $2^{++}$ | $5.25 \pm 0.25$ | $2310 \pm 110$ |
| $0^{-+}$ | $5.3 \pm 0.6$ | $2332 \pm 264$ |
| $1^{+-}$ | $6.6 \pm 0.6$ | $2904 \pm 264$ |
| $0^{+-}$ex | $<9$ | $<3960$ |
| $2^{-+}$ | $7.0 \pm 0.3$ | $3080 \pm 132$ |
| $1^{-+}$ex | $<10$ | $<4400$ |
| $3^{++}$ | $8.9 \pm 1.1$ | $3916 \pm 485$ |
| $2^{+-}$ex | $10.0 \pm 2.0$ | $4400 \pm 880$ |
| $1^{++}$ | $9.0 \pm 0.7$ | $3960 \pm 308$ |
| $2^{--}$ | $9.2 \pm 0.8$ | $4050 \pm 352$ |

Table 4. Low-lying pure gluon states in the flux tube model [49]. The model includes parameters which prevent unambiguous determination of the position of the lowest glueball, but the mass splittings are not very sensitive to the selection of these parameters.

| $J^{P C}$ | $m / \mathrm{MeV}$ |
| :--- | :--- |
| $0^{++}$ | 1520 |
| $1^{+-}$ | 2250 |
| $0^{++}$ | 2750 |
| $0^{++}, 0^{+-}, 0^{-+}, 0^{--}$ | 2790 |
| $2^{++}$ | 2840 |
| $2^{++}, 2^{+-}, 2^{-+}, 2^{--}$ | 2840 |
| $1^{+-}$ | 3250 |
| $3^{+-}$ | 3350 |

with the $q \bar{q}$ components, followed by inclusion of the effects of such admixtures.

### 6.3 Flux tube model [49]

The flux tube model is based conceptually on lattice calculations: in this model the structure of the interaction is motivated by the Hamiltonian of the lattice QCD. The model does not include the concept of a 'constituent gluon'. It is assumed that in the weak coupling limit a more satisfactory concept describing glueballs is the gluon field flux: the ground state and excitations of a flux tube determine the glueball spectrum.

The glueball masses obtained on the basis of this model are given in Table 4. In spite of the fact that the model is based conceptually on the lattice calculations, the masses of the excited glueball states are significantly different from the masses reported in Ref. [48] (see Table 3). The mass of the lowest glueball $0^{++}(1520)$ given in Table 4 is fixed by a certain selection of the parameters.

We must stress one other important property of the model: it can be used to calculate the states of radial excitations. For example, Table 4 lists a whole series of the $0^{++}$glueballs: $0^{++}(1520), 0^{++}(2750)$ and $0^{++}(2790)$. In the lattice calculation framework it is not at present possible to separate the radial excitations and it is hardly likely that such calculations will be carried out in the nearest future.

### 6.4 Glueballs as composite systems of massive effective gluons

Introduction of massive effective gluons is related closely to the problem of the analyticity of the amplitudes in the soft range. Analytic properties may be lost if gluons are regarded as zero-mass particles. A typical example is the scattering of hadrons at high energies: hadrons are described by the $t$-channel gluon exchange (Fig. 10). In the language of the hadron states this represents exchange of


Figure 10. Diagrams for high-energy meson-meson scattering with $t$-channel exchange of gluons (wavy lines). A set of ladder gluon diagrams defines the Lipatov pomeron [50].

Regge glueballs. The diagrams in Fig. 10 representing zeromass gluons have singularities at $t=0$ and these disturb the analytic properties of the scattering amplitude: in fact, the $t$ channel singularities are located at $t \geqslant 4 \mu_{\pi}^{2}$ ( $\mu_{\pi}$ is the mass of a pion). A possible mechanism of recovery of the analytic properties of the amplitude is the appearance of the effective mass of soft gluons.

The need to introduce the effective gluon mass is dictated also by the experimental data on the decay of heavy quarkonia ( $c \bar{c}$ and $b \bar{b}$ states). Matching of the description of the radiative decays of $J / \psi$ and $\gamma$, compatible with a self-consistent definition of $\alpha_{s}\left(k^{2}\right)$, requires introduction of an effective gluon mass of the order of [51, 52]

$$
\begin{equation*}
M_{G} \simeq 700-1100 \mathrm{MeV} . \tag{35}
\end{equation*}
$$

There are grounds for assuming that liquidation of the incorrect singularities in the amplitudes and recovery of the correct analytic properties result from infrared QCD divergences: in the soft range, the coupling constant $\alpha_{\mathrm{s}}\left(k^{2}\right)$ is not small and regrouping of the diagrams takes place. This mechanism of the formation of massive gluons is discussed in a series of papers by Cornwall et al. [53-55] who used a special set of gauge-invariant DysonSchwinger equations to construct a gluon propagator and a three-gluon vertex function. Numerical estimates give the following effective mass of a gluon [53]:

$$
\begin{equation*}
M_{\mathrm{G}}=500 \pm 200 \mathrm{MeV} \tag{36}
\end{equation*}
$$

The mass of the lowest glueball $0^{++}$is approximately twice the gluon mass [53]:

$$
\begin{equation*}
m\left(0^{++}\right) \simeq 1000 \pm 400 \mathrm{MeV} \tag{37}
\end{equation*}
$$

An effective gluon mass of the same order as given by formulas (35) and (36) is obtained from a self-consistent description of the spectra of low-lying mesons in the quark model [56]. If the case of low-lying hadrons, the long-range component of the confinement forces is unimportant. In discussing low-lying meson states one can treat coloured objects as normal particles. One can also use in these calculations the bootstrap concept according to which lowlying meson states are formed by forces that represent analogous meson states in crossing channels.

This approach has led to a self-consistent description of the amplitudes of low-energy processes $q \bar{q} \rightarrow q \bar{q}$ and $\mathrm{qq} \rightarrow \mathrm{qq}$ in all three channels and has resulted in calculations of the meson masses in the nonets $0^{-}, 1^{-}$, and $0^{+}$which are in resonable agreement with the experimental values. The effective gluon mass is then [56]:

$$
\begin{equation*}
M_{\mathrm{G}} \simeq 700 \mathrm{MeV} . \tag{38}
\end{equation*}
$$

In the bootstrap procedure the three parameters of the model are fixed on the basis of the masses of the four lowlying mesons ( $\pi, \eta^{\prime}, K$, and $\rho$ ). The effective mass of a gluon is defined quite rigorously in this procedure: it is close to the mass of the $\rho$ meson. This is not accidental: we know that typical hadron parameters are of the order of the inverse mass of the $\rho$ meson, $\langle r\rangle_{\text {hadron }} \propto 1 / m_{\rho}$, whereas it is the exchange of effective gluons that contributes primarily to the formation of a potential well responsible for the appearance of the bound quark states, so that we have $M_{\mathrm{G}} \propto 1 /\langle r\rangle_{\text {hadron }}$. Selection of the masses of the lowlying hadrons determines the parameters of this potential well including $\langle r\rangle_{\text {hadron }}$ and, therefore, $M_{\mathrm{G}}$.

It follows that the physics of soft interactions requires introduction of an effective gluon whose mass is of the order of $700-1000 \mathrm{MeV}$. The doubled mass of an effective gluon lies in the range where mesons are located and these are the most probable candidates for glueballs.

### 6.5 Glueballs and scalarons [57]

Zero-mass gluons lead, as pointed out above, to irregular singularities in the amplitudes of the soft processes. We can assume that recovery of the correct properties of the amplitudes is due to infrared QCD divergences: a regrouping of the Feynman diagrams takes place in this range and this leads to the appearance of a massive effective gluon.

Recovery of the correct analytic properties of the amplitudes in the soft range is modelled in Ref. [57] by the Higgs mechanism in such a way as to conserve the $\operatorname{SU}(3)$ colour symmetry in the soft range (over distances of the order of $0.2-1.0 \mathrm{fm}$ ): the experience gained with the quark model suggests that this symmetry is retained at least at the global level. It should be mentioned that the mechanism of realisation of the global $\mathrm{SU}(3)$ symmetry after spontaneous breaking of the local $\mathrm{SU}(3)$ symmetry is considered in a series of papers by various authors [58-61].

