# Fundamentals of the spectral diagnostics of gases containing a condensed dispersed phase 

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The emission from a gas containing macroscopic particles, as well as the transmission of light through such a system with account taken of multiple scattering of light by the particles, is studied. The emission from objects of different shapes is studied in the continuum and in spectral lines. The influence of the scattering particles on this emission is studied theoretically and experimentally. The emission from a closed isothermal cavity filled with gas containing a condensed dispersed phase is analyzed. Relationships are obtained that are necessary to determine the temperature of the gas and the particles, and the concentration of atoms (or molecules) from the measured intensities of the intrinsic and transmitted radiation.

## 1. INTRODUCTION

In physics and technology one often encounters gaseous emitters that contain impurities of macroscopic solid particles and liquid drops. We can give two examples. First, the brightly glowing layer near the electrode of a high-current gas discharge contains macroscopic particles of the material of the electrode being eroded. Second, the radiating flow of combustion products in thermal electric power stations, in magnetohydrodynamic generators, rocket jets, etc., contains particles of varied origin and composition. Despite the vast differences in the objects, the fundamentals of the methods of spectral diagnostics are practically identical.

In the traditional methods of spectral diagnostics of a gaseous medium, one measures the intensities of the intrinsic radiation and of that transmitted through the medium under study. The measurements are performed both in the region of the spectral lines of atoms and molecules and at frequencies of the continuous spectrum. A correct interpretation of the results of the measurements requires strict localization of the volumes from which the radiation reaches the measuring spectral instruments. Many methods of diagnostics are based on the idea that the radiation freely escapes from the medium, i.e., the medium is optically thin.

When macroscopic particles arise the emission from the gaseous medium can change radically. When the temperature of the particles is high enough, intrinsic continuous emission arises from them. The emission from the gaseous component can be substantially weakened by absorption by the particles. The overall optical density of the medium increases. Finally, the scattering in the medium increases sharply. Light scattering by macroscopic particles is usually greater by an order of magnitude than the scattering by the components of the gas phase. The substantial scattering leads to difficulties involving the localization of the volume of observation. Light arising at any part of the emitter, after scattering by the particles lying in the field of view of the instrument, can be detected.

Even if there are few particles and they scatter light relatively weakly, but strong emitters exist outside the field of view of the instrument, the effect can be large. In a layer near an electrode such emitters can be bright spots on the electrode itself, or in a flow of combustion products they can be the glowing walls surrounding the flow. Taking account of light scattering in diagnostics of a gas containing a condensed dispersed phase is a serious problem. On the other hand, the same effect is used for studying macroscopic particles in specially organized experiments-by illuminating the particles with high-power laser radiation. When there are many particles and they strongly scatter light, the influence of scattering on the measured intensity of emission can be very substantial, even in the absence of external emitters.

To find the characteristics of an object from the observable intensities of intrinsic and transmitted radiation, one must have preliminary information of two types. First, one must know precisely how the primary radiation as well as the optical characteristics of the absorption and light scattering are related to the quantities being sought. The corresponding relationships are determined by the mechanisms of population of the energy states of the emitters and the mechanisms of interaction of the radiation with matter. Second, one must know the relations of the measured intensities of the intrinsic and transmitted radiation with the primary radiation and the optical characteristics. These relations are governed by processes of radiation transport. In discussing radiation transport below, we assume that the population of the atomic (or molecular) levels, as well as the thermal and optical characteristics of the macroscopic particles do not depend on the escape or transmission of the radiation being studied. This assumption makes it possible to separate the problem of radiation transport from that of the energy state of the medium. Radiation transport is treated in this way in most of the fundamental studies (see Refs. 1 and 2); both problems are solved jointly in Ref. 3.

Of fundamental interest in the present study are the features of spectral diagnostics that arise from the macro-
scopic particles. In this regard we shall first discuss the optical characteristics of the macroscopic particles (Sec. 2 ). The presented data make it possible to estimate the effective absorption and scattering cross sections and the fundamental features of the scattering indicatrix in various cases. The same section describes the optical characteristics of atoms and the primary thermal emission from a gas containing particles.

Then we shall discuss in considerable detail radiation transport complicated by the possibility of multiple scattering, i.e., zigzag motion of photons in the medium (Sec. 3). To solve the equations of radiation transport we have chosen, developed, and applied the probabilistic method of V. V. Sobolev. This method has established the most direct and simple connection between the primary emitters and the optical characteristics, on the one hand, and with the observed emission, on the other hand. We discuss the equilibrium radiation of a closed isothermal cavity filled with a gas containing condensed dispersed particles. This radiation is separated into terms of different origins: emission from atoms, emission from particles, and finally, emission from the walls of the cavity that is scattered and directly arrives at the observer. This makes it possible in concrete problems to estimate the limiting possible influences of various circumstances on the observable radiation.

Section 4 treats the escape of radiation from a gas containing a condensed dispersed phase in the continuum and in lines. Illustrative calculations are performed of the intensities, the physical pattern of the obtained results is analyzed, and experimental studies of the influence of scattering particles on the radiation are described.

One can find from the attenuation of the radiation of an external source (a lamp or a laser) the optical density of the medium or any characteristic associated with the optical density. When the light scattering by particles is substantial in a gas containing a condensed dispersed phase, the result of the measurements can depend on the optical scheme of the experiments. Section 5 discusses the transmission of a thin beam through a gas containing a condensed dispersed phase. The results of the treatment make it possible to formulate experiments correctly and to interpret reliably the results of the measurements of attenuation.

The concrete features of measurements of temperatures and concentrations in a gas containing a condensed dispersed phase are discussed in Sec. 6. The most reliable methods of measuring the averaged characteristics are analyzed, as well as the possibilities of finding the spatial distributions upon observing along a single ray of vision.

## 2. OPTICAL CHARACTERISTICS

### 2.1. Optical characteristics of particles of a condensed dispersed phase

First we shall examine the optical characteristics of individual macroscopic particles. The interaction of radiation with single particles has been studied repeatedly. ${ }^{5-7}$ In finding the optical characteristics most often one solves the wave problem of diffraction by the particle. The frequency
of the radiation does not vary in this interaction, i.e., the scattering is monochromatic. The result of the interaction for a given polarization of the light incident on the particle depends on the structure and the shape of the particle, on the complex refractive index of the material of the particle

$$
\begin{equation*}
n=n_{1}-i n_{2} \tag{2.1}
\end{equation*}
$$

and on the diffraction parameter of the particle

$$
\begin{equation*}
D=2 \pi a_{\mathrm{p}} / \Lambda \tag{2.2}
\end{equation*}
$$

Here $a_{\mathrm{p}}$ is the characteristic dimension of the particle, and $\Lambda$ is the wavelength of the radiation.

Especially many studies have been devoted to spherical particles. In this case $a_{\mathrm{p}}=r_{\mathrm{p}}$ is the radius of the particle. In calculations one usually employs the theory of Mie, ${ }^{8}$ which was developed at the beginning of the century.

The interaction of radiation with individual particles is described by using the cross sections for absorption ( $\sigma_{\mathrm{p}}$ ), scattering ( $\sigma_{\mathrm{s}}$ ), and the scattering indicatrix ( $\chi\left(\gamma_{\mathrm{s}}\right)$ ); here $\gamma_{\mathrm{s}}$ is the scattering angle of the light. (Translator's note: misprint?) The scattering indicatrix satisfies the normalization condition

$$
\begin{equation*}
\int_{4 \pi} \chi\left(\gamma_{\mathrm{s}}\right) \mathrm{d} \omega / 4 \pi=0.5 \int_{0}^{\pi} \chi\left(\gamma_{\mathrm{s}}\right) \sin \gamma_{\mathrm{s}} \mathrm{~d} \gamma_{\mathrm{s}}=1 \tag{2.3}
\end{equation*}
$$

Here $\mathrm{d} \omega$ is the element of solid angle.
The absorption and scattering cross sections make it possible to find the total cross section of attenuation (extinction)

$$
\begin{equation*}
\Sigma_{\mathrm{p}}+\sigma_{\mathrm{p}}+\sigma_{\mathrm{s}} \tag{2.4}
\end{equation*}
$$

and the probability of survival of the radiation per single interaction with a particle

$$
\begin{equation*}
\lambda_{\mathrm{p}}=\sigma_{\mathrm{s}} /\left(\sigma_{\mathrm{p}}+\sigma_{\mathrm{s}}\right)=\sigma_{\mathrm{s}} / \Sigma_{\mathrm{p}} \tag{2.5}
\end{equation*}
$$

The extension forward of the scattering indicatrix with respect to the initial direction of the radiation incident on the particle is well described by the mean of the scattering cosine

$$
\begin{align*}
\left\langle\cos \gamma_{\mathrm{s}}\right\rangle & =\int_{4 \pi} \cos \gamma_{\mathrm{s}} \chi\left(\gamma_{\mathrm{s}}\right) \mathrm{d} \omega / 4 \pi \\
& =0.5 \int_{0}^{\pi} \chi\left(\gamma_{\mathrm{s}}\right) \cos \gamma_{\mathrm{s}} \sin \gamma_{\mathrm{s}} \mathrm{~d} \gamma_{\mathrm{s}} \tag{2.6}
\end{align*}
$$

As the indicatrix becomes extended forward, the mean cosine approaches closer and closer to unity. The mean scattering cosine is associated with the transport approximation, which is applicable in the case in which the radiation intensities depend weakly on the direction. ${ }^{9}$ In the transport approximation it is assumed that the scattering is isotropic, i.e.,

$$
\begin{equation*}
\chi_{\mathrm{tr}}\left(\gamma_{\mathrm{s}}\right)=1 \tag{2.7}
\end{equation*}
$$

while the scattering cross section is determined by the expression

$$
\begin{equation*}
\sigma_{\mathrm{tr}}=\sigma_{\mathrm{s}}\left(1-\left\langle\cos \gamma_{\mathrm{s}}\right\rangle\right) \tag{2.8}
\end{equation*}
$$

Here the scattering at different angles is taken into account with equal weight by using $\left\langle\cos \gamma_{\mathrm{s}}\right\rangle$. The smaller the scattering angle, the smaller the contribution of scattering to $\sigma_{\mathrm{tr}}$ is, while scattering exactly forward is not at all taken into account. In the transport approximation the expression for the probability of survival of the radiation has the form

$$
\begin{equation*}
\lambda_{\mathrm{tr}}=\frac{\sigma_{\mathrm{tr}}}{\sigma_{\mathrm{tr}}+\sigma_{\mathrm{p}}}=\frac{\lambda_{\mathrm{p}}\left(1-\left\langle\cos \gamma_{\mathrm{s}}\right\rangle\right)}{1-\lambda_{\mathrm{p}}\left\langle\cos \gamma_{\mathrm{s}}\right\rangle} \tag{2.9}
\end{equation*}
$$

In pure scattering (without absorption) we have

$$
\begin{equation*}
\lambda_{\mathrm{tr}}=\lambda_{\mathrm{p}}=1 \tag{2.10}
\end{equation*}
$$

The extinction cross section in the transport approximation is expressed in terms of the true extinction cross section as follows:

$$
\begin{equation*}
\Sigma_{\mathrm{tr}}=\sigma_{\mathrm{tr}}+\sigma_{\mathrm{p}}=\Sigma_{\mathrm{p}}\left(1-\lambda_{\mathrm{p}}\left\langle\cos \gamma_{\mathrm{s}}\right)\right) \tag{2.11}
\end{equation*}
$$

The literature contains extensive tables of calculated cross sections, $\sigma_{\mathrm{p}}, \sigma_{\mathrm{s}}$, and the indicatrix $\chi\left(\gamma_{\mathrm{s}}\right)$, which are required mainly in meteorology; see, e.g., Ref. 10. At present computer programs are being published, even in monographs, for calculating the optical characteristics of both spherical, homogeneous particles, and of more complicated structures. ${ }^{7}$

The results, especially on the scattering indicatrices, are marked by a great variety. For correct design of spectral experiments and choice of a suitable method of diagnostics, it is useful to have a picture of the possible limits of variations and of typical values of cross sections and indicatrices. In this regard we shall make a certain classification of the results pertaining to spherical particles. Let us study particles having different diffraction parameters of (2.2). We shall classify all the particles into three groups: $D<1, D>1$, and $D \approx 1$. We note that in the visible region of the spectrum, where $\lambda \approx 0.6 \mu \mathrm{~m}$, we have $D \approx 1$ for particles with $r_{\mathrm{p}} \approx 0.1 \mu \mathrm{~m}$.

1. Small diffraction parameter ( $D<1$ ). In the visible region of the spectrum these are submicrometer particles ( $r_{\mathrm{p}}<0.1 \mu \mathrm{~m}$ ). If in addition to the condition $D<1$ also the condition $D|n|<1$ is fulfilled, the cross sections and indicatrices are described by the expressions: ${ }^{7}$

$$
\begin{align*}
& \sigma_{\mathrm{p}}=4 D \operatorname{Im} \frac{n^{2}-1}{n^{2}+1} \cdot \pi r_{\mathrm{p}}^{2} \\
& \sigma_{\mathrm{s}}=\frac{8}{3} D^{4}\left|\frac{n^{2}-1}{n^{2}+2}\right|^{2} \cdot \pi r_{\mathrm{p}}^{2}  \tag{2.12}\\
& \chi\left(\gamma_{\mathrm{s}}\right)=0.75\left(1+\cos ^{2} \gamma_{\mathrm{s}}\right)
\end{align*}
$$

In this case we have

$$
\begin{equation*}
\left\langle\cos \gamma_{\mathrm{s}}\right\rangle=0, \quad \sigma_{\mathrm{tr}}=\sigma_{\mathrm{s}}, \quad \lambda_{\mathrm{tr}}=\lambda_{\mathrm{p}} \tag{2.13}
\end{equation*}
$$

This is the well known case of Rayleigh scattering. We find the following expression for the ratio of cross sections from (2.12):

$$
\begin{equation*}
\frac{\sigma_{\mathrm{s}}}{\sigma_{\mathrm{p}}}=\frac{2}{3} D^{3}\left|\frac{n^{2}-1}{n^{2}+2}\right|^{2}\left(\operatorname{Im} \frac{n^{2}-1}{n^{2}-1}\right)^{-1} \tag{2.14}
\end{equation*}
$$

Equations (2.14) and (2.5) imply that for small diffraction parameters $D$ and not too large refractive indices $|n|$ we have

$$
\begin{equation*}
\sigma_{\mathrm{s}} \ll \sigma_{\mathrm{p}}, \quad \lambda_{\mathrm{p}} \ll 1 . \tag{2.15}
\end{equation*}
$$

The relationship (2.15) is important in practice, for it implies that sometimes we can neglect scattering in comparison with absorption. In cases in which scattering is the object of study or the means of diagnostics, of course, one must not do this.
2. Large diffraction parameter ( $D>1$ ). In the visible region of the spectrum these are particles with $r_{p} \gtrsim 0.5-1$ $\mu \mathrm{m}$. Scattering by large particles is described by the exact theory of Mie. More simple and useful in practice is a combined treatment of extremely large particles, in which the interaction of light with a particle is separated into two parts. ${ }^{5,7}$

First, independently of the optical properties of the particle, a diffraction pattern arises as from a black circular screen of radius $r_{p}$. Here the scattering cross section is determined simply by the area of the corresponding screen

$$
\begin{equation*}
\sigma_{\mathrm{d}}=\pi r_{\mathrm{p}}^{2} \tag{2.16}
\end{equation*}
$$

while for the indicatrix we have

$$
\begin{align*}
\chi_{\mathrm{d}}\left(\gamma_{\mathrm{s}}\right)= & 0.25 D^{2}\left(1+\cos \gamma_{\mathrm{s}}\right)^{2} \\
& \times\left(2 J_{1}\left(D \sin \gamma_{\mathrm{s}}\right) / D \sin \gamma_{\mathrm{s}}\right)^{2} \tag{2.17}
\end{align*}
$$

Here $J_{1}$ is the Bessel function. The greater part of the light energy ( $84 \%$ ) scattered via diffraction is found in the region of angles determined by the direction to the first minimum of the diffraction pattern

$$
\begin{equation*}
\sin \gamma_{\mathrm{s} 1}=3.83 / D \tag{2.18}
\end{equation*}
$$

Consequently the forward directionality of the indicatrix becomes sharper as the diffraction parameter increases. We can represent the indicatrix of (2.17) approximately in the form

$$
\begin{align*}
\chi_{\mathrm{d}}\left(\gamma_{\mathrm{s}}\right)= & 0.25 D^{2}\left(1+\cos \gamma_{\mathrm{s}}\right)^{2} \exp \left(-0.3 D^{2} \sin ^{2} \gamma_{\mathrm{s}}\right) \\
& \text { when } \sin \gamma_{\mathrm{s}} \leqslant 3.83 / D \\
= & 0 \quad \text { when } \sin \gamma_{\mathrm{s}}>3.83 / D \tag{2.19}
\end{align*}
$$

Further, this rather crude representation will be used in estimates and illustrative calculations.

Second, owing to reflection and transmission of light through the particle, scattering and absorption that substantially depend on the complex refractive index of the material of the particle arise. The reflection and transmission of light are described by the method of geometric optics. Let us denote the corresponding components of the scattering cross section and the indicatrix as $\sigma_{\mathrm{g}}$ and $\chi_{\mathrm{g}}\left(\gamma_{\mathrm{s}}\right)$. For water having $n=1.33, \chi_{\mathrm{g}}\left(\gamma_{\mathrm{s}}\right)$ was calculated in Ref. 5. Mainly the scattered light here is also directed forward, i.e., $\gamma_{\mathrm{s}}<90^{\circ}$, but is not as extended as in diffraction by the indicatrix. For not too large values of $n_{1}$, the indicatrices do not change very strongly as $n$ varies, and here the following approximate description is not too bad:

$$
\begin{equation*}
\chi_{\mathrm{g}}\left(\gamma_{\mathrm{s}}\right) \approx 16.5 \exp \left[-9\left(1-\cos \gamma_{\mathrm{s}}\right)\right] . \tag{2.20}
\end{equation*}
$$

The total cross sections of absorption and geometric scattering are determined by the cross sectional area of the sphere. Therefore we have

$$
\begin{equation*}
\sigma_{\mathrm{g}}+\sigma_{\mathrm{p}}=\pi r_{\mathrm{p}}^{2} \tag{2.21}
\end{equation*}
$$

The total cross section of interaction with a large particle $\Sigma_{\mathrm{p}}$ is obtained by adding the diffraction and geometric cross sections:

$$
\begin{equation*}
\Sigma_{\mathrm{p}}=\sigma_{\mathrm{d}}+\sigma_{\mathrm{g}}+\sigma_{\mathrm{p}}=2 \pi r_{\mathrm{p}}^{2} \tag{2.22}
\end{equation*}
$$

The total scattering cross section with account taken of (2.16) has the form

$$
\begin{equation*}
\sigma_{\mathrm{s}}=\sigma_{\mathrm{d}}+\sigma_{\mathrm{g}}=\pi r_{\mathrm{p}}^{2}+\sigma_{\mathrm{g}} \tag{2.23}
\end{equation*}
$$

The probability of survival can be written as

$$
\begin{equation*}
\lambda_{\mathrm{p}}=\sigma_{\mathrm{s}} / \Sigma_{\mathrm{p}}=0.5+\left(\sigma_{\mathrm{g}} / 2 \pi r_{\mathrm{p}}^{2}\right) \tag{2.24}
\end{equation*}
$$

The overall scattering indicatrix of a large particle is obtained by combining the expressions for the probabilities of interaction of the radiation with the particle with subsequent scattering. As a result we obtain

$$
\begin{equation*}
\chi_{\mathrm{d}+\mathrm{g}}\left(\gamma_{\mathrm{s}}\right)=\sigma_{\mathrm{s}}^{-1}\left(\sigma_{\mathrm{g}} \chi_{\mathrm{g}}\left(\gamma_{\mathrm{s}}\right)+\sigma_{\mathrm{d}} \chi_{\mathrm{d}}\left(\gamma_{\mathrm{s}}\right)\right) \tag{2.25}
\end{equation*}
$$

The normalization in (2.3) preserves the meaning for each of the introduced indicatrices. Comparison of the indicatrices of (2.25) with the exact expressions calculated by the authors of Ref. 11 shows that, even when $D=5$, the indicatrices of (2.25) qualitatively describe the fundamental features of the pattern, while at larger $D$ the agreement is quite satisfactory.

Let us study the limits within which the cross sections and indicatrices vary upon changing the substance of the particles. An especially large influence on the optical characteristics is exerted by the absorptive properties of the material, involving the imaginary component of the refractive index $n_{2}$. Therefore we shall assume, first, that all the light incident on the particle is absorbed. This can happen in particles of soot ( $n_{1}=1.8, n_{2}=0.6^{12}$ ), where not only is the absorption large, but also the reflection small. In this case we have $\sigma_{\mathrm{g}}=0$ for the geometric scattering cross section and, in agreement with (2.21), we find $\sigma_{\mathrm{p}}=\pi r_{\mathrm{p}}^{2}$. We emphasize that diffractive scattering remains also in this case and is described by Eqs. (2.16) and (2.17). Second, we assume that the particle is transparent, i.e., it does not absorb light at all, which can happen if the imaginary component of the coefficient of refraction is small. In the visible region of the spectrum this pertains, e.g., to water drops or particles of aluminum oxide. Thus in aluminum oxide the real and imaginary components of the refractive index vary within the range $n_{1}=1.7-1.8, n_{2}=10^{-8}-10^{-5} .{ }^{13}$ In this case we have $\sigma_{\mathrm{p}}=0$, and according to (2.21) we have $\sigma_{\mathrm{g}}=\pi r_{\mathrm{p}}^{2}$.

After one has determined $\sigma_{\mathrm{p}}$ and $\sigma_{\mathrm{g}}$, one can find from (2.23) the scattering cross section $\sigma_{\mathrm{s}}$, from (2.24) the probability of survival $\lambda_{p}$, and from (2.25) the scattering indicatrix $\chi\left(\gamma_{\mathrm{s}}\right)$. The results are presented in the Table. We see there that, for a maximal change in the effect of
absorption, the scattering cross section $\sigma_{\mathrm{s}}$ and the probability of survival $\lambda_{p}$ vary twofold. Yet the extinction cross section, as was noted above, does not change at all. The scattering indicatrix in the case of completely scattering particles, with account taken of the fact that $\sigma_{\mathrm{d}}=\sigma_{\mathrm{g}}=\pi r_{\mathrm{p}}^{2}$ and $\sigma_{\mathrm{s}}=2 \pi r_{\mathrm{p}}^{2}$, is determined in agreement with (2.25) in the form

$$
\begin{equation*}
\chi_{\mathrm{d}+\mathrm{g}}\left(\gamma_{\mathrm{s}}\right)=0.5\left(\chi_{\mathrm{g}}\left(\gamma_{\mathrm{s}}\right)+\chi_{\mathrm{d}}\left(\gamma_{\mathrm{s}}\right)\right) \tag{2.26}
\end{equation*}
$$

The forward extension of the indicatrices is illustrated by estimates of the mean scattering cosines by (2.6) (see the Table). The estimates have employed the representations of (2.19) and (2.20). A transition from scattering to absorbing particles leads to forward extension of the indicatrices: $\left\langle\cos \gamma_{\mathrm{s}}\right\rangle$ approaches unity. With increase in the parameter $D$ the indicatrix of absorbing particles is very sharply extended. In the case of scattering particles for sufficiently large $D(D>20)$, further increase in $D$ practically does not affect $\left\langle\cos \gamma_{s}\right\rangle$, since here $\chi_{g}$, which does not depend on $D$, determines the scattering pattern in (2.25) and (2.26). The characteristics of the transport approximation also are given for $D=1000$. We see that the scattering properties of large absorbing particles are infinitesimal. For transparent particles the transport cross section is smaller than the true value by approximately a factor of 18 , while the probability of scattering is unaltered ( $\lambda_{\mathrm{p}}=\lambda_{\mathrm{tr}}=1$ ).
3. Diffraction parameter of the order of unity $(D=1)$. In this case the absorption cross section $\sigma_{\mathrm{p}}$ depends on $n_{2}$ in exactly the same way as in the previous case (see the Table). The scattering cross section is an oscillating function of the diffraction parameter $D$ or the radius $r_{\mathrm{p}} .{ }^{5-7}$ The amplitude of the oscillations decreases with increasing $r_{p}$ (or $D$ ) and with increase in the imaginary component of the refractive coefficient $n_{2}$. For small $n_{2}$ the oscillations of $\sigma_{\mathrm{s}}$ occur about the mean value $2 \pi r_{\mathrm{p}}^{2}$, while for large $n_{2}$ the cross section $\sigma_{\mathrm{s}}$ approaches $\pi r_{\mathrm{p}}^{2}$. These mean values vary within the same range as when $D \gg 1$, just as an increase in $D$ leads to forward extension of the indicatrices, however complicated their forms are. However, in contrast to the previous case, the characteristic of extension $\left\langle\cos \gamma_{s}\right\rangle$ varies over a very broad range. This range can be indicated approximately as: from $\left\langle\cos \gamma_{\mathrm{s}}\right\rangle=0$ for a Rayleigh indicatrix on the side of small $D$ to $\left\langle\cos \gamma_{s}\right\rangle=0.91-0.93$ for $D=5$, in agreement with Table I.

The ideas and estimates presented here make no claim to accuracy or total scope of the variety of characteristics of individual particles, but make it possible to orient one's self in this variety.

Let us turn to the characteristics of an ensemble of particles. Usually the absorption, scattering, and extinction coefficients of a medium are determined by simple summation per unit volume of the cross sections of the individual particles. When the particles are identical we have

$$
\begin{equation*}
k_{i}=n_{\mathrm{p}} \sigma_{i}, \quad \alpha_{j}=n_{\mathrm{p}} \Sigma_{j} \tag{2.27}
\end{equation*}
$$

These expressions pertain to the cross sections presented above; i.e., $i=\mathrm{p}, \mathrm{s}$, or tr and $j=\mathrm{p}$ or $\mathrm{tr} ; n_{\mathrm{p}}$ is the concen-

TABLE I. Optical characteristics of particles with different absorptive properties of the material.

| D | Optical characteristic | $\begin{gathered} n_{2}<10^{-5} \\ \left(\mathrm{Al}_{2} \mathrm{O}_{3}, \mathrm{H}_{2} \mathrm{O}\right) \end{gathered}$ | $n_{2} \sim 0,6$ ( soonl. $)$ |
| :---: | :---: | :---: | :---: |
| D>>1 | $\sigma_{p}$ | 0 | - $\mathrm{pr}_{\mathrm{p}}{ }^{2}$ |
| * | $\sigma_{s}$ | $2 \pi r_{p}^{2}$ | $\pi r_{p}^{p}$ |
| * | $\Sigma_{\text {p }}$ | $2 \pi r_{\text {p }}$ | $2 \pi r_{\text {P }}$ |
| * | $\lambda_{p}$ | 1,0 | 0.5 |
| * | $x\left(\gamma_{s}\right)$ | $\chi_{\text {d }+8}{ }^{(2.26)}$ | $\chi_{\text {d }}{ }^{(2.17)}$ |
| 5 | $\left(\cos \gamma_{\boldsymbol{s}}\right.$ ) | $1-8,8 \cdot 10^{-2}$ | $1-6,8 \cdot 10^{-2}$ |
| 20 | , | $1-5.7 \cdot 10^{-2}$ | $1-4 \cdot 10^{-3}$ |
| 100 | * | $1-5.6 \cdot 10^{-2}$ | $1-2 \cdot 10^{-4}$ |
| 500 | * | $1-5.6 \cdot 10^{-2}$ | 1-6.10 ${ }^{-6}$ |
| 1000 | - | 1-5,6 $\cdot 10^{-2}$ | 1-2.10 ${ }^{-6}$ |
| 1000 | ${ }_{\text {* }}$ | $5.6 \cdot 10^{-2} \sigma_{s}$ | $2 \cdot 10^{-6} \sigma_{3}$ |
| 1000 | $\lambda_{1}$ |  | $2 \cdot 10^{-6}$ |
| $D \sim 1$ | ${ }_{\text {o }}$ | 0 | $\pi r_{p}^{2}$ |
| * | ( $\sigma_{\text {, }}$ ) | $2 \pi r_{p}^{2}$ | $\pi r_{p}^{2}$ |
| - | $\left\langle\Sigma_{p}\right\rangle$ | $2 \pi r_{p}^{2}$ | $2 \pi r_{p}^{2}$ |

tration of particles. If the particles differ only in dimensions and their distribution is $f\left(r_{\mathrm{p}}\right)$ with the normalization

$$
\begin{equation*}
\int_{0}^{\infty} f\left(r_{\mathrm{p}}\right) \mathrm{d} r_{\mathrm{p}}=n_{\mathrm{p}} \tag{2.28}
\end{equation*}
$$

then we have

$$
\begin{align*}
& k_{i}=\int_{0}^{\infty} f\left(r_{\mathrm{p}}\right) \sigma_{i}\left(r_{\mathrm{p}}\right) \mathrm{d} r_{\mathrm{p}}, \\
& \alpha_{j}=\int_{0}^{\infty} f\left(r_{\mathrm{p}}\right) \Sigma_{j}\left(r_{\mathrm{p}}\right) \mathrm{d} r_{\mathrm{p}} . \tag{2.29}
\end{align*}
$$

One can perform the summation of the scattering and extinction cross sections by (2.27) and (2.29) only in the case when the scattering events by the individual particles are independent. A necessary condition is a sufficient separation of the particles from one another. ${ }^{14,15}$ The simplest considerations lead to the following. First, the distance between the particles $n_{\mathrm{p}}^{-1 / 3}$ must be much larger than the wavelength of the radiation; second, the medium must be actually not continuous, but disperse; i.e., the distance between the particles must be substantially larger than the characteristic dimension of the particles. Hence we have

$$
\begin{equation*}
n_{\mathrm{p}}^{-1 / 3} \gg \Lambda, \quad n_{\mathrm{p}}^{-1 / 3} \gg r_{\mathrm{p}} . \tag{2.30}
\end{equation*}
$$

A well grounded treatment of the problem is contained in Ref. 14. The probability of survival of the radiation in a single interaction with the particles can be written in the form

$$
\begin{equation*}
\lambda_{\mathrm{p}}=k_{\mathrm{s}} / \alpha_{\mathrm{p}} . \tag{2.31}
\end{equation*}
$$

When all the particles are identical, then (2.31) differs in no way from (2.5); yet when the particles differ in dimen-
sions and in $k_{\mathrm{s}}$, and $\alpha_{\mathrm{p}}$ is determined by (2.29), then $\lambda_{\mathrm{p}}$ characterizes a certain averaged probability of survival.

