# Quantum optics: quantum, classical, and metaphysical aspects 

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## Contents

1. Introduction ..... 1097
2. Semantic sketch of quantum optics ..... 1098
2.1 Quantum Q-language; 2.2 Metaphysical M-language; 2.3 Semiclassical C-language; 2.4 Classical C*-language;2.5 Towards the definition of the 'photon' notion
3. General description of optical experiments ..... 1103
3.1 Initial state of the field; 3.2 Evolution of the field in an optical channel; 3.3 Detection4. Amplitude interference1110
4. Correlation and anticorrelation of photon counts ..... 1111
5.1 Brown-Twiss effect; 5.2 Detection of anticcorrelations; 5.3 Anticorrelation effect
5. Intensity interference ..... 1114
6.1 Two types of intensity interference; 6.2 Brown - Twiss intensity interference; 6.3 Advanced waves interference; 6.4 Simultaneous observation of two types of interference; 6.5 Classical models
6. Conclusion ..... 1120
References ..... 1122 ..... 1122


#### Abstract

Presented below is the critical review of the modern optical experiments devoted to the demonstration of the quantum nature of light and which reveal the properties of photons. The three main languages used for the description of such demonstration experiments are described and compared: the formal quantum language Q , which enables calculation of all averaged experimental data; the classical or semiclassical language C , which allows a visual qualitative description of some effects; the metaphysical language $M$, which uses vaguely defined terms (such as photons, their duality, quantum nonlocality, etc.) and provides no new observable results, but claims to offer the most profound reflection of the quantum-optical phenomena. It is proposed to distinguish the three types of photons: Q-photon (the Fock state with $n=1$ ), C-photon (the classical wave packet), and M-photon (the hypothetical elementary particle producing discrete pulses at the photon detector output, and which has not yet been defined in the framework of any consistent theory).


[^0]Le t no one expect from us a complete history and theory of the Glass Bead Game. Even authors of higher rank and competence than ourselves would not be capable of providing that at the present time.... Still less is our essay intended as a textbook of the Glass Bead Game; indeed no such thing will ever be written.

Hermann Hesse Das Glasperlenspiel

## 1. Introduction

One of the areas of concentration of recent investigations in quantum optics is the study of different types of the twophoton interference of light (see Refs [1-9]). The specific feature of these investigations is the use of two-photon light, consisting of pair-correlated photons and, correspondingly, the detection of the coinciding in time of output pulses of two photodetectors. The possible effects in the interference of three- and more numbers of photons are being discussed [10-13]. Also, the usual (amplitude or 'one-photon') interference [14-17] and the anticoincidence of photon counts in the two output shoulders of a beamsplitter (see Refs [18, 19]) continue to attract attention.

Usually these experiments are motivated by the wish to demonstrate the essential nonclassicality of some optical phenomena and to emphasize the principal difference of the quantum and classical description of light. Many physicists also hope that such investigations would lead to the fulfilment of Einstein's unrealized dream to understand, at last, what the photon really is.

In some types of two-photon experiments it is possible to introduce a quantitative measure of the nonclassicality, the visibility $V$ of the interference pattern. The point is that the exceeding of the interference visibility $V$ over a certain
level may contradict the classical stochastic models of the light or the famous Bell inequalities [20] following from the wide range of classical models (see Refs [7, 8]). In other experiments there is no such measure, although they are frequently taken to be demonstrating the photon structure of light. Sometimes even the optical geometrical Berry phase is considered to be the property of photons (see Refs [21, 22]).

Quantum optics play a special role in the problem of the interpretation of the quantum formalism. Light, first of all, is the main source of information about the surrounding world and, naturally, investigators would like to understand the nature of the object delivering this information. Quantum optics often tackles linear problems, which simplifies its comparison with the classical models. The aforesaid simplification is because of the fact that the Maxwell equations for the classical free-space fields have exactly the same form as the corresponding Heisenberg equations. It is essential that light can be directly detected by the eye with the retina playing the role of a classical detector. Note that the modern laser techniques enable one to prepare the light field in a two-photon state (plus the vacuum component), which in some approximation is a pure state and corresponds to the macroscopic scale of coherence phenomena [23].

The purpose of this paper is to present a short review and schematic description of the main types of quantum demonstration experiments of modern optics, i.e. the experiments aiming at two goals: demonstration of the quantum properties of light and revelation of the nature of the photon. An attempt is made to estimate the achievements in solving the problems and also to classify and compare different 'languages' used for the description of demonstration experiments. Section 2 is devoted to defining the 'languages'.

A general description of some typical optical experiments in different languages is given in Section 3. For the sake of completeness the conventional two-modes amplitude interference is discussed in Section 4. Demonstration experiments, using a beamsplitter for mixing of two transversal modes (beams), and two detectors with a coincidence circuit (or analog correlometer) which reveal the correlation and anticorrelation of photons, are analyzed in Section 5. Section 6 is devoted to the more complicated, four-mode schemes. Two types of intensity interference are distinguished here: the Brown-Twiss interference and the advanced waves interference. The main conclusions of the review are concentrated in Section 7.

It should be emphasized that, for the sake of being specific, the range of considered problems is limited here to several stationary effects in the optical wave range. When saying a quantum object, I shall basically keep in mind only the electromagnetic field of this range.

## 2. Semantic sketch of quantum optics

It is possible to define the following main languages of quantum optics (Fig. 1):

- the quantum (Q), giving predictions which are in quantitative agreement with the experiment and consisting of several ‘dialects’ $\left(\mathrm{Q}_{\mathrm{H}}, \mathrm{Q}_{\mathrm{s}}, \mathrm{Q}_{\mathrm{C}}\right)$;
- the metaphysical (M), pretending to give the traditional realistic interpretation of observed phenomena and of the quantum formalism mainly with the help of new terms;
- the classical ( $\mathrm{C}^{*}$ ) and semiclassical (C), presenting the quantum theory and the observed effects in usual visual patterns, without pretensions to the universal quantitative description.

Mixing of the mentioned languages, often in a single phrase, is typical of many works. I shall try to avoid such mixing, defining different languages by the corresponding


Figure 1. Scheme of the main language families used for the description of optical phenomena.
code symbols $\mathrm{Q}, \mathrm{M}, \mathrm{C}$. The adopted classification (shown below) of languages is in many aspects a reflection of my own subjective standpoint.

The starting point here is my conviction that at present there is a necessity to distinguish quantum physics, for which a steady and fruitful interconnection between the experiments and the mathematical models is typical, and fruitless, mainly verbal quantum metaphysics, which is not controlled by the experiment but pretends to be a more profound description of the quantum phenomena. Physics, as an experimental science, cannot evidently avoid criteria similar to the Popper principle of falsification or the Bridgeman operational determinability (at least for some key notions).

On the other hand, it is desirable not to ignore, as is accepted in current quantum optics, but as far as possible to emphasize the common features of the classical and quantum models.

### 2.1 Quantum Q-language

Let us define by symbol $Q$ the language that uses conventional mathematical formalism of the quantum theory and well-established quantum models of the optical processes.

The most important constituents of the Q-language are: noncommutative algebra of the 'observables', absence of the joint probability distributions for the two noncommuting observables, and the uncertainty relation.

It is usually assumed that two main 'dialects' of the Qlanguage, the Schrodinger representation $\left(\mathrm{Q}_{\mathrm{s}}\right)$ and the Heisenberg representation $\left(\mathrm{Q}_{\mathrm{H}}\right)$, give equivalent results. However, the different time correlation functions, describing the observable optical phenomena, can be precisely defined in the general case only in the $\mathrm{Q}_{\mathrm{H}}$-language. An important property of the $\mathrm{Q}_{\mathrm{H}}$-description is the possibility of transition to the explicit Lorentz-invariant form. Moreover this language follows closely the classical description of the field evolution in the space-time coordinates, which is much more convenient and visual and performs specific calculations.

However, the $\mathrm{Q}_{\mathrm{H}}$-language leaves little room for different 'realistic' interpretations. This is probably the reason why, in the discussions of the demonstration experiments, the schematic Schrodinger description ( $\mathrm{Q}_{\mathrm{s}}$-language) with Fock's stationary vector $\left(|1\rangle_{k}\right)$, or the quasi-stationary vectors of the states and the corresponding Metaphysical language M (see below) are used. Evidently the evolution of the state vector $|\psi(t)\rangle$ in the Fock basis looks more realistic and visual than the evolution of the field operator $\boldsymbol{E}(\boldsymbol{r}, t)$ in the Heisenberg representation, where it is possible to avoid the notion of a photon.

The overwhelming ('silent') majority of physicists are completely satisfied with the Q-language, i.e. with the pragmatic approach to the quantum theory, which explains and predicts a lot of the observed phenomena very well. It will also be shown that the $\mathrm{Q}_{\mathrm{H}}$-language (unlike the $\mathrm{Qs}^{-}$ language) gives a simple universal description of the main optical effects-the classical ones and the essentially quantum ones, i.e. without classic analogies.

The $\mathrm{Q}_{\mathrm{H}^{-}}$and $\mathrm{Q}_{\mathrm{s}^{-}}$-languages are pure mathematical ones. Some rules of correspondence between the mathematical symbols and the measured quantities should be postulated to connect the languages and the experiments. These rules are closely connected with the problem of the quantum theory interpretation. I shall speak here on the Copenhagen
(it is also called orthodox or minimal) interpretation of the quantum formalism ( $\mathrm{Q}_{\mathrm{C}}$-language), according to which there is no sense in asking too many questions about its nature (I shall not try to define different versions of the $\mathrm{Q}_{\mathrm{C}^{-}}$ language). In essence, this is not an interpretation but a rejection, if one takes the interpretation as an aspiration to attribute to the quantum objects some a priori dynamical properties (except those determined by its state and 'the measuring projective postulate'). Besides, new 'meta-laws' are postulated: the complementary principle or (according to Fock's formulation) the principle of relativity to the observation means.

This principle is reflected in quantum optics as a rather vague term, photon duality, which (along with the understanding of the photon as an elementary particle) is, from my point of view, rather a metaphysical category, i.e. belongs to the M-language because there is no exact operational or Q-definition.

The most important element of the $\mathrm{Q}_{\mathrm{C}}$-language is the 'measuring projective postulate' determining the connection between the state vector of a quantum object and the results of classical measurements, i.e. bridging the gap that divides the quantum and classical worlds.

When the $\mathrm{Q}_{\mathrm{H}}$-language is used for a quantitative description of the experiment, some operators of the quantum model are regarded as observables [these are usually the different time correlation functions for the field operators $\boldsymbol{E}(\boldsymbol{r}, t)$ ]. The goal of the $\mathrm{Q}_{\mathrm{H}}$-theory is reduced to the calculation of the observables average values for the determined initial state of object $\left|\psi\left(t_{0}\right)\right\rangle$.

When the $\mathrm{Q}_{\mathrm{s}}$-language is used, it is postulated that the projections of the final state $|\psi(t)\rangle$ at the same definite vectors of the Hilbert space of the system states determine the statistics of the observed phenomena (which is established in the course of repeating trials under macroscopically identical conditions). In the schematic qualitative models of quantum optics the one-photon Fock states $|1\rangle_{\boldsymbol{k}}$, where $\boldsymbol{k}$ is the mode index, are usually chosen as these vectors.

The notion reduction of the state vector is considered to be an important constituent of the Copenhagen interpretation. This term has two main aspects: in the first one, the problem of the mechanism of an individual (from many possible) measurement result is emphasized, i.e. the problem of description in the Q-language of the nonunitary projection operation; in the second one, attention is given to the problem of what is happening with the quantum object state itself in the course of the 'measurement'. I shall keep in mind only the last aspect. The term reduction (according to the proposed definition) may be attributed to the $\mathrm{Q}_{\mathrm{C}}$-language, if one assumes that it gives only a brief, symbolic description of the experimental situation for which the correlations of the measurements (or conditional probabilities) of two or more macrodevices are detected. These devices inevitably change the state of the object (according to the Q-theory including classical C-parameters of the devices). It is the so-called back action. The third close term is the preparing of the state.

As an example, it is instructive to consider, according to Schiff [24], a trace formation by an electron in the Wilson chamber. The fair calculation of electron scattering by two atoms shows that the electron trace for the high energies should, with a high probability, be almost a straight line parallel to the electron momentum $\boldsymbol{p}$. This actually follows
from the calculations of the probability of exciting both detector-atoms with their diameter $a$ being much larger than $\hbar / p$ (it is the $\mathrm{Qs}_{\mathrm{s}}$-languages). On the other hand (for the estimates or visualization), it is possible to exclude from the very beginning the first atom in the trace and assume that the electron interaction with the first atom is the act of measurement of the transverse electron coordinate with accuracy $a$ (it is the $\mathrm{Q}_{\mathrm{C}}$-languages). It is also assumed that an instantaneous change (Q-reduction or preparation) of the electron wave function takes place, i.e. the transformation of the initial plane wave $\exp (i \boldsymbol{p r} / \hbar)$ into a narrow beam, as if the first atom is an orifice of the diameter $a$ in a nontransparent screen in the path of the plane wave.

If the reduction is accepted, as is often assumed, as some mysterious, 'other world' process, indescribable by the Q-languages in principle, it should be attributed to the metaphysical M-language because it does not lead to the observable consequences different from the predictions of the Q-theory.

In the same way the term backaction should be specified. If it is accepted as a convenient brief definition of the device action on an object which can be explicitly described by the inclusion in the Q-theory of the corresponding (classical) C-parameters, ensuring the unitarity of the evolution of the object (i.e. conservation of the commuting relations and the uncertainty relation), it is the Q-term. If it is accepted that the backaction is something more meaningful, it is the Mterm to which no experimentally verifiable (or according to the Popper formulation, falsified) predictions correspond, i.e. this term has no operational meaning.

A popular term which may, with the same specification, be attributed to the Q-languages is the entangled state term. In the simplest optical case it is a nonfactorizable two-photon four-mode state which is used for the Einstein - Podolsky Rosen (EPR) and Bell paradoxes demonstrations. According to the $\mathrm{Q}_{\mathrm{C}}$-language prescriptions some a priori individual properties cannot, contrary to the semiclassical theory, be attributed to the Q-photons. These are frequency, direction of the wave vector, and polarization. Note here that, contrary to widespread conviction, the condition of the entangling (nonfactorizability) of the field state is not obligatory for the optical demonstration experiments (see Section 6.4)

### 2.2 Metaphysical M-language

For the metaphysical description (M-language), typical is a desire (as yet unrealized) to go out of the frame of the minimal (Copenhagen) interpretation and to look behind 'the scenes', behind the 'looking glass'. It is usually achieved with the help of the M-terms, such as reduction of the state vector, photon duality, photon indistinguishability, quantum nonlocality, contextuality, contrafactuality etc., which throw the uninitiated into a state of anxiety.

The specific indications of the M-language are, according to the proposed determination, vague terms (having, as a rule, neither an explicit reflection in the formal Q-theory nor a clear operational meaning), doubtful discussions, and the absence of real predictions different from those of the quantum theory (Q-language). Many 'fashionable' notions and formulations of the M-language were adopted by frequent and noncritical repetition.

An example of a term currently popular is quantum nonlocality. Here the term 'nonlocality' has actually no connection with the conventional meaning of this word, but
defines, in a squeezed form, something else. This is a quantitative contradiction between the classical and quantum descriptions of some models (the Bell or KochenSpecker type paradoxes) connected, in essence, with the use in the Q-language of the noncommutative algebra of observables, and the absence of the notion of 'joint probability distribution' for the noncommutative observables.

The logic of the origin of the term may be presented briefly as the following (see for details Refs $[8,25]$ and the references therein). There are two statements:

1. The Q-language presents some affirmation $A$ (which can be confirmed in the experiment).
2. The classical C-language (more precise, with some of its dialect $\mathrm{C}_{\mathrm{B}}$ introduced by Bell) also presents the affirmation $A$, although only with the use of such additional notions as the negative probability or the unknown long range forces, manifesting themselves in the instantaneous mutual influence of separated measuring devices or some other even more unacceptable assumptions.

From these, despite contradiction with formal logic, the conclusion to be drawn is that the Q-theory and physical reality are nonlocal.

However, the main notion of optical metaphysics, according to me, is that of 'the photon as an elementary particle of the light field'. I shall call it M-photon, to distinguish it from the formally introduced Q-photon 'created' from vacuum by the operator $a^{+}$, and from the C-photon, a wave packet of the semiclassical C-language. I shall not touch old mechanical models of the photons in the form of balls with definite energies and momenta (see the review Ref. [26]), which are only of historical interest and are successfully used by the authors of manuals on quantum mechanics to confuse students at once and forever. According to current publications the M-photon is something objectively existing in time-space and resulting in a pulse of current at the photon detector output (see Section 2.5).

The formal Q-theory gives no a priori information about the field, except at most about its vector of the state $|\psi\rangle$. The Fock one-mode state $|1\rangle_{k}$ or the many-mode superposition of these states (quantum wave packet) are rather exotic representatives of the multitude of all possible field states, preparation of which is very complicated even with the help of modern laser techniques. In the usual realistic situations, there are, according to the Q-theory, mixed states that are quite different from the ideal ones (see Section 3.1 for details). The $\quad$ Q-theory gives no grounds for schoolboy assertions like 'light consists of photons', which is assumed to be true by the overwhelming majority of physicists.

Let us emphasize the essential difference of the situations connected with the detection of nonrelativistic electrons or other Fermi particles and of those arising from pulses at the photon detector output. In the first case, the number of particles is fixed and the notion of an elementary particle is natural. In this case it is possible to draw some conclusions (the so-called 'retrodiction') on the a priori parameters of the electron state vector. In the second case, the a priori number of the particles is, as a rule, indefinite (see Fig. 3 below) and it is possible only to ask about the state vector of the field.

The problem of inconsistency between the notion of the photon as an elementary particle, introduced in the first
pages of manuals on quantum mechanics, and the 'realistic' state of the quantized field (according to the formal Q language) is not, unfortunately, mentioned in the textbooks and original papers known to me. A paradoxical situation has arisen in quantum optics: for its main notion, $M$ photon (as an elementary particle of the light field), there is no precisely determined place in formal quantum theory.

In general there is a sharp contrast between the very high accuracy of some calculations (Q-language), sometimes coinciding with the measured quantities to seven digits (and better), and the vague verbal description of the phenomena (M-language), which drives the students to despair. Additional difficulties are created by the absence in the textbooks of a distinct border between mathematics and physics, between classical and quantum physics, and also by the poor terminology (e.g. by quantization we often mean a mathematical procedure leading to discrete Fouriertransforms, which is also useful in the classical approach).

The quantitative calculations in the Q-language, which describe the observed effects well, are not needed in the Mnotions. At the same time some Q-terms (type of reduction of the state) can be useful as compact symbol nota-tions of the definite Q-notions at the stage of forecasting some new effects and their preliminary qualitative description. It is also difficult to imagine modern physics without the 'photon' language which describes visually at a qualitative level a lot of phenomena (however, one keeps in mind the Q-photons, i.e. packets, of the semiclassical language).

### 2.3 Semiclassical C-language

The semiclassical description (C-language) is based on the semiclassical radiation theory (e.g. see Refs $[26,27]$ ). This theory considers matter in a quantum manner and the field in a classical way, i.e. as a superposition of the 'C-photons', classical wavepackets of the energy $\hbar \omega$.

Evidently almost all physicists imagine the light field consisting only of the actually existing C-photons, each of them having definite a priori properties: spectral distribution, wave-front shapes, longitudinal and transverse length, polarization. Though the demonstration experiments show convincingly that this visual picture is not adequate (e.g. see Figs 3 and 10) the classical 'heresy', learned in school, persists.

The most important (and mysterious) constituent of the semiclassical description is the postulate on $C$-reduction of the wave packet (do not confuse with the Q - and M reductions considered above), according to which the C photon can be detected only once, whereas the probability of this event (it is assumed that detection happens instantaneously) is proportional to the square of the field averaged over the detec-tion volume. Thus, the 'corpuscular' properties of the C-photon manifest themselves at the moment of detection only.

### 2.4 Classical C*-language

This is the language of classical statistical optics describing classical analogs of quantum effects. When I say classical analog, I keep in mind the phenomenon which has all the most important features of the quantum effect. Usually it is understood as analog detection at which the detected photo-current is proportional to the instantaneous intensity of the incident light at the detector.

Of course, this definition can lead to some subjectivism and nonunique classification of the phenomena. For exam-
ple, according to me, the most important feature of twophoton interference (see Section 6.3) is a definite periodicity in the dependence of two-detector correlation on the lengths of the optical paths and, therefore, it has an optical analog (see Section 6 and Refs $[28,29]$ ). On the other hand, it is possible to consider as such a feature, the high visibility $V$ of the observed interference pattern, i.e. the low value of the background signal (absence of random coincidences). However, the absence of the random coincidences seems to be unspecific for two-photon interference because it is related to the properties of the light source (not to the specific optical scheme) and can be observed in more simple two-mode experiments (see Section 5).

