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Acceleration of polarised protons to high energies in synchrotrons

Yu M Ado, V A Anferov

Contents

Abstract. The problem of accelerating polarised protons to high energies in synchrotrons is considered. The Siberian snake method is applied to preserve beam polarisation during acceleration in synchrotrons. The practical application of this method is considered. An estimate is obtained of the number of snakes required in an accelerator for preserving beam polarisation. The influence of the Siberian snakes on beam dynamics is analysed. The suggested method of calculating practical snake schemes allows one to obtain appropriate snake configurations with conventional or helical dipoles for a particular accelerator.

1. Introduction

Difficulties that appear during the acceleration of polarised particles in cyclic accelerators are connected with the existence of spin motion resonances, a large number of which are crossed during the acceleration and thus depolarise the beam. The unperturbed motion of the particle spin in the accelerator is governed by the interaction of the proper magnetic moment of the particle with an external magnetic field, and represents a precession of the particle spin around the accelerator magnetic field that forms particle closed orbits. Since the magnetic field of the synchrotron increases in the course of acceleration, a generalised spin frequency (or the number of spin rotations around the axis of precession during one particle revolution) linearly increases with energy:

$$\nu_{sp} = G\gamma \,, \tag{1}$$

where G is the anomalous magnetic moment of the particle ($G = 1.79285$ for protons) and γ is the Lorentz factor.

Precession around the vertical magnetic field is perturbed by horizontal fields of different kinds which are present along the particle trajectory. A spectrum of such horizontal perturbations determines the spin precession frequencies, by approaching which the vertical spin direction becomes unstable and depolarisation occurs. Depolarising resonances are individual harmonics of the perturbing field, which destroy the beam polarisation at frequencies of the spin precession specific to each resonance. The linear dependence of the spin frequency on energy leads to intersection of a large number of the depolarising resonances during the beam acceleration.

At the accelerator energies 10 – 20 GeV, the number of the dangerous resonances is small, so that the following methods of individual correction of spin resonances are effective: (1) compensation for dangerous harmonics caused by imperfections in the magnetic structure; (2) a jump-like change of betatron frequencies on passing through the intrinsic spin resonances.

In practice these methods, however, become invalid at an energy of 30 GeV because of the increasing number of spin resonances and their power. In high-energy regions the Siberian snakes method should be used [1, 2], which is based on installing special devices (snakes) into the magnetic structure of the accelerator. Each snake rotates the particle spin by 180° around the horizontal axis and at the same time is optically transparent for the orbital motion of particles in the accelerator. Inserting one or several snakes into the magnetic structure of the accelerator allows one to obtain an energy-independent generalised spin frequency and thus to avoid the intersection of spin resonances.

Yu M Ado, **V A Anferov** M V Lomonosov Moscow State University, Physics Department, Vorobievy Gory, 119899, Moscow
Tel. (7 095) 939 49 46
Fax (7 095) 230 23 37
E-mail: anferov@mich.physics.lsa.umich.edu

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In this review the problems arising during practical application of the Siberian snakes method in high-energy proton accelerators are discussed. As a reference frame for the description of the spin and motion of betatron particles, the accelerator's reference frame $(\boldsymbol{e}_x, \boldsymbol{e}_y, \boldsymbol{e}_z)$ is taken, where the $\boldsymbol{e}_y$ axis is tangent to the orbit of the particle, and $\boldsymbol{e}_x$ and $\boldsymbol{e}_z$ are directed radially and vertically, respectively.

2. The necessary number of the Siberian snakes

The necessary number of Siberian snakes in an accelerator is defined by the requirement of the preservation of beam polarisation, which for an accelerator with the snakes is expressed in terms of a smallness of polarisation perturbations between passages of two consecutive snakes. The perturbations in this case must be small enough not to destroy the control of beam polarisation by the snakes.

Under the condition that the snakes are equally spaced $\theta = 2\pi/N_s$ from each other, the maximal polarisation perturbation between two snakes is determined by the power of the strongest spin resonances deviating the spin out of the stable direction by an angle

$$\xi = \frac{2\pi}{N_s}\,\varepsilon_{\max}\,, \tag{2}$$

where N_s is the number of the snakes in the accelerator and $\varepsilon_{\max}$ is the width (power) of the depolarising resonance.

The influence of an individual snake on the spin can be described also as the coherent perturbation for all particles with a power $\varepsilon_s = 0.5$, as the snake rotates the spin of all particles by 180°. Therefore, N_s snakes in the accelerator will control particle spin motion under the condition

$$N_s\varepsilon_s = N_s \times 0.5 \gg \varepsilon_{\max}\,. \tag{3}$$

To get a more practically convenient estimate, one needs to determine how strong the action caused by the snakes must be to keep control over the spin motion against the action of the depolarising resonances. During the crossing of a resonance with the power at a constant rate $\alpha = \mathrm{d}\nu_{sp}/\mathrm{d}\theta$, the initial and final polarisations of the beam are connected by a relation [3]

$$P_f = P_i\left[2\exp\left(-\frac{\pi\varepsilon^2}{2\alpha}\right) - 1\right]. \tag{4}$$

The resonance begins to control the spin motion if $\varepsilon^2 \gg 2\alpha/\pi$ when the polarisation reversal occurs. By substituting $\varepsilon = 0.5N_s - \varepsilon_{\max}$ as the resonance power, one gets a criterion for estimating the number of snakes necessary to control the spin motion:

$$N_s > 2\varepsilon_{\max} + \sqrt{\frac{8}{\pi}\alpha\eta}\,, \tag{5}$$

where the parameter $\eta = -\ln(P_f/2P_i + 1/2)$ can be taken equal to 6, which corresponds to $P_f = -0.995P_i$. With this approach, the spin resonance is considered as being isolated (that is, the resonance width is much less than the distance to the neighbouring resonance). However, such an admission is not valid at high energies, since strong imperfection resonances are located close to the strongest intrinsic resonances. Therefore, one needs to consider $\varepsilon_{\max}$ in Eqn (5) as a sum of powers of the intrinsic and imperfection resonances.

As an example, consider the proton synchrotron U-70 (IHEP, Protvino) and the first-stage accelerator UNK-1, which is currently under construction. The maximum powers of their intrinsic resonances are 0.12 and 0.55, respectively (for beam accelerations to 400 GeV). With the acceleration rates $\alpha_{\text{U-70}} = 4.23 \times 10^{-5}$ and $\alpha_{\text{UNK}} = 6 \times 10^{-4}$, the number of snakes N_s must exceed 0.26 and 1.2 in U-70 and UNK-1, respectively. Therefore, for preserving polarisation one needs to use one snake in the U-70 and a couple of snakes with perpendicular axes in UNK-1.