In closing we note that the optical coefficients $k$ and $\alpha$ in the treatment that we have conducted do not depend on the direction of the radiation. This condition is fulfilled in a gas containing a condensed dispersed phase when the particles have a more complicated form, but their orientation is random.

### 2.2. Optical characteristics of the gas phase. Comparlson with the characteristics of the macroscopic particles

The fundamental optical characteristic of a gas is the absorption coefficient $k_{\mathrm{a}}$, which depends strongly on the frequency of the light $v$ in a region of spectral lines. Scattering of radiation by atoms and molecules that occurs as a result of photoexcitation with subsequent emission plays no role in the problem being discussed, owing to the assumption made in the Introduction that the transport of radiation exerts no appreciable influence on the populations of the energy levels (see also Sec. 2.3). In an ionized gas one must estimate the optical characteristics of the continuous spectrum caused by free-free and free-bound transitions of electrons, as well as scattering of radiation by electrons.

The absorption coefficient calculated per atom (we omit henceforth the word "molecule" for brevity) $\sigma_{\mathrm{a}}(v)$ has the dimensions of area and can be treated as the absorption cross section of a single atom. By analogy to (2.27) we find

$$
\begin{equation*}
k_{\mathrm{a}}(v)=n_{\mathrm{a}} \sigma_{\mathrm{a}}(v) \tag{2.32}
\end{equation*}
$$

The absolute value of $\sigma_{\mathrm{a}}$ depends on the oscillator strength of the corresponding transition in the atom. The depen-
dence on the frequency of $\sigma_{a}(v)$ in a gas containing a condensed dispersed phase is most often determined by the Doppler broadening and the interaction of the absorbing atom with the surrounding components of the gas. Here we shall not take up the concrete $\sigma_{\mathrm{a}}(v)$ dependences, for an extensive literature is devoted to this (see, e.g., the monographs ${ }^{16-20}$ and the special experimental studies in combustion products ${ }^{21,22}$ ). We shall discuss only the condition that the interaction of an absorbing atom with the particles does not influence the contour of the absorption lines $\sigma_{\mathrm{a}}(v)$. We can make a very simple, crude estimate by comparing the frequency of collision of the atoms with the surrounding components of the gas ( $v_{a-m}$ ) and with the macroscopic particles ( $\nu_{\mathrm{a}-\mathrm{p}}$ ). We shall assume that the relative velocities of the interacting particles in the two cases are the same; then we have

$$
v_{\mathrm{a}-\mathrm{p}} / v_{\mathrm{a}-\mathrm{m}}=n_{\mathrm{p}} \sigma_{\mathrm{a}-\mathrm{p}} / n_{\mathrm{m}} \sigma_{\mathrm{a}-\mathrm{m}}
$$

Here $n_{\mathrm{m}}$ is the concentration of surrounding gas molecules, and $\sigma_{\mathrm{a}-\mathrm{m}}$ and $\sigma_{\mathrm{a}-\mathrm{p}}$ are the cross sections for the interaction of an atom with the surrounding molecules and with the macroscopic particles. One need not take the influence of the particles into account if

$$
\begin{equation*}
n_{\mathrm{p}} \ll n_{\mathrm{m}} \sigma_{\mathrm{a}-\mathrm{m}} / \sigma_{\mathrm{a}-\mathrm{p}} \tag{2.33}
\end{equation*}
$$

Let us study the flow of combustion products at atmospheric pressure and at the temperature 2000 K mentioned in the Introduction. The characteristic dimension of the particles under these conditions is $r_{\mathrm{p}} \approx 1 \mu \mathrm{~m}\left(\sigma_{\mathrm{a}-\mathrm{p}} \approx 10^{-8}\right.$ $\mathrm{cm}^{2}$ ) (Ref. 56) and $n_{\mathrm{m}}=10^{18} \mathrm{~cm}^{-3}$. The cross section of the atom interaction with the surrounding molecules is determined by the value of $\sigma_{\mathrm{a}-\mathrm{m}} \approx 10^{-15} \mathrm{~cm}^{2} .^{21,22}$ Upon substituting these values into (2.33) we find

$$
n_{\mathrm{p}} \ll 10^{11} \mathrm{~cm}^{-3}
$$

This condition practically coincides with the condition of disperseness in (2.30). We shall assume that the inequality (2.33) is fulfilled.

We can write the absorption coefficient in the electronic continuum in the form

$$
k_{\mathrm{e}}=\sigma_{\mathrm{e}} n_{\mathrm{a}}
$$

where $\sigma_{\mathrm{e}}$ is the absorption cross section for light in the continuum, which depends on the temperature of the electrons and the structure of the atoms being ionized, and $n_{a}$ is their concentration. ${ }^{23}$ Analogously we have the following expression for the coefficient of light scattering by electrons:

$$
k_{\mathrm{es}}=\sigma_{\mathrm{es}} n_{\mathrm{e}}
$$

$\sigma_{\text {es }}$ is the cross section for scattering by electrons, and $n_{e}$ is their concentration.

We can approximately estimate the relative role of the particles and of the components of the gas phase by using the following expressions:

$$
\begin{align*}
& \alpha_{\mathrm{p}} / k_{\mathrm{a}}(v)=\Sigma_{\mathrm{p}} n_{\mathrm{p}} / \sigma_{\mathrm{a}}\left(v \left(n_{\mathrm{a}},\right.\right. \\
& \alpha_{\mathrm{p}} / k_{\mathrm{e}}(v)=\Sigma_{\mathrm{p}} n_{\mathrm{p}} / \sigma_{\mathrm{e}} n_{\mathrm{a}},  \tag{2.34}\\
& k_{\mathrm{s}} / k_{\mathrm{es}}=\sigma_{\mathrm{s}} n_{\mathrm{p}} / \sigma_{\mathrm{es}} n_{\mathrm{e}} .
\end{align*}
$$

The first ratio characterizes the role of the particles in the region of spectral lines, the second in the continuum, and finally, the third characterizes the relative role of the particles and the electrons in the scattering process.

For estimates of the role of macroscopic particles in a region of spectral lines and in the continuum, we should concretize the conditions. Again let us study a radiating flow of combustion products at atmospheric pressure and at the temperature 2000 K . We can find from the experimental data of Ref. 56: $n_{\mathrm{p}}=10^{5} \mathrm{~cm}^{-3}$ for $r_{\mathrm{p}}=1 \mu \mathrm{~m}$. The combustion products include atomic sodium, which emits one of the brightest lines of the visible spectrum of NaI, 589.0 nm . This line is often used in spectral diagnostics. The natural content of sodium atoms in combustion products usually does not exceed $n_{\mathrm{a}} \approx 10^{12} \mathrm{~cm}^{-3}$. The cross section $\sigma_{a}(v)$ for not too great deviations of the wavelength $\Lambda$ from the wavelength $\Lambda_{0}$ of the center of the line is described by the Voigt integral. By using information from the books, Refs. 19 and 20, we can find in the center of the line that $\sigma_{a}\left(\Lambda_{0}\right)=2 \times 10^{-12} \mathrm{~cm}^{2}$, while $\sigma_{\mathrm{a}}(\Lambda)=10^{-1} \mathrm{~cm}^{2}$ for $\left|\Lambda-\Lambda_{0}\right|=0.2 \mathrm{~nm}$. If we assume that $\Sigma_{p} \approx r_{p}^{2}=10^{-8} \mathrm{~cm}^{2}$, we obtain that the first ratio in (2.34) varies over the following range:

$$
\alpha_{\mathrm{p}} / k_{\mathrm{a}}(v)=10^{-3}-1
$$

Consequently, at the center of the strong sodium line the influence of the particles is small, while with distance from the center it becomes substantial. Evidently the influence of the particles on less strong lines will be important even in the central parts of lines. Let us estimate under the conditions of Ref. 56 the role of the particles in the continuum at $\Lambda=0.6 \mu \mathrm{~m}$. We shall assume that sodium atoms are being ionized in the combustion products. Then by the UnsoldKramers formula ${ }^{23}$ we find $\sigma_{e}=10^{-24} \mathrm{~cm}^{2}$ and obtain $\alpha_{\mathrm{p}} / k_{\mathrm{e}}(\nu)=10^{-8} \cdot 10^{5} / 10^{-24} \cdot 10^{12}=10^{9}>1$. We see that the electrons play no role here. Even smaller is the role of scattering of radiation by electrons according to (2.34), since the cross section of this process also is very small ( $\sigma_{\text {es }} \approx 6.6 \times 10^{-25} \mathrm{~cm}^{2} 19$ ), while the concentration of electrons is usually smaller than $n_{\mathrm{a}}$.

As a second example let us study the above-mentioned brightly glowing layer near the electrode of a high-current gas discharge. According to the data of the book, Ref. 24, we can obtain an estimate of the content and the dimensions of the particles near the surface of the electrode: $n_{\mathrm{p}} \approx 10^{8} \mathrm{~cm}^{-3}, r_{\mathrm{p}} \approx 1 \mu \mathrm{~m}$. This case differs from the previous one mainly in the fact that the number of particles here is $10^{-3}$ times as much. Correspondingly the influence of the particles increases, both in the spectral lines and in the continuum. As we go away from the surface, the particles disappear, but in the immediate vicinity of the surface they completely determine the optical characteristics of the medium.

Thus in the discussed examples the extinction coefficient of the particles $\alpha_{p}$ is much larger than the coefficients $k_{\mathrm{e}}$ and $k_{\text {es }}$ in the continuum. Therefore we can neglect the electronic continuum. In the region of spectral lines, where $k_{\mathrm{a}}(v) \neq 0$, we must jointly take into account the atoms and the macroscopic particles. In these regions the total extinction coefficient is obtained by summation:

$$
\begin{equation*}
\alpha(v)=\alpha_{\mathrm{p}}+k_{\mathrm{a}}(v) \tag{2.35}
\end{equation*}
$$

Here $\alpha(v)$ strongly depends on the frequency owing to the $k_{\mathrm{a}}(v)$ relationship. On the other hand, one can usually assume that inside the region of a spectral line the characteristics of the particles ( $k_{\mathrm{p}}, k_{\mathrm{s}}$, and $\alpha_{\mathrm{p}}$ ) do not depend on the frequency. We obtain for the probability of survival

$$
\begin{equation*}
\lambda(v)=k_{\mathrm{s}} / \alpha(v)=\lambda_{\mathrm{p}} \alpha_{\mathrm{p}} / \alpha(v) \tag{2.36}
\end{equation*}
$$

As we approach the centers of the lines, the scattering probability $\lambda(v)$ decreases owing to the increase in $k_{\mathrm{a}}(v)$.

### 2.3. Primary thermal emission of a gas containing a condensed dispersed phase

In a gas containing a condensed dispersed phase the intrinsic primary emission usually depends on the temperature of the gas and of the macroscopic particles. The temperatures of the gaseous and dispersed phases can differ. The temperature of the surface of the particles $\left(T_{p}\right)$ and their intrinsic primary emission depend on the processes of heat exchange, wherein radiative heat exchange plays an important role. When in diagnostics one uses simply part of the radiation naturally escaping from the volume, this in no way affects the temperature of the particles. Yet if one passes light through the volume of study for diagnostic purposes from an external emitter or from the walls surrounding this volume, or special apertures are made for extracting part of the radiation incident on the wall, this must be done with certain precautions. The light from an external source (often a laser) must not heat the particles, while the escape of intrinsic radiation must not cool them substantially. Both of these conditions are usually not too hard to fulfill, by making the power of the source and the apertures in the walls sufficiently small.

The primary intrinsic emission in the spectral lines of the gas involves the temperature of excitation $T_{a}$, which determines the relative population of the energy levels among which the radiative transitions occur. In a gas containing a condensed dispersed phase the populations most often are established in collisions of an atom with surrounding atoms and molecules of the gas phase. When both the excitation and depletion of the levels arises from collisions, the character of the collisions determines the physical meaning of the temperature $T_{\mathrm{a}}$. Thus, for example, in combustion products at atmospheric pressure the temperature of excitation $T_{\mathrm{a}}$ of alkali atoms is determined by the temperature of vibrational excitation of the molecules. ${ }^{4}$ Consequently $T_{\mathrm{a}}$ can generally differ from the temperature of thermal motion of the atoms and molecules of the gas.

Escape of radiation, external irradiation, and spatial inhomogeneities can lead to deviations from the collisional
populations. Let the rate of quenching by atoms and molecules of the upper level of the transition being discussed ( $v_{\mathrm{q}}$ ) be substantially larger than the rate of spontaneous emission ( $\nu_{r}$ ):

$$
\begin{equation*}
v_{\mathrm{q}} \gg v_{\mathrm{r}} \tag{2.37}
\end{equation*}
$$

When this condition is fulfilled, escape of radiation does not appreciably affect the populations of the levels. Here we assume that condition (2.37) is fulfilled, while the power of the external source is so small that photoexcitation by its radiation also does not affect the populations.

Let us examine the possible influencces of spatial inhomogeneity on the populations of levels by atoms. A very simple equation of balance of atoms on a level with account taken of the fulfillment of condition (2.37) has the form

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{dr}}\left(n_{m} \bar{u}_{m}\right)+\left(v_{\mathrm{ex}}-v_{\mathrm{q}}\right) n_{m}=0 \tag{2.38}
\end{equation*}
$$

Here $n_{m}$ is the concentration of atoms in level $m, \bar{u}_{m}$ is the directional velocity arising either from the flow of the gas mixture or from diffusion, $v_{\mathrm{ex}}$ is the rate of collisional excitation of level $m$. The magnitude of $n_{m}$ is determined only by collisions if the first term is small in comparison with the rest. One obtains from (2.38) the characteristic length of the inhomogeneities, in which the first term plays the same role as one of the collisional terms:

$$
l_{m}=\bar{u}_{m} v_{\mathbf{q}}^{-1}
$$

In the case of a diffusional flux we find

$$
n_{m} \bar{u}_{m}=-D_{m} \mathrm{~d} n_{m} / \mathrm{dr}
$$

Here $D_{m}$ is the diffusion coefficient of excited atoms. Whei: the concentration $n_{m}$ varies substantially at distances equal to or less than $l_{m}$, we can expect deviations from the "collisional" populations. This defines the smallest dimension of a region $\left(l_{\mathrm{a}}\right)$ that can be characterized by a single atomic temperature. Namely, the condition must be satisfied that

$$
l_{\mathrm{a}} \geqslant l_{m} .
$$

If all the described conditions are fulfilled, then we can apply the Kirchhoff law locally in space to each of the phases of the discussed two-temperature medium. From an elementary volume of thickness $\left|\mathrm{dr} r^{\prime}\right|$ primary radiation escapes in all directions here with the intensity

$$
\begin{equation*}
\left(k_{\mathrm{a}} I_{\mathrm{a}}^{0}+k_{\mathrm{p}} I_{\mathrm{p}}^{0}\right)|\mathrm{d} \mathrm{r}| \tag{2.39}
\end{equation*}
$$

Here we have

$$
\begin{equation*}
I^{0}(v, T)=\frac{1}{\exp (h v / k T)-1} \tag{2.40}
\end{equation*}
$$

while $I_{a}^{0}$ and $I_{\mathrm{p}}^{0}$ are the intensities of the emission from an absolutely black body at the temperature of the gas ( $T_{\mathrm{a}}$ ) and the particles ( $T_{\mathrm{p}}$ ), respectively, $k_{\mathrm{a}}$ and $k_{\mathrm{p}}$ are the absorption coefficients of the atoms and the particles, $v$ is the frequency of the radiation, $h$ and $K$ are the Planck and Boltzmann constants, and $c$ is the velocity of light in a vacuum. The expressions (2.39) and (2.40) determine the relation between the primary radiation and the temperatures $T_{\mathrm{a}}$ and $T_{\mathrm{p}}$ by suitable measurements. Moreover, the
absolute values of the coefficients $k_{\mathrm{a}}$ and $k_{\mathrm{p}}$ are proportional to the concentrations of the absorbing components of the gas and of the macroscopic particles. The dependence of $k_{\mathrm{a}}$ on the frequency is determined by processes of broadening of the spectral lines. In turn, the broadening depends both on the temperature of the gas and on the concentrations of the components of the gaseous medium. Therefore the primary intensity in (2.40) at different frequencies contains in itself varied information on the gas and the particles. The extraction of this information from the observed intensities of the radiation is the problem of spectral diagnostics.

In closing we briefly summarize the general information.

1. When the diffraction parameter is small ( $D \ll 1$ ), the absorption and scattering cross sections are much smaller than the geometric cross section of the particle, The probability of scattering here is also small: $\lambda_{\mathrm{p}} \approx D^{3}<1$. Yet if $D \approx 1$ or $D>1$, the optical cross sections are determined in order of magnitude by the geometric cross section, the probability of scattering varies over the small range $\lambda_{p} \approx 0.5-1$, and the scattering indicatrix is extended forward as the diffraction parameter increases.
2. In the continuous spectrum of a gas containing a condensed dispersed phase, the extinction coefficients in characteristic cases are determined in practice only by the particles of the dispersed phase. In the region of spectral lines the extinction coefficients of the macroscopic particles and the gas can be comparable in order of magnitude.
3. A gas containing a condensed dispersed phase can be treated as a two-temperature medium. Here the primary radiation is described by the Kirchhoff law as applied separately to the gas and to the macroscopic particles.

## 3. RADIATION TRANSPORT

The fundamental characteristic of the radiation in many problems of spectral diagnostics is the intensity, i.e., the energy flux per unit intervals of area, time, solid angle, and frequency. The intensity inside the emitter and at its output is determined by solving the transport equations of the radiation, which have their meaning in the geometricoptics approximation. The possibility of using this approximation in a gas containing scattering particles is discussed in detail in Ref. 14. Here we note only that one must in any case take account of the main condition: the mean free path of a photon

$$
\begin{equation*}
l_{\mathrm{ph}}=\alpha^{-1} \tag{3.1}
\end{equation*}
$$

must be substantially greater than the wavelength $\Lambda$. In the centers of spectral lines, where $\sigma_{\mathrm{a}}\left(\boldsymbol{v}_{0}\right)$ can attain large values, geometric optics is applicable if the concentrations $n_{a}$ of atoms are not too great. An analysis of the data presented in Ref. 15 implies that in diagnostics of a gas containing a condensed dispersed phase, as a rule, one need not take account of refraction in the propagation of the radiation. And finally, here we shall not discuss effects of polarization of the radiation, since they exert little influence on the overall intensity, ${ }^{15}$ while diagnostics based on polarization effects is not the topic of this study.


FIG. 1. Diagram of an emitter of arbitrary shape.

### 3.1. Transport equations of radiation and the primary surface sources

We shall initially discuss the radiation transport equations in a rather general case. Let the gas volume containing the condensed dispersed phase be bounded by an arbitrary convex surface (Fig. 1). The medium can be optically inhomogeneous, i.e., the optical characteristics can vary in space. Let $I(\mathbf{r}, \vec{\Omega})$ be the intensity of the radiation at the point of radius vector $r$ in the direction of the unit vector $\vec{\Omega}$. For brevity we shall omit the designation of the frequency $v$. The steady-state transport equation has the form: ${ }^{25}$

$$
\begin{equation*}
\vec{\Omega} \nabla I(\mathbf{r}, \vec{\Omega})+\alpha(r) I(\mathbf{r}, \vec{\Omega})^{\prime}=\alpha(r) \varepsilon(\mathbf{r}, \vec{\Omega}) \tag{3.2}
\end{equation*}
$$

Equation (3.2) is the equation of balance of the radiant energy per unit volume, $\alpha I$ is the loss and $a \varepsilon$ the gain of energy; here $\varepsilon$ is a function of the sources:

$$
\begin{align*}
\varepsilon(\mathbf{r}, \vec{\Omega})= & g_{v}(\mathbf{r}, \vec{\Omega}) \\
& +\int_{4 \pi} I\left(\mathbf{r}, \vec{\Omega}^{\prime}\right) \lambda(\mathbf{r}) \chi\left(\mathbf{r}, \vec{\Omega}^{\prime} \rightarrow \vec{\Omega}\right) \mathrm{d} \omega^{\prime} / 4 \pi \tag{3.3}
\end{align*}
$$

The meaning of $\varepsilon$ is the accession of energy per length of ray tube equal to the mean free path of a photon in (3.1) owing the primary emission of the medium $\left(g_{v}\right)$ and owing to scattering of the radiation that has arrived from all directions (the second term on the right-hand side of (3.3). We obtain from Eq. (3.2) and the boundary conditions that fix the radiation intensity of the surface $I\left(r_{s}, \vec{\Omega}\right)$ the following:

$$
\begin{align*}
I(\mathbf{r}, \vec{\Omega})= & \int_{\mathbf{r}_{S \downarrow!}}^{\mathbf{r}} \varepsilon\left(\mathbf{r}^{\prime \prime}, \vec{\Omega}\right) \exp \left(-\tau\left(\mathbf{r}^{\prime \prime} \rightarrow \mathbf{r}\right)\right) \alpha\left(\mathbf{r}^{\prime \prime}\right) \mathrm{d} \mathbf{r}^{\prime \prime} \\
& +I\left(\mathbf{r}_{S \uparrow \downarrow}, \vec{\Omega}\right) \exp \left[-\tau\left(\mathbf{r}_{S \uparrow \downarrow} \rightarrow \mathbf{r}\right)\right] \tag{3.4}
\end{align*}
$$

Here $\mathbf{r}_{S_{+\downarrow}}$ is the radius vector of the point of intersection of the surface $S$ with the straight line passing through the point $\mathbf{r}$ in the direction ( $-\vec{\Omega}$ ) (see Fig. 1); the integration is performed along this straight line;

$$
\begin{equation*}
\tau\left(\mathbf{r}_{1} \rightarrow \mathbf{r}_{2}\right)=\int_{\mathrm{r}_{1}}^{\mathbf{r}_{2}} \alpha(\mathbf{r}) \mathrm{dr} \tag{3.5}
\end{equation*}
$$

is the optical density between the points $r_{1}$ and $r_{2}$, and the integration is performed along the straight line $\mathbf{r}_{1}-r_{2}$. In solving Eqs. (3.3) and (3.4), either $I$ is determined from (3.4), from which $\varepsilon$ is eliminated by using (3.3), or conversely, $I$ is eliminated from (3.3) by using (3.4), and one starts by solving the equation for $\varepsilon$. Most of the fundamental studies have been devoted to solving the equation for $\varepsilon$. The equation for $\varepsilon$ can be also written directly on the basis of physical considerations analogously to the way that this was done in Ref. 26 in describing neutron transport, or in Ref. 27 in describing radiation transport in a case more general than the present one. The result has the form

$$
\begin{equation*}
\varepsilon(\mathbf{r}, \vec{\Omega})=V \varepsilon\left(\mathbf{r}^{\prime}, \vec{\Omega}^{\prime}\right)+g(\mathbf{r}, \vec{\Omega}) \tag{3.6}
\end{equation*}
$$

Here $V \varepsilon$ is the scattering integral (operator) that determines the scattering at the point $\mathbf{r}$ of radiation arriving from all parts of the volume; $g(\mathbf{r}, \vec{\Omega})$ is the intensity of the primary sources; and here we have

$$
\begin{align*}
V \varepsilon\left(\mathbf{r}^{\prime}, \vec{\Omega}^{\prime}\right)= & \frac{\lambda(\mathbf{r})}{4 \pi} \int_{v} \mathrm{~d}^{3} \mathbf{r}^{\prime}\left\{\varepsilon\left(\mathbf{r}^{\prime}, \overrightarrow{\Omega^{\prime}}\right) \alpha\left(\mathbf{r}^{\prime}\right)\right. \\
& \left.\times \exp \left[-\tau\left(\mathbf{r}^{\prime} \rightarrow \mathbf{r}\right)\right] \chi\left(\mathbf{r}, \vec{\Omega}^{\prime} \rightarrow \vec{\Omega}\right)\left(\mathbf{r}^{\prime}-\mathbf{r}\right)^{-2}\right\}, \tag{3.7}
\end{align*}
$$

$g(\mathbf{r}, \vec{\Omega})=g_{v}(\mathbf{r}, \vec{\Omega} Q)+g_{S}(\mathbf{r}, \vec{\Omega})$,

$$
\begin{align*}
g_{S}(\mathbf{r}, \vec{\Omega})= & \frac{\lambda(\mathbf{r})}{4 \pi} \int_{S} \mathrm{~d}^{2} \mathbf{r}_{S}^{\prime} I\left(\mathbf{r}_{S}^{\prime}, \vec{\Omega}_{S}^{\prime}\right) \cos \left(\mathbf{n}_{S}^{\prime} \vec{\Omega}_{S}^{\prime}\right)  \tag{3.8}\\
& \times \exp \left[-\tau\left(\mathbf{r}_{S}^{\prime} \rightarrow \mathbf{r}\right)\right] \chi\left(\mathbf{r}, \vec{\Omega}_{S}^{\prime} \rightarrow \vec{\Omega}\right)\left(\mathbf{r}_{S}^{\prime}-\mathbf{r}\right)^{-2} . \tag{3.9}
\end{align*}
$$

Here $\mathrm{d}^{3} \mathbf{r}^{\prime}$ and $\mathrm{d}^{2} \mathbf{r}_{S}^{\prime}$ are respectively the elements of volume and surface area. We note that here, in addition to $g_{v}$, in the primary sources that enter into (3.3) appeared the intensity of the primarily scattered radiation of the surface $g_{S}$ of (3.9). Let us explain the meaning of $g_{S}$ : from an element of surface area $d^{2} \mathbf{r}_{S}^{\prime}$, the flux $I\left(\mathbf{r}_{S}^{\prime}, \vec{\Omega}_{S}^{\prime}\right) \mathrm{d}^{2} \mathbf{r}_{S}^{\prime} \cos \left(\mathbf{n}_{S} \vec{\Omega}_{S}^{\prime}\right) /\left(\mathbf{r}_{S}^{\prime}-\mathbf{r}\right)^{2}$ proceeds toward a unit area in the vicinity of $\mathbf{r}$, the fraction $\exp \left[-\tau\left(\mathbf{r}_{S}^{\prime} \rightarrow \mathbf{r}\right)\right]$ arrives at $\mathbf{r}$ without interacting with the medium, and the first scattering event at $\mathbf{r}$ in the direction $\vec{\Omega}$ at the length $\alpha^{-1}$ occurs with probability $\lambda(\mathbf{r}) \chi\left(\mathbf{r}, \vec{\Omega}_{S}^{\prime} \rightarrow \vec{\Omega}\right)$. Taking account of what we have described and integrating over $S$ yields Eq. (3.9). Analogously one can also explain the meaning of the scattering integral in (3.7). The difference is only that the initial flux in the scattering integral is a volume flux ( $\mathrm{d}^{3} \mathbf{r}^{\prime} \alpha\left(\mathbf{r}^{\prime}\right) \varepsilon\left(\mathbf{r}^{\prime}, \vec{\Omega}^{\prime}\right)$ ), and the integration is performed over the volume. The expression for the intensity of the intrinsic primary sources $g_{v}$ in the two-
temperature medium being studied (a gas containing a condensed dispersed phase) is written by using (2.31), (2.35), and (2.39):

$$
\begin{align*}
& g_{\nu}(\mathbf{r})=g_{\mathrm{a}}(\mathbf{r})+g_{\mathrm{p}}(\mathbf{r})  \tag{3.10}\\
& g_{\mathrm{a}}(\mathbf{r})=k_{\mathrm{a}}(\mathbf{r}) I_{\mathrm{a}}^{0}(\mathbf{r}) / \alpha(\mathbf{r}) \\
& g_{\mathrm{p}}(\mathbf{r})=\alpha_{\mathrm{p}}(\mathbf{r})\left(1-\lambda_{\mathrm{p}}(\mathbf{r})\right) I_{\mathrm{p}}^{0}(\mathbf{r}) / \alpha(\mathbf{r})
\end{align*}
$$

In a gas containing a condensed dispersed phase the primary volume sources are composed of sources arising from the emission from atoms ( $g_{\mathrm{a}}$ ) and the emission from particles ( $g_{\mathrm{p}}$ ). We only must emphasize that the existence of any of the components influences the magnitude of the primary sources of the other component via the overall extinction coefficient, since $\alpha(r)=\alpha_{p}(r)+k_{\mathrm{a}}(r)$. The thermal emission is isotropic; therefore $g_{v}$ does not depend on $\vec{\Omega}$. Outside the spectral lines, where $k_{\mathrm{a}}=0$, we have

$$
\begin{equation*}
g_{\nu}(\mathbf{r})=g_{\mathrm{p}}(\mathbf{r})=\left(1-\lambda_{\mathrm{p}}(\mathbf{r})\right) / I_{\mathrm{p}}^{0}(\mathbf{r}) \tag{3.11}
\end{equation*}
$$

When the temperatures of the gas and of the particles are the same at each point, i.e., $I_{\mathrm{a}}^{0}(\mathrm{r})=I_{\mathrm{p}}^{0}(\mathbf{r})=I^{0}(\mathbf{r})$, we find the following expression with account taken of (2.36):

$$
\begin{equation*}
g_{v}(\mathbf{r})=(1-\lambda(\mathbf{r})) I^{0}(\mathbf{r}) \tag{3.12}
\end{equation*}
$$

To concretize the scattering operator $V \varepsilon$ and the intensity of the primary scattering $g_{S}$, we must fix the form of the surface $S$, the scattering indicatrix $\chi$, and the distribution $I\left(\mathbf{r}_{S}^{\prime}, \vec{\Omega}_{S}^{\prime}\right)$. Let us study isotropic scattering ( $\chi=1$ ) in three very simple geometries: an infinite plane layer, an infinite cylinder, and a sphere. We shall assume that the surface emits in the same way everywhere isotropically: $I\left(\mathbf{r}_{S}^{\prime}\right)=I_{S 0}$. To find the distances $\mathbf{r}^{\prime}-\mathbf{r}, \mathbf{r}_{S}^{\prime}-\mathbf{r}$, the volume elements $d^{3} r^{\prime}$, and the area elements of the surface $d^{2} r_{S}^{\prime}$ in the plane layer and the cylinder, we can use Figs. 2 and 3. Then we should substitute the values that are found into (3.7) and (3.9) and perform the necessary integrations. In the plane layer as a result of these operations we have
$V \varepsilon\left(\mathbf{r}^{\prime}, \vec{\Omega}^{\prime}\right)=V \varepsilon\left(t^{\prime}\right)=0.5 \int_{0}^{t_{0}} \varepsilon\left(t^{\prime}\right) E_{1}\left(\left|t-t^{\prime}\right|\right) \lambda\left(t^{\prime}\right) \mathrm{d} t^{\prime}$,
$g_{S}(t)=g_{S \Sigma}(t)=g_{S 0}(t)+g_{S_{0}}(t)$,
$g_{S O}=0.5 I_{S O} \lambda(t) E_{2}(t)$,
$g_{S_{0}}=0.5 I_{S 0} \lambda(t) E_{2}\left(t_{0}-t\right)$.
Here $t$ and $t_{0}$ are the optical coordinate and the optical density of the layer (see Fig. 2):

$$
\begin{equation*}
t=\int_{0}^{X} \alpha(X) \mathrm{d} X, \quad t_{0}=\int_{0}^{l} \alpha(X) \mathrm{d} X \tag{3.15}
\end{equation*}
$$

$E_{k}(y)$ is an integral exponential function: ${ }^{28}$

$$
\begin{equation*}
E_{k}(y)=\int_{0}^{1} \exp (-y / z) z^{k-2} \mathrm{~d} z \tag{3.16}
\end{equation*}
$$

Also, $g_{S 0}, g_{S_{0}}$, and $g_{S}$ are the surface sources due to the primary scattering of the radiation by the surfaces $t=0$, $t=t_{0}$, and the two surfaces together.


