Physics – Uspekhi **37** (1) 57 – 68 (1994)

PHYSICS OF OUR DAYS PACS numbers: 04.50. + h

Possible existence of asymptotic freedom of gravitational interactions in nature

M A Markov

Contents

1. **Introduction** **57**
2. **Problem of the very early universe** **60**
3. **Possible existence of special 'ultramicrouniverse' physics in the Planck length range** **66**

References **68**

Abstract. The possibility of existence of asymptotic freedom of gravitational interactions, i.e. that gravitational interactions become weaker on increase in the mass density, is discussed. In spite of all differences between the 'string' approach and the formalism discussed in the paper, a future full picture of the Universe may possibly have many features in common with the 'string' approach.

1. Introduction

The asymptotic freedom of gravitational interactions will be understood here to be weakening of the gravitational interaction on increase in the mass density.

In Einstein's equations the gravitational constant $\varkappa_0$ is replaced with the function

$$\varkappa = \varkappa_0 \psi \left(\frac{\rho}{\rho_0} \right) , \tag{1}$$

where ρ is the density of the mass of matter in a reference system comoving with matter and ρ_0 is the limiting maximum mass density at which the function ψ vanishes:

$$\psi \to 0 \quad \text{when} \quad \rho \to \rho_0 \, .$$

Variation of such a modified Einstein action function with respect to the metric g_{ik} gives rise to equations which for $\rho \ll \rho_0$ becomes identical with Einstein's equations describing, let us say, the Friedman universe, but in the limit $\rho \to \rho_0$ the traditional term on the right-hand side of Einstein's equations tends to zero (because $\varkappa \to 0$). However, the additional term which appears automatically as a result of variation of the modified action function is of the de Sitter type, namely $F(\rho/\rho_0)\Lambda$, where $F(\rho/\rho_0) \to 1$ when $\rho \to \rho_0$, and the constant Λ term of the de Sitter metric:

$$\Lambda = \frac{1}{l^2_{\min}} \, .$$

In both quantum and classical physics the universal constants (c, which is the limiting velocity of signal propagation, e representing the constant minimum electric charge; and $\hbar$, the Planck constant representing the minimum value of the action) together with the gravitational constant $\varkappa_0$ can be used to form two limiting quantities: the shortest length and the highest density. One of these quantities, which I shall denote by A, is purely classical, i.e. it does not contain the Planck constant $\hbar$, namely

$$A = l_{\min} = \frac{e\varkappa_0^{1/2}}{c^2} \sim 10^{-34} \text{ cm} \, ,$$

$$= \rho_0 = \frac{c^6 e^2}{\varkappa_0^2} \sim 10^{95} \text{ g cm}^{-3} \, . \tag{2}$$

The second, B, contains the Planck constant:

$$B = l_{\min} = \left(\frac{\hbar\varkappa_0}{c^3} \right)^{1/2} \sim 10^{-33} \text{ cm} \, ,$$

$$= \rho_0 = \frac{c^5}{\varkappa_0^2 \hbar} \sim 10^{94} \text{ g cm}^{-3} \, . \tag{3}$$

The task here will be to describe a certain *imagined* world in which asymptotic freedom of gravitation is realised not in quantum but in classical physics, and to discuss not only the possible existence of asymptotic freedom of gravitational interactions in the imagined world, but also the corresponding formalism, in which the main role is played by the hypothesis of the existence in such a world of the highest mass density ρ_0 and the shortest length $l_{\min}$, associated with this formalism. In this approach it is natural to adopt the classical values of ρ_0 and $l_{\min}$ for such an imagined world. In this case it is obvious that our imagined classical universe with its special formalism including asymptotic freedom could predate the appearance of quantum mechanics and could claim to describe the real Universe and predict its initial inflationary phase and the de Sitter metric at the time of appearance of the Universe. However, it is widely held that the future theory of gravitation and cosmology, providing a

M A Markov Institute for Nuclear Research, Russian Academy of Sciences, 7a prospekt 60-letiya Oktyabrya, 117312 Moscow, Russian Federation, Tel. (7-095) 132 62 19

Received 1 September 1993
Uspekhi Fizicheskikh Nauk **164** (1) 63-75 (1994)
Translated by A Tybulewicz

satisfactory descrip-tion of nature, will be developed within the framework of quantum theory (or, for example, of string theory). Therefore, in this communication the limiting values used in the mathematical apparatus of the theory will be described by the expressions for ρ_0 and $l_{\min}$ containing the Planck constant. Bearing in mind that the values of these quantities are arbitrary in the given formalism† and anticipating later discussion, one can say that the formalism developed in a number of papers for collapsing objects containing only gravitating matter leaves the collapse of such an object at the limiting length of the order of $l_{\min}$. When free gravita-tional radiation appears in the universe the asymptotic freedom stops the collapse only in the presence of a strong free gravitational field if it is converted into gravitating matter. I shall consider various ways in which this occurs, particularly in the case of free relic gravitational radiation and in the case of the appearance of Kasner perturbations of the metric. If the collapse of a black hole stops at a distance of $\sim l_{\min}$ from a classical singularity, the hole unavoidably becomes a source of new universes in another space (R'' space) which is in the absolute future relative to the time at which the given black hole has formed [22, 23].

Formation of black holes in a given universe (in the course of evolution of stars or collapse of a closed universe) causes it to split into a number of universes, and the whole Universe represents an overall set of many universes which develop in their own space and time [33]. There is an ongoing discussion of possible wormhole-type links and of the possibility of existence, at the limit of the shortest lengths, of a characteristic 'ultramicrophysics' in which the classical ideas on space and time break down. One cannot exclude the possibility that lengths shorter than $l_{\min}$ do not exist in regions where the mass density reaches the limit of the highest values. In classical language this can be interpreted 'physically' by assuming that a region of this kind represents an absolutely hard medium in the form of a sphere along which a signal travels at an infinitely high velocity, so that for example near a classical singularity we have

$$l_{\min}\big|_{v=\infty} = t = 0 \,.$$

In other words, time and space seem to be absent here. At the end of this paper I shall cite a study of a two-dimensional black hole considered using string theory. Its author reaches the conclusion that in the exact solution near a classical singularity the metric becomes Euclidean instead of Lorentzian and that the whole Universe can be regarded overall as a set of many universes. One cannot exclude the possibility that our later description of the imagined universe is characterised by asymptotic freedom of gravitational interactions and has certain characteristic features of the future theory of gravitation which matches the properties of gravitation found in nature.

However, it may not be necessary to change the classical equations of gravitation. It may be that the quantised form of these equations will automatically solve the cosmological difficulties of the classical equations. These ideas are valid and they are frequently put forward. First of all, it is useful to recall the fate of such ideas in the face of known difficulties of other (nongravitational) fields.

† I must mention the remarkable proximity of the numerical values of what we shall call the classical and quantum values of the limiting densities (ρ_0) and lengths ($l_{\min}$) which can be derived from the universal constants.

As is known, all the other fields *considered separately* face difficulties of infinite values of the energy of their point sources. The appearance of the Dirac equations must raise the question: do these difficulties remain in the Dirac electrodynamics? We shall later obtain a general answer: the quantum theory of any of the known fields does not solve the difficulty under discussion.

One can frequently hear the conclusion that, in contrast to other fields, we have no quantum theory of the gravitational field. Strictly speaking, we have a quantum theory of *all* fields considered separately, including the gravitational field, but only in the case of *weak fields*. This statement means that near a point source of, for example, an electromagnetic field, when the field becomes strong, we must allow for the interaction of the electromagnetic field of an electron with all types of elementary particles that exist in nature. These virtual pro-cesses make their own contribution to the self-energy of the electron. Then, in principle, one cannot exclude gravitons. This situation can be described by the following statement: "Every, i.e. any, so-called elementary particle consists — in this sense — of all the other particles" [1].

In many cases attempts have been made to remove divergent expressions from the modern theory of fields by various modifications of specifically classical equations of fields in their relativistic form, for example, by introducing (into the relevant wave equations) relativistically invariant cutoff factors, which reduce correspondingly the contribution of high frequencies to the self-energy of a source.

Other classical relativistically invariant procedures for avoiding these difficulties have been proposed and they range from what is known as the 'lambda-limiting' process to the use of indefinite metrics.

All these attempts to solve the problem of divergences in the theory of fields have been unsuccessful.

It is these attempts that have led to the propagation of a signal at superluminal velocities in the region of a singularity. An infinitely hard core seems to appear in the region of a singularity.

These 'insiduous' failures resulted in perplexity: a conflict with relativistic theory arose even when, for example, explicitly relativistic expressions for the form factors were introduced. However, it later became clear that the difficulty arose entirely from consideration of the problem within the framework of just one time t, whereas the field is a system comprising an infinite number of particles.

