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The role of nonquasineutrality in unstable plasma oscillations

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Abstract. Nonquasineutrality of perturbations has a destabilising influence on a plasma which is not in thermodynamic equilibrium and has a density gradient across a homogeneous axial magnetic field. This destabilising effect applies not only to short but also to long drift – dissipative instability waves. If long drift waves are excited, nonquasineutrality can also destabilise a plasma simultaneously with ion inertia. In strong magnetic fields the absence of quasineutrality of perturbations is important even when the density of charged particles is high.

Beginning with theoretical studies of the ion sound in a current-carrying plasma it has been assumed that in the case of long, compared with the Debye radius, waves ($kr_{\mathrm{D}} \ll 1$, where k is the wavenumber and r_{D} is the Debye radius) the density perturbations in an unstable plasma are quasineutral, $n_{1\mathrm{e}} = n_{1\mathrm{i}}$, and nonquasineutrality can be ignored (see, for example, Ref. [1]). Ion-acoustic waves then appear in a plasma and the velocity of these waves is $v_{\mathrm{s}} = (T_{\mathrm{e}}/m_{\mathrm{i}})^{1/2}$, where T_{e} is the electron temperature and m_{i} is the ion mass.

However, the concept of quasineutral perturbations, adopted in the theory of the ion-acoustic instability of a current-carrying plasma, cannot be applied to a plasma in a magnetic field.

A drift – dissipative instability is excited in a plasma which is not in a thermodynamic equilibrium and has a density gradient perpendicular to an external magnetic field. In the pioneering theory of A V Timofeev [2] it is assumed that ion inertia is a factor which destabilises such a plasma and that density perturbations are quasineutral. Drift and ion-acoustic oscillations are then excited spontaneously in a plasma.

A P Zhilinskii has since shown [3] that ion inertia is not the only plasma-destabilising factor. Nonquasineutrality of perturbations over distances comparable with the scale of the oscillations may lead to the excitation of another branch of drift – dissipative instability — oscillations of a low-density plasma. Zhilinskii assumes [3] that quasineutrality of oscillations may be absent only when short (comparable with the Debye radius) waves are excited. Attempts to separate oscillations into quasineutral and nonquasineutral, depending on the value of kr_{D} compared with unity, have also been made later (see, for example, Ref. [4]).

In fact, nonquasineutrality of pertubations may play an important role not only in the case of short waves ($\lambda \approx r_{\mathrm{D}}$). It has been shown [5] that it leads to the self-excitation of oscillations of a low-density plasma and to the appearance of long, compared with the Debye radius, waves ($kr_{\mathrm{D}} \ll 1$). When long drift waves are excited, nonquasineutrality of perturbations may also destabilise a plasma in conjunction with ion inertia [6 – 9]. The degree of influence of the first of these two factors depends on the plasma density and on the magnetic field intensity. In fact, when these two destabilising factors act simultaneously, in the simplest case of a plasma with magnetised ions and with no account taken of collisions of ions with neutral atoms the dispersion equation is [10]

$$\omega - \omega^* = \mathrm{i}\frac{\omega^2}{D_{\mathrm{e}} k_z^2} k_y^2 (r_{\mathrm{ie}}^2 + r_{\mathrm{D}}^2) - \omega k_y^2 (r_{\mathrm{ie}}^2 + r_{\mathrm{D}}^2) \; . \qquad (1)$$

Here ω is the frequency of the oscillations; $\omega^* = k_y \kappa c T_{\mathrm{e}}/eB$ is the drift frequency; B is the magnetic induction; e is the electron charge; c is the velocity of light; κ^{-1} is the characteristic dimension of a plasma inhomogeneity; $k_y = 2\pi/\lambda_y$; λ_y is the azimuthal wavelength; k_z is the projection of the wave vector along the magnetic field; D_{e} is the electron diffusion coefficient; $r_{\mathrm{ie}} = v_{\mathrm{s}}/\omega_{\mathrm{i}}$ is the ion Larmor radius; ω_{i} is the ion cyclotron frequency. When $r_{\mathrm{ie}} \gg r_{\mathrm{D}}$, Eqn (1) is identical with the dispersion equation for drift oscillations, and nonquasineutrality of perturbations does not influence unstable oscillations. In the opposite limit, when $r_{\mathrm{ie}} \ll r_{\mathrm{D}}$, this equation is identical with the dispersion equation for oscillations of a low-density plasma, and the excitation of oscillations is then governed by the departure from quasineutrality of perturbations. In the former case the oscillation increment, which can be readily obtained from Eqn (1) by replacing ω with $\omega + \mathrm{i}\gamma$, is

$$\gamma = \frac{D_{\mathrm{e}} k_z^2}{2 k_y^2 r_{\mathrm{ie}}^2} \left(\frac{\omega^*}{\omega} - 1 - k_y^2 r_{\mathrm{ie}}^2 \right) \; . \qquad (2)$$

