Squeezed light and nonclassical motion in mechanics

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From the fact of observation of squeezed light follows the possibility of a nonperturbing or nondemolition measurement (observation) of a single microscopic object in a widely distributed quantum-mechanical state. Macroscopic bodies—objects of classical mechanics—are usually found in states with narrow wave packets, although quantum mechanics does not forbid them to be in states with wide wave packets. The absence of macroscopic bodies in such states requires explanation. In this article it is shown that in virtue of a special, geometrical nature of the gravitational field the absence in nature of macroscopic bodies in widely distributed states can be due to the focusing action of the self-gravitational field of a macroscopic body on its wave packet. Thus, the gravitational field can play an important role in the relaxation of the classical limit of quantum mechanics.

In the present paper we would like to draw attention to those consequences for the quantum-mechanical description of the motion of macroscopic bodies which follow from the fact of observation of squeezed light. This observation turned out, without any doubt, to be one of the most important achievements of optics in recent years which will have numerous scientific extensions and practical applications. However, as we see it, the most important consequence of observing squeezed light is the change in the quantum-mechanical description of the motion of macroscopic bodies required by the fact of such an observation. Essentially, in observing squeezed light radical changes occurred in the procedure of measuring the parameters of macroscopic quantum states, and they occurred imperceptibly.

In Sec. 1 a simple example is given of a quantum-mechanical description of rectilinear and uniform motion of a macroscopic body and a concept is introduced of concentrated and distributed wave packets of such bodies. In Secs. 2 and 3 using the examples of a mechanical oscillator and a body moving in a gravitational field it is shown that the concentrated and distributed wave packets arise always in going over to macroscopic bodies. Attention is drawn to the fact that the distributed wave packets require a statistical or ensemble interpretation while for concentrated wave packets this is not obligatory.

In Sec. 4 squeezed light is discussed. Attention is drawn to the nonstandard nature of the measuring procedure of squeezed light and it is shown that observation of it is observation of a single object—the field of a selected mode of an optical resonator of a parametric generator occurring, in particular, in a distributed state (wide wave packet). It is noted that from the identity of the description of a mechanical and an electromagnetic oscillator follows the possibility in principle of observing an individual mechanical object in a distributed quantum mechanical state. Since from the theoretical point of view concentrated and distributed wave packets are equally valid, the question can be posed as to why macroscopic bodies in a distributed quantum-mechanical state are not observed in the world surrounding us. In Sec. 5 the reflection of an electromagnetic pulse from a macroscopic body (mirror) occurring in a distributed state is investigated. This investigation shows that the variance of a quantum-mechanical state of an individual macroscopic body can be determined in a nonperturbing manner by the same means that have been developed for observing squeezed light.

Thus, it is shown that the observation of squeezed light indicates the possibility of observing individual macroscopic bodies in distributed quantum mechanical states and requires an explanation of the absence of such bodies in the world surrounding us.

In conclusion attention is drawn to the significant divergence in the quantum-mechanical description of macroscopic objects in optics and in mechanics and a possible method of overcoming this divergence is discussed.

1. QUANTUM MECHANICAL DESCRIPTION OF MOTION OF MACROSCOPIC BODIES

Thus, let us turn to the quantum-mechanical description of the motion of macroscopic bodies. We shall take macroscopic bodies to mean bodies of considerable mass (for example, 1 g; subsequently a possible estimate of this quantity will be made more precise). Thus, we examine the quantum-mechanical description of motion of bodies which, as is well known, obey the laws of classical mechanics.

Let us first examine a simple example—uniform and rectilinear motion of a macroscopic body of mass m in free space with a velocity v. It is described by a wave packet[1] of the form

$$\Psi(\mathbf{r},t)=\frac{C}{(1+it\tau^{-1})^{3/2}}\exp\left[-\frac{\mathbf{r}^2-2iv\mathbf{r}\tau+iv^2\tau t}{2a^2(1+it\tau^{-1})}\right] \quad (1.1)$$

of a characteristic dimension a associated with the time τ of spreading out of the packet by the relationship

$$\tau = ma^2/\hbar. \tag{1.2}$$

For a macroscopic body $m=1$ g with a characteristic size of the packet of the order of the dimensions of an atom ($a=10^{-8}$ cm) the packet (1.1) describes a well defined rectilinear trajectory over a large time interval $\tau=10^{11}$s ($=3 \cdot 10^3$ years).

However, quantum mechanics does not forbid large values of the parameter a comparable, let us say, with the optical wave length or even with the geometrical dimensions of a macroscopic body of arbitrary mass. Narrow wave packets much smaller then the geometrical dimensions of the body or smaller even than a characteristic optical wave length (10^{-6} m), can be naturally ascribed to macroscopic classical objects, since in this case the existence of the packet can be in general neglected, taken to be a point. But to what do broad wave packets correspond? No macroscopic classical objects with a large indefiniteness of position so far have been noticed in the world surrounding us. If such objects existed then there would have been no such science as classical mechanics with its strictly defined trajectories. This circumstance characteristic for quantum mechanics has been noted a long time ago and was reflected in the accepted interpretation of the distributed solutions of the Schrödinger equation. These solutions received an ensemble, i.e., statistical interpretation. It was accepted that in measuring the coordinates of a particular body described, for example, by the packet (1.1) one can obtain any value of it, but, if such measurements were to be carried out on many bodies with identically prepared states, the distribution of the probability of obtaining in an experiment of some one value of the coordinate is described by the square of the modulus of the wave function $\Psi(\mathbf{r},t)$. Thus, for example, in the textbook by A. Messiah[2] in discussing the distributed solutions of the Schrödinger equation it is stated: "In the classical approximation the function Ψ describes the "liquid" made up of classical noninteracting particles of mass m (statistical ensemble)...". An analogous treatment of the distributed solution is given in the textbook by L. Schiff.[1]

In this methodological note attention is drawn to a certain discontinuity in the quantum-mechanical description of macroscopic objects in optics and in mechanics which appears in an analysis of experiments with squeezed light.

