Nonlinear lenses and their applications

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Work on the physics of nonlinear optical lenses is reviewed. The foundations of the theory of optical systems with thin nonlinear lenses are examined. The results of investigations of nonlinear lenses under different conditions of laser excitation are presented and analyzed. The basic applications of nonlinear lenses in optical measurements and for control of laser radiation are discussed.

INTRODUCTION

The property of curved interfaces between optical media to focus (defocus) light rays has been used in optics for many centuries (lenses, microscopes, telescopes, etc.). In the last few decades graded-index optical elements have appeared. The focusing properties of these elements are determined by a specially selected continuously nonuniform spatial distribution of the index of refraction. The appearance of lasers opened a new page in the development of optical systems. Due to the changes occurring in the index of refraction in strong laser fields the standard slab of matter bounded by plane-parallel surfaces acts on a light beam as a graded-index lens whose optical power increases with the power of the laser beam. Such a nonlinear lens can give rise to self-focusing of the laser beam and damage to the material due to catastrophic growth of the radiation intensity occurring when the power increases, the focal point of the photoinduced lens moves into the nonlinear medium.[1-3] In a sufficiently thin layer of matter, however, the redistribution of the radiation intensity occurring in the process of nonlinear refraction of the light rays is negligibly small, and just as a standard thin lens, the nonlinear layer acts only on the phase of the light wave.[4] Nonlinear lenses differ from standard lenses mainly by the fact that the focusing properties of nonlinear lenses depend on the light intensity. This opens up new possibilities for application of lens optics for controlling the characteristics of light fields.

Nonlinear lenses have already found diverse applications in optics and laser physics. Due to its simplicity and high sensitivity the "nonlinear lens" method is widely used for measuring weak absorption in liquids and gases[5-16] as well as for studying the properties of semiconductors,[17-33] liquid crystals,[34-36] and other nonlinear materials. Important information about the mechanisms of nonlinearity and the relaxation of the nonlinear response of optical materials exposed to pulsed laser radiation can be obtained by studying the dynamics of nonlinear lenses.[21,25-30]

Nonlinear lenses are important in the optics of high-power lasers. Such lenses are especially strongly manifested in high-power cw lasers, where these lenses are formed due to the thermal nonlinearity in the active elements and the feedthrough optics. Nonlinear lenses are important for the optical channels of pico—and femptosecond laser systems, where due to the high intensity of the light even comparatively weak electronic nonlinearity of optical elements can result in significant deformations of the wavefront.

Nonlinear deformations of the transverse distribution of a beam by photoinduced lenses can be used for solving diverse problems of control of laser radiation. This field has been developing rapidly in the last few years. A nonlinear lens, coupled with a spatial filter, can be used for stabilizing radiation power,[37-40] changing the duration of light pulses,[39,41] controlling the radiation of cw lasers by pulses from an auxiliary low-power laser,[42] in the optics of bistable devices[43-45] and nonlinearly optical laser-power and laser-energy limiters (self-limiters),[24,37,46-59] and for Q-switching[103] and modelocking.[110,111]

Thus nonlinear lenses have very diverse applications in modern laser physics and technology. There are more than 100 works in this field. However, a review of works on nonlinear lenses and their applications still does not exist. We attempt to fill this lacuna. This review consists of three sections. The basic theoretical ideas on the focusing properties and aberrations of nonlinear lenses are examined in Sec. 1. Experiments with nonlinear lenses are analyzed in Sec. 2. In Sec. 3 the material on application of nonlinear lenses in optical measurements, lasers, and for light control is generalized and analyzed.

1. OPTICAL SYSTEMS WITH "THIN" NONLINEAR LENSES

1.1. Optical power of a nonlinear lens

Consider a light beam propagating along the z axis along the normal to a plane-parallel slab of a transparent material with index-of-refraction variations $\Delta n(x,y,z)$ (Fig. 1). The bending which the wave beam undergoes in such a slab is determined by the phase increment

$$\Delta\Phi(x,y)=\int_0^l k\Delta n(x,y,z)\mathrm{d}z, \tag{1.1}$$

where $k=2\pi/\lambda$ is the wave number and λ is the wavelength of the light, l is the thickness of the material, and the integration extends along the light rays. In a uniform nonlinear medium the change in the index of refraction is de-

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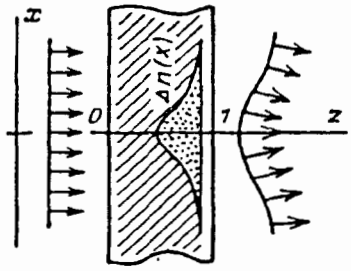

FIG. 1. Transformation of the wave front of the beam with bell-shaped transverse distribution of the intensity during passage through a nonlinear medium.

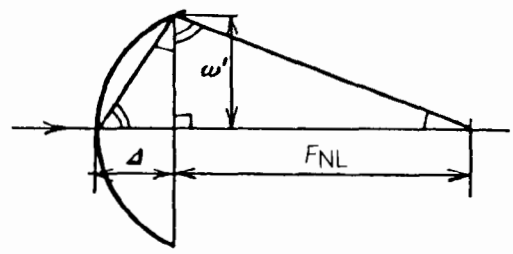

FIG. 2. Approximate calculation of the focal length F_{NL} of a nonlinear lens in terms of the transverse size ω' and sag Δ of the wavefront.

termined by the radiation intensity $I(x,y,z)$. To a first approximation it is usually assumed that this relation is linear:

$$\Delta n = n_2 I, \tag{1.2}$$

where n_2 is the coefficient of nonlinearity of the refractive index of the material. The relation (1.2) is strictly valid for a cubic (in the field) noninertial nonlinearity.[1)] In this case the sagging of the wavefront in a sufficiently thin nonlinear layer repeats the transverse intensity distribution $I(x,y)$ in the light beam, i.e., a thin nonlinear layer of the linearly transforms the intensity profile into the transverse distribution of the phase. For beams with a bell-shaped profile this is equivalent to passage through a thin lens whose focal length depends on the beam power.

In the general case the linear intensity dependence of the nonlinear correction of the refractive index holds only approximately, and higher-order terms must be included in the expansion of the function $\Delta n(I)$ in powers of I. In addition, for a number of mechanisms of nonlinearity the inertia and nonlocal nature of the nonlinear response of the material are important. These factors distort the nonlinear sagging of the wavefront with respect to the intensity distribution. In particular, they can decrease or increase the transverse scale of the nonlinear phase shift compared with the beam width. In spite of the difficulty of describing the nonlinear response of a material in the general case, most effects considered in this review can be explained by assuming a simple power-law intensity dependence of the nonlinear correction to the index of refraction

$$\Delta n = n_\alpha I^{\alpha/2}, \tag{1.3}$$

where n_α is the effective coefficient of nonlinearity of the refractive index and α is the order index of the nonlinearity. This expression is strictly valid for noninertial nonlinearity of fixed order ($\alpha = 2, 4, 6, \ldots$). If components of nonlinear polarization of several orders contribute simultaneously to the nonlinear change in the index of refraction, then α can also be a positive nonintegral number. The expression (1.3) with $\alpha = 1/3$ for single-photon and $\alpha = 4/3$ with two-photon absorption of light describes well the nonlinearity of the index of refraction of many semiconductors, if Auger recombination of photoexcited carriers is taken into account.[25,20] For this reason, the theoretical analysis of nonlinear lenses is performed here with the help of the expression (1.3) for both even and odd integral and positive nonintegral values of the order of nonlinearity α.

Different formulas are presented in the literature for the focal length of nonlinear lenses in different approximations.[4,17,61,63] The focal length of a nonlinear lens with a bell-shaped transverse radiation intensity distribution and nonlinearity of the type (1.3) can be estimated from simple geometric considerations (Fig. 2):

$$F_{NL} = \omega'^2/\Delta, \tag{1.4}$$

where $2\omega'$ and Δ are the transverse and longitudinal magnitudes of the nonlinear sag of the wavefront. The nonlinear sag Δ is proportional to the nonlinear phase increment on the beam axis

$$\Phi_{NL} = k n_\alpha \int_0^l (I(0,0,z))^{\alpha/2} dz \tag{1.5}$$

and is equal to $\Delta = \Phi_{NL}/k$. The width of the sag, as is easily seen from Eq. (1.3), is approximately $\alpha^{1/2}$ times smaller than the width of the light beam. For this reason, in a power-law approximation of the nonlinear increment to the index of refraction (1.3) the focal length of the nonlinear lens is given by the expression

$$F_{NL} = k\omega^2/\alpha\Phi_{NL}, \tag{1.6}$$

where ω is the half-width of the beam at the exit of the nonlinear slab.

Under the standard experimental conditions the optical power of nonlinear lenses can reach a significant magnitude with quite sharp focusing of the beam in the nonlinear medium. Thus for $\Phi_{NL} = \pi$, wavelength of 1 μm, and beam radius of 100 μm, the optical power of a photoinduced lens with cubic nonlinearity ($\alpha = 2$) is 100 D. For liquid carbon disulfide (with orientational nonlinearity $n_2 = 10^{-11}$ CGSE), a medium widely employed in nonlinear optics, such a nonlinear lens is induced in a slab of the order of 1 mm thick with laser beam power of about 6 MW, easily realizable in pulsed lasers. For thermal nonlinearity, strong nonlinear lenses can be observed even with low-power cw-gas-laser radiation.[64–66]

The sign of a nonlinear lens with a bell-shaped beam profile is determined by the sign of the nonlinear correction to the index of refraction. When the correction to the index of refraction is positive $\Delta n > 0$ the nonlinear lens is positive

(converging) and when $\Delta n<0$ the nonlinear lens is negative (diverging). It is evident from Eq. (1.5) that the optical power of a nonlinear lens increases with beam intensity as $I^{\alpha}/2$, and under otherwise equal conditions it increases more for layers with high order of nonlinearity α. At the same time the relation $\omega'=\omega/\alpha^{1/2}$ implies that the fraction of the beam cross section affected by the nonlinear lens decreases with increasing α. Due to this effect the axial and peripheral zones of the beam are focused by the nonlinear lens at different distances. This means that nonlinear lenses with large values of α have strong aberrations. The considerations presented here determine the basic features of nonlinear lenses for different orders α of the nonlinearity. These features will be studied in greater detail in Sec. 1.5. We note also that nonlinear lenses can arise in ordinary lenses when high-power light fluxes pass through them.[107]

