Radiative processes involving fractal structures*

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Radiative processes of the flame of a candle and of fractal discharges associated with the presence in them of fractal aggregates are examined. A brief description is given of the modern fractal concept of ball lightning. An analysis is given of a specific physical object—a fractal tangle.

1. INTRODUCTION

The development of fractal concepts in physics which began with the remarkable books by Mandelbrot[1,2] have filled many fields of physics with new content. As a result of research in this direction one can now understand that the condensed state of matter can represent not only a continuous medium, but can also have a porous fractal structure, particularly, if the process of formation of the condensed system proceeds under nonequilibrium conditions. Therefore fractal concepts are regarded as physics concepts of the XX century (cf., for example, Ref. 3).

Below we shall examine a number of problems related to physical objects with a fractal structure. The fractal properties of such systems were the object of detailed research (cf., for example, the books and the reviews of Refs. 4–16). One of the properties of fractal systems is associated with their interaction with electromagnetic waves. As a rule fractal systems have higher specific radiative parameters, than solid systems. Below, two examples are given of this kind—radiation from a candle and fractal discharges, and it is shown that the formation of fractal aggregates in such systems sharply increases the power of the radiation produced in them.

We note that the level of experimental investigation of the systems being considered does not correspond to the level of understanding of the processes taking place within them. Indeed, an analysis to detect the presence within them of fractal aggregates and the measurement of their parameters is carried out only in special cases. Therefore one of the principal aims of the present article is associated with drawing attention to this side of the problem.

Along with the radiation from systems containing the simplest fractal structures—fractal aggregates, a brief analysis is given also of a more complicated fractal structure—a fractal tangle, which has a number of specific properties including radiative ones. A fractal tangle as a physical object has been introduced as a model of the core of ball lightning. However, in exhibiting specific physical properties, in particular a combination of the properties of a gas, a liquid and a solid this object has an independent significance not related to the problem of ball lightning.

2. FRACTAL AGGREGATES IN HIGH-TEMPERATURE PROCESSES

2.1. Formation of fractal aggregates in the evaporation of materials

The general scheme of formation of fractal aggregates in the gas phase appears as follows. First the vapor of the material is introduced into the gas under the action of high fluxes of energy on the surface. Then, in the process of cooling and relaxation of the vapor, atoms and molecules condense forming clusters in the volume which grow into macroparticles. If the density of the material within the volume is sufficiently great, the unification of the particles continues even after they have become solid and lead to the formation of fractal aggregates.

The properties of fractal aggregates and the nature of processes occurring in the course of their formation have been brought together in the monographs and reviews of Refs. 4–16. Below, we shall present briefly information on these questions which we shall then later use. The correlation function for a fractal aggregate—a bound system of solid particles—has the form

$$\Phi(r)=\langle\rho(\mathbf{r}')\rho(\mathbf{r}'-\mathbf{r})\rangle/\langle\rho(\mathbf{r}')\rangle=Ar^{-\alpha}; \qquad (1)$$

here $\rho(\mathbf{r})$ is the density of particles at a given point, the averaging $\langle\ldots\rangle$ is carried out over the positions of the particles in the aggregate. Formula (1) means that if one draws a sphere of radius r, the center of which is one of the particles, and counts the number of particles on the sphere, and repeats this operation many times, changing the particles at the centre of the sphere, then the average density of the particles on the sphere varies according to $r^{-\alpha}$ as the radius r of the sphere varies. From this it follows that the average density of the particles within the sphere in three-dimensional space also varies according to the law

$$\rho(r)\sim r^{-\alpha}\sim r^{D-3},$$

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TABLE I. Methods of obtaining fractal aggregates.

Method used	Material	Fractal dimensionality	References
Electrical explosion of a wire	Fe, Zn, SiO_2	1,60 ± 0,07	[17]
Heating of a tungsten spiral in vacuum	Co, Ni, Al	1,80 ± 0,05	[18—23]
Laser irradiation of metals	Al, Ti, Fe, Ag, Pt	1,82 ± 0,05	[24, 25]
Explosion of material	C	1,9	[26]
Combustion of SiH_4	SiO_2	1,8 – 2,0	[27—29]

where $\alpha = 3 - D$, D is the fractal dimensionality of the aggregate. From the formulas given above it follows that a fractal aggregate of radius R contains the following number of individual particles

$$n = (R/a)^D, \tag{2}$$

where a is the average radius of a particle.