2. MECHANICAL OSCILLATOR

We recall the principal elements of the quantum description of a mechanical oscillator (Fig. 1). This simplest system is described by the Hamiltonian[3]

$$H=\frac{1}{2m}P^2+\frac{1}{2}\varkappa Q^2, \tag{2.1}$$

where P,Q are canonically conjugate variables–the operators of the momentum and the coordinate obeying the commutation relation

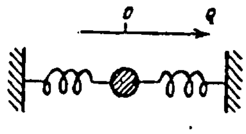

FIG. 1. Mechanical oscillator.

$$QP-PQ=i\hbar, \tag{2.2}$$

m is the mass and $\varkappa$ is the stiffness of the restoring force of the oscillator. In the variables $p=Pm^{-1/2}$, $q=Qm^{1/2}$ the Hamiltonian has the more commonly accepted form

$$H=\tfrac{1}{2}(P^2+\omega^2q^2)\left(\omega^2=\frac{\varkappa}{m}\right). \tag{2.3}$$

An important role is played by the operators of creation a^+ and annihilation a of excitations of the oscillator:

$$a^+=\frac{1}{(2\hbar\omega)^{1/2}}(\omega q-ip),$$

$$a=\frac{1}{(2\hbar\omega)^{1/2}}(\omega q+ip). \tag{2.4}$$

A coherent state of the oscillator $|z\rangle$ (Fig. 2a) represents the eigenstate of the annihilation operator $a|z\rangle=z|z\rangle$; for $z=z_0e^{-i\omega t}$ it is the solution of the time-dependent Schröinger equation and in the coordinate representation is described by the wave function

$$\Psi_{\mathrm{coh}}(q)=A\exp\left\{-\frac{\omega}{2\hbar}\left[q-\left(\frac{2\hbar}{\omega}\right)^{1/2}z\right]^2\right\}. \tag{2.5}$$

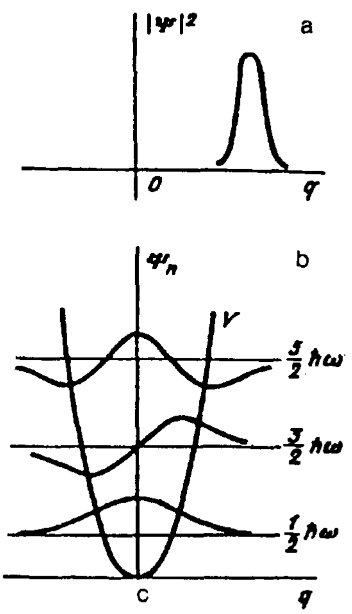

FIG. 2. Quantum-mechanical states of an oscillator. a—Coherent state. b—Stationary states.

The coordinate distribution for this state is shown in Fig. 2a; in the course of time it represents a harmonic right-left motion with frequencies ω. Since the width of this distribution, equal to

$$\Delta Q_{\rm coh}=(\hbar/2m\omega)^{1/2},$$

is small compared to the amplitude of oscillations for $|z|\gg 1(Q\approx 1$ cm, $\Delta Q/Q\approx 10^{-14})$ this state is a typical concentrated state and describes in a natural manner the classical oscillations of a mechanical oscillator.

However there also exist steady states of the oscillator $|n\rangle$ (Fig. 2b; n is an integer), which represent the eigenstates of the operator of the number of particles (of the number of excitations of the oscillator) $a^+a|n\rangle=n|n\rangle$, where n is an integer. The state

$$|n(t)\rangle=e^{-in\omega t}|n\rangle$$

is also a solution of the time-dependent Schrödinger equation and in the coordinate representation is described by the wave function

$$\Psi_{\rm stat}(q)=AH_n((\omega/\hbar)^{1/2}q)\exp(-\omega q^2/2\hbar), \qquad (2.6)$$

where H_n is the Hermite polynomial of the nth degree. The indefiniteness in the coordinate in this state for a sufficiently large energy $E=n\hbar\omega(n\gg 1)$ is great $\Delta Q\approx\omega^{-1}(E/m)^{1/2}$, of the order of the amplitude of the oscillations in the coherent state at the same energy $(n=|z|^2)$.

Thus, stationary states are typical distributed states. They are macroscopic since they can have a high energy $(n\gg 1)$ and correspond to an oscillator of large mass m. In classical mechanics there are no motions of individual objects corresponding to such states and therefore they require an ensemble, statistical interpretation about which we have spoken earlier.

3. MACROSCOPIC BODY IN A GRAVITATIONAL FIELD

The motion of a material point of mass m in the gravitational field $V(\mathbf{r})$ which varies slowly in space can be described by the Gaussian wave packet[4]

$$\Psi(\mathbf{r},t)=C(t)\exp\left[-(\rho,F\rho)+\frac{i}{\hbar}(\mathbf{p}_0(t)\rho+E(t))\right], \qquad (3.1)$$

where $\rho=r-r_0(t)$ and the quantities $r_0(t)$ and $p_0(t)$ obey the classical Hamiltonian equations

$$\mathbf{r}_0=\mathbf{p}_0/m,\ \mathbf{p}_0=-\mathrm{grad}_{\mathbf{r}_0}U(\mathbf{r}_0); \qquad (3.2)$$

the quantity $F(t)$ represents a symmetric matrix of dimensionality 3×3 with complex time-dependent elements, with the real part of the matrix F determining the geometrical size of the wave packet.