1.2. Thin nonlinear lenses

It was tacitly assumed in the forgoing discussion that the radiation intensity distribution $I(x,y,z)$, unperturbed by nonlinear effects, in a nonlinear slab can be used in the calculations. This is valid only when the focal length of the nonlinear lens is much greater than the thickness of the nonlinear layer:

$$F_{\rm NL}\gg l. \tag{1.7}$$

In the opposite case the nonlinear lens can change the amplitude profile of the light beam even in the nonlinear slab itself. For a sufficiently thick slab the nonlinear change in amplitude gives rise, in turn, to an additional phase increment, which itself gives rise to a new change in the amplitude, and so on. Such feedback, which is characteristic for nonlinear systems, makes nonlinear lenses less controllable at high initial radiation power. Such internal feedback plays an especially important role in media with positive nonlinearity, for which it gives rise to self-focusing.[61,67–69] This effect consists of the fact that when the radiation power exceeds some critical magnitude the beam collapses in the nonlinear focal point. The focal point of a nonlinear lens induced by the laser pulse moves into the slab from the back side, and this ultimately gives rise to damage in the material due to catastrophic growth of the radiation power density. It is important to note that for such thick positive nonlinear lenses the focal length can be much shorter than predicted by the expression (1.5). Conversely, in thick slabs with negative nonlinearity the change in the amplitude due to self-defocusing of the beam results in a significant decrease of the radiation intensity in the nonlinear medium and corresponding weakening of the nonlinear lens.[26,53]

In what follows a nonlinear lens in which the nonlinear changes of the beam intensity profile in the nonlinear slab can be neglected is considered to be thin. In contrast to the self-focusing regime, the thin nonlinear lens regime admits a comparatively simple analytical description, gives stable behavior of light beams, and does not result in damage to optical elements. For this reason, only thin nonlinear lenses are of practical significance for problems of optical measurements in the control of light fields. Thick nonlinear lenses can be of definite interest for such problems only in the case of negative refractive-index nonlinearity.

1.3. Thin nonlinear lens condition

It is obvious that for a slab of nonlinear material prescribed thickness a nonlinear lens is thin only in a certain range of light-beam powers. For this reason the thin nonlinear lens condition should have the form of a restriction on the thickness of the nonlinear layer

$$l<z_{\rm NL}, \tag{1.8}$$

where $z_{\rm NL}$ is the characteristic nonlinear-refraction length of light beams, which depends on the beam power P. We now consider the thin-nonlinear-lens condition for a Gaussian transverse field distribution, which is the most important case for practical applications,

$$E(r,z=0)=U_{\rm in}\exp\left[-\left(\frac{r}{\omega_{\rm in}}\right)^2+ikn_0\frac{r^2}{2R_{\rm in}}\right]; \tag{1.9}$$

where $U_{\rm in}$, $\omega_{\rm in}$, and $R_{\rm in}$ are, respectively, the on-axis amplitude, the half-width, and the radius of curvature of the wavefront of the beam at the entrance ($z=0$), and $r=(x^2+y^2)^{1/2}$. Such a distribution approximates well the structure of the laser beam of the fundamental mode of a stable laser resonator.

The field E in the material is described by a nonlinear parabolic equation[61]

$$2ikn_0\frac{\partial E}{\partial z}+\Delta_{\perp}E=2k^2n_0\Delta n\cdot E,$$

where n_0 is the linear index of refraction of the nonlinear material and $\Delta_{\perp}$ is the transverse Laplacian. It is well known that in the absence of nonlinearity ($\Delta n=0$) the solution with the boundary condition (1.8) is a Gaussian beam with z-dependent parameters. In particular, as a result of diffraction, the width ω of a collimated beam ($R_{\rm in}\to\infty$) varies as

$$\omega(z)=\omega_{\rm in}\left[1+\left(\frac{z}{z_0}\right)^2\right]^{1/2}, \tag{1.10}$$

where $z_0=kn_0\omega_{\rm in}^2/2$ is the diffraction length.[70] Then, following the approach of Ref. 71, the following expression can be derived for the nonlinear sag of the wavefront:

$$\Delta\Phi(x,y)=\Phi_{\rm NL}\exp\left[-\alpha\left(\frac{r}{\omega(l)}\right)^2\right], \tag{1.11}$$

where the on-axis nonlinear phase shift $\Phi_{\rm NL}$ is calculated according to Eq. (1.5) and is expressed in terms of tabulated integrals with integer values of the parameter α. Thus for cubic nonlinearity ($\alpha=2$) we obtain

$$\Phi_{\rm NL}=\frac{P}{P_{\rm cr}}\,{\rm arctg}\,\frac{l}{z_0}, \tag{1.12}$$

where $P=(cn_0/8)(U_{\rm in}\omega_{\rm in})^2$ is the beam power and we have introduced the critical self-focusing power[61,67–68]

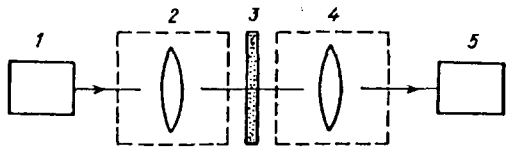

FIG. 3. General optical scheme for studying nonlinear lenses. *1*—laser, *2*—forming optical system, *3*—slab of nonlinear material, *4*—transforming optical system, *5*—system for recording the field distribution of the beam.

$$P_{\rm cr}=\frac{cn_0}{4k^2n_2}. \tag{1.13}$$

This gives the following estimate for the relative nonlinear change in the beam amplitude:[72]

$$\left|\frac{\Delta U_{\rm NL}}{U_{\rm L}}\right|<\frac{\alpha+2}{2}|\Phi_{\rm NL}|\frac{l}{z_0}. \tag{1.14}$$

A necessary and sufficient condition for the nonlinear lens to be thin is $|\Delta U_{\rm NL}|\ll|U_{\rm L}|$, where $U_{\rm L}$ is the field amplitude calculated neglecting the nonlinearity of the medium. In particular, given the criterion $|\Delta U_{\rm NL}|<|U_{\rm L}|/10$ we obtain from Eqs. (1.12) and (1.14) the thin nonlinear lens condition in the form

$$l<z_{\rm NL}=z_0\left(\exp\frac{10P_{\rm cr}}{P}-1\right)^{1/2}. \tag{1.15}$$

For finite radius of curvature $R_{\rm in}$ of the wavefront at the entrance into the slab we obtain from the condition (1.15), using the so-called lens transformation, the thin nonlinear lens condition in the more general form

$$l<\frac{z_{\rm NL}}{1+(z_{\rm NL}/R_{\rm in})}. \tag{1.16}$$

1.4. Beam transformation in an optical system with a thin nonlinear lens

Nonlinear lens investigations described in the literature were conducted for diverse configurations of the relative arrangement of the laser beam, the nonlinear lens, and the recording plane. In spite of this, the seemingly most diverse (with respect to the value of specific parameters) optical systems with nonlinear lenses can be studied from a unified standpoint and they can be compared with the help of the dimensionless complex β introduced in Ref. 73 for an optical system with a nonlinear lens.

For this, we consider the general optical arrangement of an experiment with a nonlinear lens. The arrangement contains a laser source and an optical system with a nonlinear lens. In a general optical system with a nonlinear lens it is possible to separate in the general case, besides a nonlinear element, two linear auxiliary subsystems (Fig. 3). The first subsystem forms the beam at the entrance into the nonlinear material with the required parameters (the forming optical system). The second subsystem performs the linear transformation of the beam after the nonlinear lens (the transforming optical system).

In experiments with nonlinear lenses the source is usually a single-mode laser with a nearly Gaussian transverse field structure. The radiation of such a laser has the property that its Gaussian transverse distribution is conserved during propagation in linear optical systems. For this reason, the field distribution at the entrance to the nonlinear element is also Gaussian and can be described by the expression (1.9). The parameters of this distribution $U_{\rm in}$, $\omega_{\rm in}$, and $R_{\rm in}$ can be easily calculated in terms of the parameters of the exit beam of the laser and the ABCD matrix of the forming optical system with the help of the well-known ABCD rule for linear optical systems.[70] We describe the transformation of the beam in the transforming optical system (see Fig. 3) by a modified Fresnel integral[70]

$$E(\mathbf{r})=\frac{k}{2iB}\iint\limits_{(\infty)}\mathrm{d}\mathbf{r}'^2E'(\mathbf{r}')\times\exp\left[\frac{ik}{2B}(Ar'^2+Dr^2-2\mathbf{r}\mathbf{r}')\right], \tag{1.17}$$

where A, B, and D are the elements of the ray matrix of the transforming optical system and $E'(\mathbf{r}')$ is the field at the exit of the nonlinear element. When the restrictions (1.15) and (1.16) on the thickness of the slab are satisfied, the field $E'(\mathbf{r}')$, as shown above (Sec. 1.3), has the form

$$E'(\mathbf{r}')=U\exp\left[-\left(\frac{r'}{\omega}\right)^2+\frac{ikn_0}{2R}r'^2+i\Phi_{\rm NL}e^{-\alpha(r'/\omega)^2}\right], \tag{1.18}$$

where U, ω, and R are the parameters of the Gaussian beam at the exit of the layer and are calculated neglecting the nonlinearity of the medium. Switching to dimensionless transverse coordinates $\vec{\rho}'=\mathbf{r}'/\omega$ at the entrance and $\vec{\rho}=(k\omega/B)\mathbf{r}$ at the exit of the transforming system, we obtain

$$E_{\rm out}(\vec{\rho})=U\iint\limits_{(\infty)}\mathrm{d}\vec{\rho}'^2\exp\left[-\rho'^2+i\Phi_{\rm NL}e^{-\alpha\rho'^2}+\frac{ik\omega^2}{2}\left(\frac{1}{R}+\frac{A}{B}\right)\rho'^2-i\vec{\rho}\vec{\rho}'\right]. \tag{1.19}$$

Hence one can see that for a prescribed nonlinearity (prescribed value of α) the character of the nonlinear deformations of the beam in an optical system with a nonlinear lens is determined by the single dimensionless parameter

$$\beta=\frac{k\omega^2}{2}\left(\frac{1}{R}+\frac{A}{B}\right). \tag{1.20}$$

In the simplest case, when the wavefront at the exit from the nonlinear element at low power is planar and the transmitted beam is transformed as a result of propagation in free space over a distance z, the parameter $\beta=k\omega^2/2z$. Therefore the transformation of the laser beam with nonlinear sag of the wavefront (1.19) in optical systems with

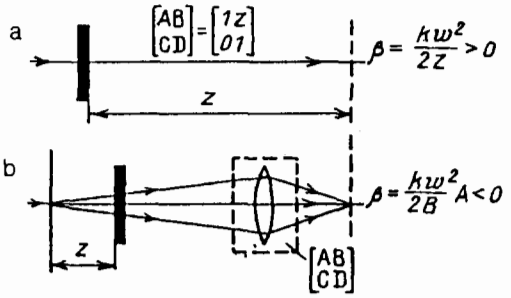

FIG. 4. Ray *ABCD* matrix and β-complex for a collimated beam in a nonlinear slab with free propagation over a distance z (a) and realization of negative equivalent distances with the help of a transforming optical system (b).

an overall spherical wavefront with radius R occurs, as in free propagation of the same beam with an initially planar wavefront, at the equivalent distance

$$z_{\text{eqv}} = \left(\frac{1}{R} + \frac{A}{B}\right)^{-1}. \tag{1.21}$$

In contrast to real free propagation, z_{eqv} can be both positive and negative, depending on the plane from which the image is transferred to the exit of the optical system (Fig. 4). Such an analysis makes it much easier to interpret the effects observed in these systems.

