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How Gamow calculated the temperature of the background radiation or a few words about the fine art of theoretical physics

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Contents

Abstract. In a paper published in 1953, i.e. more than a decade before the observational discovery of the cosmic microwave background radiation, George Gamow predicted theoretically the temperature of this radiation. He estimated it to be 7 K, which is very close to the subsequently measured value of about 3 K. Gamow found the present temperature of the background radiation on the basis of general formulas of cosmological dynamics. This prediction was in no way related to primordial nucleosynthesis. This circumstance has and is still causing misunderstanding in those cases in which the authors have raised doubts about Gamow's results, although an actual error has never been demonstrated. A detailed analysis makes it possible to understand how Gamow's calculation is possible. The problem lies in the fact that Gamow makes a certain additional implicit assumption which allows him to dispense with information on nucleosynthesis. This assumption is discussed in the context of the state of cosmology in the period from the fifties to the seventies, and of the current status of this branch of science.

1. Introduction

About 100 years ago the possibility of the thermal death of the Universe was actively discussed by physicists. Later, 50 years ago, the thermal birth of the Universe had begun to attract attention: in the midforties George Gamow proposed the idea of a 'hot' beginning of the Universe. In this way thermodynamics and, together with it, nuclear physics have entered cosmology, because right from the beginning it has been assumed that nuclear reactions, which have determined the observed chemical composition of cosmological matter, should occur in the hot and dense early Universe. Gamow's theory is frequently called 'Big Bang' cosmology. For Gamow, the science of evolution of the Universe, founded mainly by Friedmann † — who was Gamow's teacher at Leningrad University — consisted only of the dynamics and geometry of the Universe.

One of the results of Big Bang cosmology is the theoretical prediction of the relic radiation, which is the name suggested by I S Shklovskii, or a cosmic microwave background, which is the name usually employed in the West. This radiation represents a thermodynamic-equilibrium distribution of photons which fills uniformly the whole observable Universe. It was discovered by the direct observations of American radioastronomers A A Penzias and R W Wilson in 1965 (Nobel Prize 1978), but was predicted long before that by Gamow (who perhaps should be called a Russian – American theoretician?). In modern physics and cosmology the background radiation has been the object of investigation and also a means for the study of the large-scale structure of the Universe and its evolution (see, for example, the books of Peebles [1], Zel'dovich and Novikov [2], and Weinberg [3]). The notes below deal with this background or relic radiation and how Gamow calculated its present temperature. More specifically, we shall consider one fairly short paper [4] written by Gamow on the subject in 1953.

The paper in question is entitled "Expanding universe and the origin of galaxies" and was published in *Kongelige Danske Videnskabernes Selskab, Matematik-Fysiske*

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† Also spelt Friedman or, in Russian, Fridman.

Skrifter. Gamow was a member of the Danish Academy of Sciences and his election was proposed by Bohr. We do not know why he preferred to publish in this Danish journal and not, for example, in *Physical Review* to which he normally contributed. It is unlikely that there would have been any diffi-culties in publication in the USA. True, in the case of a different paper which was also written in 1953, he did indeed have a problem: none of the American journals wanted to publish it and he published it in the following year (1954) in the same Danish journal; however, this was not a paper on physics but on genetics (by the way, Gamow regarded this genetics paper as his most important contribution to science; see, for example, the booklet of Frenkel' and Chernin [5]).

There are at least three reasons why it is worth recalling the cosmological paper of 1953 in connection with the ninetieth anniversary (in 1994) of Gamow's birth. First, this is a very simple paper. Second, there is still some mystery about this paper, which is capable of surprising the reader and placing him at a loss (see Section 3). The third and final reason is that undoubtedly the paper is illuminating both in the historical and particularly in the methodological sense. Moreover, Gamow himself liked it very much.

Very briefly, in his paper Gamow took two numbers — the age of the Universe and the average density of matter in the Universe — and found a third number, the background radiation temperature.

It is well known that estimates of quite important cosmological parameters follow from the background radiation temperature: the specific entropy of the Universe (which is sometimes used in discussions of the thermal death), the charge asymmetry of the Universe, the concen-trations of neutrino and other background (relic) particles, etc. [1 – 3].

However, the question is: can this third quantity (and then all its consequences) be found by combination of the two numbers with which Gamow started? If we look up cosmological monographs [1 – 3], we can see that the radiation temperature is found by calculations relating to the primordial nucleosynthesis, which is the process of nuclear conversion leading to the appearance of nuclei heavier than the proton throughout the Universe (see Section 2). It is then necessary to know at least one more number, which is the comological abundance of helium.

The mystery of Gamow's work and its paradox is how could he get his result using just two numbers?

Gamow's problem is presented to the interested reader below in Section 5; this is preceded by Section 4 where the necessary preliminary information, given also in Gamow's paper, is provided. The reader who solves the paradox in Section 5 can omit Section 6, where 'guiding ideas' are given, and he can also bypass Section 7, where the 'answer' is provided, and go direct to the concluding comments in Section 8.