There are available different methods of obtaining fractal aggregates (cf. Table I). Let us examine briefly some of them. In the first experiment of Forrest and Witten[17] the fractal aggregates are formed as a result of passing a strong electric current along a wire containing definite components. The evaporated material was cooled in the process of expansion into space, and this led to condensation and, subsequently, to aggregation into clusters of the solid particles being formed. The average radius of the particles in the structure amounted to 3–4 nm and the characteristic size of the aggregate was several microns. This work was the first experimental investigation of fractal aggregates and laid methodological foundations for the analysis of such objects.

This method of obtaining aggregates was modified in the cycle of papers of Refs. 18–23 where the fractal aggregates in the final analysis were precipitated on the surface and their high efficiency for the absorption of thermal radiation was utilized. We now analyze one of the papers of this cycle (Ref. 23) where the formation of fractal aggregates of cobalt was investigated. The small particles of cobalt were obtained in an atmosphere of argon on evaporating the metal by the traditional method[18] using a heated tungsten spiral under the conditions of convectional transport of the vapor. The pressure of argon was in the range of 0.25–10 torr. The metal particles similar to soot were collected on a copper grid covered by carbon and were investigated with the aid of an electron microscope. The average thickness of the metallic deposit amounted to 10–200 μm, and the volume of the particles of cobalt in this layer was estimated to be 10^{-4}–10^{-2}, i.e., the deposit had a porous structure and most of its volume was occupied by pores. The average radius of the particles in these formations increased with an increase in the pressure of argon, the fractal dimensionality of the formed aggregates amounted to 1.75–1.9 at an argon pressure of 0.9–8 torr, and the average radius of the particles was less than 8 nm. At argon pressures greater than 8 torr the radius of the particles in the aggregate exceeded 8 nm and the fractal dimensionality of the aggregate amounted to 1.9–2.05. We note that the fractal dimensionality of the aggregates formed on evaporation of metal into an inert gas is somewhat higher than in the case of its evaporation in a vacuum.

Another method of obtaining fractal aggregates utilizes the irradiation of different metals by laser radiation.[24,25] Then near the surface there is formed a weakly ionized vapor of the metal which is under high pressure (the temperature amounts to several thousand degrees, and the pressure is several tens of atmospheres). This vapor, expanding into space, is cooled and condenses on the ions. When the temperature of the particles being formed becomes lower than the melting temperature of the material the metal particles unite into fractal aggregates. Experiments have been conducted for different metals Al, Ti, Fe, Ag, Pt, in a number of buffer gases (air, argon, helium) and at different pressures of the order of 1 atm. The qualitative nature of the results in these cases remains the same.

The experience of obtaining fractal aggregates shows that the process of their formation requires relatively high energy fluxes which evaporate the material bringing it into the form of a gas or a high density vapor. The subsequent relaxation of the vapor associated with its expansion and cooling leads to the formation of fractal aggregates. Different types of energy can be utilized to achieve this aim. In particular, in Ref. 26, fractal aggregates of carbon were formed as a result of exploding carbon containing materials. The average radius of particles in the structure was equal to 3 nm, the average radius of the fractal aggregates was 20 nm, and their fractal dimensionality amounted to approximately 1.9.

Fractal aggregates can be obtained by burning a mixture in a gaseous medium, if in the flame or near it conditions for condensation and formation of solid particles are provided. This method is used in burning $SiCl_4$ in a hydrogen-oxygen flame.[27–29] The powder formed as a result of this contains fractal aggregates consisting of $\sim 10^3$ particles of 8–10 nm radius. The fractal dimensionalities of the aggregates lie in the interval 1.8–2.0. It is of interest that the particles themselves entering into the composition of the fractal aggregates have an internal structure. This is established by measuring the specific surface of the aggregates,[29] which exceeds by a factor of 1.8–3.0 the specific surface of particles if they are regarded as being solid.

The density of a fractal aggregate decreases as its dimensions increase, and simultaneously its strength decreases.[30] As a rule, the maximum size of fractal aggregates amounts to microns, and the maximum number of particles in it is $\sim 10^4$.