At present nobody has doubts that the classical optical models are limited. Nevertheless the search for the features common with those of classical optics facilitates understanding of the essence of new effects. It corresponds to the traditional conservatism of physics, to the law of the Occam's razor', to the principles of reductionism, to the evristic rule 'from the simple to the complex'. Besides, the flagrant contradiction between the instinctive realistic convictions (they are sometimes called naive realism) of the overwhelming majority of physicists and the Copenhagen $\mathrm{Q}_{\mathrm{C}}$-language, which defies some a priori properties of quantum objects of observation, makes natural the desire to restrict themselves, at the faintest opportunity, to the classical notions, 'do not mention the name of God in vain'. (Unfortunately, the opposite, the desire to emphasize without any need the quantum or 'other world' nature of the phenomenon, frequently happens.)

According to the proposed definition, C - and $\mathrm{C}^{*}-$ languages, contrary to the M-language, do not have pretensions to scientific reflection of the reality at the quantum level. It is evident that only the Q-language has such pretensions. The $\mathrm{M}-\mathrm{C}$-, and $\mathrm{C}^{*}$-languages and their separate terms in the best case play auxiliary, heuristic, comforting, or mnemonic roles.

### 2.5 Towards the definition of the 'photon' notion

Q. As is generally known, the Maxwell equations in the case of the Hamiltonian description are reduced to the system of equations for a multitude of noninteracting linear oscillators. (In the case of the free field these equations are of the same form in both classical and quantum theories.) As a result, the theory of the free field is reduced to the investigation of every possible initial state and their properties for the system of oscillators. In the quantum case, to every state of the system corresponds a point in the space of this system.

In the case of one mode the Hilbert space is covered, for example, by a complete system of Fock $(|n\rangle)$ vectors or the coherent $(|z\rangle)$ basis vectors. One of the popular directions in modern quantum optics is the construction and investigation of new classes of the states and their properties, i.e. of the new subspaces of the quantum oscillator Hilbert space. In the framework of the Q-language (the formal mathematical system of the postulates and the quantum theory theorems) all points of this space are equivalent (except, perhaps, point $|\mathrm{vac}\rangle=|n=0\rangle=|z=0\rangle$ ) and therefore, the Fock state $|n=1\rangle$ with determined energy $\hbar \omega$ is not distinguished in any way.

What is the reason for this special role of the state $|n=1\rangle$ corresponding to the metaphysical notion of the $\mathrm{M}-$ photon? First, it may be assumed that the measurable
observables are to be conserved in the closed system. In optics, this condition distinguishes the energy of the field and, accordingly, the Fock vectors $|1\rangle,|2\rangle,|3\rangle, \ldots$ as 'distinguished' ones. Second, vector $|n=1\rangle$ is distinguished by the weakness of the field interaction with matter in light sources and photon detectors. Many-photon states with $n>1$ are rarely important. (This lucky property of our world reveals itself in the linearity of most optical effects.) Thus, the notion of the photon is tightly connected with the detection process, and a paradoxical formulation can be suggested: a photon has a fleeting existence only at the moment of its absorption in the detector.

However, it is possible to imagine a world made of matter with forbidden one-photon transitions, as, for example, in the case of certain pairs of states (of the type $1 s-2 s$ in a hydrogen atom). In this world the main state should be $|2\rangle$ and the elementary particle should be considered as a 'biphoton'.

In the framework of orthodox quantum theory ( $\mathrm{Q}_{\mathrm{C}^{-}}$ language) questions such as "what is a photon?" and "what are its properties?"' are senseless. It is possible only to ask about the properties of a given pure or mixed field state, the projection of which at vectors $|n=1\rangle_{k}$ determines the count statistics of photon detectors and other measuring devices.

In the Q-language the following definition of a photon is possible. The photon is the objective reality corresponding to the stationary Fock vector $|n=1\rangle_{k}$ or to the quasistationary one-photon wave packet $\left|\psi_{1}\right\rangle$. However, this definition is not very good because, according to the Qtheory, the state $\left|\psi_{1}\right\rangle$ is practically never realized in optics (see Section 3.1 for further details).

If this rare possibility is excluded one should conclude that the photon appears for a moment from nonexistence only at the moment of its absorption by the detector! (Thus the names of operators $a^{+}$and $a$ ought to be swapped.) Let us remember in this connection the well known aphorism reflecting the Copenhagen school creed: "A quantum phenomenon is a phenomenon only if it is a recorded phenomenon'". (This aphorism is attributed to John Wheeler, but in response to my direct question he denied his authorship. Evidently the aphorism belongs to N Bohr.) When applied to the problem under discussion it may be reformulated in the following way: "a photon is a phenomenon only if it is a recorded photon'".
M. The metaphysical language is based on the conviction that the notion 'photon' corresponds not only to the mathematical symbols $|1\rangle$ or $\left|\psi_{1}\right\rangle$, but also to some 'real' physical substance with some a priori properties (elements of the physical reality according to the well-known Einstein formulation), and that any electromagnetic field of radiation consists of the set of these independent (neglecting the rather weak nonlinearity of the vacuum) substances, similar to that for the ideal gas consisting of noninteracting atoms.

It is usually assumed that the final revelation of 'real' properties of the M-photon is only a matter of time and applied efforts. Great hopes are pinned on the introduction of new, frequently vaguely determined, terms and notions, leaving room for the subsequent interpretations and refining. This optimistic point of view (which up to now is followed by a significant number of physicists) has endured since the introduction by Einstein in the beginning of the century of the light quantum notion, in spite of the apparent absence of any progress in this direction. Nevertheless many still hope
that the investigation of new interference schemes based on the use of many-photon light is the 'way to the holy grail'?

Note that the generally accepted standard visual formulation, such as a 'photon was emitted (or absorbed) by an atom', or a 'photon as a whole is transmitted through the semitransparent mirror or is reflected by it' belongs to the semiclassical C-language, because simultaneously (maybe unconsciously) something, similar to the real wave packet, is assumed.

In the M-language the photon is frequently defined as something which was the direct cause of a separate pulse at the photon detector output. To clear up the precise meaning of this M-definition from the point of view of the Qlanguage compare two experiments: in the first, nonrelativistic electrons are detected (or other Fermi particles); and in the second, discrete photon detectors are used.

Let in both cases the following a priori information be known: the sources prepare quantum objects in the pure states $|\psi\rangle$ with definite spin (polarization) components, and the detectors 'see' only one mode of the corresponding wave fields, the de Broglie electron wave or the Maxwell electromagnetic one. This means that the cross section and the specific time of the detector $T_{\text {det }}$ are much less than the corresponding scale of the field inhomogeneity (and therefore the quantum detection efficiency $\eta$ is less than unity).

The appearance of one pulse at the electron detector output in these circumstances is naturally and uniquely interpreted as the consequence of the hitting of one a priori existing electron on the surface of the detector during interval $T_{\text {det }}$. Two or more electrons are unable to be in this interval according to the Pauli exclusion principle. The superposition of the one-electron state with the vacuum of type $|\psi\rangle=c_{0}|0\rangle+c_{1}|1\rangle$ is forbidden by the charge conservation law: the electron is either present or absent (this is an example of the superselection rules).

In the case of the Bose-particle detector, possible conclusions which follow from the detection of an individual event are much more diverse. Suppose that initially $\eta=1$. Then a pulse in the one-photon detector according to the projection postulate means that the projection of $|\psi\rangle$ at the Fock vector $|n\rangle$ with $n=1$ for the corresponding mode is different from zero: $c_{1}=\langle 1 \mid \psi\rangle \neq 0$. Hence, $|\psi\rangle=|1\rangle$ or $|\psi\rangle$ is the state with an indefinite photon number, e.g. a coherent state (see Section 3.1).

Further, taking into account the inevitable nonideality of the detector leads to $\eta<1$. This gives an additional possibility that the field was in the stationary state $|n\rangle$ with any $n>1$ [the probability of this equals to $\eta(1-\eta)^{n-1}$ ]. Only repeated trials with the identically prepared vector $|\psi\rangle$ can refine the procedure of the initial state reconstruction (retrodiction).

Therefore, detection of a separate pulse at the photon detector output in the general case does not allow us to state that it was produced by a single photon: the field can be in a multitude of states different from the one-photon state $|1\rangle$.

Therefore, we arrive at the conclusion that the photon as an elementary particle of the optical field has no reasonable distinct definition and consequently is, according to the suggested definition, a metaphysical category.

More detailed substantiation of the above accepted 'linguistic' classification will be given later with the specific examples. Of course, a variety of other languages and 'dialects', which cannot be overviewed here, are actually
used, so the choice was inevitably influenced by the author's point of view. In particular, the alternative theories of de Broglie, Bohm and others are not considered at all. The same relates also to the different versions of the statistical interpretation of the quantum theory (see review Ref. [30]), to the general theory of quantum measurements (see the monograph by Braginsky and Khalili [31]), and to the $\mathrm{C}_{\mathrm{B}^{-}}$ language of Bell's dichotomic classical observables [20] (see also Refs [7, 8, 25]).

## 3. General description of optical experiments

An experimental set up usually consists of three main parts: a light source, an optical channel, and a detecting device. Thus the theory should describe the properties of light emitted by the source, changing of those properties in the course of light propagation through a linear optical channel, and connection of the properties (changed) of light with the detecting device readings.

According to the above discussion, it is convenient to consider the formal quantum description (Q-language) consisting of three main stages: (1) choice (or calculation) of the required initial state of the field at the optical channel input, (2) calculation of the field evolution in the optical channel, and (3) choice of the operators for the observables corresponding to the experimental measuring procedure. [Sometimes the field evolution is excluded with the help of a spectral expansion over the eigenfunctions of the optical channel (see Ref. [24]). However, the last approach is less universal and visual.] In this connection, consider the well-known quantum trichotomy of the $Q_{C}$-language including a classical device (preparation of the initial state of a quantum object), the object itself (the dynamics of which is governed by the Schrodinger or Heisenberg equations), and a classical measuring device.

A description of these three stages is presented below in different languages.

### 3.1 Initial field state

Q. Since it is not to be involved in calculations, let us assume that the initial field state is known. This state can be either mixed or pure. The latter occurs in two cases: (1) when the motion of charges in the source are describable in the framework of a classical consistent theory ('semi-quantum' emission theory), as is done, for example, in the phenomenological theory of parametric scattering [32]; and (2) when the wave function of the system 'source + field' are factorable.

The state vector or density matrix of the field carries the information which is not required for description of most quantum-optics experiments, where usually it is sufficient to know only the intensity correlation function of the first and second orders. (A rare exception is an interesting experiment [33], where the field density matrix was reconstructed from the photon statistics.) In many cases the observable effects are specified by one number: $g=\left\langle a^{+2} a^{2}\right\rangle / N^{2}$. Where $\langle\ldots\rangle=\langle\psi| \ldots|\psi\rangle, N=\langle n\rangle=\left\langle a^{+} a\right\rangle, n=a^{+} a$ is the Q-photon number operator, and $a\left(a^{+}\right)$is the operator of $\mathrm{Q}-$ photon annihilation (creation) for the individual field mode. Number $g$ is called the photon bunching parameter or $a$ fourth normalized moment.

Number $g$ is simply connected to the other frequently used parameters describing the photon number fluctuations,
the dispersion $\sigma^{2}=\left\langle n^{2}-N^{2}\right\rangle$, and the Fano factor $\Phi=\sigma^{2} / N:$

$$
\begin{equation*}
g-1=\frac{\sigma^{2}-N}{N^{2}}=\frac{\Phi-1}{N} \tag{1}
\end{equation*}
$$

At $g>1$ the term photon bunching (the Brown-Twiss effect) is accepted; at $g<1$ the term antibunching is used. In respect of the directly observed photon counts the terms superpoissonian statistics and subpoissonian statistics at $\Phi>1$ and $\Phi<1$ are used, respectively.

The correlation of the field intensities in the modes $A$ and $B$ can be specified by the moment $G_{A B}=\left\langle a^{+} a b^{+} b\right\rangle$. This moment being normalized by the average photon numbers gives the parameter $g_{A B}=G_{A B} / N_{A} N_{B}$ (which is often called a second order coherence degree $[34,35]$ ), where $N_{A}=\left\langle a^{+} a\right\rangle, N_{B}=\left\langle b^{+} b\right\rangle$. It is convenient also to introduce the correlation coefficient normalized, as is accepted, by the dispersion:

$$
\begin{align*}
& K=\frac{G_{A B}-N_{A} N_{B}}{\sigma_{A} \sigma_{B}} \\
& =\frac{G_{A B}-N_{A} N_{B}}{\left(G_{A A}+N_{A}-N_{A}^{2}\right)^{1 / 2}\left(G_{B B}+N_{B}-N_{B}^{2}\right)^{1 / 2}} \tag{2}
\end{align*}
$$

In the experiments under consideration sources of three main types corresponding in the quantum theory to three main types of the initial state for one mode of the field are normally used.

1. The chaotic (thermal or Gaussian) state is described by the equilibrium density matrix corresponding to some effective (brightness) temperature. In this case $g=2$.
2. The coherent state $|z\rangle$ [where $z=A \exp (\mathrm{i} \varphi)$ is the classical parameter of the state] in the Fock base vectors $|z\rangle$ is of the form [34, 35],

$$
\begin{equation*}
|\psi\rangle=\sum_{n=0}^{\infty} c_{n}|n\rangle \tag{3}
\end{equation*}
$$

Here $c_{n}=\left(z^{n} / \sqrt{n!}\right) \exp \left(-|z|^{2} / 2\right)$. Thus, the Q-photon number is indefinite and its statistics corresponds to the Poissonian distribution with $\left\langle\left(a^{+}\right)^{k} a^{m}\right\rangle=\left(z^{*}\right)^{k} z^{m}$ and $g=1$.

In the case of an ideal one-mode laser beam it is possible to consider parameter $\varphi(t)$ as a classical stochastic function of time with a homogeneous distribution and some definite coherence scale $t_{\mathrm{coh}} \sim 1 / \Delta \omega_{\mathrm{coh}}$. In this approach vector $|z\rangle$ is related to the individual mode with a fixed frequency, but the field state due to the classical stochasticity becomes mixed and describes the radiation with the final spectral width $\Delta \omega_{\text {coh }}$.
3. The squeezed vacuum $|F\rangle$ state describes the quantum noises of a parametric amplifier-converter. Here $F$ is the increment of the parametric gain proportional to the amplitude of laser pumping. In fact, the field state at the amplifier output is also mixed because there is a random classical parameter, the phase of the pumping field $\varphi_{0}(t)$.

If $|F| \ll 1$ this state describes two-photon light, emitted by the parametric amplifier (effect of parametric scatte ring), and

$$
\begin{align*}
|F\rangle & \approx\left|\psi_{2}\right\rangle \equiv|\mathrm{vac}\rangle+\sum F_{k l} a_{k}^{+} a_{l}^{+}|\mathrm{vac}\rangle \\
& =|\mathrm{vac}\rangle+\sum F_{k l}|1\rangle_{k}|1\rangle_{l} . \tag{4}
\end{align*}
$$

Here function $F_{k l}$ is the probability amplitude of finding one photon in both $\boldsymbol{k}$ and $\boldsymbol{l}$ modes. Its Fourier transform describes in C-language a two-photon wave packet depending on two points in time-space.

In the two-mode approximation,

$$
\begin{equation*}
\left|\psi_{2}\right\rangle=|\mathrm{vac}\rangle+F a^{+} b^{+}|\mathrm{vac}\rangle=|\mathrm{vac}\rangle+F|1\rangle_{A}|1\rangle_{B} \tag{5}
\end{equation*}
$$

where operators $a, b$ and indices $A, B$ relate to the signal and idle modes with the frequencies connected by the relation $\omega_{A}+\omega_{B}=\omega_{0}$ ( $\omega_{0}$ is the pumping frequency). In the nondegenerate case modes $A$ and $B$ differ in at least one of the parameters: frequency, wavevector direction, polarization type.

The results of two-detector experiments are reducible in Q-language to the measuring of parameter $g_{A B}$ for the output light. The high visibility of intensity interference, provided two-photon light is used [1-6], proves that inequality $g_{A B} \gg 1$ [36]. In a typical experiment $g_{A B} \sim 10^{8}$, which leads to the negligible low influence of random coincidences. A degenerate parametric amplifier at the low amplification level is described by the state with $g \gg 1$.

In the past, the two-quantum transitions in atomic beams were used as the sources of two-photon light [1,2], whereas now the more convenient parametric frequency downconverters [3-6] are used. Note that the phenomenological descriptions of both types of transformers have much in common [37, 38].

At large pumping amplitudes the components with large (even) numbers of Q -photons $(4,6,8, \ldots)$ are visible and it is possible to observe (with the help of a homodyne detector) the squeezing effect (in the degenerate case it is decrease in dispersion of one of the quadratic components at the expense of an increase in the dispersion of the second component [39]).

For the nondegenerate two-mode parametric amplifiers at any $F$ [28],

$$
\begin{align*}
& N_{A}=N_{B}=N=\sinh ^{2} F, \quad g_{A A}=g_{B B}=2, \\
& g_{A B}=1+\operatorname{coth}^{2} F=2+N^{-1}, \quad K=1 \tag{6}
\end{align*}
$$

Therefore the numbers of the signal and idle photons are completely correlated, although the statistics of each 'half' of the parametric scattering field from either the signal or the idle mode (they can be separated with the help of frequency, polarization, or angular filters) is a chaotic one. These circumstances were evidently not taken into account in the paper devoted to investigations of the M-photon properties [18].

An unusual peculiarity of the state $|F\rangle$ is the nonzero nonstationary moment (also called anomalous). If the operators' temporal dependence is taken into account it takes the form

$$
\begin{align*}
M=\langle a b\rangle & =[N(N+1)]^{1 / 2} \exp \left(-\mathrm{i} \omega_{0} t-\mathrm{i} \varphi_{0}\right) \\
& =0.5 \sinh (2 F) \exp \left(-\mathrm{i} \omega_{0} t-\mathrm{i} \varphi_{0}\right) . \tag{7}
\end{align*}
$$

To study a classical analog of the squeezed vacuum state it is useful to consider a generalization of the state $|F\rangle$ obtained by the action at the parametric amplifierconverter input of an additional chaotic radiation with an average number of Q-photons per mode $N_{0}$ (Fig. 2). With the help of the parameter $N_{0}$, this state enables one to trace the continuous transition of the essentially quantum


Figure 2. Scheme for the squeezed vacuum and squeezed classical light preparation. The classical pumping field P excites the nonlinear crystal (hatched) which emits the signal (a) and the idle (b) field. The crystal is also subjected to the action of the initial fields $a_{0}$ and $b_{0}$. At $a_{0}=b_{0}=0$ a spontaneous emission is observed and at $a_{0}=b_{0} \neq 0$ the induced radiation is added. If $N_{0}=\left\langle a_{0}^{+} a_{0}\right\rangle=\left\langle b_{0}^{+} b_{0}\right\rangle \gg 1 / 2$, the classical squeezed light is emitted by the crystal.
light to classical light [28]. At $N_{0} \ll 1 / 2$ it is the squeezed vacuum. With $N_{0} \gg 1 / 2$ it is the state with a close classical analog, the so-called classically squeezed light $[13,28,40$, 41]. In this case instead of Eqns (6), (7) one gets [28]

$$
\begin{align*}
& N_{A}=N_{B}=N=N_{0} \cosh (2 F) \\
& G_{A A}=G_{B B}=2 N^{2}=N_{0}^{2}[1+\cosh (4 F)] \\
& M=N_{0} \sinh (2 F), \quad G_{A B}=N^{2}+M^{2}=N_{0}^{2} \cosh (4 F) \\
& g_{A B}=1+\tanh ^{2}(2 F)=2-\left(\frac{N_{0}}{N}\right)^{2} \tag{8}
\end{align*}
$$

Let us, apart from three relatively easily realizable states, examine two 'exotic' pure states.
4. The stationary one-photon state of a single mode $|1\rangle_{k}=a_{k}^{+}|\mathrm{vac}\rangle$ and the quasi-stationary multimode superposition of these states are very popular in theoretical works. The latter has the following form,

$$
\begin{align*}
& \left|\psi_{1}(t)\right\rangle \equiv \sum_{k} F_{k} \exp \left(-\mathrm{i} \omega_{k} t\right)|1\rangle_{k} \\
& \sum_{k}\left|F_{k}\right|^{2}=1, \quad \omega_{k}=c k \tag{9}
\end{align*}
$$

and is called a one-photon wave packet. Only this state of a rather specific form is the sole element of the Q -language corresponding to the M-photon.