To conclude, we note that the given estimate does not take into account higher-order perturbations, known as snake resonances. However, by choosing betatron frequencies distant from half-integer values, the condition (5) coincides to good accuracy with estimates accounting for the snake effects [5].

3. Calculation of practical schemes of Siberian snakes

The magnetic field normal to the direction of the motion of the particles rotate the spin by an angle that is independent of energy. So for high-energy accelerators the Siberian snake is usually considered as a combination of dipole magnets conserving the beam orbit at the snake exit.

Let us estimate the number of dipoles necessary for construction of the Siberian snake. The requirement of conservation of the closed orbit of the beam yields four conditions on the parameters of the magnets of the snake: the position and direction of the orbit in the horizontal and vertical planes must remain unchanged, that is $\Delta x = \Delta x' = \Delta z = \Delta z' = 0$. The requirement of the snake rotating spin by 180° and two angles defining the horizontal axis of the snake, provide three additional conditions. In total, seven conditions arise.

On the other hand, each dipole magnet has two free parameters: direction of the field and the angle of the spin rotation by the magnet; that is, the field integrals. As the number of free parameters must be equal to or greater than the number of imposed conditions, the minimal number of magnets in the snake is equal to four. The general solution of all seven conditions, which are nonlinear equations, is a very difficult problem. Therefore, we will consider schemes of snakes where the majority of these conditions is satisfied automatically by choosing a special symmetry of the snake magnetic field.

3.1 Choice of the snake magnetic field symmetry

Consider now how the conditions of the restoration of beam orbit are simpified at a specific choice of the snake magnetic field symmetry. The change of the beam orbital direction in a transverse magnetic field is proportional to the integral of such a field. Hence, the conservation of orbit direction after the passage of the snake is achieved under the condition that the integral of the snake field is equal to zero:

$$\Delta v = \frac{e}{m\gamma c}\boldsymbol{e}_y \times \int_0^{L_s} \boldsymbol{B}\,\mathrm{d}y = 0\,, \tag{6}$$

where L_s is the snake length. In addition to this condition, optical transparancy of the snake also requires recovering the orbit position after the passage of the snake, which is defined by the equation

$$\Delta\boldsymbol{r} = \frac{e}{pc}\int_0^{L_s}\mathrm{d}l\int_0^{l}\mathrm{d}y(B_z\boldsymbol{e}_x - B_x\boldsymbol{e}_z) = 0\,. \tag{7}$$

The snake magnetic field configuration can have two types of symmetry. The magnetic field projection on the horizontal and vertical directions is either symmetric relative to centre of the the snake, or is antisymmetric:

$$B_z(y) = B_z(L_s - y) \quad \text{or} \quad B_z(y) = -B_z(L_s - y) \ . \tag{8}$$

Obviously, choosing the antisymmetric configuration of the field automatically provides the restoration of orbital direction.

The expression for the orbital shift of the beam in the horizontal plane can be rewritten by separating contributions from symmetrical parts of the snake:

$$\Delta x = \frac{e}{pc}\left\{\int_0^{L_s/2} \mathrm{d}l \int_0^l \mathrm{d}y\left[B_z(y) - B_z(L_s - y)\right] + \frac{L_s}{2}\int_0^{L_s/2} \mathrm{d}y\left[B_z(y) + B_z(L_s - y)\right]\right\} . \tag{9}$$

It is easy to note that by choosing the symmetric configuration of vertical field projections, the orbital shift of the beam in the horizontal plane vanishes if the integral of the snake field is equal to zero as well; that is, the conditions of orbital direction and position recovery coincide in this case.

In the case of an antisymmetric projection of the field, the expression for the orbital shift is simplified and becomes equal to twice the value of the orbital deviation at the centre of the snake:

$$\Delta x = \frac{2e}{pc}\int_0^{L_s/2} \mathrm{d}l \int_0^l \mathrm{d}y\, B_z(y) = 2\Delta x\left(\frac{L_s}{2}\right) . \tag{10}$$

Therefore, the restoration of beam orbit in one of the planes is achieved if:

(1) the corresponding projection of the magnetic field is symmetric with respect to the snake centre, and the integral of this projection along the snake's length is equal to zero;

(2) the projection of the field is antisymmetric and orbital deviation at the snake centre is equal to zero.

Now consider the influence of the field symmetry on the orientation of the spin axis. It is convenient to write the rotation of the spin in a dipole magnetic field directed along $\boldsymbol{n}_B$ in the spinor representation [7]:

$$\hat{R} = \exp\left\{-\frac{\mathrm{i}}{2}(\boldsymbol{n}_B\cdot\boldsymbol{\sigma})\varphi\right\} = \hat{I}\cos\frac{\varphi}{2} - \mathrm{i}(\boldsymbol{n}_B\cdot\boldsymbol{\sigma})\sin\frac{\varphi}{2} , \tag{11}$$

where $\hat{I}$ is the identity matrix, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ is a vector constituted from the Pauli matrices, $\varphi = \int B\,\mathrm{d}l/1.746$ is the spin rotation angle of a proton by a dipole magnet.

We will show now that if both projections of the magnetic field of the snake are chosen symmetric relative to the snake centre, the snake axis lies in a plane perpendicular to the direction of the particle motion. This is not difficult to achieve if one considers that three consecutive rotations† $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{a})$ constitute transformation of a rotation with an axis lying in the plane of vectors $\boldsymbol{a}, \boldsymbol{b}$:

$$\hat{R} = \exp(\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{a})\exp(\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{b})\exp(\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{a}) \ . \tag{12}$$

Thus, for an arbitrary symmetric configuration of dipole magnets, the resulting rotational spin axis and fields of individual magnets lie in the same plane:

$$\hat{R} = \hat{R}_1\hat{R}_2\ldots\hat{R}_n\hat{R}_n\cdots\hat{R}_2\hat{R}_1 \ . \tag{13}$$

†Here the vector direction defines the rotational axis, and its length defines the rotational angle.