This wave packet satisfies the Schrödinger equation in the case when in the expansion of the potential energy near the point r_0

$$U(\mathbf{r}_0+\rho)=U(\mathbf{r}_0)+(\rho\,\mathrm{grad}_{\mathbf{r}_0}U(\mathbf{r}_0))+\tfrac{1}{2}(\rho,U''\rho)+\dots \qquad (3.3)$$

one can neglect terms of the third and higher degrees in terms of ρ denoted by multiple dots. The matrix of the second derivatives U'' of dimensionality 3×3 is symmetric; its real elements

$$U''_{\alpha\beta}(t)=\frac{\partial^2 U(\mathbf{r})}{\partial x_\alpha \partial x_\beta}\Big|\mathbf{r}=\mathbf{r}_0(t) \qquad (3.4)$$

depend on the time through the vector $\mathbf{r}_0(t)$. As long as the dimensions of the wave packet are so small that terms of the third and higher degrees in the potential energy are insignificant, it moves along the classical trajectory (3.2). In this case the matrix $F(t)$ which determines, in particular, the dimensions of the packet must satisfy the matrix equation

$$i\hbar\dot{F}=\frac{2\hbar^2}{m}F^2-\tfrac{1}{2}U'', \qquad (3.5)$$

reminiscent of the Riccati equation, while the quantities $C(t)$ and $E(t)$ are equal to

$$C(t)=C_0\exp\left[-\frac{i\hbar}{m}\int_0^t \mathrm{d}t(F_{11}+F_{22}+F_{33})\right],$$

$$E(t)=\int_0^t \mathrm{d}t\left(\frac{p_0^2}{2m}-U(\mathbf{r}_0)\right). \qquad (3.6)$$

The trajectory of the motion of the body in the gravitational potential (3.2) does not depend on the mass m, since p_0 and $U(\mathbf{r}_0)$ are proportional to m. Separating out explicitly the proportionality of $U(r_0)$ to the mass m we obtain from (3.5) the equation

$$i\dot{\Phi}=2\Phi^2-\tfrac{1}{2}V(\Phi=F\hbar m^{-1},\quad mV=U''), \qquad (3.7)$$

in which all the quantities are independent of the mass m. From this it can be seen that as the mass m increases (or $\hbar$ decreases) F also increases, and this means that the dimensions of the wave packet decrease and it becomes more and more concentrated near the classical trajectory (3.2). Consequently, the wave packet (3.1) describes the motion of a macroscopic body in a natural manner within the framework of quantum mechanics.

But this is not the only method of transition to the description of a classical motion of macroscopic bodies within the framework of quantum mechanics. Another method is associated with the widespread method of the eikonal[2], or, in other words, with the Hamilton–Jacobi equation. For sufficiently large m and, consequently, for short de Broglie wavelengths $\lambda=\hbar/(2mE)^{1/2}$ (E is th energy of the state) and the steady-state solution of the Schrödinger equation can be sought in the form

$$\Psi(\mathbf{r},t)=A(\mathbf{r})\exp[i(S(\mathbf{r})\lambda^{-1}-Et\hbar^{-1})], \qquad (3.8)$$

where $S(\mathbf{r})$ is a function of the coordinates that varies little over lengths of the order of λ which is called the eikonal and obeys the equation

$$(\mathrm{grad}S(\mathbf{r}))^2=1-(U/E). \qquad (3.9)$$

The function of $A(\mathbf{r})$ also varies little over lengths of the order of λ and satisfies the equation

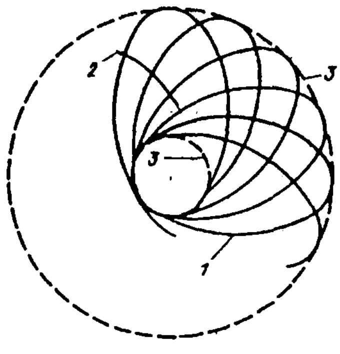

FIG. 3. System of orbits in a gravitational field forming the eikonal solution of the Schrödinger equation. 1—trajectory 2—wave front 3—caustic lines.

$$(\text{grad } S(\mathbf{r})\text{grad } A(\mathbf{r}))+\tfrac{1}{2}A\Delta S=0. \tag{3.10}$$

It can be easily shown that for the gravitational field U and E are proportional to m, and then S does not depend on m. Lines orthogonal to the wave surfaces $S(\mathbf{r})=\text{const}$ are classical trajectories of a material point. The eikonal function $S(\mathbf{r})$ is broad both in the longitudinal and in the transverse (with respect to the trajectories) directions. In the transverse direction it can be limited only by possible caustic surfaces. Figure 3 shows schematically the eikonal solution describing the motion of a macroscopic body in a centrally-symmetric gravitational (newtonian) field. The eikonal function occupies the region between two concentric circles and essentially there are no restrictions on the width of this ring region. If such a distributed solution is regarded as relating to a single specific body, then there are no corresponding motions in classical mechanics.

Thus, in the case of a quantum-mechanical description of the motion of a macroscopic body in a gravitational field, just as in previously discussed cases, both concentrated solutions are possible that go over naturally into the solutions of classical mechanics, and also distributed solutions which for correspondence with the classical approach require a statistical, ensemble interpretation.

In conclusion of this section we note that transition to the classical description in quantum mechanics is not as automatic as, say, in the theory of relativity. There it is sufficient for the ratio of the velocity of the body to the velocity of light to become small in order for classical mechanics to arise from relativistic mechanics. In quantum mechanics only a part of the states (and specifically, concentrated wave packets) go over into classical motion as the mass of the body increases. But the other part of the states—the distributed solutions—go over into the classical motion only in a statistical, ensemble sense.

4. SQUEEZED LIGHT AND OBSERVATION OF IT

We now turn to a discussion of squeezed light. Similarly to laser light, squeezed light is generated in an optical resonator. The field in the resonator represents a superposition of fields of individual modes, or resonances, with, as can be seen in practice, it being possible in an optical resonator to excite an individual mode. Since the spatial distribution of the field of a mode is determined by the boundary conditions, the mode can be regarded as a system with one degree of freedom, the coordinate of which is the electric field of the mode at some chosen point of the resonator. Then the quantum theory of field of the mode coincides with the theory of a mechanical oscillator,—this field is described by the same Hamiltonian (2.3) in which the coordinate q is the electric field at the chosen point of the resonator. In particular, different states of the field are possible—coherent and steady state ones—mentioned above.