A consistent relativistically invariant theory of many particles is known to require the introduction of many times. Such a many-time formalism was first proposed in the well-known work of Dirac, Fock, and Podolsky relating to the quantum electrodynamics of a finite number of particles. In the classical (nonquantum) electrodynamics of a finite number of particles this formalism is given in one of my papers [2]. I unsuccessfully sought a many-time formalism for the description of an infinite number of photons for the electromagnetic field itself. The appearance of a paper by Tomonaga [3] demonstrated that this problem was solved by him. In the Tomonaga formalism and then in the formalism of Schwinger [4] each point (x, y, z) of a space-like surface is attributed its own time $t(x, y, z)$. I sought unsuccessfully a generalisation of the Dirac – Fock – Podolsky formalism to the spectrum of photons, namely quanta in a gas of photons which should have a continuous structure. It is possible that such a formalism, equivalent to the Tomonaga formalism, does exist.

The appearance of the Tomonaga – Schwinger equation accounted for the justifiable failure of all the previous attempts to liquidate divergences in the theory of fields within the framework of one time for all space-like points.

The historical value of the Tomonaga – Schwinger equation is therefore that all the solutions of the field equation modified in this way are not solutions of a consistently relativistic equation: they do not satisfy the condition of integrability of the Tomonaga – Schwinger equation, namely the condition of commutativity of the corresponding functions on a space-like surface, i.e. they do not satisfy *the requirement of finite velocity of signal propagation*.

In other words, these approaches to solve the problem of singularity of fields are fully covered by the requirements of *consistent relativistic invariance of the many-(infinite-)time formalism of the Tomonaga – Schwinger equations.*†

The so-called renormalisation method is highly developed and is being used very successfully: this method makes it possible to extract finite values from a singularity of values in field theory and such finite values can then be used satisfactorily in further calculations. Unfortunately, this method of extraction of finite values from infinite ones works only within a framework of one field, i.e. when the contribution of all the other fields is ignored. However, the main point is that this procedure is 'imposed from outside' on the natural formalism of field theory. This objection to the renormalisation procedure is supported by the well-known comment of Dirac in the preface to the Russian edition of his book, where he states that he avoided presentation of the renormalisation methods because, in his opinion, they would disappear in a future consistent theory of fields. Moreover, there is a suspicion that the validity of the renormalisation methods has not yet been fully established.

I mentioned earlier that the self-mass of a source of any field includes contributions also of other fields, not excluding the contribution of gravitation. However, allowance for the gravitational field requires the knowledge of the physical laws valid over distances of the order of the Planck length

$$l \sim 10^{-33}\ \text{cm}\ .$$

In any case it is possible that the existence in nature of the asymptotic freedom of gravitational interactions in strong gravitational fields, which will be discussed later, will help one to understand better the physical meaning and success of the renormalisation methods. In a number of papers published in the last decade I considered the characteristic features of theories of gravitation augmented by asymptotic freedom of gravitational interactions. Here, I shall summarise some of the results, removing unjustified pretensions and sometimes even erroneous statements.

* * *

For the simplest and 'poorest' in its physical content imagined world, I proposed ten years ago (1982) a modification of one Einstein's equation, particularly in the simplified form [5, 17]:

$$\left(\frac{\dot{R}}{c}\right)^2 + 1 = \frac{8\pi R^2 \varkappa_0}{3c^2}\left[\rho\left(1 - \frac{\rho^2}{\rho_0^2}\right) + \Lambda'\left(\frac{\rho}{\rho_0}\right)^2\right] . \qquad (4)$$

This expression describes the history of an imagined isotropic and homogeneous closed universe filled with dust-like matter whose density is ρ at the pressure $p = 0$.

During the epoch of this universe when the density is $\rho \ll \rho_0$, the right-hand side of Eqn (4) is reduced to the first term $8\pi R^2 \varkappa_0 \rho / 3c^2$. During this epoch of the universe it is of the Friedman type. On collapse of the universe, when the increasing density of matter tends to ρ_0, the first term on the right-hand side of Eqn (4) vanishes in the limit $\rho = \rho_0$ and from the second term only the constant Λ', with the dimensions of the density of matter, remains. Selection of the expression for this constant in the form

$$\Lambda' = \frac{3c^2}{8\pi\varkappa_0}\Lambda$$

converts this term to $R^2\Lambda$, where the constant Λ has the dimensions of $1/l^2$ (l is in centimetres).

In the course of the process $\rho \to \rho_0$ the universe becomes of the de Sitter type and it does not have a singularity in its history.‡ An equation of the type (4) describes an imagined universe with properties that seem to be very far from the properties of our real Universe. However, we shall see later that some properties of this imagined universe are very useful in the search for a formalism describing our real Universe. It should be mentioned that Eqn (4) is discussed in my paper [5]. Putting forward the hypothesis of the existence of a limiting matter density as the solution to the problem of singularity of a collapsing universe (1982), I was not aware (for accidental reasons) that this problem had been considered earlier by Wheeler [6] and that he rejected it 'outright' because the corresponding Einstein's equation admits the possibility that sound may have a higher velocity than light. The equation proposed by me is free of this difficulty.

On the other hand, it is clear that any modification of the theory which leads to a finite curvature during gravitational collapse unavoidably results in a finite value of the density of the collapsing matter.

The model of a universe described by Eqn (4) represents a universe which oscillates perpetually between the values $R_{\max}$ and $R_{\min}$. The nature of these oscillations is very illuminating and useful in the search for a theory capable of

† It is worth mentioning that in 1943 my doctoral dissertation was devoted to the many-time formalism in electrodynamics, and the main result of an analysis of the attempts to solve the problem of divergence of point sources of fields by introducing relativistically invariant factors is in conflict with the following system of equations for particles

$$\hbar\frac{\partial\psi}{\partial t_s} = H_s(t_s r_s), \quad s = 1, 2, 3, \ldots .$$

In this case the system of equations simply does not have such solutions since the conditions for the existence of the solution requires a finite velocity. Therefore, introduction of relativistically invariant form factors is in conflict with a consistent relativistic theory of electrodynamics. Therefore, I was unable to write down this infinite system of equations in the form a single equation, as was done by Tomonaga. My thesis was not published because we were at war. The failure of causality in a small region has the danger that it might lead to observable failures of causality in macroregions. However, we cannot exclude the possibility that the existence of such limiting shortest lengths in cosmology does not lead to the appearance of observable propagation of a signal at a velocity exceeding c in macrophysics. I shall naturally consider this problem later. More probably it does not lead to violation of the natural sequence of the past and future.

‡ This is valid provided one ignores the possibility of the appearance in the real Universe of a free gravitational field (for example, of the Kasner type) or even of relic radiation.

describing the real Universe throughout its history. In a subsequent series of papers dealing with the development of the physical ideas contained in Eqn (4) it has been shown that the modification of Einstein's equation in the form given by Eqn (4) is based on the physical hypothesis that the gravitational interaction of matter decreases at high densities. In fact, the constant representing the gravitational interaction is replaced in Eqn (4) by the function $\varkappa = \varkappa_0[1-(\rho^2/\rho_0^2)]$. At $\rho = \rho_0$ the gravitational interaction vanishes completely. This property of matter can be called asymptotic freedom of gravitational interactions. Calculations show that the collapse of a closed universe with a bare mass M_0 causes $\varkappa$ to vanish only if $M_0 = \infty$. In other words, $\varkappa$ cannot have negative values. A better understanding of a possible role of asymptotic freedom of gravitational interactions in the solu-tion of the singularity problem of collapsing systems filled with gravitating matter is provided in the paper by Markov and Mukhanov [7]. A Λ-like term is introduced 'by hand' into Eqn (4) and in the limit $\rho \to \rho_0$ this term leads to the de Sitter equation. The action S is considered in the form [7]

$$S = \frac{c^4}{16\pi G_0}\int (R + 2\varkappa\varepsilon) g^{1/2} \mathrm{d}^4 x \ .$$

This form of the action is analogous to the expression given for the action in Einstein's theory. However, here $\varkappa$ is not a constant but represents a function of the energy density ε:

$$\varkappa = \frac{8\pi G_0}{c^4}\psi\left(\frac{\varepsilon}{\varepsilon_0}\right) \underset{\varepsilon\to\infty}{\longrightarrow} 0 \ , \tag{5}$$

where the energy density is given by the Fock expression [8], whereas the mass density ρ^* obeys the equation of continuity

$$(\rho^* u^i) = 0 \ .$$

Variation of S with respect to the metric g_{ik} leads to the equation

$$R^i{}_k - \frac{1}{2}R\delta^i{}_k = \left(\frac{\varepsilon\partial\varkappa}{\partial\varepsilon} + \varkappa\right)T^i{}_k - \varepsilon^2\frac{\partial\varkappa}{\partial\varepsilon}\delta^i{}_k \ , \tag{6}$$

where

$$T^i{}_k = (\varepsilon + p)u^i u_k - p\delta^i{}_k \ .$$

Eqn (6) can be rewritten in the form

$$R^i{}_k - \frac{1}{2}R\delta^i{}_k = G(\varepsilon)T^i{}_k + \Lambda(\varepsilon)\delta^i{}_k \ , \tag{7}$$

where

$$T^i{}_k = (\varepsilon + p)u^i u_k - p\delta^i{}_k \ . \tag{8}$$

Eqn (6) is a generalisation of Einstein's theory of gravitation to the case of the existence in nature of asymptotic freedom of gravitational interactions, described mathematically by Eqn (1). This is illuminating because the asymptotic freedom leads automatically to the appearance of a Λ-like term, as is evident if one writes down this equation in the form (7). On increase in the energy density ε the first term on the right-hand side of Eqn (7) tends to zero and the equation assumes the de Sitter form. I demonstrated [5] that when the pressure is $p = 0$ in an isotropic dust-filled universe, Eqn (6) describes a universe oscillating between the values $R_{\max}$ and $R_{\min}$.