Gamow speaks not only of the background radiation in his paper [4], but also of the gravitational instability of the hot Universe. This is also a very interesting but separate topic, which is outside the scope of my note.

2. Big Bang

The history of the prediction and discovery of the background radiation has now its own extensive literature, with perhaps the best account in the famous book of Steven Weinberg *The First Three Minutes* [6], which has become widely known and has been translated into many languages, including Russian (the translation was edited by Ya B Zel'dovich). We shall not recount the history again and mention just one episode.

In the first report of Penzias and Wilson [7] and in the paper by Dicke and his Princeton colleagues which accompanied the former (and which gave the correct cosmological interpretation of the discovery of the background radiation) [8], there is surprisingly no mention whatever of the Big Bang and of Gamow's work. Later, things fell into place but not immediately and not without struggle. For example, there is a story that in 1967 (a year before his death) Gamow chaired one of the sessions at the Fourth Texas Symposium on Relativistic Astrophysics. This session was specifically devoted to the background radiation and, to the accompaniment of laughter and applause, Gamow said: ''If I have lost a nickel and somebody took it away, how can I prove that it is my nickel? However, I have lost my nickel exactly where it has been found later'' [9].

When in 1965 Penzias sent a draft of his new paper, written after Ref. [7], on the background radiation, Gamow answered by a brief note in which he pointed out that the subject was first discussed not by Dicke (as it would appear to be stated in the preprint), but in Gamow's own paper in 1946 [10], and that the temperature of the background radiation at the present epoch was first estimated by his students R A Alpher and R Herman in 1948 [11]. Their value was 5 K and then in 1953 Gamow himself obtained 7 K [4]. This note was published by Penzias [9].

The temperature of the background radiation has now been measured exceptionally accurately. In 1965 Penzias and Wilson reported 3.1 ± 1 K, but the latest (summer 1993) measurements carried out on board the COBE American satellite was 2.726 ± 0.001 K [12]. In cosmology, where the most important parameters are frequently known to the nearest order of magnitude or still are not yet fully determined observationally, this is an unusual and happy experimental situation.

However, equally surprising is the success of the theoreticians — Alpher, Herman, and Gamow — who were able to calculate the background radiation temperature many years before the experimental measurements of this quantity.

The fact that Gamow and his students Alpher and Herman obtained an estimate of 5 K or 7 K and not 3 K, is not in conflict with the direct measurements and cannot be regarded as a discrepancy: just the opposite, the agreement is exceptionally good. After all, the calculations have been carried out on the basis of numerical parameters known very unreliably to the cosmologists in the forties and fifties: these parameters have not been known even to within one order of magnitude.

The controversy about the priority on the subject of the Big Bang and the background radiation is possibly of interest in the history of science and even then not so much for itself but rather as an indication of the attitude of the participants of the controversy, and especially that of

Gamow as a 'victor' (universally regarded as such, although only posthumously: Weinberg's best seller *The First Three Minutes* published in the USA in 1977 is usually taken as the last word on the subject), to the papers which I shall consider below. It is particularly relevant that Gamow regarded his paper of 1953 and the work of his students [11] as an argument in the priority dispute.

Alpher and Herman [11] developed the general concept of Gamow on the 'hot' beginning of the Universe and of the nuclear reactions in the early Universe, and found the temperature of 5 K on the basis of the cosmological primordial nucleosynthesis, as is now stated in textbooks (usually over two to three pages). Alpher and Herman carried out a very difficult, time-consuming, and pioneering investigation in a brilliant manner. Their calculations have since been repeated and sometimes corrected by many theoreticians; however, they have always been confirmed. At present, the problem is tackled on the basis of the latest ideas in the physics of elementary particles. This is done by numerical simulation and computer calculations are continuously revealing new and nontrivial details and variants of the kinetics of nuclear transformations in the early Universe, first studied by Gamow and his students. Apparently, for many years there will be something to calculate in the Big Bang theory.

3. 'Vicious circle' and 'baroque graphics'

Gamow's paper [4] was very different: he used an exceptionally simple method with three steps to obtain almost the same quantitative result of 7 K. Gamow often said that he is not very good with sums and he could not make two additions in succession without an error. Long and cumbersome calculations, needed in dealing with cosmological nucleosynthesis, were for the young and he produced a result which looked like sleight-of-hand. Like a magician, he produced his results — one, two, three! — out of a hat, greatly surprising his collaborators at the time (who many years later were still bewildered by this [13]), as well as later researchers, professional cosmologists, and historians of science (as discussed later).

In a recent (1990) paper, Alpher and Herman [13] write about the mysterious ways of their teacher as follows: "... in a Danish journal he estimated a 7 K background temperature by means of a strange linear extrapolation of matter and radiation densities ..." and later they say: "Again three years later he persisted with yet another arcane calculation, obtained 6 K." (They mean here Gamow's review in Ref. [14].)