But these two kinds of states are not the only possible ones both in the mechanical and in the electromagnetic oscillator. Squeezed states unite in themselves properties of concentrated and distributed states.[5,6] They represent the eigenstates of the operator $\mu a+\nu a^+$

$$(\mu a+\nu a^+)|\zeta\rangle=\zeta|\zeta\rangle(|\mu|^2-|\nu|^2=1), \tag{4.1}$$

where μ, ν, ζ are complex parameters with the first two of them satisfying the relationship $|\mu|^2-|\nu|^2=1$. In the case of the relationship $\mu=\mu_0 e^{i\omega t}$, $\nu=\nu_0 e^{-i\omega t}$ they are solutions of the time-dependent Schrödinger equation. Among the squeezed states the most representative one is the state of squeezed vaccuum corresponding to $\zeta=0$. In the coordinate representation it is described by the Gaussian wave packet

$$\Psi_\zeta(q)=A\exp\left(-\frac{\omega q^2}{2\hbar}\frac{\mu+\nu}{\mu-\nu}\right). \tag{4.2}$$

The variance of the squeezed state

$$D^2=\frac{\hbar}{2\omega}[|\mu|^2+|\nu|^2-2|\mu|\cdot|\nu|\cos(\Psi_0+2\omega t)] \tag{4.3}$$

varies with double the frequency of the oscillator and can be both smaller than the variance of the coherent state:

$$D^2_{\min}=\frac{\hbar}{2\omega}(|\mu|-|\nu|)^2<D^2_{\text{coh}}=\frac{\hbar}{2\omega},$$

and also greater than it

$$D^2_{\max}=\frac{\hbar}{2\omega}(|\mu|+|\nu|)^2>D^2_{\text{coh}}=\frac{\hbar}{2\omega}$$

(Fig. 4). The moments, when the variance is small, have served as the basis for the name—squeezed states. Squeezed states are characterized by the squeezing coefficient

$$K=D_{\text{coh}}/D_{\min}+(D_{\max}/D_{\min})^{1/2}$$
$$=|\mu|+|\nu|=(|\mu|-|\nu|)^{-1}. \tag{4.4}$$

Energy restrictions exist on the squeezing coefficient; in the case that the number of photons in the oscillator is equal to N the maximum possible squeezing coefficient is equal to

$$K_{\max}=(N+1)^{1/2}+N^{1/2}. \tag{4.5}$$

The main properties of squeezed states can be qualitatively understood from Fig. 4 which shows the oscillations

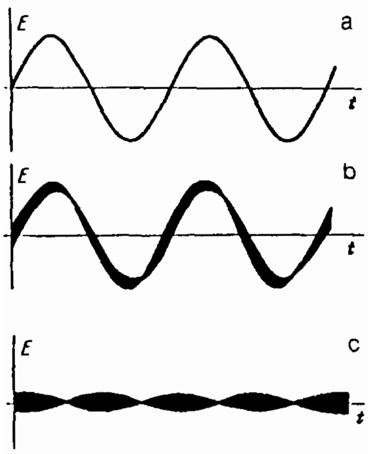

FIG. 4. Dependence on the time of the field and of its indeterminacy. a—Coherent state. b—Squeezed state. c—Squeezed vacuum.

of the coordinate of a mechanical oscillator, or of the field of an electromagnetic oscillator, as a function of the time. In Fig. 4a the oscillations correspond to a coherent state, with the constant small variance of the state being represented by the thickness of the line in the sinusoid. In the case of the usual laser intensity, say, with an energy of 1 J stored in the resonator, the ratio of the variance to the amplitude of the oscillations is small $=10^{-9}-10^{-10}$. In Fig. 4b oscillations are shown in the squeezed state—here the variance changes with time and at certain instances is comparable with the amplitude of the oscillations. Fig. 4c corresponds to the state of squeezed vacuum—now there are practically no oscillations with the principal frequency ω and there are only changes in the variance with doubled frequency. We emphasize that in spite of the absence of oscillations and the use of the term squeezed vaccuum this is a highly excited macroscopic state with high energy.

Squeezed light has been observed in a number of laboratories.[7,8] One of the simplest schemes for observing it (Fig. 5) represents a degenerate parametric generator pumped by the second harmonic of a neodymium laser. In the parametric generator the first harmonic arises again, but not already in the state of a squeezed vacuum. The present scheme differs from the early schemes of observing parametric generation of light[9,10] only by the special receiver-analyser of the squeezed state. This receiver compares the signal of the parametric generator with a reference (laser) signal and measures the variance of the squeezed light. In Fig. 6 the results of measurements are schematically shown. It can be seen that the variance of the signal of the parametric generator undergoes two oscillations during a period of the high frequency field—the period of its change is π, and not 2π. It can also be seen that for certain values of the phase θ the variance of the squeezed light becomes less than the variance of the coherent state (dotted line). This is exactly what indicates that the light is in a squeezed state.

We call attention to the nonstandard nature of the measuring procedure. A parameter of the state—its dispersion is measured directly; and the measurement occurs in a