FIG. 2. Diagram of a plane layer.

For an infinite, optically homogeneous cylinder ( $\alpha$ $=$ const, $\lambda=$ const) we assume that

$$
\begin{equation*}
x=\rho / R, \quad t_{0}=2 \alpha R \tag{3.17}
\end{equation*}
$$

Then we obtain

$$
\begin{equation*}
V \varepsilon\left(\mathbf{r}^{\prime}, \vec{\Omega}^{\prime}\right)=V \varepsilon(x)=\left(\lambda t_{0} / 2 \pi\right) \int_{0}^{1} x^{\prime} \varepsilon\left(x^{\prime}\right) J\left(x, x^{\prime}\right) \mathrm{d} x^{\prime} \tag{3.18}
\end{equation*}
$$



FIG. 3. Diagram of a cylindrical emitter.

$$
\begin{align*}
J\left(x, x^{\prime}\right)= & \int_{0}^{\pi} \mathrm{d} \varphi^{\prime}\left[K _ { i 1 } \left(0 . 5 t _ { 0 } \left(x^{2}+x^{\prime 2}\right.\right.\right. \\
& \left.\left.-2 x x^{\prime} \cos \varphi^{\prime}\right)^{1 / 2}\right)\left(x^{2}+x^{\prime 2}\right. \\
& \left.\left.-2 x x^{\prime} \cos \varphi^{\prime}\right)^{-1 / 2}\right]  \tag{3.19}\\
g_{S}(x)= & \frac{I_{S 0} \lambda}{\pi} \int_{0}^{\pi} \mathrm{d} \varphi_{S}^{\prime}\left[\left(1-x \cos \varphi_{S}^{\prime}\right)\right. \\
& \left.\times \frac{K_{i 2}\left(0.5 t_{0}\left(1+x^{2}-2 x \cos \varphi_{S}^{\prime}\right)^{1 / 2}\right)}{1+x^{2}-2 x \cos \varphi_{S}^{\prime}}\right]
\end{align*}
$$

Here $K_{i k}$ is the $k$-fold integral of the zero-order cylinder function. As the arguments vary from 0 to $\infty$, the integrals $K_{i 1}$ and $K_{i 2}$ monotonically decline to zero from $\pi / 2$ in the case of $K_{i 1}$, and from $I$ in the case of $K_{i 2} \cdot{ }^{28}$ Finally, in an optically homogeneous sphere we can analogously find

$$
\begin{align*}
V \varepsilon\left(x^{\prime}\right)= & 0.5 \lambda t_{0} \int_{0}^{1} \varepsilon\left(x^{\prime}\right) x^{\prime} \mathrm{d} x^{\prime}\left(E_{1}\left(0.5 t_{0}\left|x-x^{\prime}\right|\right)\right. \\
& \left.\quad-E_{1}\left(0.5 t_{0}\left(x+x^{\prime}\right)\right)\right)  \tag{3.21}\\
g_{S}= & \left(I_{S 0} \lambda / 4 \pi\right)\left[(1+x) E_{2}\left(0.5 t_{0}(1-x)\right)\right. \\
& -(1-x) E_{2}\left(0.5 t_{0}(1+x)\right)+\left(2 / t_{0}\right) \\
& \left.\times\left\{\exp \left[-0.5 t_{0}(1-x)\right]-\exp \left[-0.5 t_{0}(1+x)\right]\right\}\right] \tag{3.22}
\end{align*}
$$

Thus the equations for describing radiation transport in a plasma containing a condensed dispersed phase have been presented in general form. In the special cases of isotropic scattering and simple geometric shapes, the scattering integrals and the primary sources $g_{S}$ due to scattering of the emission from the surfaces have been obtained. The scattering integrals $V \varepsilon$ in the particular cases of (3.13), (3.18), and (3.21) do not differ from the known values. ${ }^{29}$

### 3.2. Components of the equillbrium emission of a closed isothermal cavity filled with a gas containing a condensed dispersed phase

Equation (3.6) implies that a function of the sources arising from the summation of certain primary sources
equals the sum of the functions of the sources corresponding to each of the primary sources being added. We can write this in the form

$$
\begin{equation*}
\varepsilon\left(\sum_{i} g_{i}\right)=\sum_{i}\left(\varepsilon\left(g_{i}\right)\right) \tag{3.23}
\end{equation*}
$$

Here we have omitted all the arguments of the function $\varepsilon$, while only the dependence of $\varepsilon$ on $g$ is emphasized. The well known additivity of (3.23) arises from the linearity of the transport equation (3.2), and hence, of Eq. (3.6). Equation (3.23) also implies that the integral term in the expression for the intensity (3.4) is also summed in the summation of the primary sources. We emphasize that the emitter can be arbitrary in shape, optically inhomogeneous, and can scatter anisotropically. We shall employ the general property of additivity to find the relationships between certain functions of the sources $\varepsilon$ and to determine the terms in the equilibrium radiation in the case in which the emitter is a thermal one surrounded by the closed surface $S$, while the temperature of the gas, of the particles, and of the surface are the same and invariant within the cavity. In this case the emitter amounts to a model of a black body. In such an emitter, as is implied by general physical considerations, the intensity of the radiation everywhere inside the cavity is the same, is isotropic, and equals the intensity of the radiation of an absolutely black body:

$$
\begin{equation*}
I=I^{0} \tag{3.24}
\end{equation*}
$$

Moreover, we obtain from (3.2) for $\alpha \neq 0$ and constant $I$ :

$$
\begin{equation*}
\varepsilon=I=I^{0} \tag{3.25}
\end{equation*}
$$

When $\alpha=0$ the value of $\varepsilon$ can be an arbitrary finite quantity, but since the total effect of input of energy is a zero effect, we can assume that also in this case (3.25) holds.

The primary sources are determined by the intrinsic thermal emission $g_{v}$ of (3.12) and by the intensity of the primary scattered radiation $g_{S}$ according to (3.9) with

$$
\begin{equation*}
I\left(\mathbf{r}^{\prime}, \vec{\Omega}_{S}^{\prime}\right)=I_{S O}=I^{0} \tag{3.26}
\end{equation*}
$$

Let $\varepsilon_{v}$ and $\varepsilon_{S}$ be functions of the sources, which are the solution of Eq. (3.6) for $g=g_{v}$ and $g=g_{S}$, respectively. The total source function is determined according to (3.23) and satisfies the expression (3.25). That is, we have

$$
\begin{equation*}
\varepsilon=I^{0}=\varepsilon_{v}+\varepsilon_{S} \tag{3.27}
\end{equation*}
$$

or

$$
\begin{equation*}
1=\frac{\varepsilon_{v}}{I^{0}}+\frac{\varepsilon_{S}}{I_{S 0}} \tag{3.28}
\end{equation*}
$$

The equations (3.27) and (3.28) amount to the conditions for radiative equilibrium inside the isothermal cavity. These equations establish the connection between the source functions $\varepsilon_{v}$ and $\varepsilon_{S}$ under very general assumptions on the optical characteristics $\alpha, \lambda$, and $\chi\left(\gamma_{S}\right)$. At the same time they make it possible to represent the equilibrium source function in the form of the sum of two terms of
different origins: one arises from the primary volume emitters $\left(g_{v}\right)$, and the other from the primary surface sources ( $g_{S}$ ).

We emphasize that here the scattering is taken into account only in the volume. Incidence of light on the wall removes it from the treatment; its subsequent fate is not important, since it is replaced by the assigned radiation intensity $\Gamma^{0}$. This approach is analogous to that adopted in studying the absorption of light by atoms; the emission from the atoms was described independently of this absorption.

Let us use (3.27) to determine the terms of the intensity $I^{0}$ of the radiation leaving the cavity (or incident from inside on the surface). To do this, we must substitute $\varepsilon=\varepsilon_{v}+\varepsilon_{S}$ in (3.4). Let $I_{v}$ be the intensity arising only from the primary volume sources $g_{v}$, and correspondingly from the source function $\varepsilon_{v} ; I_{S}$ arises only from the primary surface sources, and correspondingly from the source function $\varepsilon_{S}$. For the sake of definiteness we shall study the yield of radiation at the point $\mathbf{r}_{S}$ of the surface $S$ (see Fig. 1). That is, we assume in (3.4) that $\mathbf{r}=\mathbf{r}_{S}$. Then, instead of (3.4) we find

$$
\begin{equation*}
I=I^{0}=I_{v}+I_{S}+I^{0} \exp \left[-\tau\left(\mathbf{r} \rightarrow \mathbf{r}_{S_{\uparrow}}\right]\right. \tag{3.29}
\end{equation*}
$$

Here $\tau\left(\mathbf{r}_{S} \rightarrow \mathbf{r}_{S \dagger \downarrow}\right)$ is the optical thickness of the cavity in the direction of observation. The intensity of emission of a gas containing a condensed dispersed phase, in view of the classification of the primary sources into atomic sources and radiation sources of particles of the dispersed phase in (3.1) can also be represented in the form of the sum

$$
I_{v}=I_{v a}+I_{v p}
$$

Then, instead of (3.29) we obtain the following with account taken of

$$
\begin{align*}
& I_{a}^{0}=I_{p}^{0}=I_{S 0} \\
& I^{0}=I_{v a}+I_{v \mathrm{p}}+I_{S}+I^{0} \exp \left[-\tau\left(\mathbf{r}_{S} \rightarrow \mathbf{r}_{S \uparrow \downarrow}\right)\right],  \tag{3.30}\\
& \frac{I_{v \mathrm{a}}}{I_{\mathrm{a}}^{0}}+\frac{I_{v \mathrm{p}}}{I_{\mathrm{p}}^{0}}+\frac{I_{S}}{I_{S 0}}=1-\exp \left[-\tau\left(\mathbf{r}_{S} \rightarrow \mathbf{r}_{S \uparrow \downarrow}\right)\right]
\end{align*}
$$

Each term on the left-hand side of the last equation is determined only by the optical characteristics in agreement with (3.12), (3.9), and (3.10). The last terms on the righthand sides in (3.29) and (3.30) are the intensities of the radiation of the opposite wall that has reached the observer without interacting with the medium. Each of the terms in the intensity, just like the terms in the source functions $\varepsilon$, can be calculated separately, as will be used below.

Of course, the obtained resolutions into components of (3.28) and (3.29) are valid in the special cases discussed above of isotropic scattering, where the scattering operators and the primary surface sources were determined in a plane layer by the expressions (3.13) and (3.14), in a cylinder by (3.18) and (3.20), and in a sphere by (3.21) and (3.22). An equation similar to (3.28) was derived mathematically in the special case of a plane layer and of a homogeneous, isotropically scattering medium. ${ }^{30}$ In Ref. 31 the physical meaning of the equation was elucidated.

Let us take up the question of what the anisotropy of scattering and inhomogeneity of the medium in a isothermal cavity is manifested in. Equation (3.27) implies that, although individually the primary scattering at $\mathbf{r}^{\prime}$ of a surface emitter and the scattering of the light that has arrived at $\mathbf{r}^{\prime}$ from the volume can be anisotropic, the overall scattering of light at $\mathbf{r}^{\prime}$ is isotropic. This becomes clear if we recall that the light coming from a bounded surface $S$ is equivalent to the light coming from an unbounded isothermal medium that might lie outside the cavity. The total light reaching $\mathbf{r}$ is isotropic, while anisotropy of scattering cannot alter this property. The fraction of the light lost (absorbed) at $\mathbf{r}$ is ( $1-\lambda(\mathbf{r})$ ), and at equilibrium it exactly is compensated by the corresponding thermal emission in (3.12).

The relationship between the absorbed and scattered light in an inhomogeneous medium can vary, since $\lambda(r)$ varies, but the sum of the emitted and scattered light is determined only by the temperature and yields a source function equal to $1^{\circ}$. When $\lambda=0$, the problem is markedly simplified, since there is simply no scattering, while $\varepsilon=g_{v}$. When $\alpha(r)$ varies in space, then the absolute value of the absorbed and scattered light varies, but the compensation of all the losses by the corresponding volume sources is still conserved. When $\alpha=0$, there is no absorption, scattering, or emission inside the medium, and the light simply passes from the walls through the medium without interacting with it, and we have $I=I^{0}$.

The separation of the equilibrium radiation into components obtained here can be used in rather varied ways. Let us present two examples from spectral diagnostics: 1) One obtains from (3.29) and (3.30) the limiting values of the intensities $I_{S}, I_{v a}$, and $I_{v p}$, which it is important to know in solving problems of diagnostics. Thus, for example, one can estimate the maximum possible influence of the scattering of the emission from the walls from the measured intensity. 2) In numerical calculations of the intensities $I_{S}, I_{v a}$, and $I_{v p}$, Eqs. (3.29) and (3.30) make it possible to estimate the error of the calculations.

### 3.3. The probabilistic method of solving the transport equations

The purpose of the spectral diagnostics of a gas containing a condensed dispersed phase is primarily to determine the characteristics that enter into the primary volume sources of (3.10)-(3.12). Yet in experiments one measures the intensities of the output radiation. In radiation transport theory the relationships between the primary sources and the measurable intensity is established by two different methods. In the overwhelming number of studies the distribution of the source functions $\varepsilon$ or of the intensities $I$ inside the entire studied volume is determined. After solving this problem, one can also find the output intensity. This approach yields the concrete results required in diagnostics only in the simplest cases, although various studies have used different calculational procedures. Usually one studies a plane layer containing an optically homogeneous, isotropically scattering medium, while here actually one can take account only of single scattering.

In the other method of solving the problem, which was proposed in 1951 by V. V. Sobolev, ${ }^{32}$ first one determines the probability of escape of radiation from the object with account taken of multiple scattering, after which one establishes the direct connection between the primary emitters and the observed intensity. The method is based on the fact that the propagation of radiation in the medium is a statistical process, and the elementary interaction of the radiation with matter has a probabilistic meaning. Therefore one can write: $\alpha d r$ is the probability of interaction with the medium on the element of path dr ; $\exp \left(-\tau\left(\mathbf{r} \rightarrow \mathbf{r}^{\prime}\right)\right)$ is the probability that the radiation will proceed from the point $\mathbf{r}$ to the point $\mathbf{r}^{\prime}$ without interacting with the medium; $\lambda$ is the probability of scattering in a single interaction with the medium. $\chi\left(\vec{\Omega} \rightarrow \vec{\Omega}^{\prime}\right) \mathrm{d} \vec{\Omega}^{\prime} / 4 \pi$ is the probability that radiation in the direction $\vec{\Omega}$ after single scattering will proceed in the direction $\vec{\Omega}^{\prime}$ inside the element of solid angle $\mathrm{d} \vec{\Omega}^{\prime}$.

To explain the physical meaning of the method, first we shall write the expression for the intensity of radiation leaving at an arbitrary point $\mathbf{r}_{S}$ in the direction $\vec{\Omega}$ in the absence of scattering ( $\lambda=0$ ). Instead of (3.4), with account taken of (3.3), we obtain (see Fig. 1)

$$
\begin{align*}
I\left(\mathbf{r}_{S}, \vec{\Omega}\right)= & \int_{\mathbf{r}_{S \downarrow}}^{\mathbf{r}_{S}} \mathrm{dr} \mathbf{r}^{\prime \prime}\left\{g_{v}\left(\mathbf{r}^{\prime \prime}, \vec{\Omega}\right) \exp \left[-\tau\left(\mathbf{r}^{\prime \prime} \rightarrow \mathbf{r}_{S}\right)\right]\right. \\
& \left.\times \alpha\left(\mathbf{r}^{\prime \prime}\right)\right\}+I\left(\mathbf{r}_{S_{\uparrow}}, \vec{\Omega}\right) \exp \left[-\tau\left(\mathbf{r}_{S \uparrow \downarrow} \rightarrow \mathbf{r}_{S}\right)\right] \tag{3.31}
\end{align*}
$$

The first term in (3.31) is the radiation of the volume, and the second is that of the opposite surface. Both terms include $\exp (-\tau)$, the probability that the radiation will reach the point of observation $\mathbf{r}_{S}$ without interacting with the medium. The radiation that interacted with the medium was absorbed by it. At the point $\mathbf{r}_{S}$ only the radiation can lie in the direction $\vec{\Omega}$ that appeared on the straight line ( $\mathbf{r}_{S \uparrow 1}-\mathbf{r}_{S}$ ) and from the outset had the direction $\Omega$.

The probabilistic method makes it possible to write the intensity of the output radiation in a form similar to (3.31) also in the presence of scattering, i.e., to express the intensity of the output radiation in terms of the primary sources and the probability of exit in quadratures. But to do this we must find the probability of escape. In the presence of scattering the exponentials $\exp (-\tau)$ continue to describe the probability of escape or transmission of the radiation without interacting with the medium. But the total probability of escape can substantially exceed $\exp (-\tau)$, since even after interaction of the radiation with the medium, the scattered (surviving) radiation can reach the boundary of the medium. Here it can undergo many scattering events, or in other words, the photons can move in zigzag, rather chaotic trajectories. The light emitted at the point $r$ in the direction $\vec{\Omega}$ can leave the medium at an arbitrary point of the surface $S$ and in an arbitrary direction. And conversely, light can arrive at the point of observation $\mathbf{r}_{S}$ in the direction $\vec{\Omega}$ that was emitted in an arbitrary direction at any point of the object. Already this implies that, if one can find the probsability of escape from various points of the
object, in calculating the escaping intensity the integration must be performed, not along the straight line ( $\mathbf{r}_{S \dagger \downarrow}-\mathbf{r}_{S}$ ), but over the entire volume.

In concrete cases one can introduce and use the probabilities of events that somewhat differ from one another. The differences are determined by what radiation is being considered at the beginning of its passage to the surface or at the end of this path. For example, let

$$
p\left[\left(\mathbf{r}, \vec{\Omega}^{\prime \prime \prime}\right) \rightarrow\left(\mathbf{r}_{S}, \vec{\Omega}\right)\right] \mathrm{d}^{3} \mathbf{r d} \vec{\Omega}^{\prime \prime \prime} \mathrm{d}^{2} \mathbf{r}_{S} \mathrm{~d} \vec{\Omega}
$$

be the probability that a photon having a direction inside the element of solid angle from ( $\vec{\Omega}^{\prime \prime \prime}$ to $\vec{\Omega}^{\prime \prime \prime}+\mathrm{d} \vec{\Omega}^{\prime \prime \prime}$ ) is the probability that a photon with a direction inside the element of solid angle from $\vec{\Omega}^{\prime \prime \prime}$ to $\vec{\Omega} \vec{\Omega}^{\prime \prime \prime}+d \vec{\Omega}^{\prime \prime \prime}$ that interacts with the medium in the volume element $d^{3} r$ will reach the surface $S$ and will fall inside the element of area $\mathrm{d}^{2} \mathrm{r}_{S}$ in the neighborhood of the point $\mathbf{r}_{S}$ inside the element of solid angle from $\vec{\Omega}$ to $\vec{\Omega}+\mathrm{d} \vec{\Omega}$. Incidence of a photon on the surface can occur with intermediate scattering of any multiplicity. Analogously one can determine the probability of escape of a photon that does not interact with the medium, but was emitted by it as the result of intrinsic emission or scattering in the volume element $d^{3} r$ in a direction from $\vec{\Omega}^{\prime \prime \prime}$ to $\vec{\Omega}^{\prime \prime \prime}+\mathrm{d} \vec{\Omega}^{\prime \prime \prime}$. Let us denote this probability as:

$$
q\left[\left(\mathbf{r}, \overrightarrow{\Omega^{\prime \prime \prime}}\right) \rightarrow\left(\mathbf{r}_{S}, \vec{\Omega}\right)\right] \mathrm{d}^{3} \mathbf{r} \mathbf{d} \vec{\Omega}^{\prime \prime \prime} \mathrm{d}^{2} \mathbf{r}_{S} \mathrm{~d} \vec{\Omega}
$$

The probabilities of escape within finite intervals of areas and solid angles are obtained by integrating over $\mathbf{r}_{S}$ and $\vec{\Omega}$. One can study also the probability of escape, not at a certain part of the surface, but only in a given direction, independently of the site of escape (see Sec. 4.1).

At first glance it seems very difficult to write the equations describing any of these probabilities, owing to the complex behavior of the radiation with possible multiple scattering. However, in fact, the process of writing these equations differs little from the process of writing the equations of radiation transport. The difference lies only in the fact that in writing the equations for the source functions it was necessary to take account of the possible arrival of light at the point $\mathbf{r}$ under study from all directions. In treating the probability of escape, we must take account of the possible escape of light from the point $\mathbf{r}$ in all directions, i.e., in opposite directions. Let us show this.

Let us write the integral equation for the probability density of escape of a photon that interacts at $\mathbf{r}$ with the medium. The initial direction of the photon $\vec{\Omega}^{\prime \prime \prime}$ can be arbitrary; let us take as this direction ( $-\vec{\Omega}$ ). The probability $p(\mathbf{r},-\vec{\Omega})$ that the radiation will reach the surface is composed of the probabilities of two mutually exclusive events: passage of the light after single scattering at the point $r$ directly to the surface $S$, and with intermediate scattering events. The probability of passage with intermediate scattering events is defined as the product of the probabilities of a number of successive events:

1) Scattering at $\mathbf{r}$ in the direction of the volume element $d^{3} r^{\prime}$ in the neighborhood of the arbitrary point $r^{\prime}$ (see Fig. 1). The solid angle that this volume subtends at $\mathbf{r}$ is $\mathrm{d}^{2} \mathbf{r}^{\prime} /\left(\mathbf{r}-\mathbf{r}^{\prime}\right)^{2}$. Therefore the corresponding probability equals $\lambda(\mathbf{r}) \chi\left(-\vec{\Omega},-\vec{\Omega}^{\prime}\right) \mathrm{d}^{2} \mathbf{r}^{\prime} / 4 \pi\left(\mathbf{r}-\mathbf{r}^{\prime}\right)^{2}$.
2) Passage to $r^{\prime}$ without interacting with the medium: $\exp \left[-\tau\left(\mathbf{r}-\mathbf{r}^{\prime}\right)\right]$.
3) Interaction with the medium within $d^{3} \mathbf{r}^{\prime}: \alpha\left(\mathbf{r}^{\prime}\right) d \mathbf{r}^{\prime}$.
4) Escape from $\mathbf{r}^{\prime}: p\left(\mathbf{r}^{\prime},-\overrightarrow{\Omega^{\prime}}\right)$.

To take account of all the possibilities of this approach one must perform the integration over the entire volume, i.e., over all $\mathbf{r}^{\prime}$. Upon taking account of the presented ideas, we obtain the equation for the probability of escape:

$$
\begin{align*}
p(\mathbf{r},-\vec{\Omega})= & \frac{\lambda\left(\mathbf{r}^{\prime}\right)}{4 \pi} \int_{v} \mathrm{~d}^{3} \mathbf{r}^{\prime}\left\{p\left(\mathbf{r}^{\prime},-\vec{\Omega}^{\prime}\right)\right. \\
& \times \exp \left[-\tau\left(\mathbf{r} \rightarrow \mathbf{r}^{\prime}\right)\right] \alpha\left(\mathbf{r}^{\prime}\right) \chi\left(\mathbf{r},-\vec{\Omega} \rightarrow-\vec{Q}^{\prime}\right) \\
& \left.\times\left(\mathbf{r}-\mathbf{r}^{\prime}\right)^{-2}\right\}+p_{1}(\mathbf{r},-\vec{\Omega}) \tag{3.32}
\end{align*}
$$

Here we have omitted the symbols for the site and direction of escape of the radiation; it is only important that they are the same in all the terms of the equation. Here $p_{1}$ is the probability of passage of the light to the surface without intermediate scattering events. The index 1 means that one takes account only of a single scattering at the beginning of the path $\mathbf{r}$. The probability $p_{1}$ substantially depends on precisely where and in which direction the photon escapes from the medium. Let us write the expression for the probability of escape $p_{1}$ in the neighborhood of an arbitrary point $\mathbf{r}_{S}^{\prime}$, on the surface (see Fig. 1), while taking account of two consecutive events:

1) Scattering at $\mathbf{r}$ in the direction of the surface element $\mathrm{d}^{2} \mathbf{r}_{S}^{\prime}$, (see Fig. 1):

$$
\lambda(\mathbf{r}) \chi\left(\mathbf{r},-\vec{\Omega} \rightarrow-\vec{\Omega}_{S}^{\prime}\right) \mathrm{d}^{2} \mathbf{r}_{S}^{\prime} \cos \left(\vec{\Omega}_{S}^{\hat{}} \mathbf{n}_{S}^{\prime}\right) / 4 \pi\left(\mathbf{r}-\mathbf{r}_{S}^{\prime}\right)^{2}
$$

2) Passage to $r_{S}^{\prime}$, without interacting with the medium: $\exp \left[-\tau\left(\mathbf{r} \rightarrow \mathbf{r}_{S}^{\prime}\right)\right]$.

In the case of escape in any part of the surface $S$, we obtain after multiplying the probabilities of the two stated events and integrating over the surface:

$$
\begin{align*}
p_{1}(\mathbf{r},-\Omega)= & \lambda(\mathbf{r}) \int_{S} \mathrm{~d}^{2} \mathbf{r}_{S}^{\prime} \exp \left[-\tau\left(\mathbf{r} \rightarrow \mathbf{r}_{S}^{\prime}\right)\right] \\
& \times \cos \left(\vec{\Omega}_{S}^{\hat{S}} n_{S}^{\prime}\right) \chi\left(\mathbf{r},-\vec{\Omega} \rightarrow-\vec{\Omega}_{S}^{\prime}\right) / \\
& 4 \pi\left(\mathbf{r}-\mathbf{r}_{S}^{\prime}\right)^{2} \tag{3.33}
\end{align*}
$$

Let us compare Eq. (3.32) with the equation for the source function (3.6). The scattering integrals in the two cases are the same if the condition is satisfied that

$$
\begin{equation*}
\chi\left(\mathbf{r},-\vec{\Omega} \rightarrow-\vec{\Omega}^{\prime}\right)=\chi\left(\mathbf{r}, \vec{\Omega}^{\prime} \rightarrow \vec{\Omega}\right)^{2} \tag{3.34}
\end{equation*}
$$

Moreover, if in Eq. (3.6) the free term $g$ is determined only by the primarily scattered radiation of the surface (3.9), while in Eq. (3.32) the free term is determined by Eq. (3.33), then the fulfillment of the condition (3.34) ensures coincidence also of the free terms of the equations. The first studies of V. V. Sobolev ${ }^{32,33}$ and I. N. Minin ${ }^{34}$ called attention to the fact that the obtained equations for the probabilities of escape $p$ and $q$ from a plane layer in a given direction coincide with the equations for the source functions and intensities in the case in which sufficient conditions are found for the source functions and intensities created by an external emitter in a medium illuminated
from outside to equal the probabilities of escape $p$ and $q$ in the region of location of the external emitter. The fundamental condition is: the reciprocity relationships must be fulfilled in a single interaction of the radiation with matter. This condition was fulfilled in Refs. 32-34. In the case in which $\alpha$ does not depend on the direction of the radiation, the reciprocity relationship acquires the form of (3.34). The condition (3.34) implies that the scattering indicatrix must not change if the directions of the incident and scattered light change place. It was shown ${ }^{35}$ that in the wave zone, i.e., sufficiently remote from scattering particles, the reciprocity principle is fulfilled. The indicatrices presented in Sec. 2.1 pertain to the wave zone, and depend only on the absolute value of the scattering angle $\gamma_{\mathrm{s}}$ (or on $\cos \gamma_{\mathrm{s}}$ ). That is, they satisfy the condition (3.34). Relationships exist between the probability densities $p$ and $q$ when they characterize the escape at the very same site and in the very same direction. The relationships between $p$ and $q$ in a plane layer were derived by I. N. Minin, ${ }^{15,34}$ and in the general case in Ref. 27. Here we shall discuss a very simple relationship, which will be used below. The probability that radiation that interacts with the medium will escape from it is determined by the product of the probabilities of two consecutive events: the probability of scattering at the point $\mathbf{r}$ in the arbitrary direction $\lambda(\mathbf{r}) \chi\left(\mathbf{r}, \overrightarrow{\Omega^{\prime \prime \prime}} \rightarrow \vec{\Omega}^{\prime}\right) \mathrm{d} \vec{\Omega}^{\prime} /$ $4 \pi$ and the probability of subsequent escape of the already scattered radiation $q\left(\mathbf{r}, \vec{\Omega}^{\prime}\right)$. Here we must take account by integration of the possibility of initial scattering at $r$ in any direction:

$$
\begin{equation*}
p\left(\mathbf{r}, \vec{\Omega}^{\prime \prime \prime}\right)=(\lambda(\mathbf{r}) / 4 \pi) \int_{4 \pi} \chi\left(\mathbf{r}^{\prime}, \overrightarrow{\Omega^{\prime \prime \prime}} \rightarrow \vec{\Omega}^{\prime}\right) q\left(\mathbf{r}, \overrightarrow{\Omega^{\prime}}\right) \mathrm{d} \vec{\Omega}^{\prime} \tag{3.35}
\end{equation*}
$$

Here we have omitted in the notation for $p$ and $q$ the site and direction of the escape of the photon. For (3.35) to be correct it is only required that they be the same for $p$ and $q$.