Symmetry conservation over long distances influences the hadron sector: in addition to glueballs, the scalar particles with $I=0, J^{P C}=0^{++}$appear as well, and we shall call them scalarons. The mechanism of the appearance of scalarons can be demonstrated by considering the example of the simplest QCD-motivated Lagrangian, which includes coloured Higgs particles:

$$
\begin{equation*}
L=L_{\mathrm{QCD}}+L_{\mathrm{H}} . \tag{39}
\end{equation*}
$$

Here $L_{\mathrm{QCD}}$ is the standard QCD Lagrangian and $L_{\mathrm{H}}$ is the Lagrangian for the coloured Higgs fields $\phi$ :

$$
\begin{equation*}
L_{\mathrm{H}}=-\frac{1}{2} \operatorname{Tr}\left(\mathrm{D}^{\mu} \phi\right)^{+} \mathrm{D}_{\mu} \phi+V(\phi) \tag{40}
\end{equation*}
$$

where

$$
\begin{aligned}
& V(\phi)=-a \operatorname{Tr} \phi^{+} \phi+\frac{1}{3} b\left(\operatorname{Tr} \phi^{+} \phi\right)^{2} \\
&+B\left[\operatorname{Tr} \phi^{+} \phi \phi^{+} \phi-\frac{1}{3}\left(\operatorname{Tr} \phi^{+} \phi\right)^{2}\right] \\
&-\sqrt{\frac{2}{3}} c\left(\operatorname{det} \phi^{+}+\operatorname{det} \phi\right)
\end{aligned}
$$

A generalised derivative $\mathrm{D}_{\mu}=\partial_{\mu}-(\mathrm{i} / 2) g \lambda_{a} A_{\mu}^{a}$ is determined by the gluon field $A_{\mu}^{a}$ and the complex scalar Higgs field $\phi_{k l}$ is a triplet in accordance with the local colour $\mathrm{SU}(3)_{\mathrm{c}}$ group (index 1 ) and a triplet in accordance with a certain global $\operatorname{SU}(3)$ group (index $k$ ). Spontaneous breaking of the local colour group is accompanied, for $a>0$, $b>0$, and $c>0$, by conservation of the global colour $\mathrm{SU}(3)$ group. The new basis includes eight massive gluons [colour $\operatorname{SU}(3)$ octet, $J^{P}=1^{-}$] with the mass

$$
\begin{equation*}
m_{\mathrm{G}}^{2}=\frac{1}{6} g^{2} v^{2} \tag{41}
\end{equation*}
$$

Here, $\left\langle\phi_{k l}\right\rangle=\delta_{k l} v / \sqrt{6}$ and the vacuum average $v$ is governed by the equality $b v^{2}=3 a+c v$. There are two massive scalarons [ $\mathrm{SU}(3)$ singlets, $J^{P}=0^{+}$] with the masses

$$
\begin{equation*}
M_{1}^{2}=c v, \quad M_{2}^{2}=\frac{2}{3} b v^{2}-\frac{1}{3} c v \tag{42}
\end{equation*}
$$

as well as eight massive coloured Higgs mesons $[\mathrm{SU}(3)$ octet, $J^{P}=0^{+}$] with the mass

$$
\begin{equation*}
m_{\mathrm{H}}^{2}=\frac{2}{3}\left(B v^{2}+c v\right) \tag{43}
\end{equation*}
$$

The mass of a Higgs meson $m_{\mathrm{H}}$ can be as large as you please if we assume that $B \rightarrow \infty$ and this removes effectively such an auxiliary particle from consideration. The effective theory, developed in order to describe the low-energy interaction of hadrons in the soft range, then operates with quarks and massive gluons as components. The vector gauge field $A_{\mu}^{a}$ retains in this theory the same self-interaction structure as in QCD and colour is retained as the exact global symmetry. The quark - gluon - antiquark vertex has a structure governed by the Lagrangian $L_{\mathrm{QCD}}$ so that the quark components are triplets with the colour global $\mathrm{SU}(3)$ group. The effective theory is renormalisable and the vacuum average $v$ is a scalar in accordance with the new (global) colour symmetry, but not in accordance with the initial local $\operatorname{SU}(3)_{c}$ symmetry. The 'payoff' for conservation of the colour symmetry as the global one is the existence of two scalar Higgs mesons (scalarons) with masses given by formula (42).

Introduction of the effective Lagrangian, described in expression (40), represents realisation of the assumption that there are three ranges of application of the basic QCD Lagrangian:
(1) short distances $r<0.1 \mathrm{fm}$, where perturbative calculations can be made;
(2) intermediate distances $0.1 \mathrm{fm}<r<1 \mathrm{fm}$ where the coupling constant $\alpha_{\mathrm{s}}$ is not small but the effects of colour neutralisation are not yet significant;
(3) long distances, where the confinement effects are important.

It is assumed that the effective Hamiltonian is applicable in the range (2) where the use of constituent quarks and effective gluons is justified. This Lagrangian cannot be used in the range (1) because the constituent quarks, effective gluons, and coloured Higgs mesons give rise to a structure complex from the point of view of the basic QCD quarks and gluons. The Lagrangian of expression (40) is unsuitable also when the distances are long, because it does not contain the infrared divergences that are characteristic of the basic QCD Lagrangian: it is these divergences that are assumed to ensure the confinement of coloured objects. Therefore, the Lagrangian of expression (40) is by its nature applicable to the phenomena that correspond to the range of intermediate distances. It is assumed that low-lying glueballs are formed by the interactions occurring in the range (2) exactly in the same way as low-lying $q \bar{q}$ mesons and $q q q$ baryons. The possibility of using the Lagrangian defined by expressions (39) and (40) in the quark model for the description of the soft interactions was pointed out in Refs [62, 63].

We can readily estimate the parameters of the model for which the group of resonances described by expression (4) is extra to the $q \bar{q}$ systematics and can be classified as scalarons and lower glueballs. For example, if

$$
\begin{align*}
& a=1.33 \mathrm{GeV}^{2}, \quad \mathrm{c}=0.33 \mathrm{GeV} \\
& b=0.56, \quad g^{2}=0.38, \quad B>3 \tag{44}
\end{align*}
$$

Table 5. Model of massive gluons [57]: two-gluon S-, P-, and D-wave glueballs and scalarons.

| States | Possible interpretation of $(4)$ and (5) states as GG glueballs and scalarons | Glueball states, predicted by expression (46) |
| :--- | :--- | :--- |
| GG,$L=0$ | $0^{++}(1505), 2^{++}(1710)$ |  |
| GG,$L=1$ | $0^{-+}(?), 1^{-+}(1910), 2^{-+}(?)$ | $0^{-+}(1730), 2^{-+}(2260)$ |
| GG,$L=2$ | $0^{++}(2100), 1^{++}(?), 2^{++}(?), 2^{++}(?), 3^{++}(?), 4^{++}(?)$ | $1^{++}(2280), \ldots$ |
| Scalarons | $0^{++}(980), 0^{++}(1740)$ |  |

(all the dimensional quantities are in gigaelectronvolts), we have

$$
\begin{align*}
& m_{\mathrm{G}}=750 \mathrm{MeV}, \quad \mathrm{~m}_{\mathrm{H}}>4500 \mathrm{MeV} \\
& M_{1}=1000 \mathrm{MeV}, \quad \mathrm{M}_{2}=1750 \mathrm{MeV} \tag{45}
\end{align*}
$$

The doubled gluon mass is 1500 MeV and the scalar $\mathrm{f}_{0}(1505)$ resonance is interpreted as the lowest glueball $0^{++}$, the resonances $f_{0}(980)$ and $f_{0}(1740)$ are scalarons, and the coloured Higgs bosons are auxiliary particles necessary for the regularisation of the diagrams corresponding to short distances. Table 5 gives a possible interpretation of the resonances described by expressions (4) and (5), as glueballs and scalarons. This table includes also the predicted masses of the P-wave GG glueballs $0^{-+}$and $2^{-+}$and of the D-wave $1^{++}$glueball. These masses are estimated from the phenomenological formula

$$
\begin{equation*}
M=m+a J(J+1)+b S(S+1)+c L(L+1) \tag{46}
\end{equation*}
$$

where $S$ and $L$ are the total spin and momentum of the GG state; $m, a, b$, and $c$ are free parameters. This formula is an expansion of the mass term of the operators $\hat{J}^{2}, \hat{S}^{2}$, and $\hat{L}^{2}$, which is valid in the case of relatively small mass splittings. In this case it is unimportant whether the formula for the mass splittings is given by expression (46) or whether it describes the behaviour of the square of the mass $M^{2}$. The predicted mass of the $0^{-+}$glueball is 1730 MeV and it is very close to the mass of the S-wave $2^{++}$glueball. The masses of the $2^{-+}$and $1^{++}$glueballs are then in the range $2250-2300 \mathrm{MeV}$ and the masses of the remaining $D$-wave glueballs occur in the region of 2500 MeV or higher.

Thus, the model in which glueballs consist of massive gluons with $M_{\mathrm{G}} \approx 700-1000 \mathrm{MeV}$ gives rise to a systematics that describes qualitatively the group of resonances given by expressions (4) and (5). Colour conservation in the form of the global $\mathrm{SU}(3)$ symmetry in the soft range requires, in addition to the glueball sector, the existence of two more $0^{++}$resonances (scalarons), which can explain the superfluous scalar mesons in the group described by expression (4).

Unfortunately, a qualitative description of glueballs in terms of the models with purely gluon degrees of freedom, i.e. ignoring the quark states, is hardly possible. The problem of mixing of the $q \bar{q}$ and glueball states is discussed in the next section.