On the other hand, if the vertical component of the snake field is antisymmetric, whereas the horizontal component is symmetric relative to the snake centre, the resulting rotational axis of spin lies in the horizontal plane. By using straightforward calculations, one can show that an addition to the rotation around the horizontal axis of an antisymmetric combination of turns around the vertical axis, or of a symmetric combination of turns around axis $\boldsymbol{e}_x$, does not change the horizontal axis of rotation.

$$\exp\left(\frac{\mathrm{i}}{2}\alpha\sigma_z\right)\exp\left\{\mathrm{i}\beta(\sigma_x\cos\varphi + \sigma_y\sin\varphi)\right\}\exp\left(-\frac{\mathrm{i}}{2}\alpha\sigma_z\right) = \exp\left\{\mathrm{i}\beta[\sigma_x\cos(\varphi-\alpha) + \sigma_y\sin(\varphi-\alpha)]\right\} , \tag{14}$$

$$\exp\left(\frac{\mathrm{i}}{2}\alpha\sigma_x\right)\exp\left\{\mathrm{i}\beta(\sigma_x\cos\varphi + \sigma_y\sin\varphi)\right\}\exp\left(\frac{\mathrm{i}}{2}\alpha\sigma_x\right) = \hat{I}(\cos\beta\cos\alpha - \sin\beta\sin\alpha\cos\varphi) + \mathrm{i}\sigma_x(\cos\beta\sin\alpha + \sin\beta\cos\alpha\cos\varphi) + \mathrm{i}\sigma_y\sin\beta\sin\varphi \ . \tag{15}$$

Only combinations like Eqns (14) and (15) can appear for any configuration of dipole magnets with antisymmetric vertical and symmetric horizontal components of the field around the snake centre. Hence, the resulting rotational axis is horizontal.

Therefore, the choice of magnetic field symmetry determines one of the angles of the orientation of the snake axis, in addition to simplifying the conditions of the orbital restoration.

Finally, only three out of seven conditions for obtaining an optically transparent snake for both types of field symmetries remains: the condition of the spin rotation by 180°, the angle of orientation of the spin rotational axis (either in the horizontal plane or in the plane transverse to the particle velocity), and the requirement of the restoration of beam orbit at the snake exit.

In what follows we will refer to as symmetric the schemes of snakes with both symmetric components of the field, and as antisymmetric those with symmetric horizontal and antisymmetric vertical components of the field.

3.2 Discrete schemes of Siberian snakes

For analysis of different schemes of Siberian snakes it is convenient to use vector diagrams in which each magnet of the snake corresponds to a vector with a length proportional to the spin rotation angle, and with a direction aligned with the spin rotation (i.e. with the magnetic field direction). The sum of vectors in such a diagram is determined by the integral of individual field components and thus must be equal to zero. Note that this requirement coincides with the condition of orbital restoration for a symmetric configuration.

The condition of the spin rotation by 180° and of orientation of the spin rotational axis can be deduced from the spin transformation matrix in the snake. Consecutive multiplication of spin rotation transformation matrices in individual magnets results in a matrix of rotation of the spin by 180° around a horizontal axis $\boldsymbol{n}_s = (\cos\varphi_s, \sin\varphi_s, 0)$:

$$\hat{S} = \hat{R}_n\ldots\hat{R}_1 = \exp\left\{\frac{\mathrm{i}}{2}(\boldsymbol{n}_s\cdot\boldsymbol{\sigma})\pi\right\} = \mathrm{i}\begin{pmatrix} 0 & \exp(-i\varphi_s) \\ \exp(i\varphi_s) & 0 \end{pmatrix} . \tag{16}$$

To obtain the matrix $\hat{S}$, it is unnecessary to multiply the matrices of all the magnets of the snake: it is sufficient to obtain the spin transformation matrix in a half of the snake. Then, with the symmetry of the snake field taken into account, the matrix $\hat{S}$ can be represented as the product

$$\hat{S} = \left[A\hat{I} - \mathrm{i}(\boldsymbol{\sigma}\cdot\boldsymbol{a})\right]\left[A\hat{I} - \mathrm{i}(\boldsymbol{\sigma}\cdot\boldsymbol{b})\right]$$

$$= \hat{I}\left[A^2 - (\boldsymbol{a}\cdot\boldsymbol{b})\right] - \mathrm{i}\left(\boldsymbol{\sigma}\cdot\left\{A\left(\boldsymbol{a}+\boldsymbol{b}\right) + \left(\boldsymbol{a}\times\boldsymbol{b}\right)\right\}\right) , \qquad (17)$$

where $\boldsymbol{b} = (a_x, a_y, -a_z)$ in the case of the antisymmetric snake field configuration, and $\boldsymbol{b} = (a_x, -a_y, a_z)$ in the case of the symmetric snake. The condition of the rotation by 180° is zero trace of the matrix $\hat{S}$:

$$A^2 - \boldsymbol{a}\cdot\boldsymbol{b} = 0 . \qquad (18)$$

As an example, consider several schemes of Siberian snakes. The minimum number of snake magnets is four, as was shown above. The snake shown in Fig. 1 illustrates this possibility. This snake is antisymmetric and has three free parameters: magnet orientation angles α, β and the projection of the spin rotation vector of any magnet on the horizontal axis $\psi_x = \psi_1 \cos\alpha = \psi_2 \cos\beta$.

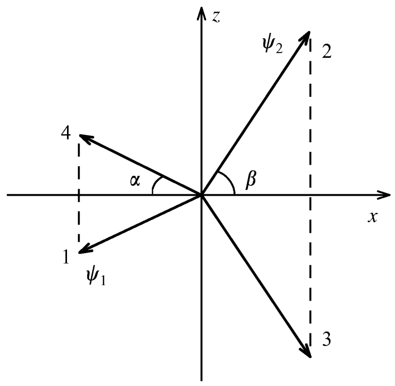

Figure 1. Scheme of a four-magnet snake.

One of the free parameters can be excluded by requiring restoration of closed orbit. For an antisymmetric snake this condition is satisfied at zero shift of the beam orbit in a horizontal plane at the snake centre:

$$\Delta x\left(\frac{L_s}{2}\right) = \frac{\psi_x \tan\alpha}{G\gamma}\left\{\frac{l_1}{2} + l_{\mathrm{gap}} + \frac{l_2}{2}\frac{\tan\alpha}{\tan\beta}\right.$$

$$\left. - \left(\frac{\tan\beta}{\tan\alpha} - 1\right)\left[\frac{l_{\mathrm{gap}}}{2} + \frac{l_2}{2}\left(1 - \frac{\tan\alpha}{\tan\beta}\right)\right]\right\} = 0 , \qquad (19)$$

where the lengths of the snake magnets are determined by a choice of magnetic field induction B from the condition that the field integral, which is needed for the rotation of the spin by an angle ψ, is equal to $\int B\,\mathrm{d}l = 1.746\,\psi$.