2.2. Emission from small particles and fractal aggregates

Let us examine the radiation from a hot gas containing small particles. For simplicity we shall regard these particles as being spherical. Moreover, for the spectral region being utilized—optical and infrared, and for the dimen-

sions of the particles under discussion the condition of smallness of the dimensions of the particles compared to the wavelength of radiation λ definitely holds

$$\lambda \gg r_0, \tag{3}$$

where r_0 is the characteristic size of the particles.

The cross section for the absorption of a photon by a small spherical particle of radius r is equal to[31]

$$\sigma_{abs}(\omega)=\frac{12\pi\omega r^3\varepsilon''}{c[(\varepsilon'+2)^2+\varepsilon''^2]}, \tag{4}$$

where $\omega=2\pi c/\lambda$ is the frequency of the electromagnetic wave, c is the velocity of light, ε', ε'' are the real and the imaginary parts of the permittivity of the material of the particle. We represent the cross section for the absorption of electromagnetic waves in the form[31]

$$\sigma_{abs}=\pi r^2\frac{r}{\lambda}f(\omega), \tag{5}$$

where $f(\omega)=12\varepsilon''[(\varepsilon'+2)^2+\varepsilon''^2]^{-1}$. From this we go on to the spectral radiated power p_ω, which in accordance with Kirchhoff's law which establishes the connection between the rates of emission and absorption processes, is equal to[31]

$$p_\omega=c\sigma_{abs}(\omega)U(\omega)/4, \tag{6}$$

where $U(\omega)$ is the spectral density of blackbody radiation, i.e., the energy per unit volume and per unit frequency interval:

$$U(\omega)=\hbar\omega^3/\pi^2c^3[\exp(\hbar\omega/T)-1]. \tag{7}$$

Assuming the frequency dependence of the permittivity to be weak we determine from this the total radiated power from a small particle:[32,34]

$$P_{rad}=\int I(\omega)d\omega=4\pi r^3 f(\omega_0)k\sigma T^5/\hbar c; \tag{8}$$

here σ is the Stefan–Boltzmann constant, ω_0 is the effective frequency of the radiation and the parameter k is equal to

$$k=\int_0^\infty x^4(e^x-1)^{-1}dx\left[\int_0^\infty x^3(e^x-1)^{-1}dx\right]^{-1}=3.83.$$

The chosen method of representing the radiated power of the particle enables one to compare it with the radiated power of an absolutely black particle $P_0=4\pi r^2\sigma T^4$. It is seen that

$$P_{rad}/P_0=kfrT/\hbar c=3.02fr/\lambda_m, \tag{9}$$

i.e., this ratio is proportional to the small parameter (3)r/λ_m and the wavelength $\lambda_m=1.27\hbar c/T=0.29$ cm · K/T in accordance with Wien's law corresponds to the maximum spectral radiated power for an absolutely black body. We note that the energy of a photon corresponding to the maximum of the spectral radiated power of a small particle somewhat (by a factor of 1.2) exceeds the maximum energy of a photon for blackbody radiation at a given temperature.

Let us analyze the result that we have obtained from the point of view of radiation from a hot gas containing small particles. It is essential that the radiated power is proportional to the volume of the particle. This means that the radiated power per unit volume is proportional to the mass of the particles per unit volume and does not depend on the size distribution of the particles. Therefore in order to determine the radiated power per unit volume it is sufficient to use the mass of the material contained within the volume in the form of large clusters.

Further, in order to estimate the radiated power of the plasma we shall use the spectral parameters of soot.[33] In this case[32,34] $f=0.9\pm0.1$ (we note that if we formally assume that ε', $\varepsilon''\geqslant 0$, then the maximum value of the parameter $f(\omega)$ amounts to $f=3$ and corresponds to $\varepsilon'=0$, $\varepsilon''=2$. In this case the indicated error characterizes the change of the given parameter in the optical frequency region. Correspondingly, the radiated power per unit volume of the radiating particles is given by formula[32,34]

$$J_{rad}V=\gamma T^5, \tag{10}$$

where V is the volume of the radiating material which is in the state of particles-clusters, and the parameter γ for soot is equal to

$$\gamma=2.5\pm0.2\ \text{W/cm}^3\text{K}^3. \tag{11}$$

We use the obtained relation for making estimates. Let us find out what contribution to the energy balance of clusters is introduced by radiation. In order to do this we write the equation for the energy balance per unit volume of plasma associated with emission of radiation in the form

$$C_p\frac{dT}{dt}=-J_{rad}=-C_pT\tau_{rad}^{-1}. \tag{12}$$

The quantity τ_{rad} determined in this way in accordance with formulas (10) and (11) is equal to $\tau_{rad}=C_p/4\gamma T^4$. Comparing this with the observed times of heat transfer one can determine the contribution of the radiation to the heat balance of a hot gas.