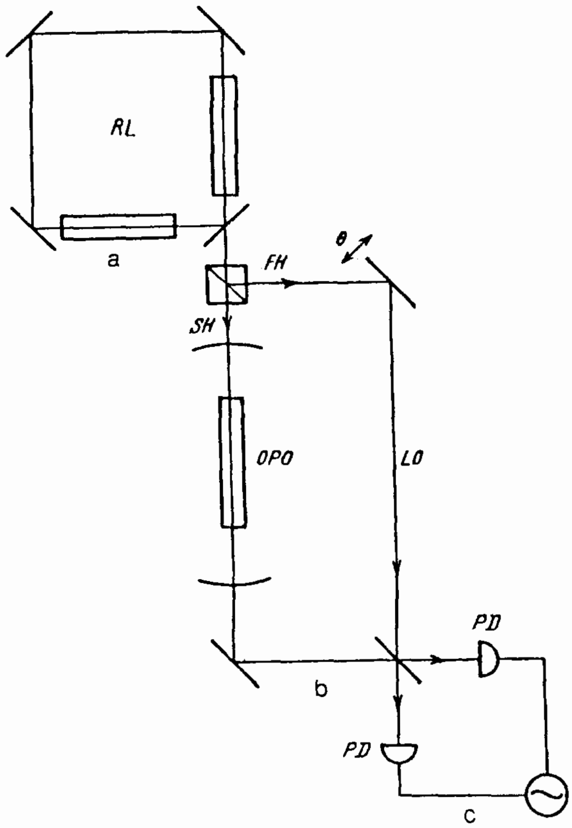

FIG. 5. Laboratory arrangement for observing squeezed light: a—Pumping system. b—Parametric generator. c—Receiver-analyzer.

nonperturbing (or a nondemolition) manner. Indeed, the measurements are made on the beam exiting from the resonator of the parametric generator and this beam is not returned to the resonator independently of the fact whether measurements on it have performed or not. If such measurements are made, then the information concerning this cannot in any way enter the resonator and, consequently,

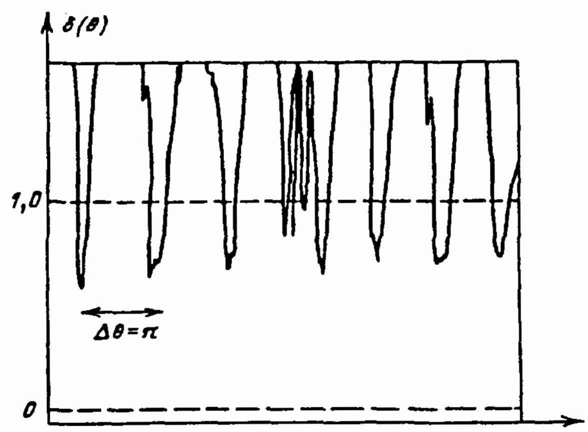

FIG. 6. Variance of squeezed light as a function of the phase of the reference signal.

the state of the field in the resonator is not perturbed by the measurement. At the same time, on the basis of the results of measurements made on the beam exiting from the resonator, one can obtain information concerning the state of the field in this resonator.

We note that as a result of the dynamic equilibrium between pumping and losses the field in a resonator of the parametric generator is in a steady state. Consequently, having once made a measurement one can be confident that the field in the resonator is in a state, say, of a squeezed vacuum after an hour or two after the measurement. Thus, the observation of squeezed light shows that there exists a macroscopic (the number of photons is great) quantum mechanical object—the electromagnetic field of the selected mode of the resonator, the variance of the state of which can be measured, leaving the object in the same state in which it was prior to the measurement.

The greatest attention of investigators of squeezed light has been attracted to the instants of time when its variance is less than the variance of the coherent state. This is natural, since quantum-mechanical indeterminacies, like noise, hinder the exact measurements of the corresponding quantities. The smaller are these indeterminacies, the more accurately can the corresponding quantity be measured, and this is the merit of states with a small variance in the squeezed light. However, from the investigator's point of view the states with a large variance are the most interesting ones; they are less "classical" and more "quantum-mechanical". For the subject of this article, they are important since observation of them, taking into account what has been said above concerning the measurements of squeezed light, testifies concerning the possibility of measuring the variance of a distributed state of an individual object without destroying this distributed state.

Naturally, if such measurements are possible on an electromagnetic oscillator, they are possible also on a mechanical oscillator; we shall comment on this thought in the next section.

5. MACROSCOPIC BODIES IN DISTRIBUTED STATES AND THE NONPERTURBING MEASUREMENT OF THEIR POSITION

Thus, in experiments with squeezed light something greater has been shown than simply the possibility of generating them. It is also shown that the quantum-mechanical state of a specific macroscopic object (the field of a certain selected mode of the resonator) can be determined in a nonperturbing manner. In particular, it can be shown that this object is in a distributed state with a large, macroscopic, indeterminacy of the coordinate, equal approximately to the amplitude of the oscillations in a coherent state when the energies of both states are equal.

If such a nonperturbing determination of the state is possible with respect to the electromagnetic oscillator, then, of course, it is possible also with respect to the mechanical oscillator and even also with respect to a body freely moving in space, since it is only a particular case of an oscillator corresponding to its zero frequency. Such a conclusion, naturally, follows from the identity of the the-

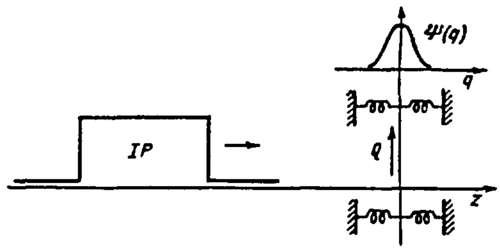

FIG. 7. Reflection of light from a mirror with a large quantum-mechanical indeterminacy.

oretical description of the electromagnetic and mechanical oscillators, but the possibility of a nonperturbing determination of the state of a mechanical oscillator will be demonstrated more directly below. One can ask why macroscopic bodies in distributed states are not observed in the world surrounding us. Quantum mechanics does not forbid such states (see Sec. 1), and in some cases they might even be the preferred ones.

One of the possible and widespread explanations of this consists of the assertion that in each observation of a body in such a state its localization occurs, i.e., the transition from a distributed state into a concentrated one (reduction of the wave packet). As we shall see, the problem examined below concerning the reflection of an electromagnetic signal from a mirror with a large quantum-mechanical indeterminacy of its coordinate does not confirm this assertion[11] and reduces the problem of investigating the state of a macroscopic body to the same measurements which are made with squeezed light.