In order to compute the second parameter of the snake, we use condition (18) of the spin rotation by 180°, in which one needs to substitute the expressions for A and $\boldsymbol{a}$ determined by a given configuration of the snake:

$$A = \cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2} + \cos(\beta-\alpha)\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} ,$$

$$\boldsymbol{a} = \begin{pmatrix} \cos\beta\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2} - \cos\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} \\ -\sin(\beta-\alpha)\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \\ \sin\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} - \sin\beta\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \end{pmatrix} . \qquad (20)$$

By varying the last remaining parameter, we obtain schemes of snakes with different angles of spin rotational axis orientation φ_s. Table 1 contains the solutions obtained numerically for the snake made of four magnets. The magnetic field strength was chosen to be $B = 1.7$ T, with a distance between the magnets $l_{\mathrm{gap}} = 0.4$ m. Note that such a snake scheme is not the optimal one and gives too high values of the total integral of the snake field and of deviations of a closed orbit inside the snake. This is caused by the fact that the magnetic field direction in the second magnet is practically opposite to that in the first magnet, and hence the resulting spin rotation is small in comparison with the spin rotation by individual magnets.

Table 1. Examples of four-magnet snakes ($l_{\mathrm{gap}} = 0.4$ m, $B = 1.7$ T)

$\alpha/°$	$\beta/°$	$\psi_1/°$	$\psi_2/°$	$\varphi_s/°$	$\int B\,\mathrm{d}l/\mathrm{T\,m}$
40	65.09	161.12	293.04	101.2	27.68
35	61.25	159.77	272.12	108.1	26.32
32.5	59.12	162.37	266.78	112.3	26.15
30	56.8	167.44	264.85	117.4	26.35
25	51.53	191.72	279.29	133.6	28.71

More optimal configurations of snakes can be obtained by increasing the number of snake magnets. For example, a smaller field integral is given by an antisymmetric scheme of the snake made of five magnets shown in Fig. 2. With account taken of the chosen field symmetry, the number of free parameters is three: rotational angles of the spin in the first and second magnets, ψ_1 and ψ_2, and the orientation angle of the first magnet α. The spin rotation angle in the third magnet is $\psi_3 = 2\psi = 2\psi_1\cos\alpha$.

The algorithm for constructing the five-magnet snake is the same as that for the four-magnets snake. The orbital recovery condition permits one to express the spin rotation angle in the second magnet through that in the first magnet:

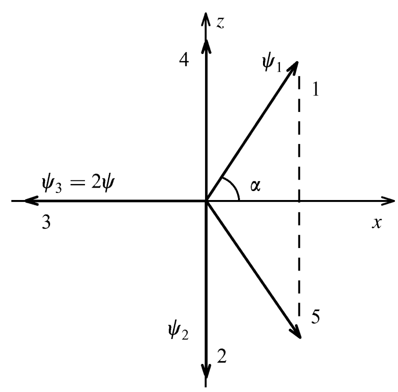

Figure 2. Scheme of a five-magnet snake.

$$\frac{l_1}{2}+l_{\text{gap}}+\frac{l_2}{2}\frac{\psi_1 \sin\alpha}{\psi_2}$$
$$-\left(\frac{\psi_2}{\psi_1 \sin\alpha}-1\right)\left[\frac{l_3}{2}+l_{\text{gap}}+\frac{l_2}{2}\left(1-\frac{\psi_1 \sin\alpha}{\psi_2}\right)\right]=0\ . \quad (21)$$

The second free parameter is excluded with the help of condition (18) of the spin rotation by 180° in the snake, where one needs to use the expressions

$$A=\cos\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2}+\sin\alpha\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right)$$
$$+\cos\alpha\sin\frac{\psi}{2}\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2}\ ,$$

$$\boldsymbol{a}=\begin{pmatrix}\cos\alpha\cos\frac{\psi}{2}\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2} \\ \quad-\sin\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\cos\frac{\psi_2}{2}+\sin\alpha\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) \\ \sin\frac{\psi}{2}\left(\sin\alpha\sin\frac{\psi_1}{2}\cos\frac{\psi_2}{2}-\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\right) \\ \quad-\cos\alpha\cos\frac{\psi}{2}\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2} \\ \cos\frac{\psi}{2}\left(\cos\frac{\psi_1}{2}\sin\frac{\psi_2}{2}-\sin\alpha\cos\frac{\psi_2}{2}\sin\frac{\psi_1}{2}\right) \\ \quad-\cos\alpha\sin\frac{\psi}{2}\sin\frac{\psi_1}{2}\sin\frac{\psi_2}{2}\end{pmatrix}. \quad (22)$$

Different solutions for the five-magnet snake are presented in Table 2.

Table 2. Examples of five-magnet snakes ($l_{\text{gap}}=0.4$ m, $B=1.7$ T).

$\alpha/°$	$\psi_1/°$	$\psi_2/°$	$\psi/°$	$\varphi_s/°$	$\int B\,dl$/T m
49.82	101.7	148.3	65.61	75.28	19.24
47.97	102.6	144.3	68.69	71.23	19.23
45.79	104.4	140.3	72.8	66.02	19.35
43.13	108	136.9	78.82	58.48	19.73
41.38	111.6	135.7	83.74	52.23	20.17
39.83	116.2	135.9	89.21	45	20.8
39.17	118.8	136.7	92.1	41	21.18
38.46	122.4	138.2	95.84	35.62	21.72
37.94	126	140.2	99.38	30.27	22.28

Another example of the antisymmetric snake with an economic field integral is the seven-magnet scheme described in Ref. [8]. It is determined by the sequence (−H, −V, mH, 2V, $-m$H, −V, H), where H is a dipole with vertical field rotating the spin by an angle ψ_x, V is a dipole that rotates the spin around the radial axis by an angle ψ_z, and $m \gtrsim 2$ is a numerical parameter.

