Physics – Uspekhi **37** (5) 515 – 516 (1994)

PACS number: 03.50.De

Details of the motion of charged nonrelativistic particles in a variable field

B M Bolotovskii, A V Serov

Abstract. It is shown that a particle in an alternating field of force does not in the general case oscillate around its initial position but undergoes a systematic drift. The velocity of the drift depends on initial the conditions.

Let us consider a varying electric field $E(t)$, which depends on time t. Let the time-average value of the electric field be equal to zero. The field of a monochromatic plane wave satisfies such a condition. Let us place in a such field a particle at rest with mass m and charge e, and consider the character of its motion. It is natural to suppose that the particle will oscillate around the rest point. Such a statement can be found in some texts (see for example Ref. [1]), but in general is not true. As it turns out, in an alternating field the particle is not only oscillating but undergoes a systematic drift. Let us consider a simple example. Let the electric field E depend on time according to the harmonic law with frequency ω:

$$E = E_0 \cos(\omega t + \varphi) , \tag{1}$$

where φ is the field phase at the intial time.

The equation of motion of the particle in this field has the following form:

$$m\ddot{x} = eE_0 \cos(\omega t + \varphi) . \tag{2}$$

The general solution of this equation is

$$x(t) = -eE_0(m\omega^2)^{-1} \cos(\omega t + \varphi) + At + B , \tag{3}$$

where A and B are arbitrary constants, which do not depend on time and are determined from initial conditions. Let us require that the particle is initially at rest at the origin, i.e.

$$x(t=0) = 0 , \quad \dot{x}(t=0) = 0 . \tag{4}$$

Then the constants A and B are easily found:

$$A = -eE_0(m\omega)^{-1} \sin\varphi, \quad B = eE_0(m\omega^2)^{-1} \cos\varphi , \tag{5}$$

and the solution (3) is written in the following form:

$$x(t) = -eE_0(m\omega^2)^{-1} \cos(\omega t + \varphi) - eE_0 t(m\omega)^{-1} \sin\varphi + eE_0(m\omega^2)^{-1} \cos\varphi . \tag{6}$$

B M Bolotovskii, A V Serov P N Lebedev Institute of Physics, Russian Academy of Sciences, 53 Leninskii prosp., 117924 Moscow, Russia
Tel. (7-095) 132 62 35

Received 21 January 1994
Uspekhi Fizicheskikh Nauk **164** (5) 545–547 (1994)
Translated by A G Nargizyan; revised by J R Briggs

Formula (6) presents the law of motion of a charged particle in the alternating field described by Eqn (1). We shall not consider the magnetic field, though it is present in every electromagnetic wave. If we consider that the particle undergoes nonrelativistic motion, the magnetic field can be neglected. From Eqn (6) it follows that in an alternating field the particle undergoes systematic directed motion with the velocity V, the value of which is equal to the constant A:

$$V = A = -eE_0(m\omega)^{-1} \sin\varphi . \tag{7}$$

We can see from Eqn (7) that systematic motion is absent only for the case $\sin\varphi = 0$, i.e. for definite values of the initial phase. For all other values of the phase φ the particle moves systematically along the direction of the electric field. It is evident that the drift will be directed in the positive or negative directions along the x axis depending on the value of the initial phase φ. If all values of the phase x have the same probability then the drift velocity averaged over the phase φ is equal to zero. In this case there are two groups of particles drifting in opposite directions.

If the initial velocity of the particle is not equal to zero then in the variable field of Eqn (1) the initial velocity will be appended with the drift velocity. In fact, if we require the fulfillment of the following conditions instead of Eqn (4):

$$x(t=0) = 0 , \quad \dot{x}(t=0) = V_0 , \tag{8}$$

then for the constants A and B in solution (3) we have

$$A = -eE_0(m\omega)^{-1} \sin\varphi + V_0 ,$$
$$B = eE_0(m\omega^2)^{-1} \cos\varphi . \tag{9}$$

Because the constant A has the meaning of drift velocity, as seen in Eqn (3), we see that in this case the initial velocity is appended by the drift velocity. With regard to Eqn (7) the limitation of nonrelativistic velocities is given by the condition

$$eE_0(m\omega)^{-1} < c , \quad \text{or} \quad eE_0\lambda(2\pi mc^2)^{-1} < 1 , \tag{10}$$

where $\lambda = 2\pi c/\omega$ is the wavelength corresponding to the frequency ω.

