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Can the asymptotic freedom of the gravitational interactions violate the energy dominance of classical cosmology?

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Abstract. An arbitrary choice of the functional dependence describing weakening of the gravitational interactions as a result of an increase in the density of matter during collapse makes it possible to show that the asymptotic freedom of gravitational interactions violates the energy dominance.

In my paper on ''Possible existence of asymptotic freedom of gravitational interactions in nature'' published recently in this journal [1] I pointed out that, within the framework of the model of isotropic universes, the collapse of these universes stops at the Planck length. This result was obtained neglecting anisotropic perturbations, which can naturally occur in the course of collapse. This troubled me greatly. Further investigations revealed that none of the models I proposed for isotropic universes violate the energy dominance of classical cosmology. In other words, the collapse stoppage is predicted only in those models of universes which are in fact not realised. However, making an arbitrary choice of the function describing weakening of the gravitational interactions, which accompanies an increase in the density of matter, it is possible to propose even simpler functions and in this case the energy dominance is violated in the process of collapse at high matter densities.

This paper, which answers the question raised in Ref. [1], is therefore a natural continuation of my earlier paper.

It is appropriate to mention here a paper by R Penrose [2], published a long time ago (1965), in which he put forward four scenarios in which the problem of the singularity can be solved.

Unfortunately, I became aware of Penrose's paper only recently after the publication of my paper in *Uspekhi Fizicheskikh Nauk* [1].

These four scenarios are identified by the letters (a), (b), (c), and (d) in Penrose's paper:

(a) a negative local energy exists;
(b) Einstein's equations are violated;
(c) the space–time manifold is incomplete;
(d) the concept of space–time loses its meaning at very high curvatures, possibly because of quantum phenomena.

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All these possible scenarios are related in one way or another to the idea of the asymptotic freedom of the gravitational interactions discussed here. I began the discussion of the asymptotic freedom with a paper on ''Limiting density of matter as a universal law of nature'' [3]†.

The homogeneous isotropic dust-filled closed universe can be described by an equation whose simplest form (used in later papers) is

$$\left(\frac{\dot{R}}{c}\right)^2 + 1 = \frac{8\pi R^2\kappa_0}{3c^2}\left[\rho\left(1-\frac{\rho^2}{\rho_0^2}\right) + \Lambda'\left(\frac{\rho}{\rho_0}\right)^2\right], \quad (1)$$

where ρ_0 is the maximum density of matter which appears in the course of collapse of a universe;

$$\Lambda' = 2\rho_0\,, \quad \Lambda'\,\frac{8\pi\kappa_0}{3c^2} = \Lambda$$

(Λ is the de Sitter universe term), and $1-(\rho^2/\rho_0^2)$ is some function $\psi(\rho)$ which represents weakening of the gravitational interactions as the density of matter in a universe increases in the course of its collapse.

This equation has been used to illustrate the collapse stoppage at a selected value

$$\rho_0 \approx \frac{c^5}{h\kappa^2} \sim 10^{94} \ \ \mathrm{g\ cm^{-3}} \quad (2)$$

at a distance from a classical singularity which lies in the range of the Planck length for a very weak dependence on the total bare mass of the closed universe. This equation is not derived by variation of any action functional. Moreover, the Λ term is selected ad hoc. An attempt to modify Einstein's theory of gravitation is made in Ref. [5] by replacement of the gravitational constant κ_0 in the action function S with some function

$$\kappa = \kappa_0\psi(\varepsilon), \quad (3)$$

where ε is the energy density. No constraints are imposed on the ψ function, except that it should decrease as the energy density increases in the course of collapse of a universe. Unexpectedly, such variation of the modified action functional yields Einstein's modified equations with an additional de Sitter term of the type $\Lambda\delta_k^i$, where $\Lambda = -\varepsilon\,\mathrm{d}\psi/\mathrm{d}\varepsilon$. This equation is obtained for

$$T_k^i = (\varepsilon + p)u^i u_k - p\delta_k^i\,. \quad (4)$$

† The same idea was put forward somewhat later by Rosen [4].

In the simple case of a dust-filled universe with $\psi = 1 - (\rho^2/3\rho_0^2)$ the equation obtained resembles Eqn (1):

$$\left(\frac{\dot{R}}{c}\right)^2 + 1 = \frac{8\pi R^2 \kappa_0}{c^2} \left[\rho\left(1 - \frac{\rho^2}{\rho_0^2}\right) + \Lambda' \left(\frac{\rho}{\rho_0}\right)^3\right], \qquad (5)$$

where $\Lambda' = 2\rho_0/3$.