1.5. General properties of optical systems with thin nonlinear lenses

We now consider the nonlinear transformation of laser beams in an optical system with thin nonlinear lenses. In order to analyze such systems we investigate the diffraction integral (1.19) as a function of the parameters β and Φ_{NL}.

The configurational parameter β is the main parameter used for classification of different types of transformations of laser beams in optical systems with a thin nonlinear lens. According to its definition β has the same sign as the equivalent-propagation distance z_{eqv}, defined in Eq. (1.21), and the modulus is equal to the Fresnel number for an aperture of radius ω. The observation plane of the transverse distribution of the field will correspond to the near field of the beam in the case $|\beta| \gg 1$ and the far-zone of the beam in the case $|\beta| \ll 1$. The on-axis nonlinear phase increment Φ_{NL} characterizes the optical power of the nonlinear lens, and this power will be manifested differently in the near and far fields.

The properties of a nonlinear lens can also depend significantly on the nonlinearity order parameter α, which in accordance with Eq. (1.19) determines the transverse scale ω' of the nonlinear lens (see Fig. 2).

In the present section we examine some general properties of nonlinear lenses for characteristic combinations of the parameters α, β, and Φ_{NL}. We note first that the intensity distribution of the transmitted beam does not depend on the signs of β and Φ_{NL} separately, but rather it depends on the sign of their product. Indeed, as is evident from Eq. (1.19), when the signs of β and Φ_{NL} are changed simultaneously we arrive, to within a factor, at the field distribution which is the complex conjugate of the initial distribution and rotated with respect to it by 180°.[27,73] Such a symmetric property makes it possible to reduce the analysis of an optical system with a nonlinear lens to two cases: $\beta\Phi_{\text{NL}} < 0$, which, as will be shown below, corresponds to nonlinear expansion of the emerging beam, and $\beta\Phi_{\text{NL}} > 0$, for which nonlinear pinching of the beam occurs at the exit of an optical system with a nonlinear lens. For this reason, we assume below, without loss of generality, that Φ_{NL} is positive. All results obtained also hold for negative nonlinearity, if β is replaced by $-\beta$.

1.5.1. Aberration-free approximation

For some optical nonlinearities (for example, saturation nonlinearity[20]) the parameter α in Eq. (1.19) can take on values that are small compared to 1. The scale of the transverse nonlinear change in the phase of the field is then so much larger than the beam diameter that $\exp(-\alpha\rho^2)$ can be replaced by the first two terms of its series expansion: $\exp(-\alpha\rho^2) \approx 1 - \alpha\rho^2 - \ldots$. Therefore, in this approximation a nonlinear lens is a quadratic phase corrector. It then follows from Eq. (1.19) that a Gaussian beam at the exit of an optical system with a nonlinear lens remains Gaussian with Φ_{NL}-dependent parameters:

$$E_{\text{out}} = U \exp \frac{i\Phi_{\text{NL}} - \dfrac{\rho^2}{1 + i(\alpha\Phi_{\text{NL}} - \beta)}}{1 + i(\alpha\Phi_{\text{NL}} - \beta)}. \tag{1.22}$$

It is evident from Eq. (1.22) that for $\beta < 0$ the beam expands as Φ_{NL} increases (the width of the beam doubles at $\Phi_{\text{NL}} \approx (3+\beta^2)^{1/2}/\alpha$). For $\beta > 0$, as Φ_{NL} increases, the exit beam is pinched from zero to $\approx \beta/\alpha$, when the maximum pinch ratio $(1+\beta^2)^{1/2}$ is reached. As Φ_{NL} increases further, the nonlinear pinching of the beam is replaced by expansion.

Thus strong nonlinear deformations of the transverse intensity profile of the transmitted beam should be expected when $\Phi_{\text{NL}} > |\beta/\alpha|$, and in the near field ($|\beta| \gg 1$) the nonlinear lens is much stronger with pinching than expansion of the beam. In the far field ($|\beta| \ll 1$) nonlinear expansion of the beam prevails over pinching.

1.5.2. Weak nonlinear lenses

For $\alpha > 1$ the aberrations of nonlinear lenses can no longer be neglected, so that for nonlinearities which are most often encountered in practice (cubic nonlinearity ($\alpha = 2$); fifth-order nonlinearity ($\alpha = 4$), associated with two-photon absorption in semiconductors;[31] etc.) the transmitted beam can be calculated accurately only numerically. For small nonlinear phase shifts ($\Phi_{\text{NL}} \ll 1$), however, the following analytical expression can be obtained for the diffraction integral (1.19):[73]

$$E_{\text{out}} = U\left[\frac{\exp\left(-\dfrac{\rho^2}{1-i\beta}\right)}{1-i\beta} + i\Phi_{\text{NL}}\frac{\exp\left(-\dfrac{\rho^2}{s-i\beta}\right)}{s-i\beta}\right], \tag{1.23}$$

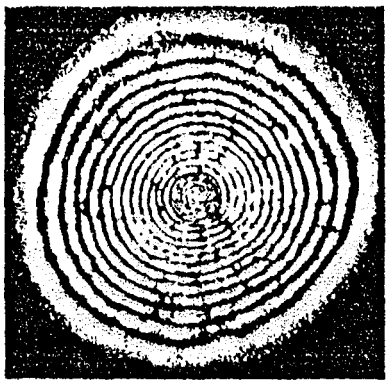

FIG. 5. Typical pattern of aberration rings in a laser beam which has passed through an optical system with a strong nonlinear lens.

where $s=1+\alpha$ is the order of the nonlinearity of the material. The differences from the aberration-free approximation lie primarily in the fact that the interference of the two terms in Eq. (1.23) determines the distortion of the Gaussian envelope of the transverse intensity distribution in the form of aberration rings. This distortion intensifies with increasing order α of nonlinearity. In addition, for large and small values of $|\beta|$ the intensity profile of the exit beam remains virtually Gaussian, while maximum distortions appear for the intermediate values $|\beta| \sim \alpha/2$.

1.5.3. Strong nonlinear lenses. Aberration rings

In the presence of strong nonlinear sags of the wavefront ($\Phi_{NL} \gg 1$) a characteristic ring structure appears in the transverse intensity distribution of radiation which has passed through the nonlinear medium (Fig. 5). The appearance of aberration rings is explained by interference of light rays arriving at the point of observation $\vec{\rho}$ from the points $\vec{\rho}_1'$ and $\vec{\rho}_2'$ with different phases $\Phi(\vec{\rho}_1')$ and $\Phi(\vec{\rho}_2')$ (Fig. 6), when

$$\Phi(\vec{\rho}_1') - \Phi(\vec{\rho}_2') = n\pi, \quad n=0, \pm 1, \pm 2, \ldots, \tag{1.24}$$

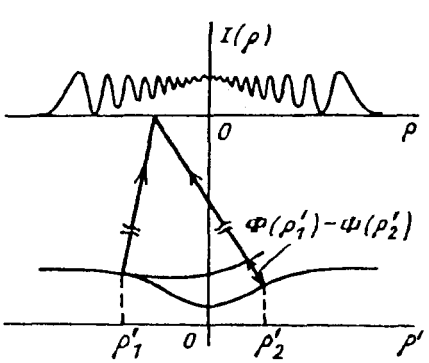

FIG. 6. Formation of aberration structure of the transverse intensity distribution of a transmitted beam with interference of light rays emanating from different points ρ_1' and ρ_2' on the wavefront of the initial beam. The figure shows rays whose phase difference at ρ gives a minimum of the field.

light rings forming for even integers n and dark rings forming for odd integers n. Thus a prerequisite for ring formation is that due to aberrations the light rays must intersect off-axis. However, amplitude-phase relations of the interfering rays play a determining role in the formation of the aberration ring structure. In order to perform a qualitative analysis of the phenomenon it is convenient to calculate the diffraction integral (1.19) approximately by the stationary-phase method (this is equivalent to the geometric-optics approximation). We obtain from the condition of stationary phase of the integrand that the rays emanating from the points of the initial distribution whose coordinates $\vec{\rho}'$ satisfy the equation

$$\vec{\rho} = 2\vec{\rho}'(\beta - \alpha\Phi_{NL}\exp(-\alpha\rho'^2)), \tag{1.25}$$

and the phases of the corresponding rays are

$$\Phi(\vec{\rho}') = \Phi_{NL}\exp(-\alpha\rho'^2)(1+2\alpha\rho'^2) - \beta\rho'^2, \tag{1.26}$$

make the main contribution to the field of the exit beam at the point $\vec{\rho}$. Hence it follows, in particular, that in the far field ($\beta \to 0$) radiation from two points $\vec{\rho}_1'$ and $\vec{\rho}_2'$ is directed into each point on the aperture of the exit beam, the distance $|\vec{\rho}_1' - \vec{\rho}_2'|$ between these two points being maximum for rays arriving near the axis ($|\rho| \ll 1$) and minimum for rays forming the edge of the transmitted beam ($|\vec{\rho}| \sim \rho_{max} = \Phi_{NL}$) (Fig. 6). Therefore the difference of the phases and amplitudes of pairs of interfering rays increases as the observation point $\vec{\rho}$ moves away from the edge toward the center of the beam. For this reason, the widest and darkest rings form in the peripheral region, and the period and amplitude of the spatial "beats" of the intensity decrease toward the beam axis (see Fig. 6). For large values of Φ_{NL} the total number of rings in the far field is given approximately by the expression[34]

$$N = \Phi_{NL}/2\pi. \tag{1.27}$$

According to the expressions (1.25) and (1.26), aberration rings form in the near field ($|\beta| > 1$) only if Φ_{NL} exceeds a definite threshold ($\Phi_{NL.th.} = |\beta/\alpha|$), and rings appear in the region of defocusing of the transmitted beam for any sign of the parameter β. For negative values of β the light rays intersect, due to aberrations, only at the edge of the beam, so that there are no rings near the axis. For positive values of β rings form, as Φ_{NL} increases, only after the nonlinear paraxial focal point bypasses the observation plane and the additional nonlinear focusing of the beam is replaced by defocusing of the beam. The total number of rings in the near field, naturally, is less than in the far field, and the laws governing ring formation are much more complicated, since, as follows from Eq. (1.25), ring appearance in the near field is now associated with the interference of not two but three rays.[2)]