## 7. Mixing of glueball and quark - antiquark states

The problem of mixing of gluon and quark-antiquark components is very important in the physics of glueballs. The search for glueballs would be greatly simplified if glueballs were pure gluon ensembles. First, one could treat
with greater confidence the model calculations of the glueball masses. It is worth recalling once again that the calculations carried out so far ignore the influence of the $\mathrm{q} \overline{\mathrm{q}}$ component. Second, the decay of pure (without any admixture) gluon states would have obeyed the rules of quark democracy: equality of the amplitudes of the $\mathrm{GG} \rightarrow \mathrm{q} \overline{\mathrm{q}}$ transitions for light quarks of all flavours. In this case the reduced decay widths (representing the partial widths divided by the phase volume $\phi$ ) should be the same for all transitions such as $\pi \pi, \eta \eta, \mathrm{K} \overline{\mathrm{K}}$ :

$$
\begin{equation*}
\frac{2}{3} \frac{\Gamma(\pi \pi)}{\phi(\pi \pi)}=\frac{\Gamma(\eta \eta)}{\phi(\eta \eta)}=\frac{1}{2} \frac{\Gamma(\mathrm{~K} \overline{\mathrm{~K}})}{\phi(\mathrm{K} \overline{\mathrm{~K}})} \tag{47}
\end{equation*}
$$

Here

$$
\begin{aligned}
& \Gamma(\pi \pi)=\Gamma\left(\pi^{+} \pi^{-}\right)+\Gamma\left(\pi^{0} \pi^{0}\right) \\
& \Gamma(\mathrm{K} \overline{\mathrm{~K}})=\Gamma\left(\mathrm{K}^{+} \mathrm{K}^{-}\right)+\Gamma\left(\mathrm{K}^{0} \overline{\mathrm{~K}}^{0}\right)
\end{aligned}
$$

However, there are important reasons to assume that mixing of the GG and $q \bar{q}$ states is not a small effect. This means that glueballs cannot be regarded as pure gluon ensembles and, for exactly the same reason, all the mesons in Table 1 cannot be regarded as pure $q \bar{q}$ states. Arguments supporting mixing of the $\mathrm{q} \overline{\mathrm{q}}$ and GG states are provided by the rules of the $1 / N$ expansion [29, 30]: $N=N_{\mathrm{c}}=N_{\mathrm{f}}=3$, where $N_{\mathrm{c}}$ is the number of colours and $N_{\mathrm{f}}$ is the number of light flavours. Experience gained with quark diagrams shows that this expansion works quite well.

By way of illustration we shall consider a simple model of the formation of $q \bar{q}$ mesons and GG glueballs. We shall assume that the bound states $q \bar{q}$ and $G G$ form as a result of gluon exchange (Figs 11a and 11c, respectively):

a
b




Figure 11. Ladder diagrams (a, c) describing the $q \bar{q}$ bound state (b) and a glueball (d), and diagrams showing how mixed $G G$ and $q \bar{q}$ states are obtained (e, f, g).
according to the rules of the $1 / N$ expansion, diagrams of this type make a large contribution to the formation of hadrons. The sum of all possible ladder diagrams (Fig. 11a) results in a $\mathrm{q} \overline{\mathrm{q}}$ meson (Fig. 11b) and the ladder diagrams in Fig. 11c result in a glueball (Fig. 11d). Inclusion of the $q \bar{q}$ loop in the gluon ladder (Fig. 11e) makes a contribution of the same order after summation over all the flavours: $\mathrm{q} \overline{\mathrm{q}} \rightarrow \sum_{\mathrm{f}} \mathrm{q}_{\mathrm{f}} \overline{\mathrm{q}}_{\mathrm{f}}$. The same order of magnitude is retained in the diagram shown in Fig. 11f: this means that according to the rules of the $1 / N$ expansion the transition (Fig. 11 g )

$$
\begin{equation*}
\text { glueball } \rightarrow \sum_{\mathrm{f}} \mathrm{q}_{\mathrm{f}} \overline{\mathrm{q}}_{\mathrm{f}}-\text { mesons } \rightarrow \text { glueball } \tag{48}
\end{equation*}
$$

is not suppressed. It should be stressed that the vertex of the transition of a glueball to one specific quark - antiquark state is of the order of $1 / \sqrt{N}$ : the glueball $\rightarrow \mathrm{q}_{\mathrm{f}} \overline{\mathrm{q}}_{\mathrm{f}} \rightarrow$ glueball transition is characterised by a vertex of the order of $1 / N$. Only the summation over all the flavours removes this smallness: $\sum_{\mathrm{f}}=N_{\mathrm{f}}$. Therefore, an admixture of one specific $\mathrm{q}_{\mathrm{f}} \bar{q}_{\mathrm{f}}$ state in a glueball represents a small contribution $(\sim 1 / N)$ and the admixture of a glueball in one specific $\mathrm{q}_{\mathrm{f}} \overline{\mathrm{q}}_{\mathrm{f}}$ state is equally small. However, the sum of the admixtures of the $\sum_{f} q_{f} \bar{q}_{f}$ states in a glueball is no longer small.

Two comments should be made about possible deviations from the $1 / N$ expansion rules. First, these rules postulate that the creation of $\mathrm{q} \overline{\mathrm{q}}$ is independent of the quark flavours, as is true of nonperturbative QCD. However, in the range of the soft interactions the $\mathrm{GG} \rightarrow \mathrm{s} \overline{\mathrm{s}}$ transition is suppressed compared with the transitions to nonstrange quarks: $G G \rightarrow u \bar{u}$ or $G G \rightarrow d \bar{d}$. This is supported by the experimental results on the creation of hadrons at high and very high energies: in a gluon net forming a pomeron the probability of creation of an ss̄ pair is suppressed compared with uū and the suppression parameter $\lambda$ is of the order of $0.4-0.5$ (see, for example, Ref. [64] and the literature cited there). A pomeron is equivalent to a set of $t$-channel exchanges of Regge glueballs. Therefore, the probability of a glueball transforming into $s \bar{s}$ is reduced by the same factor if we compare it with the probabilities of analogous transitions to nonstrange quarks:

$$
\begin{equation*}
\frac{W(\text { glueball } \rightarrow \mathrm{s} \overline{\mathrm{~s}})}{W(\text { glueball } \rightarrow \mathrm{n} \overline{\mathrm{n}})} \simeq \frac{1}{2} \tag{49}
\end{equation*}
$$

Here, $n$ represents a $u$ or d quark. When this point is taken into account, the ratios given by expression (47) for the partial widths of the decay of a glueball in the pseudoscalar mesons can be rewritten as follows:

$$
\begin{equation*}
\frac{\Gamma(\pi \pi)}{\phi(\pi \pi)} \simeq 5 \frac{\Gamma(\eta \eta)}{\phi(\eta \eta)} \simeq 2 \frac{\Gamma(\overline{\mathrm{~K}} \mathrm{~K})}{\phi(\mathrm{K} \overline{\mathrm{~K}})} \tag{50}
\end{equation*}
$$

Quark democracy then breaks down and the pion decay channel predominates.

The second comment relates to the number of meson states which are present as admixtures in a glueball: their number need not be $N=3$. F or example, the $0^{++}$glueball may have admixtures of just two terms of the $q \bar{q}$ multiplet characterised by $1^{3} \mathrm{P}_{0}\left(0^{++}\right)$: s $\bar{s}$ and $(u \bar{u}+d \bar{d}) / \sqrt{2}$. The third state $(u \bar{u}-d \bar{d}) / \sqrt{2}$ has the isospin of 1 and does not mix with a glueball. However, the $0^{++}$glueball may be mixed with mesons from other multiplets, for example, with radial excitations such as $2^{3} \mathrm{P}_{0}(\mathrm{q} \overline{\mathrm{q}})$. As usual, mixing is enhanced in the states with similar masses and, therefore, the inclusion of $q \bar{q}$ admixtures in a glueball is determined
by those specific $q \bar{q}$ states which are close to a glueball. For example, we can expect $f_{0}(1505)$ and $f_{0}(1590)$ to be formed as a result of strong mixing of the GG and $q \bar{q}$ states.

In conclusion, we must stress once again that in each specific $q \bar{q}$ state an admixture of the glueball component is reduced by a factor proportional to $1 / N$ and, therefore, should not perturb greatly the structure of the $q \bar{q}$ states. However, the reverse, namely the perturbation of the glueball structure, cannot be regarded as small.

## 8. State $\left(I, J^{P C}\right)=\left(0,0^{++}\right)$: the puzzle of the amplitude structure in the region of $\mathbf{1} \mathbf{G e V}$

The mass range $1000-1500 \mathrm{MeV}$ is the key to the interpretation of the spectroscopy of exotic states. An exceptionally important role is played by the $\left(0,0^{++}\right)$ states: it is these states that contribute the largest number of puzzles. The problems appear at a very early stage when amplitudes in the $600-1000 \mathrm{MeV}$ range are discussed.