In the range $\rho \ll \rho_0$ Eqn (5) describes a Friedmann universe which in the course of collapse is transformed into a de Sitter universe. In other words, the early universe, at $\rho \approx \rho_0$ includes, in the course of its expansion an inflationary de Sitter phase which has been the subject under discussion in the course of the last few years. However, the ideas on the evolution of the Universe in this form could have been proposed theoretically back in the time of Friedmann (after 1992) if the idea of a nonconstant (in our sense) value of κ_0 had been put forward.

The writing down of Eqn (5), where the Λ term appears, raises the question whether, in the course of collapse, there may be a reversal of the sign on the right of Eqn (5), which would imply that in the course of collapse at high densities attraction changes to repulsion or, equivalently, a negative energy appears on the right of the equation (as suggested by Penrose).

Penrose proposed that (a) a negative local energy exists. The expression on the right-hand side of Eqn (5) can be written as follows:

$$\left[1 - \frac{\rho^2}{3\rho_0^2}\right]. \qquad (6)$$

The bracket we are talking about can reverse its sign only for $\rho^2 > 3\rho_0^2$, which is forbidden by the limiting value of ρ_0. This means that the stoppage of collapse in a universe described by Eqn (5) is entirely due to the neglect of anisotropic perturbations, and for this ψ function the asymptotic freedom does not violate the energy dominance in the course of collapse. This applies also to Eqn (1), which was derived ad hoc. The question is whether an arbitrary choice of the function $\psi(\rho)$ can identify such functions of $\psi(\rho)$ which in the course of collapse would result in violation of the energy dominance at high densities. It can easily be shown that such functions do indeed exist, for example,

$$\psi(\rho) = 1 - \frac{\alpha\rho}{\rho_0}, \qquad (7)$$

where $\alpha > 1$. The bracket on the right-hand side of the modified Einstein equation is of the form

$$\left[\rho\left(1 - \frac{2\alpha\rho}{\rho_0}\right) + \alpha\left(\frac{\rho^2}{\rho_0^2}\right)\rho_0\right] = \left[\rho\left(1 - \frac{\alpha\rho}{\rho_0}\right)\right]. \qquad (8)$$

In the course of collapse at values of ρ smaller than ρ_0, but larger than ρ_0/α, the bracket (8) becomes negative. It remains negative also for $\rho = \rho_0$, if $\alpha > 1$. In this case we cannot identify the numerical value of α necessary, for example, to 'quench' the Kasner perturbations by repulsion. This question can be answered by direct calculations. A self- consistent future theory of gravitation, if indeed it includes, according to my understanding,

(1) the asymptotic freedom,

(2) the limiting density law,

should include also a specific form of the function ψ.

It would be a miracle if that function were identical with the ad hoc function (7), although it is said that nature prefers simplicity. A function ψ simpler than that given by Eqn (7) would be difficult to imagine. It is very probable that violation of the energy dominance can also be predicted by modifying the left-hand side of the Einstein equation, in accordance with what obviously Penrose has in mind by his scenario (b). There are grounds for assuming that in this case some transformations make it possible to reduce this form of violation of the Einstein equations to that form of violation in which the right-hand side of the Einstein equation is modified. This possibility is discussed in Ref. [6] and very briefly also in Ref. [1].

The cosmological problem discussed here so far is within the framework of classical (nonquantum) theory. If we consider the collapse of a universe which after the stoppage at some distance $l_{\min}$ from a classical singularity begins to expand again, then distances l smaller than $l_{\min}$ do not cause any problems.

However, the situation is more complex in the case of the collapse of black holes. In this case the matter which escapes within the Schwarzschild sphere can move only towards the singularity. However, if the limiting density law and the associated finite values of all the curvatures exist, the question is how do subsequent events proceed in a black hole? The answer is given in Refs [7, 8], according to which a black hole is a source of new universes which appear in their own space in a time which represents the absolute future relative to the time of the appearance of the black hole. However, according to my earlier papers, new universes should appear in their own spaces at a distance $l_{\min}$ from the classical singularity. The question is, what does space represent in a circle of radius $l_{\min}$ around the classical singular points? It will be assumed that this question is answered in Ref. [9] which deals with a two-dimensional theory of strings, according to which a region with Euclidean rather than Lorentzian metric appears near a classical singularity. Here, special physics may apply, the physics of the ultramicroworld discussed in Ref. [1]. These last comments essentially raise the possibilities labelled (c) and (d) by Penrose. In the abstract of Ref. [1] I expressed the hope for a contribution from string theory. Specifically, this should be supplemented by the question: can the introduction of a dilaton field, which plays an important role in the theory of strings, be regarded as an attempt to replace the gravitational constant with some function of this field?

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