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# Thermodynamics of chaotic systems: An introduction
by C Beck, F Schlogel

**Thermodynamics of chaotic systems: An introduction** by C Beck, F Schlogel (Cambridge: Cambridge University Press, 1993) 286 pp

Since the classical work of Boltzmann and Poincare, the complex motion in nonlinear dissipative systems has been described in two ways: either by kinetic theory methods (thermodynamics of irreversible nonlinear processes) or on the basis of the dynamical theory of Poincare, developed initially for Hamiltonian systems.

Until recently these theories have been developing practically independently. The rapid growth of the statistical theory of open systems and particularly the theory of self-organisation makes it imperative to synthesise now these two scientific directions. This is the aim of *Thermodynamics of Chaotic Systems*.

In this book the thermodynamic concepts serve to provide an analysis of nonlinear dissipative dynamical systems with complex behaviour. The book provides an elementary and readable introduction to the subject which undoubtedly will be useful to a wide spectrum of readers, not only to physicists, but also specialists from other disciplines. No special mathematical knowledge is expected of the reader.

The main aim of the book is to stress interesting and deep analogies between thermodynamic methods in nonlinear chaotic dynamics and the generally accepted statistical-mechanics concepts. The book consists of five parts.

Part I ''Essentials of nonlinear dynamics'' provides a brief elementary account of the main concepts and phenomena from the theory of nonlinear dynamical systems.

Part II ''Essentials of information theory and thermodynamics'' presents the main concepts used in thermodynamic analysis of chaotic systems. The authors define the Shannon information measure, the information gain (Kullback entropy), and the Renyi information, and they discuss in detail the general properties of these quantities. The results from thermodynamics, essential for the understanding of the later material, are given in this part.

In Part III ''Thermodynamics of multifractals'' the thermodynamic method is used to analyse the distribution of probabilities in the case of complex fractal structures. An important concept of 'escort distributions' is introduced: they represent canonical distributions from statistical thermodynamics. They can be used to derive the main thermodynamic relationships for chaotic systems.

Part IV ''Dynamical analysis of chaotic systems'' deals with the time evolution of chaotic dynamical systems. Important quantities representing nonlinear systems, such as the Renyi dimensions, the dynamical Renyi entropy, and the generalised Lyapunov (spelt Liapunov in this book) exponents are analysed on the basis of the free-energy density in the 'thermodynamic limit'. This concept is used for mapping in the limiting case when the size of the cells in the phase space tends to zero and the number of iterations per map becomes infinite.

The last part (V) of the book, ''Advanced thermodynamics,'' deals with the possible unification of the various thermodynamic concepts in one theory. This is possible for the special class of 'hyperbolic' maps. For these maps the free energy is sufficient to derive all the main characteristics: The Renyi dimensions, the Renyi entropy, and the generalised Lyapunov exponents. The authors derived here the traditional, for thermodynamics, variational principle of the minimum free energy.

The final chapter of Part V provides an account of the theory of phase transitions in chaotic systems, which — as in conventional statistical mechanics — correspond to non-analytic behaviour of the free energy at a critical point. A classification of phase transitions is provided and the transition mechanisms are explained.

Undoubtedly the book will be useful not only to researchers, but also to students in the physics, chemistry, and biology departments.

Naturally, this relatively small book (only 290 pages) cannot provide a full account of the current status of the statistical theory of open systems with complex ('chaotic') behaviour.

In fact, Part IV deals with dynamical aspects of the time evolution of chaotic dynamical systems. This leads to the following question: what is the relationship between the concepts of evolution and 'self-organisation'? When speaking of self-organisation, we have in mind a process leading to more complex and more highly organised structures. One can therefore ask: does every evolution process represent self-organisation? The answer is naturally negative, because the 'intrinsic tendency to self-organisation' is not a common property of physical or biological systems. Evolution can lead only to degradation. A physical example is the transition to an equilibrium state, which according to Boltzmann and Gibbs, is the most chaotic. Therefore, self-organisation is only one of the possible evolution paths.

The following comments must be made.

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The kinetic theory (i.e. the statistical theory of nonequilibrium processes) is traditionally constructed without recourse to the fundamental ideas of dynamical theory, the concepts of dynamical chaos, the K entropy, and mixing. However, these concepts can throw light on the causes of irreversibility. The first step in this direction was made in 1950 by N S Krylov. The dynamical instability of the motion of atoms in a Boltzmann gas leads to mixing and, therefore, opens up a way for the transition form reversible equations of the Hamiltonian mechanics to irreversible Boltzmann kinetic equation. This is a manifestation of the constructive role of the dynamical instability of the motion of atoms in the development of the statistical theory of nonequilibrium processes in open systems (see Refs [1–3] and the literature cited there).

Can the dynamical instability of motion of macroscopic characteristics also play a constructive role? Will it lead to dissipative structures or to chaos? The answers to these questions require a criterion of the relative degree of order of nonequilibrium states of open systems. I demonstrated in 1983–1984 (see Refs [1–7]) that the Boltzmann–Gibbs–Shannon entropy, renormalised to a given average effective energy (effective Hamiltonian) conserves this criterion. I formulated this criterion in the form of the 'S-theorem' [1–7]. The letter 'S' stands for self-organisation. This stresses that the S-theorem is the criterion of self-organisation. The S-theorem criterion can also be used to check the choice of the controlling parameters.

It is important to stress that there is a direct relationship between, on the one hand, the experimental and renormalised distribution functions and, on the other, the escort distribution functions and the Renyi information (or entropy), i.e. the main concepts of the book under review [7].

*Thermodynamics of Chaotic Systems* by Christian Beck and Friedrich Schlogl provides a bridge between the dynamical theory of chaotic motion and the statistical theory of open systems, particularly the theory of self-organisation.

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