A full theoretical description of reflection is quite complicated and we shall examine the simplest variant, when it is assumed that the mirror has only two degrees of freedom (Fig. 7). One—the transverse oscillator—describes the motion of the negative charges (bound electrons) along the mirror (displacement Q, conjugate momentum P, surface mass density ρ, surface charge density σ). The motion of the negative charges along the mirror is what actually leads to the reflection of the electromagnetic signal. The second degree of freedom—a longitudinal oscillator—describes the motion of the mirror along the direction of propagation of radiation (coordinate q, conjugate momentum p, surface mass density μ). It is also assumed that only those waves are present which have normal incidence on the mirror. The mirror is assumed to be infinitely thin and oriented perpendicular to the z axis along which the electromagnetic waves are propagated. The electric field and the displacement Q of the charges are directed along the x axis. The region of space occupied by the field has the same shape as the mirror; the area of the mirror is equal to s. Then the quantum system mirror+field is described by the Hamiltonian

$$H=\frac{1}{2\rho s}\left(P-\frac{\sigma s}{c}A(q)\right)^2+\frac{1}{2}sKQ^2+\frac{1}{2}s\int \mathrm{d}z\left[\frac{1}{4\pi}\left(\frac{\partial A}{\partial z}\right)^2\right.$$

$$+4\pi c^2 \prod{}^2\Big] + \frac{1}{2\mu s}p^2 + \frac{1}{2}s\varkappa q^2, \qquad (5.1)$$

where K and $\varkappa$ are the stiffness, respectively, of the transverse and longitudinal oscillators, $A(z)$ is the vector potential of electromagnetic field and $\Pi(z)$ is the momentum canonically conjugate to it.

The oscillators of the field (plane waves) are at the initial moment in coherent states, so phased that the field forms a rectangular pulse filled with a high frequency,

$$\langle E_{\text{in}}(z,t)\rangle = E_0 \sin \omega_0 \tilde{t} \prod (\tau_0, \tilde{t}), \qquad (5.2)$$

where $\Pi(z)$ is a function describing the rectangular shape of the pulse of duration $2\tau_0 = 4\pi n_0/\omega_0$ ($2n_0$ is the number of periods in the pulse), $\tilde{t} = t - \bar{t} - (z/c)$ and $\bar{t}$ is the moment of arrival of the pulse at the origin. The transverse oscillator in the initial moment is in the ground vacuum state, and the longitudinal oscillator is in the squeezed state described by the wave function

$$\Psi(q) = (2\pi\bar{q}^2)^{-1/4}\exp(-q^2/4\bar{q}^2), \qquad (5.3)$$

where $\bar{q}^2 = \hbar/(2s\mu\nu K^{1/2})$; K is the squeezing coefficient. The additional parameter in the wave function (5.3)—the squeezing coefficient—will enable us later to vary the parameters μ,ν of the longitudinal oscillator without changing its distribution (5.3); in particular, the transition will become possible to motion in free space ($\nu \to 0$).

The investigation of this problem completely within the framework of quantum mechanics shows that the average value of the field of the reflected signal in the steady-state regime and at resonance ($\Omega = \omega_0$) is equal to

$$\langle E_{\text{R}}(z,t)\rangle = E_0 e^{-2\bar{q}^2/\lambda\!\!\!^{-2}} \sin[\Omega(\tau - \bar{t})], \qquad (5.4)$$

where $\lambda\!\!\!^{-} = \lambda/2\pi = c/\Omega$ and $\tau = t + (z/c)$, while since the average value of the square of the field of the reflected signal which is proportional to the energy density of the electric field in it, is equal to

$$\langle E_{\text{R}}^2(z,t)\rangle = \frac{1}{2}E_0^2\{1 - e^{-8q^{-2}/\lambda\!\!\!^{-2}}\cos[2\Omega(\tau - t)]\}. \qquad (5.5)$$

As we can see with a small indeterminacy in the position of the mirror ($\bar{q} \ll \lambda\!\!\!^{-}$) the reflected signal preserves the properties of coherent light; in particular, $\langle E_{\text{R}}\rangle^2 \approx \langle E_{\text{R}}^2\rangle$. But when the indeterminacy of the position of the mirror is great ($\bar{q} \gg \lambda\!\!\!^{-}$), the average value of the field is close to zero, while the average value of the square of the field retains its finite value; only the oscillations of the energy density of the field of doubled frequency are close to zero. One can say that the amplitude coefficient of reflection tends to zero as the indeterminacy of the position of the mirror increases, while the coefficient of reflection with respect to power retains its finite value. This means that the reflected signal is in an essentially quantum state, although it is a macroscopic one, since only in such a state is the inequality $(\langle E\rangle^2) \ll (\langle E^2\rangle)$ possible.

One can also show that the length of the reflected signal is approximately by $\bar{q}$ greater than the length of the incident signal.

The consequence of this investigation is the conclusion that there exists an experimental possibility of observing a distributed quantum mechanical state of a macroscopic body (mirror) by means of probing this body by an electromagnetic pulse without an essential change in the state of this body. Indeed, as we have seen, the reflected pulse carries information concerning the distribution characterization of the state of the mirror and, it being macroscopic, can be analyzed by the existing experimental means similar to those which are used for analysis of squeezed light.[7,8] At the same time it can be shown[11] that the process of reflection does not lead to an essential change in the quantum-mechanical state of the mirror. Indeed, the phenomenon of recoil accompanying the reflection of the pulse from the mirror leads to a small shift of the longitudinal distribution as the whole, but in the first order of perturbation theory does not affect its width, i.e., its variance. Consequently, if one repeatedly probes the mirror by a light pulse the measured value of the variance will turn out to be the same as in the first probing. This, more than anything else, convinces us that no reduction occurs of the wave packet in probing the mirror. But the possibility of observing a single object (mirror) in a distributed state contradicts the usual interpretation of such a state as describing an ensemble of objects.[1,2]

One should also note that the reflected pulse $\langle E_{\text{R}}^2\rangle$ is lengthened in comparison with the incident pulse due to the partial reflection of the incident pulse from different layers of the distribution of the longitudinal coordinate of the mirror. A human eye with its low time resolution cannot, of course, notice such a lengthening of the pulse (approximately by $\bar{q}$). However, in the case of oblique incidence of light from a point source onto the mirror partial reflection from different layers of the longitudinal distribution will go over into the angular distribution of the rays. Since the angular resolution of the human eye is sufficiently high, the body with a large quantum-mechanical indeterminacy will appear simply as somewhat smeared out. Consequently, if bodies with a large quantum-mechanical indeterminacy existed in the world surrounding us they would have been visible to a literally unaided eye.