The most direct way for the investigation of the structure of the $\left(0,0^{++}\right)$state is extraction of the $\pi \pi$ amplitude by the method of separation of the $t$-channel pole diagrams. Much research has been carried out in the last 30 years. Let us turn to the phase $\delta_{\mathrm{S}}^{0}$ of the $\pi \pi$ scattering process found by the Chew-Low method (Fig. 12). This phase is not small (it amounts to about $60^{\circ}$ ) already in the range $M_{\pi \pi} \approx 600-700 \mathrm{MeV}$, rising to $90^{\circ}$ in the range $800-900 \mathrm{MeV}$. It then increases steeply, passing through $180^{\circ}$ at $M_{\pi \pi} \approx 1000 \mathrm{MeV}$, and at $1100-1200 \mathrm{MeV}$ it is again close to the 'resonance' value $270^{\circ}$. A simple interpretation of this behaviour is the existence of two resonances, one of which is wide and the other narrow.


Figure 12. Phase $\delta_{\mathrm{S}}^{0}$ of the $\pi \pi \rightarrow \pi \pi$ amplitude. The continuous curve is the description of the phase by means of expressions (51)-(53).

The question arises of how strong the arguments are for interpreting the structure of the $\pi \pi$ amplitude as representing two resonances.

The analytic structure of the $\pi \pi$ amplitude considered as a function of $s$ is complicated by the presence of the $K \bar{K}$ threshold singularity and by the large amplitude of the


Figure 13. Inelasticity parameter $\eta_{\mathrm{S}}^{0}$ of the $\pi \pi \rightarrow \pi \pi$ amplitude. The continuous curve is the description of the phase by means of expressions (51)-(53).
$\pi \pi \rightarrow K \bar{K}$ transition: the inelasticity parameter $\eta$ of the $\pi \pi \rightarrow \pi \pi$ amplitude is plotted in Fig. 13.

What is the influence of the threshold $\mathrm{K} \overline{\mathrm{K}}$ singularity on the poles of the $\pi \pi$ amplitude?

The values of $\delta_{\mathrm{S}}^{0}$ and $\eta_{\mathrm{S}}^{0}$, plotted in Figs 12 and 13, were obtained by the Chew-Low method, i. e. by separation of the pole component of the diagrams in Fig. 14 in the limit $t \rightarrow \mu_{\pi}^{2}$. We can formulate a different problem of


Figure 14. Pole diagrams describing peripheral creation of $\pi \pi$ and $K \bar{K}$.
investigating the process illustrated in Fig. 14a by variation of $t$ from low to moderately large values. Such an investigation is reported in Ref. [65]: the S-wave spectra are shown in Fig. 15. The most remarkable result is the rapid reduction in the wide bump with increase in $|t|$, whereas a narrow $f_{0}(980)$ resonance is seen clearly even for $|t| \approx 0.5 \mathrm{GeV} c^{-2}$. It follows that the wide bump has a fairly open structure and that $f_{0}(980)$ includes a very hard component in the pion part of the wave function.

Interpretation of the $f_{0}(980)$ resonance as the meson molecule or a deuson (a combination of the words deuteron and meson) [13, 66] is now popular. It is assumed that these 'molecules' do not fit the $\mathrm{SU}(3)$ flavour systematics.

Does the existence of the hard component in the $\mathrm{f}_{0}(980)$ resonance agree with its interpretation as the $\mathrm{K} \overline{\mathrm{K}}$ molecule?

The most recent experimental results make it possible to consider the analytic structure of the $\pi \pi$ amplitude. We shall now consider the results based on a simultaneous analysis [65] of the following data:
(1) the $\pi^{0} \pi^{0}$ spectra of the $\pi^{-} p \rightarrow \pi^{0} \pi^{0} n$ reaction with momenta transferred in the range $|t|<1 \mathrm{GeV}^{2} c^{-2}$ [14];
(2) the S-wave amplitude $\pi \pi$ with $I=0$, deduced by the Chew-Low method [2];
(3) the spectra of mesons produced by three-particle reactions $\mathrm{p} \overline{\mathrm{p}} \rightarrow \pi^{0} \pi^{0} \pi^{0}, \mathrm{p} \overline{\mathrm{p}} \rightarrow \eta \eta \pi^{0}[16]$ and $\mathrm{p} \overline{\mathrm{p}} \rightarrow \eta \pi^{0} \pi^{0}$ [19].

The method for analysis of the experimental results, listed as items (2) and (3) above, is discussed in Section 5. When creation of the $\pi^{0} \pi^{0}$ system with nonzero momentum transfer is included in the analysis, it is necessary to take account of dropping of a $t$-channel pion off the mass shell in the $\pi^{-} \pi^{+} \rightarrow \pi^{0} \pi^{0}$ block (Fig. 14a). The isoscalar S-wave $\pi \pi \rightarrow \pi \pi$ amplitude in the $K$-matrix representation is
$\langle\pi \pi| A|\pi \pi\rangle=\frac{K_{\pi \pi}+\mathrm{i}\left(K_{\pi \mathrm{K}} K_{\mathrm{K} \pi}-K_{\pi \pi} K_{\mathrm{KK}}\right)}{1-\mathrm{i} K_{\pi \pi}-\mathrm{i} K_{\mathrm{KK}}+\left(K_{\pi \mathrm{K}} K_{\mathrm{K} \pi}-K_{\pi \pi} K_{\mathrm{KK}}\right)}$.
Here, $\quad K_{\pi \pi} \equiv\langle\pi \pi| K|\pi \pi\rangle, \quad K_{\mathrm{K} \pi} \equiv\langle\mathrm{K} \overline{\mathrm{K}}| K|\pi \pi\rangle, \quad$ and $\quad K_{\mathrm{KK}}$ $\equiv\langle\mathrm{K} \overline{\mathrm{K}}| K|\mathrm{~K} \overline{\mathrm{~K}}\rangle$. Fitting of the data listed under points (1)-(3) above was carried out by a procedure


Figure 15. Mass spectra of the $\pi \pi$ system formed by the $\pi^{-} p \rightarrow n \pi^{0} \pi^{0}$ reaction.


Figure 16. Argand diagrams of the $p \bar{p} \rightarrow \pi^{0} \pi^{0} \pi^{0}$ (a), $p \bar{p} \rightarrow \pi^{0} \pi^{0} \eta$ (b), and $p \bar{p} \rightarrow \eta \eta \pi^{0}$ (c) reactions.
involving parametrisation of the $k$-matrix elements in a two-pole form:

$$
\begin{equation*}
K_{a b}=\sqrt{\rho_{a}}\left(f_{a b}+\frac{g_{a} g_{b}}{M_{1}^{2}-s}+\frac{G_{a} G_{b}}{M_{2}^{2}-s}\right) \frac{s-m_{\pi}^{2} / 2}{s} \sqrt{\rho_{b}} \tag{52}
\end{equation*}
$$

where the indices $a b$ apply to the $\pi \pi$ and $\mathrm{K} \overline{\mathrm{K}}$ states, $\rho_{K}=\left[\left(s-4 m_{K}^{2}\right) / s\right]^{1 / 2}$, whereas $f_{a b}, g_{a}$, and $G_{a}$ are constants. The factor $\left(s=m_{\pi}^{2} / 2\right) / s$ is introduced into the $K$ matrix to ensure that Adler's self-consistency condition [67] is satisfied automatically. Such a description of the experimental results gives the following values of the parameters (all the dimensional quantities are given in gigaelectronvolts):

$$
\begin{array}{ll}
M_{1}=0.757, & M_{2}=1.162 \\
g_{\pi}=0.885, & G_{\pi}=0.885 \\
g_{\mathrm{K}}=0.474, & G_{\mathrm{K}}=-0.333 \\
f_{\pi \mathrm{K}}=0.72, & f_{\pi \pi}=f_{\mathrm{KK}}=0 \tag{53}
\end{array}
$$

These experimental results and the parameters given above were used to plot the diagrams in Figs 12, 13, and 15, and
the Argand diagrams for the amplitudes of the $p \bar{p} \rightarrow$ (three mesons) processes, which are shown in Fig. 16. The above parameters describe well all the experimental data.

The resultant $\pi \pi$ amplitude has a very complex structure. We shall consider this structure by excluding one particular decay channel at any given stage. This can be done by varying $G_{a}, g_{a}$, and $f$.

As $G_{a}, g_{a}$, and $f$ tend to zero, the poles of the $K$ matrix become the poles of the amplitude. Thus $M_{1}$ and $M_{2}$ are the masses of the resonances expected if the widths of the $\pi \pi$ and $\mathrm{K} \overline{\mathrm{K}}$ decay channels are small. The transitions

$$
\begin{equation*}
\text { (resonance 1) } \rightarrow\binom{\pi \pi}{\mathrm{K} \overline{\mathrm{~K}}} \rightarrow(\text { resonance } 2) \tag{54}
\end{equation*}
$$

lead, as shown below, to a strong mixing of the initial ('pure') states 1 and 2, and the mixed (physical) states acquire masses very different from $M_{1}$ and $M_{2}$.