Here, $R_{\min}$ is related to the constant ε_0 in my paper [5]. If ρ_0 in Eqn (4) and ε_0 in Eqn (5) are expressed in terms of the universal constants c, $\hbar$, and $\varkappa_0$, then $R_{\min}$ (i.e. the length over which the collapse stops and a new period of inflation of the universe begins) proves to be the Planck length

$$l_{\mathrm{Pl}} \sim \left(\frac{\hbar\varkappa}{c^3}\right)^{1/2} \sim 10^{-33}\ \mathrm{cm} \ .$$

In the view of the close similarity of the models of universes described by Eqns (4) and (6), I shall (for the sake of simplicity) use more frequently Eqn (4) to illustrate the special nature of the physical properties of a gravitational field† that has the asymptotic freedom, which is discussed above and which does not allow us to use the approach in question in developing a currently acceptable cosmology; I shall use also this equation to consider whether there is any possibility of overcoming such problems.

2. Problem of the very early universe

According to current ideas, the very early universe arises in the form of a de Sitter universe, the energy of which, represented by the Λ term, decays in the course of expansion of the universe into all forms of particles, and the de Sitter universe transforms into one of the Friedman type with all its characteristics. The serious study of the various modifications of what is known as the 'inflationary' period in the history of the universe started with the paper of Guth [10].

Eqns (4) and (6) describe a variant of such an inflationary process. It should be mentioned that the title of the paper by Markov and Mukhanov [7] was "de Sitter-like initial state of the universe as a result of asymptotic disappearance of gravitational interactions of matter''. In publishing a paper presenting the properties of Eqn (4) in 1982 I was not aware that over a decade earlier (in 1970) Gliner published a paper on "Vacuum-like states of a medium and Friedman cosmology'' [11], where he wrote: "The purpose of this note is to show that the vacuum-like state of a physical medium (with the energy tensor $T_{ik} = \Lambda g_{ik}$) can be the starting point of any of the three Friedman models''.‡

In modern variants of the very early universe it is postulated that primordial matter has the properties of a scalar. The scalar may, in principle, have a variety of origins. For example,

† Eqn (4) is not obtained by variation of the action of the type (6) with respect to the metric. However, one can obtain an analogue of Eqn (4) by taking the function $\varkappa$ in the form

$$\varkappa = \varkappa_0\left(1 - \frac{\varepsilon^2}{3\varepsilon_0^2}\right) \ . \tag{3'}$$

Then the function $G(\varepsilon)$ in Eqn (7) becomes

$$G(\varepsilon) = \varepsilon\frac{\partial\varkappa}{\partial\varepsilon} + \varkappa = 1 - \frac{\varepsilon^2}{\varepsilon_0^2} \ ,$$

and the term Λ' is then

$$\Lambda'(\varepsilon) = -\varepsilon^2\frac{\partial\varkappa}{\partial\varepsilon} = +\frac{2}{3}\left(\frac{\varepsilon}{\varepsilon_0}\right)^2\varepsilon_0 \ .$$

In this way we obtain an analogue of Eqn (4) in the form

$$\left(\frac{\dot{R}}{a}\right)^2 + 1 = \frac{8\pi R^2\varkappa_0}{c^2}\left[\rho\left(1 - \frac{\rho^2}{\rho_0^2}\right) + \Lambda'\left(\frac{\rho}{\rho_0}\right)^3\right] \ , \tag{4'}$$

where

$$\Lambda' = \frac{2}{3}\rho_0 \ .$$

‡ One can also say that Gliner revived the ancient cosmology of Anaxagoras, postulating some sort of primordial form of matter "incapable of any motion'' [12].

we cannot exclude that this scalar is a gas of black holes.† In this case one may expect an equation of the type (4) to be suitable for the description of the very early universe. A cold gas of black holes may create, in a natural manner, in the course of Hawking evaporation, a hot phase of a Friedman universe to which Eqn (4) with $p = 0$ no longer applies, but in principle Eqn (6) (with $p \neq 0$) is still valid. In the realisation of this black-hole hypothesis it is important to know in greater detail the properties of black holes. Specifically, we have to know whether a black hole disappears completely in the process of Hawking evaporation. Moreover, we have to know whether black holes with arbitrarily small masses can exist. There are grounds for assuming that the smallest mass of a black hole is determined by the universal constants c, $\hbar$, and $\varkappa_0$, namely $m_{\min} \sim (c/\varkappa_0)^{1/2} \sim 10^{-5}$ g and the radius $r \sim (\hbar\varkappa/c^3)^{1/2}$ cm. In 1965 I suggested [13] the possible existence of an upper limit to the mass spectrum of elementary particles: if there is in nature a fundamental length

$$l_0 = \left(\frac{\hbar\varkappa}{c^3}\right)^{1/2} \sim 10^{-33}\ \text{cm}\ ,$$

then the energy of a particle with the wavelength $\lambda = l_0$ is

$$E = \frac{\hbar c}{l_0} = \left(\frac{\hbar c}{\varkappa}\right)^{1/2} c^2 = m_{\max} c^2\ ,$$

and its maximum mass is

$$m_{\max} = \left(\frac{\hbar c}{\varkappa}\right)^{1/2} \sim 10^{-5}\ \text{g}\ .$$

This leads to the hypothesis of the existence in nature of a heaviest elementary particle with a mass of 10^{-5} g, called the maximon. This hypothesis is completely unrelated to the theory of black holes developed later. Subsequently several writers considered the possible existence of such a particles in nature and have proposed different names for it. The theory of Hawking evaporation of black holes (put forward in 1973)‡ led to the realisation that the evaporation unavoidably creates a black hole with parameters typical of a maximon. This object may be called also an elementary black hole. It should either disappear instantaneously, releasing all its energy in one radiative event, or it may remain stable. There have been several papers§ justifying the stability of an elementary black hole [14].

It would seem that there is a serious objection against the stability of elementary black holes, which was put forward by Hawking [15]. According to Hawking, if any number of black holes appears in any closed volume, then they all should disappear at $t \to \infty$.

Without going into the details of Hawking's thermodynamic arguments, it should be pointed out that in a closed volume filled with an arbitrarily large number of stable elementary black holes, they should disappear by the coalescence of two or several into a large black hole, which should emit radiation and decay to an elementary stable black hole. Therefore, out of the initial arbitrarily large number of elementary stable black holes only one elementary black hole remains. The primordial scalar matter of the very early universe can therefore be also a gas of stable elementary black holes (maximons), since there is a real possibility of conversion of a gas of stable elementary black holes in the course of inflation into hot matter of a Friedman universe.¶ A gas of black holes considered as primordial scalar matter is of the greatest interest in the case of a closed universe, for which the phase of inflation of the finite maximum expansion is followed by a collapse phase, which may be called the very late phase of the universe. It is natural to assume that the very late universe should return to its initial de Sitter phase. In other words, all the variety of fields and particles during the Friedman phase of the universe should return again to is primordial scalar form. Under conditions of a high density of all the forms of matter in the final phase of the collapse, one can expect conversion of all the matter in the final phase back to a scalar gas of black holes. The asymptotic freedom of gravitational interactions leads to the formalism of the de Sitter symmetry of the very early and very late closed universe. The primordial matter in the form of black holes is an example of the physics of such symmetry.