In the seven-magnet scheme the first and the last magnets serve to correct the orbital deviations in the horizontal plane. They influence the spin rotation by turning the snake axis by an angle ψ_x around the vertical axis. Hence, only five inner magnets provide the spin rotation by 180°. This condition can be easily obtained:

$$\cos^2\psi_z+\sin^2\psi_z\cos\psi_{\text{m}}=0\ . \quad (23)$$

The orientation of the snake axis φ_s taking into account the turn by the extreme magnets is determined from the relation

$$\tan(\varphi_s+\psi_x)=-\tan\frac{\psi_{\text{m}}}{2}\cos\psi_z\ . \quad (24)$$

The angle of the spin axis rotation in the extreme magnets can in turn be found from the condition of the orbital restoration during the snake passage:

$$\frac{l_1}{2}+2l_{\text{gap}}+l_2+\frac{l_3}{2}\frac{\psi_x}{\psi_{\text{m}}}$$
$$-\left(\frac{\psi_{\text{m}}}{\psi_x}-1\right)\left[l_2+l_{\text{gap}}+\frac{l_3}{2}\left(1-\frac{\psi_x}{\psi_{\text{m}}}\right)\right]=0\ . \quad (25)$$

Examples of solutions for such a snake are presented in Table 3.

Table 3. Examples of seven-magnet snakes ($l_{\text{gap}}=0.4$ m, $B=1.7$ T)

$\psi_x/°$	$\psi_z/°$	$\psi_{\text{m}}/°$	$\varphi_s/°$	$\int B\,dl$/T m
50.77	122.31	113.57	0	24.92
42.40	106.82	95.24	30	21.41
40.01	94.97	90.43	45	19.53
40.15	80.01	91.78	60	17.79
56.97	50.02	134.66	90	17.78

As we already made clear, a special class among different schemes of snakes may be composed from those with a symmetric field configuration. The spin rotational axis in these schemes is always normal to the direction of beam motion; thus, it can be made radial by turning the snake as a whole around the radial axis. One of the best-known schemes of this type is a snake consisting of eight similar magnets, with each turning the spin by 90°. It is easy to show that for the sequence of magnets (V, −H, −V, H, H, −V, −H, V) the spin rotational axis is radial. The full field integral for the eight similar magnets snake is $4\pi \times 1.746 = 21.76$ T m.

Other variants of the symmetric schemes of the snakes are possible. One can use fewer magnets, which, however, leads to increasing the snake magnetic field integral. For example, in the symmetric six-magnet snake shown in Fig. 3, the field integral is 23.574 T m. The axis of this snake is not horizontal and it is necessary to turn the entire snake by an angle of $\beta = 30.455°$ around the longitudal direction in order to obtain the radial spin rotational axis. The parameters of magnets for such a snake are presented in Table 4.

To conclude this section, it should be noted that use of the symmetric snake configurations is restricted by large (in comparison with antisymmetric schemes) values of the

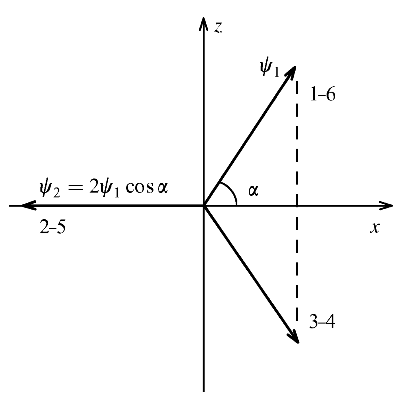

Figure 3. Scheme of a six-magnet snake. The magnets are the same in pairs 1-6, 2-5, 3-4.

Table 4. Parameters of a six-magnet symmetric snake.

Angle of spin rotation by the first magnet	$\psi_1 = 125.82°$
Angle of spin rotation by the second magnet	$\psi_2 = 135.172°$
First magnet field orientation angle	$\alpha = 57.51°$
Angle by which the snake must be turned around the longitudal axis to obtain radial axis of the snake	$\beta = 30.455°$
Full integral of the snake field ($\int B\,\mathrm{d}l$)	23.574 T m
Direction of the snake axis φ_s	radial

beam orbital deviations and the snake field integrals. In addition, it is preferable to use antisymmetric schemes with an angle of spin rotational orientation of 45° in accelerators with an odd number of snake pairs. Then all the snakes in the accelerator have a similar structure, and the condition of orthogonality of the snake axes in each pair is reached by inverting the sequence of magnets in the 'second' snakes (orientation angle of the spin rotation thus becomes equal to −45°). Hence, for the UNK-1, where it is necessary to use two snakes, antisymmetric schemes of snakes consisting of five or seven magnets with an angle of orientation of the spin rotation of 45°, are preferable.

3.3 Helical schemes of Siberian snakes

By increasing the number of the snake magnets in order to obtain a more optimal scheme, in the limit of infinitely many magnets, we necessarily come to use a spiral magnetic field. Such fields are also called helical. They are widely used for obtaining a circularly polarised synchrotron radiation. We consider the possibility of obtaining compact schemes of Siberian snakes with the help of such fields.

The spin transformation matrix in a magnet with a helical field can be obtained by considering a system of N dipoles, with each dipole being turned relative to the preceding one by an angle $\delta = kL/N$. By assuming the magnetic field at every point to be equal to B_0, one can write the spin rotation in each magnet as:

$$\hat{R}_n = \exp\left\{-\frac{\mathrm{i}}{2}\theta(\sigma_x\cos\alpha_n + \sigma_z\sin\alpha_n)\right\}$$

$$= \exp\left(\frac{\mathrm{i}}{2}\sigma_y\alpha_n\right)\exp\left(-\frac{\mathrm{i}}{2}\sigma_x\theta\right)\exp\left(-\frac{\mathrm{i}}{2}\sigma_y\alpha_n\right), \quad (26)$$

where $\theta = B_0L/(1.746N) = \kappa L/N$ is the spin rotation angle in the magnet with orientation α_n.

The spin transformation matrix in the system of N dipoles is a product of the spin rotation of each magnet:

$$\hat{R}_N\ldots\hat{R}_1 = \exp\left(\frac{\mathrm{i}}{2}\sigma_y\alpha_\mathrm{f}\right)\left[\exp\left(\frac{\mathrm{i}}{2}\sigma_x\theta\right)\exp\left(-\frac{\mathrm{i}}{2}\sigma_y\delta\right)\right]^{N-1}$$

$$\times\exp\left(\frac{\mathrm{i}}{2}\sigma_x\theta\right)\exp\left(-\frac{\mathrm{i}}{2}\sigma_y\alpha_\mathrm{i}\right). \quad (27)$$