Sometimes $F_{k}$ is interpreted as a wave function of the photon in the momentum representation. (Let us emphasize that Eqn (9) formally is the wave function of the whole field, not of a separate photon.) The Fourier transform of function $F_{k}$ gives a visual space-time pattern of the photon.

In the case of the one-photon state of the single mode $N=g=1$ and $\Phi=\sigma=0$, i.e. there is maximum possible antibunching and no fluctuations.

It is essential for our discussion that actual preparation of the optical field in the state $\left|\psi_{1}\right\rangle$ is very difficult, even with the help of modern laser techniques. It is very likely that the one-photon states of the form of Eqn (9) were actually not realized up to now in any of the numerous demonstration experiments aimed at studying properties of a single M-photon. Thus, in the work described in Ref. [18] only the signal radiation of a parametric downconverter (described according to Eqn (6) in terms of chaotic statistics) was used. However, in the work described in Ref. [16], the two-photon states and, respectively, twodetector coincidence circuits were used (although, if one
of the detectors is considered as being a preparing component of the experimental set up it is formally possible to speak about a one-photon state).

Note a curious paradox: it is easier to prepare a twophoton state such as that described by Eqn (6) than a onephoton state in the form of Eqn (9). The reason is the following: preparation of an atom in a stationary excited state is connected with great difficulties and in the general case its initial state is nonstationary. After some time the atom returns to the ground state and the field to a nonstationary state with the indefinite Q-photon number which can be schematically presented in the form analogous to Eqn (3).

If there are many atoms or molecules being independently excited (for example, in the gas discharge), the incoherent mixture of states of the form of Eqn (3) should be chosen. (Some paradoxes related to the many-photon emission of the heated matter were discussed in Ref. [32].)
5. It is instructive to examine one more specific state of the field having, at first glance, paradoxical properties. In $\mathrm{Q}_{\mathrm{C}}$-language terms it has two peculiarities: it is a state with the definite energy $\hbar \omega_{C}$ but with an undetermined Q-photon number $n$. Besides, in this state the cube of the field $\left\langle E^{3}\right\rangle$ is not zero [32, 42].

Assume that in the initial moment a noncentrosymmetric molecule was in the excited state 1 (Fig. 3) and the field was in the vacuum state. At $t \rightarrow \infty$ the molecule goes over to the ground state 3 , giving the energy $\hbar \omega_{C}$ to the field. It is essential that the transition to the final state can proceed along two paths: the direct one $1 \Rightarrow 3$ with the creation of one $\mathrm{Q}-\mathrm{photon}$ and the cascade $1 \Rightarrow 2 \Rightarrow 3$, creating two Q-photons $\hbar \omega_{A}$ and $\hbar \omega_{B}$ with the frequencies being related by $\omega_{A}+\omega_{B}=\omega_{C}$.


Figure 3. One-photon and two-photon quantum transitions from the excited state 1 to the ground state 3 can be simultaneously allowed in a noncentrosymmetrical molecule (crystal). As a result the field with definite energy $\hbar \omega_{31}=\hbar \omega_{C}$ but indefinite photon number is formed. Besides, the average cube of the field $\left\langle E^{3}\right\rangle$ is nonzero.

The final state of the field can be schematically presented in the form,

$$
\begin{align*}
&|\psi\rangle=\alpha|0\rangle_{A}|0\rangle_{B}|1\rangle_{C} \exp \left(-\mathrm{i} \omega_{C} t\right) \\
&+\beta|1\rangle_{A}|1\rangle_{B}|0\rangle_{C} \exp \left(-\mathrm{i} \omega_{A} t-\mathrm{i} \omega_{B} t\right), \tag{10}
\end{align*}
$$

where $\alpha, \beta$ are the coefficients depending on the molecular properties (see for more details Ref. [32]). It is easy to prove that this state is not the eigenstate for the operators of the photon number, $a^{+} a, b^{+} b, c^{+} c$ and the sum of these operators.

One finds from Eqn (10) $\left\langle a_{A}^{+} a_{B}^{+} a_{C}\right\rangle=\alpha \beta^{*}$, which leads to the nonzero third moment at the point $r$ :

$$
\left\langle E^{3}\right\rangle \propto 2 \operatorname{Re}\left\{\left\langle a_{A}^{+} a_{B}^{+} a_{C}\right\rangle \exp \left[\mathrm{i}\left(\boldsymbol{k}_{C}-\boldsymbol{k}_{A}-\boldsymbol{k}_{B}\right) \boldsymbol{r}\right]\right\} .
$$

This effect can be observed with the aid of the inverted transition in a molecule detector [32]. It can be easily modeled in C-language terms. The cubic power of the field consisting of the superposition of three photon-packets has the stationary component:

$$
\begin{aligned}
& E^{3} \propto \cos \left(\omega_{A} t+\varphi_{A}\right) \cos \left(\omega_{B} t+\varphi_{B}\right) \cos \left(\omega_{C} t+\varphi_{C}\right) \\
& =\frac{1}{4} \cos \left(\varphi_{A}+\varphi_{B}-\varphi_{C}\right)+f(t)
\end{aligned}
$$

However, this model contradicts the first peculiarity: the packet numbers should be equal to one or two but not to three.

A study of the pure state given by Eqn (10) shows that the assertions such as 'light consists of photons', which assume a definite number $n$ of these constituent elements of the light make no sense in the Q-language because the measured photon number $n$ in this state in some experiments is one, but in some others, it is two. In the $\mathrm{Q}_{\mathrm{C}^{-}}$ language this result is interpreted in the following way: a priori the field has no definite $n$. It is also evidently impossible to use the C - and $\mathrm{C}^{*}$-languages, i.e. to present the field in this state as a classical stochastic field with three independent frequency components.

Similar arguments are applicable also to the more general nonstationary states of the type given by Eqn (3).
C. In the semiclassical theory a photon is accepted as a wave packet, i.e. the quasi-monochromatic and quasi-plane classical field $E(\boldsymbol{r}, t)$ with energy $\hbar \omega$, where $\omega$ is some central frequency of the radiation spectrum ( as a rule, a monochromatic field is under consideration). The spectrum of this field is determined by function $F_{k}$ from Eqn (9). Let us emphasize that we consider the C-language as a useful palliative, which gives only a qualitative description of the limited range of the optical effects.

It is accepted that a single packet with the exponential envelope curve with the time constant $1 / 2 A$, where $A$ is the Einstein coefficient, is emitted in a spontaneous onequantum transition in an atom (molecule, crystal). In the general case the field is formed by a superposition of several such packets with different parameters, including time localization. Statistically independent C-photons are emitted in spontaneous transitions, whereas in the case of induced emission the C-photons are bunched with phase conservation.

In the case of the stationary stochastic field the longitudinal and transverse sizes of the packets are described by some distribution and their average values are determined by the corresponding coherence scales. The C-photon number in the coherent volume corresponds to an important notion of the Q-language, to the operator of the Q-photon number $n_{k}=a_{k}^{+} a_{k}$ in mode $\boldsymbol{k}$. In C-theory, $n_{k}$ is a random discrete quantity with some distribution $P\left(n_{k}\right)$. Inequality $\left\langle n_{k}\right\rangle=N_{k} \gg 1$ is often accepted as a condition of applicability of the classical theory. In the C-language this corresponds to a large number of packets in an individual coherent volume.

A packet with the doubled energy $2 \hbar \omega$ corresponds to the two-photon one-mode Q-state. Two mutually coherent packets with average energies $\hbar \omega_{1}$ and $\hbar \omega_{2}$ can be put into the correspondence to the nondegenerate two-photon twomode state. (For more details on the two-photon wave packets notion see Refs [23, 37, 43].)

The semiclassical theory gives plausible results for many elementary processes: spontaneous and induced emission, photoelectric effect, etc. [44]. However, it is unlikely now that anybody accepts it seriously. Here we consider the semiclassical theory as no more than a convenient and helpful language for the qualitative and graphic presentation of some phenomena, remembering at the same time that it is not applicable to the effects described in the Qlanguage by the states given by Eqns (5) and (10).

C". The possible 'states' of the stationary radiation field in classical stochastic optics are determined by the multidimensional nonnegative distribution function $P(\boldsymbol{a})=$ $P\left(a_{1}, a_{2}, a_{3}, \ldots\right)$ which allows us to calculate various moments $\left\langle\left\langle a_{k}\right\rangle\right\rangle,\left\langle\left\langle a_{k} a_{l}\right\rangle\right\rangle,\left\langle\left\langle a_{k}^{*} a_{l}\right\rangle\right\rangle, \ldots$ (if, of course, they do exist) and the averaged values from other functions of the state $P(\boldsymbol{a})$,

$$
\begin{equation*}
\langle\langle f(\boldsymbol{a})\rangle\rangle=\int \ldots \int \mathrm{d}^{2} a_{1} \mathrm{~d}^{2} a_{2} \ldots P(\boldsymbol{a}) f(\boldsymbol{a}) \tag{11}
\end{equation*}
$$

We introduce here the notation $\langle\langle\ldots\rangle\rangle$ for the operation of the classical averaging of the observables.

Commutative algebra is used in the $\mathrm{C}^{*}$-language, so there is no difference between the ordered and nonordered moments (contrary to the Q-language). This results in the different forms of some mathematical inequalities for the moments in the $\mathrm{Q}-$ and $\mathrm{C}^{*}$-theories. For example, in the quantum theory $G \equiv\left\langle a^{+2} a^{2}\right\rangle=\left\langle n^{2}\right\rangle-N$ (here $N=\langle n\rangle$ ), and provided that the dispersion is positively determined $\left(\sigma^{2}=\left\langle(n-N)^{2}\right\rangle=\left\langle n^{2}\right\rangle-N^{2} \geqslant 0\right)$, one gets the following inequality:

$$
\begin{equation*}
G \geqslant N^{2}-N . \tag{12}
\end{equation*}
$$

For the normalized moment one gets $g \geqslant 1-N^{-1}$, so $g$ can be less than unity.

At the same time, in the $\mathrm{C}^{*}$-theory the second term in Eqn (12) is absent $G_{\text {class }}=\left\langle\left\langle a^{* 2} a^{2}\right\rangle\right\rangle=\left\langle\left\langle n^{2}\right\rangle\right\rangle$, so

$$
\begin{equation*}
g_{\text {class }}=\frac{\left\langle\left\langle n^{2}\right\rangle\right\rangle}{N^{2}} \geqslant 1 \tag{13}
\end{equation*}
$$

Thus, if the fluctuations are absent $(\sigma=0), g=1$ in the classical theory and $g=1-N^{-1}$ in the quantum theory. In this connection inequality $g<1$ is accepted as one of the criteria of a one-mode light field being nonclassical. The states, for which this inequality is satisfied, are called nonclassical. In this case, the term effect of photon antibunching is also used.

Let us now consider two modes. The second moments for the intensities should satisfy the Cauchy - Schwartz and Cauchy inequalities:

$$
\begin{equation*}
G_{A B}=\left\langle n_{A} n_{B}\right\rangle \leqslant\left(\left\langle n_{A}^{2}\right\rangle\left\langle n_{B}^{2}\right\rangle\right)^{1 / 2} \leqslant \frac{1}{2}\left(\left\langle n_{A}^{2}\right\rangle+\left\langle n_{B}^{2}\right\rangle\right) . \tag{14}
\end{equation*}
$$

Proceed to the normally ordered moments:

$$
\left\langle n_{A}^{2}\right\rangle=G_{A A}+N_{A}, \quad\left\langle n_{B}^{2}\right\rangle=G_{B B}+N_{B}
$$

Here, the second terms, which are absent in the $\mathrm{C}^{*}$-theory, can be interpreted as a revelation of the quantum (photons or shot) noise in the energy measurements. Now Eqn (14) takes the form,

$$
\begin{align*}
& G_{A B} \leqslant\left[\left(G_{A A}+N_{A}\right)\left(G_{B B}+N_{B}\right)\right]^{1 / 2} \\
& \leqslant \frac{1}{2}\left(G_{A A}+N_{A}+G_{B B}+N_{B}\right) . \tag{15}
\end{align*}
$$

In the symmetric case $G_{A A}=G_{B B}=G, N_{A}=N_{B}=N$ so

$$
\begin{equation*}
G_{A B} \leqslant G+N, \tag{16}
\end{equation*}
$$

or for the normalized moments $g_{A B} \leqslant g+N^{-1}$. At the same time there is no quantum noise in the $\mathrm{C}^{*}$-theory and the inequality $\left\langle\left\langle n_{A} n_{B}\right\rangle\right\rangle \leqslant\left\langle\left\langle n_{A}^{2}\right\rangle\right\rangle$ coincides with the inequality $\left(G_{A B}\right)_{\text {class }} \leqslant\left(G_{A A}\right)_{\text {class }}$.

Let the $A$ and $B$ modes be in a chaotic state: $G=2 N^{2}$, $g=2$. Then in the frame of the Q-theory one gets the restriction

$$
\begin{equation*}
g_{A B} \leqslant 2+N^{-1} \tag{17}
\end{equation*}
$$

In the case of the squeezed vacuum state, Eqn (17) is converted into an equality [see Eqn (6)]. Moreover, in typical experiments $g_{A B} \sim N^{-1} \sim 10^{-8}$, i.e. the classical inequality $\left\langle\left\langle n_{A} n_{B}\right\rangle\right\rangle / N^{2} \leqslant 2$ is broken by eight orders of magnitude!

According to Eqn (7) in the case of the squeezed state there is a nonstationary moment $\langle a b\rangle$. It is not difficult to obtain the following form of the Cauchy-Schwartz inequality for operators $a, b$ :

$$
\begin{equation*}
|\langle a b\rangle|^{2} \leqslant\left\langle a^{+} a\right\rangle\left\langle b b^{+}\right\rangle=N_{A}\left(N_{B}+1\right) \tag{18}
\end{equation*}
$$

The sign of equality is reached here in the case of the squeezed vacuum. Thus $|\langle a b\rangle|^{2} / N_{A} N_{B}=g_{A B}-1$ and $N_{A}=N_{B}$, so Eqn (17) again follows from Eqn (18).

Inequality (18) in the C-theory takes the following form:

$$
\begin{equation*}
|\langle\langle a b\rangle\rangle|^{2} \leqslant\left\langle\left\langle a^{*} a\right\rangle\right\rangle\left\langle\left\langle b b^{*}\right\rangle\right\rangle=N_{A} N_{B} \tag{19}
\end{equation*}
$$

According to Eqn (8) the classically squeezed light satisfies this restriction, and the sign of equality is reached at high gain. For typical conditions $N_{A}=N_{B}=N=10^{8}$ the difference of the right-hand sides of Eqn (18) and Eqn (19) normalized by $N^{2}$ reaches $10^{8}$.

Therefore, for some field states the normal-ordered moments $G=\left\langle a^{+2} a^{2}\right\rangle=\left\langle: n^{2}:\right\rangle \quad$ and usual moments $\left\langle a^{+} a a^{+} a\right\rangle=\left\langle n^{2}\right\rangle=G+N$ differ essentially, which leads to a contradiction with the C-description, for which there is no mentioned difference. Besides, in the case of the squeezed vacuum state, owing to the difference of $a^{+} a$ and $a a^{+}$the classical inequality (19) is violated.

### 3.2 Evolution of the field in an optical channel

The evolution of the field can be conveniently described in the spectral representation with the help of expansion over the system of orthogonal functions - for example, the spherical ones or the plane waves:

$$
\begin{align*}
& E(\boldsymbol{r}, t)=E^{(+)}(\boldsymbol{r}, t)+E^{(-)}(\boldsymbol{r}, t) \\
& E^{(+)}(\boldsymbol{r}, t)=\sum_{k} u_{k}(\boldsymbol{r}) a_{k} \exp \left(-\mathrm{i} \omega_{k} t\right) \\
& E^{(-)}(\boldsymbol{r}, t)=\sum_{k} u_{k}^{*}(\boldsymbol{r}) a_{k}^{+} \exp \left(\mathrm{i} \omega_{k} t\right)=\left[E^{(+)}(\boldsymbol{r}, t)\right]^{+} \tag{20}
\end{align*}
$$

Here $E^{(+)}$and $E^{(-)}$are the positive- and negativefrequency components of the field. For the sake of simplicity we take into account only one type of polarization and assume that there are no field sources in the space region under consideration. In the case of expansion over the plane waves,

$$
\begin{equation*}
u_{k}(\boldsymbol{r})=\mathrm{i}\left(\frac{2 \pi h \omega_{k}}{L^{3}}\right)^{1 / 2} \exp (\mathrm{i} \boldsymbol{k} \boldsymbol{r}) \tag{21}
\end{equation*}
$$

where $\omega_{k}=c k ; L$ is the periodicity length.

In the classical theory $a_{k}$ determines the amplitude (in dimensionless units) and the phase of the plane wave at the point $(\boldsymbol{r}=0, t=0)$, and in the quantum theory $a_{k}$ is the operator of the photon annihilation in the Schrodinger representation. Let us pay attention to the important, although widely unknown circumstance. In free space the Maxwell equations for the classical fields and the Heisenberg equations for the field operators have an identical form; therefore the classical and quantum Green functions (propagators) coincide.
$\mathbf{Q}_{\mathbf{H}}$. The influence of a linear optical system (beamsplitters, lenses etc.) on the characteristics of light is conveniently described by the Heisenberg representation with the help of the phenomenological scattering matrix of the system connecting the plane wave amplitudes at the input and output of an optical channel [32, 42]:

$$
\begin{equation*}
a_{k}^{\prime}=\sum_{l} D_{k l} a_{l} \tag{22}
\end{equation*}
$$

where the prime marks the transformed quantities. In vector notations $\boldsymbol{a}^{\prime}=\boldsymbol{D} \boldsymbol{a}$. If the parametric frequency converters are excluded, then $D_{k l} \sim \delta\left(\omega_{k}-\omega_{l}\right)$. (For a more general case see Ref. [29].) If there is no dissipation matrix, $D$ is unitary and uniquely connected to the evolution operator [21, 45]. Thus, all possible $K$-mode optical systems, neglecting the losses, realize a group of the unitary matrices $U(K)$.

It is essential that the linear transformations (22) describing the functioning of the optical system are of the same form in the classical and quantum theories (if in the latter the Heisenberg presentation is used), and so the main parts of our formulae are applicable to both theories if the corresponding redefinition of the symbols is done. Actually matrix $D$ is the phenomenological classical Green function in the spectral representation.

We shall be mainly interested in the mixing of two transverse modes (beams) of amplitudes $A$ and $B$ by means of the beamsplitter without any dissipation, the Mach Zehnder interferometers, and so on. (The same formalism describes transformation of the polarization in an individual beam; see Ref. [9].) It is assumed that the radius of each beam is much smaller than the coherence radius. Besides, for the sake of simplicity we shall examine only one longitudinal mode (spectral component) in each beam. This one-mode approximation is justified if the coherence time of the radiation $\tau_{\text {coh }} \sim 1 / \Delta \omega$ (the C-photon length divided by $c$ ) is much larger than the typical detection time $T_{\text {det }}$.

Let $a$ and $b$ be the photon creation operators or the classical amplitudes of the fields in two transverse modes. If the common phase factor is not taken into account, the beamsplitter performs the transformation,

$$
\begin{equation*}
a^{\prime}=t a+r b, \quad b^{\prime}=-r^{*} a+t^{*} b \tag{23}
\end{equation*}
$$

Here $t$ and $r$ are the phenomenological amplitude transmission and reflection coefficients for the beamsplitter or for the whole optical channel satisfying, in the absence of losses, relation $|t|^{2}+|r|^{2} \equiv T+R=1$. Thus the scattering matrix of the two-mode optical system may be presented in the form,

$$
D=\left(\begin{array}{cc}
t & r  \tag{24}\\
-r^{*} & t^{*}
\end{array}\right)
$$

Matrix $D$ has the property of unitarity, i.e. $D^{+} D=D D^{+}=I$, and belongs to the $S U(2)$ group.