I shall leave for later discussion the main flaw of the model described by Eqn (4), which is the impermissible neglect of the fate of the free gravitational field that unavoidably appears in the real Universe in the course of collapse. Bearing in mind, for example, the creation of the Kasner metric, it would be of some interest to consider in

†The creation of matter (from the primordial Λ-matter) in the form of black holes has a number of nontrivial consequences which apply to the nature of the particles after decay of the primordial black holes; for example, one cannot exclude the possibility that the maximum mass in the mass spectrum of elementary particles is the mass of the smallest black hole. Moreover, the baryon asymmetry may be related to decay of the primordial black holes.

‡It is perhaps relevant to the history of the theory of Hawking evaporation of black holes that it started with the following idea of Hawking, formul-ated almost in the form of a law: the mass of a black hole can only increase. However, at this time Frolov and I were studying the behaviour of the mass of charged matter that has escaped into the Schwarzschild sphere. We found that such an object, which we did not yet call a black hole (1970), should release an electric charge reducing the charge to $\varepsilon \sim 137e$ [40] and reducing correspondingly the mass; here, e is the electron charge. This was in conflict with the Hawking law as then established. This conflict troubled us greatly. I carefully looked for an error in our calculations and reached only, as it seemed to me, a possible critical conclusion that the Hawking law is valid in classical physics, but it breaks down in quantum physics. In my paper presented at the Warsaw conference in 1973 [31] I dared to interpret this creation process reducing the mass of a black hole, as creation of a pair — in the electric field of a black hole — when one particle escapes to infinity and the other drops into the hole reducing its charge and mass. This paper failed to elicit any response.

§One cannot exclude the possibility that only the class of cold black holes is stable [1].

¶It should be pointed out that the appearance of a gas of nonelementary black holes in the course of collapse is not a new hypothesis. However, the statement that in the initial and final phases of the history of the universe this gas consists of elementary black holes is a new hypothesis, which may raise objections. In fact, if the primordial scalar matter is a gas of elementary black holes, which in the course of inflation should largely disappear by merging into larger black holes and in the process of Hawking evaporation should form the whole matter of a Friedman universe, the question arises whether there is sufficient time for this merging process to occur. True, the ideas put forward in the case of Dirac magnetic monopoles (namely fast reduction in their density in the process of exponential expansion of the universe) may not be decisive here, because in the process of exponential expansion of the very early universe considered on the basis of the model described by Eqn (4) the matter density of the black holes remains practically at the Planck value. The lifetime of such a state of the early universe depends on the value of its bare mass m_0. During this time the distance between maximons is practically zero.

greater detail the role of the asymptotic freedom in this 'toy' model of a closed universe.

It is well known that the covariant divergence of the left-hand side of Einstein's equation vanishes. Vanishing of the divergence of the right-hand side of Eqn (4) leads to an equation whose integration yields the relationship between the mass density ρ and the volume R^3 of a closed universe which is isotropic and filled with dust:

$$\frac{\rho(\rho + \rho_0)^3}{|\rho_0 - \rho|} = \frac{M_0 \rho_0^2}{2\pi^2 R^3} . \tag{9}$$

It is assumed here that the constant $\Lambda' = 2\rho_0$ and M_0 is the bare mass of the closed universe.

Here, $\dot{R} = 0$ leads, in accordance with Eqn (4) to the maximum, $R_{\max}$, and the minimum, $R_{\min}$, radius of the universe.

If I also venture to select ρ_0 in Eqn (4) in the Planck form, then the maximum density is

$$\rho_0 = \rho_{\mathrm{Pl}} = \frac{c^5}{\hbar \varkappa_0^2} \sim 10^{94} \text{ g cm}^{-3} .$$

We then have

$$R_{\min} \approx l_{\mathrm{Pl}} \left(1 + \frac{m_0}{M_0}\right) ,$$

where

$$m_0 \approx \left(\frac{\hbar c}{\varkappa_0}\right)^{1/2} \sim 10^{-5} \text{ g} .$$

In other words, for all the closed universes with the bare mass $M_0 \gg m_0 \approx 10^{-5}$ *g the minimum dimensions of the universe are given by the Planck length.* If, for example, the bare mass of our Universe is $M_0 \approx 10^{55}$ g, then its smallest size at the end of the collapse is

$$R_{\min} = l_{\mathrm{Pl}}(1 + 10^{-60}) . \tag{10}$$

We can ask the question: is this property retained also in the model which will describe the real closed Universe?

Another question is: will the classical properties of space be retained in the future correct theory down to the Planck length?

In other words, are the corrections to the Planck length in the form $l_{\mathrm{Pl}} m_0 / M_0$ physically distinguishable in the range of the lengths discussed here?

In the model under discussion described by Eqn (4) the length at which the collapse ends and the process of new expansion of the universe begins is governed by the bare mass of the universe. Conversely, the point of 'rebound', i.e. the beginning of a new cycle of the oscillating universe, determines the bare mass M_0 in this new cycle. If the lengths l_{Pl} and $l_{\mathrm{Pl}}[1 + (m_0/M_0)]$ are indistinguishable in this region of space, then in the next period of the oscillation a universe may appear with any bare mass

$$M_0 \gg m_{\mathrm{Pl}} .$$

This problem is considered again at the end of the paper after discussing the possibility of existence of 'ultramicrouniverse' physics when $l < l_{\mathrm{Pl}}$.

We recall that the model of a perpetually oscillating universe of the type described by Eqn (4) with a constant value of M_0 may be unrelated to the real Universe, because the entropy of the latter should increase from one oscillation to the next. Something must happen to the universes in the course of their collapse to avoid this fundamental difficulty of an increase in the entropy. A major shortcoming of the model (4) is the neglect of the unavoidable fluctuations of the mass density in the process of collapse. The maximum density in the model (4), at which the collapse ceases, also depends on the bare mass of the closed universe:

$$\rho_{\max} = \rho_{\mathrm{Pl}} \left(1 - \frac{m_{\mathrm{Pl}}}{M_0}\right) . \tag{11}$$

It may happen that in the course of fluctuation in some region of a collapsing universe the value $\rho_{\max}$ corresponding to the end of collapse appears at a time earlier than given by Eqn (11). The question is: does it mean that in some regions of a collapsing universe an anticollapse process may appear earlier than in other parts of the universe? Does not this give rise to decay of the initial universe into a series of universes with a smaller bare mass?

Before considering a more detailed discussion of such possibilities, it is useful to note that inflation of a universe in the model (4) differs considerably from inflation of the known models because the duration of the de Sitter phase of the very early and very late universe is governed again by the bare mass (M_0) of the closed universe. In the case when $M_0 \sim 10^{55}$ g, the de Sitter phase, during which the mass density remains practically constant during inflation, extends from $R \approx l_{\mathrm{Pl}} \sim 10^{-33}$ cm to $R \sim 10^{-13}$ cm. If the pressure is finite, $p \neq 0$, it extends to $R \sim 10^{-3}$ cm [16]. The considered modification of the Einstein's equations is based, as stressed repeatedly above, on the assumed existence of asymptotic freedom of gravitational interactions. It is necessary to stress one con-sequence of this hypothesis which is important for under-standing the physical consequences of asymptotic freedom for the nature of the particles representing the matter, manifested in Eqns (4) and (4′). It follows from these equa-tions that the first term on the right-hand side vanishes at ε_0 when the density goes to the limit $\varepsilon \to \varepsilon_0$. This circumstance led to an incorrect comment in several of my papers that the formalism of Eqns (4), (4′), and (6) includes both creation and annihilation of particles. This would apply particularly to Eqn (4). However, it follows from the text [7] that the action was varied by Fock's method [8], which uses the expression for the number density n of particles that satisfy a continuity equation, which is typical of any classical hydrodynamic model:

$$(nu^i)_i = 0 . \tag{12}$$

The situation under discussion can be described as follows: particles do not disappear in the process of collapse, but each of them becomes increasingly 'density-free' (its mass decreases) or, more correctly, it 'weighs' less because of weakening of the gravitational interaction. At the moment when the collapse ends, the initial number of particles is conserved in the form of their almost zero-density 'souls'.

It follows from the above discussion that in order to describe the very early and very late phases of a universe it is the hydrodynamic form which should be used for the energy-momentum tensor; in particular, this applies to dust-like matter. If it is found that our Universe is not closed, then the problem of collapse of the very late universe disappears completely. However, the problem of collapse is still important in this case. In an open universe the collapse of massive stars and, ultimately, black holes is unavoidable.

Returning to the idea of the existence in nature of asymptotic freedom of gravitational interactions and its possible role in solving the singularity problem in the process of gravitational collapse, we recall that the singularity problem with which we are dealing can be solved only on condition that the asymptotic freedom is either directly or indirectly related to a free gravitational field. More exactly, we have to consider whether the energy pseudotensor of the gravitational field is necessarily transformed under the conditions of collapse of the universe into a tensor in some material form and then appears on the right-hand side as a correction to the total matter tensor.