It seems that Alpher and Herman do not like very much what was printed 'in a Danish journal'. I confirmed this impression by direct exchange of letters. In a letter to me (dated 25 September 1991) Alpher and Herman expressed their respect and admiration for Gamow, but in their opinion Gamow's paper did not add anything positive to their results "but, rather confused the issue".

Let me disagree with this opinion, particularly because neither in their review [13] nor in this personal letter did Alpher and Herman point out any error in Gamow's calculations.

Attempts to find this error were made by compilers and commentators in a collection of classical papers on cosmology published in 1986 by Columbia University [15]. This is what they say: "Gamow used a baroque graphical extrapolation method ... If one does the calculation straightforwardly one will find that the answer depends on knowing the present radiation temperature, which renders the method circular. Somehow Gamow managed to set a present proton temperature of 7 K out of this argument."

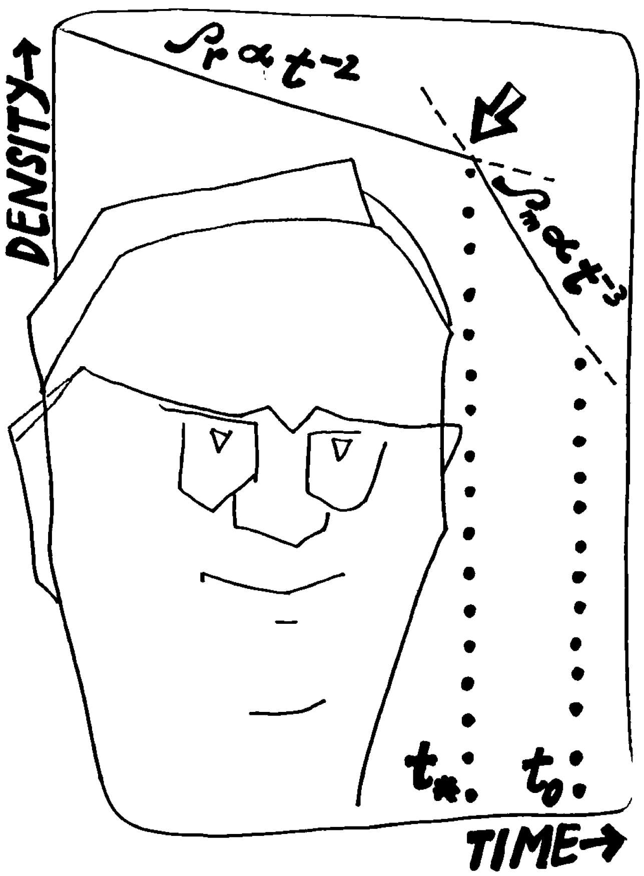

Figure 1. Gamow (drawing based on a photograph taken in the sixties) and the dependence of the density on time in an expanding hot Universe: the two asymptotes are matched at the demarcation point t_*. Gamow called this diagram 'divine creation curve'.

The reader will probably agree that such words as strange, baroque, arcane are unusual in scientific language. It has frequently happened that authors of scientific texts have lifted their heads in a helpless gesture: this has been done somehow, but we do not understand how.

Weinberg also discusses the paper of interest to us. He has published not only the bestseller *The First Three Minutes* [6], but also a solid monograph on cosmology [3], apart from his work rewarded with the Nobel Prize in 1979. In his bestseller [6] he considers the letter, mentioned above, from Penzias to Gamow and of the answering note in which Gamow speaks of the theoretical prediction of radiation 'with an approximately correct temperature of 7 K'. In this connection, Weinberg makes the following critical comment: "However, a close look at this 1953 paper shows that Gamow's prediction was based on mathematically fallacious arguments having to do with the age of the Universe and not on his own theory of cosmic nucleosynthesis."

This expression used by Weinberg should be rather understood that Gamow's reasoning seems logically incorrect to Weinberg and that the letter's objection is not so much to the specific number, which is the age of the Universe used by Gamow and adopted in the cosmology of the fifties, as to the way in which Gamow is reasoning (this was mentioned by L B Okun'). The extent to which the actual age of the Universe is important to the topic in hand will be considered in Section 8.

4. What is 'given'

Gamow's work is based on two clearly formulated initial assumptions. The first is the cosmological model used in the calculations. Gamow adopts Friedmann's open model and assumes that the present epoch corresponds to the asymptotic inertial stage of expansion of the Universe. The distances between the bodies in space and the radius of curvature of the Universere as a whole then increase simply proportionally to time and the relative velocities of all the bodies remain constant:

$$R \propto t\,, \; v \propto \mathrm{const}\,. \tag{1}$$

This is obviously the simplest variant of cosmological dynamics.