With the help of Eqn (23) it is possible to express the output (transformed) moments through the input ones determined by the properties of the light source. If the input beams are mutually incoherent in the first order $\left(\left\langle a^{+} b\right\rangle=0\right)$, then the output intensities have the form,

$$
\begin{equation*}
N_{A}^{\prime}=T N_{A}+R N_{B}, \quad N_{B}^{\prime}=R N_{A}+T N_{B} . \tag{25}
\end{equation*}
$$

From $T+R=1$ it follows that $N_{A}^{\prime}+N_{B}^{\prime}=N_{A}+N_{B}$.
Let relations $\left\langle a^{+2} b^{2}\right\rangle=\left\langle a^{+2} a b\right\rangle=\left\langle b^{+2} b a\right\rangle=0$ be satisfied at the input. Then Eqn (23) leads to the following relations:

$$
\begin{align*}
G_{A A}^{\prime} & =T^{2} G_{A A}+R^{2} G_{B B}+4 T R G_{A B}, \\
G_{B B}^{\prime} & =R^{2} G_{A A}+T^{2} G_{B B}+4 T R G_{A B}, \\
G_{A B}^{\prime} & =T R\left(G_{A A}+G_{B B}\right)+(T-R)^{2} G_{A B}, \tag{26}
\end{align*}
$$

where $G_{A A}=\left\langle a^{+2} a^{2}\right\rangle, G_{B B}=\left\langle b^{+2} b^{2}\right\rangle, G_{A B}=\left\langle a^{+} b^{+} b a\right\rangle$. If $T+R=1$ these transformations possess the property of invariance ('the law of correlations and fluctuations sum conservation'):

$$
\begin{equation*}
G_{A A}^{\prime}+G_{B B}^{\prime}+2 G_{A B}^{\prime}=G_{A A}+G_{B B}+2 G_{A B} \tag{27}
\end{equation*}
$$

Let the input moments of the $A$ and $B$ beams be equal $\left(N_{A}=N_{B}=N, G_{A A}=G_{B B}=G\right)$. In this case, the output moments are also symmetric and from Eqns (26) one finds the following relations between the moments normalized by $N^{2}$ :

$$
\begin{align*}
& g^{\prime}=(1-x) g+2 x g_{A B}, \quad g_{A B}^{\prime}=x g+(1-2 x) g_{A B}, \\
& g^{\prime}+g_{A B}^{\prime}=g+g_{A B} \tag{28}
\end{align*}
$$

Here the following notations are used

$$
\begin{aligned}
& x=2 T R=2 T(1-T)=0,5 \sin ^{2}(2 \alpha) \\
& 1-2 x=\cos ^{2}(2 \alpha), \quad T=\cos ^{2} \alpha, \quad R=\sin ^{2} \alpha
\end{aligned}
$$

(In the case of the polarization-dependent beamsplitter, $\alpha$ is the angle of the prism rotation, and in the case of the Mach-Zehnder interferometer, it is one half of the optical path difference; see Section 4.)

The fluctuation transformations into (anti)correlation and back production by the beamsplitter are described by the relations (28). According to Eqn (28) the increase (decrease) of the output intensity fluctuations due to the change of the beamsplitter parameters (for the unchanged statistics of the incident light) is accompanied by the decrease (increase) of the correlations between the output intensities: $\partial g_{A B}^{\prime} / \partial x=-\partial g^{\prime} / \partial x$.

Actually, by the term 'beamsplitter' one can designate any four-pole described by a $S U(2)$ matrix with the first row elements $t$, $r$ and mixing two modes (distinguished by the polarization or the propagation direction) - the Nicol prism, the Mach-Zehnder interferometer etc.- or the succession of elements of this kind described by the product of the corresponding matrices. If parameters $t, r$ are subjected to dispersion the given relationships are valid in the spectral representation only.

The fluctuations and correlations can also be characterized by the dispersion $\sigma_{A}^{2}=G_{A A}+N_{A}-N_{A}^{2}$, and the correlation coefficient $K$ normalized by the dispersion [see Eqn (2)]. For the symmetric excitation,
$K=\left(g_{A B}-1\right) \frac{N^{2}}{\sigma^{2}}, g=\frac{\sigma^{2}}{N^{2}}+1-\frac{1}{N}, g_{A B}=K \frac{\sigma^{2}}{N^{2}}+1$,
so from Eqn (28) one gets
$\frac{\sigma^{\prime 2}}{\sigma^{2}}=\frac{1+K}{1+K^{\prime}}$.
Hence, it again follows that the correlation increase at the output is accompanied by the fluctuation decrease, and vice versa.

Exactly the same relationships follow from the classical theory. Thus, the quantum specific by no way reveals itself in the course of the field propagation through an optical channel. It can reveal itself only in the peculiarities of the input linear scheme and in the detection process.

Transformations (26) describe a number of optical effects observed with the help of the parametric sources of two- photon light and which are called photon anticorrelations, two-photon interference, and so on (see below). Actually, however, these effects have no connection with the M-photon properties but reflect the statistics of the used light source and the properties of the transformation (26), which in their turn follow from the transformations of the $S U(2)$-type [Eqn (23)], common to the $\mathrm{Q}^{-}$and $\mathrm{C}^{*}$-languages.

The inverse to the transformations given by Eqn (22) determine the input moments through output ones, i.e. give a solution of the inverse problem - a reconstruction of the incident light properties from the detected data. The pragmatic value of such experiments lies in this very possibility. Moreover, these experiments can be used for measuring the optical channel parameters: dispersion, group-delay [4, 36], detector efficiency [46].

Let us now examine the nonunitary transformations [37, 45]. Let the input mode b not be excited. In the classical theory it means that

$$
\begin{equation*}
b=0, \quad a^{\prime}=t a, \quad b^{\prime}=-r^{*} a, \tag{31}
\end{equation*}
$$

and in the quantum theory it means that the initial state is of the form $|\psi\rangle=|\psi\rangle_{A}|v a c\rangle_{B}$. Transformations (31) are nonunitary. They break the commutation relations. For example, now $\left[a^{\prime}, a^{\prime+}\right]$ is equal to $T$ instead of unity. However, they can be used in the preliminary normally ordered expressions. This approach simplifies significantly the computations and gives the same results as the computations with the complete expressions (23), which take into account the initial state $|\psi\rangle_{A}|\mathrm{vac}\rangle_{B}$ only at the end of the computations.

Frequency or space filtration are also nonunitary operations. Thus, any detector 'sees' a limited number of modes which can be taken into account by the quantum efficiency factor $\eta(\boldsymbol{k})$. The screens and diaphragms surrounding the detector restrict its emission diagram $\eta(\vartheta, \varphi)$, and the photocathode does the same with its frequency characteristics $\eta(\omega)$.

Moreover, the optical channel usually includes additional frequency filters characterized by some amplitude characteristics $D(\omega)$. All these factors can be taken into account in the complete scattering matrix $D$ of the whole system including the detectors, although this transforms the matrix into a nonunitary one, i.e. nonconserving the commutation relations. Similar to the case discussed above of two modes mixing, this does not prevent this matrix from being used in calculations with preliminary-ordered operator functions. As a result, the classical and quantum descriptions retain similar forms even if the dissipation is taken into account. Such a simple description of the
nonunitary transformations is unlikely to be possible in the Schrodinger representation.

An analogous formalism is also applicable to the case of more complicated (multimode) optical schemes which may contain resonators and linear parametric downconverters [29]. With the help of the nonunitary scattering matrix the functioning of a lens can also be described [47].

Let the effective detector cross-section be much less than the coherence square of the incident radiation. Under this condition the detector sees only one transverse mode with the output amplitude,

$$
\begin{equation*}
A(t, z)=T^{-1 / 2} \sum_{\omega} D(\omega) a(\omega) \exp [-\mathrm{i} \omega(t-z / c)] \tag{32}
\end{equation*}
$$

Here $T$ is the periodicity interval which does not enter the final expression; $z$ is the coordinate of the observation point along the beam axis. Note, that for $D \propto \sqrt{\omega}$ the operator $A(t)$ is proportional to the positive-frequency term of the electric field operator $E^{(+)}(t)$ but for $D=\sqrt{\eta}$ it takes into account the quantum efficiency of the detector, and the meaning of $r(t)=A^{+}(t) A(t)$ is the photo electron flow per unit temporal interval [the dimension of $A(t)$ is $\left.\mathrm{s}^{-1 / 2}\right]$.

If the frequency transmission band of the filter is much less than $1 / T_{\text {det }}$ (where $T_{\text {det }}$ is the time constant of the detector), it is possible to limit the consideration to only one (central) longitudinal mode with the amplitude $a\left(\omega_{0}\right)=a$. In this case the field dynamics is lost; however, this is not of principal interest because it is the same in both quantum and classical theories. These considerations are used as a basis for the applicability conditions of the frequently used one-mode approximation (for each beam).

Qs. In the quantum theory there is an alternative possibility for the field evolution description: the description in the Schrodinger representation (the classical analog is the Fokker-Planck equations application) where the optical system transforms not the operators $a$ and $b$ but the field state vector $|\psi\rangle$. For transition to this representation let us express the initial state vector at the beamsplitter input $|\psi\rangle$ as the result of action of some combination of the creation and annihilation operators at the vacuum state $\left.|\psi\rangle=f\left(a^{+}, b^{+}\right) \mathrm{vac}\right\rangle$.

The transformation inverse of Eqn (23) is described by the Hermitian conjugated matrix $D^{-1}=D^{+}$:

$$
\begin{equation*}
a=t^{*} a^{\prime}-r b^{\prime}, \quad b=r^{*} a^{\prime}+t b^{\prime} \tag{33}
\end{equation*}
$$

Substitution of the Hermitian conjugated expressions in function $f$ determines the field state vector at the beamsplitter output:

$$
\begin{equation*}
|\psi\rangle^{\prime}=f\left(t a^{\prime+}-r^{*} b^{\prime+}, r a^{\prime+}+t^{*} b^{\prime+}\right)|\mathrm{vac}\rangle . \tag{34}
\end{equation*}
$$

In particular, if there is a vacuum state at the input of beam B, the initial state vector can be presented in the form $|\psi\rangle=f\left(a^{+}\right)|\mathrm{vac}\rangle=f\left(a^{+}\right)|0\rangle_{A}|0\rangle_{B}$, which gives at the output $\left|\psi^{\prime}\right\rangle=f\left(t a^{\prime+}-r^{*} b^{\prime+}\right)|0\rangle_{A}^{\prime}|0\rangle_{B}^{\prime}$.

Let one $Q-$ photon be at the $A$-beam input, i.e. $f\left(a^{+}\right)=a^{+}=t a^{\prime+}-r^{*} b^{\prime+}$. Then

$$
\begin{align*}
\left|\psi^{\prime}\right\rangle & =\left(t a^{\prime+}-r^{*} b^{\prime+}\right)|0\rangle_{A}^{\prime}|0\rangle_{B}^{\prime} \\
& =t|1\rangle_{A}^{\prime}|0\rangle_{B}^{\prime}-r^{*}|0\rangle_{A}^{\prime}|1\rangle_{B}^{\prime} \tag{35}
\end{align*}
$$

According to the projection postulate, the probability amplitudes for observing the photons in exit beams $A$ and
$B$ are of the form $\left\langle\psi^{\prime} \mid 1\right\rangle_{A}^{\prime}=t^{*},\left\langle\psi^{\prime} \mid 1\right\rangle_{B}^{\prime}=-r$. From here one finds the probabilities themselves:

$$
\begin{align*}
\left|\left\langle\psi^{\prime} \mid 1\right\rangle_{A}^{\prime}\right|^{2} & =|t|^{2}=T \\
\left|\left\langle\psi^{\prime} \mid 1\right\rangle_{B}^{\prime}\right|^{2} & =|r|^{2}=R=1-T \tag{36}
\end{align*}
$$

Sometimes the transition amplitudes from the initial to some final state are considered, which leads to the same result:

$$
\begin{align*}
\langle\psi \mid 1\rangle_{A}^{\prime} & ={ }_{A}\langle 1 \mid 1\rangle_{A}^{\prime}={ }_{A}\langle 1| a^{\prime+}|\mathrm{vac}\rangle \\
& ={ }_{A}\langle 1| t^{*} a^{+}+r^{*} b^{+}|\mathrm{vac}\rangle=t^{*}, \\
\langle\psi \mid 1\rangle_{B}^{\prime} & ={ }_{A}\langle 1 \mid 1\rangle_{B}^{\prime}={ }_{A}\langle 1| b^{\prime+}|\mathrm{vac}\rangle \\
& ={ }_{A}\langle 1|-r a^{+}+t b^{+}|\mathrm{vac}\rangle=-r . \tag{37}
\end{align*}
$$

Let the input state now be a two-photon one with one photon each in modes $A$ and $B$ :

$$
\begin{equation*}
|\psi\rangle=a^{+} b^{+}|\mathrm{vac}\rangle=|1,1\rangle, \quad|m, n\rangle=|m\rangle_{A}|n\rangle_{B} \tag{38}
\end{equation*}
$$

In this case $f(x, y)=x y$ and according to Eqn (34) one gets

$$
\begin{align*}
|\psi\rangle^{\prime} & =\left(t a^{\prime+}-r^{*} b^{\prime+}\right)\left(r a^{\prime+}+t^{*} b^{\prime+}\right)|\mathrm{vac}\rangle \\
& =\operatorname{tr}|2,0\rangle+(T-R)|1,1\rangle-t^{*} r^{*}|0,2\rangle \tag{39}
\end{align*}
$$

In particular, at $T=R=0.5$ this gives

$$
\begin{equation*}
|\psi\rangle=0.5(|2,0\rangle-|0,2\rangle) \tag{40}
\end{equation*}
$$

In the multimode case the transformed state vector can also be expressed analogously with the aid of the phenomenological scattering matrix of the channel [21].

The comparison of the presented relationships shows that the $\mathrm{Q}_{\mathrm{H}}$-language is more advantageous than the $\mathrm{Q}_{\mathrm{S}^{-}}$ language in terms of the compactness, universality, and closeness to the $\mathrm{C}^{*}$-language.

The description of field evolution in other languages will be discussed later together with the examination of specific effects.

### 3.3 Detection

$\mathbf{Q}_{\mathbf{H}}$. The currently accepted 'standard model' of photon detection (see Ref. [35]) evidently describes well all the experimental observations. The dependencies of the output counting rate for two or more detectors $R_{A}, R_{B}, \ldots$ and the coincidence counting rate for detector pairs $R_{A B}, \ldots$ on the different parameters of the source and the detecting devices are, in fact, the main result of the modern demonstration experiments of quantum optics. The corresponding averages are obtained as a result of time averaging (sometimes for hours) under the stationary macroscopic conditions.

In order to compare with the theoretical averages over the quantum ensemble $\langle\ldots\rangle$ one needs to accept the ergodic hypothesis, i.e. to assume that the source repeatedly produces the field just in the same state. (This problem has been poorly investigated in quantum optics.) The counting rates are determined by the formulae,

$$
\begin{align*}
& R_{A}=\left\langle r_{A}(t)\right\rangle=\left\langle A^{+}(t) A(t)\right\rangle=\eta_{A} N_{A} \Delta v_{A}, \\
& R_{B}=\left\langle r_{B}(t)\right\rangle=\left\langle B^{+}(t) B(t)\right\rangle=\eta_{B} N_{B} \Delta v_{B} \tag{41}
\end{align*}
$$

where $\Delta v_{A, B}=\Delta \omega_{A, B} / 2 \pi$ are the effective frequency bands of the emission for the beams, and $N_{A, B}$ are the average photon numbers in the central longitudinal modes. The
quantum efficiencies of the detectors $\eta_{A}$ and $\eta_{B}$ are included in the definitions of operators $A$ and $B$ [see Eqn (32)].

Let us examine an idealized case, where between the characteristic times of detectors, coincidence circuit, and field, there is the following relationship: $T_{\text {det }} \ll T_{\text {coin }} \ll \tau_{\text {coh }}$. The anticoincedence counting rate can be presented in the form of

$$
\begin{equation*}
R_{A B}=T_{\text {coin }}\left\langle A^{+}(t) A(t) B^{+}(t) B(t)\right\rangle=R_{\text {acc }} g_{A B} \tag{42}
\end{equation*}
$$

Here $R_{\text {acc }}=T_{\text {coin }} R_{A} R_{B}$ corresponds to the 'random' coincidences observed for the independent beams $A$ and $B, g_{A B}=g_{A B}(0)$, where

$$
\begin{equation*}
g_{A B}(\tau)=\frac{1}{R_{A} R_{B}}\left\langle A^{+}(t) A(t) B^{+}(t+\tau) B(t+\tau)\right\rangle \tag{43}
\end{equation*}
$$

is the normalized intensity correlation function for the $A$ and $B$ beams.

Note that the notion of the correlation second-order function $g_{A B}(\tau)$, the Fourier transform of which determines the spectrum of the intensity fluctuations, comes from the $\mathrm{Q}_{\mathrm{H}}$-language. The same is true for the first-order correlation function $\left\langle A^{+}(t) A(t+\tau)\right\rangle$ determining the usual field spectrum.

Thus, the parameter $g_{A B}$ determines the coincidence counting rate normalized by the random counting rate. Note that parameter $g_{A B}-1$ is proportional to the correlation coefficients of the Q-photon flows in two beams [see Eqn (2)]. Note also that the opposite inequality usually holds in the experiments: $T_{\text {coin }} \gg T_{\text {det }} \gg \tau_{\text {cohh }}$. In this case one should change in Eqn (42) $g_{A B}$ by $1+\left(g_{A B}-1\right) \tau_{\text {coh }} / T_{\text {coin }}$. This quantity is usually close to unity. An exception is the two-photon light for which $g_{A B}-1=N^{-1} \gg 1$.

In the general case the observed statistics of the photon detector readings is expressed through the field correlation function and, in principle, the inverse problem can also be solved: reconstruction of the field state at the optical channel output through the observed statistic. Thus, the detectors can be considered as classical devices for the observation of a quantum object - incident light. However, this quantum object is connected by the classical laws to the initial field at the optical channel input [see Eqn (26) for the couplings of the moments], so the optical channel may be regarded as a classical part of the detecting device. In this approach various optical schemes can be used only for the investigations of the statistical properties of light sources.

In some experiments with analog detectors, the fluctuation spectrum of the detector photocurrent, which is proportional to the cube of the Fourier transform of the nonordered intensity correlation function, is measured:

$$
\begin{equation*}
\langle r(0) r(\tau)\rangle=R \delta(\tau)+R^{2} g(\tau) \tag{44}
\end{equation*}
$$

where $r(t)=A^{+}(t) A(t)$ and $R=\langle r(t)\rangle=\eta N \Delta v$ is the average photoelectron flow ( $N$ is the average photon number in the central longitudinal mode, $\Delta v$ is the effective spectral bandwidth),

$$
\begin{equation*}
g(\tau)=\frac{1}{R^{2}}\left\langle A^{+}(0) A^{+}(\tau) A(\tau) A(0)\right\rangle \tag{45}
\end{equation*}
$$

is the normalized normally ordered autocorrelation function.

The first term in Eqn (44) describes the so-called shot, or photon, noise. It is due to the noncommutivity of the
field operators: $\left[A(t), A^{+}\left(t^{\prime}\right)\right] \approx \delta\left(t-t^{\prime}\right)$. The function $g(\tau)-1$ describes the so-called excessive noise, which is absent in the case of the coherent state, but in the case of the nonclassical field states it can be compensated by the shot noise.

Note that on the right-hand side of Eqn (44) it is possible to take into account nonunitary transformations, for example, the quantum efficiency (not unity) of the photodetector $A \Rightarrow \eta^{1 / 2} A$. After this substitution the operator $r(t)$ describes the flow of photoelectrons, not of photons.

Qc. The photocurrent fluctuations can be considered as a revelation of the quantum fluctuations connected with the field energy measurement. These fluctuations should be absent in the case of the state with the definite energy, for example, for the one-photon state (with $g=0$ ). However, the conventional field states have no definite energy, which reveals itself as photocurrent fluctuations. The coherent state $(g=1)$ gives the shot noise only, which is, therefore, a characteristic quantum effect connected with the quantum fluctuations of energy and having no classical analog in stochastic optics (in the C*-language the field amplitude of an ideal laser does not fluctuate). At the same time, the excessive (above Poissonian) photocurrent noises in the case, for example, of thermal radiation allow the use, for their explanation, of a simple classical analogy: they are caused by the thermal fluctuations of the field intensity.

As was mentioned above, the coincidence detection in the case of two-photon light leads to a very high contrast $g_{A B}-1$, connected with the low probability of the random coincidence. In C-language this can be explained visually by the fact that for $N_{A}=N_{B} \ll 1$ the signal and idle packets, belonging to the different pairs, rarely overlap.
M. In the case of the metaphysical approach the detecting process plays a special role. The observed macroevent (the appearance of a pulse at the detector output, or a silver grain in the photoemulsion) is, in the M-language, accepted as a proof of the a priori existence of the quantum of light with definite but not yet completely studied properties, which is somewhere (maybe emitted at a remote star), flies through the interstellar space and the laboratory channel, crossing simultaneously two screen slits or two interferometer shoulders, and again unites as a whole at the moment of detection (do not confuse with the C-photon; see below).