In the simplest case we are dealing with, for example, the fate of relic gravitons, which obviously exist at present in our Universe, and of their fate in the process of collapse. If, contrary to our hope, relic radiation does not appear in any material form on the right-hand side of Eqn (4), the existence of a free gravitational field in the final phase of the collapse unavoidably leads to a singularity.

Estimates have been published of the cross section of creation of electron – positron pairs by two gravitons. The probability of this process rises on collapse of a universe and, if we venture to continue these estimates to graviton wavelengths comparable with the Planck length, then conversion of relic graviton waves into gravitating matter is possible and this would allow us to avoid the danger of appearance of singularities by the asymptotic freedom of gravitational interactions.

The question also arises whether strong free gravitational radiation can generate black holes. According to Isaacson, the integral of the pseudotensor already has the properties of a tensor under the conditions of flat space at infinity. In the case of short wavelengths the condition at infinity may be weaker [18]. Therefore, gravitating objects might form from a gravitational field and for these objects we should allow for the asymptotic freedom in Eqn (4). It should be noted that a collection of waves travelling in different directions has a rest mass [18].

When strong Kasner fields appear during the very late phase of a universe, once again we may hope that particles are created under the conditions of very strong anisotropy. This process is also supported by familiar considerations. I have in mind the process of isotropisation of the Kasner anisotropy in the very early universe as a result of rapid creation of material particles. Recent ideas indicate the need for quantisation of a strong gravitational field if not to solve the problem of collapse using unmodified Einstein's equations then at least because of the need to have a complete theory of the interaction of all the fields.

I shall not discuss in detail the possibility of modification of the left-hand side of Einstein's equations and instead I shall refer the reader to a recent paper [19]. On examination of the postulated close identity of the physical content of these two apparently so different modifications of Einstein's equations we may consider the frequently discussed modification of Einstein's equations, when the initial action function S is replaced by the expression

$$R \to R + \beta R^2 \, ,$$

where β is a constant.

The new form of the left-hand side of the equation can be transformed to its old form.† Then, on the right-hand side of this equation we find that $T^0{}_0$ appears in the form

$$T^0{}_0 = \varkappa_0 \varepsilon \left(1 - \frac{\varepsilon}{\varepsilon_0} + \ldots\right) = \varkappa_0 \varepsilon f\left(\frac{\varepsilon}{\varepsilon_0}\right) .$$

Then, $\varkappa_0 f(\varepsilon/\varepsilon_0)$ plays the role of the 'gravitational constant' which tends to zero on increase in the energy density ε. In other words, in this example of the possible modification of the left-hand side of Einstein's equations the mathematical formalism reveals realisation of asymptotic freedom. Incidentally, some variants of this modification are given also in the paper where Eqn (4) is proposed. It may be pointed out that in the example demonstrating the role of asymptotic freedom at the end of the collapse process the function f is taken in the form [7]

$$f = \left(1 + \frac{\varepsilon}{\varepsilon_0}\right)^{-2} \sim 1 - \frac{\varepsilon}{\varepsilon_0} + \ldots$$

A more detailed discussion of the postulated close identity of the physical content of these two possible modifications of Einstein's equations will be appropriate when a continuation of the paper referred to by Mukhanov and Brandenberger [19] is published.

It is worth recalling here the content of a series of papers dealing with the collapse of black holes on the assumption that the singularity problem of the collapse of a universe is solved and that there is in nature a limiting value of the curvature of space, so that (for example) the maximum value of the Riemann tensor is given by

$$R_{\mu\nu\delta\gamma} R^{\mu\nu\delta\gamma} = \frac{1}{l_0^4} \, ,$$

where l_0 is the Planck length

$$l_0 = l_{\mathrm{Pl}} = \left(\frac{\hbar\varkappa}{c^3}\right)^{1/2} \sim 10^{-33} \ \mathrm{cm} \, .$$

A massive star, which cools in the process of emission of radiation, transforms into a neutron star. If its mass is sufficiently large, a neutron star continues to be compressed by gravitational forces and a moment may arrive at which all its mass is inside the Schwarzschild sphere. This creates a black hole. The collapse of a black hole differs significantly from the collapse of a closed universe. In the case of a closed universe we can, at least in principle, consider a perpetually oscillating model in which collapse changes to expansion, and this possibility does not exist for a black hole. The point is this: a complete space-time description inside the Schwarzschild sphere is given by the Kruskal diagram [20]. In the complete Kruskal space-time diagram the path of a free particle should begin from a singularity or from infinity and it should end at infinity or at a singularity.

† This was done, for example, by V K Mal'tsev at an Anglo – Soviet seminar held in Moscow in 1990. My example ($R \to R + \beta R^2$) of the appearance of an effect of the type of asymptotic freedom should be regarded only as a hint of the possibility of appearance of asymptotic freedom in the course of modification of the left-hand side of Einstein's equation. However, this modification does not remove the singularity that appears during collapse.

The Kruskal diagram inside the Schwarzschild sphere has two regions, one of which (T_-) is that which a particle reaches when it crosses the Schwarzschild surface from outside. The motion of the particle must terminate in this region on reaching the singularity. However, in the case of a particle which appears at a singularity there is a region T_+ in which the path of the particle created at the singularity must travel in the T_+-region from an internal singularity to the Schwarzschild surface and, crossing it, move into the space outside the Schwarzschild sphere. This is known as the second space (R''), in contrast to the R' space, which a particle enters when it passes below the Schwarzschild sphere and travels toward the singularity in the T_--region, utilising the whole of its lifetime for its motion to the singularity. If somehow a particle arriving from the R'-space could appear near the singularity in the T_+-region, then crossing the Schwarzschild surface it would find itself in the second (R'') space, where time is again measured from zero. In other words, the second space R'' is in the absolute future relative to the first space.

We recall that over twenty years ago (more exactly, in 1966) a paper on the "Change from contraction to expansion and physical singularities during contraction" was published by I D Novikov [21]. Classical physics was used in this paper to consider the fate (i.e. the 'path') of a collapsing electrically charged star when specifically the electric field of the matter stops the collapse of the start. According to Novikov, in this case, after the collapse has stopped in the T_--region of the Kruskal diagram, a black hole begins to expand into the T_+-region and reached the other R'' space, which lies in the absolute future of the initial Schwarzschild R' space.

However, this specific case of an electrically charged star is inappropriate because if we allow for the creation of particles by the electromagnetic field of the star, the collapse of the star simply does not stop.

However, if some modification of Einstein's equations keeps the values of all the curvatures finite in the process of collapse of a black hole, in other words, if the matter in a black hole does not travel to a singularity, then using Novikov's ideas [21] we can conclude that in this case the matter of the black hole may be in the T_+-region and in the course of its motion inside the Schwarzschild sphere in the direction away form the singularity it should cross the Schwarzschild surface and find itself in the second R'' space, forming a new universe in this space.

At a conference on the occasion of the centenary of the birth of A A Friedman (1988) I presented a paper on this subject: "Collapse of stars as a possible source of closed and simiclosed worlds" [22].

In 1990 a paper [23] by three authors also on this subject appeared in *Physics Letters*: it was entitled 'Black holes as a possible source of closed and semiclosed worlds'. We must bear in mind that the development of the new worlds occurs in the R'' spaces in the absolute futures relative to the times of the universes in which the formation of the black holes takes place. For example, it was stated in this connection [22] that our Universe may have appeared from a black hole formed in some universe existing in the absolute past, i.e. in the time of existence of the black hole, and time is measured from zero in the new universe.

Since many black holes are also formed during the existence of a universe and in the course of its collapse, I shall consider more specifically the process of decay of the very late universe into a multitude of universes, discussed in a somewhat different connection of the problem of entropy in a perpetually oscillating model of a universe.

If these ideas on the appearance of universes are valid, then many new phenomena may be realised in cosmology. We may recall that Jeans in 1928 suggested [24] that at "the centers of nebulae matter flows into our world from an external space". Novikov [25] and Ne'eman [26] put forward the idea that galaxies are special parts of our Universe which for some reason have lagged in their development behind the whole universe. This lag in time could be explained if the galaxies have poured into our Universe form other universes by the processes described above.

From this point of view the central region of the nucleus of a galaxy at the initial moment of its formation should have the de Sitter metric and be characterised by its own inflationary process. From this point of view the amount of matter in the initial state of a galaxy may vary greatly. In other words, microgalaxies could also appear. It may be that the most frequent projectiles from the absolute past into our Universe are the smallest white holes and even single maximons. However, if it is found that the nuclei of the observed galaxies are of very different nature, the possibility of appearance of projectiles from the past in our Universe in the form of separate maximons or groups of them at different moments of history of our Universe still exists, and they could form a medium of dark matter [27] as maximons in the shape of elementary white holes.