Let us consider first the pure states 1 and 2 . The masses of these states are located at 800 and 1200 MeV , respectively, and the constants representing the coupling of these states to the $\pi \pi$ and $\mathrm{K} \overline{\mathrm{K}}$ channels are not small. Let us see what half-widths would be obtained for these
coupling constants had the states 1 and 2 been physical particles:

$$
\begin{align*}
& \frac{\Gamma_{1}}{2}=\frac{g_{\pi}^{2} \rho_{\pi}\left(M_{1}^{2}\right)}{2 M_{1}} \approx 470 \mathrm{MeV} \\
& \frac{\Gamma_{2}}{2}=\frac{G_{\pi}^{2} \rho_{\pi}\left(M_{2}^{2}\right)}{2 M_{2}}+\frac{G_{\mathrm{K}}^{2} \rho_{\mathrm{K}}\left(M_{2}^{2}\right)}{2 M_{2}} \approx 350 \mathrm{MeV} \tag{55}
\end{align*}
$$

This is indeed greater than the half-width. However, we know that mixing of the 'particles' 1 and 2 has the effect that only one physical state retains a large half-width whereas the other state, $\mathrm{f}_{0}(980)$, becomes narrow. We can follow the dynamics of the motion of zeros of the denominator of the right-hand side of expression (51), which determine the complex mass of the resonances, when mixing is 'activated'. Such 'activation' can be induced by the following simple procedure: the substitutions $f \rightarrow x f$, $g_{a} \rightarrow x g_{a}$, and $G_{a} \rightarrow x G_{a} \rightarrow x G_{a}$ are made in the denominator of expression (51) and then $x$ is varied from 0 (no mixing) to 1 (real case corresponding to a description of the experimental data).

The 'motion' of the $\pi \pi$-amplitude poles due to variation of $x$ from 0 to 1 is shown in Fig.17. At $x=o$ the transitions of the 'particles' 1 and 2 to the $\pi \pi$ and KK states are missing and the poles are located on the real axis: $\sqrt{s}=M_{\pi \pi}=M_{1}=760 \mathrm{MeV}$ and $M_{\pi \pi}=M_{2}=1160 \mathrm{MeV}$. In the range $x>0$, the poles shift to the complex plane. An increase in $x$ shifts the poles of the 'particle' 1 in the direction of large masses approaching the $\mathrm{K} \overline{\mathrm{K}}$ threshold and the mass of the 'particle' 2 decreases.


Figure 17. 'Motion' of poles of the $\pi \pi$ amplitude, described by expression (51), due to 'inclusion' of the coupling constants $g_{\pi}, g_{\mathrm{K}}$ and $G_{\pi}, G_{\mathrm{K}}$. The positions 0 correspond to the poles for 0 coupling constants, and the positions 1 are the physical values of the coupling constants given by the set of expressions (53).

Let us discuss in greater detail the positions of the poles when $x=0.5$. We then have the standard situation with the usual two Breit-Wigner poles. On the second sheet the pole associated with the first resonance is located at $M_{1}^{I I}(x=0.5) \simeq 900-\mathrm{i} 150 \mathrm{MeV}$, i. e. relatively close to the physical region. The conjugate pole, which is also associated with the first resonance and is due to doubling of the poles as a result of the KK threshold singularity, is quite far from the physical region: it is located on the third sheet at $M_{1}^{I I I}(x=0.5) \simeq 890-\mathrm{i} 200 \mathrm{MeV}$. Two poles associated with the second resonance are located at $M_{2}^{I I I}(x=0.5) \simeq 1130-\mathrm{i} 180 \mathrm{MeV}$ (not far from the physical region) and at $M_{2}^{I I}(x=0.5) \simeq 1120-\mathrm{i} 190 \mathrm{MeV}$ (far from the physical region).

In the realistic case $(x=1)$ both poles of the first resonance are relatively close to the physical region and they approach the $\mathrm{K} \overline{\mathrm{K}}$ threshold:

$$
\begin{equation*}
M_{1}^{I I}=990-\mathrm{i} 41 \mathrm{MeV}, \quad M_{1}^{I I I}=975-\mathrm{i} 75 \mathrm{MeV} \tag{56}
\end{equation*}
$$

The poles of the second resonance are quite far from the physical region. The nearest pole is located at

$$
\begin{equation*}
M_{2}^{I I I}=1163-\mathrm{i} 476 \mathrm{MeV} \tag{57}
\end{equation*}
$$

It is necessary to determine the reason for the proximity of the $M_{1}$ poles to the physical region. There are two possible reasons:
-existence of the $\mathrm{K} \overline{\mathrm{K}}$ component in a bound state:
-mixing of the states 1 and 2.
The role of the $\mathrm{K} \overline{\mathrm{K}}$ component can be estimated by excluding it completely and assuming that $g_{\mathrm{K}}=G_{\mathrm{K}}=$ $\mathrm{f}=0$. Then the $\pi \pi$ amplitude has poles at

$$
\begin{align*}
& M_{1}^{I I}\left(g_{\mathrm{K}}=G_{\mathrm{K}}=\mathrm{f}=0\right)=985-\mathrm{i} 65 \mathrm{MeV} \\
& M_{2}^{I I}\left(\mathrm{KK}=\mathrm{G}_{\mathrm{K}}=\mathrm{f}=0\right) \simeq 1100-\mathrm{i} 1000 \mathrm{MeV} \tag{58}
\end{align*}
$$

i. e. the pole $M_{1}^{I I}$ shifts very little. In general, this is the expected situation: the states 1 and 2 are strongly coupled to the $\pi \pi$ channel and their coupling to $\mathrm{K} \overline{\mathrm{K}}$ is an order of magnitude weaker, i.e.

$$
\begin{equation*}
\frac{g_{\mathrm{K}}^{2}}{g_{\pi}^{2}} \simeq \frac{G_{\mathrm{K}}^{2}}{G_{\pi}^{2}} \sim \frac{1}{10} \tag{59}
\end{equation*}
$$

We can similarly demonstrate that it is interference between the states 1 and 2 which is responsible for the small width of $f_{0}(980)$. We can exclude the state 2 by assuming that $G_{\pi}=G_{\mathrm{K}}=0$. This gives the following position of the pole $M_{1}^{I I}$ :

$$
\begin{equation*}
M_{1}^{I I}\left(G_{\pi}=G_{\mathrm{K}}=0\right)=750-\mathrm{i} 125 \mathrm{MeV} \tag{60}
\end{equation*}
$$

The pole has the normal hadron width $\Gamma / 2=125 \mathrm{MeV}$ and it is quite far from the $K \bar{K}$ threshold, although this threshold singularity is present in the $\pi \pi$ amplitude when the condition given by expression (60) applies; this is due to the transitions that are associated with the nonzero constants $g_{\mathrm{K}}$ and $f$. The 'experiments' involving exclusion of the $\mathrm{K} \overline{\mathrm{K}}$ channel and of the state 2 demonstrate unambiguously that the narrow $\mathrm{f}_{0}(980)$ resonance is formed by the following transitions: state $1 \rightarrow \pi \pi \rightarrow$ state 2 .

Two poles described by expression (56) can be interpreted as the $\mathrm{f}_{0}(980)$ resonance. This resonance is essentially governed by the $\pi \pi$ channel: we can see that the position of the appropriate pole does not change greatly when the $K \bar{K}$ channel is excluded from the phenomenological amplitude described by expression (51). This circumstance, together with the results reported in

Ref. (14) indicating the existence of a hard component in the $f_{0}(980)$ resonance, suggest that the interpretation of this resonance as the $\mathrm{K} \overline{\mathrm{K}}$ molecule is unlikely to be correct.

According to the classification in Table 1, the wide resonance governed by the pole of expression (57) is $\mathrm{f}_{0}(1000)$. In this table the resonance in question is interpreted as a term in the $3 \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ nonet, whereas $\mathrm{f}_{0}(980)$ is a candidate for the class of exotic mesons. It is proposed to consider the resonance of $\mathrm{f}_{0}(980)$ listed in Table 5 as a scalaron. Our analysis demonstrates the arbitrary nature of such a classification: the pure states 1 and 2 become strongly mixed and each of the physically observed resonances is a mixture of a number of components. In accordance with the proposed schemes (Tables 1 and 2), we have

$$
\begin{align*}
& \mathrm{f}_{0}(1000) \rightarrow \text { mixture: } 3 \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}} \text {-state/scalaron; } \\
& \mathrm{f}_{0}(980) \rightarrow \text { mixture: scalaron } 3 \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}} \text {-state } \tag{61}
\end{align*}
$$

In exactly the same manner the pure $3 \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ state is an $\mathrm{f}_{0}(1000) / \mathrm{f}_{0}(980)$ mixture and a pure scalaron is an orthogonal combination of the resonances $f_{0}(980)$ and $\mathrm{f}_{0}(1000)$.