Here α_i and α_f are the angles of the initial and final orientation of the magnetic field, respectively. Going to the limit $N \to \infty$ in such a way that $\theta N \to \kappa L$, $\delta N \to kL$ while θ and δ tend to zero, one obtains the spin transformation matrix in a helical field:

$$\hat{R}_\mathrm{h} = \exp\left(\frac{\mathrm{i}}{2}\sigma_y\alpha_\mathrm{f}\right)\exp\left\{-\frac{\mathrm{i}}{2}L(\sigma_x\kappa + \sigma_y k)\right\}$$

$$\times\exp\left(-\frac{\mathrm{i}}{2}\sigma_y\alpha_\mathrm{i}\right), \quad (28)$$

where we have made use of the relation:

$$\exp A\,\exp B = \exp\big(A + B + [A,B] + [A,[A,B]] + \ldots\big).$$

By inspecting the shape of the obtained spin transformation matrix in a helical field, one can easily note that if the initial field orientation lies in a horizontal plane and the field makes an integer number of turns, the spin rotational axis is horizontal. The spin rotation angle in such a magnet is

$$\psi = L\sqrt{\kappa^2 + k^2} = 2\pi n\sqrt{\frac{\kappa^2}{k^2} + 1}. \quad (29)$$

As the horizontal projection of the field is symmetric, the orbital restoration in a vertical plane is reached automatically. Therefore, the simplest scheme of the helical Siberian snake consists of a magnet with a helical field that makes one rotation, and of a pair of different-sign dipoles that are needed for compensating orbital deviations in the horizontal plane.

The magnetic field configuration and profile of deviations of a closed orbit at the energy $T = 10$ GeV are shown in Fig. 4; Table 5 contains the parameters of this snake. The obtained snake has a field integral of 15.4 T, which is much

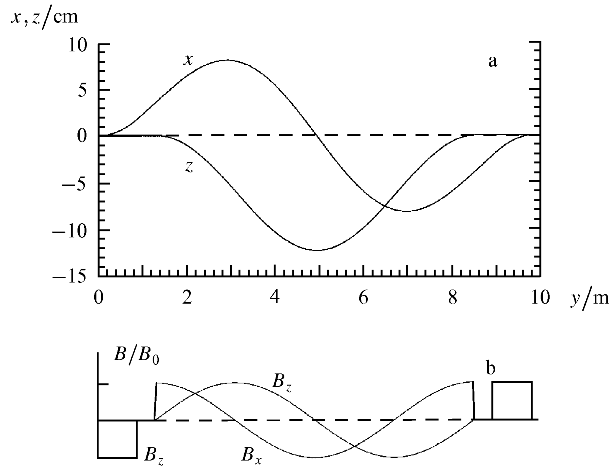

Figure 4. (a) The beam orbital deviation at an energy of $T = 10$ GeV. (b) Magnetic field configuration in the Siberian snake consisting of one helical magnet and two orbit correction dipoles. The field of the snake is $B_0 = 1.7$ T. Maximal orbital deviation in the snake is $z_{max} = 12.28$ cm, $x_{max} = 8.14$ cm.

Table 5. Parameters of a Siberian snake consisting of one helical magnet and two correction dipoles

Field in the magnets of the snake	1.7 T
Distance between the magnets of the snake	0.4 m
Length of correction dipoles	0.927 m
Length of the magnet with helical field	7.215 m
Length of the snake	9.869 m
Maximal orbital deviation at $T = 10$ GeV	12.28 cm
Direction of the snake axis φ_s (away from the radial)	86.5°

smaller in comparison with the discrete schemes of snakes. The compactness together with small orbital deviations make the use of this snake very effective in accelerators with an injection energy of about 10 GeV.

Note that the snake has a fixed axis in a nearly longitudal direction; however, simple changes in the configuration allow one to obtain different directions of the snake axis. In order to do this, one needs to install two correcting dipoles with different signs at both sides of the snake, instead of one. The condition of orbital restoration in the horizontal plane is imposed in the both cases.

Behaviour of the orbit in a helical field is described by the system of equations

$$x'' = \frac{e}{pc} B_z = \frac{e}{pc} B_0 \sin(ky + \alpha_i) ,$$

$$z'' = -\frac{e}{pc} B_x = -\frac{e}{pc} B_0 \cos(ky + \alpha_i) . \qquad (30)$$

Solution to this system is written in terms of the initial coordinates and velocities of the particles:

$$x = \rho(ky \cos\alpha_i - \sin(ky + \alpha_i) + \sin\alpha_i) + x_0' y + x_0 ,$$

$$z = \rho(ky \sin\alpha_i + \cos(ky + \alpha_i) - \cos\alpha_i) + z_0' y + z_0 . \qquad (31)$$

Obviously, minimal deviations of the orbit occur if the beam is moving inside the helical field along a spiral orbit with a radius

$$\rho/\mathrm{m} = \frac{B_0}{B\rho k^2} = \frac{0.974(\kappa/k)^2}{\gamma B_0/\mathrm{T}} .$$

For this purpose, one needs to provide the corresponding initial conditions of the beam motion by using correction dipoles at the entrance to the helical magnet. However, additional efforts for decreasing orbital deviations inside the snake lead to increase of the snake field integral. Different schemes of this type were considered by E Courant [6].

Another interesting possibility for the composition of the compact helical schemes is the use of dipole field insertions into the helical field such that the field direction remains a continuous function of the longitudal coordinate.

Consider as an example an antisymmetrical snake consisting of one helical magnet with a horizontal dipole field inserted at the snake centre. Magnetic field configuration inside such a snake is shown in Fig. 5. The insertion length choice $l = 2/k$ provides the orbital recovery beyond the snake. The condition of the spin rotation by 180° for such a snake can be written as

$$\tan\left(\frac{3\pi}{4}\sqrt{1+\xi^2}\right)\tan\xi = \frac{2}{\sqrt{1+\xi^2}} , \qquad (32)$$

where $\xi = \kappa/k$. Eqn (32) defines the snake orientation angle $\varphi_s = 39.47°$ as well. The snake parameters are listed in Table 6.

It is also easy to note that the scheme described above relates to one family with the simplest helical snake considered at the beginning of this section. As the length of the insertion decreases, the use of two correction dipoles and decreasing the angle of the field turn in both parts of the snake are required to provide optical transparency of the snake. In the limiting case, we obtain a snake with zero length of insertion (i.e. an ordinary helical dipole) and with two dipoles correcting the orbital deviations in the horizontal plane.