The geometric-optics approximation employed above can be be used to describe the field at the light-shadow boundary as well as small-angle components of radiation in the far field, where diffraction must be taken into account. Diffraction calculations[73] show that, in particular, for values of Φ_{NL} far above the ring-formation threshold the aberration part of the radiation with comparatively large di-

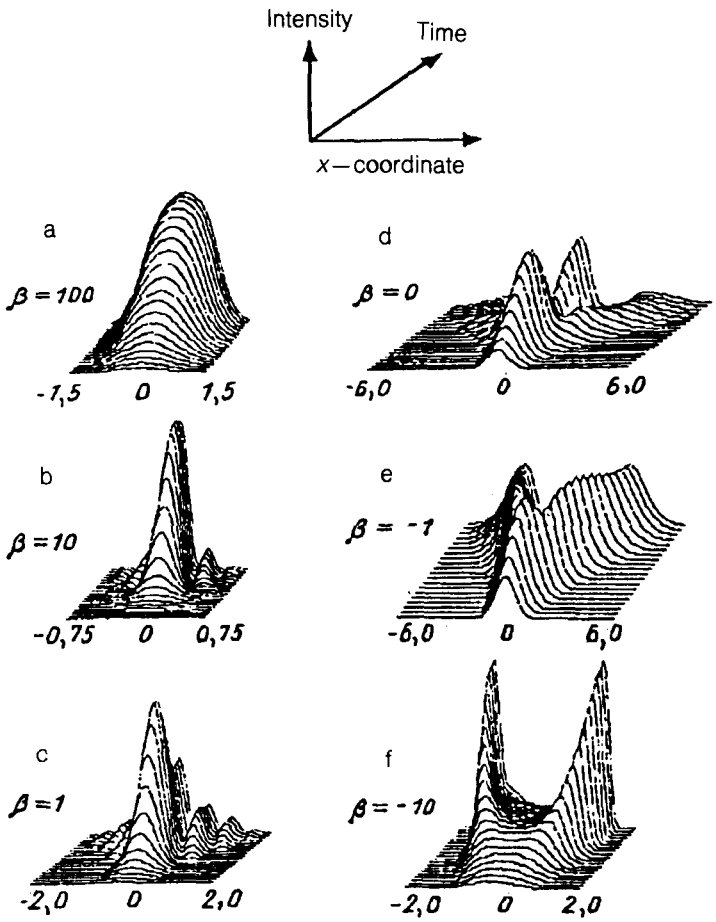

FIG. 7. Evolution of the space-time distribution of the intensity of light pulses which have passed through an optical system with a thin layer of a cubically nonlinear material for different values of β (shown in Figs. a–f).

vergence drifts away from the central "core" of the transmitted beam (the region bounded by the first dark ring), so that as Φ_{NL} increases, the transverse distribution of the field in the "core" approaches a Gaussian form. It is interesting that in the case of cubic nonlinearity ($\alpha=2$) the total radiation power in the "core" is virtually independent of the intensity at the entrance and is equal, in order of magnitude, to the critical power for self-focusing.

1.6. Photoinduced lenses in a slab of a cubically nonlinear medium

In this section we examine the case of greatest practical interest: transformation of Gaussian beams in optical systems containing a slab of material with cubic nonlinearity ($\alpha=2$). This nonlinearity is usually associated with electronic and electronic-nuclear mechanisms and is observed in the overwhelming majority of optical media exposed to sufficiently short light pulses. The nonstationary correction to the index of refraction, taking into account the finite relaxation time of the nonlinearity, can be written as the following more general expression instead of Eq. (1.2):[61,109]

$$\Delta n=n_2\int_{-\infty}^{t} I(t')\exp\left(\frac{t'-t}{\tau}\right)dt'. \qquad (1.28)$$

It is easy to see that the characteristics of quasistationary nonlinear lenses also extend to the case of inertial nonlinear response, described by the expression (1.28), of the material. Because of the inertial nature of the response nonlinear distortions accumulate toward the end of the light pulse, while the quasistationary nonlinear lens described by the expression (1.2) relaxes on the trailing edge of the pulse. For this reason, in this section we study, without loss of generality, the action of a nonlinear lens only on the leading edge of the light pulse in the quasistationary case (1.2). Even in this simple case the detailed picture of the transformation of radiation can be obtained only by numerical analysis of the diffraction integral (1.19). Figure 7 displays the results of computer modeling of nonlinear lenses under the assumption that the light pulses are Gaussian as a function of both the transverse coordinate and the time (Fig. 7). The intensity I of the incident beam was chosen so that the nonlinear phase increment at the maximum of the pulse would be 4π. The transverse coordinate x is scaled to the half-width $\omega_L=2|B|(1+\beta^2)^{1/2}/k\omega$ of the beam at low power, and the intensity is scaled to $|z_0/(Bk\ln_2)|$. Figures *a–f* display the manifestations of the same nonlinear lens as a function of the value of the configurational parameter.[27,72] In accordance with the analysis made above of the general properties of optical systems with a nonlinear lens, the changes in the intensity distribution of the transmitted beam appear when Φ_{NL} becomes comparable to $|\beta|$. If, however, $\Phi_{NL}\ll|\beta|$, then the spatiotemporal distribution of the intensity of radiation which has passed through the optical

system with a nonlinear lens is virtually identical to the intensity distribution of the incident beam (Fig. 7a). For $\beta>0$ the radiation is focused, the most striking effects of nonlinear pinching being observed for large values of $|\beta|$ (in the near field). In this case strong nonlinear beam pinching is replaced abruptly by nonlinear expansion, consisting of redistribution of the light power from the region near the axis into the first side maximum (first aberration ring) (Fig. 7b).

As the observation plane moves out of the near field into the far field (i.e., as β decreases) the nonlinear pinching of the beam becomes increasingly weaker, and defocusing starts at weaker intensities at the entrance, the light power being redistributed between the side maxima of the transverse intensity distribution (aberration rings) more uniformly (Figs. 7c, d). The increase in the transverse scale of the distributions, the broadening of the central peak, and especially the broadening of the side peaks with decreasing β are interesting. The maximum nonlinear "spreading" of the beam is observed in the far field ($\beta\to 0$) (Fig. 7d).

It is evident from the figures that the nonmonotonic (oscillating) character of the defocusing with increasing light intensity is connected with the formation of the side intensity maxima (aberration rings) and redistribution of power in them in favor of the higher-order peaks. It is also evident that in the first minimum of the intensity oscillations on the axis the transmitted intensity decreases as β decreases (Figs. 7b–d). The calculations show that this tendency also remains for negative values of β right down to values $\beta\approx -1.1$, when the on-axis amplitude of the field in the first minimum reaches zero (Fig. 7e). As β decreases further, the on-axis intensity of the radiation at the first minimum starts to increase and the first peak decreases, so that the nonlinear oscillations of the intensity near the axis are smoothed (Fig. 7f). Comparing Figs. 7c and d to Figs. 7e and f, we can see that as β becomes negative, the character of the formation of the aberration rings also changes radically. For $\beta>0$ rings form on the beam periphery and become narrower as the light intensity increases. For $\beta<0$ nonlinear "dispersion" of aberration rings away from the axis occurs, the higher order rings now arising closer to the axis. The nonlinear defocusing of the beam as a whole is expressed in the fact that most of the power in the beam is concentrated in the first (farthest away from the axis) aberration ring (Fig. 7f). Figure 7 displays the effect of the nonlinear lens only on the leading edge of the light pulse. The qualitative character of the nonlinear lenses on the leading edge of the pulse is identical for quasistationary (1.2) and inertial (1.28) responses of the medium. In these figures the computed radiation profiles can be extended to the trailing edge of the pulse either symmetrically with respect to the pulse maximum (for quasistationary nonlinear lenses) or with further accumulation of nonlinear distortions (for inertial nonlinear lenses) (see also Secs. 2.2 and Fig. 9 below). The transformations of laser beams by a nonlinear lens have been analyzed for some important particular cases in a number of works (see, for example, Refs. 11, 15, 16, 25, 47, and 72).

1.7. Thermal nonlinear lenses

As the duration of the laser action increases up to magnitudes comparable to the thermodiffusion time ($10^{-3}-10^{-1}$ sec), the photoinduced nonlinear lenses are usually associated with the nonuniform laser heating of the absorbing medium. In order to find the nonlinear correction to the index of refraction in this case it is necessary to solve the nonstationary heat-conduction equation with prescribed boundary conditions and heat source. This problem has been solved analytically in Refs. 75–78 for a rectangular heat pulse with a Gaussian spatial profile (this corresponds to a rectangular radiation pulse in the form of a colimated Gaussian beam). In this case the nonlinear correction to the index of refraction can be expressed in terms of a combination of exponential integral functions:

$$\Delta n\approx 0.06\,\frac{\mathrm{d}n}{\mathrm{d}T}\,\frac{bP}{\chi}\left\{\mathrm{Ei}\left(-2\,\frac{r^2}{\omega^2}\right)-\mathrm{Ei}\left(-\frac{2r^2}{\omega^2[1+(t/\tau)]}\right)\right\}, \tag{1.29}$$

where P and ω are the power and half-width of the radiation beam, b and χ are the absorption coefficient of the radiation and the coefficient of thermal conductivity of the medium, and τ is the characteristic time over which a thermal lens is established

$$\tau=\frac{\omega^2\rho c_P}{8\chi} \tag{1.30}$$

and depends on the specific heat capacity c_P of the medium. In the transitional regime with $\tau_p\ll\tau$ the nonlinear correction to the index of refraction increases with time in accordance with the absorbed energy and can be described by the expression (1.28), in which n_2 is a combination of the thermophysical parameters of the medium.[16] Thus the theory developed in Sec. 1.6 for cubic nonlinearity is applicable to nonstationary thermal nonlinear lenses. For radiation pulses with duration comparable to or greater than the thermodiffusion time τ, the nonlinear phase increment can be calculated with the help of the expression (1.29) (for thin thermal lenses) or the transfer equation for the field can be solved directly and simultaneously with the heat-conduction equation (for thick thermal nonlinear lenses).[78] Different aspects, associated with the thermal nonlinearity, of the theory of nonlinear lenses are examined in Refs. 79–83.