In the latter case the quantity r_{ie}^2 in Eqn (2) should be replaced with r_{D}^2. However, if $r_{\mathrm{ie}} \approx r_{\mathrm{D}}$, then r_{ie}^2 in Eqn (2) should be replaced with $(r_{\mathrm{ie}}^2 + r_{\mathrm{D}}^2)$. Consequently, the question of whether it is necessary to allow for nonquasineutrality of perturbations depends in this case not on the product kr_{D}, but on the ratio $r_{\mathrm{ie}}/r_{\mathrm{D}}$. For long drift – dissipative waves ($kr_{\mathrm{D}} \ll 1$) nonquasineutrality of perturbations can be ignored only if $r_{\mathrm{ie}} \gg r_{\mathrm{D}}$. Whether one should take into account one or other of the two plasma-destabilising factors — ion inertia or nonquasineutrality of

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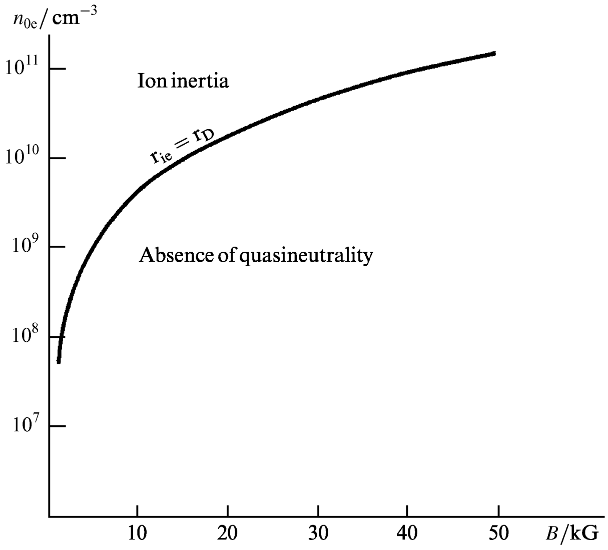

Figure 1. Boundary corresponding to the equality $r_{ie} = r_D$.

perturbations — depends on the relationship between the steady-state electron density n_{0e} and the magnetic field B. The equality $r_{ie} = r_D$ corresponds to the condition

$$n_{0e} = \frac{B^2}{4\pi c^2 m_i} \ . \tag{3}$$

The continuous curve in Fig. 1 corresponds to condition (3). It is evident from this figure that as the magnetic field is increased, the equality $r_{ie} = r_D$ is reached at increasing values of the density n_{0e}. For example, if a hydrogen plasma is subjected to a field $B = 1\,\mathrm{kG}$, condition (3) is obeyed when $n_{0e} = 5.3 \times 10^7\,\mathrm{cm}^{-3}$. However, in a field of $B = 50\,\mathrm{kG}$ the corresponding electron density is $n_{0e} = 1.3 \times 10^{11}\,\mathrm{cm}^{-3}$ and these values of the magnetic field and the electron density are reached in a wall plasma in devices designed to achieve controlled thermonuclear fusion. In fact, nonquasineutrality of perturbations acting as a plasma-destabilising factor proves important in the development of unstable drift–dissipative oscillations also at electron densities somewhat higher than those given by Eqn (3).

A strong departure from quasineutrality of long-wavelength perturbations in an unstable plasma in a magnetic field has been confirmed experimentally [6].

Therefore, in the case of long-wavelength drift oscillations ($kr_D \ll 1$) a plasma subjected to a magnetic field, under conditions such that the ion Larmor radius at the electron temperature is comparable with the Debye radius, may be destabilised not only by ion inertia but also by nonquasineutrality of perturbations.

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