It follows from the above idea on the nature of dark matter consisting of maximons alone that maximons without an electric charge hardly interact with matter. The celestial bodies are transparent to these maximons [13]. Therefore, our current Universe would represent celestial bodies moving without hindrance in a maximon medium. In practice neither neutrinos nor other forms of radiation interact with dark matter. Only coalescence of two or more colliding maximons into one black hole of larger mass is possible.

In principle, coalescence of two or several elementary black holes is possible and this may be followed by Hawking emission and such emission of neutrinos b y small black holes form the nuclei of galaxies could be detected in neutrino experiments using detectors of the DUMAND type [28]. Here, the energies $E_\nu \sim 10^{16}$ eV and $E_\nu > 10^{19}$ eV are of special interest. It should be stressed that the process of appearance of matter in our Universe arriving from other universes can be regarded as a process that mimics the creation of matter, specifically in our Universe. There are many models of universes postulating continuous creation of matter in them. The question is: is there a concrete possibility to realise these models within the framework of the ideas put forward above [41]?

The possibility of the appearance of 'semiclosed universes' was put forward by Frolov, Markov, and Mukhanov [23]. A closed universe is characterised by the fact that the mass of its matter (the so-called bare mass of the particles) is reduced so much by the gravitational forces acting between the particles of the matter that the total energy of the closed universe is zero.

In the case of a closed universe there is no external space and no external observer who might conclude that a universe exists. A closed or, in other words, 'shut' universe is electrically neutral. But a shut or closed universe has one striking property. If just one electron is introduced into such a universe (for example into our Universe, if it is closed), it ceases to be closed and an external space appears where the

Coulomb electric field is generated. The universe opens up a throat and the whole universe behaves as a particle with a very small mass of the order of the maximon mass.†

Twenty years ago (in 1973), inspired by the paper of O Klein [29] on the possibility of semiclosed universes, I published a paper on the 'Micro-macrosymmetric universe' [30] which I introduced with two first stanzas from Bryusov's poem 'The world of electron' (1922):

Maybe these electrons
Are planets of five continents:
Art, knowledge, war, kingdoms,
And memories of forty aeons spent!

Maybe, then, each atom
Is a universe — a hundred worlds;
All here is there in microform
But some not here lies there unfurled.

(English version by Sean Harrop)

The paper under discussion is based on an internally self-consistent possibility put forward by O Klein and on general considerations is related to the familiar phrase of Dirac with which he ended one of his papers: "It would be strange if nature does not make use of this possibility". Such a possibility has in fact arisen.

One is speaking here of a universe with arbitrarily large throats and which contains the exterior mass of a given universe. It may be that this mass could be emitted by an object, as in the cause of an ordinary black hole. I even called this object a black hole of the second kind [31]. If the mass of a throat can be emitted as radiation completely, then a closed universe would form. If a very small mass of a throat remains, then such a universe behaves in the exterior space as, for example, an elementary particle and there are grounds for assuming that it is in the form of a maximon. We therefore cannot exclude the possibility that our Universe is filled with an enormous number of particles and some of them are in fact universes.

Summarising the above, we can say that the Universe as a whole can be a special structure consisting of a set of universes developing in their own spaces and times. This Universe cannot be pictured on a sheet of paper as, for example, a set of universes [32]. It is more likely to resemble a Russian 'matryoshka' doll. However, inside each of the component dolls there is a set of other dolls, etc. Such a picture of the Universe as a whole is discussed in one of my papers [33]. Naturally, such a Universe as a whole has no beginning or end.

> "*What really interests me is whether God had any choice when He created the World.*"
>
> A Einstein

Einstein's equation contains a large number of different cosmological solutions. The current state of the Universe is described quite satisfactorily by the solutions of Einstein's equation found by A A Friedman. However, these solutions are unsuitable for the description of the very early Universe. Moreover, in the case of the very early Universe the natural solutions are of the de Sitter type. The question arises: should one limit God's choice to these two types of solutions? In the study of this problem it is convenient to write down Einstein's equations in a form similar to the integral Yang–Feldman equation in electrodynamics [34]:

$$g_{\mu\nu} = \frac{8\pi\varkappa}{c^4}\int G^{\alpha\beta}_{\mu\nu} T_{\alpha\beta}(y)[-g(y)]^{1/2}\mathrm{d}^4 y + A_{\mu\nu}\,, \qquad (13)$$

where $G^{\alpha\beta}_{\mu\nu}$ is an analogue of a Green function and $A_{\mu\nu}$ is a free gravitational field. If in this equation we assume that $A_{\mu\nu} = 0$, then $g_{\mu\nu}$ differs from zero only if $T_{\alpha\beta} \neq 0$.

In other words, in this theory ($A_{\mu\nu} = 0$) space exists only if $T_{\alpha\beta} \neq 0$, i.e. the very existence of space is connected with the existence of matter. The *principle of existence of space*‡ ($A_{\mu\nu} = 0$) directly makes meaningless the creation of a universe 'from nothing', the existence of 'empty spaces', beginning from Minkowski space, and even of asymptotically flat metrics of the Schwarzschild type. For many years it has been thought that the de Sitter space is also empty. This was based on the view that matter corresponding to this case should have a state

$$\varepsilon + p = 0\,,$$

where ε is the energy density and p is the pressure density.

Such matter is not encountered in nature. However, we have gradually realised that we are dealing here with the very early universe.

This state of matter is possible. The condition $A_{\mu\nu} = 0$ does not imply either that gravitational waves cannot appear. It is only that in a given four-dimensional universe these waves must always have a material source. The equation with $A_{\mu\nu} = 0$ has been considered by a number of authors from the point of view of the presence or absence of Mach's principle in Einstein's theory. In view of the mathematical complexity of investigations of Eqn (13), the work on this topic has not brought definite results. I suggested to V K Mal'tsev to limit the investigation to conformally flat spaces:

$$g_{\mu\nu}(x) = \varphi^2(x)\eta_{\mu\nu}\,,$$

where

$$\eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

In contrast to the work of other authors, instead of dealing with ten quantities $g_{\mu\nu}$, we need to find one function $\varphi^2(x)$.

I shall determine the function $\varphi^2(x)$ by employing Einstein's equation in the form

$$R = \frac{8kT}{c^4}\,, \quad T = T_{\mu\nu}g^{\mu\nu}\,.$$

In such cases it is possible to show that the permissible metrics of this formalism include the solution of both the Friedman and de Sitter types [35]. We cannot exclude the possibility that God's choices that Einstein wrote about are limited to such a chain of the Friedman and de Sitter universes.

† A universe is also open if it contains one neutrino. A closed universe does not exhibit rotation (spin). A closed universe with one neutrino behaves as a neutrino with the maximon mass.

‡ It has not yet been demonstrated that Eqn (13) with $A_{\mu\nu} = 0$ contains Mach's principle, but it does contain the principle of existence of space only in the presence of matter.

However, the most interesting result of this simplified (scalar) formalism of gravitation is that in the $A_{\mu\nu} = 0$ case in the presence of a massive central body ($m > 0$) a test body, considered as such, does not interact in accordance with Newton's law with the massive central body M, i.e. in the $A_{\mu\nu} = 0$ case the motion of the test body is not governed by the mass m of the central body. The value of the mass simply drops out from the equation of motion of the test body. However, if the mass of the test body m is included in the value of T together with the mass of the central body (M_0), then the Newtonian interaction appears between them. In other words, the gravitational Newtonian interaction appears between the 'central' and test bodies only if we allow for the influence of the mass of this 'central' body and of the mass of the test body on the metric. It is possible that this is one of the characteristic features of the physics in which Mach's principle is valid, although in a very much impoverished scalar theory of gravitation. However, this result gives us some grounds for believing that a more thorough investigation of the integral equation (13) may lead to the proof of the existence of Mach's principle.

The realisation of Mach's principle in the integral formalism of the theory of gravitation is attractive because a free gravitational field appears in this theory only because of the matter tensor.

If in the integral formalism we introduce the condition of asymptotic freedom, then the resultant free gravitational field may prove, because of the matter tensor, to be under the influence of a factor representing the asymptotic freedom.

The circumstance that the resultant energy pseudotensor of the gravitational field is in this formalism integrated automatically over the whole four-dimensional space leaves us with the hope (see Isaacson [36]) that the energy (mass) of a gravitational field appearing in this way is indeed characterised by asymptotic freedom. In other words, the singularity problem, which has occupied us on the preceding pages and which we have attempted to solve in particular using the framework of physical effects, may be solved in general even in classical physics if Mach's principle is realised. However, these intuitive ideas have not yet been confirmed mathematic-ally and most probably they do not correspond to reality, i.e. if in this case there is no 'complete isaacsonisation' of gravita-tional radiation [36]: the pseudotensor of the emitted field acquires the properties of a tensor for each emission event. In any case a rigorous answer to this question could yield an equation of the type (13) in which $\varkappa$ would be given by the function (1).