1.8. Thin nonlinear lenses in materials with quadratic nonlinearity

In crystals the nonlinear correction to the index of refraction can be caused by the quadratic nonlinearity of the material.[84] Nonlinear lenses associated with quadratic nonlinearity are observed under conditions of three-wave interactions in crystals with small detuning from phase matching. Self-focusing of laser beams under conditions of three-wave interactions in a thick slab of a material with quadratic nonlinearity was investigated in Refs. 85 and 86. Thin nonlinear lenses in crystals with quadratic nonlinearity and their effect on second-harmonic generation were studied in Refs. 87 and 88. We now consider a shortened

equation for the complex amplitudes of the laser-pump wave A_1 and the second-harmonic wave A_2 with 00e-interaction:

$$i\frac{\partial A_1}{\partial z}=\sigma A_2 A_1^* e^{-i\Delta z},\quad i\frac{\partial A_2}{\partial z}=\sigma(A_1)^2 e^{i\Delta z}, \tag{1.31}$$

where the wave detuning $\Delta=2k_1-k_2$, k_1 and k_2 are the wave numbers of laser pumping and the second harmonic, z is the coordinate in the direction of light propagation, and the nonlinear coupling constant σ is proportional to the components of the quadratic nonlinearity tensor $\chi^{(2)}$ of the crystal. The solutions of these equations for the pump phase Φ_1 and the second-harmonic phase Φ_2 have the form[86]

$$\Phi_1=\Phi_{\rm NL},\quad \Phi_2=\frac{\Delta\cdot z}{2}-\frac{\pi}{2}, \tag{1.32}$$

where the nonlinear phase increment of the light for the pump wave is expressed in terms of the known solutions for the amplitudes of the interacting waves[88]

$$\Phi_{\rm NL}=\frac{\Delta}{2}\int_0^l dz\left|\frac{A_2(z)}{A_1(z)}\right|^2. \tag{1.33}$$

It is evident from these expressions that in the case of nonzero detuning from synchronism the pump wave is subjected to phase self-action. The sign of the nonlinear phase increment depends on the sign of the wave detuning Δ, so that in the same crystal the nonlinear lens can be both positive and negative (see Fig. 12). The magnitude of the nonlinear phase increment can be very significant, especially in the case of large local transformation of the pump into the second harmonic. In particular, for crystals of the KDP group the effective coefficient of nonlinearity of the index of refraction of the quadratic nonlinearity can exceed n_2 of highly nonlinear materials, such as liquid carbon disulfide ($n_2\sim 10^{-11}$ CGSE).[88]

The nonlinear lens associated with the quadratic nonlinearity of the crystal affects the transverse distribution of the pump beam in the far field. In practice, however, the most important effect of this lens is the change in the spatiotemporal structure of the second-harmonic field already in the nonlinear crystal itself. Indeed, it follows from Eq. (1.31) that the energy of the waves interacting in the crystal is transferred in a direction determined by the quantity

$$\sin(2\Phi_1-\Phi_2+\Delta\cdot z)=\cos\left(2\Phi_{\rm NL}+\frac{\Delta\cdot z}{2}\right). \tag{1.34}$$

Therefore, the direction of transfer obviously changes when $|\Phi_{\rm NL}|$ increases by an amount of the order of $\pi/2$. The energy transfer induced between the waves by the photoinduced lens results in distortion of the amplitude distribution and decrease of the coefficient of conversion into the second harmonic.[88]

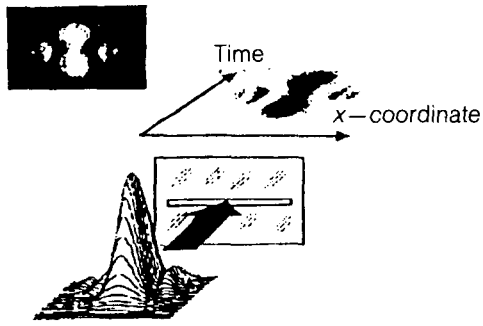

FIG. 8. Recording of the space-time distribution of the intensity of ultrashort laser pulses with the help of a streak camera.

2. EXPERIMENTS WITH NONLINEAR LENSES

2.1. Methods of experimental study of nonlinear lenses

A typical experimental arrangement for observation and investigation of the properties of nonlinear lenses is displayed in Fig. 3. The radiation of the laser 1 is focused by the forming optics 2 in the slab 3 of nonlinear material. This gives the required optical power density. The transmitted beam is transformed by the optical system 4 and enters the recording system 5. Depending on the problems studied, either only the time-integrated spatial distribution of the transmitted beam or the temporal evolution of the distribution (for pulsed radiation) was studied in different works. In a number of works nonlinear lenses, induced by IR laser pulses, were recorded in the visible region of the spectrum with the help of a probe beam from an auxiliary cw laser.[15,16]

For pulsed radiation the record of the time-averaged transverse structure of the transmitted beam does not give adequate information about the optical nonlinearity, especially if the duration of the laser action becomes comparable to and shorter than the characteristic times of establishment and relaxation of the nonlinear response. Methods of fast scanning of radiation pulses separated by spatial filters from different sections of the transmitted beam are widely used for investigation of nonlinear lenses induced by short (10^{-7} sec) laser pulses. In experiments with submicrosecond pulses spatial filtering of the transmitted radiation was performed with the help of circular or annular diaphragms with the electric signal from a fast photodetector, placed behind the diaphragm, time-resolved on an oscilloscope.[25,29] For pulses shorter than 1 nsec slit diaphragms were arranged along the diameter of the light beam and the transmitted radiation was resolved on the screen of a streak camera (Fig. 8).[23,24,73,91] This is the most informative approach for investigation of the dynamics of nonlinear lenses and makes it possible to compare in detail the experimental data to theoretical predictions. In addition, for pulses shorter than 1 nsec the nonlinear response of the material can be considered to be spatially local to a high degree of accuracy. For this reason, in what follows, when the experimental data are compared to the theoretical calculations we consider primarily streak-camera ex-

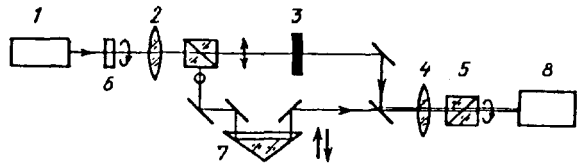

FIG. 9. Arrangement of the experimental apparatus for investigations of thin nonlinear lenses. *1*—laser, *2*—forming optics, *3*—slab of nonlinear material, *4*—transforming optical system, *5*—polarizer, *6*—$\lambda/2$ phase plate, *7*—regulatable optical delay line, *8*—fast photorecorder.

periments with subnanosecond laser pulses. Among other methods for investigating dynamical nonlinear lenses, we call attention also to a method based on successive excitation of a nonlinear lens by a high-power short laser pulse and probing of the relaxation of the nonlinear response by weak, time-delayed probe pulses.[21] In this case photorecorders with high temporal resolution are not required, but the information obtained is averaged.

2.2. Investigations of nonlinear lasers in the field of ultrashort laser pulses

Photoinduced lenses, which vanish over a time shorter than 10^{-11} sec after passage of the light pulse, were observed back in the mid-1970s with the help of streak cameras.[89,91] Nonlinear deformations of the spatiotemporal structure of picosecond laser pulses were observed for nonlinear lenses in semiconductors,[27,28,92] glasses[91] and liquid carbon disulfide.[27,90] The temporal evolution, due to thin nonlinear lenses induced by laser pulses in the active elements of the system, of the transverse distribution of subnanosecond pulses at the exit of a multicascade neodymium-glass laser system was investigated in Ref. 89. Dynamical nonlinear lenses in the field of ultrashort laser pulses were studied systematically in later works.[27,72,90,93] The β-complex introduced above is convenient for analyzing the results of these works. As noted in Sec. 1.5, optical systems with a nonlinear lens exhibit symmetry, consisting in the fact that the nonlinear transformation of radiation in such systems does not change when the signs of the parameters β and Φ_{NL} are changed simultaneously, and when β and Φ_{NL} have the same signs, nonlinear pinching of the exit beam and nonlinear expansion—in the opposite case—occur. This symmetry property was first observed experimentally in Ref. 27, where nonlinear lenses induced by ultrashort pulses in media with both positive (liquid carbon disulfide) and negative (semiconductors) nonlinearity with different tunings of the transforming optical system, were investigated.

The polarization interferometer arrangement[90,93] shown in Fig. 9 is convenient for experimental investigations of nonlinear lenses. A thin element, in which a nonlinear lens is induced, is placed in one arm of the interferometer. The second arm of the interferometer is used to produce a reference comparison channel, in which only linear transformation of the radiation of the forming (2) and transforming (4) optics occurs. The ratio of the intensities in the investigated channel and the comparison channel can be varied continuously over wide limits by rotating the $\lambda/2$ phase plate 6. In addition, the time interval between the pulses in the two channels can be established by regulating the optical delay line (7). This organization of the optical arrangement makes it possible to investigate the nonlinear transformation of both the amplitude and phase structure of the pulsed laser radiation in optical systems with a nonlinear lens.

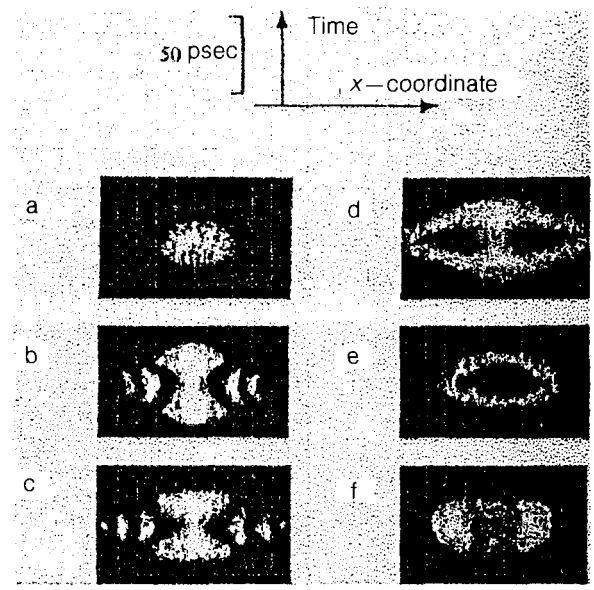

FIG. 10. Evolution of the space-time intensity distribution of light pulses which have passed through an optical system with a thin slab of liquid carbon disulfide for different values of β: 100 (a), 10 (b), 3 (c), 0 (d), -1 (e), and -10 (f).