The probability densities $p$ and $q$ can be used to write expressions for the output intensity. To elucidate the features of the method, let us write the expression for the intensity by using the probability of escape of a photon emitted by the medium ( $q$ ).

The intensity of the primary emission from a unit volume in the neighborhood of the point $r$ in the direction $\vec{\Omega}^{\prime \prime \prime}$ can be written in the form $g\left(r, \vec{\Omega}^{\prime \prime \prime}\right) \alpha(r)$. Let $q\left[\left(\mathbf{r}, \vec{\Omega}^{\prime \prime \prime}\right) \rightarrow\left(\mathbf{r}_{S}, \vec{\Omega}\right)\right] \mathrm{d}^{3} \mathbf{r d} \vec{\Omega}^{\prime \prime \prime} \mathrm{d}^{2} \mathbf{r}_{S} \mathrm{~d} \vec{\Omega}$ be the probability that the radiation emitted or scattered in $d^{3} \mathbf{r}$ within the interval of solid angles from $\vec{\Omega}^{\prime \prime \prime}$ to $\vec{\Omega}^{\prime \prime \prime}+\mathrm{d} \vec{\Omega}^{\prime \prime \prime}$ will reach the surface and fall inside the area element $d^{2} r_{S}$ in the neighborhood of $\mathbf{r}_{S}$ inside the element of angle from $\vec{\Omega}$ to $\vec{\Omega}+\mathrm{d} \vec{\Omega}$. Then we directly find the following expression for the intensity of the output radiation:

$$
\begin{align*}
I\left(\mathbf{r}_{S}, \vec{\Omega}\right)= & \int_{4 \pi} \mathrm{~d} \vec{\Omega}^{\prime \prime \prime} \int_{v} \mathrm{~d}^{3} \mathbf{r}\left\{g ( \mathbf { r } , \vec { \Omega } ^ { \prime \prime \prime } ) \alpha ( \mathbf { r } ) q \left[\left(\mathbf{r}, \vec{\Omega}^{\prime \prime \prime}\right)\right.\right. \\
\rightarrow & \left.\left.\left(\mathbf{r}_{S}, \vec{\Omega}\right)\right] \cos \left(\vec{\Omega} n_{S}\right)\right\}+I\left(\mathbf{r}_{S+1}, \vec{\Omega}\right) \\
& \times \exp \left[-\tau\left(\mathbf{r}_{S} \rightarrow \mathbf{r}_{S+1}\right)\right] \tag{3.36}
\end{align*}
$$

The factor $\cos \left(\vec{\Omega} \mathbf{r}_{S}\right)$ appeared within the integrand because, in the definition of $q$, the area of the surface was indicated toward which the radiation escapes, while the intensity must be calculated per unit of area perpendicular to the flux. The second term takes account of the arrival of light directly from the opposite wall.

Thus, in the probabilistic method first one must find the probabilities of escape by solving equations of a type like (3.32). Then the intensity can be determined by (3.36). In the other methods first one must find the source functions from equations of the type of (3.6), and then determine the intensity by (3.4), having set $\mathbf{r}=\mathbf{r}_{S}$. The advantages of the probabilistic method in diagnostics involve the fact that the equation (3.6) for $\varepsilon$ contains the optical characteristics of the medium and arbitrary primary sources $g(r)$, while in an equation of the type of (3.32) there are no primary sources of radiation, while all the terms are determined only by the optical characteristics of the medium ( $\alpha, \lambda, \chi$ ). This facilitates extremely finding the probabilities as compared with finding the source functions. The expression (3.36) is more complicated than (3.4), since it requires integration over the volume, but real objects are usually rather simple in shape, which markedly simplifies the problem. On the other hand, the direct relationship in the quadratures of the observed intensity $I$ with the sought function $g$ plays an important role in obtaining reliable results. When the integral terms in the equations for $\varepsilon$ and the probabilities coincide, as was the case above, the probability can be treated as a special solution of the transport equation (3.6) with a given free term. ${ }^{31}$ This solution is a function of the response function of the escaping radiation to the appearance of a single source inside the medium.

The equations for the probability of escape of a photon from a plane layer for isotropic scattering were derived in Ref. 33, and used in Ref. 36 for calculating the contours of spectral lines as required in spectral diagnostics. Temperature inhomogeneity in the layer and multiple scattering have been taken into account without complications. A similar problem was solved in Ref. 37 by the ordinary method, but it was possible with difficulty to take account of only single scattering.

Thus the probabilistic method was chosen for describing the transport of radiation in a gas containing a condensed dispersed phase. The method makes it possible most directly to relate the measured intensity of the radiation to the sought characteristics of the object, even in the case when light scattering by particles is substantial. The relationships needed in the calculations were derived in rather general form.

## 4. EMISSION FROM A GAS CONTAINING A CONDENSED DISPERSED PHASE

The present Sec. 4 is devoted to describing the output of the intrinsic emission of a gas containing a condensed dispersed phase. The primary sources $g_{v}$ in [(3.10)-(3.12)] and the intensity of the thermal emission of the walls are isotiopic. This often leads to a weak dependence of the emission intensity on the direction, despite the anisotropy
of scattering by the particles. Under these conditions one can use the transport approximation and assume the scattering to be isotropic. In Secs. 4.1-4.3 this is done in studying the output of radiation from objects of very simple shapes. Sections 4.4 and 4.5 theoretically and experimentally discuss the case, of practical importance in diagnostics, in which one must take account of the anisotropy of scattering.

### 4.1. Escape of radiation for isotropic scattering

Let us study the radiation from an infinite plane layer, an infinite cylinder, and a sphere. We shall assume that the distribution of primary volume sources in the layer depends only on the coordinate $X$ in the layer (see Fig. 2), and in the cylinder and the sphere only on the radius $\rho$ (see Fig. 3). If in addition the walls emit homogeneously, we can expect that the radiation will escape the object in all regions of its surface in the same way. The intensity of the escaping radiation here can depend only on the direction. Let us introduce a concept of the probability of escape suitable for describing the radiation in these cases: let $q(\mathbf{r} \rightarrow \vec{\Omega}) \mathrm{d}^{3} \mathbf{r d} \vec{\Omega} / 4 \pi$ be the probability that a a photon that has appeared in the neighborhood of $r$ inside $d^{3} r$ will leave the volume being studied in the direction $\vec{\Omega}$. In contrast to the probabilities discussed in Sec. 3.3, here we do not indicate an initial direction of the photon appearing at $r$ and have not stipulated the site of escape of the photon from the medium. We have fixed only the direction $\vec{\Omega}$ of this escape, which involves the features of the objects being studied. Yet the absence in the definition of $q$ of an initial direction implies that the probability that has been introduced is averaged over the initial directions. The probability averaged over the initial directions involves the evident relationship introduced earlier:

$$
\begin{equation*}
q(\mathbf{r} \rightarrow \vec{\Omega}) / 4 \pi=\int q\left(\mathbf{r}, \overrightarrow{\Omega^{\prime \prime \prime}} \rightarrow \vec{\Omega}\right) \mathrm{d} \vec{\Omega}^{\prime \prime \prime} / 4 \pi \tag{4.1}
\end{equation*}
$$

Analogously we can introduce the probability $p(\mathbf{r} \rightarrow \vec{\Omega})$ averaged over the initial directions. However, in isotropic scattering the probability of escape of a photon interacting with the medium does not depend on the initial direction. That is, the probability $p$ averaged over the initial directions equals this probability itself. With account taken of (4.1) and $\chi=1$, we find instead of (3.35):

$$
\begin{equation*}
p(\mathbf{r} \rightarrow \vec{\Omega})=\lambda(\mathbf{r}) q(\mathbf{r} \rightarrow \vec{\Omega}) / 4 \pi \tag{4.2}
\end{equation*}
$$

Now we shall derive the equation for the probability $q$ by using the general formula (3.32). In Eq. (3.32) we shall omit the symbols for the initial directions $-\vec{\Omega}$ and $-\vec{\Omega}^{\prime}$, since nothing depends on them in the case being studied. Let us substitute (4.2) and $\chi=1$ into (3.32) and divide both sides of the equation by $\lambda(r) / 4 \pi$ :

$$
\begin{align*}
q(\mathbf{r} \rightarrow \vec{\Omega})= & \frac{1}{4 \pi} \int_{v} \mathrm{~d}^{3} \mathbf{r}^{\prime}\left\{q\left(\mathbf{r}^{\prime} \rightarrow \vec{\Omega}\right) \exp \left[-\tau\left(\mathbf{r} \rightarrow \mathbf{r}^{\prime}\right)\right] \alpha\left(\mathbf{r}^{\prime}\right)\right. \\
& \left.\times \lambda\left(\mathbf{r}^{\prime}\right)\left(\mathbf{r}-\mathbf{r}^{\prime}\right)^{-2}\right\}+q_{0}(\mathbf{r} \rightarrow \vec{\Omega}) \tag{4.3}
\end{align*}
$$

Here $q_{0}(\mathbf{r} \rightarrow \vec{\Omega})$ is the probability of immediate escape (without interacting with the medium) of a photon that
has arisen in the neighborhood of $\mathbf{r}$. Let us write in general form the expression for $q_{0}(\mathbf{r} \rightarrow \vec{\Omega})$ while paying attention to the following. The direction of escape $\vec{\Omega}$ in emitters of different geometric shapes is usually determined by some directing angles on the surface, which can differ from the directing angles inside the emitter. The corresponding elements of solid angle also can differ.

The probability of immediate escape is determined by the probabilities of two consecutive events: 1) The probability of traveling after appearance at $\mathbf{r}$ inside the element of solid angle $\mathrm{d} \vec{\Omega}_{0}$, which corresponds on the surface to the element $d \vec{\Omega}$. This probability is $\mathrm{d} \vec{\Omega}_{0} / 4 \pi$. 2) The probability of reaching the surface without interacting with the medium: $\exp [-\tau(\mathbf{r} \rightarrow \vec{\Omega})]$. Here the direction to the surface is indicated in the argument of the optical density, rather than the point on the surface. As a result we obtain

$$
\begin{align*}
& q_{0}(\mathbf{r} \rightarrow \vec{\Omega}) \mathrm{d} \vec{\Omega} / 4 \pi=\exp [-\tau(\mathbf{r} \rightarrow \vec{\Omega})] \mathrm{d} \vec{\Omega}_{0} / 4 \pi  \tag{4.4}\\
& q_{0}(\mathbf{r} \rightarrow \vec{\Omega})=\exp [-\tau(\mathbf{r} \rightarrow \vec{\Omega})] \mathrm{d} \vec{\Omega}_{0} / \mathrm{d} \vec{\Omega}
\end{align*}
$$

The probability $q(\mathbf{r} \rightarrow \vec{\Omega})$ can be found in concrete geometric shapes by solving Eq. (4.3). To obtain the intensity emerging in the direction $\vec{\Omega}$, one needs the power of the primary sources emitted on all sides of the volume element $4 \pi \alpha(\mathbf{r}) g(\mathbf{r}) \mathrm{d}^{3} \mathbf{r}$, multiply it by the probability density of escape, and integrate over the volume. Upon adding the intensity of the light that has come directly from the opposite region of the wall, we obtain

$$
\begin{align*}
I(\vec{\Omega})= & \int_{v} 4 \pi \alpha(\mathbf{r}) g(\mathbf{r}) q(\mathbf{r}, \vec{\Omega}) \mathrm{d}^{3} \mathbf{r}\left[4 \pi S \cos \left(\mathbf{n}_{S} \vec{\Omega}\right)\right]^{-1} \\
& +I\left(\mathbf{r}_{S \uparrow \downarrow}, \vec{\Omega}\right) \exp \left[-\tau\left(\mathbf{r}_{S \uparrow \downarrow} \rightarrow \mathbf{r}_{S}\right)\right] \tag{4.5}
\end{align*}
$$

Here $S$, the area of the bounding surface, appeared because the probability $q$ describes the emergence over the entire surface. The second term does not differ from that in (3.36).

Let us apply the obtained expression to an infinite plane layer, an infinite cylinder, and a sphere. ${ }^{38}$ The scattering operators for $q$ [the integral term in (4.3)] equal the scattering operators of (3.13), (3.18), and (3.21), respectively, in the layer, the cylinder, and the sphere (in the cylinder and the sphere the assumption was made that the medium is homogeneous). This result is ensured by the fulfillment of the reciprocity principle (3.34). We shall write the quantities $q_{0}$ and 1 in each case by using the general expressions (4.4) and (4.5).

In the plane layer the direction inside the layer and at its surface is determined by the angle $\Phi$ between the chosen direction and the normal to the surface $X=0$ (Fig. 2). Therefore we have here $\mathrm{d} \vec{\Omega}_{0}=d \vec{\Omega}$. The optical density $\tau$ in the plane case is simply expressed in terms of the optical coordinate $t$ and $\cos \Phi=\eta$, namely: $\tau=t / \eta$. We can refine the definition of the probability of escape in this way: $q(t \rightarrow \eta) / 4 \pi$ is the probability that a photon that has appeared at the optical depth $t$ in a unit volume will escape through the boundary $X=t=0$ at the angle $\Phi$ in a unit of solid angle. Then $q\left(\left(t_{0}-t\right) \rightarrow \eta\right)$ is the same characteristic
of escape through the boundary $X=1, t=t_{0}$ (see Fig. 2) The probabilities of immediate escape here are:

$$
\begin{align*}
& q_{0}(t \rightarrow \eta)=\exp (-t / \eta) \quad(\eta>0)  \tag{4.6}\\
& q_{0}\left(\left(t_{0}-t\right) \rightarrow \eta\right)=\exp \left[\left(t_{0}-t\right) / \eta\right] \quad(\eta<0)
\end{align*}
$$

The equation for the probability of escape has the form [with account taken of (3.13)]:

$$
\begin{align*}
q(t \rightarrow \eta)= & 0.5 \int_{0}^{t_{0}} \lambda\left(t^{\prime}\right) q\left(t^{\prime} \rightarrow \eta\right) E_{1}\left(\left|t-t^{\prime}\right|\right) \mathrm{d} t^{\prime} \\
& +\exp (-t / \eta) \tag{4.7}
\end{align*}
$$

Equation (4.7) differs from the equation that was derived in the pioneering study ${ }^{33}$ only in that the probability of survival $\lambda$ can be a variable and therefore is placed in the integrand.

If the probability $q$ is found by solving Eq. (4.7), then, by using (4.5) we can find the intensity of the escaping radiation. In the case of the infinite plane layer we can take as the volume $v$ the volume of a rectangular parallelepiped with the base $S$ (see Fig. 2). This dissecting out of a finite column is possible, since all the radiation escaping from it to other parts of the surface is fully compensated by the incidence from the other volumes.

Let us set in (4.5) $\mathrm{d}^{3} \mathrm{r}=S \mathrm{~d} X$ and integrate over the height:

$$
\begin{align*}
I(0, \eta)= & \int_{0}^{t_{0}} g(t) q(t, \eta) \mathrm{d} t \eta^{-1}+I\left(t_{0}\right) \exp \left(-t_{0} / \eta\right) \\
& (\eta>0)  \tag{4.8}\\
I\left(t_{0}, \eta\right)= & -\int_{0}^{t_{0}} g(t) q\left(t_{0}-t\right) \mathrm{d} t \eta^{-1}+I(0) \exp \left(t_{0} / \eta\right) \\
& (\eta<0) \tag{4.9}
\end{align*}
$$

The primary sources $g(t)$ in (4.8) and (4.9) are determined by the specifics of the problem. The surface primary sources with homogeneous isotropic emission from both surfaces are determined by Eq. (3.14).

The direction of the radiation escaping from an infinite cylinder is determined by two angles on the surface (see Fig. 3): $\theta$, the angle between the generator of the cylinder and the direction $\vec{\Omega}$, and $\Phi$, the angle between the projection of $\vec{\Omega}$ on the radial plane and the normal to the surface. The direction inside the cylinder is characterized by the same angle $\theta$ and the angle $\psi$ between the chosen direction and the radius at the point being studied. The elements of solid angle on the surface and inside the cylinder are related as follows:

$$
\begin{equation*}
\mathrm{d} \vec{\Omega}_{0}=\mathrm{d} \vec{\Omega} \cos \Phi \cdot\left(x^{2}-\sin ^{2} \Phi\right)^{-1 / 2} \tag{4.10}
\end{equation*}
$$

Here we have $x=\rho / R$. Let us take account of the fact that escape is possible from each point inside the cylinder at two points of the surface ( $B$ and $C$ in Fig. 3) with identical $\Phi$ and $\Theta$. We shall assume the medium to be homogeneous. Upon paying attention to (3.5), we can obtain from (4.4)

$$
\begin{aligned}
q_{0}(x, \Phi, \Theta)= & \cos \Phi \cdot\left(x^{2}-\sin ^{3} \Phi\right)^{-1 / 2}\left[\operatorname { e x p } \left\{-t_{0}[\cos \Phi\right.\right. \\
& \left.\left.-\left(x^{2}-\sin ^{2} \Phi\right)^{1 / 2}\right] / 2 \sin \theta\right\} \\
& +\exp \left\{-t_{0}\left[\cos \Phi+\left(x^{2}\right.\right.\right. \\
& \left.\left.\left.\left.-\sin ^{2} \Phi\right)^{1 / 2}\right] / 2 \sin \theta\right\}\right]
\end{aligned}
$$

for $x^{2}>\sin ^{2} \Phi$,

$$
\begin{equation*}
q_{0}(x, \Phi, \theta)=0 \quad \text { when } x^{2} \leqslant \sin ^{2} \Phi \tag{4.11}
\end{equation*}
$$

Here we have noted that the probability of escape depends only on the relative radius $x$ and the angles $\theta$ and $\Phi$. The second equation of (4.11) describes the fact that, for a given $\Phi$, immediate escape at the surface is possible only for a sufficiently large distance ( $x$ ) from the center.

Equation (4.3) for the probability of escape with account taken of (3.18) acquires the form

$$
\begin{align*}
q(x, \Phi, \theta)= & \left(\lambda t_{0} / 2 \pi\right) \int_{0}^{1} x^{\prime} q\left(x^{\prime}, \Phi, \theta\right) J\left(x, x^{\prime}\right) \mathrm{d} x^{\prime} \\
& +q_{0}(x, \Phi, \Theta) \tag{4.12}
\end{align*}
$$

where $J\left(x, x^{\prime}\right)$ is determined by Eq. (3.19), while $q_{0}$ is determined by the expressions of (4.11).

Let us find the expression for the intensity of radiation escaping from the cylinder. Here as the volume equivalent to its neighbors we shall take a cylinder of height $H$. Then we have

$$
\begin{equation*}
S=2 \pi R H, \quad \mathrm{~d}^{3} \mathbf{r}^{\prime}=H \rho \mathrm{~d} \varphi \mathrm{~d} \rho \tag{4.13}
\end{equation*}
$$

Upon integrating over $\varphi$ from 0 to $2 \pi$, and over $\rho$ from 0 to $R$, we can find the first term of (4.5). The optical density along the chosen direction to the opposite surface in the cylinder is

$$
\begin{equation*}
\tau\left(\mathbf{r}_{S \rightarrow \mathbf{r}_{S_{\uparrow} \downarrow}}\right)=t_{0} \cos \Phi / \sin \theta \tag{4.14}
\end{equation*}
$$

Taking all this into account, we find

$$
\begin{align*}
I(\theta, \Phi)= & \left(t_{0} / 2 \sin \theta \cos \Phi\right) \int_{0}^{1} g(x) x q(x, \Phi, \theta) \mathrm{d} x \\
& +I_{S 0} \exp \left(-t_{0} \cos \Phi / \sin \theta\right) \tag{4.15}
\end{align*}
$$

In this case the surface primary sources are determined by Eq. (3.20).

The direction on the surface of the sphere can be characterized by the angle $\Phi$ between $\vec{\Omega}$ and the normal to the surface, while the direction inside the sphere is characterized by the angle $\psi$ between the chosen direction and the radius (see the lower part of Fig. 3). The relationship between the elements of solid angle on the surface and inside the sphere has the form

$$
\begin{equation*}
\mathrm{d} \vec{\Omega}_{0}=\mathrm{d} \vec{\Omega} \cos \Phi / x\left(x^{2}-\sin ^{2} \Phi\right)^{1 / 2} \tag{4.16}
\end{equation*}
$$

Finally in the case of the sphere we find

$$
\begin{aligned}
q_{0}(x, \Phi)= & {\left[\cos \Phi / x\left(x^{2}-\sin ^{2} \Phi\right)^{1 / 2}\right] } \\
& \times\left[\exp \left\{-0.5 t_{0}\left[\cos \Phi-\left(x^{2}-\sin ^{2} \Phi\right)^{1 / 2}\right]\right\}\right. \\
& \left.+\exp \left\{-0.5 t_{0}\left[\cos \Phi+\left(x^{2}-\sin ^{2} \Phi\right)^{1 / 2}\right]\right\}\right]
\end{aligned}
$$

when $x^{2}>\sin ^{2} \Phi,=0$
when $x^{2} \leqslant \sin ^{2} \Phi$.
The equation for the probability of escape with account taken of (3.21) acquires the form

$$
\begin{align*}
q(x, \Phi)= & 0.5 \lambda t_{0} \int_{0}^{1} q\left(x_{1}^{\prime} \Phi\right) x^{\prime} \mathrm{d} x^{\prime}\left(E_{1}\left(0.5\left|x-x^{\prime}\right|\right)\right. \\
& \left.-E_{1}\left(0.5\left(x+x^{\prime}\right)\right)\right)+q_{0}(x, \Phi) \tag{4.18}
\end{align*}
$$

where $q_{0}(x, \Phi)$ is determined by Eq. (4.17).
Let us find the intensity of radiation escaping from the sphere. To do this we shall study the entire surface of the sphere $S=4 \pi R^{2}$. Integrating over the volume of the sphere, we find from (4.5)

$$
\begin{align*}
I(\Phi)= & \left(t_{0} / 2 \cos \Phi\right) \int_{0}^{1} g(x) x^{2} q(x, \Phi) \mathrm{d} x+I_{\mathrm{S} 0} \\
& \times \exp \left(-t_{0} \cos \Phi\right) \tag{4.19}
\end{align*}
$$

Here $g_{S}$ is determined by Eq. (3.22).
Let us adopt the method of successive approximations, which makes it possible to solve the equations for $q$ in the cases discussed here. The solution is represented in the form of the series:

$$
\begin{align*}
& q(\mathbf{r} \rightarrow \vec{\Omega})=\sum_{k=0}^{\infty} q_{k}(\mathbf{r} \rightarrow \vec{\Omega}) \lambda_{\max }^{k}  \tag{4.20}\\
& q_{1}=V q_{0}, \quad q_{k}=V q_{\mathrm{k}-1}=V^{k} q_{0} \tag{4.21}
\end{align*}
$$

Here $\lambda_{\text {max }}$ is the largest value of $\lambda$ in the inhomogeneous gas; $q_{0}$ is determined by Eqs. (4.4), (4.6), (4.11), and (4.17) in the different cases; and the $V$ are the integral scattering operators that enter into Eqs. (4.3), (4.7), (4.12), and (4.18). The $k$ th term of the series in (4.20) amounts to the probability of escape with $k$ scattering events. To establish the convergence of the series in (4.20), it suffices to estimate the difference between the infinite sum in (4.20) and the sum of a finite number of terms of the series. If we use $m$ terms of the series, then the residual is

$$
\begin{align*}
& q(\mathbf{r} \rightarrow \vec{\Omega})-\sum_{k=0}^{m} \lambda_{\max }^{k} q_{\mathrm{k}}(\mathbf{r} \rightarrow \vec{\Omega}) \\
&=\sum_{k=m+1}^{\infty} \lambda_{\max }^{k} V^{k} q_{0}<q_{0 \max }=\sum_{k=m+1}\|V\|^{k} \lambda_{\max }^{k} \tag{4.22}
\end{align*}
$$

Here $\|V\|$ is the norm of the operators. If $\|V\| \lambda_{\text {max }}<1$, then one can use the formula for a geometric progression and, instead of (4.22), obtain
$q(\mathbf{r} \rightarrow \vec{\Omega})-\sum_{\mathrm{k}=0}^{m} \lambda_{\max }^{k} q_{k}(\mathbf{r} \rightarrow \vec{\Omega}) \leqslant q_{0 \text { max }} \frac{\|V\|^{m+1} \lambda_{\max }^{m+1}}{1-\|V\| \lambda_{\max }}$.

This implies that the series of (4.23) converges when $\lambda_{\text {max }}\|V\|<1$. An estimate of the norm in a plane layer was made in Ref. 37, and it was shown that $\|V\|<1$. Hence the series converges when $\lambda \leqslant 1$. The spherical case does not differ from the plane case.

In the following Secs. 4.2 and 4.3 we shall use the relationships obtained here to perform illustrative calculations. Below it will be useful to classify the optical densities into those governed by particles ( $t_{\mathrm{p}}$ ) and atoms ( $t_{\mathrm{a}}$ ). Using (2.35) instead of (3.15), we find

$$
\begin{align*}
& t_{0}(v)=t_{\mathrm{a}}(v)+t_{\mathrm{p}} \\
& t_{\mathrm{a}}(v)=\int_{0} k_{\mathrm{a}}(v) \mathrm{d} X  \tag{4.24}\\
& t_{\mathrm{p}}=\int_{0}^{l} \alpha_{\mathrm{p}} \mathrm{~d} X
\end{align*}
$$

In the cases of the cylinder and the sphere, in line with (3.17), the expressions (4.24) also hold when $l=2 R$. Using (4.24), we can rewrite (2.36) and (3.10) in the case of an optically homogeneous medium:

$$
\begin{align*}
& \lambda=\lambda_{\mathrm{p}} t_{\mathrm{p}} /\left(t_{\mathrm{a}}+t_{\mathrm{p}}\right),  \tag{4.25}\\
& g_{v}(x)=g_{\mathrm{a}}+g_{\mathrm{p}}, \\
& g_{\mathrm{a}}=\frac{t_{\mathrm{a}} I_{\mathrm{a}}^{0}(x)}{t_{\mathrm{a}}+t_{\mathrm{p}}} \\
& g_{\mathrm{p}}=\frac{t_{\mathrm{p}}\left(1-\lambda_{\mathrm{p}}\right) I_{\mathrm{p}}^{0}(x)}{t_{\mathrm{a}}+t_{\mathrm{p}}} \tag{4.26}
\end{align*}
$$

Here $x$ is the relative coordinate. We emphasize that (4.25) and (4.26) are valid both for isotropic and anisotropic scattering. If we use the transport approximation in Eqs. (4.25) and (4.26), instead of $\lambda_{p}$ we must introduce $\lambda_{\mathrm{tr}}$, which is related to the true $\lambda_{\mathrm{p}}$ by Eq. (2.9), while instead of $t_{\mathrm{p}}$ we must use $t_{\mathrm{tr}}$, which is determined by the expression

$$
t_{\mathrm{tr}}=\int_{0}^{l} \alpha_{\mathrm{tr}} \mathrm{~d} X
$$

Here $\alpha_{\mathrm{tr}}$ is expressed in terms of $\Sigma_{\mathrm{tr}}$ by using (2.29), while, when all the particles are identical, one can use (2.27). In this case we find by using Eq. (2.11):

$$
\begin{equation*}
t_{\mathrm{tr}}=\int_{0}^{l} a_{\mathrm{p}}\left(1-\left\langle\cos \gamma_{s}\right) \lambda_{\mathrm{p}}\right) \mathrm{d} X \tag{4.27}
\end{equation*}
$$

Finally, in the case in which $\lambda_{\mathrm{p}}$ does not vary inside the sphere, we obtain the relationship between the transport value and the true optical density:

$$
\begin{equation*}
t_{\mathrm{tr}}=t_{\mathrm{p}}\left(1-\lambda_{\mathrm{p}}\left\langle\cos \gamma_{s}\right\rangle\right) \tag{4.28}
\end{equation*}
$$

Thus we have derived the integral equations and that determine the probability of escape for isotropic scattering and the expressions for the escaping intensity in terms of the primary sources and the probabilities of escape. The relationships that we have obtained can be used for calculating the intensities of radiation from a layer, a cylinder, and a sphere.


FIG. 4. Variation inside a plane layer ( $x=t / t_{\mathrm{p}}$ ) of the probabilities of escape in the direction of the normal ( $\eta=1$ ) through the boundary $x=1$ for $t_{\mathrm{p}}=2$ (solid curves) and primary surface sources (dashed curves). $0-q_{0}, 1-q_{1}, 2-q_{2}, 3-q_{3}, 4-q_{4}, 5-q$ for $\lambda_{\mathrm{p}}=0.5$, curves $6-8-g_{S} / I_{s 0}$ for $\lambda_{\mathrm{p}}=0.5 ; 6-t_{\mathrm{p}}=1, g_{S}=g_{S 0}$ [according to (3.14)], 7- $t_{\mathrm{p}}=1, g_{S}=g_{S \Sigma} 0$, $8-t_{\mathrm{p}}=2, g_{S}=g_{S \Sigma}$.

### 4.2. Emission from the condensed dispersed phase in the continuum

4.2.1. This section will discuss examples of application of the relationships derived in Sec. 4.1 for describing the intensity of the output radiation in the regions of the spectrum where there are no atomic and molecular lines, i.e., $k_{\mathrm{a}}=t_{\mathrm{a}}=0, t_{0}=t_{\mathrm{p}}, \lambda=\lambda_{\mathrm{p}}$. The goal of the discussion is to distinguish the features of the radiation that must be taken into account in spectral diagnostics. Further, it is assumed in the calculations that $\lambda$ does not vary inside the medium.

In obtaining the intensities first one determines the probability of escape by solving Eqs. (4.7), (4.12), or (4.18) by the method of successive approximations in different cases. Figures 4 and 5 show the probabilities of escape from various depths in a plane layer and a cylinder. In the layer we use the relative optical coordinate $x=t / t_{0}$ and study the escape through the boundary $t=t_{0}$ (or $x=1$ ) in the cylinder $x=\rho / R$.