This investigation allows us to conclude that the combined contribution of the two resonances represents the $\sigma$ meson [68], which plays such an important role in low-temperature physics of hadrons and in nuclear physics.

## 9. Multiplet $\mathbf{1}^{\mathbf{3}} \mathbf{P}_{\mathbf{0}} \mathbf{q} \overline{\mathbf{q}}$

The following dilemma is encountered in determination of the basic ${ }^{3} \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ nonet.
(1) We can assume, as in Table 1, that the basic $q \bar{q}$ nonet is formed by the states
$1^{3} \mathrm{P}_{0}: \mathrm{a}_{0}(980), \mathrm{f}_{0}(1000) / \mathrm{f}_{0}(980), \mathrm{f}_{0}(1240), \mathrm{K}_{0}(1430)$.
Then the second-lowest scalar energy state $\mathrm{f}_{0}(980) / \mathrm{f}_{0}(1000)$ is extra, i.e. this state should be regarded as exotic.
(2) However, we can also follow Refs [8, 11], and assume that the basic $1^{3} \mathrm{P}_{0}$ multiplet is located relatively high on the mass scale above the other P-wave multiplets:

$$
\begin{equation*}
1^{3} \mathrm{P}_{0}: \mathrm{a}_{0}(1415), \mathrm{f}_{0}(1335), \mathrm{f}_{0}(1590) / \mathrm{f}_{0}(1505), \mathrm{K}_{0}(1430) \tag{63}
\end{equation*}
$$

In this case there are three extra mesons in the region of 1000 MeV .

$$
\begin{equation*}
\mathrm{a}_{0}(980), \mathrm{f}_{0}(980), \mathrm{f}_{0}(1000) \tag{64}
\end{equation*}
$$

In scheme (2) it is difficult to accept that all the resonances listed above are exotic: it would be desirable to remove the wide resonance $\mathrm{f}_{0}(1000)$ on the basis that the position of the resonance pole lies deep inside the complex plane and analytic continuation of the $\pi \pi$ amplitude to such a distant region is multivalued. However, we must take account of the circumstance that the $\pi \pi$-scattering phase $\delta_{\mathrm{S}}^{0}$ becomes 'resonant' twice in the range $800-1200 \mathrm{MeV}$, intersects the values $90^{\circ}$ and $270^{\circ}$, and in the Argand diagram the amplitude forms two loops when energy is increased. In scheme (2) such 'quasiresonant' behaviour of the amplitude near 1000 MeV can be attributed to a strong low-energy interaction between the pions. An awkward fact encountered in this interpretation is that at present there are no obvious sources of such a strong low-energy interaction. One should perhaps mention here that the past attempts to
account for bumps in the hadron cross sections have ignored the particle resonances. In the early fifties an extensive discussion took place about whether the $\Delta$ resonance should be regarded as a particle, and somewhat later there was a similar discussion of the bump in the $\pi \mathrm{N}$ cross section in the region of 1700 MeV . We now know that in all these cases the interpretation of such irregularities in the cross sections in the form of particle resonances was correct in respect of their quark - gluon structure.

In scheme (2) if $f_{0}(1000)$ is removed as the particle resonance, it is necessary to identify the nature of the $\mathrm{a}_{0}(980)$ and $\mathrm{f}_{0}(980)$ resonances as exotic mesons. Interpretation of these resonances in the form of the $\mathrm{K} \overline{\mathrm{K}}$ molecules is being very actively pursued $[13,66]$.

The structure of the poles responsible for $\mathrm{a}_{0}(980)$ has been investigated $[19,20]$. The results of these investigations indicate that the denominator of the amplitudes for the $K \bar{K}$ and $\pi \eta$ channels is as follows:

$$
\begin{align*}
& \left(M^{2}-s\right) \operatorname{det}|\hat{D}|=M^{2}-s-\mathrm{i} g_{\mathrm{K}}^{2} \rho_{\mathrm{K}}-\mathrm{i} g_{\pi \eta}^{2} \rho_{\pi \eta} \\
& M=1.012, \quad g_{\mathrm{K}}=1.52, \quad g_{\pi \eta}=1.87 \tag{65}
\end{align*}
$$

Here, $\rho_{K}$ and $\rho_{\pi \eta}$ are the phase volumes of the $K \bar{K}$ and $\pi \eta$ systems (all the dimensional quantities are in gigaelectronvolts). The zeros of expression (65) correspond to the poles of the amplitudes. Both poles are located near the $\mathrm{K} \overline{\mathrm{K}}$ threshold, on the second and third sheets:

$$
\begin{align*}
& a_{0}(980): M^{I I} \\
&=1014-\mathrm{i} 41 \mathrm{MeV}  \tag{66}\\
& M^{I I I}
\end{align*}=957-\mathrm{i} 106 \mathrm{MeV} .
$$

We can see that the situation relating to these poles is similar to that found for $\mathrm{f}_{0}(980)$ : they are located near the real axis and, together with the threshold singularity, they give rise to relatively narrow cusps in the cross sections. The narrowness of the visible width supports the interpretation of $\mathrm{a}_{0}(980)$ as the $\mathrm{K} \overline{\mathrm{K}}$ molecule. However, the large values of the $g_{\mathrm{K}}$ and $g_{\pi \eta}$, which are the constants of the coupling with the $K \bar{K}$ and $\pi \eta$ channels, do not agree with this interpretation. Another contradiction is that the probabilities of creation of $\mathrm{a}_{0}(980)$ in hadron reactions are not small, which is not natural in the case of the molecular structure of $\mathrm{a}_{0}(980)$.

Similar arguments against interpretation of $\mathrm{f}_{0}(980)$ as the $K \bar{K}$ molecule are used in the preceding section.

The $\mathrm{f}_{0}(980)$ and $\mathrm{a}_{0}(980)$ resonances are regarded in Ref. [10] as 'minions' (particles more compact than known particles) whose distinguishing property is their weak coupling to light quarks and, consequently, to ordinary hadrons. In fact, a detailed analysis shows that the constants representing the coupling of these two resonances to the $\pi \pi, \pi \eta$, and $K \overline{\mathrm{~K}}$ hadron channels are not small, which is in conflict with the interpretation of $\mathrm{f}_{0}(980)$ and $\mathrm{a}_{0}(980)$ as 'minions'.

Summarising the above discussion, we can say that adoption of scheme (2) raises difficult problems:
(a) according to this scheme, the ${ }^{3} \mathrm{P}_{0}$ nonet lies above other P-wave mesons, which requires revision of the existing quark models involving introduction of a new type of the interquark interactions;
(b) the scheme includes the $\mathrm{a}_{0}(980), \mathrm{f}_{0}(980)$, and $f_{0}(1000)$ group of states which does not fit the $q \bar{q}$ classification and does not agree with the versions of exotics accepted at present.

Scheme (1) is characterised by a normal distribution of the P -wave mesons considered on the basis of the currently discussed quark - antiquark interactions: the scalar meson nonet is the lowest. This scheme includes only one exotic state: $\mathrm{f}_{0}(980) / \mathrm{f}_{0}(1000)$. We can assume that this is the white Higgs meson of the effective Lagrangian described by expression (40) or it is a scalaron [57], which appears because of conservation of the colour symmetry in the soft interaction range.

In scheme (1), the second $\left(0,0^{++}\right)$meson with the dominant $s \bar{s}$ component is $f_{0}(1240)$. This resonance is observed in the $\mathrm{K}_{\mathrm{S}}^{0} \mathrm{~K}_{\mathrm{S}}^{0}$ channel [21]. Figure 18 shows the dependence of the square of the S-wave $\mathrm{K}_{\mathrm{S}}^{0} \mathrm{~K}_{\mathrm{S}}^{0}$ amplitude, which has a clear peak in the region of a resonance. However, it should be pointed out that this is the only observation: the $\mathrm{f}_{0}(1240)$ resonance has to be confirmed. At present, the Crystal Barrel collaboration has data on the $\mathrm{p} \overline{\mathrm{p}} \rightarrow \mathrm{K} \overline{\mathrm{K}} \pi$ annihilation at rest with the statistics reaching hundreds of thousands of events. The data are now being analysed [69]. We hope that in the near future the situation relating to the creation of $\mathrm{f}_{0}(1240)$ and other scalar resonances in the $K \bar{K}$ channel will become much clearer.


Figure 18. $\left|S_{0}\right|^{2}$ and $\left|\phi_{S}-\phi_{D}\right|$, obtained by fitting the data for the amplitude of the $\pi \pi \rightarrow \mathrm{K}_{\mathrm{S}}^{0} \mathrm{~K}_{\mathrm{S}}^{0}$ reaction with $I=0$. The continuous curves represent the results of fitting with $\mathrm{f}_{0}(1240)$, and the dashed curves those obtained ignoring $\mathrm{f}_{0}(1240)$.