Magnets with helical field can be used for construction of symmetric snakes as well. An example can be given by a

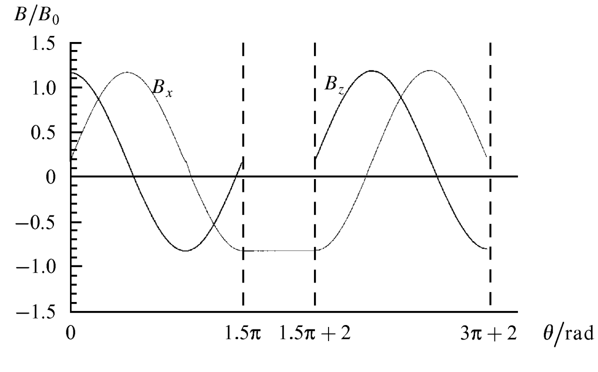

Figure 5. Magnetic field configuration in an antisymmetric snake consisting of one magnet with helical field and an insertion with a constant dipole field. As a longitudal coordinates $\theta = ky$ is taken. The spin rotational axis is inclined by an angle $\varphi_s = 39.47°$ to the radial direction.

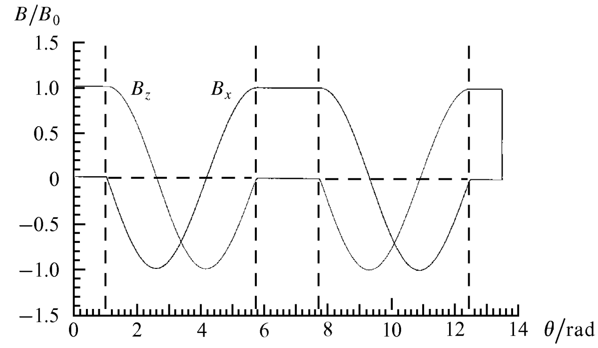

Figure 6. Magnetic field configuration of a symmetric snake consisting of two magnets with helical field and three dipole field insertions. As a longitudal coordinate $\theta = ky$ is used. To get the radial direction of the spin rotational axis, the snake must be turned by an angle of 94.57° around the longitudal axis.

Table 6. Parameters of antisymmetric and symmetric helical snakes with insertion.

Parameters	Antisymmetric	Symmetric
Value of κ/k	0.8819	0.8341
Period length in the helical magnet (field $B = 1.7$ T)	5.69 m	5.383 m
Full integral of the snake field ($\int B\,dl$)	17.59 T m	19.55 T m
Direction of the snake axis φ_s (away from the radial)	39.47°	94.57°
Maximal orbital deviation	$z = 1.5\rho(\pi + 1), x = 2\rho$, where $\rho = 0.758/(\gamma B_0/\mathrm{T})$	$z = x = 1.5(\pi + 1)\rho$, where $\rho = 0.678/(\gamma B_0/\mathrm{T})$

system, which consists of two helical magnets separated by the insertion of a horizontal dipole field with a length $l = 2/k$, and is completed by two insertions of a vertical field with a length $l = 1/k$ (Fig. 6). Each of the helical magnets turns the field by 3/4 of a revolution, but with the opposite rotational directions of the field. The condition of spin rotation by 180° for such a system can be written in the form

$$\left[(1+\xi^2)\tan^2\xi - \xi^2\right]\cos\left(\frac{3\pi}{2}\sqrt{1+\xi^2}\right) + 2\xi\sqrt{1+\xi^2}\tan\xi\sin\left(\frac{3\pi}{2}\sqrt{1+\xi^2}\right) = 1\,, \quad (33)$$

The spin rotational axis of the snake is close to the vertical direction. Therefore, in order to obtain the radial axis, the entire snake must be rotated by 94.57°. The snake parameters are listed in Table 6.

To conclude this section, we note that all helical schemes of snakes have discrete analogues, which can easily be obtained by approximating parts of the helical field by dipole magnets. Nevertheless, helical snakes have definite advantages in comparison with the discrete schemes in the low-energy region. This is illustrated by the compactness of the helical snakes and by low orbital deviations of the beam inside the snake. These are the reasons that determine the snake scheme choice for U-70-like accelerators, whose free-space deficit and low injection energy make it impossible to use discrete schemes [4].

4. Effect of the Siberian snakes on beam dynamics in an accelerator

4.1 Focusing effect of the snake

An additional focusing of the beam appears by passing the Siberian snake, which is caused by a longitudal magnetic field B_y arising at the trajectory of particles as result of a varying transverse field of the snake $B_\perp$. We will use an approach suggested in Ref. [9] to estimate this effect.

The equation of motion of a particle moving in a magnetic field is written in general form as

$$m\gamma\dot{v} = \frac{e}{c}\left(v_\parallel \times \boldsymbol{B}_\perp + v_\perp \times \boldsymbol{B}_\parallel\right)\,. \quad (34)$$

For relativistic particles moving in the accelerator, $v_\parallel = v\boldsymbol{e}_y$. Therefore, by passing from the time derivative to the derivative with respect to the longitudal coordinate $\dot{v} = vv'$, one can rewrite Eqn (34) in the form

$$m\gamma c v' = e\boldsymbol{e}_y \times \boldsymbol{B}_\perp + e\frac{v_\perp}{v} \times \boldsymbol{e}_y B_y\,. \quad (35)$$

By using Eqn (35) to describe the dynamics of the transverse motion of the particle in the snake field, we can distinguish two components of the motion: the rapid one, describing the beam orbital deviations inside the snake, and the slow one, describing the beam focusing by the snake. As the transverse velocity $v_\perp$ is small in comparison with the longitudal one, and the longitudal magnetic field inside the snake is small with respect to the transverse one, the rapid component of the motion can be written as

$$v_\perp^{(f)} = \frac{e}{m\gamma c}\boldsymbol{e}_y \times \int_0^y \boldsymbol{B}_\perp\,dy\,. \quad (36)$$

The equation of motion for the slow component of the transverse motion is obtained by averaging expression (35):

$$\langle v'_\perp\rangle = \frac{e}{pc}\langle v_\perp \times \boldsymbol{e}_y B_y\rangle\,, \quad (37)$$

where we have taken into account that, for an optically transparent snake,

$$\langle B_\perp\rangle = \frac{1}{L_s}\int_0^{L_s} B_\perp\,dy = 0\,.$$

As B_y is a rapidly varying field, the focusing results from a correlation between B_y and $v_\perp$. Thus we can substitute expression (36) for the transverse velocity of the rapidly moving $v_\perp^{(f)}$ into the right-hand side of Eqn (37) to get

$$\boldsymbol{r}'' = \left(\frac{e}{pc}\right)^2\left\langle B_y\int_0^y \boldsymbol{B}_\perp\,dy\right\rangle\,. \quad (38)$$