The curves $0-4$ in the two diagrams show the functions $q_{k}$ that determine the $k$-fold scattering in the expansion in (4.20). The quantities $q_{k}$ decrease with increasing $k$; hence the norms of the scattering operators $\|V\|$ are smaller than unity. There are substantial differences in the behavior of the probabilities of escape from a plane layer and a cylinder. In the case of the cylinder there is a singularity on the $q_{0}$ curve as $x \rightarrow 0$ and $\Phi=0$, while with increasing $x$ the value of $q_{0}$ monotonically decreases in agreement with (4.11). This is reflected in the behavior of all the rest of the curves. The singularity arises from the fact that a photon that arises near the axis of the cylinder can escape through the surface without being scattered only when $\Phi \rightarrow 0$. With increasing $x$ the photon can directly escape in an ever larger range of angles $\Phi$, which leads to a decrease in $q_{0}$. We note that singularities exist only in the probability den-


FIG. 5. Variation inside a cylinder of the probabilities of escape in the direction of the normal to the surface ( $\Phi=0, \theta=\pi / 2$ ) for $t_{\mathrm{p}}=2$. $0-q_{0}$, $1-q_{1}, 2-q_{2}, 3-q_{3}, 4-q_{4}, 5-q$ for $\lambda_{p}=0.5,6-q$ for $\lambda_{p}=0.75$.
sities, while the intensities are always finite as a result of the integration over (4.15).

In a plane layer, as we approach the surface $x=1$, on the whole the probabilities increase, although there are some nonmonotonicities (curves $1-4$ in Fig. 4).

The probabilities of escape from a sphere behave qualitatively in the same way as for a cylinder.

To calculate the intensities of the escaping radiation, we must define the primary sources. Let us study thermal sources $g_{p}$ for $t_{\mathrm{a}}=0(4.26)$ and surface sources of (3.14)-in a layer, (3.20)-in a cylinder, and (3.22)-in a sphere. The variation of the thermal sources $g_{p}$ according to (3.11) or (4.26) inside the medium is caused by the variation of the temperature, and correspondingly, of $I_{\mathrm{p}}^{0}(x)$. In the calculations we shall adopt three very simple variants: 1) the temperature is constant, 2) the temperature increases in the direction toward the surface of observation, 3) the temperature decreases in the direction toward the surface of observation. In cases 2) and 3) the temperature variations are such that $I^{0}(x)$ varies linearly. In the stated variants we have

$$
\begin{equation*}
I_{1}^{0}(x)=I_{\max }^{0}, \quad I_{2}^{0}(x)=I_{\max }^{0} x, \quad I_{3}^{0}=I_{\max }(1-x) \tag{4.29}
\end{equation*}
$$

We shall denote the corresponding primary volume sources as $g_{\mathrm{v} 1}, g_{\mathrm{v} 2}$, and $g_{\mathrm{v} 3}$.

In the cases of the sphere and the cylinder $g_{\mathrm{v} 2}$ and $g_{\mathrm{v} 3}$ correspond to linear increase and decrease of $I^{0}(x)$ as we go away from the center or the axis, respectively. In the layer the reference point, and hence the increase or decrease in $I^{0}(x)$, begins from the surface opposite to the observer. That is, the character of the inhomogeneity differs from that in the cylinder and the sphere.

The primary surface sources are shown in the case of the layer in Fig. 4 by dashed curves: curve 6 describes the scattering of radiation by one surface $\left[t=x=0, g_{S}=g_{S 0}\right.$ according to (3.14)], while curves 7 and 8 describe scat-


FIG. 6. Dependence of the intensity of the radiation escaping a plane layer in the direction of the normal $(\eta=1)$ toward the surface $t=t_{0}=t_{\mathrm{p}}$ for various primary sources. $I_{u p} / I_{\max }^{0}$ are the relative intensities for volume sources determined by the formulas of (4.26) for $t_{\mathrm{a}}=0$ and by the formulas of (4.29) for $I_{\mathrm{p}}^{0}(x)$. The $I_{S} / I_{S 0}$ are the relative intensities for surface sources $g_{S 00}, g_{S 0}$, and $g_{S \Sigma}$ determined by (3.14). The number after the symbol for the source on the curves is the value of $\lambda_{p}$.
tering of radiation by both surfaces [ $g_{S \Sigma}$ according to (3.14)]. Increase in the optical density leads to a sharper decline in $g_{S}$ with distance from the surface. Qualitatively, in the cylinder and the sphere $g_{S}(x)$ are analogous to $g_{S \Sigma}$ in the layer.

The calculations of the intensities of the radiation escaping the layer, the cylinder, and the sphere must be performed by Eqs. (4.9), (4.15), and (4.19), respectively. The result depends on the optical density $t_{\mathrm{p}}$, the probability of survival $\lambda_{p}$, the shape of the emitter, and the character of the primary sources $g$. Figures 6 and 7 show the dependences on $t_{\mathrm{p}}$ of the intensity of radiation escaping in the direction of the normal to the surface from the layer, cylinder, and sphere for $\lambda_{\mathrm{p}}=0$ and $\lambda_{\mathrm{p}}=0.5 ; I_{\nu \mathrm{p}}$ is the intensity arising from the primary volume sources, and $I_{S}$ from the surface sources. We shall use these calculations to reveal the features of the different emitters.
4.2.2. $I_{S O}=0, I_{\mathrm{p}}^{0} \neq 0$-the primary sources are determined by Eq. (4.26) when $t_{\mathrm{a}}=0$ for $g_{\mathrm{p}}$, where $I_{\mathrm{p}}^{0}(x)$ is described by one of the formulas of (4.29). The bounding surfaces are either absent or transparent or completely absorb the light incident on them, but here do not themselves emit, e.g., because their temperatures are low.

Here a very simple case is an emitter without scattering, i.e., with $\lambda_{\mathrm{p}}=0$. In the absence of scattering the relative intensity does not depend on the shape of the emitter, while only the character of the inhomogeneity along the direction of observation is defined. In the homogeneous case (curves $v 1 ; 0$ in Figs. 6 and 7) the intensity of the radiation is described by the well known formula, which is also implied by (3.4) when $I_{S}=0$ :

$$
\begin{equation*}
I_{u} / I_{\max }^{0}=1-\exp \left(-t_{\mathrm{p}}\right) \tag{4.30}
\end{equation*}
$$



FIG. 7. Dependence of the intensity of the radiation escaping in the direction normal to the surface. The symbols for the primary sources are the same as in Fig. 6, except that $S$ is the intensity for the sources determined by (3.22) for a sphere; sp-sphere, slab-plane layer, cylcylinder.

In inhomogeneous emitters $I_{v p} / I_{\text {max }}^{0}$ is smaller than in (4.30) for all $t_{\mathrm{p}}$, while in the case of decreasing temperature, nonmonotonicity (self-reversal) arises as one approaches the observer in the course of $I_{v p} / I_{\max }^{0}(v 3 ; 0$ in Fig. 6).

When scattering arises $\left(\lambda_{\mathrm{p}} \neq 0\right)$, the $I_{v} / I_{\max }^{0}$ variation is substantially altered, while this alteration depends on the shape of the emitter. In the homogeneous case ( $v 1 ; 0.5$ ) for all $t_{\mathrm{p}}$ the relative intensities decline. The decrease is caused by the change in $g_{v}$ owing to the coefficient ( $1-\lambda_{p}$ ), since the probabilities $q$ only increase with increasing $\lambda_{p}$ according to (4.20).

The influence of the shape of the object on the decrease in intensity in the presence of scattering by particles can be explained as follows. When the primary intensity of the radiation is fixed in the object at every point, this radiation in the steady-state case is compensated by the losses by absorption and by escape through the surface.

For a given optical density and with an increasing role played by scattering, the role of escape through the surface is increased. Depending on the ratio of the area of the surface and the volume, the steady state is established at some particular level of density of radiant energy.

In going from the plane layer to the cylinder and the sphere, the role of escape through the surface increases and the intensity decreases (see Fig. 7).

The ratio $I_{v p} / I_{\text {max }}^{0}$ is often called the emissive capacity ("emittance" in the English-language literature). In a homogeneous thermal emitter, when one can apply the Kirchhoff law to the emitter as a whole, the ratio $I_{v p} / I_{\text {max }}^{0}$ also characterizes the emittance of the particles; consequently both of these definitions pertain to the curves $v 1$ (see Sec. 6.2). We emphasize only again that the emittance in the absence of scattering, independently of the shape of the homogeneous emitter, is related to the optical
density by Eq. (4.30), while in the presence of scattering a dependence on $\lambda_{p}$ and on the shape of the emitter arises.
4.2.3. $I_{\mathrm{p}}^{0}=0, I_{S 0} \neq 0$-nonradiating macroscopic particles of the dispersed phase surrounded by glowing surfaces. This situation can occur either when the temperature of the particles is so small that they do not emit, or the particles are purely scattering particles $\left(\lambda_{p}=1\right)$. The primary sources here are only surface sources ( $g_{S}$ ), and are determined by the formulas (3.14), (3.20), and (3.22) in a layer, a cylinder, and a sphere, and can differ from zero only if $\lambda_{\mathrm{p}} \neq 0$. The relative intensities $I_{S} / I_{S 0}$ of the radiation of the walls scattered by the particles are given in Figs. 6 and 7 ( $S 0, S t_{0}, S \Sigma$, and $S$ ). In calculating these intensities the second term in Eqs. (4.9), (4.15), and (4.19) was omitted. That is, no account was taken of the light that arrived directly from the opposite surface lying in the field of view of the observer. The $I_{S} / I_{50}\left(t_{\mathrm{p}}\right)$ dependences are nonmonotonic, which involves the behavior of the primary sources with varying $t_{\mathrm{p}}$ (see Fig. 4).
4.2.4. $I_{S 0} \neq 0, I_{\mathrm{P}}^{0} \neq 0$-thermal emitter surrounded by radiating walls. In this case the intensities can be obtained by adding the intensities $I_{v \mathrm{p}}$ and $I_{S}$ obtained in the previous treatments. If the field of view of the observer includes the opposite surface, then we must add also the intensity coming directly from it [the second terms in (4.9), (4.15), and (4.19)].

If the temperature everywhere in the volume is the same and equals the temperature of the surface ( $I_{\max }^{0}=I_{S 0}=I^{0}$ ), then one can apply to the emitter the equilibrium relationships derived in Sec. 3.2; in particular, the intensity arising from the volume and surface emitters must satisfy the condition (3.30) for $I_{v p}=0$ :

$$
\begin{equation*}
I_{u \mathrm{p}} / I_{\mathrm{p}}^{0}+\left(I_{S} / I_{S 0}\right)=1-\exp \left(-t_{\mathrm{p}}\right) \tag{4.31}
\end{equation*}
$$

A consequence is directly obtained from Eq. (4.31). If $\lambda_{\mathrm{p}}=0$. then we have $I_{S}=0$ and $I_{i \mathrm{p}} / I^{0}=1-\exp \left(-t_{\mathrm{p}}\right)$. If $\lambda_{\mathrm{p}}=1$, then we have $I_{v}=0$ and $I_{S} / I_{S 0}=1-\exp \left(-t_{\mathrm{p}}\right)$. This means that in the two cases of extremes in scattering the relative intensities the relative intensities of volume and surface sources are the same. Consequently the curves $v 1 ; 0$ in Figs. 6 and 7 describe not only homogeneous emitters without scattering at $\lambda_{\mathrm{p}}=0$, but also purely scattering media with isothermal walls (in our notation- $S$; 1 ). For arbitrary $0 \leqslant \lambda_{p} \leqslant 1$ the same curve describes the total relative intensity of the intrinsic radiation of the particles and the scattered radiation of the walls ( $\Sigma$ ).

This can be used to estimate the error in calculations that have been performed. For the test we must add the intensities of the homogeneous thermal ( $I_{v 1} / I_{\max }^{0}$ ) and surface ( $I_{S} / I_{S 0}$ ) sources, add $\exp \left(-t_{\mathrm{p}}\right)$, and compare the result with unity. This procedure was performed with the curves ( $v 1 ; 0.5$ ) and ( $S ; 0.5$ ) in Figs. 6 and 7. It turned out that the deviation from unity does not exceed 0.07 , which defines the greatest total error of the performed intensity calculations.

Let us take up the dependence of the output radiation on the angle for volume homogeneous sources. We note here two circumstances:

First, at low $\Phi$ the dependence on the angle is very weak, which does not contradict the original justification for using the transport approximation.

Second, one observes a certain angular redistribution of the radiation when scattering arises. In general, the effect depends both on the optical density and on the shape of the emitter. When scattering arises, the output of radiation at glancing angles to the surface of the layer and the cylinder declines, since here the geometric paths of the radiation are longer, and hence, the probability of both a change of direction upon scattering and of absorption are greater. Consequently the output in directions close to the normal increases relatively. The effect is strongest in a plane layer and is absent in a sphere.

Thus the presented data indicate a strong dependence of the intensity of the continuum on the probability of survival $\lambda_{p}$, the shape of the emitter, and the presence of glowing walls, even in the case in which they do not fall within the field of view of the observer.

### 4.3. Emission in spectral lines

4.3.1. The present section 4.3 discusses the intensity of radiation in a region containing lines, where $k_{\mathrm{a}}(v) \neq 0$. In the development of the foundations of spectral diagnostics and in the practical use of diagnostic methods, it is convenient to construct the dependence of the intensities on the optical density, ${ }^{19,39,40}$ rather than on the frequency $v$. This makes it possible to study and employ the general laws of behavior of the lines. To transform from optical density to the frequency $v$, e.g., in comparing calculated with experimental results, one must use Eq. (4.24) with a known $k_{\mathrm{a}}(v)$. Simultaneous measurements of the intensities and optical densities prove useful. Further, for definiteness one studies a single line; the generality of treatment is not limited by this when the relationships are constructed not versus the frequency, but versus the optical density.

There is a substantial contrast in the construction of a spectral line $I\left(t_{\mathrm{a}}\right)$ from the constructions of $I\left(t_{\mathrm{p}}\right)$ performed in Sec. 4.2. The change in $t_{\mathrm{p}}$ occurred without change in the absorptive and scattering properties of the individual particles. Each curve was constructed for an invariant probability of survival $\lambda_{\mathrm{p}}$. The variation of $t_{\mathrm{p}}$ here can occur either because of variation of the thickness of the object or variation of the concentration of particles $n_{\mathrm{p}}$. In constructing a spectral line $I\left(t_{\mathrm{a}}\right)$ the variation of the atomic optical density $t_{\mathrm{a}}(v)$ is caused by the variation of the absorption coefficient of each atom $k_{\mathrm{a}}(v)$ according to (2.32) and (4.24), while the fundamental characteristic of the scattering $\lambda_{\mathrm{p}}$ varies along with the variation of $t_{\mathrm{a}}(v)$ [see (4.25)] with invariant characteristics of the particles $\lambda_{\mathrm{p}}$ and $t_{\mathrm{p}}$.

The calculations of the probabilities of escape were performed by solving Eqs. (4.7), (4.12), and (4.18), and those of the intensities $I$ by Eqs. (4.9), (4.15), and (4.19) in a plane layer, a cylinder, and a sphere. The construction of each dependence on $t_{\mathrm{a}}$ was performed for given values of $\lambda_{\mathrm{p}}, t_{\mathrm{p}}$, and $I_{\mathrm{p}}^{0}$. Each value of $t_{\mathrm{a}}$ corresponded to its own value $t_{0}$ according to (4.24) and its own $\lambda$ by (4.25). These values of $t_{0}$ and $\lambda$ enter into the calculations of $q$ and $I$.


FIG. 8. Variation inside the spectral region of an atomic line of the probability of scattering and the probability of escape of the radiation from the middle of a plane layer in the direction of the normal to the surface. $g\left(t_{0} / 2, \eta=1\right): 1-t_{p}=0 ; 2-t_{p}=1 ; 3-t_{p}=1, \lambda_{p}=1 ; 4-t_{p}=1$, $\lambda_{\mathrm{p}}=1$.

As an example, Fig. 8 shows the variation inside a spectral line of the probability of survival $\lambda$ and the probabilities $q$ of escape from the middle of a plane layer. With increasing $t_{\mathrm{a}}$, i.e., with approach to the center of the line, the probability of escape decreases, both in the absence of particles (1) and in their presence (2, 3). Absorbing particles (2) decrease the probability of escape $q$, while scattering particles (3) decrease $q$ only at large $t_{a}$, while at small $t_{\mathrm{a}}$ (tail of the line) the effect can be the opposite.

In calculating the intensities one studies further the primary volume sources in (4.26) and the surface sources in (3.14), (3.20), and (3.22). It was assumed in the calculations in volume sources that $I_{\mathrm{p}}^{0}$ does not depend on $x$, while $I_{\mathrm{a}}^{0}(x)$ in the various cases varies as in the formulas of (4.29).

Let us study the different conditions of emission.
4.3.2. $I_{S 0}=I_{\mathrm{p}}^{0}=0, I_{\mathrm{a}}^{0} \neq 0$. In such an emitter there is only the primary emission of the atoms, $g_{v}(x)=g_{\mathrm{a}}(x)$ according to (4.26). The surrounding surfaces either are absent or only absorb light, while the particles scatter and absorb, but do not emit light. This can occur either at a low enough temperature of the particles or if the particles are purely scattering ( $\lambda_{\mathrm{p}}=1$ ). $I_{v a}$ is the corresponding intensity. Figures 9 and 10 show the results of calculations in a layer, a cylinder, and a sphere. The influence of temperature inhomogeneity is qualitatively everywhere the same: $I_{\mathrm{va}} / I_{\max }^{0}$ decreases when inhomogeneity arises, while, when $I^{0}$ decreases in the direction toward the observer, selfreversal arises. When the particles only absorb light ( $\lambda_{\mathrm{p}}=0$ ), they decrease the intensity for all $t_{\mathrm{a}}$. When the particles only scatter light ( $\lambda_{\mathrm{p}}=1$ ), they decrease the intensity at large optical densities of the atoms $t_{\mathrm{a}}$. At small $t_{\mathrm{a}}$


FIG. 9. Variation inside the spectral region of an atomic line of the radiation intensity $I_{\mathrm{va}}$ through the boundary $t=t_{0}$ of a plane layer in the direction of the normal to the surface when the primary sources arise from the thermal emission of atoms [ $g=g_{\mathrm{a}}$ according to (4.26)]. Solid curves- $I_{\mathrm{a}}^{0}=I_{1}^{0}$ in (4.29); dashed curves- $I_{\mathrm{a}}^{0}=I_{2}^{0}$; dot-dash curves- $I_{3}^{0}$; on the curves the first number is $t_{\mathrm{p}}$, and the second number is $\lambda_{\mathrm{p}}$.
the intensity increases in the layer and the cylinder; in the sphere this type of increase is not observed (cf. the solid curve and the dotted curve for $t_{\mathrm{p}}=\lambda_{\mathrm{p}}=1$ in Fig. 10). Particles with $0<\lambda_{\mathrm{p}}<1$ exert an intermediate action on $I\left(t_{\mathrm{a}}\right)$.

Thus the presence of particles in a radiating gas, even if the particles themselves do not emit, leads to a substantial change in the form of the line contour. The diminution of intensity by absorbing particles needs no explanation. Yet the influence of scattering particles $\left(\lambda_{p}=1\right)$ requires explanations, since the particles per se should not alter the number of photons in the medium. Their influence arises


FIG. 10. Variation inside the region of an atomic line of the radiation intensity $I_{v a}$ in the direction of the normal to the surface when the primary sources arise from the thermal emission of atoms $\left[g=g_{a}\right.$ according to (4.26)]. Cylinder ( $\theta=\pi / 2, \Phi=0$ )-notation the same as in Fig. 9 ; sphere-curve for the homogeneous case is drawn with dots.


FIG. 11. Variation inside the spectral region of an atomic line of the radiation intensity of a cylinder ( $\theta=\pi / 2, \Phi=0$ ) for various primary sources. Dashed curves- $g=g_{S}$ according to (3.20), $t_{\mathrm{p}}=1$; dot-dash- $g=g_{\mathrm{p}}$ according to (4.26), $t_{\mathrm{p}}=1$; solid curves: $\mathrm{a}-$-for $g=g_{\mathrm{a}}$, $t_{\mathrm{p}}=0$. The volume sources $g_{\mathrm{a}}$ and $g_{\mathrm{p}}$ correspond to an invariant temperature $\left[I^{0}=I_{1}^{0}\right.$ according to (4.29)].
from the increase in path length (or time) that the radiation spends in the medium owing to the zigzag motion of the photons caused by scattering. At large optical densities of atoms, this behavior of the photons leads to an increase in the probability that they will be absorbed by atoms to decrease the intensity. Of course, at large enough atomic optical densities ( $t_{\mathrm{a}}>5$ ) this effect should disappear, since the radiation is absorbed by atoms faster than it is scattered by the particles. At low optical densities of the atoms (tails of the line), the zigzag motion of the photons leads to a redistribution of the radiation in direction, just as in the continuum. Precisely this led to the increase in $q$ (curve 3 in Fig. 8) at low $t_{\mathrm{a}}$ in the plane layer.
4.3.3. Now let us turn to other sources of the emission from a gas containing a condensed dispersed phase. First, let $I_{\mathrm{a}}^{0}=I_{50}=0, I_{\mathrm{p}}^{0} \neq 0$-in this case there is only the primary radiation of the macroscopic particles [ $g_{p}(x)$ according to (4.26)]. The atoms only absorb the radiation of the particles. Let us denote the corresponding intensity as $I_{2 \mathrm{p}}$. In Fig. 11 the dot-dash curves show the variation of the corresponding intensity in the region of the spectral line in a cylinder for homogeneous initial emission from the particles. As the optical density of atoms increases, the intensity $I_{u p}$ caused by the emission from the particles declines owing to absorption by the atoms. Second, let us assume that $I_{\mathrm{a}}^{0}=I_{\mathrm{p}}^{0}=0 ; I_{50} \neq 0$-here there are only the primary surface sources. The corresponding intensities $I_{S}$ also decline with increasing $t_{\mathrm{a}}$ owing to absorption by atoms (Fig. 11; dashed curves, without taking direct account of arrival of light from the opposite wall).

Just as in the case of the radiation from particles (Sec. 4.2), the combination of different primary sources yields an intensity obtained by summing the intensities $I_{v a}, I_{v p}$, and $I_{S}$ arising from the different sources. Figure 11 shows


FIG. 12. Typical diagram of the arrangement of a diagnostic instrument. $L$-radiation source, 1,7 -focusing optics; 2,6 -optical viewing tubes; 3-surface surrounding the volume under study; 4, 5-the part of the surface adjoining the tube; 8-spectral instrument.
the contour of the line in a homogeneous emitter in the absence of particles (a) described by the form $I_{\mathrm{a}}=I-\exp \left(t_{\mathrm{a}}\right)$, and the contour $\Sigma$, which is obtained for identical temperatures of the atoms, the particles, and the surface. The equilibrium relationship (3.30) in the case in which both the atoms and the particles emit and Eq. (4.24) holds can be written as:

$$
\begin{equation*}
\left(I_{v a} / I_{\mathrm{a}}^{0}\right)+\left(I_{v p} / I_{\mathrm{p}}^{0}\right)+\left(I_{S} / I_{S 0}\right)=1-\exp \left(-t_{\mathrm{a}}-t_{\mathrm{p}}\right) . \tag{4.32}
\end{equation*}
$$

The correctness of the equation does not depend on the magnitude of $\lambda_{\mathrm{p}}$. As $\lambda_{\mathrm{p}}$ varies, only the individual terms vary, but not the sum. As we said above, e.g., when $\lambda_{\mathrm{p}}=1$ we have $I_{u \mathrm{p}}=0$, while when $\lambda_{\mathrm{p}}=0$ we find that $I_{S}=0$ with an invariant sum. Thus the particles substantially affect the radiation in the spectral lines. The strongest agents are the intrinsic emission from the particles and the scattering of the emission from the walls where the optical density of the atoms is smaller, i.e., either everywhere in optically thin layers, or in the tails of the lines for which the optical density of the centers is large $\left(t_{\mathrm{a}}\left(v_{0}\right)>1\right)$.

### 4.4. Taking account of anisotropic scattering of the radiation of the surfaces in experiments in diagnostics

In studying a gas containing a condensed dispersed phase one often must deal with closed volumes. Special apertures are made in the walls surrounding the volume, fitted with optical viewing tubes. A typical diagram of the setup of spectral equipment is shown in Fig. 12. The equipment is set up so that surfaces that might emit and reflect light do not lie in the field of view of the recording instrument. The entry into the instrument of radiation from the walls scattered by particles usually can be taken into account as was done in Secs. 4.2 and 4.3 by using the transport approximation, and hence, assuming isotropic scattering. Exceptions include cases in which the directionality of the scattering governs the pattern. Such cases include the possible entry into the spectral instrument of singly scattered radiation of the walls. Multiple scattering of the radiation usually erases the traces of directionality of scattering. If the particles are large $(D \gg 1)$ and their indicatrices are extended forward, then the spectral instrument can receive the singly scattered radiation of those


FIG. 13. Diagram of the scattering of radiation of a plane wall in a cylindrical field of view.
regions 5 of the wall opposite the instrument 8 that are adjacent to the viewing tube. We shall derive below the relationships ${ }^{41}$ needed to take account of the effect, and also the conditions that must be fulfilled to eliminate it experimentally.

Let the field of view of the spectral instrument be a cylinder with radius $R$ and length $l$ equal to the depth of the object, while the entrance aperture of the instrument is defined by a cone with an opening angle $\beta_{\text {entr }}$ and is everywhere the same inside the cylinder (Fig. 13). The condition of constant aperture is fulfilled if the entrance optics lies far enough from the object.

The recording instrument can receive only the light that passes through the field of view or is scattered (or emitted) in it inside the entrance aperture. Let us study the infinite plane surface of a wall with a window of radius $\rho_{S 0}$ ( $\rho_{S 0} \geqslant R$ ). We shall asume the medium to be optically homogeneous. In the cylindrical system of coordinates shown in Fig. 13, the coordinates of the points on the surface can take on the values: $z_{S}^{\prime}=0, \rho_{0 S} \leqslant \rho_{S}^{\prime} \leqslant \infty, 0 \leqslant \varphi_{S}^{\prime} \leqslant 2 \pi$. Let the intensity of the radiation of the wall depend only on the radius $\rho_{n S}^{\prime}$ : $I\left(\rho_{S}^{\prime}\right)$. Let us find the intensity of the primary sources $g_{S}$ at an arbitrary point of the field of view $\mathrm{r}(\rho, \varphi, z)$ by using Eq. (3.9). We can find from Fig. 13:

$$
\begin{align*}
& \left|\mathbf{r}_{S}^{\prime}-\mathbf{r}\right|^{2}=\rho_{S}^{\prime 2}+\rho^{2}-2 \rho \rho_{S}^{\prime} \cos \varphi_{S}^{\prime}+z^{2} \\
& \cos \left(\mathbf{n}_{S}^{\prime} \Omega_{S}^{\prime}\right)=z /\left|\mathbf{r}_{S}^{\prime}-\mathbf{r}\right|  \tag{4.33}\\
& \mathrm{d}^{2} \mathbf{r}_{S}^{\prime}=\rho_{S}^{\prime} \mathrm{d} \rho_{S}^{\prime} \mathrm{d} \varphi_{S}^{\prime}
\end{align*}
$$

Substituting these expressions into (3.9) and taking account of the indicated limits of integration, we obtain

$$
\begin{align*}
g_{S}\left(\mathrm{r}, \vec{\Omega}_{\mathrm{e}}\right)= & \frac{\lambda z}{4 \pi} \int_{\rho_{S O}}^{\infty} \rho_{S}^{\prime} \mathrm{d} \rho_{S}^{\prime} I\left(\rho_{S}^{\prime}\right) \int_{0}^{2 \pi} \mathrm{~d} \varphi_{\mathrm{S}}^{\prime} \chi\left(\vec{\Omega}_{\mathrm{s}}^{\star} \vec{\Omega}_{\mathrm{e}}\right) \\
& \times \frac{\exp \left[-\alpha\left(\rho_{S}^{\prime 2}+\rho^{2}-2 \rho \rho_{S}^{\prime} \cos \varphi_{S}^{\prime}+z^{2}\right]^{1 / 2}\right.}{\left(\rho_{S}^{\prime 2}+\rho^{2}-2 \rho \rho_{S}^{\prime} \varphi_{S}^{\prime}+z^{2}\right)^{3 / 2}} \tag{4.34}
\end{align*}
$$

Let us study in greater detail the scattering angle $\gamma_{S}=\vec{\Omega} \hat{S} \vec{\Omega}_{e}$. The direction of the unit vector in the cylindrical system is fixed by the angle $\beta$ between the vector and a straight line parallel to the axis of the cylinder, and by the azimuth $\varphi$. That is, one can write $\vec{\Omega}(\beta, \varphi)$. The direction of the vector $\vec{\Omega}_{S}^{\prime}\left(\beta_{1}, \varphi_{1}\right)$ depends on the mutual arrangement of the points $\mathbf{r}_{S}^{\prime}$ and $\mathbf{r}^{\prime}$. We see from Fig. 13 that $\cos \beta_{1}=z /\left|\mathbf{r}_{S}^{\prime}-\mathbf{r}\right|$, while we can find $\varphi_{1}$ from the triangle $O A B$ formed in the plane $z=0$ by the projections of the vectors $\mathbf{r}$ and $\mathbf{r}-\mathbf{r}_{s}^{\prime}$ :

$$
\sin \varphi_{1}=\sin \varphi_{S}^{\prime} \cdot \rho_{S}^{\prime} /\left(\rho^{\prime 2}+\rho^{2}-2 \rho \rho^{\prime} \cos \varphi_{S}^{\prime}\right)^{1 / 2}
$$

Thus the scattering angle depends on the coordinates $\mathbf{r}$ and $\mathbf{r}_{S^{\prime}}$, on the direction into the entrance aperture $\vec{\Omega}_{e}\left(\beta_{e}, \varphi_{e}\right)$, and is determined by the known trigonometric formula

$$
\cos \gamma_{S}=\cos \beta_{1} \cdot \cos \beta_{\mathrm{e}}+\sin \beta_{1} \cdot \sin \beta_{\mathrm{e}} \cdot \cos \left(\varphi_{\mathrm{e}}-\varphi_{1}\right)
$$

When we measure an almost parallel beam of light along the axis ( $\beta_{e} \rightarrow 0, \cos \beta_{e} \rightarrow 1$ ), we find

$$
\begin{align*}
\cos \gamma_{s}=\cos \left(\vec{\Omega}_{S}^{\hat{\Omega}} \vec{\Omega}_{e}\right) & =\cos \beta_{1} \\
& =\cos \left(\mathbf{n}_{S}^{\hat{S}} \vec{\Omega}_{S}^{\prime}\right)=z /\left|\mathbf{r}_{S}^{\prime}-\mathbf{r}\right| \tag{4.35}
\end{align*}
$$

The probability that the light, after scattering in the direction $\vec{\Omega}_{e}$ at the point $\mathbf{r}$ will reach the boundary $z=l$ even without subsequent interactions with the medium is determined by the expression

$$
\begin{equation*}
q=q_{0}=\exp \left[-\alpha(l-z) / \cos \beta_{\mathrm{e}}\right] \tag{4.36}
\end{equation*}
$$

In contrast to the infinite emitters discussed previously, here we study the output of radiation, not through a lateral surface of the cylinder, but along the axis of the cylinder.