## 10. Where is the $\mathbf{0}^{-+}$glueball?

It follows from the above discussion that there are many arguments in support of the fact that the glueballs $0^{++}(1505), 2^{++}(1710)$, and $1^{-+}(1910)$ are observed in the mass range $1500-2000 \mathrm{MeV}$. The sequence in which these glueballs appear is that predicted by the models dealing with pure gluon states. However, there is an observation which does not fit this 'idyllic pattern': in the $1500-2000 \mathrm{MeV}$ mass range there is no resonance which could be regarded as the $0^{-+}$glueball. Moreover, the


Figure 19. Spectra of $K^{+} K^{-} \pi^{0}$ [71] and $K_{S}^{0} K^{ \pm} \pi^{\mp}$ [70] formed by the $\mathrm{J} / \psi \rightarrow \gamma \mathrm{K} \overline{\mathrm{K}} \pi$ radiative decay.
spectra of the radiative decay of $\mathrm{J} / \psi$ have the $0^{-+}$ resonance at much lower masses, near 1440 MeV .

Let us now turn to the data on the $\mathrm{J} / \psi \rightarrow \gamma+\mathrm{K} \overline{\mathrm{K}} \pi$ reaction plotted in Fig. 19 [70, 71]. The $\mathrm{K} \overline{\mathrm{K}} \pi$ spectrum has a peak (with not quite the correct profile) in the $1400-$ 1500 MeV range, but there is no evidence of a resonance at $1600-2000 \mathrm{MeV}$. The bump at $1400-1500 \mathrm{MeV}$ corresponds primarily to the $\mathrm{K}^{*} \overline{\mathrm{~K}}$ (or $\overline{\mathrm{K}}^{*} \mathrm{~K}$ ) mode: another mode which may be responsible for this bump is $\mathrm{a}_{0}(980) \pi[72-74]$ (Fig. 20). The quantum numbers for both modes are $I, J^{P C}=0,0^{-+}$. The experimental data support a two-pole (two-resonance) structure of the bump (we shall discuss this in greater detail below): one of these resonances, designated $\eta^{\prime}(1440)$, is included in Table 1 as a member of the $2^{1} \mathrm{~S}_{0}$ nonet.

The Particle Data Group compilation [1] includes the $\eta(1760)$ resonance which could be a candidate for the $0^{-+}$ glueballs. However, our recent re-analysis of the experimental data on $\mathrm{J} / \psi \rightarrow \gamma+\pi^{+} \pi^{-} \pi^{+} \pi^{-}$[24] failed to confirm the results of the previous fit: the bump in the region of 1760 MeV (Fig. 21) has the quantum numbers $2^{++}$. This bump, designated in Ref. [24] as $2^{++}(1780)$, is most likely due to the contributions of $f_{2}(1710)$ and $f_{2}(1810)$. Stronger production of $\mathrm{f}_{2}(1710)$, observed in the $\mathrm{J} / \psi \rightarrow \gamma+\overline{\mathrm{K} K}$ radiative decay, and likely to be a good candidate for the $2^{++}$glueballs, is not surprising. However, significant production of the $f_{2}(1810)$ resonance, observed earlier by the GAMS group [12], should indicate an important


Figure 20. Mass distribution of $\pi^{+} \pi^{-} \eta$ in the $J / \psi \rightarrow \gamma \pi^{+} \pi^{-} \eta$ reaction before (a) and after (b) separation of $\mathrm{a}_{0}(980) \pi$ events. The peaks correspond to the $\mathrm{J} / \psi \rightarrow \gamma \eta(1295)$ and $\mathrm{J} / \psi \rightarrow \gamma \eta(1440)$ transitions.

No. of events


Figure 21. Spectrum of four pions in the $J / \psi \rightarrow \gamma \pi^{+} \pi^{-} \pi^{+} \pi^{-}$radiative decay.
admixture of the GG component to this resonance [in Table 1 the $f_{2}(1810)$ resonance is classified as a member of the $2^{3} \mathrm{P}_{2}$ multiplet]. Two other peaks in the $\pi^{+} \pi^{-} \pi^{+} \pi^{-}$ spectrum have the quantum numbers $0^{++}$; the parameters of all three peaks observed in the $\mathrm{J} / \psi \rightarrow \gamma+\pi^{+} \pi^{-} \pi^{+} \pi^{-}$ decay are as follows:

| $J^{P C}$ | Mass | Width | Decay mode |
| :--- | :--- | :--- | :--- |
| $0^{++}$ | 1505 | 148 | $\sigma \sigma(92 \%)+\rho \rho(8 \%)$ |
| $2^{++}$ | 1780 | 150 | $\sigma \sigma(30 \%)+\rho \rho(70 \%)$ |
| $0^{++}$ | 2104 | 200 | $\sigma \sigma(88 \%)+\rho \rho(12 \%)$ |

Here, $\sigma$ represents the combined contribution of $\pi \pi$ in the S wave characterised by $I=0$, i. e. $\mathrm{f}_{0}(1000)+\mathrm{f}_{0}(980)$.

It is thus clear that there is no experimental support for the existence of the $0^{-+}$glueball in the range $1550-1950 \mathrm{MeV}$ : it is in this mass range, in the lowest and exotic $1^{-+}$glueballs, that one would naturally expect the existence of such a bound GG state.

We are thus faced with the question of where is the $0^{-+}$ glueball? If the bump in the spectra of the $\mathrm{J} / \psi$ radiative decay in the range $1400-1500 \mathrm{MeV}$ includes the $0^{-+}$ glueball, why is its mass less than 1500 MeV , i.e. why is this glueball the lightest? The answers to these questions are most likely to be given by the hypothesis put forward by Gershtein, Likhoded, and Prokoshkin (GLP hypothesis) over ten years ago [75]: the lowest pseudoscalar $\eta$ mesons contain considerable admixtures of the gluon component, i. e. of the $0^{-+}$glueball component. Therefore, in accordance with the GLP hypothesis, admixtures of the $0^{-+}$glueball state are contained in $\eta(550)$ and $\eta^{\prime}(960)$. It is therefore natural to assume that such admixtures are found in the pseudoscalar mesons of radial excitations, i.e. in $\eta(1295), \eta^{\prime}(1440)$, etc. Then the $0^{-+}$'glueball', with a very large admixture of the $q \bar{q}$ component, may drop to the range of lower masses in the region of $1400-1500 \mathrm{MeV}$. The result is that what we call $\eta(1440)$ is in fact a two-pole structure: each pole corresponds to a state which represents a mixture of the $\mathrm{q} \overline{\mathrm{q}}$ and GG components and certain fractions of the GG component have 'escaped' also to $\eta(550), \eta^{\prime}(960)$, $\eta(1295)$, and other pseudoscalar $q \bar{q}$ states.

The experimental data on the radiative decay of $J / \psi$ confirm the GLP hypothesis of a significant admixture of the GG component in $\eta$ and $\eta^{\prime}$. Let us therefore compare the partial probabilities $\left(\Gamma_{i} / \Gamma\right)$ in the $\mathrm{J} / \psi \rightarrow \gamma+0^{-+}$ meson decays [1, 72]:

$$
\begin{array}{rlrl}
\mathrm{J} / \psi & \rightarrow \gamma \eta & & (0.86 \pm 0.08) \times 10^{-3} \\
& \rightarrow \gamma \eta^{\prime} & & (4.3 \pm 0.03) \times 10^{-3} \\
& \rightarrow \gamma \eta(1295) \rightarrow \gamma(\eta \pi \pi) & & (0.52 \pm 0.16) \times 10^{-3} \\
& \rightarrow \gamma \eta(1440) \rightarrow \gamma(\eta \pi \pi) & (1.4 \pm 0.4) \times 10^{-3} \\
& \rightarrow \gamma \eta(1440) \rightarrow \gamma(\mathrm{K} \overline{\mathrm{~K}} \pi) & & (0.91 \pm 0.18) \times 10^{-3}
\end{array}
$$

The radiative creation of $\eta$ and $\eta^{\prime}$ is of the same order as of $\eta(1440)$ and it is not suppressed compared with other radiative decays of $J / \psi$.

If a pole associated with the $0^{-+}$glueball is shifted to the mass range $1400-1500 \mathrm{MeV}$, then - as pointed out above - we can expect two poles in this range: the second pole should be associated with the first radial excitation of the $\eta^{\prime}$ meson (see Table 1 and Fig. 1). The experimental data are in agreement with the existence of two pseudoscalar resonances in the range $1400-1500 \mathrm{MeV}$ : the data on the $\mathrm{J} / \psi \rightarrow \gamma(\pi \pi \eta)$ decay [72-74] indicate the existence of a peak near 1400 MeV , whereas the $\mathrm{J} / \psi \rightarrow \mathrm{K} \overline{\mathrm{K}} \pi$ channel [70] is in good agreement with the hypothesis of creation of two resonances with the masses

1420 and 1490 MeV . The hadron creation of $\eta(1440)$ also suggests the existence of two resonances with the masses 1410 and 1490 MeV [76].