The fact that the particle experiences systematic drift in an alternating field may seem strange at first. But a simple consideration helps one to understand this fact. Let the initial phase be $\varphi = -\frac{1}{2}\pi$ in formula (1) for the field acting on the particle. Then the force acting on the particle has the following form:

$$F = eE_0 \sin(\omega t) . \tag{11}$$

During the first half-period this force is positive, therefore the particle acceleration is also positive and the velocity of the particle is continuously increasing (we assume that according to the initial conditions the particle was at rest at the moment $t = 0$). During the second half-period the force is negative, hence the acceleration is negative too and the velocity of the particle is decreasing. We can see that at the end of the period the velocity is equal to zero, as it was at the beginning. But during one period the particle has moved a distance $2\pi e E_0/m\omega^2$. And for every successive period the particle will move the same distance. Let us consider the kinetic energy W of the particle moving in the field of a wave. It is evident that

$$W = \frac{1}{2} m\dot{x}^2 , \tag{12}$$

where the velocity of particle $\dot{x}$ determined by formula (6):

$$\dot{x} = eE_0(m\omega)^{-1} \sin(\omega t + \varphi) - eE_0(m\omega)^{-1} \sin\varphi . \tag{13}$$

Now substitute this expression for the velocity of the particle in Eqn (12) for the kinetic energy and perform averaging over the period of the wave. We obtain

$$\bar{W} = e^2E^2(4m\omega^2)^{-1} + e^2E^2(2m\omega^2)^{-1} \sin^2\varphi , \tag{12'}$$

where the bar over the W means average over time. The first term of this formula gives the average energy of the oscillating particle in the wave field. The second term describes the average energy of the systematic drift. One can see that at $\varphi = \frac{1}{2}\pi$ the energy of the systematic drift is twice as large as the energy of the oscillations.

The presence of systematic drift of the particle in the wave field changes the qualitative picture of the light scattering by the free particles. Usually in the theory of Thomson scattering it is supposed that the charged particle in the wave field undergoes oscillations with the wave frequency and has no systematic motion [2]. The presence of systematic shift leads to a change of angular distribution and yields a shift to the scattered frequency. These effects are of the order of A/c and in most cases can be neglected. But, for the case of the scattering of highly monochromatic laser radiation on charged particles, the effect of frequency shift can be detected and measured.

The considered phenomenon can influence the motion of charged particles formed at the photoionisation of atoms in a beam of laser radiation. The cause of the ionisation is not necessarily the laser radiation. Ionisation can occur because of an additional source of radiation. The electrons knocked out in the process of the ionisation interact with the beam of laser radiation. Usually it is assumed that the interaction is caused by the so-called Gaponov–Miller force [3],

$$F = -e^2(4m\omega^2)^{-1}\nabla\bar{E}^2 . \tag{14}$$

We can see from Eqn (14) that the force in the laser beam with axial symmetric distribution of field strength E pushes the particles out of the strong-field region. And this force depends only on the distance of the particle from the beam axes and does not depend on field polarisation. It is evident that such a consideration does not take into account the systematic drift of the particle in the wave field. Taking the drift into account yields the result that the azimuthal distribution of particles pushed out from the light beam is nonisotropic in contrast with the case when only the axially symmetric Gaponov–Miller force is taken into account. In addition, when we take into account the drift, the energy of particles leaving the light beam will depend on the angle between the velocity of the particle at the moment of ejection and the direction of the beam polarisation.

It is evident that particle drift under the influence of periodic forces occurs for all kinds of force and not only for the electric force. Therefore, it may be mentioned that Grishchuk [4] considered the drift of a massive particle under the action of a gravitational wave. Braginskii and Grishchuk [5] suggested that this phenomenon could be used for detecting gravitational waves.

An alternating frictional force can also make bodies drift systematically. If a conveyor belt performs alternating translational movement then, under appropriate conditions, bodies that are lying on it can be made to move in a desired direction.

References

1. Matveev A N *Mekhanika i Teoriya Otnositel'nosti (Mechanics and the Relativity Theory)* (Moscow: Vysshaya Shkola, 1976) p. 251
2. Landau L D, Lifshitz E M *Teoriya Polya* (Moscow: Nauka, 1973) [English translation: *The Classical Theory of Fields* (Oxford: Pergamon Press, 1975)]
3. Gaponov A V, Miller M A *Zh. Exp. Teor. Fiz.* **34** 242 (1958) [*Sov. Phys. JETP* **7** 168 (1958)]
4. Grishchuk L P *Zh. Exp. Teor. Fiz.* **66** 833 (1974) [*Sov. Phys. JETP* **39** 402 (1974)]
5. Braginskii V B, Grishchuk L P *Zh. Exp. Teor. Fiz.* **89** 744 (1985) [*Sov. Phys. JETP* **62** 427 (1985)]