Gamow uses for his purpose the expression for the density of matter in this model. Matter is assumed to be 'ordinary', i.e. nonrelativistic. The relevant formula is obtained as a result of division of the mass by the volume, since in Friedmann's world the density is uniform:

$$\rho_{\mathrm{m}} = \frac{M_{\mathrm{m}}}{(4\pi/3)R^3} \propto t^{-3}\,. \tag{2}$$

Here, M_{m} is the mass of matter in a spherical volume of radius R (which, in particular, can be the radius of curvature).

The above two formulas describe the asymptotic behaviour, when time tends to infinity, of the exact Friedmann solution for the open model, which can be written in the following parametric form:

$$\begin{aligned} R &= R_{\mathrm{m}}(\cosh\eta - 1)\,, \\ t &= \frac{R_{\mathrm{m}}}{c}\,(\sinh\eta - \eta)\,, \\ \rho_{\mathrm{m}} &= \frac{6R_{\mathrm{m}}}{\kappa R^3}\,, \end{aligned} \tag{3}$$

where η is the parameter of the solution; c is the velocity of light in vacuum; $\kappa = 8\pi G/c^2$ is the Einstein gravitational constant; G is the Newtonian gravitational constant; $R_{\mathrm{m}} = GM_{\mathrm{m}}/c^2$ is an arbitrary constant of the solution, related to the mass of matter M_{m} inside a given volume. The asymptotic expressions given by Eqns (1) and (2) correspond to the limit $\eta \to \infty$ in Eqn (3).

Gamow assumed the following numerical values of the present age of the world and the present density of matter:

$$\begin{aligned} t_0 &= 10^{17}\ \mathrm{s} \approx 3 \times 10^9\ \text{years}\,, \\ \rho_{\mathrm{m}}(t_0) &= 10^{-30}\ \mathrm{g\,cm^{-3}}\,. \end{aligned} \tag{4}$$

These values, like the general concept of Friedmann's open Universe, had been generally accepted in the cosmology of the fifties. The values given by Eqn (4) led Gamow to the final formula for the density of matter in an expanding universe:

$$\rho_{\mathrm{m}} = \rho_{\mathrm{m}}(t_0)\left(\frac{t_0}{t}\right)^3 = 10^{21} t^{-3}\ \mathrm{g\ cm^{-3}}\,. \tag{5}$$

The second initial assumption adopted by Gamow in the paper under discussion is related not to the dynamics but to the thermodynamics of the Universe. This had not been (up to 1965) in any sense generally accepted: it was the idea of a 'hot' beginning of the Universe. Gamow assumed specifically that the temperature of matter in the Universe throughout all the epochs of history was different from zero and at the very beginning of the expansion it could be very high. A thermodynamic equilibrium existed at the time and, therefore, together with matter the Universe also contained blackbody radiation with the same temperature. In the course of cosmological expansion the radiation cooled, but did not disappear and consequently was conserved in the Universe right down to our epoch. This was the theoretical prediction of the relic or fossil (background) radiation, which Gamow made first in a paper in 1946.

At a given temperature T the thermodynamic-equilibrium radiation has the energy density

$$\varepsilon_{\mathrm{r}} = aT^4\,, \tag{6}$$

where a is the Stefan–Boltzmann constant.

The energy density corresponds, in accordance with the formula $E_0 = mc^2$, to the mass density of the radiation†

$$\rho_{\mathrm{r}} = \frac{\varepsilon_{\mathrm{r}}}{c^2}\,. \tag{7}$$

In the course of adiabatic expansion the radiation temperature falls in accordance with the law

$$T \propto R^{-1}\,, \tag{8}$$

which corresponds to the adiabatic exponent $\gamma = \frac{4}{3}$. For this reason the mass density of the radiation varies in an expanding universe as

$$\rho_{\mathrm{r}} \propto T^4 \propto R^{-4}\,. \tag{9}$$

If we compare Eqns (2) and (9), we can see that the relationship between the density of matter and the density of radiation changes in the course of cosmological expansion as follows:

$$\frac{\rho_{\mathrm{r}}}{\rho_{\mathrm{m}}} \propto R^{-1}\,. \tag{10}$$

In the early Universe the radiation has a higher density than that of matter:

$$\frac{\rho_{\mathrm{r}}}{\rho_{\mathrm{m}}} \to \infty \quad \text{for} \quad R \to 0\,, \;\; t \to 0\,. \tag{11}$$

The dynamics of the expansion during the early epochs of predominance of the radiation is described by Gamow with the aid of the asymptotic relationship

$$R \propto t^{-1/2}\,, \tag{12}$$

which follows from the exact solution for an open universe filled with radiation [it is clear that in the limit of Eqn (11) the presence of matter in the Universe can be ignored if we are dealing with the dynamic problem]. This, like the

† It should be explained that the energy density and the mass density of the radiation are defined here in a reference system in which the gas of photons has no anisotropy and no general translational motion.