3. Possible existence of special 'ultramicro-universe' physics in the Planck length range

In the preceding pages it was assumed that in a region close to a singularity the classical properties of space-time are retained fully in the range of lengths shorter than the Planck length ($l \sim 10^{-33}$ cm) and times shorter than $t \sim 10^{-41}$ s.

Over fifty years ago (in 1940) it was proposed to tackle the difficulties of the singularity of fields by the idea of nonlocality in the form of noncommutativity of fields $\varphi(x)$ and of the coordinate; in other words, noncommutativity of coordinates [36], which also leads to superluminal signal propagation. Over distances of the order of the Planck length considered here it might be desirable to modify the view that instantaneous signals are inadmissible in cosmology. I discussed this earlier [37].

If the velocity of a signal over a path l_{Pl} is infinite, then time regarded as a distance divided by velocity loses its meaning: $l/v(\to\infty) = t = 0$. If the T_-- and T_+-regions in the Kruskal diagram are separated by an absolutely hard core of radius equal to the Planck length, then in the process of collapse the matter in a black hole reaching the distance l_{Pl} from the singularity in the T_--region finds itself instantaneously at the distance l_{Pl} from the singularity in the T_+-region. In other words, the critical length seems not to exist in space.

It is not essential to represent space in the form of some lattice. For example, the classical properties of space may be lost only under the conditions in the region of the limiting matter density. Incidentally, in measurements at the limit of short lengths an observer unavoidably uses the wavelength, i.e. a quantum of the limiting density of matter. Such an interpretation of the physics of the limiting length should probably not lead to the observed violations of causality in the macroworld, i.e., to violations of the macroscopic link between the present and future.

The above discussion demonstrates a rather sceptical view on the subject of solution of these problems of gravitational collapse in the framework of the future quantum theory of strong gravitational fields. However, the natural existence of the Planck length $l_{\mathrm{Pl}} = (\hbar\varkappa/c^3)^{1/2}$ in the final expression for the curvature of the Riemann tensor

$$R_{\mu\nu\lambda\delta}R^{\mu\nu\lambda\delta} = \frac{1}{l_{\mathrm{Pl}}^4} ,$$

or rather the presence of the Planck constant in this formalism, seems to provide evidence against such scepticism. I considered earlier [37] the possibility of the appearance of a fundamental length associated with violation of the classical characteristics of space *itself*, which *do not apply in quantum theory* (for example, noncommuta-tivity of the coordinates, the expression for which could contain $\hbar$). It follows from the above that one cannot exclude the possibility that the very early and very late universe can both be described by hydrodynamic matter in the form of a cold gas of black holes.

Naturally, it is very tempting to extend this picture of this theory of a classical universe and its history to the limit of very small dimensions on the asumption that the limiting density is described by an expression consisting of the universal constants c, $\varkappa$, and $\hbar$:

$$\rho_0 \sim \frac{c^5}{\hbar\varkappa^2} \sim 10^{94}\ \mathrm{g\ cm^{-3}} ;$$

the limit of small dimensions in the history of a universe is then indeed governed by the Planck length:

$$R_{\min} \sim \left(\frac{\hbar\varkappa}{c^3}\right)^{1/2} \sim 10^{-33}\ \mathrm{cm} .$$

Obviously, and this must be stressed, we are now entering the realm of hypotheses which are not yet supported by any formalism. Naturally, we cannot exclude the possibility that quantisation of strong nonlinear gravitational fields in the future theory will automatically lead to the existence in nature of such a limiting length, but it is also possible that this hope may be dashed. Obviously, the appearance of the Planck constant in the expression for $R_{\min}$ makes it natural to relate the appearance of $R_{\min}$ to the formalism of quantum mechanics. However, some caution is also necessary: the quantum theory of all other fields does not contain such a limiting length. The formalism of quantum mechanics is formulated in terms of classical ideas on space irrespective of how short the length is.

On the other hand, a certain audacity is necessary to say that our ideas on space are valid down to very small dimensions. We cannot exclude *a priori* a situation in which the physics of quantum mechanics is modified significantly if a limiting length is usually related to the concept of space as a certain lattice. We are speaking here rather of a characteristic absence of lengths shorter than the Planck length only in the regions where the limiting density of matter appears. It should be stressed once again that in the classical interpretation the absence of lengths less than the Planck value could be illustrated by a small region of the infinitely hard sphere type in which a signal travels at an infinite velocity, so that the concepts of the duration of the signal and the time of its propagation are absent [37]. If universes appear by evolution of black holes, the situation is determinate [7], but we cannot exclude in principle the possibility that all the laws of conserv-ation break down. If convenient, we can use this picture to interpret space near a classical singularity in the spirit of a black box, which is discussed in the well-known paper of Wheeler [6]. We cannot exclude the possibility that in the 'ultramicrouniverse', when the limiting density is reached, the state of 'lawlessness' reigns and the Planck constant itself appears as a combination of the universal constants

$$h = \frac{c^5}{\varkappa_0^2 \rho_0} \, .$$

The proposed model of the Universe as a whole postulates a set of universes created and evolving in their own spaces and times. If in some sense this corresponds to reality, then the question arises what these separate universes represent. For example, we do not know whether our Universe is open or closed. Moreover, we must not avoid the following questions: can a completely closed universe exist if, in principle, the matter of black holes can appear in it from universes existing in the absolute past relative to the times of the given universe, or simply because the existence of the limiting length prevents a universe from becoming completely closed?

In the papers dealing with our model of the Universe as a whole, frequent use is made of a gas of black holes (instead of the usually discussed scalar field) as the primordial matter in the inflationary phase of the very early universe. Such primordial matter has a number of attractive features, but this hypothesis is not organically related to my model of the Universe as a whole. However, it seems to me that it should be discussed. Moreover, it is worth discussing also dark matter in the form of stable elementary holes or particles whose mass is

$$m = \alpha \left(\frac{\hbar c}{\varkappa} \right)^{1/2} .$$

However, α is such that the matter in these particles is not, for example, inside the Schwarzschild sphere. It is interesting to note that such particles, including stable elementary black holes, practically do not interact with ordinary matter, but interact quite strongly with one another:

$$E = \frac{m^2 \varkappa}{r} = \frac{\alpha^2 hc}{r} \, ;$$

if $\alpha \sim 1$, two such particles interact with one another by a mechanism two orders of magnitude stronger than the Coulomb interaction between two electrons. However, the most interesting scientific event which attracted my attention is a preprint of a paper by V P Frolov et al. on ''Wormholes as device for a study of black hole interior'' (Nordita, preprint No. 93/8a). There are objections against the Bohr interpretation of quantum theory in the case of such objects as closed universes or in the case of the physics of matter inside the Schwarzschild sphere. The existence of wormholes fits into the general pattern of the thought model discussed above. In this sense, multiple universes represent a certain connected organism.

It should be mentioned also that if asymptotic freedom of gravitational interactions does indeed lead to a finite value of the limiting curvature, then on the basis of the results of Frolov, Markov, and Mukhanov [23] it should be possible, in principle, to create new universes in the laboratory if black holes can be formed in the laboratory by compressing a certain amount of matter to less than its gravitational radius. However, it should be stressed that this possibility exists also in any other cosmological theory characterised by a limiting value of the curvature. The existence of asymptotic freedom of gravitational interactions can alter significantly the theory of elementary particles. This applies, for example, to electrodynamics if the modern nature of this theory with its logarithmic divergence is retained. The existence in nature of the limiting energy density naturally changes the formalism of electrodynamics. The fundamental importance of the role of the gravitational field in the theory of elementary particles was pointed out by me in 1947 [47] and in 1955 by Landau and Pomeranchuk [43].

We cannot exclude the possibility that in fact all the elementary particles with a finite rest mass are characterised by a logarithmic divergence if we ignore the gravitational field.

The appearance of a free gravitational field in a universe filled with matter has caused a lot of problems, as is evident from the above. Attempts to solve these problems have given rise to a series of hypotheses which have a right to be considered.

However, if these hypotheses fail, there is one idea which may for some reason be unacceptable.