We present below the results of experimental investigations of fast nonlinear lenses, obtained on an apparatus of this type in Refs. 27, 72, and 94. The second-harmonic radiation of an aluminum-yttrium garnet laser with neodymium (wavelength 0.53 μm), operating in the passive longitudinal modelocking regime and generating the fundamental transverse mode TEM_{00}, was employed in these works. The radiation passing through the optical system with a thin nonlinear lens entered the streak camera 8 with time resolution of 4 psec. Figure 10 displays the evolution of the transverse intensity distribution of a picosecond pulse after the pulse passes through the optical system with a slab of liquid carbon disulfide. The short duration of the light pulses (50–100 psec) excluded nonlocal inertial nonlinearities (strictional, thermal, etc.). In addition, Δn_{NL} of carbon disulfide is positive and associated with cubic (in the field) polarizability of the medium. The high value of the nonlinear refractive index of carbon disulfide made it possible to obtain with comparatively low-power (<1 MW) laser radiation a nonlinear phase increment in carbon disulfide up to $(3\text{–}4)\cdot\pi$ with a 5 mm thick slab and beam diameter in the slab less than 100 μm. Comparing the streak pictures in Fig. 10 to the corresponding computed intensity profiles (see Fig. 7) shows that the experiment agrees well with the theory. We note also that the

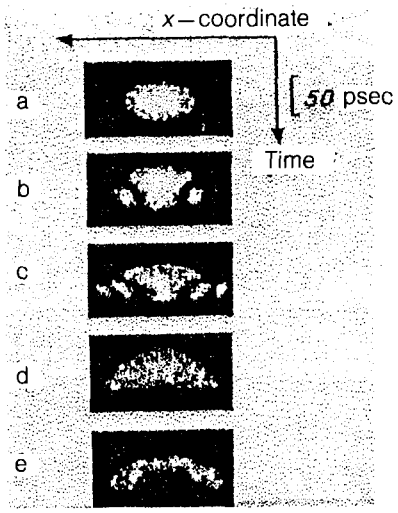

FIG. 11. Evolution of the space-time intensity distribution of light pulses which have passed through an optical system with a thin slab of the semiconductor CdS for different values of β: -100 (a), -10 (b), -3 (c), 0 (d), 1 (e), and 10 (f).

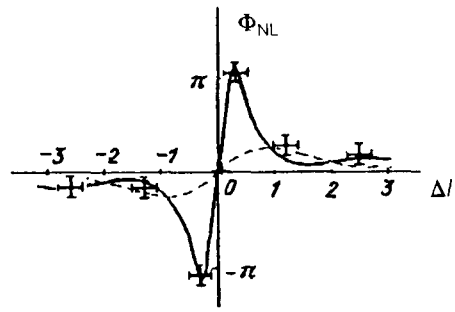

FIG. 12. Nonlinear phase increment Φ_{NL} for laser pulses in materials with quadratic nonlinearity as a function of the normalized wave detuning Δl from synchronism of second-harmonic generation (l is the length of the nonlinear crystal). The dashed line represents the value of Φ_{NL} calculated in the fixed-intensity approximation.[87] The solid line was constructed using Eq. (1.33). The experimental data (crosses) are presented for pumping pulses in the case of second-harmonic generation in a CDA crystal by YAG:Nd-laser pulses with peak intensity $I \sim 100$ MW/cm^2.[88]

relaxation of the nonlinear lens on the trailing edge of the pulse is also virtually inertialess (see Fig. 10). This agrees with the relaxation time of the orientational nonlinearity of carbon disulfide (2 psec).

Similar investigations have been performed for nonlinear lenses in plates of single crystals and optical ceramics of group A_2B_6 wide-bandgap semiconductors (CdS, ZnS, ZnSe).[27,72,94] The mechanisms of optical nonlinearity in semiconductors are very diverse, but for subnanosecond laser pulses the most important ones are electronic impurity recharging transitions[22] as well as photoexcitation of nonequilibrium bound (excitons) and free carriers.[60] Filling of vacant impurity levels in weakly doped semiconductors occurs in comparatively weak light fluxes, so that for sufficiently high radiation intensity the photoinduced electron-hole plasma makes the main contribution to the optical nonlinearity. The sign of the nonlinearity is negative, and the relaxation time exceeds 10^{-9} sec.[60] The nonlinearity order parameter of wide-bandgap semiconducting compounds increases with the gapwidth as a result of the fact that the contribution of single-photon saturating absorption to carrier photogeneration decreases and the contribution of multiphoton processes increases. Nonetheless, as one can see from Fig. 11, the above-studied mechanisms of nonlinear transformation of beams remain qualitatively the same in this case also.

The main qualitative difference between nonlinear lenses in semiconductors with picosecond excitation and nonlinear lenses in carbon disulfide lies in the fact that in semiconductors the nonlinear phase increment is determined by the moment of the envelope of the intensity $I(t)$ of the laser pulse:

$$\Phi_{NL} \approx \int (I(t))^{\alpha/2} dt \qquad (2.1)$$

where, generally speaking, $\alpha \neq 2$, and the nonlinear lens does not relax on the trailing edge of the pulse (Fig. 11).

Nonlinear lenses in materials with quadratic nonlinearity were investigated in Ref. 88. It was shown by the methods of dynamic interferometry that the nonlinear correction to the index of refraction at the pump frequency in crystals of the KDP group can exceed the nonlinear correction to the index of refraction of liquid carbon disulfide at the same intensities. The sign of the nonlinear lens can be controlled by changing the sign of the wave detuning from synchronism by rotating the crystal (Fig. 12). Considering that the response time of the electronic nonlinearity lies in the femtosecond range, these features of nonlinear lenses in materials with quadratic nonlinearity make such materials promising for applications in ultrafast devices for controlling light with light. A distinguishing feature of nonlinear lenses in materials with quadratic nonlinearity is that they appear mainly near synchronism of three-wave parametric interactions and can significantly change the transverse distribution of the intensity of interacting waves already directly in the thin nonlinear layer. This effect was observed experimentally in Ref. 88, when in a number of crystals under conditions of generation of the second harmonic of picosecond pulses the temporal structure of the second-harmonic pulse acquired with increasing laser power the form of a "nucleus" in a "shell" (Fig. 13).

2.3. Experiments with thermal nonlinear lenses

The first investigations of nonlinear lenses were made in experiments with thermal optical nonlinearity in liquids, when changes in the size and shape of a laser beam which has passed through a cell with a weakly absorbing liquid were observed with increasing laser power.[5,76–78] Similar effects were observed on thermal nonlinearities in liquid crystals,[35] ferroceramics,[66] and semiconductors.[19,20] The accumulation of nonlinear changes in the index of refraction over a long time interval (of the order of the relaxation time constant of the nonlinearity, which for many thermal nonlinearities is several seconds and longer) made

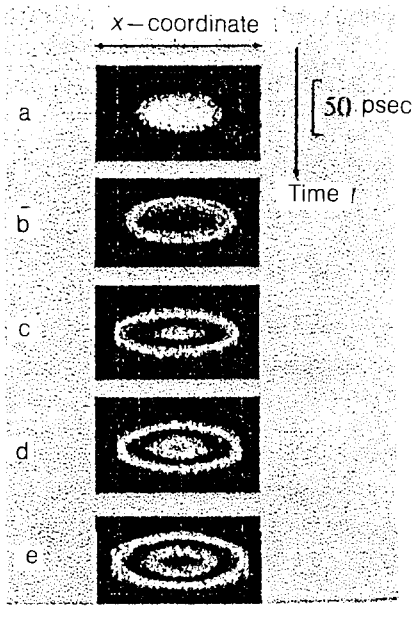

FIG. 13. Deformation of the space-time intensity distribution of picosecond pulses of the second harmonic of a neodymium laser due to an induced lens on a quadratic nonlinearity in a CDA crystal with small detuning of synchronism and different peak intensities: $I\,(MW)=50$ (a), 100 (b), 150 (c), 200 (d), and 250 (e).

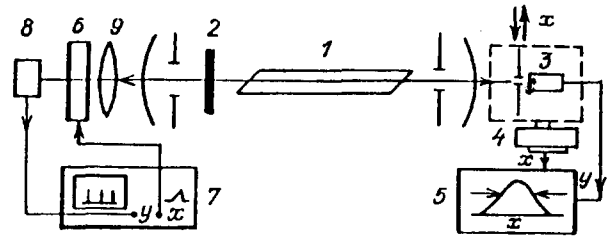

FIG. 14. Arrangement for measuring small coefficients of absorption ($<10\ \mathrm{cm}^{-1}$) by the intracavity thermal nonlinear lens method.[5] *1*—gas laser, *2*—cell with the experimental material, *3–5*—apparatus for monitoring the transverse intensity distribution of the laser mode (photomultiplier 3, scanning over the transverse cross section of the beam by means of a stepping motor 4 and display on a plotter 5), *6–9*—system for monitoring the spectral composition of radiation based on a scanning interferometer 6.

it possible to observe thermal nonlinear lenses even in radiation from low-power cw gas lasers. In this case the nonlinear wavefront sag Φ_{NL} in the material can be much greater than the wavelength of the light wave, so that more than 100 aberration rings can be observed in the far field of the beam.[34,35,66] The total number of rings is estimated from the nonlinear phase increment Φ_{NL} on the beam axis according to the formula (27) for different types of thermal nonlinearities.[34] The details of the spatial distribution of the transmitted beam can, however, differ strongly in different types of materials. Thus a characteristic feature of thermal self-(de)focusing of an axisymmetric laser beam in liquids is vertical astigmatism of the transmitted beam. This astigmatism increases with the radiation intensity and is associated with convection in the presence of strong laser heating.[76–78] Thermal nonlinear lenses in the elements of cw and pulsed laser cavities can significantly affect the mode structure[7,75] and the lasing dynamics of the laser,[65] and they are also manifested in the fine structure of the radiation spectrum.[83,95] Thermal lenses, together with nonlinear lenses in liquid crystals with orientation of the director,[35] are the simplest and most striking demonstrations of the nonlinear lens effect.[64]