solution for matter of Eqn (3), is usually written in the parametric form:

$$R = R_{\mathrm{r}} \sinh \eta \,, \quad t = \frac{R_{\mathrm{r}}}{c} (\cosh \eta - 1) \,, \quad \rho_{\mathrm{r}} = \frac{3R_{\mathrm{r}}^2}{\kappa R^4} \,, \tag{13}$$

where R_{r} is an arbitrary constant which plays the same role in the solution as the constant R_{m} in Eqn (3). The origin of time is selected in the system of equations (13) so that time begins at the beginning of the cosmological expansion. We can readily see that in the limit $\eta \to 0$ and $t \to 0$, Eqn (12) is a consequence of Eqn (13). Solution (12) is the parabolic expansion law, which — in Newtonian language (see, for example, Ref. [1]) — corresponds to the dynamics with zero total energy † ($E = 0$). This law is a general asymptotic expression for two other types of dynamics: elliptic ($E < 0$) and hyperbolic ($E > 0$).

In the same limit the expression for the radiation density contained in solution (13) becomes

$$\rho_{\mathrm{r}} = \frac{3}{32\pi G t^2} = 4.5 \times 10^5 \, t^{-2} \text{ g cm}^{-3} \,. \tag{14}$$

(In this, Gamow's last formula, there is an unimportant misprint: the original text has 4.4, instead of 4.5.)

It should be pointed out that all the formulas given above have been known in cosmology before Gamow: in one form or another they can be found in the work of Friedmann, Lemaitre, Tolman, Einstein, and de Sitter. Gamow now applies them to a completely new problem, which follows from his idea of the hot beginning of the Universe: he wants to find the present temperature of the background radiation.

5. Three simple steps

Gamow achieved his aim thus. First he found a 'demarcation point' (Gamow's term) in the history of the Universe. This is the moment $t = t_*$ at which the earlier epoch of the predominance of radiation changes to the epoch of the predominance of matter. This moment can be found from Eqns (5) and (14) and from the condition $\rho_{\mathrm{r}}(t_*) = \rho_m(t_*)$:

$$t_* = 2.2 \times 10^{15} \text{ s} = 73 \times 10^9 \text{ years} \,. \tag{15}$$

At this moment the densities of both components of the cosmological medium are

$$\rho_{\mathrm{r}} = \rho_{\mathrm{m}} = 9.4 \times 10^{-26} \text{ g cm}^{-3} \,, \quad t = t_* \,. \tag{16}$$

Gamow then made the second step: from Eqn (16) he found, with the aid of the Stefan – Boltzmann law (6), the radiation temperature at the 'demarcation point':

$$T_* = T(t_*) = 320 \text{ K} \,. \tag{17}$$

The third and last step gave the final result. Gamow took the temperature at the 'demarcation point' and used it to find the present temperature by means of Eqns (1) and (8):

$$T_0 = T(t_0) = T_* \left(\frac{t_*}{t_0} \right) = 7 \, \text{K} \,. \tag{18}$$

The aim he set out was achieved and, as we can see, this was done by the simplest mathematics. "Elementary, my dear Watson", as Gamow said in a similar situation in one of his popular science books.

6. Exact but useless solution

The cosmological model used by Gamow describes a universe filled with matter and radiation. He avoided all mathematical complications by employing only the asymptotic formulas which follow from this model. However, it is not difficult to derive also the exact formulas. If he did not wish to integrate the relevant equations of the Friedmann cosmology (because, after all, they are nonlinear), Gamow could have found the solution in the literature, in the papers of Lemaiötre or in the work of his students Alpher and Herman. I shall now give this exact solution in a parametric form, similar to Eqns (3) and (13) [16]:

$$R = R_{\mathrm{m}} (\cosh \eta - 1) + R_{\mathrm{r}} \sinh \,, \quad t = \frac{R_{\mathrm{m}}}{c} (\sinh \eta - \eta) + \frac{R_{\mathrm{r}}}{c} (\cosh \eta - 1) \tag{19}$$

Here each of the equalities represents a sum of the relevant expressions from the solutions which apply only to matter [Eqn (3)] or only to radiation [Eqn (13)], although the equations from which the solution is obtained are nonlinear. It should be pointed out that in Eqn (19) the constants of integration are defined somewhat differently than in the case of the solutions represented by Eqns (3) and (13)

$$R_{\mathrm{m}} = \frac{\rho_{\mathrm{m}}}{2\kappa^{1/2} (\rho_{\mathrm{c}} - \rho)^{3/2}} \,, \quad R_{\mathrm{r}} = \frac{\rho_{\mathrm{r}}^{1/2}}{\kappa^{1/2} (\rho_{\mathrm{c}} - \rho)} \,. \tag{20}$$

Here,

$$\rho = \rho_{\mathrm{m}} + \rho_{\mathrm{r}} \,, \quad \rho_{\mathrm{c}} = \frac{3}{8\pi G} H^2 \tag{21}$$

is the critical density, and H is the Hubble constant. To be specific, the solution is written in such a way that R in Eqn (19) is the radius of curvature of the co-moving three-dimensional space.