However, the Q-theory gives no grounds for this or some other similar conclusion (see Section 2.5). It does not allow a unique retrodiction on the basis of a single observation, even with the a priori information on the purity of the state, similar to that in vector algebra where reconstruction of the vector based at its single projection is impossible. Thus, the M-photon materializes (like a phantom) only at the moment of its detection, to disappear again at the same moment.

In the case of the $n$-photon field an appearance of the pulse in one detector is accompanied by an even more mysterious, superluminal process-M-reduction-i.e. by the conversion of the field state into the $(n-1)$-photon state (do not confuse with the Q-reduction described for $n=2$ in Ref. [23]).
C. In the semiclassical theory the probability of the photocurrent pulse appearing at the detector output is proportional to the square of the wave-packet envelope describing the C-photon. This is why the pulse appears most frequently at the moment when the envelope maximum crosses the photocathode surface.

In the case of a coherent field state the photon packets and, correspondingly, the photocurrent pulses are distributed in time according to the Poissonian distribution, and in the case of a thermal radiation they are distributed according to the geometrical Bose-Einstein distribution. The periodic preparation of the one-photon state should give rise to equal intervals between the packets and to more uniform pulse distribution, i.e. to the sub-Poissonian statistics.

The average pulse counting rate $R=\eta N / \tau_{\text {coh }}$, i.e. $\eta N$ determines the average number pulse during the coherence time $\tau_{\text {coh }}$. At $N \ll 1$ the intervals between the packets are larger than the length of the packets themselves, i.e. packet overlapping rarely occurs.

If the subsequent amplifier has the time constant $T_{\text {amp }}$, much larger than the average interval between the pulses $1 / R$, then the pulses are smoothed, i.e. the detection becomes an analog one. And in this case there are shot noises plus (in the case $g>1$ ) excessive photocurrent noises in the amplifier band $1 / T_{\text {amp }}$. If $g<1$ the noises are below the level determined by the Schottky formula for the detector average current.

C*. In the classical theory $r(t)=A^{*}(t) A(t)$ multiplied by the factor $\sqrt{\hbar \omega}$ in the summation [see Eqn (32)] has the orientation of the light intensity in the beam which, for example, can be measured by a calorimeter. If the calorimeter inertia is sufficiently low then it is possible to estimate the statistics of the field intensity from the fluctuations of its readings. If, however, one takes into account the discrete nature of the charge the shot noises, according to the Schottky formula, should be added. These noises are not connected with the noncommutativity of the operators and cannot be suppressed at any field state.

It is possible to investigate the correlation and interference of the intensities of two beams by means of two such detectors. In this case a classical analog of the BrownTwiss effect should be observed.

## 4. Amplitude interference

Two-beam interferometers may be divided into two classes: polarization type and conventional type (like the Michelson or Mach-Zehnder interferometers). Their phenomenological descriptions are identical (see, e.g. Ref. [9]) and therefore only the latter will be discussed here.

Let us briefly examine in the Q-language the conventional interference of two modes using the example of the Mach-Zehnder interferometer depicted schemati-cally in Fig. 4.
Q. The interferometer scattering matrix is equal to the product of three matrices, describing the input and output mirrors and the difference of the optical passes $k\left(z_{1}-z_{2}\right)=2 \alpha$ between the mirrors:

$$
D=\frac{1}{2}\left(\begin{array}{cc}
1 & -1 \\
1 & 1
\end{array}\right)\left(\begin{array}{cc}
\exp (\mathrm{i} \alpha) & 0 \\
0 & \exp (-\mathrm{i} \alpha)
\end{array}\right)\left(\begin{array}{cc}
1 & 1 \\
-1 & 1
\end{array}\right)
$$



Figure 4. Schematic representation of the Mach-Zehnder interferometer.

$$
d=\left(\begin{array}{cc}
\cos \alpha & \mathrm{i} \sin \alpha  \tag{46}\\
\mathrm{i} \sin \alpha & \cos \alpha
\end{array}\right)
$$

Thus for the system as a whole, $t=\cos \alpha, r=\mathrm{i} \sin \alpha$, i.e. the interferometer possesses the properties of a beamsplitter with easily changeable (but frequency dependent) transmission $T=\cos ^{2} \alpha$ and reflection $R=\sin ^{2} \alpha$ coefficients. The signs in Eqn (46) are chosen so as to satisfy $t(0)=1$.
$\mathbf{Q}_{\mathbf{H}}$. Using the Heisenberg representation, one easily finds with the help of Eqn (23) the output intensity of beam $A$ :

$$
\begin{align*}
N_{A}^{\prime} & =\left\langle a^{\prime+} a^{\prime}\right\rangle=\left\langle\left(t^{*} a^{+}+r^{*} b^{+}\right)(t a+r b)\right\rangle \\
& =N_{A} \cos ^{2} \alpha+N_{B} \sin ^{2} \alpha-\operatorname{Im}\left\langle a^{+} b\right\rangle \sin (2 \alpha) \tag{47}
\end{align*}
$$

Here $N_{A}=\langle\psi| a^{+} a|\psi\rangle$, and $N_{B}=\langle\psi| b^{+} b|\psi\rangle$. If there exists a vacuum state at the input of beam $B$, one gets the usual result:

$$
\begin{equation*}
N_{A}^{\prime}=T N_{A}=N_{A} \cos ^{2} \alpha \tag{48}
\end{equation*}
$$

Note that this expression describes the interference independently of the initial field state which influences the average intensity $N_{A}$ only.

Formula (47) and the analogous expressions for $N_{B}^{\prime}$ and $\left\langle a^{\prime+} b^{\prime}\right\rangle$ give unique algebraic couplings between the input and output field moments in the spectral representation. The Fourier transforms of these couplings determine the relationships between the correlation functions. This procedure is easily generalizable for the higher moments [9]. The corresponding results can be symbolically presented in the form

$$
\begin{equation*}
G_{m n}^{\prime}=D^{* m} D^{n} G_{m n} \tag{49}
\end{equation*}
$$

It should be emphasized that these phenomenological relationships are of the classical type.

Qs. Let us now use the Schrodinger representation. According to Eqn (34) one has
$|\psi\rangle^{\prime}=f\left[a^{\prime+} \cos \alpha+\mathrm{i} b^{\prime+} \sin \alpha, \mathrm{i} a^{\prime+} \sin \alpha+b^{\prime+} \cos \alpha\right]|\mathrm{vac}\rangle$.

This expression should lead to the result equivalent to Eqn (47) at the arbitrary initial state [21]. However, for the sake of simplicity, we will assume that it is the one-photon state: $|\psi\rangle=|1,0\rangle$. Then $f=a^{+}=a^{\prime+} \cos \alpha+\mathrm{i} b^{\prime+} \sin \alpha$. In the result

$$
\begin{equation*}
|\psi\rangle^{\prime}=\cos \alpha|1,0\rangle+i \sin \alpha|0,1\rangle \tag{51}
\end{equation*}
$$

and thus

$$
\begin{equation*}
N_{A}^{\prime}=\left\langle\psi^{\prime}\right| a^{+} a\left|\psi^{\prime}\right\rangle=T=\cos ^{2} \alpha \tag{52}
\end{equation*}
$$

which is in agreement with Eqn (48).
The examination performed above shows that in the Heisenberg representation the interference is described in exactly the same manner as in classical optics: by the superposition of two fields with the amplitudes $a$ and $b$. The Schrodinger representation camouflages this analogy, therefore the effect is frequently interpreted as the result of the quantum interference of two transition amplitudes from the initial and the final state [see Eqn (31)] in two indistinguishable ways.
M. The last term is related to the M-language because its meaning is quite mysterious. In the translation into the Q language it means that there are no additional macroscopic parts in the interferometer, which break the coherence and change the evolution (calculated above) of the field. For example, it is possible to insert in one of the interferometer shoulders a beamsplitter which directs part of the beam to an additional detector. Similar multidetector schemes can be easily described in the $\mathrm{Q}_{\mathrm{H}}$-language (see Sections 5 and 6 ). The other languages add nothing to the calculations which predict all statistic characteristics of the photocounts, i.e. the average pulse counting rates and their correlation or anticorrelation.

In the explanation of interference in the M -language photon indistinguishability is frequently discussed. Here, in fact, the effects analogous to the classical ones and connected with the notion coherence of the fields are discussed. Analogously, the photon distinguishability in the translation from the 'newspeak' reduces to incoherence. Another popular new term, quantum eraser, is used in quantum optics at the reversible breaking and consequent restoration of the field coherence in two modes. For observations of the attenuated total internal reflection the M-term, photon tunneling, is used.

## 5. Correlation and anticorrelation of photocounts

Let us proceed to the demonstration experiments with a beamsplitter and a two-detector coincidence circuit.

### 5.1 Brown-Twiss effect

Let us first consider the case when, in the scheme presented in Fig. 5, mode $B$ is not excited. The counting rate $R_{A B}(\tau)$ is detected as the function of the difference $c \tau$ of optical passes between the beamsplitter and the photodetector. When $\tau \gg \tau_{\text {coh }}$ the coincidence counting rate becomes independent of the delay: $R_{A B}(\infty)=\mathrm{const}=R_{\text {acc }}=$ $R_{A} R_{B} T_{\text {coin }}$ (Fig. 6), where $T_{\text {coin }}$ is the coincidence circuit 'gap'. The events in channels $A$ and $B$ are independent and there are 'random' coincidences only. When $\tau \ll \tau_{\text {coh }}$ the extreme value of $R_{A B}(0)$ is usually observed.
$\mathbf{Q}_{\mathbf{H}}$. According to Eqn (42) $R_{A B}(0)=g_{A B}^{\prime} R_{\text {acc }}$. With the help of Eqn (26) at $G_{A B}=G_{B B}=0$ one finds $G_{A B}^{\prime}=T R G_{A A}$ and $g_{A B}^{\prime}=g_{A A}$.

Thus, a beamsplitter, two detectors, and a coincidence circuit enable one to measure parameter $g_{A A}$ characterizing the bunching or antibunching of intensities in the incident light:


Figure 5. Schematic representation of the Brown-Twiss experiment. The input beam of the amplitude $a$ is incident at the beamsplitter and is divided into two mutually coherent beams. The intensity correlation in the output beams, as a function of the relative delay time $\tau$, is measured by two detectors $A$ and $B$ in the coincidence circuit.


Figure 6. The normalized intensity correlation $g$ versus the relative delay $\tau$ for the different states of incident light: one-photon (a), coherent (b), thermal (c), two-photon (d).

$$
g_{A A}=g_{A B}^{\prime}=\frac{R_{A B}(0)}{R_{\mathrm{acc}}}
$$

The relationship $g_{A B}^{\prime}=g_{A A}$ means that the beamsplitter transforms the input intensity fluctuations into the output intensity correlations. Fig. 6 schematically depicts the Brown-Twiss effect for the one-photon (a), coherent (b) and two-photon light.

The exceeding of $g_{A A}$ by unity in the case of thermal radiation was first discovered by Brown and Twiss about forty years ago [48, 49]. This experiment is considered as one of the first in quantum optics. The discovered effect caused a lively discussion at that time; however, in the Qand $\mathrm{C}^{*}$-languages it gives only the evidence of 'excessive' fluctuations in the used light source. More astonishing from the classical point of view is the photon antibunching effect $\left(g_{A A}<1\right)$ discovered in 1977 in the light of the resonant fluorescence of individual atoms [50].

Qs. In the Schrodinger representation we shall consider only the case (a) presented in Fig. 6. For the one-photon state of the input mode $A$ the output state vector, according to Eqn (35), is of the form
$\left|\psi^{\prime}\right\rangle=\left(t a^{\prime+}-r^{*} b^{\prime+}\right)|0\rangle_{A}^{\prime}|0\rangle_{B}^{\prime}=t|1\rangle_{A}^{\prime}|0\rangle_{B}^{\prime}-r^{*}|0\rangle_{A}^{\prime}|1\rangle_{B}^{\prime}$
Here the Q-photon belongs to the two output modes. As a result the coincidence probability is equal to zero: $\left.\left|\left\langle\psi^{\prime} \mid 1\right\rangle_{A}^{\prime}\right| 1\right\rangle\left._{B}^{\prime}\right|^{2}=0$, i.e. the effect of complete anticorrelation of photocounts takes place (Fig. 6a). This also follows directly from $g_{A A}=\langle 1| a^{+2} a^{2}|1\rangle=0$.

Qc. The Copenhagen language does not allow us to pose 'inappropriate' questions. The photocount statistics,
according to the measurement postulate and the standard photodetection model, is determined by the statistics of the field incident at detectors, and in the Heisenberg representation the problem is completely resolved by the phenomenological couplings [Eqn (26)] between the input and output moments and the statistics of the incident light.
M. The Brown-Twiss effect at $g_{A B}^{\prime}>1$ is explained sometimes by the M-photon tendency to bunching. The opposite inequality probably corresponds to antibunching, i.e. to the M-photon repulsion, respectively. Such speculative conclusions (explanation of the statistics property of a separate quantum ensemble by the individual properties of its constituent hypothetical particles) are characteristic for metaphysics.

In the experiments [51] the output intensity correlation was observed in the course of modulation of the beamsplitter transmissivity by the noise radio-frequency signal. The results of the experiment were interpreted as a tendency of the M-photon to obey sometimes the types of statistics intermediate between the Bose and Fermi statistics. In the Q- and C-languages similar experiments with modulated optical parameters are described by the couplings [Eqn (26)], where $T$ and $R$ are the functions of time.
C. If two-photon packets belong to the common coherence volume it seems natural that the beamsplitter is able to divide and direct them to the different detectors so that the detection is almost simultaneous. (Sometimes the appearance of such 'tight pairs' of C-photons in the thermal radiation is explained by the induced emission in the light source.) And vice versa, if the photons arrive at the beamsplitter one by one at equal temporal intervals, there will be no coincidences. In the experiments [50] the latter was caused by the finite time (order of the reciprocal Rabi frequency) of the consequent atom excitation for the resonant fluorescence.

C*. In the first Brown - Twiss experiments analog detectors were actually used and the correlation of the two current noises was detected. A simple classical interpretation of this effect is possible: the input intensity fluctuation at the beamsplitter should lead to the output intensity correlations and, correspondingly, to the photocurrent correlation.

### 5.2 Detection of anticorrelations

Let us now consider a different detection scheme, which is, evidently, close to the one used in Ref. [18]. In fact, this is again the problem of the count anticorrelation in the output beams (Fig. 6a), although for rather different electronic pro-cessing of the output pulses. Let us be interested not in the coincidences but in their absence, i.e. in the anticoincidences.
$\mathbf{Q}_{\mathbf{H}}$. The probability of the pulse to be detected by detector $A$, depicted in Fig. 5, during some time $\Delta t$ much larger than the pulse duration is, according to Eqn (41), equal to $P_{A}(1)=R_{A}^{\prime} \Delta t=\eta_{A} N_{A}^{\prime} \Delta v \Delta t$ (the events connected with two or more pulses in a single detector are not taken into consideration). This probability is, evidently, equal to the sum of mutual probabilities, taking into account the alternative events: appearance or absence of the pulse in detector $B$ during the same interval, i.e.

$$
\begin{equation*}
P_{A}(1)=P_{A B}(1,1)+P_{A B}(1,0) . \tag{54}
\end{equation*}
$$

The coincidence probability here is of a familiar form [see Eqn (42)]:
$P_{A B}(1,1)=R_{A B} \Delta t=g_{A B}^{\prime} R_{A}^{\prime} R_{B}^{\prime}(\Delta t)^{2}=g_{A B}^{\prime} P_{A}(1) P_{B}(1)$.

From Eqns (54) and (55) one finds the sought-for anticorrelation probability $P_{A B}(1,0)$, normalized by the probability of the count in detector $A$ :

$$
\begin{equation*}
\frac{P_{A B}(1,0)}{P_{A}(1)}=1-\frac{P_{A B}(1,1)}{P_{A}(1)}=1-g_{A A} R_{B}^{\prime} \Delta t \tag{56}
\end{equation*}
$$

(we took into account the equality of the normalized input and output moments, $g_{A B}^{\prime}=g_{A A}$ ).

Thus, for a sufficiently low counting rate in the $B$ channel $\left(R_{B}^{\prime} \ll 1 /\left(g_{A A} \Delta t\right)\right.$ the anticorrelations are observed: the pulses in the $A$ channel are not accompanied by the pulses in the $B$ channel. This affirmation is true for any incident light state which influences the bunching parameter $g_{A A}$ only and, consequently, the threshold value of the counting rate $R_{B}^{\prime}=1 /\left(g_{A A} \Delta t\right)$ above which the probability drops. In particular, in the case of the one-photon state, $g_{A A}=0$, and therefore the coincidences are absent at any counting rate (of course, inside the limits of the accepted assumptions).

It follows from the aforementioned that the detection of coincidences $P_{A B}(1,0)=P_{A}(1)$ themselves is not the proof of light being nonclassical (in the sense of $g<1$ ). The only pragmatic result of these experiments is the measurement of the incident light bunching parameter $g_{A A}$, similarly to the case of the coincidences detection.
M. Observation of coincidences is accepted as the proof of M-photon indivisibility in the M-language, of its corpuscular nature. Usually, the following condition (which is not fulfilled in any experiment) is added: the incident light should be one-photon.

Moreover, if a tunnel beamsplitter is used for beamsplitting [19] i.e. attenuated total internal reflection (which is a wave effect), as was done in Ref. [18], then contrary to the traditional point of view, each anticoincidence detection demonstrates the duality of the M-photon in the same experimental situation. Note, however, that the performance of any beamsplitter, for example, of a semitransparent mirror or a polarization prism is also based on the wave nature of light.
C. The absence of coincidences in the case of the onephoton source follows in the semiclassical theory from the postulate on the C-reduction of a photon packet at the moment of detection. If, however, the coincidences are detected, they are explained by the incidence at the beamsplitter of two photons separated by the temporal interval less than $\Delta t$.
$\mathbf{C}^{*}$. In pure classical theory it is, evidently, possible to consider only the analog mode of detection of two photocurrents and their correlation and anticorrelation in detectors $A$ and $B$. Anticorrelation, when only one input mode is excited, apparently cannot be observed.

### 5.3 Anticorrelation effect

Now, in the scheme depicted in Fig. 5, let both input modes be excited.
$\mathbf{Q}_{\mathrm{H}}$. According to Eqn (26) the output moments are uniquely determined by the input moments and by the intensity transmission coefficient $T$. Let the input moments in modes $A$ and $B$ be the same and $T=0.5$. Then

$$
\begin{equation*}
g^{\prime}=0.5 g+g_{A B}, \quad g_{A B}^{\prime}=0.5 g \tag{57}
\end{equation*}
$$

Thus, at $g_{A B}<0.5 g$ (weak correlation of the input beams) the transformation results in the bunching decrease: $g^{\prime}<g$. If $g<2$ (i.e. the bunching in the input beams is less than the thermal one) the output beams are anticorrelated: $g_{A B}^{\prime}<1$ (independently of the initial correlation).

As will be shown below, the effect of photocount anticorrelations at the beamsplitter output has a simple classical analogy (contrary to the case of a single incident beam)-namely, the anticorrelation of the continuous intensities - and, correspondingly, the photocurrents are due to the energy conservation law and the phase fluctuations in the incident light.

In the quantum theory $g_{A B}^{\prime}$, as well as $g$, is allowed to be zero; however, in the $\mathrm{C}^{*}$-theory from $g_{\text {class }} \geqslant 1$ follows $g_{A B}^{\prime} \geqslant 0.5$, and the sign of equality is reached if the fluctuations are absent, when $g=1$. This limitation, however, does not prevent complete anticorrelation ( $K=-1$ ) of the output intensities.

In the case of modes differing by the polarization type they can be mixed with the help of the Nicol prism (with $T=\cos ^{2} \alpha$, where $\alpha$ is the prism orientation angle). In this case the examined effect reveals itself as a hidden polarization of the single output beam [9]. In reality, the equality $N_{A}^{\prime}=N_{B}^{\prime}$ means that at any $\alpha$ the beam is not polarized in the conventional sense of the term. However, if fluctuations or intensity correlations in the output modes (with orthogonal polarizations) are detected the beam shows a transverse structure with a fourfold symmetry axis.

Let us now consider several types of initial field statistics.