It is thus possible that the pole associated with the $0^{-+}$glueball has dropped on the mass scale because of strong mixing of the low-lying $\eta$ mesons with the $q \bar{q}$ components. As a result, there are large admixtures of the GG component in $\eta(550)$ and $\eta^{\prime}(960)$. Then, in the region of $1400-1500 \mathrm{MeV}$ there are two $0^{-+}$resonances; one of them is a memento or remnant of the $0^{+-}$glueball and the other is the radial excitation of the $\eta^{\prime}$ meson $\left(2^{1} S_{0} q \bar{q}\right)$ mixed with the GG component.

## 11. Process $J / \Psi \rightarrow \gamma+G G$ as a measure of the gluon component admixture in mesons

The rules of the $1 / N$ expansion say that admixtures of the gluon components may not be dominant, but they are significant in the $q \bar{q}$ mesons. Moreover, the gluon states themselves include large admixtures of the quark-antiquark components because of the enhancement due to the large number of flavours (represented by the factor $N_{\mathrm{f}}$ ). The J/ $\psi \rightarrow \gamma+$ GG process followed by the GG $\rightarrow$ mesons transition (Fig. 22) provides an opportunity for direct determination of the distribution of the GG component between the hadron states.



Figure 22. One-meson radiative decay of $J / \psi$ (a) and multimeson decays (b, c).

Figs 21 and 23 give the spectra of $\pi^{+} \pi^{-} \pi^{+} \pi^{-}$and $K \bar{K}$ formed by the $\mathrm{J} / \psi \rightarrow \gamma+\pi^{+} \pi^{-} \pi^{+} \pi^{-}$and $\mathrm{J} / \psi \rightarrow \gamma+\mathrm{K} \overline{\mathrm{K}}$ decay processes. In both cases there is enhanced creation of the resonances in the region $1500-2000 \mathrm{MeV}$ (this process is illustrated in Fig. 22a). The background formation of two kaons by the process $\mathrm{J} / \psi \rightarrow \gamma+\mathrm{K} \overline{\mathrm{K}}$ (Fig. 22b) is the strongest in the range $2000-2500 \mathrm{MeV}$. According to the results of an analysis given in Ref. [24], the nonresonant creation of mesons in the reaction $\mathrm{J} / \psi \rightarrow \gamma+\pi^{+} \pi^{-} \pi^{+} \pi^{-}$is not very great for $M_{\pi \pi \pi \pi}<1900 \mathrm{MeV}$ and begins to play a significant role only for $M_{\pi \pi \pi \pi}>2000 \mathrm{MeV}$. In the spectrum of $\mathrm{K} \overline{\mathrm{K}} \pi$ formed by the $\mathrm{J} / \psi \rightarrow \gamma+\mathrm{K} \overline{\mathrm{K}} \pi$ decay (Fig. 19) there is strong production of $\eta(1440)$ in the range of small masses and then the spectrum shows a smooth rise without any visible structures (contribution of the processes in Figs 22 b and 22c), reaching a maximum in the range $2300-2900 \mathrm{MeV}$. It follows that the GG component is concentrated in the one-meson states, mainly in the resonances at $1500-2000 \mathrm{MeV}$, in two-meson states in the range of effective masses in excess of 2000 MeV , and in three-meson states corresponding to effective masses greater than or of the order of 2300 MeV .


Figure 23. Spectra of $K \bar{K}$ mesons formed by the $J / \psi \rightarrow \gamma K \bar{K}$ decay [71]


Figure 24. Relative probabilities $\mathrm{BR}=\Gamma_{\mathrm{x}} / \Gamma$ of creation of X mesons by $\mathrm{J} / \psi \rightarrow \gamma+\mathrm{X}$ radiative decay. Here, BR is the branching ratio.

Examination of the separate probabilities of meson production by the $\mathrm{J} / \psi \rightarrow \gamma+$ meson decays is illuminating
(Fig. 24). This figure shows clearly that the GG gluon component is distributed between a large number of mesons. Therefore, the problem of finding successor resonances of the glueballs remains basically the problem of meson systematics: it involves identification of the states that do not fit the quark classification.

## 12. Conclusions

Recent experimental investigations have begun to draw outlines of the physics of glueballs.

There are two states, scalar $\mathrm{f}_{0}(1505)$ and tensor $f_{2}(1710)$, which clearly do not fit the $q \bar{q}$ systematics. These are the states which, somewhat arbitrarily, can be called the lowest glueballs. This conclusion is arbitrary because the glueballs can, in accordance with the rules of the $1 / N$ expansion, mix readily with the adjacent $\mathrm{q} \overline{\mathrm{q}}$ states. Therefore, the $\mathrm{f}_{0}(1505)$ and $\mathrm{f}_{2}(1710)$ resonances should represent the 'relics' of the GG glueballs after their mixing with the $q \bar{q}$ components. Conversely, pure glueball states should be a superposition of real resonant states: they can be represented conveniently by the Fock columns. Therefore, the lowest pure scalar glueball (S-wave GG state) should be a superposition of $f_{0}(1505)$ and the adjacent scalar resonances:

$$
0^{++}(\mathrm{GG})_{\mathrm{S}}=\left[\begin{array}{c}
\mathrm{f}_{0}(1505)  \tag{68}\\
\mathrm{f}_{0}(1590) \\
\cdots
\end{array}\right]
$$

Similarly, the lowest pure tensor glueball state is a superposition of $f_{2}(1710)$ and of the nearest neighbours:

$$
2^{++}(\mathrm{GG})_{\mathrm{S}}=\left[\begin{array}{c}
\mathrm{f}_{2}(1710)  \tag{69}\\
\mathrm{f}_{2}(1810) \\
\cdots
\end{array}\right]
$$

The Fock columns, represented by expressions (68) and (69), include only the nearest neighbours of the $f_{0}(1505)$ and $f_{2}(1710)$ resonances. However, the GG components become mixed not with just these states. Much work will be required before a full picture of mixing of the $0^{++}(\mathrm{GG})_{\mathrm{S}}$ and $2^{++}(\mathrm{GG})_{\mathrm{S}}$ states with the $\mathrm{q} \overline{\mathrm{q}}$ states becomes available.

The radiative decays of the $J / \psi$ meson definitely indicate that a pseudoscalar glueball is distributed between the low-lying $\eta$ states:

$$
0^{-+}(\mathrm{GG})_{\mathrm{P}}=\left[\begin{array}{c}
\eta(1450)  \tag{70}\\
\eta^{\prime}(1420) \\
\eta(1280) \\
\eta^{\prime}(958) \\
\eta(550) \\
\cdots
\end{array}\right] .
$$

It is possible that mixing is responsible for the downward shift of the $0^{-+}$glueball state on the mass scale.

Strong mixing of the GG and $\mathrm{q} \overline{\mathrm{q}}$ components is the main reason which makes it difficult to identify unambiguously the glueball states. Therefore, studies of glueballs with exotic quantum numbers, i.e. of glueballs without the $q \bar{q}$ component, will be very important. There are grounds for assuming that there is such an exotic resonance and that it is a peak in the $\eta \eta^{\prime}$ system at 1910 MeV :

$$
\begin{equation*}
1^{-+}(\mathrm{GG})_{\mathrm{P}}=\mathrm{X}(1910) \tag{71}
\end{equation*}
$$

However, no direct measurements of the quantum numbers of this resonance have yet been made. Confirmation of the
exotic quantum numbers of this resonance are exceptionally important in the task of constructing a scheme of pure glueball states.

There is an extra $0^{++}$state in the region of 1000 MeV and it does not fit either the $q \bar{q}$ or the GG classification. It may represent a white scalar Higgs particle (scalaron) which appears because of conservation of the colour symmetry over moderately long distances. The $f_{0}(980)$ and $f_{0}(1000)$ resonances observed in the region 1000 MeV are superpositions of the extra $0^{++}$state (scalaron) and the ${ }^{3} \mathrm{P}_{0} q \bar{q}$ state. The two states are coupled strongly to the $\pi \pi$ channel (i.e. to nonstrange quarks) and this is the reason for their strong mixing in the observed resonances. The scalaron is, in its turn, a superposition of the $f_{0}(980)$ and $f_{0}(1000)$ resonances:

$$
\mathrm{S}\left(0^{++}\right)=\left[\begin{array}{c}
\mathrm{f}_{0}(980)  \tag{72}\\
\mathrm{f}_{0}(1000)
\end{array}\right]
$$

The small width of $f_{0}(980)$ is the result of mixing of the $\mathrm{S}\left(0^{++}\right)$and ${ }^{3} \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ states and it is not due to the presence of the $\mathrm{K} \overline{\mathrm{K}}$ threshold: even in the case of complete 'exclusion' of this threshold singularity there is little change in the position, on the second sheet, of the pole corresponding to $\mathrm{f}_{0}(980)$.

The particle known as the $\sigma$ meson, which plays an important role in low-energy physics of the hadron interactions and also in nuclear physics, is a combined contribution of a scalaron and of the ${ }^{3} \mathrm{P}_{0} \mathrm{q} \overline{\mathrm{q}}$ state.

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