It is easily seen that the exact solution of Eqn (19) contains both asymptotes used by Gamow for long and short times, measured from the onset of expansion. Here everything is the same as in Gamow's case, but the present temperature of the background radiation cannot be obtained in any way from the solutions represented by Eqns (19) – (21).

† We recall that the nonrelativistic Newtonian analogue of the Friedmann solutions is constructed for a sphere of finite radius and both the kinetic energy (in a reference system in which the centre of the sphere is at rest) and the potential energy (with its zero at infinity) are defined for this sphere. It is remarkable that in this case all the local properties of nonrelativistic models and in particular the laws of behaviour of the densities of matter and radiation can be described literally by the same formulas as in Friedmann's solutions (see, for example, Refs [1 – 3]). The total energy E is taken to be the sum of the kinetic and potential energies of any sphere of finite radius 'cut out' from the overall distribution of matter which has no boundaries.

In fact, the structure of solution (19) is evidently such that the matter and the radiation occur symmetrically, and the constants R_{m} and R_{r} representing each of the components of the cosmological medium are independent. They are arbitrary in the sense that, if we wish, we can select any values of the densities of matter and radiation for the present epoch. In other words, for the present density of matter and the present age of the Universe the solution admits, in principle, any present value of the radiation density (and, consequently, of the radiation temperature) which can differ arbitrarily from the real observed temperature, Thus, the exact solution does not establish any relationships between the radiation and matter and, consequently, does not solve Gamow's problem.

However, how could Gamow find the solution on the basis of approximate asymptotic formulas? Do Gamow's calculations rely on some additional implicit assumption which introduces the required relationship between the matter and radiation? We recall that a definite relationship of this type appears in calculations of cosmological nucleosynthesis when the correct helium yield (30 mass%, according to the observation) is taken into account.

In Gamow's paper everything is so simple and transparent so that apparently one can see through the reasoning and there is nothing there except the obvious. It would seem that Gamow is boldest in the use of two asymptotes, matched at the 'demarcation point' in place of the exact solution. However, this approach is used very widely in theoretical physics and usually leads to clear and reasonable results. In the hand of an experienced theoretician the approach is an effective means of analysis which rapidly achieves its target. The matching method naturally cannot provide absolutely exact numbers and is suitable only for approximate estimates. But the crux of the matter is not the precision of the result, but whether it can or cannot be obtained.

7. Matching method

All that Gamow does looks very natural. Who would doubt that at the beginning of the cosmological expansion of the Universe radiation predominates and that its dynamics is described by the parabolic law? It is equally reasonable to assume that in the opposite limit matter predominates and the dynamics becomes inertial (if the density is less than the critical value, as assumed at Gamow's time and is still frequently assumed at present). The first thing that one wants to do under such circumstances is to match the asymptotes and see what the result is. This is precisely what Gamow did.

However, if the result (especially so grandiose as the background radiation temperature) is obtained, it would be desirable to see how it was reached. Gamow left it to his readers as an independent exercise. Judging by what happened (Section 3), not all succeeded in this exercise. Let us therefore try and examine carefully the matching procedure adopted by Gamow with the aid of some leading ideas suggested by the exact solution of Eqns (19)–(21).

First of all, we have to allow for the fact that each of the asymptotes used by Gamow represents in fact the result of going to the limit with respect to two generally independent parameters. The first is the ratio of the densities $\rho_{\mathrm{r}}/\rho_{\mathrm{m}}$ and the second is the ratio of the energies $|E_{\mathrm{g}}|/E$, where $E_{\mathrm{g}} = -GM/R$ is the gravitational potential energy per unit mass, M is the mass inside a sphere of radius R, $E = \frac{1}{2}(\mathrm{d}R/\mathrm{d}t)^2 + E_{\mathrm{g}}$ is the total energy per unit mass. Here, Newtonian cosmological dynamics is used again (the reader is referred once more to Refs [1–3]). The asymptote of very short times corresponds to the conditions

$$\frac{\rho_{\mathrm{r}}}{\rho_{\mathrm{m}}} \gg 1\,, \quad \frac{|E_{\mathrm{g}}|}{E} \gg 1\,, \tag{22}$$

and the asymptote of very long times corresponds to

$$\frac{\rho_{\mathrm{r}}}{\rho_{\mathrm{m}}} \ll 1\,, \quad \frac{|E_{\mathrm{g}}|}{E} \ll 1\,. \tag{23}$$

If the limits $t \to 0$ and $t \to \infty$ are considered, then obviously both conditions in each pair, given by Eqns (22) and (23), are satisfied simultaneously. However, if these asymptotic expressions are applied to some intermediate value of time between zero and infinity, the problem of simultaneously satisfying these conditions requires a separate analysis.

