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***BOOK REVIEWS***

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# **Chaotic and fractal dynamics** by F C Moon

**Chaotic and fractal dynamics** by F C Moon (New York: Wiley – Interscience, 1992) 408 pp

Chaotic vibrations are nonordered motions which arise in fully determined nonlinear dynamic systems of different natures and are not associated with the systems being acted upon by random external forces, including random noise. Such motions were observed earlier (turbulent motion of a liquid), but they have been detected quite recently also in electric nonlinear systems and even in nonlinear systems with one degree of freedom. Random vibrations have received a mathematical description appropriate to their physical nature only recently. It involves a new mathematical language and a new concept — the fractal manifold. Hence the name of the book — *Chaotic and Fractal Dynamics*.

The book consists of the following chapters: 1. Introduction. A new age of dynamics. 2. How to identify chaotic vibrations. 3. Models for chaos. Maps and flows. 4. Chaos in physical systems. 5. Experimental methods in chaotic vibrations. 6. Criteria for chaotic vibrations. 7. Fractals and dynamical systems. 8. Spatio-temporal chaos. The monograph includes four special appendices: (a) glossary of terms in chaotic and nonlinear vibrations; (b) numerical experiments in chaos; (c) chaotic toys; (d) books on nonlinear dynamics, chaos, and fractals. The monograph is completed by a list of literature references, and author and subject indexes.

Francis Moon, the author, is a Professor at Cornell University and is a well known scientist in the field of theoretical and applied mechanics. He lectures by invitation on nonlinear dynamics, chaos, and fractals in more than 60 universities in many countries and, judging from the description of the subject of the book, is a good lecturer and teacher.

The book begins with the answer to the question 'why chaotic and fractal dynamics?'. The author emphasises that modern nonlinear dynamics, the origins of which date back to Isaac Newton who discovered the Second Law of Mechanics three centuries ago and who was the founder of dynamics, is passing through revolutionary changes. This is based on the discovery of the fact that unpredictable or chaotic, apparently random vibrations arise in nonlinear dynamic systems described by nonlinear equations with regular (nonrandom) coefficients and vibrating under the influence of regular external forces. In other words, the solutions of these equations are very sensitive to slight changes in the initial conditions, i.e. they exhibit a strong dependence on the initial conditions. Another important property of chaotic vibra- tions is the loss of information about the initial conditions. Chaotic vibrations 'forget' the beginning. Appreciation of the fact that chaotic dynamics is characteristic of all nonlinear physical phenomena gave rise to a feeling of revolution in modern nonlinear dynamics and generally in physics.

Three classical types of dynamic motion were known in 'prechaotic' times: equilibrium, periodic motion or a limiting cycle, and quasiperiodic motion. These states have been called attractors because in the presence of damping the system is 'attracted' to one of the states listed.

Chaotic vibrations constituted a new class of motions which do not reduce to any of those listed. This class of motions is frequently associated with the state which has been called the strange attractor. Classical geometrical regions in the phase space, namely a point and a closed curve or surface in a three-dimensional space, correspond to the classical attractors. It was found that the strange attractor is associated with a new geometrical object — the fractal manifold. The concept of the fractal was formulated by Benoit Mandelbrot. The geometry of the fractal is unusual. Fractals solve the problem of the description of objects for which the traditional process involving the measurement of lengths, areas, and volumes does not yield, at first sight, reasonable results. The strange attractor is in fact such an object. The problem is solved by the application of the Hausdorff – Besicovitch measures and dimensionalities. The theory of such a dimensionality was constructed in the 1920s. The Hausdorff dimensionality may be fractional and Mandelbrot's initial definition of fractals refers to manifolds whose Hausdorff dimensionality is greater than their topological dimensionality.

The book is aimed at a reader taking his first steps in the understanding of chaotic and fractal dynamics. The author logically constructs a system of definitions, concepts, and descriptions of nonlinear dynamic chaotic vibrating systems and in the first place physical systems. He devotes four chapters to this task. Together with the usual definitions and concepts characterising linear and nonlinear vibrating systems, definitions characteristic of the description of chaos in systems, such as, for example, maps and flows, the Poincaré and Henon maps, the 'horseshoe' map, the Lorenz attractor, etc. are described in these chapters. The experimental methods for the observation of chaotic vibrations are discussed in detail in the fifth chapter, while the sixth deals with criteria of the appearance of chaos in nonlinear dynamic vibrating systems. A quantitative measure of chaotic vibrations (the Lyapunov indices) and fractal dimensionality are discussed in detail in the seventh and eighth chapters. Three

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alternative definitions of the fractal dimensionality of strange attractors are considered: flow, correlation, and information dimensionalities. The relation between different definitions of the fractal dimensionality and the Lyapunov indices is discussed. Next, the concept of the multifractal and the possibility of the experimental measurement of the fractal dimensionality of chaotic vibrations in dynamic systems are considered. It is emphasised that one of the aims of the book is to indicate the possibility of applying the new mathematical methods to the experimental study of the behaviour and properties of nonlinear dynamic systems.

This is a good textbook. Each of its chapters is provided with exercises and problems. The glossary in the appendix makes it possible to become rapidly acquainted with new concepts and definitions pertaining to the subject of chaotic vibrations in nonlinear dynamic vibrating systems.

Up to the last decade of the twentieth century, the dominant idea was that order arises from the surrounding chaos and that this order can be recognised only from a predictable periodic structure. This view is now being displaced by another concept of chaotic phenomena. They arise in accordance with regular laws behind which is not a formless chaos but a chaos with a latent order — fractal structures. This new concept has already influenced, and is influencing greatly, the development of modern physics and science as a whole.

The world surrounding us constitutes in a broad sense a nonlinear dynamic system or an enormous number of such systems. Modern nonlinear dynamics introduced into current use new geometrical and topological ideas such as fractals and the fractal manifold, without the adoption of which a deeper understanding of nature would be impossible. Francis Moon's book *Chaotic and Fractal dynamics* will be useful to a wide range of specialists, postgraduates, and students at higher-educational establishments.

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