1. In the case of thermal input beams $g=2$, $\sigma^{2}=N+N^{2}$, and therefore at the output $g^{\prime}=1+g_{A B}$, $g_{A B}^{\prime}=1$, i.e. the beams become uncorrelated independently of the initial correlation. The bunching (fluctuations) increases if there is initial correlation $\left(g_{A B}>1\right)$ and decreases if there is initial anticorrelation.
2. In the case of two independent input beams, $g=g_{A B}=1$, and bunching (fluctuation increase) and anticorrelation occur at the output: $g^{\prime}=1.5, g_{A B}^{\prime}=0.5$ and $K=-1 /\left(1+2 N^{-1}\right)$.
3. In the case of a two-mode squeezed vacuum, at the input [according to Eqn (6)] $g=2, \sigma^{2}=N+N^{2}$ (thermal fluctuation in the modes) and $g_{A B}=2+N^{-1}, K=1$ (complete correlation of the modes independently of the amplification) and at the output $g^{\prime}=3+N^{-1}, g_{A B}^{\prime}=1$, $\sigma^{2}=2 \sigma^{2}$, i.e. the correlation is suppressed and the dispersion doubled. The invariant [Eqn (27)] occurs here in the form $4 N^{2}+N$.

For the arbitrary transmission coefficient $T=\cos ^{2} \alpha$ one finds from Eqn (26) the interference dependence on $\alpha$ :

$$
\begin{equation*}
g_{A B}^{\prime}=1+\left(1+N^{-1}\right) \cos ^{2}(2 \alpha) \propto 1+V \cos (4 \alpha) \tag{58}
\end{equation*}
$$

with the visibility $V=(1+N) /(1+3 N)$ approaching $1 / 3$ in the case of classically squeezed light $(N \gg 1)$ and approaching 1 in the case of low squeezing $(N \ll 1)$.

If a large delay is introduced in one of the incident beams, the beams become independent. When the initial correlation is conserved: $g_{A B}^{\prime}=g_{A B} \quad[$ it follows from Eqn (58) at $\alpha=0]$. Thus the 'contrast' of the effect (changing of the counting rate by introduction of a delay) at $T=R=0.5$ is equal to $2+N^{-1}$.

The anticorrelation effect was observed in many works with the help of two-photon light when $N \ll 1$ (see Refs [4, 6]). It can be used for measurements of the group of femtosecond delays $[4,6]$.
4. In the case of a symmetric two-photon state, at the input $|\psi\rangle=|1,1\rangle N=1, g=\sigma^{2}=0$ (complete antibunching), $g_{A B}=1$, and at the output $N^{\prime}=g^{\prime}=\sigma^{\prime 2}=1, g_{A B}^{\prime}=0$, i.e. the fluctuations are present and the correlations are suppressed.

Qs. The anticorrelation effect is 'explained' in the most simple way in Q-photon terms in case 4 (two-photon state at the input). Remember that in the $\mathrm{Q}_{\mathrm{S}}$-language the transformation being performed by the $50 \%$ beamsplitter according to Eqn (40) is described by the following formula,

$$
\begin{equation*}
|\psi\rangle=|1,1\rangle \Rightarrow|\psi\rangle^{\prime}=0.5(|2,0\rangle-|0,2\rangle) \tag{59}
\end{equation*}
$$

Thus, for $T=0.5$ the output state vector does not contain component $|1,1\rangle$ with one Q-photon in each beam, which results in the coincidence of photocounts. The anticorrelation effect allows the $2 J$-photon generalization: in the case of state $|J, J\rangle$, at the input the transmission coefficient $T$ has $J$ values reducing to zero component $|J, J\rangle$ in the output state vector [9]. This effect reflects the property of the $S U(2)$ matrix with the $(2 J+1)^{2}$ dimension: its central element is the Legendre polynomial $P_{J}(T-R)$, i.e. the effect is the consequence of the model symmetry, but not of its quantum specific.
M. The M-language is usually used for discussing the action of a beamsplitter on the state of the form $|\psi\rangle=|1,1\rangle$, and in the course of the discussion some conclusions about the M-photon properties are also derived. For example, the absence in $|\psi\rangle^{\prime}$ of component $|1,1\rangle$, leading to the coincidences, is explained by the wave 'component' of the M-photon, and the presence in $|\psi\rangle^{\prime}$ of components $|2,0\rangle$ and $|0,2\rangle$ is explained by the corpuscular 'component'.
C. In the semiclassical theory, case 4 (two-photon input state $|1,1\rangle$ ) can be modeled in the following way. Two photon packets with random relative phase $\varphi$ strike a semitransparent mirror from time to time, simultaneously from both sides. Each packet is a piece of a sinusoid with random phase and amplitude equal to unity.
$\varphi=0$ results in $\left|a^{\prime}\right|=\sqrt{2}, b^{\prime}=0$, i.e. both photons go through channel $A$, which corresponds to the output state $|2,0\rangle$ and to the absence of coincidences. $\varphi=\pi$ results in $a^{\prime}=0,\left|b^{\prime}\right|=\sqrt{2}$, i.e. both photons go through channel $B$, which corresponds to the output state $|0,2\rangle$ and also to the absence of coincidences. However, for all other values of the phase there is the finite field amplitude in both output channels and some probability of the photocount coincidence.

Thus, C-language does not allow complete anticorrelation of the discrete photocounts.

C** Let two quasimonochromatic waves with equal and stable incident amplitudes, which we assume to be equal to unity, and with independently drifting phases $\alpha(t)$ and $\beta(t)$ be incident at a beamsplitter. Assuming $a=\exp [\mathrm{i} \alpha(t)]$ and $b=\exp [\mathrm{i} \beta(t)]$, one finds the amplitudes at the output of a $50 \%$ beamsplitter: $a^{\prime}=(a+b) / \sqrt{2}$ and $b^{\prime}=(-a+b) / \sqrt{2}$. The output intensity can be presented in the form

$$
n_{A}^{\prime}=\left|a^{\prime}\right|^{2}=1+\cos \varphi(t), n_{B}^{\prime}=\left|b^{\prime}\right|^{2}=1-\cos \varphi(t)
$$

where $\varphi=\alpha-\beta$.
Thus, depending on the existing difference of phases $\varphi(t)$, the intensity is being redistributed in a random manner between two output channels. The total energy $n_{A}+n_{B}=2$ is conserved and this is why $\mathrm{d} n_{A}^{\prime} / \mathrm{d} t=-\mathrm{d} n_{B}^{\prime} / \mathrm{d} t$, i.e. the intensities are always changing in opposite directions, which leads to their anticorrelation.

If at the input, $g=g_{A B}=1, \sigma=0$, then at the output for the homogeneously distributed $\theta$ one has $N_{A}^{\prime}=N_{B}^{\prime}=1$, and

$$
\begin{align*}
& g^{\prime}=\left\langle\left\langle n_{A}^{\prime 2}\right\rangle\right\rangle=\left\langle\left\langle n_{B}^{\prime 2}\right\rangle\right\rangle=1+\left\langle\left\langle\cos ^{2} \varphi(t)\right\rangle\right\rangle=1.5, \\
& g_{A B}^{\prime}=\left\langle\left\langle n_{A}^{\prime} n_{B}^{\prime}\right\rangle\right\rangle=1-\left\langle\left\langle\cos ^{2} \varphi(t)\right\rangle\right\rangle=0.5, \\
& \sigma^{2}=\left\langle\left\langle n_{A}^{\prime 2}\right\rangle\right\rangle-N_{A}^{\prime 2}=0.5 \tag{61}
\end{align*}
$$

Therefore the intensity correlation coefficient $K^{\prime}=\left(G_{A B}^{\prime}-N_{A}^{\prime} N_{B}^{\prime}\right) / \sigma^{2}=-1$, i.e. complete intensity anticorrelation occurs.

Let one plane wave with amplitudes $a$ and $b$ of the orthogonal polarization be presented. The fluctuations of their relative phase $\varphi(t)$ result in the fluctuations of the polarization state of the input field. Now the role of the semitransparent mirror is played by the polarization prism that transforms the polarization fluctuation into the anticorrelated fluctuations of the output intensities.

Taking the initial fluctuations into consideration will complicate the description of the effect, but will not change its essence. A beamsplitter is a phase detector transforming the fluctuations of the relative phase of the input signals into the anticorrelated fluctuations of the output intensities.

## 6. Intensity interference

The effect of intensity interference (see, e.g. Ref. [35], page 107) is closely linked with the Brown-Twiss effect discussed above. It turns out that, under some conditions, the observed intensity (anti)correlation at the optical channel output is dependent on the definite combination of optical paths. The simplest scheme consists of four beams which are mixed either directly at the detector surfaces, or with the help of beamsplitters (or, in the polarization sensitive case, with Nicol prisms) and two detectors working in the coincidence mode (or with an analog correlometer). The mathematical descriptions of the polarization and usual interferometers have much in common (in fact, they are isomorphic; see Refs [9, 13]). There are a number of experimental versions of the observation of the effect, which differ in optical schemes and in the statistics of the light sources used [9, 13].

If two-photon or squeezed light (quantum or classical) is used, the effect is unusually dependent on the optical path
lengths. Recently some interesting experiments involving two-photon parametric light have been described [3-6]. The experiments with the sources (nonexistent now) of the three- and four-photon light are discussed [10-13].

We shall discuss below several typical schemes, which will allow us to compare different languages applicable to the description of the effect .

### 6.1 Two types of intensity interference

Let us consider the scheme depicted in Fig. 7. The controllable phase retardation $\alpha=k_{a}\left(z_{a_{1}}-z_{a_{2}}\right)$ between the input fields $a_{1}$ and $a_{2}$, each of frequency $\omega_{a}$, is introduced before mixing. Analogously, the phase retardation $\beta=k_{b}\left(z_{b_{1}}-z_{b_{2}}\right)$ between the field $b_{1}$ and $b_{2}$ of the frequency $\omega_{b}$ is also introduced.


Figure 7. Schematic representation of the four-mode intensity interferometer. Fields $a_{1}$ and $a_{2}$, and also $b_{1}$ and $b_{2}$, are mixed by the beamsplitters. The observed intensity (anti)correlation of the output fields $a$ and $b$ is periodically dependent on the phase delays $\alpha$ and $\beta$. Depending on the incident field statistics, two types of statistics are possible: with the phase $\alpha+\beta$ (two-photon interference or advanced wave interference) and with the phase $\alpha-\beta$ (the Brown-Twiss intensity interference). Both effects are trivially explained in the framework of the classical approach: they arise as a result of transformation by the beamsplitter of the incident field relative to phase fluctuations into the amplitude anticorrelation.
$\mathbf{Q}_{\mathbf{H}}$. For the output amplitude at $t=r=1 / \sqrt{2}$, taking into account the delays, one gets

$$
\begin{align*}
& a=\frac{1}{\sqrt{2}}\left[a_{1} \exp (-\mathrm{i} \alpha / 2)+a_{2} \exp (\mathrm{i} \alpha / 2)\right] \\
& a_{0}=\frac{1}{\sqrt{2}}\left[-a_{1} \exp (-\mathrm{i} \alpha / 2)+a_{2} \exp (\mathrm{i} \alpha / 2)\right] \\
& b=\frac{1}{\sqrt{2}}\left[b_{1} \exp (-\mathrm{i} \beta / 2)+b_{2} \exp (\mathrm{i} \beta / 2)\right] \\
& b_{0}=\frac{1}{\sqrt{2}}\left[-b_{1} \exp (-\mathrm{i} \beta / 2)+b_{2} \exp (\mathrm{i} \beta / 2)\right] \tag{62}
\end{align*}
$$

i.e. the scattering matrix for the upper part of the scheme shown in Fig. 7 is of the form,

$$
D(\alpha)=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}
\exp (-\mathrm{i} \alpha / 2) & \exp (\mathrm{i} \alpha / 2)  \tag{63}\\
-\exp (-\mathrm{i} \alpha / 2) & \exp (\mathrm{i} \alpha / 2)
\end{array}\right]
$$

For the transition to the $\mathrm{Q}_{\mathrm{S}}$-language, one requires the inverse matrix,
$D^{-1}(\alpha)=D^{+}(\alpha)=\frac{1}{\sqrt{2}}\left[\begin{array}{cc}\exp (\mathrm{i} \alpha / 2) & -\exp (\mathrm{i} \alpha / 2) \\ \exp (-\mathrm{i} \alpha / 2) & \exp (-\mathrm{i} \alpha / 2)\end{array}\right]$.

In matrices $D(\beta), D^{-1}(\beta)$, coupling amplitudes $b_{1}, b_{2}$, and $b, b_{0}$ are of the analogous form.

The general scattering matrix of the $4 \times 4$ dimension is of the block form:

$$
D=\left[\begin{array}{cc}
D(\alpha) & 0 \\
0 & D(\beta)
\end{array}\right]
$$

These enable one to find the photon number operators at the output of the $a$ and $b$ modes:
$n_{a}=a^{+} a=\frac{1}{2}\left[n_{a_{1}}+n_{a_{2}}+a_{1}^{+} a_{2} \exp (\mathrm{i} \alpha)+a_{2}^{+} a_{1} \exp (-\mathrm{i} \alpha)\right]$,
$n_{b}=b^{+} b=\frac{1}{2}\left[n_{b_{1}}+n_{b_{2}}+b_{1}^{+} b_{2} \exp (\mathrm{i} \beta)+b_{2}^{+} b_{1} \exp (-\mathrm{i} \beta)\right]$.
The coincidence probability is determined by the moment $G_{a b}=\left\langle: n_{a} n_{b}:\right\rangle$. Averaging the product of $n_{a}$ and $n_{b}$, provided the moments such as $\left\langle a_{1}^{+} a_{2} b_{1}^{+} b_{2}\right\rangle$ equal zero, one gets [compare with Eqn (26)]

$$
\begin{align*}
& G_{a b}(\alpha, \beta)=G_{0}+G_{+} \cos (\alpha+\beta)+G_{-} \cos (\alpha-\beta),  \tag{66}\\
& G_{0}=\frac{1}{4}\left(G_{a_{1} b_{1}}+G_{a_{2} b_{2}}+G_{a_{1} b_{2}}+G_{a_{2} b_{1}}\right), \quad G_{k l}=\left\langle: n_{k} n_{l}:\right\rangle, \\
& m G_{+}=\frac{1}{2}\left\langle a_{1}^{+} b_{1}^{+} a_{2} b_{2}\right\rangle, \quad G_{-}=\frac{1}{2}\left\langle a_{1}^{+} b_{2}^{+} a_{2} b_{1}\right\rangle \tag{67}
\end{align*}
$$

(it is assumed that moments $G_{ \pm}$are real). Here the combination of moments $G_{0}$ describes the already familiar redistribution of the fluctuations and correlations [see Eqn (26)] and the terms containing $G_{+}$and $G_{-}$describe the fluctuations of two types: with phases $\alpha+\beta$ and $\alpha-\beta$, respectively. For $k_{a}-k_{b}$ one gets

$$
\alpha+\beta=k\left(z_{a_{1} b}-z_{a_{2} b}\right), \quad \alpha-\beta=k\left(\Delta z_{1}-\Delta z_{2}\right)
$$

where $z_{a_{i} b}=z_{a_{i}}+z_{b_{i}}, \Delta z_{i}=z_{a_{i}}-z_{b_{i}}$.
The term advanced wave interference was proposed by me for the interference with phase $\alpha+\beta$, because its main peculiarity, the dependence of the intensities correlator on the sums of optical paths $z_{a_{i} b}$ between the detector and the $i$ th source, is convenient to interpret with the help of fictitious advanced waves propagating from one detector to the $i$ th source, and after 'reflecting' from it, to the second detector [13, 52]. The existence of two (or more) paths $z_{a_{i} b}$ leads to the interference with phase $z_{a_{i} b}-z_{a_{j} b}$.

The old term, Brown-Twiss intensity interference, was conserved by me for the interference with phase $\alpha-\beta$.

It is convenient to define the visibilities of these effects as:

$$
\begin{align*}
& G_{a b}(\alpha, \beta) \propto 1+V_{+} \cos (\alpha+\beta)+V_{-} \cos (\alpha-\beta) \\
& V_{ \pm}=G_{ \pm} / G_{0} \tag{68}
\end{align*}
$$

### 6.2 Brown-Twiss intensity interference

$\mathbf{Q}_{\mathbf{H}}$. If usual light sources $G_{+}=0$ are used, the harmonic dependence of the coincidence rate (or the correlator of the analog photocurrents) on $\alpha-\beta$ (due to $G_{-}$) is also called intensity interference. Condition $G_{-} \neq 0$ can be easily realized by connecting modes $a_{1}$ and $b_{1}$ to the output of one beamsplitter and modes $a_{2}$ and $b_{2}$ to the output of the other (Fig. 8), provided that the condition $\omega_{a}=\omega_{b}$ is satisfied.


Figure 8. Schematic representation of the intensity interference observation for the phase $\alpha-\beta$ (the Brown-Twiss intensity interference). The input fields $c$ and $d$ may be independent.

Assume $t=r=1 / \sqrt{2}$. Then

$$
\begin{align*}
& a_{1}=\frac{c+c_{0}}{\sqrt{2}}, \quad a_{2}=\frac{d+d_{0}}{\sqrt{2}}, \\
& b_{1}=\frac{-c+c_{0}}{\sqrt{2}}, \quad b_{2}=\frac{-d+d_{0}}{\sqrt{2}} . \tag{69}
\end{align*}
$$

According to Eqn (31) $c_{0}=d_{0}=0$. Therefore,

$$
\begin{equation*}
G_{+}=\frac{\left\langle c^{+2} d^{2}\right\rangle}{8}, \quad G_{-}=\frac{\left\langle c^{+} d^{+} d c\right\rangle}{8}=\frac{G_{c d}}{8} . \tag{70}
\end{equation*}
$$

If the unconventional light sources, with the unusual statistics, are excluded (see Section 6.4), $G_{+}=0$.

In the case of symmetric excitation, when

$$
N_{c}=N_{d}=N, \quad G_{c c}=G_{d d}=g N^{2}, \quad G_{c d}=g_{c d} N^{2},
$$

one has

$$
\begin{align*}
& G_{0}=N^{2} \frac{g+g_{c d}}{8}, \quad G=N^{2} \frac{g_{c d}}{8} \\
& V_{-}=\frac{G_{c d}}{G_{c d}+G_{a a}}=\frac{1}{1+g / g_{c d}} . \tag{71}
\end{align*}
$$

Thus, for high visibility of interference the bunching parameter $g$ should be small, i.e. the intensity fluctuation in each input beam should be minimum and the correlation of beams $g_{c d}$ should be maximum. In the classical theory $G_{c c} \geqslant G_{c d}$, therefore $V_{\text {class }} \leqslant 1 / 2$.

We will now examine different types of input fields (see Fig. 8).

1. Let the modes with amplitudes $c$ and $d$ be selected from the fields of two independent sources, for example, two stars or two lasers. From their independence it follows that $g_{c d}=1$ so

$$
\begin{equation*}
V_{-}=\frac{1}{1+g}, \tag{72}
\end{equation*}
$$

and the visibility is determined by the intensity fluctuations of the sources. Thus, in the cases of independent coherent or thermal input fields, the visibility of the intensity interference is $1 / 2$ or $1 / 3$, respectively.
2. If the squeezed vacuum state is at the input, then from Eqn (6) it follows that $g=g_{c d}-N^{-1}=2$, and

$$
\begin{equation*}
V_{-}=\frac{2 N+1}{4 N+1}=\frac{\cosh (2 F)}{2 \cosh (2 F)-1}, \tag{73}
\end{equation*}
$$

where $F$ is the parametric amplification coefficient proportional to the pumping amplitude. For weak pumping $\left(N=F^{2} \ll 1\right.$, spontaneous parametric scattering) $\quad V_{-}=1$; for strong pumping (parametric superluminescence) $V_{-}=1 / 2$. The high visibility of the intensity interference, provided that parametric two-photon light is used, can be considered to be an essentially nonclassical effect connected with the inequality $G_{c c} \ll G_{c d}$ for the input field moments. (For typical experiments, $G_{c c} / G_{c d} \sim 10^{-8}$.)
3. If the squeezed classical light is incident at the input, then according to Eqn (8),

$$
\begin{equation*}
V_{-}=\frac{\cosh (4 F)}{1+2 \cosh (4 F)} \tag{74}
\end{equation*}
$$

Thus, for weak pumping (i.e, noncorrelated Gaussian noise at the interferometer input) $V_{-}=1 / 3$, as in the case of the thermal sources, and for strong pumping (complete correlation of the input intensities) $V_{-}=1 / 2$, as in the case of the independent lasers with the Poisson statistics or in the case of the strong squeezed vacuum.

Note that, though the correlation coefficient of signal and idle modes $K$ is equal to unity for the squeezed vacuum $\left(F \ll 1, N_{0} \ll 1\right)$ and for the classical squeezed light ( $F \gg 1, N_{0} \gg 1$ ), the absence of random coincidences results in $100 \%$ visibility in the first case only.