For example, at some infinite value of time it may be found that $\rho_{\mathrm{r}}/\rho_{\mathrm{m}} \ll 1$, but $|E_{\mathrm{g}}|/E \gg 1$. In this specific case both the asymptotes used by Gamow are invalid and a solution of the $R \propto t^{2/3}$ type is to be used (this is the Einstein–de Sitter solution for matter under zero pressure).

Gamow excludes this possibility. His matching of the asymptotes implies that there have been no epochs in the history of the Universe when $\rho_{\mathrm{r}}/\rho_{\mathrm{m}} \ll 1$, but $|E_{\mathrm{g}}|/E \gg 1$. The matching means that the two conditions (22) are simultaneously satisfied at short times and that at the 'demarcation point' they simultaneously change to the two conditions (23). In other words, it is assumed that there is a certain coincidence in the history of the Universe: a transition from the epoch of predominant radiation occurs simultaneously with the transition from the parabolic to the inertial expansion. This is the additional implicit condition which makes Gamow's problem soluble.

This can be readily followed on the basis of the exact solution given by Eqns (19)–(21). According to this solution, the matter and radiation densities are equal for the following radius of curvature:

$$R = R_{\mathrm{rm}} = \frac{1}{2}\frac{R_{\mathrm{r}}^2}{R_{\mathrm{m}}}\,. \tag{24}$$

On the other hand, the dynamic regime changes at

$$R = R_{\mathrm{pi}} = 2.7\,R_{\mathrm{r}} + 2.4\,R_{\mathrm{m}}\,, \tag{25}$$

where condition (25) is equivalent to the condition $\eta = 1$.

If it is assumed, following Gamow, that the second event is also simultaneous, then the following condition should be obeyed:

$$R_{\mathrm{rm}} = R_{\mathrm{pi}} \quad \text{or} \quad R_{\mathrm{r}} = 5.7\,R_{\mathrm{m}}\,. \tag{26}$$

Eqn (26) introduces into the problem an explicit relationship between the characteristics of matter and radiation, which in the exact solution are represented by the constants R_{r} and R_{m}. It can readily be seen that when this condition is satisfied, the exact solution yields a specific value of the present temperature of the background radiation and this value is very close to that obtained by Gamow (for the same values of the cosmological density and age of the Universe).

As demonstated above, in Gamow's paper there are no arbitrary constants separately describing the characteristics of matter and radiation in the exact solution. The additional condition of the type given by Eqn (26) is not formulated explicitly by Gamow anywhere. Moreover, he does not use this condition explicitly. What he does not use, he does not formulate. "This only is the witchcraft that I have used", Gamow could say on the subject because he loved quotations, explicit or implicit, and was a master of riddles, jokes, and inoffensive hoaxes both in science and everyday life (this is clear from his autobiography [17] and from the memorial collection [9] and the booklet [5] dedicated to him).

8. Four comments

The mystery of Gamow's paper is now solved and it remains to make just a few brief comments.

(1) Condition (26) does not have to be exact to achieve the aim that Gamow set himself in his paper. It is in fact sufficient that the ratio $|R_{\rm rm} - R_{\rm pi}|/R_{\rm rm}$ should not be too large compared with unity. This automatically limits, to a greater or lesser extent, the range of the exact solution which comes out from the asymptotes employed by Gamow (see Section 7). In particular, the equality $R_{\rm m} = R_{\rm r}$, close to Eqn (26), is not in conflict with anything and the adoption of this equality simplifies very greatly the exact solution [18].

(2) The additional implicit condition, which makes Gamow's problem solvable, does not follow directly from any independent physical or astronomical ideas. It is quite arbitrary.

However, this condition does not contradict anything in science and is fully consistent with the status of cosmology in the fifties. We can say that it does not contradict today's cosmology either if we avoid looking at things in too narrow a way. (This seems appropriate when we are speaking of the Universe as a whole.)

Can we say then that the additional condition is justified post factum, when the measurements give a temperature very close to that predicted?

As is always true in such cases, the agreement between the theoretical result and the experiments or observations can be regarded as a success by a theoretician, but it does not prove experimentally all the assumptions made in the theory. In particular, Gamow's result — which is so close to the observational data — does not mean at all that the history of the Universe consisted of just the two epochs described by him and separated by the 'demarcation point'. Many now favour, for example, a cosmological model in which the dynamics of the Universe is assumed to have obeyed the parabolic law from the beginning of expansion to the present epoch. This is true of the inflation theory very popular in the last decade. In this case the predominance of radiation is followed by an epoch which continues even today (and then for an indefinite time) and is characterised by the inequalities

$$\frac{\rho_{\rm r}}{\rho_{\rm m}} \ll 1\,, \quad \frac{|E_{\rm g}|}{E} \gg 1\,.$$

This epoch is excluded completely from Gamow's cosmological picture (see Section 7). Under such circumstances Gamow's method of matching the asymptotes does not work and the radiation temperature cannot be determined by his approach. Therefore, calculations of nucleosynthesis based on the inflation model also give a correct value of the modern temperature of the background radiation.