To find the flux incident after single scattering into the entrance aperture, we must multiply the intensity scattered per unit volume ( $\alpha g_{S}$ ) by the probability of escape in (4.36), and integrate over the field of view of the instrument and the solid angle of the entrance aperture. The volume elements of the cylinder and the entrance solid angle are defined by the expressions

$$
\begin{aligned}
& \mathrm{d}^{3} \mathrm{r}=\rho \mathrm{d} \rho \mathrm{~d} \varphi \mathrm{~d} z \\
& \mathrm{~d} \vec{\Omega}_{\mathrm{e}}=\sin \beta_{\mathrm{e}} \mathrm{~d} \beta_{\mathrm{e}} \mathrm{~d} \varphi_{\mathrm{e}}
\end{aligned}
$$

As a result we find

$$
\begin{align*}
W_{\text {scat } 1}= & 2 \pi \alpha \int_{0}^{l} \mathrm{~d} z \int_{0}^{R} \rho \mathrm{~d} \rho \int_{0}^{\beta_{\text {entr }}} \mathrm{d} \beta_{\mathrm{e}} \sin \beta_{\mathrm{e}} \\
& \times \int_{0}^{2 \pi} \mathrm{~d} \varphi_{\mathrm{o}} g_{S}\left(\mathbf{r}, \vec{\Omega}_{\mathrm{e}}\right) \exp \left[-\alpha(l-z) / \cos \beta_{\mathrm{e}}\right] \tag{4.37}
\end{align*}
$$

Here we have integrated over $\varphi$. When we take account of (4.34), Eq. (4.37) is a sixfold integral. The multiplicity of the integral is determined by the scattering angle $\gamma_{S}$ entering into the indicatrix. $\gamma_{S}$ depends on from where ( $\rho_{S}^{\prime}, \varphi_{S}^{\prime}$ ) the light came to the point $\mathbf{r}$, where the point lies ( $\rho, z$ ), and to where the light goes after scattering ( $\beta_{e}, \varphi_{e}$ ). Consequently one can integrate over these variables. We obtain from (4.37) upon taking account of the fact that $\cos \beta_{e} \rightarrow 1$ :

$$
\begin{equation*}
W_{\text {scat } 1}=2 \pi^{2} \beta_{\mathrm{entr}}^{2} \alpha \int_{0}^{l} \mathrm{~d} z \int_{0}^{R} \rho \mathrm{~d} \rho g_{S}(\mathbf{r}) \exp [-\alpha(l-z)] \tag{4.38}
\end{equation*}
$$

In the case of an almost parallel beam we can assume that the intensity of the scattered radiation inside the small solid angle $\left(\pi \beta_{\text {entr }}^{2}\right)$ is invariant, therefore, by dividing (4.38) by $\pi \beta_{\text {entr }}^{2}$ and $\pi R^{2}$, we find

$$
\begin{equation*}
I_{\text {scat } 1}=2 \alpha R^{-2} \int_{0}^{l} \mathrm{~d} z \int_{0}^{R} \rho \mathrm{~d} \rho g_{S}(\mathbf{r}) \exp [-\alpha(l-z)] \tag{4.39}
\end{equation*}
$$

In the special case being discussed we shall perform some illustrative calculations. Let us introduce the relative coordinates $x=\rho / R, x_{S}^{\prime}=\rho_{S}^{\prime} / R$, and $z^{\prime}=z / l$ and the notation for the square of the distance in these coordinates

$$
y_{1}=x_{S}^{\prime 2}+x^{2}-2 x_{S}^{\prime} x \cos \varphi_{S}^{\prime}+\left(z^{\prime} l / R\right)^{2}
$$

Then, when $I\left(\rho_{S}^{\prime}\right)=I_{S 0}$, we can represent (4.34) in the form

$$
\begin{align*}
g_{S}\left(x, z^{\prime}\right) / I_{S 0}= & \lambda z^{\prime}(l / R)(4 \pi)^{-1} \int_{x_{S 0}}^{\infty} x_{S}^{\prime} \mathrm{d} x_{S}^{\prime} \int_{0}^{2 \pi} y_{1}^{-3 / 2} \mathrm{~d} \varphi_{S}^{\prime} \\
& \times \exp \left(-\frac{t_{0}}{l / R} y_{1}^{1 / 2}\right) \chi\left(\gamma_{S}\right) . \tag{4.40}
\end{align*}
$$

Here we have $t_{0}=\alpha l, x_{S 0}=\rho_{S O} / R$. Instead of (4.39) we have

$$
\begin{align*}
I_{\text {scat } 1} / I_{S O}= & 2 t_{0} \int_{0}^{1} \mathrm{~d} z^{\prime} \int_{0}^{1} x \mathrm{~d} x \\
& \times \exp \left[-t_{0}\left(1-z^{\prime}\right)\right] g_{S}\left(x, z^{\prime}\right) I_{S 0}^{-1} \tag{4.41}
\end{align*}
$$

We see from the presented relationships that the effect of single anisotropic scattering depends on the probability of survival $\lambda$, the optical density of the object $t_{0}$, the indicatrix $\chi$, the relative length of the cylinder $l / R$, and the relative magnitude of the window on the wall opposite the observer $x_{S 0}$. The meaning of these influences lies in the following. The probability of scattering within the field of view of the cylinder and the probability of escape from it are determined by the quantities $\lambda$ and $t_{0}$, while the area of the surface from which light can enter the field of view of the instrument at angles to the axis so small that the single scattering there brings the light into the instrument de-
pends on the extension of the indicatrix $\chi\left(\gamma_{s}\right)$ and on the quantities $l / R$ and $\rho_{S 0} / R$. Actually, the more extended the indicatrix is, the smaller the scattering angle and the narrower the ring on the surface adjoining the field of view ( 5 in Fig. 12) from which light can enter the cylinder at the required angle. With increasing $l / R$ the width of this ring increases-the more remote points of the surface can give rise to light at the required angles in the lower part of the cylinder. Finally, with increase in the window on the illuminating wall $S$, the regions close to the cylinder whose light enters the cylinder at small angles become more remote. Hence it is possible to decrease the influence of anisotropic scattering by increasing $\rho_{S O} / R=x_{S 0}$.

Further the results of calculation asccording to (4.41) will be presented, which are useful to compare with calculation in the transport approximation. To do this, for each chosen scattering indicatrix $\chi$ one should determine by (2.6) the mean cosine $\left\langle\cos \gamma_{s}\right\rangle$. Then one must find the probability of survival $\lambda_{\mathrm{tr}}$ by using Eq. (2.9) for known $\lambda_{\mathrm{p}}$, and find $t_{\mathrm{tr}}$ for known $\lambda_{\mathrm{p}}$ by (4.28). The calculation of the intensities $I_{\mathrm{tr}}$ in the transport approximation will be performed by the formulas for an infinite plane layer, when the primary sources $g_{S \Sigma}$ are determined by Eq. (3.14), and the intensity by (4.9) with $g=g_{S \Sigma}$. In these formulas the quantities $\lambda_{\mathrm{tr}}$ and $t_{\mathrm{tr}}$ should be used instead of $\lambda_{\mathrm{p}}$ and $t_{\mathrm{p}}$.

The calculation in the transport approximation gives the contribution of all the surfaces to the observed scattering of the radiation. If $I_{\mathrm{tr}} \gg I_{\text {scat } 2}$, there is no need to take separate account of the anisotropic scattering $I_{\text {scat } 1}$. Yet if the inequality is reversed one must take account of $I_{\text {scat } 1}$.

The illustrative calculations were performed in the case of large particles $(D \gg 1)$. Particles of opposite scattering properties were studied (see the Table above): 1) of absolutely absorbing particles, in which the indicatrix $\chi_{\mathrm{d}}$ is determined by (2.17) or approximately by (2.19); here $\lambda_{\mathrm{p}}=0.5 ; 2$ ) absolutely scattering particles with an indicatrix $\chi_{\mathrm{d}+\mathrm{g}}$ according to (2.25) and $\lambda_{\mathrm{p}}=1$.

Figure 14 shows the intensities calculated by (4.41) as functions of the relative thickness of the object $l / R$ and of the relative radius of the window opposite the observer $x_{S O}=\rho_{S O} / R$. The solid curves demonstrate the strong growth of the anisotropic radiation with increasing $l / R$. In curve $l$ the diffraction indicatrix is very narrow; the direction to the first minimum with $D=50$ is determined by (2.18) by the angle $\gamma_{\mathrm{s} 1}=4.3^{\circ}$. Here single anisotropic scattering with an object extended sufficiently in the direction of observation yields substantial extra light. According to the formulas of the transport approximation for $t_{\mathrm{p}}=0.5$, we obtain $\left\langle\cos \gamma_{\mathrm{s}}\right\rangle_{1}=0.9994, \lambda_{\mathrm{tr} 1}=5 \times 10^{-4}, t_{\mathrm{tr} 1}=0.25$. Evidently such a small value of $\lambda_{\text {tr }}$ yields a very small intensity of scattering ( $I_{\mathrm{tr} 1} / I_{S 0}=6 \times 10^{-5}$ ). Consequently it is necessary here to take account of anisotropic scattering.

When the particles only scatter light, the intensity of the extra illumination by anisotropic scattering becomes larger for two reasons: $\lambda_{\mathrm{p}}$ becomes larger and the indicatrix is less extended than in the diffraction case for the same $D$ (curve 2). The intensity is increased even to a greater extent in the transport approximation. Here we have:


FIG. 14. Dependence of the intensity of radiation singly scattered by particles on the relative length $l / R$ of the object (solid curves) and on the relative radius of the window opposite the observer $x_{S 0}=\rho_{S O} / R$ (dashed curves) for $t_{\mathrm{p}}=0.5, t_{\mathrm{a}}=0.1-\chi=\chi_{\mathrm{d}}, D=50, x_{S 0}=1 ; 2-\chi_{\mathrm{d}+\mathrm{g}}, D=50$, $x_{S 0}=1 ; 3-\chi_{\mathrm{d}}, D=5, l / R=1 ; 4-\chi_{d+g}, D=5, l / R=10$.
$\left\langle\cos \gamma_{s}\right\rangle_{2}=0.95, \quad \lambda_{\mathrm{tr1}}=1, \quad t_{\mathrm{tr} 2}=0.0244$. Here $I_{\mathrm{tr} 2} / I_{S 0}$ $=2 \times 10^{-2}$-as is shown in Fig. 14 by tr2. Nevertheless, even for not very large $l / R$, it is necessary to take account of anisotropic scattering in this case as well. We emphasize that both curves 1 and 2 are calculated under the condition that the windows on both sides of the object of study are the same ( $x_{S 0}=1$ ).

The dashed curves 3 and 4 in Fig. 14 demonstrate the dependence on the relative dimension of the window for different indicatrices and different values of $l / R$. Increase in the dimension of the opposite window diminishes the extra illumination $I_{\text {scat } 1}$. The diffraction indicatrix of curve 3 is not very strongly extended forward ( $D=5, \gamma_{\mathrm{s} 1} \approx 50^{\circ}$ ). In the transport approximation for curve 3 we have $\left\langle\cos \gamma_{s}\right\rangle_{3}=0.93, \quad \lambda_{\mathrm{tr} 3}=0.064, \quad t_{\mathrm{tr} 3}=0.267, \quad I_{\mathrm{tr} 3} / I_{S 0}$ $=9.3 \times 10^{-3}$ (tr3 in the diagram). The diagram implies that the transport approximation yields an intensity of extra illumination substantially larger than those obtained upon taking account of single scattering for all $x_{50} \geqslant 1$. Consequently in this case one can use the transport approximation for taking account of the scattered radiation of the walls. We note only that curve 3 has a broad window: $l / R=1$.

In the case of curve 4 (scattering particles) where $l / R$ $=10$, the effect of anisotropic scattering is substantially larger. The transport approximation yields $\left\langle\cos \gamma_{S}\right\rangle_{4}$ $=0.91, \lambda_{\mathrm{tr} 4}=1, t_{\mathrm{tr} 4}=0.044, I_{\mathrm{tr} 4} / I_{\mathrm{SO}}=3.4 \times 10^{-2}$. As we see from Fig. 14 (curve 4, tr 4), in order to use only transport calculations, one must have a large ratio between the dimensions of the windows on the opposite walls ( $x_{50}>7$ ). For a smaller ratio one must take account of anisotropic scattering. To estimate the error in the calculations in the transport approximation, one can compare $I_{\mathrm{tr}} / I_{S 0}$ with the


FIG. 15. Variation inside a spectral line of the intensity of emission from the walls. $D=50, t_{\mathrm{p}}=0.5, \quad l / R=25, \quad x_{S 0}=1 ; \quad 1-\chi=\chi_{\mathrm{d}}, \quad \lambda_{\mathrm{p}}=0.5$; $2-\chi=\chi_{d+8}, \lambda_{\mathrm{p}}=1 ; 3-\chi=1, t_{\mathrm{tr}}=0.0244, \lambda_{\mathrm{tr}}=1$.
corresponding equilibrium values, when according to (4.31) the equation must be fulfilled that $I_{\mathrm{tr}} / I_{S 0}$ $=1-\exp \left(-t_{\mathrm{tr}}\right)$ when $\lambda_{\mathrm{p}}=\lambda_{\mathrm{tr}}=1$. A calculation by this formula yielded a value of $I_{\mathrm{tr}} / I_{S 0} 20 \%$ larger than that calculated, which characterizes the error of the numerical calculation. Comparison of the calculated intensity of anisotropic scattering $I_{\text {scat } 1}$ with the quantity $1-\exp \left(-t_{\mathrm{p}}\right)$ $=0.39$ shows that the calculated values are much smaller than the equilibrium values. This should have been expected, since in the anisotropic calculations it is very important to remove the region of the wall lying at the site of the opposite aperture-it plays a large role and renders the problem especially non-equilibrium. In isotropic scattering this separating out is not so essential, since all parts of the infinite walls are equivalent in scattering.

Now let us turn to the influence of single anisotropic scattering on the emission in a spectral line. Equations (4.40) and (4.41) imply that the intensity of the scattered radiation is proportional to the product $\lambda t_{0}$, which is constant within the line in agreement with (4.25):

$$
\lambda t_{0}=\lambda\left(t_{\mathrm{a}}+t_{\mathrm{p}}\right)=\lambda_{\mathrm{p}} t_{\mathrm{p}}
$$

Consequently $\lambda_{\mathrm{p}} t_{\mathrm{p}}$ determines the absolute magnitude of $I_{\text {scat } 1}$, together with the geometric dimensions of the field of view and the windows, Yet the dependence on $t_{\mathrm{a}}$ (or on the frequency) is determined in (4.40) and (4.41) by the exponentials, into whose index $t_{0}=t_{\mathrm{p}}+t_{\mathrm{a}}(v)$ enters. Evidently, as $t_{\mathrm{a}}(v)$ increases, the intensity of the scattered radiation must decline, owing to absorption of the incident and scattered radiation of the walls by atoms.

Figure 15 shows the variation inside a spectral line of the intensity of anisotropic and singly scattered radiation (curves 1 and 2). Curve 3 was obtained in the transport approximation in the case of the indicatrix of curve 2 . The


FIG. 16. Contours of spectral lines of cesium at 455.5 nm (a) and sodium at 589.0 nm (b). 1 -in the absence of particles, 2-in the presence of extra illumination, 3-calculation. $\Delta \Lambda=\left|\Lambda-\Lambda_{0}\right|$.
dependences that have been obtained imply that the optically thin parts of the lines ( $t_{\mathrm{a}}<1$ ) are altered most strongly by the anisotropic extra illumination. If the optical density in the center of the line $t_{\mathrm{a}}\left(v_{0}\right)$ is small, then the entire line as a whole can be strongly changed by the light of the scattering radiation of the walls. Thus relationships were derived that are necessary for taking account of single anisotropic scattering in a situation often encountered in experiments in diagnostics. The scattering by particles of light into the instrument depends on the relative thickness of the field of view, the scattering indicatrix, and the relative dimensions of the optical windows.

### 4.5. Experimental study of the influence of scattering particles on the emission from a gas flow

In Refs. 42, 43, and 45 the influence of scattering particles on the emission from a gas in the continuum and in spectral lines was studied experimentally. The object of study was a homogeneous cylindrical flow of a dust-gas mixture at atmospheric pressure. The diameter of the cylinder was 40 mm . The mixture was prepared in the supply system of a burner often used for carrying out methods of diagnostics. ${ }^{42,43}$ In studying the influence of external illumination on the emission in the continuum, the fuel (propane) was not fed into the supply system of the burner, and the dust-air mixture was at room temperature. In studying the spectral lines, propane was supplied and a flame was formed at a temperature $\approx 2000 \mathrm{~K}$. Sodium and cesium were specially introduced into the flow and their lines were studied. The dust was prepared from scattering particles of aluminum oxide $\mathrm{Al}_{2} \mathrm{O}_{3}$ (see the Table above). The dimensions of the particles in different experiments varied from 1 to $100 \mu \mathrm{~m}$. The intensities of the radiation in the lines and in the continuum, the optical density, and the flame temperature were measured. These measurements employed two mutually perpendicular optical branches, each of which was analogous to that shown in Fig. 12. Tungsten ribbon lamps SI-10-300 were used as the source $L$, while a DSF-452 spectrograph and a MDR-6 monochromator were used as the spectral instruments.

In the experiments a part of the wall 3 , including the window and the adjacent glowing region 5 were simulated by optical means (see Fig. 12). To do this, a small opaque screen was placed between the lamp $L$ and the lens 1 , which created a shadow on the boundary of the object-a simulation of the window. The peripheral regions of the lamp filament gave bright illumination together with a shadow-a simulation of region 5. The dimensions of the different regions could be altered by varying the diameter of the lens 1 . Here the direct incidence of the light of the lamp $L$ into the instrument was eliminated. First we shall study the results on the spectral lines. Figure 16 shows the contours of the resonance lines of cesium at 455.5 nm and of sodium at 589.0 nm . The lines sharply differ in their atomic optical densities. At the centers of the lines we have: $t_{\mathrm{a}}(455.5) \approx 0.3, t_{\mathrm{a}}(589.0)=3 \times 10^{2}$. The curves 1 pertain to the case in which particles are absent. The appearance of completely scattering particles of $\mathrm{Al}_{2} \mathrm{O}_{3}$ with $r_{\mathrm{p}}=1.8 \mathrm{~mm}$ and $t_{\mathrm{p}}=0.1-0.15$ did not lead to any appreciable change in the contours of the two lines. This result agrees with the calculated curves. Actually, in the case of scattering particles $\left(\lambda_{\mathrm{p}}=1\right)$, even when $t_{\mathrm{p}}=1$, one obtains a rather small change in the contour of the line (Fig. 10, solid curves). When $t_{\mathrm{p}} \approx 0.1$ the calculated effect, undoubtedly, is smaller than the error of the measurements indicated in Fig. 16 [we recall that the curves in Fig. 16 can be redrawn as dependences on $I\left(t_{\mathrm{a}}\right)$ ].

When, in addition to introducing the particles, the lamp $L$ was turned on and the screen mentioned above was installed, i.e., a simulation of a window and adjacent glowing wall was created, the contours of the two lines underwent changes. Here the light of the lamp $L$ was able to enter the spectral instrument only as a result of scattering by particles lying in the field of view of the instrument. The cesium line was changed as a whole, but the sodium line only in the tails (see Fig. 16). Qualitatively this effect agrees with that shown in Fig. 15: the greater the optical density $t_{\mathrm{a}}$, the smaller the influence of scattering by particles was. Moreover, Fig. 16 shows the dashed curves 3 calculated by (4.34) and (4.38) with account taken of the


FIG. 17. Intensities of scattered radiation in the continuum as functions of the diameter of the focusing lens $d_{i} ; x_{50}$-calculated relative radius of the simulated optical window. Solid curves-experiment; dashedcalculation. $1-2 \mathrm{r}_{\mathrm{p}}=1 \mu \mathrm{~m}, 2-4 \mu \mathrm{~m}, 3-100 \mu \mathrm{~m}$.
measured optical densities $t_{\mathrm{a}}$ and $t_{\mathrm{p}}$ and of the experimental conditions. We see that they describe the experiment well. The small difference in the cesium line is most likely determined by the not very precise measurement of the small optical density $t_{\mathrm{a}}$ in this case. Since only the relative dependences on $t_{\mathrm{a}}$ have been drawn here, they depend neither on the scattering indicatrix of the particles, nor on the features of the optical simulation of the window and the wall.

Now let us turn to the results of the measurements in the continuum. The appearance of $\mathrm{Al}_{2} \mathrm{O}_{3}$ particles in the flame without extra illumination from outside does not lead to the appearance of an appreciable change in the continuous spectrum. This confirms that the particles completely scatter light, $\lambda_{p}=1$, and hence $g_{p}=0$ according to (3.10). Yet when there is external illumination, continuous scattered radiation arises. The corresponding background can be seen in the tails of the lines in Fig. 16.

Figure 17 shows the experimental intensities for $\Lambda=0.6 \mu \mathrm{~m}$ scattered by particles in a dust-air mixture in the case of an optical simulation of a window and a wall. The intensities are plotted as functions of the diameter $d_{1}$ of lens $l$ in Fig. 12 for various dimensions of the particles, but with the same optical density $t_{\mathrm{p}} \approx 0.1$. For the given dimensions of all the optical elements, one can find the dimensions of the field of view of the spectrograph and the simulated window and wall. Since the filament of the lamp and the slit of the spectrograph are of rectangular shape, the field of view and the window are not axially symmetric. Nevertheless the assumption was made that this symmetry exists, and certain effective dimensions $\rho_{S 0}, R$, and the maximum radius of the simulated wall $\rho_{S}^{\prime}$ were found (see Fig. 13). The calculated values $x_{S 0}=\rho_{S 0} / R$ are plotted in Fig. 17 for different values of $d_{1}$. As the diameter $d_{1}$ decreases, simultaneously the dimension of the window $\rho_{S 0}\left(x_{S 0}\right)$ increases, and the greatest radius of the simulated
wall $\rho_{S}^{\prime}$ decreases. Both factors lead to a decrease in the measured scattered radiation. The intensity becomes less as the particles become coarser. The influence on the observed intensity of the dimensions of the region of the wall yielding extra illumination and of the dimensions of the particles agree with the pattern described in Sec. 4.4. To calculate the intensity one can use Eqs. (4.34) and (4.39). In the expression for $g_{S}$ one must take account of the fact that $I\left(\rho_{S}^{\prime}\right)$ depends on the direction, while the upper limit of integration over $\rho_{S}^{\prime}$ is finite. Such calculations were performed for $\chi=\chi_{d+\mathrm{g}}$ [Eqs. (2.19), (2.20), and (2.25)], and the results are presented in Fig. 17 by the dashed curves. Despite the crudity of certain simplifying assumptions of the calculation, the overall character of the behavior of the experimental and calculated curves is analogous. The magnitude of the effect is described well.

Thus the results of the experimental and theoretical studies of the spectral lines and the continuum in the presence of anisotropically scattering particles qualitatively and quantitatively agree with one another. We note here that the experimental information qualitatively confirms the correctness of the preliminary estimates made in Sec. 2.2.

## 5. ATTENUATION OF A LIGHT BEAM IN A GAS CONTAINING A CONDENSED DISPERSED PHASE

In the different methods of spectral diagnostics one must measure the optical density or characteristics of the medium governed by it.

The optical density is determined experimentally by studying the attenuation of a light beam created by a lamp or laser. The basis of the measurements is the fact that a fraction of the light reaches the observer without interacting with the medium, which is equal to $\exp \left(-t_{0}\right)$, where $t_{0}$ is the optical density of the medium in the direction of observation. When scattering of the radiation exists in the medium, the measurement of $t_{0}$ becomes complicated because the light of the probe beam after scattering can enter the spectral instrument being used to study the attenuation. The optical density $t_{0}$ characterizes all the interaction of the light with matter, while measurement of the scattered light leads to a decrease in the observable optical density as compared with the true value.

The influence of light scattering by particles on the attenuation of a beam has been treated in individual cases in a series of papers. ${ }^{47-50}$ In Refs. 47 and 48 the attenuation of the beam was calculated with account taken of single and multiple scattering in the case of a scattering indicatrix described by a Gaussian function. Reference 49 studied forward single scattering and investigated the influence of a small entrance aperture on the results of measurements in a homogeneous medium. In Ref. 50 results were obtained by using diffusion theory, and it was shown that this approach is suitable in the case of indicatrices not extended forward. Here we shall use the probabilistic method, which makes it possible to obtain results over a broader range of conditions.

Let the probe ray be a parallel light beam, which illuminates a cylindrical column of radius $R$ (Fig. 18). Just as


FIG. 18. Diagram of the probe beam and scattering events in it.
in Sec. 4.4 we shall assume that the field of view is a cylinder of radius $R$, and the entrance aperture of the measuring apparatus is characterized by the angle $\beta_{\text {entr }}$. Let us study the possibilities for light to enter the instrument after scattering. First (broken curve 1 in Fig. 18), a photon can escape the probe ray owing to scattering in it, then return to the beam after scattering outside the ray, and then, being scattered again in the ray, direct itself toward the instrument. To avoid such entrance of scattered light into the instrument, one should make the probe ray narrow enough that the condition is fulfilled that

$$
\begin{equation*}
t_{0}=2 \int_{0}^{R} \alpha \mathrm{~d} \rho \ll 1 \tag{5.1}
\end{equation*}
$$

Second, a photon can enter the instrument if scattering does not remove it through the lateral surfaces of the cylinder (broken curves 2 and 3 in Fig. 18). These scattering events must be taken into account.

When the condition (5.1) is fulfilled, we can assume that the optical characteristics $\alpha, \lambda$, and $\chi$ do not vary in the cross section of the ray. That is, they do not depend on $\rho$ and $\varphi$, but depend only on $z$ (or on $t$ ).

Let us find the radiation fluxes incident on the instrument within the entrance aperture $\beta_{\text {entr }}$ when condition (5.1) is fulfilled. The light flux arriving directly from the illuminator in the direction $\vec{\Omega}_{0}$ (along the axis of the cylinder at the optical depth $t$ is determined by the flux $I_{0} \pi R^{2}$ incldent on the end $(z=0)$ of the cylinder and by the probability of reaching the depth $t$ without interacting with the medium $\exp (-t)$. The probability of primary interaction in the layer from $t$ to $t+\mathrm{d} t$ is $I_{0} \pi R^{2} \exp (-t) \mathrm{d} t$. Let $p\left(z, \rho, \vec{\Omega}_{0}\right)$ be the probability that a photon with the initial direction $\vec{\Omega}_{0}$ that interacts with the medium at the depth $t$ will escape into the aperture $\beta_{\text {entr }}$. Then the flux of light scattered into the aperture can be written in the form

$$
\begin{equation*}
W_{\text {scat }}=I_{0} \pi R^{2} \int_{0}^{t_{0}} \exp (-t) \mathrm{d} t p\left(z, \rho, \vec{\Omega}_{0}\right) \tag{5.2}
\end{equation*}
$$

The total flux entering the instrument consists of the flux transmitted without interacting with the medium and the scattered flux

$$
\begin{equation*}
W=I_{0} \pi R^{2} \exp \left(-t_{0}\right)+W_{\text {scat }} \tag{5.3}
\end{equation*}
$$

Now the problem consists in finding the probability of escape $p\left(z, \rho, \vec{\Omega}_{0}\right)$. Let us use the general equation (3.32) for the probability of escape: Taking account of the fact that $\alpha$, $\lambda$, and $\chi$ do not depend on $\rho$, and of the axial symmetry, and replacing $(-\vec{\Omega})$ by ( $\vec{\Omega}_{0}$, we find instead of (3.32):

$$
\begin{align*}
p\left(z, \rho, \vec{\Omega}_{0}\right)= & \frac{\lambda(z)}{4 \pi} \int_{0}^{l} \alpha\left(z^{\prime}\right) \mathrm{d} z^{\prime} \int_{0}^{2 \pi} \mathrm{~d} \varphi^{\prime} \int_{0}^{R} \rho^{\prime} \mathrm{d} \rho^{\prime} \\
& \times\left\{\chi\left(z, \vec{\Omega}_{\hat{0}} \vec{\Omega}^{\prime}\right) p\left(z^{\prime}, \rho^{\prime}, \vec{\Omega}^{\prime}\right)\right. \\
& \left.\times \exp \left[-\tau\left(\mathbf{r} \rightarrow \mathbf{r}^{\prime}\right)\right]\left|\mathbf{r}-\mathbf{r}^{\prime}\right|^{-2}\right\} \\
& +p_{1}\left(z, \rho, \vec{\Omega}_{0}\right) \tag{5.4}
\end{align*}
$$

Here, as we see from Fig. 18, the angle of the first scattering event is determined, as before in similar problems, by the relationship

$$
\cos \left(\vec{\Omega}_{\hat{0}}^{\hat{\Omega^{\prime}}} \vec{\prime}\right)=\left(z^{\prime}-z\right) /\left|\mathbf{r}^{\prime}-\mathbf{r}\right|
$$

Here $p_{1}\left(z, \rho, \vec{\Omega}_{0}\right)$ is the probability of escape into the aperture of the instrument of radiation interacting with the medium in the region $z, \rho$ and having the initial direction $\vec{\Omega}_{0}$ without subsequent scattering (after the first event). This probability is determined by the probabilities of successive events: 1) scattering in the chosen region $\lambda(z) ; 2)$ a direction after scattering inside an element of the entrance aperture. The element of solid angle has the form $\mathrm{d} \vec{\Omega}_{e}=2 \pi \sin \beta_{\mathrm{e}} \mathrm{d} \beta_{\mathrm{e}}=-2 \pi \mathrm{~d} \eta_{\mathrm{e}}$, where $\sim \eta_{\mathrm{e}}=\cos \beta_{\mathrm{e}}$. Then the probability of scattering in the chosen direction is determined by the expression $\left.\chi\left(z, \eta_{\mathrm{e}}\right) \mathrm{d} \eta_{\mathrm{e}} / 2 ; 3\right)$ passage to the boundary $z=l$ without interacting with the medium $\exp \left(-\left(t_{0}-t\right) / \eta_{e}\right)$. To obtain the probability of escape with single scattering, we must multiply the probabilities of the successive events and integrate over the entrance solid angle:

$$
\begin{align*}
p_{1}\left(z, \rho, \vec{\Omega}_{0}\right)= & 0.5 \lambda(z) \int_{\eta_{\text {entr }}}^{1} \chi\left(z, \eta_{e}\right) \\
& \times \exp \left[-\left(t_{0}-t\right) / \eta_{\mathrm{e}}\right] \mathrm{d} \eta_{\mathrm{e}} \tag{5.5}
\end{align*}
$$

Let us simplify (5.4). Since the beam is narrow we can assume that the probability of escape does not depend on the radius $\rho$. Then it is most convenient to seek the probability of escape from the axis of the cylinder for $\rho=0$. In this case, instead of (5.4), we find in an optically homogeneous medium:

$$
\begin{align*}
p\left(z, 0, \vec{\Omega}_{0}\right)= & 0.5 \alpha \lambda \int_{0}^{l} \mathrm{~d} z^{\prime} \int_{0}^{R} \rho^{\prime} \mathrm{d} \rho^{\prime}\left[\chi\left(\vec{\Omega}_{0} \vec{\Omega}^{\prime}\right) p\left(z^{\prime}, 0, \vec{\Omega}^{\prime}\right)\right. \\
& \times \exp \left\{-\alpha\left[\rho^{\prime 2}+\left(z-z^{\prime}\right)^{2}\right]^{1 / 2}\right\} \\
& \left.\times\left[\rho^{\prime 2}+\left(z-z^{\prime}\right)^{2}\right]^{-1}\right]+p_{1}\left(z, 0, \vec{\Omega}_{0}\right) \tag{5.6}
\end{align*}
$$

Here we have integrated over $\varphi^{\prime}$, since when $\rho=0$ none of the functions entering into the integrand depends on $\varphi^{\prime}$. Moreover we have:

$$
\begin{equation*}
\cos \left(\vec{\Omega}_{\hat{0}} \vec{\Omega}^{\prime}\right)=\left(z-z^{\prime}\right) /\left[\left(z-z^{\prime}\right)^{2}+\rho^{2}\right]^{1 / 2} \tag{5.7}
\end{equation*}
$$

Although the sought probability $p\left(z, \rho, \vec{\Omega}_{0}\right)$ characterizes in Eqs. (5.4) and (5.6) the escape of a photon initially traveling along the axis, the integrand contains the probability $p\left(z^{\prime}, \rho, \Omega^{\prime}\right)$, which depends on the direction $\vec{\Omega}^{\prime}$, which is determined by the arrival of the photon at $\mathbf{r}^{\prime}$ after first scattering at $\mathbf{r}$. The problem is greatly simplified if we make the assumption

$$
\begin{equation*}
p\left(z^{\prime}, \rho, \vec{Q}^{\prime}\right)=p\left(z^{\prime}, \rho, \vec{Q}_{0}\right) \tag{5.8}
\end{equation*}
$$

That is, the probability of escape in repeated scattering events does not depend on $\vec{\Omega}^{\prime}$. This simplification agrees with the conditions of scattering in a narrow beam, when the radiation escaping sideways does not return to the beam. Here primarily the radiation remains in the beam that has been scattered at small angles to the axis. The assumption (5.8) can be applied in certain cases also when the condition (5.1) is not fulfilled. Thus, in isotropic scattering the indicatrix $\chi=1$ does not depend on the initial direction. Therefore we can set $\vec{\Omega} / / \vec{\Omega}_{0}$. In the opposite case of an indicatrix strongly extended forward, the deviations from the axial direction are small. Therefore we need not take them into account.