By determining the longitudal field of the snake from the equation $\operatorname{curl}\boldsymbol{B} = 0$ and assuming the field to be zero $B_y(\boldsymbol{r}=0) = 0$ along the undisturbed orbit of the beam, we have

$$\boldsymbol{r}'' = \left(\frac{e}{pc}\right)^2\left\langle\left(\boldsymbol{r}\cdot\frac{\partial\boldsymbol{B}_\perp}{\partial y}\right)\int_0^y \boldsymbol{B}_\perp\,dy\right\rangle \quad (39)$$

or, by using the formula for differentiating the product of two functions,

$$\boldsymbol{r}'' = \left(\frac{e}{pc}\right)^2\left\langle\boldsymbol{r}\frac{\partial}{\partial y}\left(\boldsymbol{B}_\perp\int_0^y \boldsymbol{B}_\perp\,dy\right) - (\boldsymbol{r}\cdot\boldsymbol{B}_\perp)\boldsymbol{B}_\perp\right\rangle\,. \quad (40)$$

The first term in the angle brackets gives zero contribution, since

$$\left\langle\frac{\partial}{\partial y}\left(\boldsymbol{B}_\perp\int_0^y \boldsymbol{B}_\perp\,dy\right)\right\rangle = \frac{1}{L_s}\int_0^{L_s}\frac{\partial}{\partial y}\left(\boldsymbol{B}_\perp\int_0^y \boldsymbol{B}_\perp\,dy\right)dy = \frac{1}{L_s}\left(\boldsymbol{B}_\perp\int_0^y \boldsymbol{B}_\perp\,dy\right)\Bigg|_0^{L_s} = 0\,. \quad (41)$$

Bearing this in mind, the equation describing focusing of the beam by the snake can be rewritten as

$$\boldsymbol{r}'' = -\left(\frac{e}{pc}\right)^2\langle\boldsymbol{B}_\perp^2\rangle\cdot\boldsymbol{r} = -k^2\boldsymbol{r}\,. \quad (42)$$

From here one can easily find the shift of betatron frequencies caused by inserting a snake into the accelerator's magnetic structure:

$$\Delta Q_{x,z} = \frac{1}{4\pi}\int_0^{L_s}\beta_{x,z}\left(\frac{B_{x,z}}{B\rho}\right)^2 dy\,. \quad (43)$$

Note that the influence of the snake is mostly pronounced at low energies and decreases in the course of acceleration. Therefore, perturbations caused by the snake can be compensated for, if necessary, by a doublet of quadrupoles with constant strength located at both sides of the snake. A criterion of necessity of compensation of the snake focusing effect is the condition of smallness of the mean value of the accelerator's β-function in comparison with the snake focal distance:

$$\beta \ll F_s = \frac{1}{k^2 L_s} \approx 2L_s\left[\frac{0.2998}{p/(\mathrm{GeV}/c)}\int_0^{L_s} B_\perp\,dl\right]^{-2}\,. \quad (44)$$

It is easy to estimate the focal distance of the snakes described in this review at the energy 5 GeV. The use of snakes with transverse field at lower energies is practically impossible because of the increase of closed-orbit deviations inside the snake. As the field integral in the proposed snakes is equal to approximately 20 T m, the focal distance of such snakes will be of the order of twice the snake length, that is about 20 – 30 m. The obtained estimate is comparable with the β-function value. Hence, local correction of the additional focusing of the snake is necessary. However, at energies higher than 10 GeV, the snake effect is small and a special compensation is required.

4.2 Dispersion function perturbation

As the trajectories of particle motion in the snake transverse magnetic field depend on energy, the dispersion function inside the snake undergoes a perturbation. To estimate it, we will use the equation of the dispersion function behaviour inside a dipole field:

$$\boldsymbol{D}'' = \frac{1}{\boldsymbol{\rho}}\,, \tag{45}$$

where $\boldsymbol{\rho}$ is the beam orbital curvature in the magnetic field.

A formal solution to Eqn (45) is the expression

$$\boldsymbol{D}(y) = \boldsymbol{D}_0 + \boldsymbol{D}'_0\,y + \int_0^y \mathrm{d}l \int_0^l \frac{1}{\boldsymbol{\rho}(y)}\,\mathrm{d}y\,. \tag{46}$$

Therefore, the perturbation caused by the snake can be written in the form

$$\Delta\boldsymbol{D} = \frac{e}{pc}\int_0^{L_s} \mathrm{d}l \int_0^l \boldsymbol{B}_\perp(y)\,\mathrm{d}y\,. \tag{47}$$

It easy to note that the right-hand side contains the expression for the beam orbital shift at the snake passage. Therefore, for the optically transparent snake the dispersion function of the beam is perturbed only inside the snake and this perturbation equals the shift of a closed orbit inside the snake. Orbital deviation for the considered snake schemes does not exceed 20 cm at the energy 20 GeV, whereas the unperturbed dispersion function in the accelerator can reach several metres. The smallness of the introduced perturbation makes it unnecessary to compensate for this effect.

5. Conclusions

The Siberian snakes method allows one to preserve the proton beam polarisation in the course of acceleration to the highest energies. The necessary number of snakes is defined by the criterion of polarisation preservation: the number of snakes must far exceed twice the sum of the maximal power of intrinsic depolarising resonances and maximal power of the imperfection resonances of a given accelerator. For accelerators with maximum energy less than 100 GeV only one snake is required. For higher-energy accelerators several pairs of the snakes must be used.

The performed analysis of the influence of practical schemes of snakes on beam dynamics in an accelerator shows this influence is maximal at low energies and decreases in the course of acceleration. The focusing effect of the snake becomes significant and requires an additional correction at energies less than 10 GeV. The perturbation of the accelerator's dispersion function is localised inside the snake and is determined by the beam orbital deviations inside the snake.

The given method of calculation of different schemes of snakes permits one to choose an optimal configuration for different accelerators. For accelerators with a low injection energy the use of helical schemes of snakes, which are more compact and induce less deviation of the beam orbit, seems more promising. At the Institute for High Energy Physics (Protvino) a model of a helical magnet has been constructed, devoted to use in Siberian snakes†. The test results confirmed the possibility of constructing full-scale magnets with helical fields.

For accelerators with maximal energy beyond 100 GeV several pairs of snakes are necessary. Orbital deviations are small in this case, and the control of the spin frequency defined by the reciprocal orientation of the snake axes, becomes of particular importance. The considered discrete schemes of snakes allows one to choose an optimal construction of snakes with the required orientation of the spin rotation axes for a particular accelerator.

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†A talk by E A Lyudmirskii at the OKU Institute for High Energy Physics Seminar on 14 February 1994.