I have considered above the possibility that in the process of collapse a universe can split into a series of universes if during this process the limiting densities do not appear simultaneously in the various regions. In these regions we can have nonsimultaneous stopping of the motion of the collapsing matter, which then experiences the initial phase of expansion. Let us consider a situation in which throughout the universe a moment of stopping of the collapse appears throughout the collapsing matter. If this universe contains a free gravitational field, then perhaps a situation may arise in which there are two universes and one of them (filled with matter) begins to expand, while the other (filled with free gravitational radiation) actually begins to contract and at the end is transformed into a point-like closed singularity if even in this case we assume that the law of conservation of the limiting mass density is obeyed. In fact at this point the curvature will be infinite, but the question is: has this closed object any physical meaning?

Now for the final critical comments. Naturally, physicists working in cosmology may accept the possibility of the existence in nature of asymptotic freedom of gravitational interactions and the associated possible existence of a limiting matter density. Maybe I have guessed many real features of the real Universe. However, these physicists are hardly likely to accept that the ψ-function describing the asymptotic freedom of gravitational interactions is in fact described by, for example,

$$\psi \approx \psi_0 \left(1 - \frac{\varepsilon^2}{\varepsilon_0} \right) .$$

The question arises, what is the formalism of the future consistent theory of gravitation that may give rise to the function ψ existing in nature? In my defence I can say only that the ψ-function should be in the class of functions that are of the form

$$\psi \left(\frac{\varepsilon}{\varepsilon_0} \right) \to 0 ,$$

if $\varepsilon \to \varepsilon_0$ or even if $\varepsilon \to \infty$.

Among the topics which are not considered in this review one should mention an account of new theories, namely the attempts to find new possibilities in the theory of gravitation on the basis of the characteristic properties specifically of string theory.

I have in mind, for example, the paper of M J Perry and E Teo on ''Non-singularity of the exact two-dimensional string black hole'' [39], in which it is stated that near a classical singularity a region with Euclidean (and not Lorentzian) metric appears near a classical singularity† and the whole Universe represents a set of universes linked by wormholes. We cannot exclude the possibility that in spite of all the differences between the 'string' approach and the formalism discussed above, the future full picture of the Universe will have many features that are shared with these approaches.

It may be that it will become experimentally possible to detect a reduction in the gravitational constant under the conditions of high density of matter.

† This result would seem to be close or even analogous to the possibility I discussed of the existence of a characteristic 'submicrophysics' in the range of the Planck or sub-Planck lengths.

References

1. Markov M A *O Prirode Materii* (Nature of Matter) (Moscow: Nauka, 1976) p. 140
2. Markov M A, *Phys. Z. Sowjetunion* **7** 42 (1943)
3. Tomonaga S *Prog. Theor. Phys.* **1** 27 (1946)
4. Schwinger J *Phys. Rev.* **74** 1439 (1948)
5. Markov M A *Pis'ma Zh. Eksp. Teor. Fiz.* **36** 214 (1982)
6. Wheeler J A, in *Gravitation* (Eds C Misner, K Thorne, J A Wheeler) (San Francisco: Freeman, 1973)
7. Markov M A, Mukhanov V F *Nuovo Cimento B* **86** 97 (1985)
8. Fox (Fock) V A *Teoriya Prostranstva, Vremeni i Tyagoteniya* (Theory of Space, Time and Gravity) (Moscow: Gostekhizdat, 1955)
9. Markov M A *Ann. Phys.* (*N.Y*) **155** 33 (1984)
10. Starobindky A A *Phys. Lett. B* **91** 99 (1980); Guth A H *Phys. Rev. D* **23** 347 (1981); Linde A D *Fizika Elementarnykh Chastits i Inflyatsionnaya Kosmologiya* (Physics of Elementary Particles and Inflation Cosmology) (Moscow: Nauka, 1990)
11. Gliner E B *Dokl. Akad. Nauk SSSR* **192** (1970) [*Sov. Phys. Dokl.* **15** 559 (1970)]
12. Rozhanskii I D *Anaksagor* (Anaxagoras) (Moscow: Mysl', 1983) p. 29
13. Markov M A *Prog. Theor. Phys. Suppl.* (*1*) (1965) [Commemoration Issue for 30th Anniversary of the Meson Theory of Dr H Yukawa]
14. Mal'tsev V K, Markov M A, ''Quantum miniobjects in general relativity'' *Preprint P-160* (Moscow: Institute for Nuclear Resarch, Academy of Sciences of the USSR, 1980); Bunch T S *J. Phys. A* **14** L139 (1981); Mukhanov V F *Pis'ma Zh. Eksp. Theor. Fiz.* **33** 549 (1981) [*JETP Lett.* **33** 532 (1981)]; MacGibbon J H *Nature* (*London*) **329** 308 (1987)
15. Hawking S W *Phys. Rev. D* **14** 2460 (1976)
16. Aman E G, Markov M A *Teor. Mat. Fiz.* **58** 163 (1984)
17. Markov M A, ''Macro – micro symmetric universe'', in *Proc. Conf. on Group Theoretical Methods in Physics, Yurmala, USSR, 1985*
18. Zel'dovich Ya B, Novikov I Ya *Stroenie i Evolyutsiya Vselennoi* (Structure and Evolution of the Universe) (Moscow: Nauka, 1975) p. 486
19. Mukhanov V F, Brandenberger R *Phys. Rev. Lett.* **68** 1969 (1992)
20. Kruskal M D *Phys. Rev.* **119** 1743 (1960)
21. Novikov I D *Astron. Zh.* **43** 911 (1966) [*Sov. Aston.* **10** 731 (1967)]
22. Markov M A, in *Proceedings of the Friedman Centenary Conference, Leningrad, 1988* (Singapore: World Scientific, 1988)
23. Frolov V P, Markov M A, Mukhanov V F *Phys. Rev. D* **41** 383 (1990)
24. Jeans J H *Astonomy and Cosmology* (Cambridge: Cambridge University Press, 1928)
25. Novikov I D *Aston. Zh.* **41** 1075 (1964)
26. Ne'men Y *Astrophys. J.* **141** 1303 (1965)
27. Markov M A *Phys. Lett. A* **172** 331 (1993)
28. Markov M A, Zheleznykh I M *Nucl. Instrum. Methods Phys. Res. A* **248** 242 (1986)
29. Klein O *W Heisenberg und die Physik unserer Zeit* (Brunswick, 1961) p. 345
30. Markov M A *Budushchee Nauki* (The Future of Science) (Moscow: Znanie, 1973)
31. Markov M A, in *Gravitational Radiation and Gravitational Collapse* (Proc. IAU Symposium No. 64, Warsaw, 1973, Ed. C DeWitt-Morette) (Dordrecht, Netherlands: Reidel, 1974) pp. 106 – 131
32. Linde A D *Fizika Elementarnykh Chastits i Inflyatsionnaya Vselennaya* (Physics of Elementary Particles and the Inflationary Universe) (Moscow: Nauka, 1990) p. 57
33. Markov M A *Phys. Lett. A* **151** 15 (1990)
34. Sciama D W *Mon. Not. R. Astron. Soc.* **113** 34 (1954); Al'tshuler B L *Zh. Eksp. Teor. Fiz.* **51** 1143 (1966) [*Sov. Phys. JETP* **24** 766 (1967)]; Linden-Bell D *Mon. Not. R. Astron. Soc.* **135** 413 (1967); Gilman R C *Phys. Rev. D* **2** 1400 (1970); Maltsev V K, Markov M A, ''On integral formulation of Mach principle in conformally flat space'', Preprint No. JINR E-2977 (Dubna: Joint Institute of Nuclear Research, 1976); Mal'tsev V K, Markov M A *Tr. Fiz. Inst. Akad. Nauk SSSR* **96** 11 (1977)
35. Mal'tsev V K *Teor. Mat. Fiz.* **83** 476 (1990)
36. Isaacson R A *Phys. Rev.* **166** 1623 (1968)
37. Markov M A *Zh. Eksp. Teor. Fiz.* **10** 1311 (1940)
38. Markov M A *Proc. First A D Sakharov Conf. on Physics, 1991*
39. Perry M J, Teo E, Preprint No. DAMTP R93/1
40. Markov M A, Frolov V P *Teor. Mat. Fiz.* **3** 3 (1970); Markov M A, presented at *International Conference on Cosmology and Elementary Particles, Trieste, 1970* Paper IC 71/33
41. Bondi H, Gold T *Mon. Not. R. Astron. Soc.* **108** 252 (1948); Hoyle F *Mon. Not. R. Astron. Soc.* **109** 252 (1948); Pirani F A *Proc. R. Soc. London, Ser. A* **228** 455 (1955); Hoyle F, Narlikar J V *Proc. R. Soc. London, Ser. A* **282** 191 (1964)
42. Markov M A *Zh. Eksp. Teor. Fiz.* **17** 848 (1947)
43. Landau L D, Pomeranchuk I Ya *Dokl. Akad. Nauk SSSR* **103** 489 (1955)