3. APPLICATIONS OF NONLINEAR LENSES

3.1. Applications of nonlinear lenses for measurement of weak absorption

Historically, the first[5] and still one of the most important applications of nonlinear lenses is measurement of weak absorption ($10^{-4}\ \mathrm{cm}^{-1}$). Such absorption cannot be measured by the methods of traditional absorption spectroscopy, and in order to do so it was necessary to develop special highly sensitive methods, such the optoacoustic method[96] and the method of nonlinear lenses.[5–7] The crux of the nonlinear-lens method is the determination of the optical power of the light-induced thermal lens. This power is proportional to the absorption coefficient of the liquid. As follows from the expression (1.29), the optical power of a thermal lens in the steady state[76–78]

$$D_\infty \approx 0.24\,\frac{\mathrm{d}n}{\mathrm{d}T}\,\frac{blP}{\pi\chi n_0\omega^2} \tag{3.1}$$

is directly proportional to the beam power P, the absorption coefficient b, and the thickness l of the slab of material. Thus, if the thermophysical parameters of the material are known, then it is easy to determine the absorption coefficient b by measuring D_∞. The accuracy and sensitivity of the nonlinear-lens method for measuring weak absorption were investigated in Refs. 7, 12, and 15. The theoretical threshold of sensitivity of this method is $10^{-6}-10^{-8}\ \mathrm{cm}^{-1}$ with an accuracy of the order of 30%.[15] Different methods have been developed to achieve maximum measurement sensitivity and accuracy. One method is to place the cell containing the experimental material into a laser cavity (Fig. 14). The laser radiation induces in the cell a thermal lens that changes the configuration of the laser mode. By monitoring the mode composition of the radiation with the help of a scanning interferometer and measuring the radius of the laser beam with the help of a photomultiplier controlled by a stepping motor it is possible to measure, to a high degree of accuracy, the optical power of a thermal lens and to determine with its help the absorption coefficient. As shown in Ref. 7, the maximum sensitivity of the intracavity nonlinear-lens method is achieved in cavities with a nearly confocal configuration. This method was used in Ref. 6 to measure the absorption coefficients of 27 commercial solvents. A method in which high sensitivity is achieved by using a two-beam method is now widely used. In this method a strong nonlinear lens is induced by a powerful laser pulse with a single wavelength, and this lens "read out" with radiation from a highly stable cw laser at

a different wavelength. This method is used successfully for measuring weak absorption and investigating intramolecular transitions in gases and liquids.[15]

3.2. Applications of nonlinear lenses for measuring the coefficients of nonlinearity of the refractive index of optical materials

Different modifications of the nonlinear-lens method are used for measuring the nonlinear refractive coefficients of optical materials. In Ref. 18 the nonlinear refractive coefficients n_2 and n_4 of semiconducting single crystals were determined from measurements of the diameter of a transmitted low- and high-power laser beams. When performing measurements by this method it is important to take into account the fact that the nonlinear change in the diameter of the transmitted beam with the same nonlinear phase increment will be different for different configurations of the optical measurement scheme.[20,28] As shown in Ref. 73, in such measurements schemes characterized by β values equal in modulus to the order of nonlinearity of the experimental material $S=1+\alpha$ give the highest sensitivity. This conclusion follows from the "resonance" dependence, first described in Ref. 73, of the quantity γ, determining the sensitivity of the intensity I_{out} on the beam axis in the recording plane to the nonlinear wavefront sag, on the value of β:

$$\gamma=\frac{1}{I_{\text{out}}}\frac{\mathrm{d}I_{\text{out}}}{\mathrm{d}\Phi_{\text{NL}}}\bigg|_{\Phi_{\text{NL}}=0}=-\frac{2\alpha\beta}{(1+\alpha)^2+\beta^2}. \tag{3.2}$$

Later, the so-called Z-scan method was also constructed on the basis of this resonance-like dependence.[33,97–98] This version of the nonlinear-lens method is based on measurement of the on-axis intensity of the transmitted radiation as a function of the location of the nonlinear element along the Z axis of the beam for fixed initial power (see Fig. 13). When the recording plane is located in the far field β is given by the ratio[73]

$$\beta=Z/Z_0, \tag{3.3}$$

where Z is the coordinate of the position of the nonlinear element with respect to the waist of the laser beam and Z_0 is the confocal parameter. Thus, when the coordinate Z of the position of the nonlinear element is scanned, an N-shaped "resonance" curve is observed in the transmitted signal. The order of nonlinearity $1+\alpha$ and the nonlinearity constant n_α are determined from the height and position of the extrema of this curve. Taking into account the dependence of the intensity in the nonlinear element on the position of the element, the order of nonlinearity $S=1+\alpha$ is determined in terms of the corresponding values of β at the maxima of the curve in Fig. 15 from the formula

$$S=\pm\beta_{\text{m}}[(1+3\beta_{\text{m}}^2)/(1-\beta_{\text{m}}^2)]^{1/2}. \tag{3.4}$$

Another version of the nonlinear-lens method is based on the oscillating dependence, mentioned in Sec. 1.6, of the on-axis intensity I_{out} of the transmitted beam on the intensity I_{in} at the entrance. In Ref. 25 it is shown that in the far field the first maximum of this function occurs when some

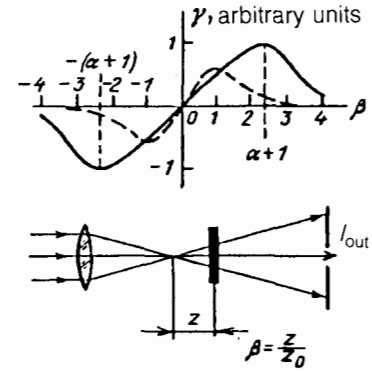

FIG. 15. Measurements of the nonlinearity of refractive indices by the z-scan method.[3] The on-axis intensity of the transmitted beam is plotted as a function of the coordinate z of the displacement of the nonlinear material from the waist of the exciting beam.

nonlinear phase increment $\Phi_{\text{NL}}^{\text{m}}$ is reached. The value of this increment can be approximated by the expression

$$\Phi_{\text{NL}}^{\text{m}}\approx\frac{\pi}{2}\frac{\alpha+1}{\alpha}. \tag{3.5}$$

Thus if the order of refractive nonlinearity $1+\alpha$ is known, then n_α can be determined with the help of the expressions (2.1) and (3.5).

In Ref. 9 the coefficients of nonlinear refraction of laser glasses were determined from the optical power of the nonlinear lens in the experimental sample, for which self-focusing of the laser beam occurs in an auxiliary sample whose nonlinearity is known.

For relative fast measurements of the coefficients of nonlinear refraction (in the presence of a standard material) it is convenient to compare the dynamics of nonlinear conversion of pulses transmitted through the experimental material and the standard material, which are installed in different arms of a polarization interferometer, whose layout is displayed in Fig. 9.[93] The ratio of the light intensities in the interferometer arms is regulated continuously by rotating the half-wave plate 6. The relative values of the nonlinear refractive coefficients of the experimental material are determined from the angle of rotation of the half-wave plate at which the nonlinear-lens dynamics in the standard material is identical to that of the experimental material.[28,93]

The thin nonlinear lens effect must also be taken into account when measuring the nonlinearity of the refractive index by the interference[89] and polarization[71] methods. These methods are not directly related to nonlinear lenses, but if the nonlinear lenses induced by laser radiation in an experimental sample is neglected, than a gross error is made when the nonlinear-refractive coefficients are determined with their help. In particular, it is shown in Ref. 71 for the case of cubic nonlinearity that in far-field measurements the nonlinear phase increment decreases by a factor of 2 due to diffraction. In the general case the true nonlinear phase increment Φ_{NL} in the sample can be determined, with the help of the conversion factor K following from the

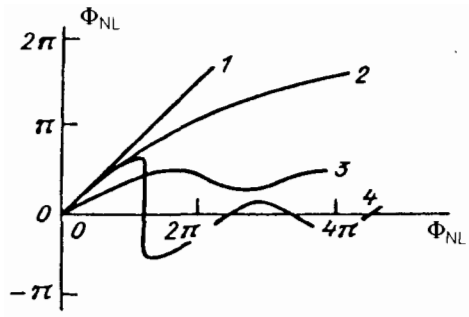

FIG. 16. Nonlinear phase increment Φ'_{NL} in the recording plane as a function of the nonlinear phase increment Φ_{NL} in the nonlinear material for different values of the configuration parameter of the scheme: $\beta=10$ (a), 3 (b), 0 (c), and -1.1 (d).

expression (1.23), from the value Φ'_{NL} measured experimentally (for example, by the interference method):

$$K=\frac{\Phi'_{NL}}{\Phi_{NL}}=\frac{S+\beta^2}{S^2+\beta^2}. \tag{3.6}$$

It is obvious that in near-field ($|\beta|\gg 1$) measurements the experimentally measured nonlinear phase increment in the sample is identical to the true value. When the recording plane lies in the far field, the nonlinear phase increment decreases monotonically due to diffraction spreading (in the far field the nonlinear phase shift decreases by a factor of S with respect to the true value of the nonlinear phase increment in the nonlinear medium).

These characteristics of the decrease in absolute magnitude of the nonlinear phase increment in a beam as the beam propagates from the near- into the far field are valid for $\Phi_{NL}<1$. For large values of Φ_{NL} the photoinduced lens effect is even more significant from the standpoint of the spatial evolution of the nonlinear phase increment. The functions $\Phi'_{NL}(\Phi_{NL})$ are displayed in Fig. 16 for different values of β. It is obvious from the figure that this dependence is nearly linear for $\beta>10$ and $\beta<3$ and as $|\beta|$ decreases, the dependence acquires characteristic oscillations. For $\beta=-1$ and $+1$ a sharp jump downwards by π occurs on the curve $\Phi'_{NL}(\Phi_{NL})$. Comparing these curves to the plots in Fig. 7 shows that the depth of the phase oscillations increases in accordance with the depth of the intensity oscillations, and the jump in phase by π occurs when the intensity on the axis of the transmitted beam becomes zero. It is interesting to note that in this case the recorded nonlinear phase increment can even change sign, while the nonlinear phase increment in the sample increases monotonically. This effect was observed experimentally in Ref. 94. In the far field ($\beta\to 0$) the observed nonlinear phase increment does not exceed $\pi/2$, even for very large values of Φ_{NL}.