Qs. Consider the case 2 at $N \ll 1$ in the Schrodinger representation. If the vacuum component is omitted, the input two-photon state is of the form:

$$
\begin{equation*}
\left|\psi_{0}\right\rangle=c^{+} d^{+}|\mathrm{vac}\rangle=|1\rangle_{c}|1\rangle_{d} . \tag{75}
\end{equation*}
$$

To find the transformed state at the output of the first beamsplitter pair, one should inverse the transformation of the operators [Eqn (69)]:
$c=\frac{a_{1}-b_{1}}{\sqrt{2}}, d=\frac{a_{2}-b_{2}}{\sqrt{2}}, c_{0}=\frac{a_{1}+b_{1}}{\sqrt{2}}, d_{0}=\frac{a_{2}+b_{2}}{\sqrt{2}}$.
Here $c_{0}$ and $d_{0}$ are the operators of the unused input modes of the input beamsplitters that are in the vacuum state. Inserting Eqn (76) in Eqn (75) one finds

$$
\begin{align*}
\left|\psi_{1}\right\rangle= & \frac{1}{2}\left(a_{1}^{+}-b_{1}^{+}\right)\left(a_{2}^{+}-b_{2}^{+}\right)|\mathrm{vac}\rangle \\
& =\frac{1}{2}(|1010\rangle+|0101\rangle-|1001\rangle-|0110\rangle), \tag{77}
\end{align*}
$$

where $|k l m n\rangle=|1\rangle_{a_{1}}|1\rangle_{b_{1}}|1\rangle_{a_{2}}|1\rangle_{b_{2}}$.
On performing a substitution according to Eqn (64) we get

$$
\begin{array}{ll}
a_{1}=\frac{a-a_{0}}{\sqrt{2}} \exp (\mathrm{i} \alpha / 2), & a_{2}=\frac{a+a_{0}}{\sqrt{2}} \exp (-\mathrm{i} \alpha / 2) \\
b_{1}=\frac{b-b_{0}}{\sqrt{2}} \exp (\mathrm{i} \beta / 2), & b_{2}=\frac{b+b_{0}}{\sqrt{2}} \exp (-\mathrm{i} \beta / 2) \tag{78}
\end{array}
$$

From the above one gets the state of four output modes,

$$
\begin{align*}
\left|\psi_{2}\right\rangle=\frac{1}{4}\{ & \exp (-\mathrm{i} \alpha)|2000\rangle+\exp (-\mathrm{i} \beta)|0200\rangle \\
& -\exp (\mathrm{i} \alpha)|0020\rangle-\exp (\mathrm{i} \beta)|0002\rangle \\
& +2 \cos [(\alpha-\beta) / 2](|0011\rangle-|1100\rangle) \\
& +2 \mathrm{i} \sin [(\alpha-\beta) / 2](|0110\rangle-|1001\rangle)\} \tag{79}
\end{align*}
$$

where $|k \operatorname{lmn}\rangle=|1\rangle_{a}|1\rangle_{b}|1\rangle_{a_{0}}|1\rangle_{b_{0}}$.
The coefficient of vector $|1100\rangle$ equals

$$
\begin{equation*}
c_{1100}=-0.5 \cos [(\alpha-\beta) / 2], \tag{80}
\end{equation*}
$$

and according to the measuring postulate corresponds to the probability amplitude of one photon for each $a$ and $b$ mode to be found. From here one finds the coincidence probability,

$$
\begin{equation*}
P_{a b}=\frac{1}{8}[1+\cos (\alpha-\beta)] . \tag{81}
\end{equation*}
$$

This expression, as well as Eqn (73) for $N \ll 1$, describes the two-photon intensity interference with the phase $\alpha-\beta$ and the visibility $V_{-}=1$. Note that calculations based on the state given by Eqn (75) do not describe the random coincidences lowering the visibility for $N \sim 1$ [compare with Eqn (73)].

### 6.3 Advanced waves interference

If squeezed or two-photon light is used, the moment $G_{+}=\left\langle a_{1}^{+} b_{1}^{+} a_{2} b_{2}\right\rangle / 2$ in Eqn (66) may be different from zero and the interference with the phase $\alpha+\beta$ will be observed. This effect is called the two-photon interference, though it can be observed (although with a lower visibility) with the help of the classical squeezed light and the analog detectors.

Let the parametric downconverter with two signal $\left(a_{1}, a_{2}\right)$ and two idle ( $b_{1}, b_{2}$ ) modes (Fig. 9), having a common pumping [52] (the signal and idle frequencies may be different), be a four-mode light source.


Figure 9. Schematic representation of the intensity interference observa-tion for the phase $\alpha+\beta$ (two-photon or advanced wave interference). Two nonlinear crystals subjected to the common pumping P form the four-mode field with nonzero correlator $\left\langle a_{1}^{+} b_{1}^{+} a_{2} b_{2}\right\rangle=\left\langle a_{1} b_{1}\right\rangle^{*}\left\langle a_{2} b_{2}\right\rangle$.
$\mathbf{Q}_{\mathbf{H}}$. Analogously to Eqn (6), one has

$$
\begin{align*}
& G_{a_{1} b_{1}}=\left\langle a_{1}^{+} b_{1}^{+} b_{1} a_{1}\right\rangle=G_{a_{2} b_{2}}=\left\langle a_{2}^{+} b_{2}^{+} b_{2} a_{2}\right\rangle=2 N^{2}+N, \\
& G_{a_{1} b_{2}}=\left\langle a_{1}^{+} b_{2}^{+} b_{2} a_{1}\right\rangle=G_{a_{2} b_{1}}=\left\langle a_{2}^{+} b_{1}^{+} b_{1} a_{2}\right\rangle=N^{2}, \\
& \left\langle a_{1}^{+} b_{1}^{+} a_{2} b_{2}\right\rangle=\left\langle a_{1} b_{1}\right\rangle^{*}\left\langle a_{2} b_{2}\right\rangle=N(N+1) . \tag{82}
\end{align*}
$$

Inserting these expressions in Eqn (67) one finds

$$
\begin{align*}
& G_{0}=\frac{1}{4}\left(G_{a_{1} b_{1}}+G_{a_{2} b_{2}}+G_{a_{1} b_{2}}+G_{a_{2} b_{1} a}\right)=\frac{1}{2} N(3 N+1), \\
& G_{+}=\frac{1}{2}\left\langle a_{1}^{+} b_{1}^{+} a_{2} b_{2}\right\rangle=\frac{1}{2} N(N+1), \tag{83}
\end{align*}
$$



Figure 10. Scheme for the observation of two-photon interference of the polarization type. The source emits photon pairs with the correlated polarizations which are detected by two detectors and the coincidence circuit. The coincidence rate is dependent on the analyzer orientation angles in accordance with the 'two-photon Malus law' $\cos ^{2}(\alpha-\beta)$ that is incompatible with the visual semiclassical ideas of photon packets with random polarizations, but follows directly from the model with the advanced wave $E_{\text {adv }}$, emitted by one of the detectors.
and thus

$$
\begin{equation*}
V_{+}=\frac{N+1}{3 N+1} . \tag{84}
\end{equation*}
$$

Thus, again the superclassical visibility $V_{+}=1$ corresponds to weak pumping and $V_{+}=1 / 3$ corresponds to strong pumping.

Let us examine the polarization version (Fig. 10) of this effect $[1,2,5,6]$, when indices 1 and 2 specify two polarization states of the same beam ( $a$ and $b$ ). The analyzers with the transmission coefficients $t_{a}=\cos \alpha$ and $t_{b}=\cos \beta$ are used as beamsplitters. The amplitudes of the input fields are [compare with Eqn (62)]

$$
\begin{equation*}
a=a_{1} \cos \alpha+a_{2} \sin \alpha, \quad b=b_{1} \cos \beta+b_{2} \sin \beta \tag{85}
\end{equation*}
$$

Consequently, the operators of the photon number at outputs $a$ and $b$ are of the form [compare with Eqn (65)]

$$
\begin{align*}
n_{a}=a^{+} a= & n_{a_{1}} \cos ^{2} \alpha+n_{a_{2}} \sin ^{2} \alpha \\
& +\left(a_{1}^{+} a_{2}+a_{2}^{+} a_{1}\right) \cos \alpha \sin \alpha, \\
n_{b}=b^{+} b= & n_{b_{1}} \cos ^{2} \beta+n_{b_{2}} \sin ^{2} \beta \\
& +\left(b_{1}^{+} b_{2}+b_{2}^{+} b_{1}\right) \cos \beta \sin \beta . \tag{86}
\end{align*}
$$

From the above discussion, at $\left\langle a_{1}^{+} a_{1}^{+} b_{1} b_{2}\right\rangle=\ldots=0$ one finds [compare with Eqn (66)]

$$
\begin{align*}
G_{a b}= & G_{a_{1} b_{1}} \cos ^{2} \alpha \cos ^{2} \beta+G_{a_{2} b_{2}} \sin ^{2} \alpha \sin ^{2} \beta \\
& +G_{a_{1} b_{2}} \cos ^{2} \alpha \sin ^{2} \beta+G_{a_{2} b_{1}} \sin ^{2} \alpha \cos ^{2} \beta \\
& +4\left(G_{+}+G_{-}\right) \cos \alpha \cos \beta \sin \alpha \sin \beta \tag{87}
\end{align*}
$$

(it is again assumed that the moments $G_{ \pm}$are real).
Inserting the values of the correlators given by Eqn (82) at $N \ll 1$ (two-photon light), $G_{a_{1} b_{1}}=G_{a_{2} b_{2}}=2 G_{+}=N$ in Eqn (87), one has
$G_{a b}=N(\cos \alpha \cos \beta+\sin \alpha \sin \beta)^{2}=0.5 N \cos ^{2}(\alpha-\beta)$.
Thus, the coincidence probability is dependent on the difference of the analyzer orientation angles only, and for the crossed analyzer there are no coincidences independent of the individual values of angles $\alpha$ and $\beta$. The latter effect shows most visually the inadequacy of the C-language (see below).

Qs. Two pairs of the signal and idle modes, having a common coherent pumping, are in the first order of the pumping described by the two-photon state,
$\left|\psi_{1}\right\rangle=\frac{1}{\sqrt{2}}(|1100\rangle+|0011\rangle)=\frac{1}{\sqrt{2}}\left(a_{1}^{+} b_{1}^{+}+a_{2}^{+} b_{2}^{+}\right)|\mathrm{vac}\rangle$,
where $|k \operatorname{lm} n\rangle=|1\rangle_{a_{1}}|1\rangle_{b_{1}}|1\rangle_{a_{2}}|1\rangle_{b_{2}}$. The vacuum component is omitted because it is of no interest in the experiment under consideration. The state given by Eqn (89) is called entangled: the signal Q-photon, as well as the idle one, belongs to two modes with indices 1 and 2 simultaneously.

Using Eqn (89) one easily finds the nonzero moments: $\left\langle a_{k}^{+} b_{k}^{+} a_{l} b_{l}\right\rangle=1 / 2$, where $k, l=1,2$. As the result one again has $V_{+}=1$.

Let us now obtain the results given above in the Schrodinger representation. With the help of Eqn (64) one finds the output state,

$$
\begin{align*}
\left|\psi_{2}\right\rangle=\frac{1}{\sqrt{2}}\{ & \cos [(\alpha+\beta) / 2](|1100\rangle+|0011\rangle) \\
& +\mathrm{i} \sin [(\alpha+\beta) / 2](|1001\rangle+|0101\rangle)\} \tag{90}
\end{align*}
$$

where $\mid$ klmn $\rangle=|1\rangle_{a}|1\rangle_{b}|1\rangle_{a_{0}}|1\rangle_{b_{0}}$. The coefficient of vector $|1100\rangle$ is equal to

$$
\begin{equation*}
c_{1100}=\frac{1}{\sqrt{2}} \cos [(\alpha+\beta) / 2] \tag{91}
\end{equation*}
$$

and according to the measuring postulate corresponds to the probability amplitude of one photon for each $a$ and $b$ mode to be found.From here one finds the coincidence probability, [compare with Eqn (81)]

$$
\begin{equation*}
P_{a b}=\frac{1}{4}[1+\cos (\alpha+\beta)] . \tag{92}
\end{equation*}
$$

This expression describes the two-photon intensity interference with the phase $\alpha+\beta$ and the visibility $V_{+}=1$.
M. Numerous papers are devoted to the description of different versions of two-photon interference in the Mlanguage. In these papers the paths chosen by the photons, the influence of their undistinguishability and distinguishability, the manifestation of the duality, the influence of the initial state entangling, etc., are discussed in detail.

An example of the statement in the M-language is given below. Let the field be in the state given by Eqn (89). When observing a photon, for example, in the $a_{1}$ mode, instantaneous reduction of the field state occurs: the second term in Eqn (89) disappears and in the first term only component $|1\rangle_{b_{1}}$ is left, i.e. the $b_{1}$ mode is prepared in the one-photon state. Detector $B$ is informed instantly about this due to the quantum nonlocality. In general, the detection of a photon in any of the four modes is at the same time a preparation of the one-photon state for the corresponding 'pair' mode. In reference to the experiment depicted in Fig. 9 it is possible to conclude that the photons are created locally: either both in crystal 1, or both in crystal 2.

Actually this type of language gives no information in addition to the computation results: in fact, it is possible to speak about the photocount coincidences observation for two equivalent detectors. (The reduction in this context is discussed in more detail in Ref. [23].) For the detection of coincidences one needs to have some coupling channel between the detectors, as there is, of course, no long-range interaction here.


Figure 11. Frequency degenerated parametric downconverter with the additional beamsplitter mixing the signal and idle modes prepares the field with nonzero correlator $\left\langle c^{+2} d^{2}\right\rangle$ and $\left\langle c^{+} c d^{+} d\right\rangle$. This field, being incident at the input of the interferometer depicted in Fig. 8, enables one to observe simultaneously both intensity interference types.

### 6.4 Simultaneous observation of two types of interference

In the examples presented above, the interferences of different types were observed only separately: either with the phase $\alpha+\beta$ or with the phase $\alpha-\beta$. Let us examine the combination of the schemes presented in Figs 8 and 11. (Recently its polarization version has been realized [53].) Here, instead of two pairs of the signal and idle modes (see Fig. 9) only one such pair is used. However the signal ( $c^{\prime}$ ) and idle $\left(d^{\prime}\right)$ modes (of the same frequency but differing in the propagation direction or in the polarization type) are preliminarily mixed by a $50 \%$ beamsplitter (see Fig. 11).

As a result the fields at the input of the interferometer depicted in Fig. 8 are of the form,

$$
\begin{equation*}
c=\frac{c^{\prime}+d^{\prime}}{\sqrt{2}}, \quad d=\frac{-c^{\prime}+d^{\prime}}{\sqrt{2}} \tag{93}
\end{equation*}
$$

Note that, according to Eqn (7), there exist the following relations:

$$
\begin{align*}
\left\langle c^{2}\right\rangle & =-\left\langle d^{2}\right\rangle=\left\langle c^{\prime} d^{\prime}\right\rangle \\
& =\left[N^{\prime}\left(N^{\prime}+1\right)\right]^{1 / 2} \exp \left(-\mathrm{i} \omega_{0} t-\mathrm{i} \varphi_{0}\right) \tag{94}
\end{align*}
$$

The transformed moments at the interferometer input, if Eqn (93) is taken into account, are of the form

$$
\begin{align*}
& G_{c c}=G_{d d} \\
&=\frac{1}{4}\left(G_{c c}^{\prime}+G_{d d}^{\prime}+4 G_{c d}^{\prime}\right)=N^{\prime}\left(3 N^{\prime}+1\right)  \tag{95}\\
& G_{c d}=G_{d c}
\end{align*}=\frac{1}{4}\left(G_{c c}^{\prime}+G_{d d}^{\prime}\right)=N^{\prime 2}, ~ l
$$

where $N^{\prime}$ is the intensity of each mode. From here, one finds the parameters determining the visibility:

$$
\begin{align*}
& G_{0}=\frac{1}{16}\left(G_{c c}^{\prime}+G_{d d}^{\prime}+2 G_{c d}^{\prime}\right)=\frac{1}{8} N^{\prime}\left(4 N^{\prime}+1\right) \\
& G_{+}=\frac{1}{32}\left(G_{c c}^{\prime}+G_{d d}^{\prime}-4 G_{c d}^{\prime}\right)=-\frac{1}{8} N^{\prime}\left(N^{\prime}+1\right) \\
& G=\frac{1}{8} G_{c d}=\frac{1}{16} G_{c c}^{\prime}=\frac{1}{8} N^{\prime 2} \tag{96}
\end{align*}
$$

Note that Eqns (93) and (96) result in a characteristic for the squeezed light property, the factorizability of the moments:

$$
\begin{equation*}
G_{+}=\frac{1}{8}\left\langle c^{+2} d^{2}\right\rangle=\frac{1}{8}\left\langle c^{+2}\right\rangle\left\langle d^{2}\right\rangle=-\frac{1}{8}\left|\left\langle c^{\prime} d^{\prime}\right\rangle\right|^{2} \tag{97}
\end{equation*}
$$

In the result, the visibilities of the two simultaneously observed interference patterns are given by

$$
\begin{align*}
& V_{+}=\frac{G_{+}}{G_{0}}=\frac{1}{2} \frac{G_{c c}^{\prime}-2 G_{c d}^{\prime}}{G_{c c}^{\prime}+G_{c d}^{\prime}}=-\frac{N^{\prime}+1}{4 N^{\prime}+1} \\
& V_{-}=\frac{G_{-}}{G_{0}}=\frac{1}{2} \frac{G_{c c}^{\prime}}{G_{c c}^{\prime}+G_{c d}^{\prime}}=\frac{N^{\prime}}{4 N^{\prime}+1} \tag{98}
\end{align*}
$$

For weak pumping $N^{\prime} \ll 1$ so the intensity interference with the phase $\alpha-\beta$ disappears $\left(V_{-}=N^{\prime}\right)$; however the two-photon interference with the superclassical visibility $\left|V_{+}\right|=1$ conserves. For strong pumping $V_{-}=-V_{+}=1 / 4$, so the coincidence rate is proportional to $1+0.5 \sin \alpha \sin \beta$.

An addition to the parametric downconverter input of the Gaussian noise of large intensity results in squeezed classical light (see Fig. 2). And according to Eqn (8) one gets

$$
\begin{align*}
& G_{0}=\frac{1}{8} N_{0}^{2}[1+2 \cosh (4 F)] \\
& G_{ \pm}=\frac{1}{16} N_{0}^{2}[1 \mp \cosh (4 F)] \tag{99}
\end{align*}
$$

so

$$
\begin{equation*}
V_{ \pm}=\frac{1}{2} \frac{1 \mp \cosh (4 F)}{1+2 \cosh (4 F)} \tag{100}
\end{equation*}
$$

Thus, for weak pumping $(F \ll 1)$, i.e. for the Gaussian noise at the input of the interferometer depicted in Fig. 8, $V_{+}=4 F^{2} / 3 \ll 1, \quad V_{-}=1 / 3$, and for strong pumping $V_{-}=-V_{+}=1 / 4$ (similar to the case $N_{0}=0$ and $F \gg 1$ ).

Qs. Let us examine the same effect in the Schrodinger representation for the initial state of the form,

$$
\begin{equation*}
|\psi\rangle=|1\rangle_{c}^{\prime}|1\rangle_{d}^{\prime}=c^{\prime+} d^{\prime+}|\mathrm{vac}\rangle \tag{101}
\end{equation*}
$$

Note that this is a factorized state: every Q-photon belongs to one mode.

At the output of the first beamsplitter (see Fig. 11) one has [compare with Eqn (75)]

$$
\begin{equation*}
\left|\psi_{0}\right\rangle=\frac{1}{2}\left(c^{+2}-d^{+2}\right)|\mathrm{vac}\rangle=\frac{1}{2}(|2,0\rangle-|0,2\rangle) \tag{102}
\end{equation*}
$$

Further, each beam $c, d$ is divided again in half at the input mirrors of the interferometer (see Fig. 8). Inserting Eqn (76) in Eqn (102) one finds the four-mode field state inside the interferometer [compare with Eqn (77)]:

$$
\begin{align*}
\left|\psi_{1}\right\rangle= & \frac{1}{2 \sqrt{2}}\left(a_{1}^{+2}-2 a_{1}^{+} b_{1}^{+}+b_{1}^{+2}-a_{2}^{+2}+2 a_{2}^{+} b_{2}^{+}-b_{2}^{+2}\right)|\mathrm{vac}\rangle \\
= & \frac{1}{2 \sqrt{2}}(|2000\rangle-2|1100\rangle+|0200\rangle \\
& \quad-|0020\rangle+2|0011\rangle-|0002\rangle) \tag{103}
\end{align*}
$$

Here $|k l m n\rangle=|k\rangle_{a_{1}}|l\rangle_{b_{1}}|m\rangle_{a_{2}}|n\rangle_{b_{2}}$.
In the $\mathrm{Q}_{\mathrm{C}}$-language the two initial photons may be distributed over four modes. The factors 2 before states of the type $|1100\rangle$ are interpreted usually in the M -language as the consequence of indistinguishability of two photons.