Let us introduce the relative coordinates

$$
\begin{equation*}
\rho^{\prime} / R=x^{\prime}, \quad z^{\prime} / l=Z^{\prime}, \quad z / l=Z . \tag{5.9}
\end{equation*}
$$

We shall omit in the arguments of $p$ the radius $\rho$ and the direction $\vec{\Omega}_{0}$. Then, instead of (5.6), taking account of the assumption (5.8), we obtain

$$
\begin{align*}
p(Z) & =0.5 \lambda t_{0} \int_{0}^{1} \mathrm{~d} Z^{\prime} p\left(Z^{\prime}\right) A\left(Z-Z^{\prime}\right)+p_{1}(Z) \\
& =\lambda V p\left(Z^{\prime}\right)+p_{1}(Z) \tag{5.10}
\end{align*}
$$

where

$$
\begin{align*}
A\left(Z, Z^{\prime}\right)= & \left(\frac{R}{l}\right)^{2} \int_{0}^{1} x^{\prime} \mathrm{d} x^{\prime} \chi\left(\gamma_{s}\right) \\
& \times \frac{\exp \left\{-t_{0}\left[x^{\prime 2}(R / l)^{2}+\left(Z-Z^{\prime}\right)^{2}\right]^{1 / 2}\right\}}{x^{\prime 2}(R / l)^{2}+\left(Z-Z^{\prime}\right)^{2}} \tag{5.11}
\end{align*}
$$

$$
\begin{equation*}
\cos \gamma_{\mathrm{s}}=\left(Z^{\prime}-Z\right) /\left[x^{\prime 2}(R / l)^{2}+\left(Z-Z^{\prime}\right)^{2}\right]^{1 / 2} \tag{5.12}
\end{equation*}
$$

We can solve Eq. (5.10) by the method of successive approximations, just as in finding $q$ in Sec. 4.1.

Let us represent the solution as a series

$$
\begin{equation*}
p(Z)=\sum_{k=1}^{m} \lambda^{k} P_{\mathbf{k}}(Z) \tag{5.13}
\end{equation*}
$$

Here $P_{1}(Z)=p_{1}(Z) / \lambda(Z)$, where $p_{1}(Z)$ is determined by Eq. (5.5), i.e.,

$$
\begin{align*}
& P_{1}(Z)=0.5 \int_{\eta_{\text {entr }}}^{1} \chi\left(Z, \eta_{\mathrm{e}}\right) \exp \left[-\left(t_{0}-t\right) / \eta_{\mathrm{e}}\right] \mathrm{d} \eta_{\mathrm{e}}  \tag{5.14}\\
& P_{k}=V P_{k-1}=V^{k-1} P_{1} \tag{5.15}
\end{align*}
$$

We find the following expression for the omitted part of the infinite series, analogously to (4.23) with $\|V\| \lambda<1$ :

$$
\begin{equation*}
p(Z)-\sum_{k=1}^{m} \lambda^{k} P_{k}(Z) \leqslant P_{1 \max } \frac{\|V\|^{m+1} \lambda^{m+1}}{1-\|V\| \lambda} \tag{5.16}
\end{equation*}
$$

In the case in which we take account only of single scattering we must set $p=p_{1}$, where $p_{1}$ is determined by (5.5).

We can calculate the magnitude of the measurable flux of scattered radiation $W_{\text {scat }}$ by using Eqs. (5.2) and (5.14) and solving Eq. (5.10) by the method of successive approximations of (5.13) and (5.15). The cited relationships imply that $W_{\text {scat }}$ depends on the optical characteristics of the medium $t_{0}, \lambda$, and $\chi\left(\gamma_{s}\right)$, on the entrance aperture of the measuring instrument, and on the relative width of the beam $R / l$ [see (5.11) and (5.12)]. Let us discuss the results of the calculations illustrating these relationships. The calculations were performed, as in Sec. 4.4, in the case of large particles $(D>1)$, when the indicatrices are determined by (2.17) and (2.19) (absorbing particles, $\chi=\chi_{\mathrm{d}}$, $\lambda_{\mathrm{p}}=0.5$ ) and by (2.25) (scattering particles, $\chi=\chi_{\mathrm{d}+\mathrm{g}}$, $\lambda_{\mathrm{p}}=1$ ). Figures 19-21 show the results for $k_{\mathrm{a}}=0, t_{0}=t_{\mathrm{p}}$, i.e., outside spectral lines. The calculation of the probability of escape with multiple scattering was performed for $m=5$, which allowed an error, according to (5.16), no greater than several percent.

Figure 19 shows the dependence of the radiation scattered into the instrument on the entrance aperture. Curves $1-3$ pertain to the same indicatrix $\chi_{d}(D=20)$. On the axis is shown the value of the angle $\gamma_{S 1}=0.192$ radian determined from (2.18). Increase in the aperture $\beta_{\text {entr }}$ up to $\gamma_{S 1}$ leads to increase in the flux of scattered radiation. The increase in $W_{\text {scat }}$ ceases with further increase in $\beta_{\text {entr }}$, since the aperture is so large that practically all the scattered radiation for the indicatrix of (2.19) has already passed into the instrument when $\beta_{\text {entr }} \approx \gamma_{S 1}$. Taking account of multiple scattering increases the calculated flux (curves 2 and 3 lie higher than curve 1). Finally, $W_{\text {scat }}$ is influenced by the relative width of the beam $R / l$ : with increasing $R / l$ more scattered light enters the instrument, since for a given optical density in a broader beam, the probability of loss of light after scattering through the lateral surface of the field of view is smaller. Curves $4-6$ pertain to the indicatrix $\chi_{\mathrm{d}+\mathrm{g}}$; taking account of multiple scattering and increase in $R / l$ affect the result in the same way as in the previous case. In contrast to it, the increase in $W_{\text {scat }}$ only slackens in the region $\beta_{\text {entr }}=\gamma_{S 1}$, while remaining substantial. This involves the fact that the more diffuse $\chi_{\mathrm{g}}$ of (2.20) and (2.25) was added to the scattering bounded in angle of (2.19). Finally, curve 7 (isotropic scattering) gives a rather small contribution to the observed flux, which is determined by the smallness of the entrance solid angle here as compared with the total angle $4 \pi$.


FIG. 19. Dependence of the light flux scattered in the instrument $W_{\text {scal }} / I_{0} \cdot \pi R^{2}$ on the entrance aperture $\beta_{\text {entr }}$ for $t_{\mathrm{p}}=1$ and $D=20$. The table gives the set of conditions for each curve:

| Curve | $R / l$ | $\chi$ | $\lambda_{\mathrm{p}}$ | $m$ |
| :--- | :---: | :--- | :--- | :--- |
| 1 | - | $\chi_{\mathrm{d}}$ | 0.5 | 1 |
| 2 | 0.01 | $\chi_{\mathrm{d}}$ | 0.5 | 5 |
| 3 | 0.1 | $\chi_{\mathrm{d}}$ | 0.5 | 5 |
| 4 | - | $\chi_{\mathrm{d}+8}$ | 1 | 1 |
| 5 | 0.01 | $\chi_{\mathrm{d}+\mathrm{8}}$ | 1 | 5 |
| 6 | 0.1 | $\chi_{\mathrm{d}+\mathrm{g}}$ | 1 | 5 |
| 7 | 0.1 | $\chi=1$ | 1 | 5 |

Figure 20 shows the fraction of the light transmitted through the gas containing particles as a function of the entrance aperture $\beta_{\text {entr }}$ in the case of large absorbing particles ( $\chi=\chi_{d}$ ) for different diffraction parameters. The larger the diffraction parameter, the lower the angles $\beta_{\text {entr }}$ close to $\gamma_{S 1}$ (marked on the axis of abscissas for $D=20,30$, and 200) at which the increase in $W$ ceases with increasing $\beta_{\text {entr }}$. On the axis of ordinates is marked the value $\exp \left(-t_{0}\right)$. Evidently, for direct measurement of this quantity in the case of extended indicatrices we thust have a small entrance angle ( $\beta_{\text {entr }}<\gamma_{S 1}$ ). Yet in the general case, when the aperture is selected and one measures $W \not W_{0} \pi R^{2}$, to find $t_{\mathrm{p}}$ we must use calculations of the dependences such


FIG. 20. Dependence of the transmission of light $W / I_{0} \cdot \pi R^{2}$ on the entrance aperture $\beta_{\text {entr }}$ for $t_{\mathrm{p}}=1, R / l=0.1, m=5, \chi=\chi_{\mathrm{d}}, \lambda_{\mathrm{p}}=0.5$ and different diffraction parameters: $1-D=20 ; 2-D=30 ; 3-D=200$.
as in Fig. 21. With increasing $t_{\mathrm{p}}$ the light flux passing into the instrument decreases, but more slowly as $\lambda_{p}$ increases. We have plotted $\exp \left(-t_{p}\right)$ in the same diagram for comparison. Evidently taking account of light entering the instrument after scattering under the conditions of Fig. 21 is fully necessary.


FIG. 21. Dependence of the transmission of light $W / I_{0} \cdot \pi R^{2}$ on the optical density $t_{\mathrm{p}}$ for $R / l=0.1, \beta_{\text {entr }}=0.2, m=5 . \quad l-\chi=\chi_{\mathrm{d}}, \quad D=20$, $\lambda_{\mathrm{p}}=0.5 ; 2-\chi=\chi_{\mathrm{d}+\mathrm{g}}, D=20, \lambda_{\mathrm{p}}=1 ; 3-\exp \left(-t_{\mathrm{p}}\right)$.

Thus we have described the passage of a thin beam in a gas containing a condensed dispersed phase. The results make it possible to determine correctly the optical density of a gas containing a condensed dispersed phase from the attenuation of the beam.

## 6. METHODS OF DIAGNOSTICS

### 6.1. General problems

This section discusses the methods of diagnostics of a gas containing a condensed dispersed phase based on measurements of the intensity of the intrinsic radiation and the attenuation of the radiation of an external emitter. As a result of such measurements one can determine the temperatures and concentrations of atoms and particles of the dispersed phase. The sought temperatures are related to the primary emission by the Kirchhoff law, while the relations of the intensities of the output radiation with the primary emitters, as determined by the process of radiation transport, are described in Secs. 3 and 4. The concentrations of atoms and particles are related to the optical density of the medium via the extinction coefficients, while the relation of the attenuation of a thin beam to the optical density was discussed in Sec. 5.

The results of the calculations of radiation transport can be used to determine the temperature if one knows the probabilities of survival $\lambda_{\mathrm{p}}$ and the scattering indicatrices $\chi\left(\gamma_{S}\right)$ of the particles. To determine the concentrations of the particles requires the extinction cross sections $\Sigma_{p}$. This information on the particles can be obtained by various methods, to which an extensive literature has been devoted (see, e.g., Refs. 7, 51-54). Here we emphasize only that if we know (even approximately) the dimensions and the refractive index of the material of the particles, then we can use the results of the calculations discussed in Sec. 2. Moreover, let us turn our attention to the dependence of the form of a spectral line on the quantities $\lambda_{\mathrm{p}}$ and $\chi$ see Secs. 4.3 and 4.4). This dependence can be used for experimental determination of $\lambda_{\mathrm{p}}$ and $\chi$.

Measurements of the intensity of the intrinsic emission and the attenuation of the emission from an external source are usually conducted according to a scheme into which the following elements enter (see Fig. 12): 1) the external source $L$; 2) the spectral instrument (8), which isolates the spectral regions within which one measures the radiation fluxes; 3) the optical system, which determines the field of view and the entrance aperture of the spectral instrument, as well as the passage through them of the light from the external source. The optical system must be constructed so that no parts of the surface limiting the volume of study, in particular, region 5 , which adjoins the optical window on the wall opposite the instrument, lie within the field of view of the spectral instrument.

Let us take up in somewhat greater detail the individual elements of the optical system. For many years ribbon
tungsten lamps have been used as the radiation source $L$, since they operate stably and can be calibrated with high accuracy. When the temperature of the gas substantially exceeds 3000 K , one must use more powerful sources to decrease the error of measurement of the intensity. One usually places between the source $L$ and the lens $l$ a rotating disk with apertures, which periodically obscures the light flux from the source $L$ to the measuring system. In studying difficultly accessible objects, one incorporates fiber optics in the optical system, which enables increasing the distance between the lamp, the object, and the spectral instrument.

The spectral apparatus usually amounts to traditional diffraction instruments with some particular spectral resolution. In the past decade the output part of the instruments, which converts light signals into electrical signals, has substantially changed. For many years, mainly photomultipliers were used for this conversion, which are able to receive light in individual spectral intervals cut off by the output slit of the monochromator. At present photomultipliers are also widely used. But, in addition, ever more often matrices of photodiodes are placed at the output of the spectral apparatus, which permit one simultaneously to record the light signals in different regions of the spectrum. Such systems, coupled with computers, sharply enhance the potentialities of diagnostics in a gas containing a condensed dispersed phase, since here one must often perform measurements at different frequencies. If the object is not stable, simultaneity of measurements becomes important.

Let $I_{\mathrm{r}}$ and $I_{L}$ be the intensities of the emitter being studied and of the external source $L$, and $J_{\mathrm{r}}$ and $J L$ be the measured electric signals proportional to these intensities.

The absolute value of $I_{\mathrm{r}}$ of the intensity being measured is calculated by the relationship

$$
\begin{equation*}
I_{\mathrm{r}}=I_{\mathrm{L}} J_{\mathrm{r}} / J_{\mathrm{L}} \tag{6.1}
\end{equation*}
$$

Here the light from the external source and the object must pass into the instrument from identical regions of space and within identical solid angles. $J_{L}$ is measured in the absence of radiation from the object.

Let $J_{\mathrm{r}+L}$ be the signal measured in the presence of emission from the medium while illuminated by the radiation from the source $L$. Then the relative attenuation of the radiation is described by the physically evident combination of signals:

$$
\begin{equation*}
\left(J_{\mathrm{r}}+J_{L}-J_{\mathrm{r}+L}\right) / J_{L}=1-\left(J_{\mathrm{r}+L}-J_{\mathrm{r}}\right) / J_{L} \tag{6.2}
\end{equation*}
$$

The fraction of transmitted radiation is

$$
\begin{equation*}
\left(J_{\mathrm{r}+L}-J_{\mathrm{r}}\right) / J_{L} \tag{6.3}
\end{equation*}
$$

Equations (6.2) and (6.3) contain only differences of signals that include the emission from the object of study. Therefore, in measurements of the attenuation or the fraction of transmitted radiation, the intrinsic radiation can come not only from the regions illuminated by the source $L$.

In the general case, to establish the relation of the observed attenuation in (6.2) or transmission of the beam
in (6.3) to the optical density $t_{0}$, one must take account of the entrance into the spectral instrument of the scattered radiation of this beam [see Eqs. (5.2) and (5.3)]. Yet when the scattered light of the beam does not enter the instrument, we have the following expressions for the attenuation or transmission of light, respectively:

$$
\begin{align*}
& 1-\left(J_{\mathrm{r}+L}-J_{\mathrm{r}}\right) / J_{L}=1-\exp \left(-t_{0}\right)  \tag{6.4}\\
& \left(J_{\mathrm{r}+L}-J_{\mathrm{r}}\right) / J_{L}=\left(I_{\mathrm{r}+L}-I_{\mathrm{r}}\right) / I_{L}=\exp \left(-t_{0}\right) \tag{6.5}
\end{align*}
$$

The relationships (6.4) and (6.5) hold either in the case in which scattering in the medium is simply absent $(\lambda=0)$, or when scattering exists $(\lambda \neq 0)$, but the entrance aperture of the spectral instrument is small enough and does not transmit the scattered light of the probe beam, i.e., in (5.3) we have $W_{\text {scat }}<W$. In (6.4) and (6.5) it is assumed that $t_{0}$ and $I_{L}$ do not vary appreciably over the measured interval.

The intensity $I_{\mathbf{r}}$ depends in a scattering medium on the radiation $I_{S 0}$ of the surfaces surrounding the volume being studied (see Secs. 3 and 4). One can find the intensity $I_{S 0}$ experimentally. To do this, one must in addition measure the signal $J_{\mathbf{r}+S}$ arising from the emission from the volume and the opposite wall. The spectral instrument must be set up so that the opposite wall lies in its field of view instead of the optical aperture in front of $L$; here the optical density in the light path from the wall being observed to the instrument must be the same as when one observes the probe beam from $L$.

In the case in which the spectral instrument does not measure the scattered radiation of the region of the opposite wall that lies in the field of view of the instrument, analogously to (6.5) we find

$$
\begin{equation*}
\left(I_{\mathrm{r}+S}-I_{\mathrm{r}}\right) / I_{S 0}=\exp \left(-t_{0}\right) \tag{6.6}
\end{equation*}
$$

Eliminating $t_{0}$ from (6.5) and (6.6), we find

$$
\begin{equation*}
I_{\mathrm{S} 0}=I_{L}\left(J_{\mathrm{r}+} S^{-}-J_{\mathrm{r}}\right) /\left(J_{\mathrm{r}+L}-J_{\mathrm{r}}\right) \tag{6.7}
\end{equation*}
$$

Thus measurement of the signals $J_{\mathrm{r}}, J_{\mathrm{r}+L}, J_{L}$, and $J_{\mathrm{r}+S}$ makes it possible to find the intensity $I_{\mathrm{r}}$, the optical density $t_{\mathrm{p}}$, and the intensity of emission from the surface $I_{S 0}$.

### 6.2. Measurement of the temperature of particles of the condensed phase

This section discusses certain possibilities of measuring the temperature of the particles averaged over the direction of observation. That is, we assume that the temperature does not vary inside the object. Let the measurements be performed in the spectral regions where there is a continuum arising only from the particles, i.e., $k_{\mathrm{a}}=0$, while $t_{0}=t_{\mathrm{p}}$. The primary sources in these regions, in agreement with (3.8) and (3.11), have the form

$$
\begin{align*}
& g=g_{\mathrm{p}}+g_{S}  \tag{6.8}\\
& g_{\mathrm{p}}=\left(1-\lambda_{\mathrm{p}}\right) I_{\mathrm{p}}^{0} \tag{6.9}
\end{align*}
$$

Each of the primary sources $g_{\mathrm{p}}$ and $g_{\mathrm{S}}$ corresponds to a term in the overall intensity. Therefore we have

$$
\begin{equation*}
I_{\mathrm{r}}=I_{v \mathrm{p}}+I_{S} \tag{6.10}
\end{equation*}
$$

Calculations of radiation transport make it possible to find the relative emissive capability of the object (the "emittance") in the form

$$
\begin{equation*}
I_{\mathrm{r}} / I_{\mathrm{p}}^{0}=A_{\mathrm{p}} \tag{6.11}
\end{equation*}
$$

Here $A_{\mathrm{p}}$ depends in the general case on the optical characteristics of the particles $\left[t_{\mathrm{p}}, \lambda_{\mathrm{p}}\right.$, and $\chi\left(\gamma_{S}\right)$ ], the shape of the emitter, and on the emission from the walls ( $I_{S 0} / I_{\mathrm{p}}^{0}$ ). When the temperature of the walls equals the temperature of the particles ( $I_{S 0} / I_{\mathrm{p}}^{0}=1$ ) or the walls are absent, (6.11) amounts to a formulation of the Kirchhoff law and $A_{\mathrm{p}}$ is the absorptivity of the object. Let $T_{L}$ be the brightness temperature of the source $L$. Then $I_{L}=I_{L}^{0}$, and we find from (6.1):

$$
\begin{equation*}
I_{\mathrm{p}}^{0} / \Gamma_{L}^{0}=J_{\mathrm{r}} / J_{L} A_{\mathrm{p}} \tag{6.12}
\end{equation*}
$$

This is the fundamental relationship with which one calculates the temperature of the particles from the measured signals $J_{\mathrm{r}}$ and $J_{L}$ and the found value of $A_{\mathrm{p}}$; here $I^{0}$ is determined by the Planck function (2.40). When $h v \gg k T$ one obtains from (6.12) the following expression for the temperature:

$$
\begin{equation*}
T_{\mathrm{p}}=T_{L}\left[1-(k T / h v) \ln \left(J_{\mathrm{p}} / J_{L} A_{\mathrm{p}}\right)\right]^{-1} \tag{6.13}
\end{equation*}
$$

## Let us study how to find $A_{\mathrm{p}}$ in various cases.

6.2.1. Scattering by the particles is so small that it can be neglected $\left(\lambda_{\mathrm{p}} \rightarrow 0\right)$. In this case we have $g_{S}=0$, and we find from (6.8)-(6.10) $g=I_{\mathrm{p}}^{0}, I_{\mathrm{r}}=I_{u \mathrm{p}}$. As was obtained earlier (see Figs. 6 and 7), the relative emittances are described in the absence of scattering by the expression $I_{\mathrm{r}} / I_{\mathrm{p}}^{0}=1-\exp \left(-t_{\mathrm{p}}\right)$. If we also take account of the fact that in the absence of scattering all the attenuation of the radiation is governed only by absorption, we can write

$$
\begin{equation*}
A_{\mathrm{p}}=1-\exp \left(-t_{\mathrm{p}}\right)=\left(J_{\mathrm{r}}+J_{L}-J_{\mathrm{r}+L}\right) / J_{L} \tag{6.14}
\end{equation*}
$$

Equations (6.12) and (6.14) imply that measurements of the three signals $J_{\mathrm{r}}, J_{L}$, and $J_{\mathrm{r}+L}$ and of the temperature values $T_{L}$ suffice to determine the temperature of the particles $T_{\mathrm{p}}$.
6.2.2. Scattering is substantial $\left(\lambda_{\mathrm{p}} \neq 0\right)$, and radiation freely escapes from the boundaries of the object. This case can occur (see Sec. 4.2.2), e.g., in the absence of bounding surfaces, or when the surfaces are transparent. Here, as in Sec. 6.2.1, $A_{p}$, which is determined by Eq. (6.11), is the absorptivity, but the first equation in (6.14) does not hold, since the optical density $t_{\mathrm{p}}$ is determined not only by absorption but also by scattering. The absorptivity here, and hence also the relative emittance, is smaller than in the absence of scattering. These qualitative considerations are confirmed by the calculated curves in Figs. 6 and 7; Here, as in the previous case, we have $I_{\mathrm{r}}=I_{\nu \mathrm{p}}$. The curves for $\lambda_{\mathrm{p}}=0.5(v 1 ; 0.5)$ lie below the curves for $\lambda_{\mathrm{p}}=0(v 1 ; 0)$. Here the course of the curves depends on the shape of the object (see Fig. 7). In this case one cannot obviate calculation of the relative emittance $I_{v \mathrm{p}} / I_{\mathrm{p}}^{0}$ as a function of the optical density $t_{\mathrm{p}}$. Yet one can perform the calculation if one knows the characteristics of the scattering by the par-
ticles $\lambda_{\mathrm{p}}$ and $\chi\left(\gamma_{\mathrm{s}}\right)$. The second equation of (6.14) here can hold if the entrance aperture of the spectral instrument is small.
6.2.3. The scattering is substantial $\left(\lambda_{p} \neq 0\right)$, and the volume under study is surrounded by a closed surface having the same temperature as the medium ( $I_{S 0}=I_{\mathrm{p}}^{0}$ ). This is the case of an isothermal cavity treated in Sec. 3.2. We assume that optical apertures exist in the cavity, as in Fig. 12, but we can neglect their influence on the equilibrium radiation. The conditions for fulfillment of this assumption have been discussed repeatedly; see, e.g., Ref. 55. The major condition is evident: the apertures must be small enough. In diagnostic experiments the fulfillment of this condition is not always simple to carry out. We can convince ourselves of this by analyzing the results of Sec. 4.4. Actually, it is precisely the dimension of the aperture on the wall opposite the observer that strongly affected the intensity of the output radiation.

If the condition of smallness of the apertures is fulfilled, then we obtain from Eq. (3.30) with $I_{v a}=0$ and $\tau\left(\mathbf{r}_{S} \rightarrow \mathbf{r}_{S+\downarrow}\right)=t_{\mathbf{p}}:$

$$
\begin{equation*}
\left(I_{u p} / I_{\mathrm{p}}^{0}\right)+\left(I_{S} / I_{S 0}\right)=1-\exp \left(-t_{\mathrm{p}}\right) \tag{6.15}
\end{equation*}
$$

When $I_{\mathrm{p}}^{0}=I_{S 0}$ we find, while taking account of Eqs. (6.10) and (6.11), that (6.15) is equivalent to the first equation of (6.14). In other words, in a closed cavity the absorptivity of the medium is the same as in the case of a purely absorbing medium with the same $t_{\mathrm{p}}$. The result does not depend on the scattering characteristics of the medium. The physical meaning is that all the attenuation of the light entering into the volume under study is determined by the absorption in it. The part that is initially scattered is then absorbed by the particles or the wall. This involves the coincidence of the curves $I_{\mathrm{r}} / I_{\mathrm{p}}^{0}$ of the relative emittance in Figs. 6 and 7 in the case of pure absorption ( $v 1 ; 0$ ), pure scattering ( $S, 1$ ), and the total curves ( $\Sigma$ ) in all the intermediate cases. We emphasize that Eq. (6.15) and the first equation of (6.14) are valid in the case of an isothermal cavity with arbitrary characteristics of the scattering of the particles, including any anisotropy of the scattering. This is implied by the general discussion conducted in Sec. 3.2.

In studying the absorptive (or emissive) capability here, as in the previous case, we must take account of $W_{\text {scat }}$ by Eq. (5.2). We can use the second equation of (6.14) only in the case of sufficiently small $W_{\text {scat }}$.
6.2.4. The scattering is substantial $\left(\lambda_{\mathrm{p}} \neq 0\right)$, and the temperature of the walls is everywhere the same, but different from the temperature of the medium, i.e., $I_{S O} \neq I_{\mathrm{p}}^{0}$; here we have $I_{\mathrm{r}}=I_{\text {up }}+I_{S}$. In this case it is not necessary to perform the calculation of the intensity of the scattered radiation of the walls $I_{S}$. We can use the results obtained in Sec. 6.2.2 and 6.2.3, i.e., restrict the treatment to calculating $I_{u p} / I_{\mathrm{p}}^{0}$, and measuring the optical density $t_{\mathrm{p}}$ by (6.4) and the relative intensity of radiation of the walls $I_{S 0} / I_{L}$ by (6.7). Actually by using (6.15) one can obtain the following expression for $A_{\mathrm{p}}$ as determined by Eq. (6.11) (taking account of the fact that $I_{l}=I_{L}^{0}$ ):

$$
\begin{align*}
A_{\mathrm{p}}=\frac{I_{\mathrm{r}}}{I_{\mathrm{p}}^{0}}= & \frac{I_{\mathrm{v}}+I_{S}}{I_{\mathrm{p}}^{0}} \\
= & \frac{I_{\nu \mathrm{p}}}{I_{\mathrm{p}}^{0}}\left(1-\frac{I_{S S}}{I_{L}^{0}} \frac{I_{L}^{0}}{I_{\mathrm{p}}^{0}}\right)-\frac{I_{S 0}}{I_{L}^{0}} \frac{I_{L}^{0}}{I_{\mathrm{p}}^{0}} \\
& \times\left[1-\exp \left(-t_{\mathrm{p}}\right)\right] . \tag{6.16}
\end{align*}
$$

$A_{\mathrm{p}}$ contains the unknown ratio $I_{\mathrm{p}}^{0} / I_{L}^{0}$. Substituting (6.16) into (6.12), we can obtain a relationship where the only unknown is $I_{\mathrm{p}}^{0} / I_{L}^{0}$. This relationship is more complicated than (6.12), but can be fully used to determine the temperature of the particles. It often happens that the temperature of the surrounding surface is lower than the temperature of the particles $I_{S 0}<I_{\mathrm{p}}^{0}$. In this case, in agreement with (6.16), the quantity $A_{\mathrm{p}}$ lies between $I_{u \mathrm{p}} / I_{\mathrm{p}}^{0}$ and $1-\exp \left(-t_{\mathrm{p}}\right)$. The difference between these two functions sometimes can be treated as the fully admissible error in determining $A_{p}$, which is found in the argument of the logarithm in (6.13).
6.2.5. The scattering is substantial $\left(\lambda_{\mathrm{p}} \neq 0\right)$, and brightly glowing small regions exist on the surfaces. These conditions often arise near electrodes in an electric discharge. Small spots of high brilliance can be formed on the electrodes, while bright spots arise on the walls surrounding the discharge. In this case the scattered radiation of the bright formations must be calculated by taking into account the concrete distribution of the formations and the spectral apparatus, as well as the anisotropy of scattering of the particles. In these calculations the probabilistic method seems highly appropriate. It can be used similarly to what was done in Sec. 4.4 and Sec. 5.