6. CONCLUSION

Thus, on the one hand, quantum mechanics does not forbid macroscopic bodies from existing in widely distributed states, and what has been presented in the last section indicates the possibility of a nondemolition observation of individual, macroscopic bodies in such states, for example, with the same means that are utilized in experiments with squeezed light, or even simply with an unaided eye. On the other hand, macroscopic bodies in widely distributed states have never been observed by anybody, and, because of this, widely distributed solutions of the Schrödinger equation have been interpreted as describing an ensemble of bodies and not individual samples. The main purpose of this methodological note consists of attracting the attention to this breakdown in the similarity of the descriptions of macroscopic objects in optics and mechanics. This equation is not only speculative; for its clarification experimental attempts of exciting mechanical oscillators into a nonclassical widely distributed state would be very useful. As is clear from the material presented above the most promising method of such excitation is parametric resonance. As a mechanical oscillator in such experiments one could use for example a microdust particle levitating in fields formed by an oscillating potential.

One might attempt to explain the absence of macroscopic bodies in widely distributed states by saying that they at a certain initial moment were formed in a concentrated state and have been gradually spreading from that time. However, one cannot see reasons for their being formed at the initial moment specifically in the concentrated state.

Also one cannot explain the concentrated nature of wave packets of macroscopic bodies by taking into account their interaction with some kind of a field. Indeed, since bodies consist of charged particles they must interact with the electromagnetic field. However, such an interaction will lead only to the formation of an accompanying (nonradiating) field since such fields are possessed by all particles in uniform rectilinear motion. Analogous considerations refer also to other fields. In all cases the wave packet (1.1) will describe the motion of the centre of inertia of a macroscopic body and the fields accompanying it, and, consequently, the question of the large values of the parameter a in (1.1) and concerning the possibility of distributed states of macroscopic bodies cannot be solved in such a manner.

Without claiming the finality of our argument we shall describe one of the possible methods of solving this problem. It is based on simple physics considerations. We have already spoken above concerning the interaction of a body with different fields surrounding and accompanying it. In this respect the gravitational field forms an exception.[12,13] Indeed, according to the general theory of relativity the gravitational field produced by the body can be regarded as a deformation of space, its deviation from a euclidean nature, with such a deformation occurring not only in the region occupied by the body but also in its neighborhood. If the mass associated with some part of the wave packet deforms space then the remaining parts of the packet move in this deformed space. In general any portion of a wave packet moves in the space deformed by this part and also all the other of its parts. Consequently, an action of the wave packet on itself occurs, or a self-action of the wave packet due to its interaction with the gravitational field. As will be shown below, this self-action will lead to the formation of a potential well being in which the wave packet preserves its concentrated form in a steady state. Moreover, the formation of such a potential well, evidently, is energetically favorable, and this explains the concentrated nature of wave packets of macroscopic bodies.

Since in this case we are dealing with macroscopic bodies of ordinary density (of the order of 1 g/cm^3), the gravitational potential is weak and, consequently, in calculations it is sufficient to utilize the newtonian expression for the potential.

Let us consider a homogeneous sphere-like body of

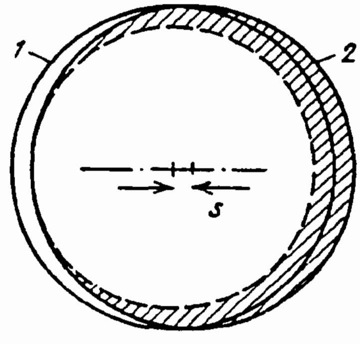

FIG. 8. Attraction of two interpenetrating spheres.

radius R with density ν. Since subsequently we shall be calculating the gravitational action of one part of the wave packet on another, and the dimensions of the wave packet are much smaller than the geometrical dimensions of the body, then as a preliminary consideration we shall examine the attraction of two interpenetrating massive spheres the centers of which are displaced by a distance s much smaller than R. As can be seen from Fig. 8, in order to calculate the force of attraction between the spheres it is sufficient to take into account the attraction to the first sphere of a layer of thickness $2s$ covering one half of the surface of the first sphere. Taking into account only the component of the forces of attraction of elementary masses along a line joining the centers of the spheres we obtain for the force of attraction between the spheres the following expression:

$$F=\frac{16\pi^2}{3}G\nu^2R^3s\int_0^{\pi/2}d\theta\sin\theta\cos\theta=\frac{8\pi^2}{3}G\nu^2R^3s. \tag{6.1}$$

Consequently the potential energy of interaction of the two spheres is equal to

$$U_s=\beta s^2,\quad \beta=\tfrac{4}{3}\pi^2G\nu^2R^3. \tag{6.2}$$

Taking this expression into account we arrive at the following Schrödinger equation describing the wave packets of macroscopic bodies:

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}=-\frac{\hbar^2}{2m}\Delta\Psi(\mathbf{r},t)+U(\mathbf{r})\Psi(\mathbf{r},t), \tag{6.3}$$

where

$$U(\mathbf{r})=\beta\int d\mathbf{r}'(\mathbf{r}-\mathbf{r}')^2|\Psi(\mathbf{r},t)|^2. \tag{6.4}$$

From this equation one can see that the effect of the gravitational self-interaction of the wave packet of a macroscopic body is analogous to the optical effect of self-focussing. The difference consists only of the fact that self-focussing occurs in two directions, perpendicular to the direction of propagation of the waves, while the gravitational self-action occurrs in all three directions.