3.3. Application of nonlinear lenses for investigation of the properties of semiconducting materials

Nonlinear lenses are a convenient tool for investigating different properties of semiconducting materials. In Ref. 18 the dispersion properties of CdS and SiC single crystals were determined with the help of nonlinear lenses. In Ref. 22 the spatial distribution of impurities in the volume of a semiconducting crystal was measured by the nonlinear-lens method. In Ref. 30 the dependence of the relaxation time constant of nonequilibrium carriers was determined as a function of the impurity concentration by studying the dynamics of nonlinear lenses in n-InP samples. In Ref. 28 the anisotropy of the nonlinear susceptibility of CdS single crystals was measured by the nonlinear-lens method. In Ref. 33 the z-scan method (Sec. 3.2), developed in Refs. 97 and 98, was used to determine the dispersion of the nonlinear susceptibility of an entire series of wide-bandgap semiconductor compounds.

3.4. Applications of nonlinear lenses for controlling laser radiation

Nonlinear lenses can be used for solving a wide spectrum of problems involving the control of laser radiation, primarily for creating different types of fast optical switches. Thus, in Ref. 42 a nonlinear lens induced in a semiconducting plate by low-power pulsed laser radiation was used to switch the transmission of a powerful *cw*-laser beam at a different wavelength. By returning part of the transmitted radiation, with the help of auxiliary mirrors, back into the nonlinear medium it is possible to change significantly the nonlinear-transmission characteristics of optical switches based on nonlinear lenses. Bistable optical devices were realized in Refs. 43–45 with the help of such feedback. In Ref. 1 it was proposed that dynamical switching of radiation from a compact beam into an aberration ring (see Fig. 7b) be used to decrease the duration of ultrashort laser pulses. The important role of nonlinear lenses in the lasing dynamics of high-power pulsed lasers with mode self-locking was pointed out in Refs. 100 and 101. Nonlinear lenses in a laser cavity can be used for controlling the lasing regime. Thus this method was used in Refs. 102 and 103 for Q-switching and generation of a giant pulse. In Ref. 34 intracavity nonlinear lenses were used for lengthening spikes and stabilizing the lasing power of pulsed lasers. In the last few years there has been a great deal of interest in using thin nonlinear lenses for modelocking. Thin lenses with electronic nonlinearity are virtually inertialess, and this makes it possible, in principle, to generate with their help pulses with duration right down to several femptoseconds. The modelocking mechanism of a nonlinear lens (Kerr Lens Modelocking[110]) is based on the increase of the transmission of an intracavity spatial filter—diaphragm with increasing radiation power. This is observed with a special arrangement of the pair "nonlinear element—diaphragm" inside the laser cavity. This method has now been realized in tunable femptosecond lasers based on sapphire with titanium, where the active element itself plays the role of a nonlinear lens.[111] Interest in laser cavities with thin nonlinear elements has stimulated the investigation of properties of light beams in periodic systems with thin layers of a self-focusing dielectric. It has been shown that some systems can be used for selection of light beams according to power.[63] The possibility of achieving

effective suppression of small-scale self-focusing in systems of this type by selecting a definite system configuration[104] has generated interest in using such systems in complex multicascade systems of high-power laser amplifiers.[105] Nonlinear lenses in feedthrough optics can significantly influence the focusing properties of the optical systems of powerful laser setups.[89,106] This is taken into account in the design of optical systems for focusing radiation in laser thermonuclear fusion setups.[107] Aberrations of nonlinear lenses can be used to obtain beams with a quasisquare intensity profile (see Fig. 7f).[108] Thus nonlinear lenses make it possible to solve different problems of passive (adaptive) control of laser beams. Some of the most important applications of nonlinear lenses are, however, apparently associated with the limitation of intensity, power, and energy of laser radiation.

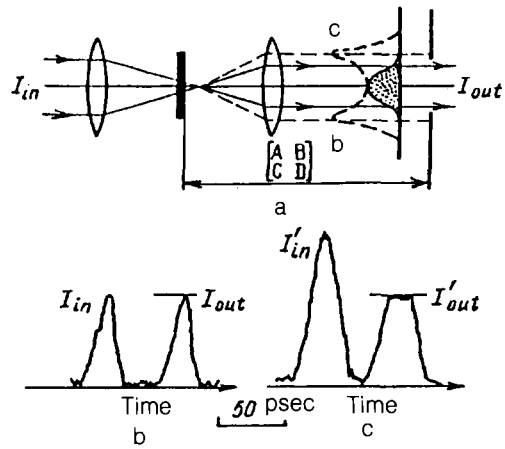

FIG. 17. Optical self-limiter of the intensity of ultrashort laser pulses (a). Bottom: experimental densitometer traces of YAG:Nd laser pulses from the screen of a streak camera for a self-limiter based on a cell with liquid carbon disulfide (b,c).

3.5. Limitation and stabilization of the intensity, power, and energy of laser radiation with the help of nonlinear lenses

One problem arising in applications of high-power laser radiation is reliable limitation of light flux below a prescribed maximum admissable level. This is important for protecting complicated and expensive elements of optical schemes from being damaged by high-power laser radiation, for protection of highly sensitive photodetectors from intense illumination, and in laser ophthalmology and other medical applications. A different aspect of this problem is connected with the possibility of stabilizing the energy parameters of radiation by limiting them. This is especially important for pulsed generators of ultrashort light pulses, whose parameters fluctuate significantly. Since the concern here is to limit the energy parameters of radiation by changing the intensity, power, or energy of the radiation itself, in what follows we shall call this effect self-limiting of the corresponding parameters of the radiation and we shall call devices based on this effect optical self-limiters.[46] Optical self-limiters are being intensely studied because they are simple, fast, reliable, and they do not consume energy.[19,20,25,26,37,46–59] The principle of operation of optical self-limiting is based on defocusing of radiation by a nonlinear lens at high power and cutoff of "excess" power with the help of a diaphragm (Fig. 17). Under certain conditions the intensity, power, or energy of the radiation transmitted through the diaphragm no longer depends on the input power. Then one talks about stabilization of the corresponding parameter. Self-limitation and stabilization of the power of cw-laser radiation with thermal defocusing in a weakly absorbing liquid was discussed in Refs. 37 and 59. Self-limitation of light was investigated under conditions of self-defocusing in semiconductors by the thermal mechanism[19,20] and by excitation of nonequilibrium carriers by short laser pulses.[24,25] In these works the inertia of the nonlinear response of semiconductors to subnanosecond pulses resulted in cutoff of the trailing edge of the transmitted pulses due to accumulation of nonlinear bending of the wavefront over the time of the pulse (see, for example, Fig. 11). As a result of this cutoff, the energy (or integral parameters of the type $\int (I(t))^{\alpha/2} dt$ with $\alpha \neq 2$) is limited. Limitation and stabilization of the transmitted energy have also been observed for other materials with inertial cubic nonlinearity.[36] Inertialess limitation of the peak power of laser pulses was achieved in Ref. 47 with deep focusing of radiation into a material with picosecond nonlinearity relaxation time. In this case the most intense part of the pulse is degraded in the nonlinear medium due to self-focusing and nonlinear light losses of different types accompanying the self-focusing.

A large number of optical self-limiters for different wavelength ranges and laser-pulse durations have now been developed (see the reviews Refs. 57 and 58). The achieved thresholds of limitation (10 nJ) with a dynamical range exceeding 10^4 (Ref. 58) make it possible to use such devices for reliable protection of elements with high photosensitivity. For purposes of stabilizing the energy parameters of radiation, however, such self-limiters are virtually never used because, as a rule, they have high losses and the optical quality of the transmitted beam is low. The problem of stabilization of the power and energy of laser pulses is especially important for subnanosecond laser pulses, for which intensity, power, and energy fluctuations are significant, and there are no active control devices, operating with the required speed, or such devices are difficult to obtain. A high-precision optical self-limiter of the intensity of picosecond pulses was proposed and realized in Ref. 72. In this self-limiter many of the drawbacks of optical self-limiters are eliminated. The main distinction of this self-limiter lies in the configuration of the elements of the optical scheme such that $\beta\Phi_{NL} < 0$ and $|\beta| > 3$ and nonlinear transformation of the beam occurs in the manner displayed in Fig. 7f. The instability of the peak intensity of picosecond laser pulses can be reduced with the help of such a device by more than an order of magnitude with a quasispherical wavefront of the transmitted radiation and weak-signal transmission coefficient of about 50% (see Fig. 17).

CONCLUSIONS

The works examined in this review illustrate the basic properties of nonlinear lenses and their applications in modern optics, laser physics, and technology. Nonlinear lenses strongly influence the characteristics of high-power lasers and multicascade laser systems. In contrast to self-focusing of a beam in the volume of an optical material, the contribution of thin nonlinear lenses to transformation of light beams can be calculated beforehand and optimized. However, a systematic theory of optical systems with thin nonlinear lenses has been constructed only for beams in the form of a fundamental Gaussian mode. This theory makes it possible to predict the basic properties of nonlinear optical filters, measuring schemes, and lasers with thin nonlinear elements. Further development of the theory of optical systems with nonlinear lenses should proceed, from our point of view, along three directions: 1) study of the evolution of light beams of arbitrary form in optical systems with a thin nonlinear lens; 2) study of laser cavities with a thin nonlinear lens; and, 3) study of nonlinear lenses in the field of femptosecond pulses, for which, together with nonlinear transformation of the phase of the field, it is also necessary to take into account the linear and nonlinear dispersions of the material. The latter problem is important for the physics of high-power femptosecond lasers and technology of formation of ultrastrong optical fields. As the theory and experimental studies of nonlinear lenses are further elaborated, their ranges of application will expand primarily for problems of control of light by light and optical measurements.

In conclusion we wish to point out that this paper could not have been written and published without the participation and support of Sergei Aleksandrovich Akhmanov. It was his idea to write this review, and his remarks and suggestions significantly influenced the content.

[1)]In the literature the third-order nonlinear refractive index is usually taken to be the coefficient of proportionality n_2' between the nonlinear correction to the index of refraction and the average squared field: $\Delta n = n_2'\langle E^2\rangle$.[60] The coefficient n_2 is related to n_2' by the relation $n_2' = n_2 c n_0/4\pi$, where c is the speed of light in vacuum and n_0 is the linear index of refraction of the medium.

[2)]In this case it would be more correct to talk about the interference of light beams from two annular zones of the beam with radii ρ_1' and ρ_2'.

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