The measurement of the optical density $t_{\mathrm{p}}$ performed in the measurements of the temperature of the particles makes it possible to find also the mean concentration of particles if one knows the extinction cross section $\Sigma_{p}$. The calculation is performed by the simple relationship

$$
\begin{equation*}
t_{\mathrm{p}}=\alpha_{\mathrm{p}} l=n_{\mathrm{p}} \Sigma_{\mathrm{p}} l . \tag{6.17}
\end{equation*}
$$

Here $l$ is the depth of the emitter.
Measurements in the continuum at several frequencies strongly differing from one another present additional possibilities, both in measuring temperatures and in measuring concentrations. If one knows the relative dependence of the extinction coefficient on the frequency, then one can find not only the concentration, but also the absolute value of the cross section $\Sigma_{\mathrm{p}}{ }^{56}$ We note that, in choosing the frequencies at which the measurements are performed in the continuum, we must take care that emission or absorption in the gas phase plays no role. Even broad tails of atomic lines can lead to a substantial error in the measurements. ${ }^{57}$

Thus one can determine by measurements in the continuum the temperature of the particles; in certain cases the effect of scattering by the particles must be calculated in advance.

### 6.3. Measurement of the temperature and concentration of atoms

Below we discuss the simplest and most reliable methods of measuring the average temperatures and concentrations of atoms. First let us take up the measurement of temperature.
6.3.1. The measurements must be performed in the regions of the spectrum where spectral lines exist, i.e., $k_{\mathrm{a}} \neq 0$ and $t_{0}(v)=t_{\mathrm{a}}(v)+t_{\mathrm{p}}$. The primary sources are determined by Eqs. (3.8) and (4.26):

$$
\begin{align*}
g(v)= & t_{\mathrm{a}}(v) I_{\mathrm{a}}^{0}\left(t_{\mathrm{a}}(v)+t_{\mathrm{p}}\right)^{-1}+t_{\mathrm{p}}\left(1-\lambda_{\mathrm{p}}\right) I_{\mathrm{p}}^{0} \\
& \times\left(t_{\mathrm{a}}(v)+t_{\mathrm{p}}\right)^{-1}+g_{S} \tag{6.18}
\end{align*}
$$

The intensity of the radiation escaping from the volume can be represented in the form:

$$
\begin{equation*}
I_{\mathrm{r}}=I_{v \mathrm{a}}+I_{v \mathrm{p}}+I_{S} \tag{6.19}
\end{equation*}
$$

Each term in the intensity (see, e.g., Fig. 11) and the probability of survival $\lambda(v)$ according to (4.25) vary upon varying the frequency.

Let us assume that the temperature $T_{\mathrm{p}}$ and the optical density $t_{\mathrm{p}}$ of the particles have been measured (see Sec. 6.2). The measurements of $t_{\mathrm{p}}$ were performed in a spectral region adjoining the region of the line. Then we can assume that $t_{\mathrm{p}}$ equals the measured value in the region of the line.

Let $\delta v$ be the minimum spectral interval that can be isolated by the spectral instrument. We shall assume that $\delta v$ is substantially smaller than the width of the atomic line being observed, i.e., the line is spectrally resolved. At different frequencies one measures the different $t_{\mathrm{a}}(v)$ and $I_{\mathrm{r}}(v)$ inside the line. Let us examine some very simple cases of measuring the temperature of a gas. We shall assume that there is no scattering, i.e., $\lambda_{p}=0$; then instead of (6.18) we have

$$
\begin{equation*}
g(v)=\left(t_{\mathrm{a}}(v) I_{\mathrm{a}}^{0}+t_{\mathrm{p}} I_{\mathrm{p}}^{0}\right) /\left(t_{\mathrm{a}}(v)+t_{\mathrm{p}}\right) \tag{6.20}
\end{equation*}
$$

and the intensity of the radiation is written in the well known form

$$
\begin{align*}
I_{\mathrm{r}}= & \left(t_{\mathrm{a}}(v) I_{\mathrm{a}}^{0}+t_{\mathrm{p}} I_{\mathrm{p}}^{0}\right)\left[1-\exp \left(-t_{\mathrm{a}}(v)-t_{\mathrm{p}}\right)\right] \\
& \times\left(t_{\mathrm{a}}(v)+t_{\mathrm{p}}\right)^{-1} \tag{6.21}
\end{align*}
$$

One must measure at the chosen frequency the signals $J_{\mathrm{r}}$, $J_{L}$, and $J_{\mathbf{r}+L}$. By using (6.1), (6.4), and $t_{0}=t_{\mathrm{a}}(v)+t_{\mathrm{p}}$, we find

$$
\begin{align*}
\left(t_{\mathrm{a}}(v) I_{\mathrm{a}}^{0}+t_{\mathrm{p}} I_{\mathrm{p}}^{0}\right) / t_{\mathrm{a}}(v)= & I_{L}^{0} J_{\mathrm{r}}(v) /\left(J_{\mathrm{r}}(v)+J_{L}\right. \\
& \left.-J_{\mathrm{r}+L}(v)\right) \tag{6.22}
\end{align*}
$$

To find $I_{\mathrm{a}}^{0}$ or the temperature $T_{\mathrm{a}}$ by (6.22), we must: 1) find $t_{0}(v)$ at the chosen frequency $v$ inside the line from the three measured signals (see (6.4)); 2) use the measured $t_{\mathrm{p}}$ to find $t_{\mathrm{a}}(v)=t_{0}(v)-t_{\mathrm{p}}, 3$ ) use the known values of $I_{\mathrm{p}}^{0}$ and $I_{L}^{0}$ for the final calculation.

Let us assume that the atomic optical density is far larger than the optical density of the particles: $t_{\mathrm{a}}(v)>t_{\mathrm{p}}$. The optical density usually increases with varying fre-
quency as the frequency approaches that of the center of the line. When the condition $t_{\mathrm{a}}(v)>t_{\mathrm{p}}$ is fulfilled, in agreement with (4.25) we find

$$
\lambda=\lambda_{\mathrm{p}} t_{\mathrm{p}} /\left(t_{\mathrm{a}}(v)+t_{\mathrm{p}}\right) \rightarrow 0
$$

This means that we can neglect scattering for sufficiently large $t_{\mathrm{a}}(v) / t_{\mathrm{p}}$. Here $g_{S \rightarrow 0}$, and $g(v) \rightarrow I_{\mathrm{a}}^{0}$. If, moreover, $t_{\mathrm{a}}(v)>1$, then we have

$$
I_{\mathrm{r}}=I_{v \mathrm{a}}=I_{\mathrm{a}}^{0}\left[1-\exp \left(-t_{\mathrm{a}}\right)\right] \rightarrow I_{\mathrm{a}}^{0}
$$

In this case we directly find from (6.1) with $I_{L}=I_{L}^{0}$ :

$$
\begin{equation*}
\Gamma_{\mathrm{a}}^{0}=I_{L}^{0}, \quad J_{\mathrm{r}} / J_{L} \tag{6.23}
\end{equation*}
$$

As we see, here we need not know $t_{\mathrm{p}}$ and $I_{\mathrm{p}}^{0}$. A similar possibility is also implied by the results of calculation given in Sec. 4.3 for a constant gas temperature inside the object (see Figs. 9-11). It is essential that the effect also occurs in the presence of walls. The result is understandable: at large $t_{\mathrm{a}}$ the influence of the emission and scattering of the particles is suppressed by the highly probable absorption by atoms. The choice of a sufficiently large $t_{\mathrm{a}}$ must be performed with caution owing to the features of the behavior of the contours described in Sec. 4.3, even for considerable values of $t_{\mathrm{a}} \approx 1-5$. The features involve the influence of the zigzag motion of the photons caused by scattering on the absorption of light by the atoms. Nevertheless, at large enough $t_{\mathrm{a}}$ these effects become so small that they can be neglected. Preliminary calculations of the lines give a basis for a correct choice of the experimental conditions.

In choosing the frequencies suitable for performing measurements of the temperature of a gas, one must take account of the possible inhomogeneity in the object. The lines can even be self-reversed; then one must shift from the center of the line and introduce certain corrections. ${ }^{4}$ It is much simpler to find the corrections for inhomogeneity than to calculate the effects of scattering, but to find them one must have preliminary information on the character of the inhomogeneity of the object. Often such information exists. One of the possibilities of experimental study of inhomogeneities is discussed in Sec. 6.4.

Only the sufficiently strong lines can satisfy the condition $t_{\mathrm{a}}\left(v_{0}\right) \gg 1$. To perform temperature measurements, often one introduces specially a substance having suitable lines. Thus, in combustion products to measure the temperature one uses the emission of resonance lines of specially introduced sodium.

In other cases, when $\lambda_{\mathrm{p}} \neq 0$ and $t_{\mathrm{a}}(v) \approx t_{\mathrm{p}}$, one must take account of scattering in the way that this is done in measuring the temperature of the particles. The details of this procedure are analogous to those described in Secs. 6.2.2 and 6.2.4, and will not be repeated here. In the case of an isothermal cavity where $I_{\mathrm{a}}^{0}=I_{\mathrm{p}}^{0}$, the treatment is the same as in Sec. 6.2.3, with the difference that one can write instead of (6.15)

$$
\begin{equation*}
I_{v}+I_{S}=I_{v \mathrm{p}}+I_{v \mathrm{a}}+I_{S}=I_{\mathrm{a}}^{0}\left[1-\exp \left(-t_{\mathrm{a}}-t_{\mathrm{p}}\right)\right] \tag{6.24}
\end{equation*}
$$

Measurement of the temperature in the absence of scattering was discussed in Refs. 57 and 58. Losses of light due to scattering were taken into account in Refs. 59 and 60. The
positive contribution of scattering of the radiation from the walls to the observable intensity was calculated under concrete conditions in Ref. 61. The results of these and other numerical calculations do not contradict the idea that, at large enough atomic optical densities $t_{\mathrm{a}}$, one can find the temperature of the gas relatively simply in the way described above.
6.3.2. Let us turn to the concentration of atoms (or molecules). We shall discuss the determination of the concentration by measuring the attenuation of radiation in a region of spectral lines. The atomic optical density is associated with the concentration of atoms by the following relationship:

$$
\begin{equation*}
t_{\mathrm{a}}(v)=n_{\mathrm{a}} \sigma_{\mathrm{a}}(v) l . \tag{6.25}
\end{equation*}
$$

This relationship is fundamental in finding the concentration of atoms by measuring the attenuation of the radiation of an external source. To find $n_{\mathrm{a}}$ one must know the absorption cross section of the line $\sigma_{\mathrm{a}}(v)$ (see Sec. 2.2).

The results of the measurements substantially depend on the width of the measurable spectral interval $\delta v$.

Let $\delta v$ be so small that the atomic optical density $t_{\mathrm{a}}(v)$ does not vary appreciably inside $\delta v$. In this case one must measure $t_{0}(v)$ at a chosen frequency inside the line, while measuring the optical density of the particles $t_{\mathrm{p}}$ near the line. Then one can find the concentration $n_{\mathrm{a}}$ from $t_{\mathrm{a}}(v)=t_{0}(v)-t_{\mathrm{p}}$. Such measurements at a single frequency inside the line are seldom employed. More often one uses measurements at two or several frequencies, which diminishes the error of the measurements. This pertains also to measurements in the absence of particles.

Let $\delta v$ be so large that it includes the entire region of the spectral line where $k_{\mathrm{a}} \neq 0$, or most of $i$. In this case the concentration of atoms is determined from the integral attenuation of the radiation in the spectral line. In the absence of macroscopic particles this method is applied rather widely. ${ }^{18,19}$

Let us assume that the scattered radiation of the source $L$ does not enter the instrument. Then in the case of a broad spectral interval $\delta v$, instead of (6.5) we should write the following expression for the transmission of light:

$$
\begin{align*}
\left(J_{\mathrm{r}+L}-J_{\mathrm{r}}\right) / J_{L}= & \int_{\delta v} I_{L}(v) \\
& \times \exp \left(-t_{0}(v)\right) \mathrm{d} v / \int_{\delta v} I_{L} \mathrm{~d} v \tag{6.26}
\end{align*}
$$

Upon taking account of Eq. (6.25), of the equation $t_{0}(v)=t_{\mathrm{a}}(v)+t_{\mathrm{p}}$, and of the fact that the optical density $t_{\mathrm{p}}$ does not vary inside the interval $\delta v$, we find

$$
\begin{align*}
\frac{J_{\mathrm{r}+L}-J_{\mathrm{r}}}{J_{L}} \exp t_{\mathrm{p}}= & \int_{v} I_{L}(v) \\
& \times \exp \left(-\sigma_{\mathrm{a}}(v) n_{\mathrm{a}} l\right) \mathrm{d} v / \int_{\delta v} I_{L} \mathrm{~d} v . \tag{6.27}
\end{align*}
$$

The right-hand side of (6.27) can be calculated for known $I_{L}(v), \delta v$, and $\sigma_{\mathrm{a}}(v)$ as a function of the product $n_{\mathrm{a}} l$. The result of calculation does not depend on the presence of particles. Yet measurement of the three signals $L_{\mathrm{r}}, J_{L}$, and $J_{\mathbf{r}+L}$ in the region of a line, and also outside the line (to determine $t_{\mathrm{p}}$ ), makes it possible in a concrete experiment to find the left-hand side of (6.27). Comparison of the result with the calculated right-hand side determines the sought concentration $n_{\mathrm{a}}$. When $I_{L}(v)$ is constant inside $\delta v$, instead of (6.27) we have

$$
\begin{equation*}
\frac{J_{\mathrm{r}+L}-J_{\mathrm{r}}}{J_{L}}\left(\exp t_{\mathrm{p}}\right)=\frac{1}{\delta v} \int_{\delta v} \exp \left(-\sigma_{\mathrm{a}}(v) n_{\mathrm{a}} l\right) \mathrm{d} v . \tag{6.28}
\end{equation*}
$$

Often in calculation, instead of the characteristics of the light transmission, one uses the so-called equivalent width of the atomic absorption line:

$$
\begin{equation*}
A_{\mathrm{g}}=\int_{\delta v}\left[1-\exp \left(-\sigma_{\mathrm{a}}(v) n_{\mathrm{a}} l\right)\right] \mathrm{d} v \tag{6.29}
\end{equation*}
$$

In the presence of particles one obtains in the measurements the following equivalent width:

$$
\begin{equation*}
A_{\mathrm{g}}=\left(1-\frac{J_{\mathrm{r}+L}-J_{\mathrm{r}}}{J_{L}} \exp t_{\mathrm{p}}\right) \delta v \tag{6.30}
\end{equation*}
$$

In the absence of particles one often determines the concentration of atoms from the width of the spectral lines of the radiation. These methods can be used also in a gas containing a condensed dispersed phase, for which the spectral lines calculated as functions of $t_{\mathrm{a}}$ are quite suitable (see Figs. 9-11). The methods of determining $n_{\mathrm{a}}$ from absorption in the gas containing a condensed dispersed phase are simpler and more reliable, since they do not require one to calculate the contours of the lines, which depend on the scattering properties of the particles ( $\lambda_{p}, \chi\left(\gamma_{s}\right)$ ) and on the emission from the walls.

In spectral diagnostics in finding temperatures and concentrations, the problem of measurement errors is usually very important. The sources of error are varied, and many studies have been devoted to these problems (see, e.g., Refs. 4 and 19). In a gas containing a condensed dispersed phase, in addition to the erorrs involving the measuring systems (error in calibrating the comparison source $L$, noise in the electrical circuits, error in the measuring instruments, etc.), an important role is played by errors determined by the object itself. Above all, these are the errors caused by the time instability of the object. ${ }^{4}$ Thus, in measuring temperature the result substantially depends on the relationship between the characteristic time of the instabilities and the time of measurement. If the time of measurement is substantially larger than the time of instability, a certain averaged value is determined that differs from the mean temperature. This involves the nonlinear dependence of the intensity of emission on the temperature. When the times of instability are comparable with the measurement times, the error in the measurements can be especially large and must be taken into account. For a reasonable interpretation of the results one must have in-
formation on the instability of the object, which usually can be obtained by measuring and analyzing the signals $J_{\mathrm{r}}$.

To summarize all that we have presented above, we can conclude that one can determine the temperature of the atoms most reliably by using the central parts of optically dense lines, and the concentration from the integral absorption in the lines.

### 6.4. Study of spatial inhomogeneity from the contour of spectral lines

In contrast to Secs. 6.2 and 6.3, here we discuss one of the possibilities of studying the spatial distribution of the characteristics of a gas containing a condensed dispersed phase. The spatial distributions in an inhomogeneous emitting gas are often studied from the emission intensity. To do this, one observes the intensities along different rays of sight. Observation in different directions has led to the development of tomographic methods. ${ }^{62}$ Observation along parallel rays of sight is often used to find distributions in a cylindrically symmetrical emitter. ${ }^{19}$ However, there is a large group of emitters to which access is highly limited, e.g., combustion products containing a condensed dispersed phase as mentioned in the Introduction. These emitters are surrounded by opaque walls in which apertures exist, which allow one to observe the radiation only along one ray of sight (see Fig. 12). The problem consists in finding the distribution of characteristics along this ray. The problem can be solved by studying various regions of the emission spectrum in which the light comes to the observer from different depths.

The probability of direct escape of light to the observer is determined by the exponential function of the optical depth in (4.6), i.e., when $\eta=0$,

$$
\begin{equation*}
q_{0}=\exp (-t(v)) \tag{6.31}
\end{equation*}
$$

It is precisely the radiation that has arrived without intermediate interaction with matter that bears the fundamental information on the primary emission of the object, although intermediate interactions are taken into account in solving the transport equations. At the frequencies at which $t_{0}(v) \gg 1$, the probability of direct arrival of light from the opposite boundary is small. But the probability of escape $q_{0}$ of light of the same frequency from small enough depths ( $X \ll l$ ) can be appreciable. Yet if the optical density $t_{0}(v)$ decreases upon changing the frequency, the depth is increased from which radiation arrives with high probability. Let the emission spectrum be characterized by a broad set of optical densities $t_{0}(v)$ at different frequencies, and let there be frequencies at which the optical density is large $\left(t_{0}(v) \gg 1\right)$. In this case one can find the distribution of characteristics of the object along the ray of sight. One of the systems for obtaining the distributions consists in the following. The dependence of the intensity on the frequency is determined experimentally. A certain form of the distribution of the sought characteristics is selected in which a set of variable parameters enters. The dependences of the intensity on the frequency are calculated for various numerical values of the parameters by solving the transport equation. The results of experiment and of calculation are
compared. The set of parameters that yields the best agreement determines the sought distribution. Another system of calculation consists in an iterative, stepwise procedure of selecting the sought distributions in the process of transition from the frequencies where the radiation comes from small depths to frequencies at which the depth of emission is greater. At each of the frequencies one compares the experimental and calculated intensities. The method described here is one of the the incorrect reverse diagnostic problems. One must take account of the features of the solution of such problems. ${ }^{62}$ In the method being discussed, in particular, for reliable results one must have a sufficiently large number of independent measurements of the intensity at different frequencies.

In Refs. 63-65, to study an inhomogeneous hightemperature gas along the ray of sight, infrared radiation in individual regions of the spectral bands was used. The choice of a sufficient number of suitable spectral regions presents substantial difficulties here. Another possibility consists in using a spectrally resolved atomic line, within which one measures $t_{\mathrm{a}}(v)$. One can use a line if the optical density in the center of the line is large: $t_{0}\left(v_{0}\right) \gg 1$. In Refs. 66 and 67 , to find the temperature distribution of a gas along the line of sight, the self-reversed contour of an atomic line was used.

References 44 and 68 analyzed the possibility of finding the temperature distribution in a gas containing a condensed dispersed phase from the form of a spectral line. The analysis was based on a description of the line shown in Sec. 4.3. A procedure was studied that makes it possible to find a set of parameters that determine the temperature distribution as well as the probability of survival $\lambda_{p}$. To find the distribution along the line of sight one must have certain preliminary information. In particular, in using the spectral line one must know the atomic absorption coefficient $k_{\mathrm{a}}(v)$. If one determines experimentally not only the contour of the lines $I_{\mathrm{r}}(v)$, but also the dependence $t_{0}(v)$, then the set of required information on $k_{\mathrm{a}}(v)$ is decreased. ${ }^{68}$

Macroscopic particles substantially influence the potentialities of the method. Below we discuss the spatial resolution and possible depth of study of inhomogeneities in a gas containing a condensed dispersed phase. Although the method is based in all cases on exact solution of the transport equations, here we shall use simple considerations for estimates.

Under concrete conditions of experiment one can usually fix the least admissible probability of direct escape $q_{0 \text { min }}$ from an arbitrary depth $X$. This quantity characterizes the smallest required fraction of the light emitted at the depth $X$ that comes to the observer without intermediate interaction with the medium. The required fraction of the radiation is determined by the features of the emitter. Thus, if the local emittance of an inhomogeneous emitter increases as one approaches the observer, then one requires a larger fraction of the light for reliable measurement. The required least probabilities in different experiments can vary roughly in the following range: $q_{0 \min } \approx 0.4-0.05$, which corresponds to a variation in the greatest accessible
optical depths according to (6.31) in the range: $t_{i}=1-3$. When $q_{0 \text { min }}$ or $t_{i}$ has been fixed, one can associate the frequency of the radiation with the greatest depth from which radiation sufficient for measurement arises. In an inhomogeneous medium the variation of the coefficients of absorption and extinction is usually considerably weaker than the variation of the emissive characteristics. Upon taking this into account, as well as the approximate character of the estimates made below, we assume that $k_{\mathrm{a}}(v)$ and $\alpha_{\mathrm{p}}$ do not vary inside the medium. Then we have

$$
\begin{equation*}
X(v)=t_{i} /\left(k_{\mathrm{a}}(v)+\alpha_{\mathrm{p}}\right) \tag{6.32}
\end{equation*}
$$

Here the given quantity $t_{i}$ determines the depths accessible to study at different frequencies.

The possibility of independent measurements of the intensity at two adjacent frequencies is determined by the measurable spectral interval $\delta v$. The increment $\delta X(v)$ corresponding to a variation of the frequency by $\delta v$ is a characteristic of the smallest regions of the object on which one can get information. We find from (6.32)

$$
\begin{align*}
& \delta X(v)=t_{\mathrm{i}} \delta v\left|\mathrm{~d} k_{\mathrm{a}}(v) / \mathrm{d} v\right|\left(k_{\mathrm{a}}(v)+\alpha_{\mathrm{p}}\right)^{-2},  \tag{6.33}\\
& \delta X(v) / X(v)=\delta v\left|\mathrm{~d} k_{\mathrm{a}}(v) \mathrm{d} v\right| /\left(k_{\mathrm{a}}(v)+\alpha_{\mathrm{p}}\right) . \tag{6.34}
\end{align*}
$$

The relative spatial resolution of (6.34) does not depend on the arbitrarily assigned magnitude of $t_{i}$. The spatial resolution of the method becomes better ( $\delta X$ is smaller) as the relative change in the atomic absorption coefficient becomes smaller and its absolute magnitude becomes greater. The absorption coefficient is maximal near the center of the line $v_{0}$. Here $X\left(v_{0}\right)$ is minimal, while $\delta X\left(v_{0}\right)$ by (6.33) determines the least thickness of the layer adjacent to the observer from which averaged information can be obtained. In other words, $\delta X\left(v_{0}\right)$ determines the minimum depth of investigation.

Now let us study the maximum depth of an object accessible to investigation. At a sufficiently large distance from the center of the line ( $k_{\mathrm{a}} \rightarrow 0$ ), the depth according to (6.32) reaches its maximum value;

$$
X_{\max }=t_{i} / \alpha_{\mathrm{p}} .
$$

Or, if we use the optical density of the particles $t_{\mathrm{p}}=\alpha_{\mathrm{p}} l$, we find

$$
X_{\max } / l=t_{i} / t_{\mathrm{p}} .
$$

If $t_{i}<t_{\mathrm{p}}$, then $X_{\text {max }}<l$, i.e., with a large enough optical density of particles, the entire depth of the object cannot be investigated.

Figure 22 illustrates what we have said; it shows schematically the variation in the atomic optical density $t_{\mathrm{a}}$ with varying frequency for $t_{\mathrm{a}}(v)=10$ (curve 1 ). The spectral line in the absence of particles is shown schematically there also (curve 2). Let the required optical density be $t_{1}=1$, which corresponds to $q_{0 \text { max }} \approx 0.37$. In this case the accessible depth of the object equals the mean free path of a photon $l_{\mathrm{ph}}$. Actually we find from (6.32) that

$$
X(v)=\alpha^{-1}(v)=l_{\mathrm{ph}} .
$$



FIG. 22. Schematic dependences on the frequency ( $\Delta v=\left|v-v_{0}\right|$ ) in the region of a spectral line: 1 -atomic optical density $t_{\mathrm{a}}(\Delta v) ; 2$-intensity of the radiation of a self-reversed line $I(\Delta v)$ in relative units for $t_{\mathrm{p}}=0$; 3-5-ratio of the mean free path of a photon to the length of the object $l_{\mathrm{ph}}(\Delta v) / l$ for various optical densities of the particles: $3-t_{\mathrm{p}}=0 ; 4-t_{\mathrm{p}}=1$; $5-t_{\mathrm{p}}=2$.

The diagram shows the variations of the mean free path (3-5) corresponding to the course of curve 1 for different values of $t_{\mathrm{p}}$. When $t_{\mathrm{p}}=0$ the value of $l_{\mathrm{ph}} / l$ increases from the minimum value at the center of the line to 1 at $t_{\mathrm{a}}(v)$, and then goes to infinity as $t_{\mathrm{a}}(v)$ decreases to zero (curve $3)$. When $t_{\mathrm{p}}=1\left(t_{\mathrm{p}}=t_{\mathrm{i}}\right)$, the greatest length $l_{\mathrm{ph}}$ equals the depth of the object (curve 4). When $t_{\mathrm{p}}>1\left(t_{\mathrm{p}}>t_{\mathrm{i}}\right)$, the greatest length $l_{\mathrm{ph}}$ is smaller than the depth of the object (curve 5). That is, despite departure of the frequency of observation outside the limits of the spectral line, light does not arrive directly from the parts of the object opposite the observer with the required probability $q_{0}=0.37$.

We note that the obtained spatial resolution must be matched to the characteristics of the object. Thus, in measurements of the temperature distribution the magnitude of $\delta X(v)$ must not be smaller than $l_{\mathrm{a}}$-the least dimension of the region that can be characterized by a single temperature (see Sec. 2.3).

Thus macroscopic particles limit the greatest depth of an object accessible to study of the spatial inhomogeneity from the contour of a spectral line. The spatial resolution is determined by the spectral resolution of the apparatus and the dependence of the atomic absorption coefficient on the frequency.

## 7. CONCLUSION

Thus the emission from a gas containing a condensed dispersed phase in the continuum and in spectral lines substantially depends on the emission, absorption, and scattering of light by the macroscopic particles. To use the measured intensities of the intrinsic and transmitted radiation in spectral diagnostics, one must be able to take re-
liable account of the influence of the particles. Taking account of scattering presents especial difficulty. Taking account of scattering is especially important in real objects that contain bright emitters lying outside the field of view of the diagnostic spectral instruments. The probabilistic method makes it possible rather simply to take account of scattering. Integral equations are written for the probabilities of escape of radiation on the basis of physical considerations. The equations are solved by the method of successive approximations. The probabilities of escape that are found are used to obtain direct connections in the quadratures between the measured intensity and the sought characteristics of the gas containing a condensed dispersed phase.

In the rather general case the equation for the probability of escape has been discussed in Sec. 3.3. Special cases are obtained from the general equation. In this way the radiation with isotropic scattering in objects of various shapes has been treated (Sec. 4.1). Analogously, by using the general equation for the probability of escape, the passage of a thin beam through a gas containing a condensed dispersed phase was described (Sec. 5). The evident physical meaning of the method makes it possible, even without the aid of the general equation, to write the required equations directly in concrete situations. This is precisely how the influence of scattering by particles of the light from the walls on the emission from a gas containing a condensed dispersed phase is described (Secs. 4.4 and 4.5).

In setting up diagnostic experiments an important role is played by preliminary estimates of the limiting possible influences of various conditions on the expected results of the measurements. For example, it can be useful to estimate the influence of scattering or absorption of light by particles on the contour of a spectral line. Such estimates in many cases can be made by using easy calculations of the individual components of the equilibrium radiation of a gas containing a condensed dispersed phase (Sec. 3.2).

Analysis of the results of calculations of the intensities of radiation obtained by the probabilistic method makes it possible to select the most reliable methods of measurements of various quantities. Thus, to measure the temperature of a gas it is recommended to use the radiation in the central parts of optically dense lines, while to determine the concentration of atoms (or molecules) the integral absorption in the lines is recommended (Sec. 6.3). Optically thin lines are subject to a strong influence of even a small amount of macroscopic particles. Therefore the use of many traditional methods of diagnostics of a gaseous medium in the case of a gas containing a condensed dispersed phase can lead to erroneous results.

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