Examine further the action of the output mirrors of the interferometer depicted in Fig. 8. Substituting Eqn (78) in Eqn (103) one finds that the state at the interferometer output consists of ten independent components [compare with Eqn (79)]:

$$
\begin{aligned}
\left|\psi_{2}\right\rangle=\frac{1}{4 \sqrt{2}}\{ & -2 \mathrm{i} \sin \alpha(|2000\rangle+|0020\rangle) \\
& -2 \mathrm{i} \sin \beta(|0200\rangle+|0002\rangle) \\
& -4 \cos \alpha|1010\rangle+4 \cos \beta|0101\rangle \\
& +4 \mathrm{i} \sin [(\alpha+\beta) / 2](|1100\rangle+|0011\rangle) \\
& +4 \cos [(\alpha+\beta) / 2](|1001\rangle+|0110\rangle)\} .(104)
\end{aligned}
$$

Here $|k l m n\rangle=|k\rangle_{a}|l\rangle_{b}|1\rangle_{a_{0}}|1\rangle_{b_{0}}$. The coefficient of vector $|1100\rangle$ is the probability amplitude of finding the photons in the output modes $a$ and $b$ :

$$
\begin{equation*}
c_{1100}=\frac{1}{\sqrt{2}} \mathrm{i} \sin [(\alpha+\beta) / 2] \tag{105}
\end{equation*}
$$

Therefore the probability itself is of the form analogous to Eqn (92):

$$
\begin{equation*}
P_{a b}=\frac{1}{4}[1-\cos (\alpha+\beta)] \tag{106}
\end{equation*}
$$

The output state $\left|\psi_{2}\right\rangle$ also belongs to the class of factorable states because it can be transformed again to the initial form [Eqn (101)] by the choice of some definite representation: in fact $\left|\psi_{0}\right\rangle$ and $\left|\psi_{2}\right\rangle$ are the same states in different representations. The transition between them is performed by the unitary transformation which is realized by the beamsplitters.

In the course of discussions on the two-photon interference effect and, in particular, about the Bell inequalities violation, special significance is attributed to the entangling (nonfactorability) of the field state. Therefore one might conclude that the state $\left|\psi_{2}\right\rangle$ is unsuitable for the demonstration of violations of these inequalities. However, this conclusion is erroneous: the above obtained $100 \%$ visibility of the two-photon interference $V_{+}$in the scheme under consideration witnesses the opposite.

If needed, it is possible to conserve the condition of state entangling, if the cases with the appearance of two photons in one mode [with the components of the form $|2000\rangle$ in Eqn (104)] are ignored. This can be done while processing the stored data. In the conventional experiments the coincidence detectors do not detect these events automatically, i.e. according to the measuring postulate they project vector $\left|\psi_{2}\right\rangle$ into the subspace formed by the vectors of the $|1100\rangle$ type only. Projecting is a nonunitary transformation so the obtained state is nonfactorable. Thus, one is compelled to conclude that, though the output state $\left|\psi_{2}\right\rangle$ is actually an unentangled one, in the course of the posteriori processing of the experimental data it becomes entangled.

Artificiality of the presented formulation shows that the entangling of the state discussed above is apparently due to the use of the $M$-language.

### 6.5 Classical models

$\mathbf{C}^{*}$. Both types of intensity interference determined above can be trivially explained in classical terms: they are the results of transformation by beamsplitters of the input field phase fluctuations into the output field intensity fluctuations (see the end of Section 5).

Let the fields $a_{k}, b_{k}(k=1,2)$ in Fig. 7 have constant amplitudes (equal to unity) and drifting phases:

$$
\begin{equation*}
a_{k}(t)=\exp \left[-\mathrm{i} x_{k}(t)\right], \quad b_{k}(t)=\exp \left[-\mathrm{i} y_{k}(t)\right] \tag{107}
\end{equation*}
$$

Phases $x_{k}(t)$ and $y_{k}(t)$ play the role of hidden parameters.

The output amplitudes are of the form [compare with Eqn (62)]

$$
\begin{align*}
& a(t)=\frac{1}{\sqrt{2}}\left[\exp \left(-\mathrm{i} x_{1}-\mathrm{i} \alpha / 2\right)+\exp \left(-\mathrm{i} x_{2}+\mathrm{i} \alpha / 2\right)\right] \\
& b(t)=\frac{1}{\sqrt{2}}\left[\exp \left(-\mathrm{i} y_{1}-\mathrm{i} \beta / 2\right)+\exp \left(-\mathrm{i} y_{2}+\mathrm{i} \beta / 2\right)\right] \tag{108}
\end{align*}
$$

From the above it follows that the expressions for the output intensities can be presented in the form [compare with Eqn (65)],

$$
\begin{equation*}
n_{a}=1+\cos (x+\alpha), \quad n_{b}=1+\cos (y+\beta) \tag{109}
\end{equation*}
$$

where $x=x_{1}-x_{2}, y=y_{1}-y_{2}$.
Let $x_{1}, x_{2}$ and $y_{1}, y_{2}$ be the independent phases. Then $\left\langle\left\langle n_{a}\right\rangle\right\rangle=\left\langle\left\langle n_{b}\right\rangle\right\rangle=1$, i.e. the conventional interference is absent. We can then form the intensity correlator [compare with Eqn (66)]

$$
\begin{equation*}
G_{a b}=\left\langle\left\langle n_{a} n_{b}\right\rangle\right\rangle=1+\frac{1}{2} \sum_{ \pm}\langle\langle\cos (x+\alpha \pm y \pm \beta)\rangle\rangle \tag{110}
\end{equation*}
$$

Consequently the stationary interference with the phases $\alpha \pm \beta$ and the visibilities $V_{ \pm}=1 / 2$ is possible for $x(t) \pm y(t)=$ const. These conditions may be called phase correlation and anticorrelation. The first condition, giving the conventional intensity interference, is satisfied if, for example, $x_{1}=y_{1}, x_{2}=y_{2}$, i.e. for $a_{1}=b_{1}$ and $a_{2}=b_{2}$ (see Fig. 8). The second condition is satisfied if $x_{1}+y_{1}=x_{2}+y_{2}$, i.e. if the nondegenerated parametric generators are used (see Fig. 9), for which the phases (as well as the frequencies) of the signal and idle waves are drifting in the opposite directions.

Note that the approach to the beamsplitters and polarization prisms as phase detectors can, with the help of the phase difference operator, be extended in the quantum description [8].

Thus, both interference types with the phases $\alpha \pm \beta$ (in particular, the two-photon interference) have close classical analogs, whose visibility, however, cannot exceed $1 / 2$.
C. Let us try to describe the polarization version of twophoton interference $[1,2,5,6]$ in terms of photon packets with the a priori determined polarization. Now in the scheme depicted in Fig. 7, symbols $a_{1}, a_{2}$ and $b_{1}, b_{2}$ are the polarization components of the beams $a$ and $b$, respectively; the dashed lines are the analyzers with the orientation angles $\alpha$ and $\beta$. According to Eqn (88) the observed coincidence probability is proportional to $\cos ^{2}(\alpha-\beta)$.

Assume that the C-photons in every pair possess the a priori definite polarization along some two directions $\alpha_{i}$ and $\beta_{i}$, and are changed at random at every trial with number $i$ (this is again an example of hidden parameters). Assuming the field amplitude to be equal to unity, one gets the input amplitudes in the $i$ th trial:

$$
\begin{array}{ll}
a_{1 i}=\cos \alpha_{i}, & a_{2 i}=\sin \alpha_{i} \\
b_{1 i}=\cos \beta_{i}, & b_{2 i}=\sin \beta_{i} \tag{111}
\end{array}
$$

At the output, according to Eqn (85) one has

$$
a=\cos \left(\alpha-\alpha_{i}\right), \quad b=\cos \left(\beta-\beta_{i}\right)
$$

From here one finds the intensities according to the Malus law:

$$
n_{a i}=\cos ^{2}\left(\alpha-\alpha_{i}\right), \quad n_{b i}=\cos ^{2}\left(\beta-\beta_{i}\right)
$$

The averaging over the hidden parameters results in the following intensity correlators:

$$
\begin{equation*}
G_{a b}=\left\langle\left\langle n_{a i} n_{b i}\right\rangle\right\rangle=\left\langle\left\langle\cos ^{2}\left(\alpha-\alpha_{i}\right) \cos ^{2}\left(\beta-\beta_{i}\right)\right\rangle\right\rangle \tag{112}
\end{equation*}
$$

For the homogeneous distribution of the polarization directions, one gets

$$
\begin{align*}
G_{a b}= & \frac{1}{4}\left[1+\left\langle\left\langle\cos \left(2 \alpha-2 \alpha_{i}\right) \cos \left(2 \beta-2 \beta_{i}\right)\right\rangle\right\rangle\right] \\
= & \frac{1}{4}\left\{1+\frac{1}{2}\left\langle\left\langle\cos \left[2\left(\alpha-\alpha_{i}+\beta-\beta_{i}\right)\right]\right\rangle\right\rangle\right. \\
& \left.+\frac{1}{2}\left\langle\left\langle\cos \left[2\left(\alpha-\alpha_{i}-\beta+\beta_{i}\right)\right]\right\rangle\right\rangle\right\} \tag{113}
\end{align*}
$$

To make this expression dependent on the difference $\alpha-\beta$ only the directions $\alpha_{i}$ and $\beta_{i}$ should be completely correlated.

Let $\alpha_{i}=\beta_{i}$, i.e. the photons in pairs are of the same polarization. Then the second term in Eqn (113) vanishes in the course of averaging and the third term is independent of the trial number:

$$
\begin{equation*}
G_{a b}=\frac{1}{2} \cos ^{2}(\alpha-\beta)=\frac{1}{4}\left\{1+\frac{1}{2} \cos [2(\alpha-\beta)]\right\} \tag{114}
\end{equation*}
$$

We again get the limiting classical visibility $50 \%$ : for the crossed polaroids the coincidence probability does not vanish, but decreases by half in comparison with the maximum one.

Thus, it is impossible to explain in the C-language, as well as in the $C^{*}$-language, the $100 \%$ visibility of the two-photon interference and, in particular, the absence of coincidences at crossed analyzers in the Clauser type experiments [1, 2, 5].

So, the $100 \%$ visibility of the intensity interference is an essentially quantum effect. It can be predicted in the framework of the C-language with the help of the model with the advanced waves $[38,52]$ and the Malus law (see Fig. 10). The unpolarized advanced wave (packet), 'emitted' in the backward direction in the time and space by one of the detectors - for example, by detector $A$ - at the moment of photon detection becomes polarized in direction $\alpha$, 'reflects' from the source, and its $\beta$-component is detected by detector $B$.

It should be emphasised that the advanced waves play here a pure 'mnemonic' role, reflecting the Q-calculation structure, so they do not belong to the M-language. Besides the two-photon interference they are useful for a qualitative description and prediction of other effects of two-photon optics [54]: two-photon diffraction [54], photon mutual focusing [23, 55], and biphoton frequency filtration [37, 38].

## 7. Conclusion

1. The main criteria in the comparison of the advantages of the alternative languages is the possibility of predicting new effects; the ability to generalize, classify, and systematize the phenomena; and universality, compactness, simplicity, and visibility. (The last item is of a historic, relative character: for Newton's contemporaries his language was probably less visual than that of Aristotle and Descartes).

The description of several optical experiments in different languages presented above has demonstrated an obvious advantage in favour of the $\mathrm{Q}_{\mathrm{H}}$-language (the quantum theory in the Heisenberg representation) for these criteria. The $\mathrm{Q}_{\mathrm{H}}$-language gives an universal quantitative description of every possible multimode interferometer through their classical scattering matrices. And different interference effects are explained just as in the classical theory: these occur because of the superposition of two or more oscillations. The $\mathrm{C}^{*}$-language (classical stochastic electro-dynamics), similar in spirit to the $\mathrm{Q}_{\mathrm{H}^{-}}$ language, gives useful classical analogies of the observed effects.

However, the possibility of description of an individual event, namely, of the appearance of an individual pulse at the detector output, which in the M-language is 'explained' by the arrival of a M-photon, is ignored here. The attempt to describe casually the individual events in space-time and the interpretation of different optical effects as the result of the mysterious propagation of M-photons via different paths is, probably, one justification for the existence of the M-language. The latter is to a certain extent based in the Qs-language (quantum theory in the Schrodinger representation), which is less convenient for the quantitative calculations of the real optical problems.

The general balance of two pairs of similar languages $\left(\mathrm{Q}_{\mathrm{H}}, \mathrm{C}^{*}\right)$ and $\left(\mathrm{Q}_{\mathrm{S}}, \mathrm{M}\right)$ moves in favor of the first pair. It appears as if the importance and perceptiveness of the Mlanguage in current publications is overestimated and the usefulness of the $\mathrm{C}^{*}$-language is underestimated.

The 'every day' C-language of the photon packets is indispensable for the visual description of the overwhelming majority of optical phenomena. The notion of the twophoton packet, which is formed with the help of advanced waves, is useful for the description and prediction of different effects of two-photon optics (biphotonics).
2. What was new in the optical demonstration experiments of the last decades? In the framework of the $\mathrm{Q}_{\mathrm{H}^{-}}$ language their results reduce to the measurements of correlation functions or of the fourth order field moments such as $G_{a a}=\left\langle a^{+2} a^{2}\right\rangle$ and $G_{a b}=\left\langle a^{+} b^{+} b a\right\rangle$ (see Section 3.2). In other words, any possible optical schemes (together with the detectors) serve as classical devices for the measurement of the moments or the correlation functions of the initial light. And the detected inequalities of the $G_{a a}<\left\langle a^{+} a\right\rangle^{2}$ type (photon antibunching effect) and $G_{a a}<G_{a b}$ type (twophoton correlation) reveal the inadequacy of the classical $\mathrm{C}^{*}$-language (see Section 3.1).

If one has confidence in the modern photodetection models and the Maxwell equations, describing propagation of light through a linear optical system, then possible manifestations of the nonclassicality in the experiments under discussion, i.e. violations of inequalities of the Cauchy-Schwartz or Bell types, are explained not by the peculiarities of the optical scheme but by the statistical properties of the used light source, which are transferred by the classical Green functions to the optical tract output. In the $\mathrm{Q}_{\mathrm{H}}$-language, all linear optical schemes are described by the classical propagators and, therefore, the quantum specificity, if it is present, is connected only with the light source used at the system input.

Further, the visibility magnitudes of the two-photon interference exceeding $1 / \sqrt{2}$ lead to violation of the Bell
inequalities and thereby deny the possibility of describing the corresponding experiments in the $\mathrm{C}_{\mathrm{B}}$-language (language of the dichotomic Bell's observables). These inequalities can be proved by means of the notion joint probability for some variables (corresponding to the noncommuting operators in the Q-theory). Their violation in the experiment should be naturally considered as evidence of nonapplicability of the notion of joint probability.

The Bell inequalities may also be proved with the help of the locality condition (i.e. in the absence of interaction of the remote detectors by means of unknown forces) and therefore their violations are usually interpreted (rather inconsistently, from my point of view; see Section 2.2) as a manifestation of the quantum nonlocality. Note that in the experiments discussed, a localized two-photon light source is used and the field propagation through the interferometers to the remote detectors is described by the classical Green functions. Therefore it is unclear how the quantum nonlocality arises.

The possibility of duplicating all two-photon experiments with the help of classical squeezed light and analog detectors is essential. Here the analogous interference dependences should be observed, but with one difference, the visibility should be lower (see Section 6.5). This kind of experiments which is easily realised, can be completely described in the C-language.
3. At the same time these experiments yielded good data for the commentary in the M-language, which, however, do not bring closer the solution to the M-photon mystery, i.e. the physical essence which gives rise to the appearance of an individual pulse at the photondetector output. Similar discussions do not involve any new experimentally verifiable or refutable conclusions and, if one follows the Popper definition, cannot be considered as scientific ones.

Apparently, in modern quantum optics there are no experimental results that contradict the standard Q-models. At the same time I do not know any experimentally observable results, which follow from the concepts and notions of the M-language, such as the a priori proper-ties-duality, distinguishability and undistinguishability, circular polarization, tendency to bunching or antibunching, etc attributed to the M-photons. It is not excluded that Bohr's words about 'the contradiction to science's spirit of mysticism' may be related to some M-terms and notions.

Taking the risk of being accused of pragmatism, operationalism and other 'heresies', I would like to note the difference between the consistent with the experiment scientific theory and its possible interpretations. The choice of the latter is, according to the definition, a matter of taste, and the importance of interpretation should not be overestimated as it takes place in current quantum optics. It seems that some 'moderate' operationalism is, however, necessary for distinguishing between physics and metaphysics. Similar considerations are also true in classical physics; however, in quantum physics the gap between mathematics and visual thinking is more evidently revealed.

I do not, of course, call into question the existence of the optical field as an objective reality (even when 'nobody sees' it), but I only suggest that exact borders be drawn between three multitudes: the firmly established objective laws, the computing algorithms with the precisely determined useful terms (Q-language); and the speculative fruitless notions and terms isolated from the experiment (M-language); and
the 'naive realism' (C-language). One should certainly not reject the 'photon' notion. However, it is worth formulating its modern status precisely.

I hope that the classification proposed above of the Q , M , and C -languages used in quantum optics, with distinct borders between them will help in a logical comprehension of the results of the demonstration optical experiments, both the known and the planned ones.
4. Nevertheless, let us try to find some excuse for the widely used M-language. It is impossible, certainly, to reject in general the importance of some indistinct intuitive notions which constitute the basis of human thinking and frequently are of evristic value. As the history of physics shows, they can be in future formally founded in the framework of some qualitative theory, the origin of which is sometimes promoted by these notions. It is not ruled out that one of the interpretations using the Mphoton notion is able to promote its rank up to the one of a scientific theory (however, the lack of progress in the founding of corresponding notions during the last 60 years is surprising). One may hope that the M-language does, nevertheless, form some base for the development and acceptance in the future of a new thesaurus that will bridge the existing gap between the quantum formalism and the traditional form of the physical realism.

Let me also recollect the well-known standpoint on the art as a 'superscience', intuitive, heuristic, 'brain righthalf' method of reality cognition. An analogous role is played by the quantum metaphysical M-language. It helps (together with the semiclassical C-language) to classify the known effects and predict on a qualitative level the results of new experimental situations. In general, the refusal, at some stage, of the axiomatic approach promotes the movement ahead. (Note in this connection the Hedel theorems.)

When we solve specific problems of quantum optics, the use of the M - or C-languages is, apparently, optimal at the first and final stages: before and after the more strict, model calculations in the Q-language.
5. Summing up, we arrive at the pessimistic vision of the modern state of 'the great quantum problem' of 20th century physics, the problem of presenting a realistic interpretation of the state vector. Despite the efforts of several generations of physicists, hundreds of articles, dozens of conferences and monographs, the invention of lots of terms, there is evidently no reasonable, commonly accepted alternative to the Copenhagen $\mathrm{Q}_{\mathrm{C}}$-language.

Quantum optics is distinguished by the fact that when observing light by a naked eye we perceive a quantum object, the light field, directly, so the interface between the classical and quantum worlds can be put somewhere in the eye retina. (In this context the experiments on the detection by a naked eye of the nonclassical light, for example, of the two-photon one, and the absolute measurements of the retina quantum efficiency with the help of such light seem to be of interest.)

Let us imagine that weak light from a star is observed by the naked eye. Let the average flow of photons $R$ be much less than, for example, 1 photon per second. If the quantum efficiency of the eye is 0.1 , then the mind registers, on average, each tenth photon. And we are sure that every sensing of a flash in the eye is caused by some preceding reason, i.e. by the arrival and absorption of the M-photon.

However, according to the only qualitative light theory based on quantum electrodynamics, there are a priori no M-photons; there is a field state only, a pure or a mixed one. The Q-theory predicts only the average rate of flashes $R$ (which is determined by the projection of the state vector at the Fock vectors $|1\rangle_{k}$ ) and the other statistic parameters of our sensing. So, what do we see at the flash moment: the M-photon or the state?

The latter suggestion is contradictory to all our instincts; however, the first one has no quantitative theoretical base. All existing mathematical models of the quantum measuring process contain two nonoverlapping multitudes of objects: the $c$-numbers and the $q$-numbers. It means that 'the iron curtain' between the classical and quantum worlds remains impenetrable and quantum optics is helpless here, as well as in other directions of quantum physics. The Mphoton remains, as 60 years ago, a 'thing in itself' and we, as before, play the role of Plato's cave inhabitants, observing only the shadows of the quantum world projections.

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