Let us find the steady-state and spherically symmetrical solution of equation (6.3). For $\Psi=\Psi(\mathbf{r})$ and $\mathrm{Im}\,\Psi=0$ we have in accordance with (6.4)

TABLE I.

m,g	R,cm	r_0,cm
10^{28}	$1,3\cdot10^{9}$	$1,25\cdot10^{-26}$
10^{12}	$6,2\cdot10^{3}$	$1,25\cdot10^{-18}$
1	0,62	$1,25\cdot10^{-12}$
10^{-12}	$6,2\cdot10^{-5}$	$1,25\cdot10^{-6}$
10^{-15}	$6,2\cdot10^{-6}$	$4\cdot10^{-5}$

$$U(r)=\alpha+\beta r^2, \tag{6.5}$$

where $\alpha=\beta\int d\mathbf{r}'^2|\Psi(\mathbf{r}')|^2$ is a nonessential constant. Thus, for the determination of $\Psi(r)$ we have the equation

$$-\frac{\hbar^2}{2m}\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\Psi}{dr}\right)+\beta r^2\Psi=E\Psi. \tag{6.6}$$

The solution of this equation has the form

$$\Psi(r)=\frac{1}{\sqrt{\pi^3}r_0^{3/2}}\exp\left(-\frac{r^2}{2r_0^2}\right), \tag{6.7}$$

where

$$r_0=\left(\frac{\hbar^2}{2\beta m}\right)^{1/4}=\left(\frac{2\hbar^2R^3}{3Gm^3}\right)^{1/4} \tag{6.8}$$

is the characteristic size of the wave packet determined by the gravitational self-action. Such a concentrated wave packet can be regarded as a soliton solution of the Schrödinger equation (6.3). In Table I some estimates are provided (translator's comment: Table I from rp98 should be inserted here followed by the following text). The columns give respectively the mass, the geometrical size of the macroscopic body with a density of $\nu=1$ g/cm^3 and the size of its wave packet. In the third line of the table data are given for a typical macroscopic body ($m=1$ g). In the first line are given data for a body with a mass of the order of the mass of the earth. The last line gives data for a mass of 10^{-15} g, for which the geometrical dimensions become approximately equal to the dimensions of the wave packet. The size corresponding to the condition $r_{cr}=r_0=R$, can be determined from (6.8):

$$r_{cr}=\left(\frac{9\hbar^2}{32G\pi^3\nu^3}\right)^{1/10}. \tag{6.9}$$

Condition (6.9) gives a quantitative criterion for division of bodies according to their sizes into macroscopic and microscopic ones. From (6.0) one can obtain also the criteria for a similar division according to masses. Bodies with a mass greater than

$$m_{cr}=\left(\frac{32\pi\nu\hbar^6}{81G^3}\right)^{1/10} \tag{6.10}$$

should be regarded as macroscopic.

Thus, for masses greater than 10^{-12} g, the dimensions of the wave packet are negligibly small not only compared with the geometrical dimensions of these masses, but also in comparison with the typical optical wavelength (10^{-4}–10^{-5}). These considerations explain the absence in the world surrounding us of macroscopic bodies in states with a large quantum-mechanical indeterminacy of their center of mass. However, this explanation is obtained at a high price, specifically by renouncing the principle of superposition and the linearity of Schrödinger's equation,[13] as can be seen from equation (6.3). It is true that one should note that the violation of the principle of superposition is not a very strong one. In particular, it completely does not affect the region of microscopic, atomic phenomena—the principal field of application of quantum mechanics.

Moreover, renunciation of the principle of superposition can turn out to be temporary. After the time when quantum theory will be unified with gravitation, the considerations discussed in this section may turn out to be something like the semiclassical theory in quantum electrodynamics, when the medium is described quantum-mechanically, and the field classically. In the quantum-mechanical description both of gravitation and also of matter the principle of superposition might become reestablished.

We also note that the potential well about which we spoke above is not a very deep one. Therefore by expending some energy a macroscopic body can be brought into the distributed state with a large dimension of the wave packet. Observation and investigation of distributed states of macroscopic bodies would be of first-rank scientific significance.

Thus, observation of squeezed light has clearly shown that there exists a practical possibility of establishing the distributed nature of a quantum-mechanical state of an individual macroscopic object. In squeezed light such an object is the field of the selected mode of an optical resonator. However, the identity of the theoretical descriptions of the electromagnetic and the mechanical oscillators, and also the specific system of probing the distributed mirror described above have shown that the distributed nature of the quantum-mechanical state of an individual macroscopic body can also be established. In our opinion this conclusion is the principal consequence of the experimental observation of squeezed light.

This conclusion leads to the question: why are macroscopic bodies not observed in the world surrounding us in distributed states, but only in concentrated states whose wave packets move according to the laws of classical mechanics? In the conclusion we have proposed a possible explanation of this fact, from which follows the important role played by gravitation in the transition from a quantum-mechanical description of motion of macroscopic bodies to a classical description.

The considerations presented above also show that macroscopic bodies can be brought into the distributed state, and it would be important to observe such states experimentally. Macroscopic bodies in such states would be a new object of investigation in physics. A natural method of obtaining such states is the parametric excitation of a mechanical oscillator.

It is not difficult to understand why the distributed states were first observed for the electromagnetic field and not in mechanical systems. The mass of the electromagnetic field is not great under ordinary intensities and cannot appreciably deform space, not even speaking of the fact that the space of the states of the field is not the usual three-dimensional space, but the abstract Hilbert space. Therefore the concentrated state of the field does not have any advantages over the distributed one.

In connection with all that has been discussed above it is justified to mention the article by A. Einstein,[14] in which for the first time the question was raised concerning the correct quantum-mechanical description of the motion of macroscopic bodies.

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