(3) Let us now recall the age of the Universe used by Gamow and see how it affects the results of calculations. The expressions from Sections 4 and 5 make it possible to present Gamow's final result as follows:

$$T_0 = 7\sqrt{\frac{\rho_0}{10^{-30}\ {\rm g\ cm^{-3}}}}\sqrt{\frac{t_0}{3\times 10^9\ {\rm years}}}\ {\rm K}\,. \tag{28}$$

In the seventies, when Weinberg's book [6] was written, it would have been necessary to replace 3×10^9 years with 13×10^9 years. The temperature would then have been twice as high as that given by Gamow, i.e. the difference is not very large.

However, in agreement with the 'historical truth', we should adopt the value used in cosmology at the time not only for the age of the Universe but also for the density of matter. This density had been assumed at the time to be $\rho = (1 - 3) \times 10^{-31}$ g cm^{-3}. The new age and density would have then given $T = 5 - 8$ K, which can be regarded as no worse that the result given by Gamow.

However, this is not all. By the seventies it had become clear that, in addition to the background photons in the Universe, there should also be neutrinos and other ultra-relativistic background particles (mostly not found in the laboratory). Their contribution to the density of the Universe might be between 5 and 50 times greater than the contribution of photons. If this is true, Gamow's formulas would give $T_0 = 2 - 5$ K, i.e. a range of temperatures which includes the current exact experimentally determined value of the background radiation temperature.

(4) The value of 3 K for the background radiation temperature appears in the literature in 1950. This value was mentioned by Gamow in an entertaining popular paper published in *Physics Today* [19]. From where did he take this value?

Here is the answer, provided by Alpher and Herman [13]: "Knowing naturally of our calculations at the time, he could take it simply from us and in his inimitable manner he could round out the result!"

The calculations of Alpher and Herman gave the value of the temperature, subject to the natural uncertainties of which Gamow would be aware, ranging from 1 K to 10 K. The logarithmic midpoint of this interval is precisely the round number 3. This was the number adopted by Gamow. There are no 'scientific' reasons to prefer it to, for example, the value 5 K, obtained at that time formally from calculations of cosmological nucleosynthesis.

However, one can guess that Gamow had a very personal relationship with theoretical physics. He regarded it as a free and even somewhat lightweight art, and from time to time he received in return greater or lesser prizes. Prizes such as the value of 3 K in *Physics Today*, reached without effort or problems, but agreeing accurately with the first measurements of Penzias and Wilson. Obviously, the value of 7 K in Gamow's paper of 1953 is also one of those prizes.

9. Conclusions

There is no error in Gamow's paper. The paper represents a rare successful product of the theoretical art. The text of the paper is still fresh and clear, and may seem even to be slightly naive.

The paper is correct. It may lack something one would like to see there, but it contains all that is required in accordance with the familiar triple formula: depth (in conception), boldness (solutions are grasped and matched), and harmony (steps one, two, three!).

In the foreword to Gamow's autobiography [17], S Ulam wrote: "My late friend, mathematician S Banach, told me once: good mathematicians see analogies between theorems or theories, but the very best see analogies between analogies. This ability to see analogies between models used in theoretical theories was possessed by Gamow to an unimaginable degree. In our days, when more and more complex mathematics is used, and it is refined beyond all measure, it is surprising to see how far Gamow could go by means of intuitive pictures and analogies taken up by comparisons from history or even from art.''

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I would like to thank the Library of the Academy of Sciences in St Petersburg: Gamow's paper of 1953 has somehow reached this Library (should it now be called the Library of the Russian Academy of Sciences?) in the form of a separate reprint, it has been well bound between hard covers, and survived fire and water during an ill-fated fire.

I am grateful to Professor Gene Byrd of the University of Alabama, Tuscaloosa, AL, for telling me about Gamow's lecture in Texas in 1967: an abstract has been carefully preserved and describes how he calculated some time ago the temperature of the background radiation with the aid of 'divine creation curves'.

Finally, I would like to mention with gratitude the discussions of cosmological problems with my colleagues and theoreticians at the Ioffe Physicotechnical Institute in Leningrad, when they were preparing a new original course of general physics (for the Polytechnic Institute) in which the first chapter was ... cosmology. And here is the question asked then by A L Efros: in cosmology, the temperature of the background radiation is calculated in such a complex manner in terms of nuclear reactions, but could one take just two asymptotes, this one and that one, match them, and find the temperature from them? The above note is essentially a detailed answer to this question. My brief answer was: first, it is not possible: second, this was done a long time ago by Gamow.

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