We now go on to the radiation from fractal aggregates. Naturally, this process is more complicated than the radiation from individual particles, and in the case to be discussed below, when the dimension of the fractal aggregate is smaller than the wavelength of the electromagnetic radiation, depends on factors including the kind of material, and also the nature of the interaction between the particles and between the atoms within the particles. The choice of these factors will determine the theoretical models[36–43] describing this process and leads to a number of interesting phenomena in the interaction of radiation with fractal aggregates.[41–43] Further, we shall use a simple and rough model for the absorption of radiation by fractal aggregates. Therefore the results that are obtained for these processes will be of a qualitative nature. The essence of the model being used amounts to the fact that the boundary conditions for the electromagnetic wave scattered by the fractal aggregate are the same ones as for a small spherical particle. Then the cross section for the absorption of an electromagnetic wave by a fractal aggregate whose dimension r_0 is small compared to the wavelength of the radiation $R_0\ll\lambda$, is determined by formula (4)

$$\sigma_{abs}=\frac{\pi c}{\omega}R_0^3 f(\omega). \tag{13}$$

Correspondingly the total radiated power of an individual fractal cluster is determined by formula (8)

$$P_{rad}=4\pi R_0^3 f(\omega_0)\sigma T^5/\hbar c, \tag{14}$$

where σ is the Stefan–Boltzmann constant. As can be seen from a comparison of formulas (8) and (14) in order to study the fractal aggregate and the radiation from particles of radius r of which the fractal aggregate is composed the radiated power of the fractal aggregate per unit mass is by a factor of $(R_0/r)^{3-D}$ greater than the radiated power of an individual particle. Thus, the assembly of solid particles—aerosols into a fractal aggregate leads to a significant increase in the yield of radiation.

2.3. Radiation from the flame of a candle

It is known that the radiation from a flame is determined by the solid particles present in it (cf., for example, Faraday's lectures[44]). Below, we shall analyze the radiative parameters of the flame of a candle basing this on measurements carried out in the Petrozavodsk State University.[45,46] In doing so we shall assume that the radiating particles are carbon soot particles, formed in the incomplete burning of hydrocarbons at a certain stage of the burning process. Then by comparing the results of calculation and experiment one can determine the efficiency and the nature of the formation of soot in the flame of a candle.

The simplifying factor in the analysis of radiation from the flame is associated with the fact that the spectral radiated power P is proportional to the mass of the radiating particles and does not depend on their size distribution if the radiation is produced by particles and not by structures. Therefore, having adopted the point of view that the radiation is produced by solid particles present in the flame, one can define the radiation parameters of the flame and compare them with experimental data.

The spectral radiated power of a spherical solid particle of radius r per unit volume is equal in accordance with formula (6) to

$$P_\omega=j_\omega\sigma_{abs}(\omega), \tag{15}$$

where

$$j_\omega=\hbar\omega^3[\exp(\hbar\omega/T)-1]^{-1}/4\pi^2c^2$$

is the spectral flux of radiation for an absolutely black body, $\sigma_{abs}(\omega)$ is the cross section for the absorption of an electromagnetic wave by a particle (4).

We shall represent the temperature distribution of the flame in the hot zone in the form

$$T=T_0-\alpha r^2, \tag{16}$$

where T_0 is the temperature of the hottest point, r is the distance to it. In calculating the spectral radiated power in the optical region of the spectrum we use the existence of the small parameter $T/\hbar\omega$, where $\hbar\omega$ is the photon energy. Realistically $T/\hbar\omega\sim 0.1$. In such a case the strongest dependence of the radiated power on the temperature is determined by the factor $\exp(-\hbar\omega/T)$. Taking this into account we obtain for the spectral radiated power

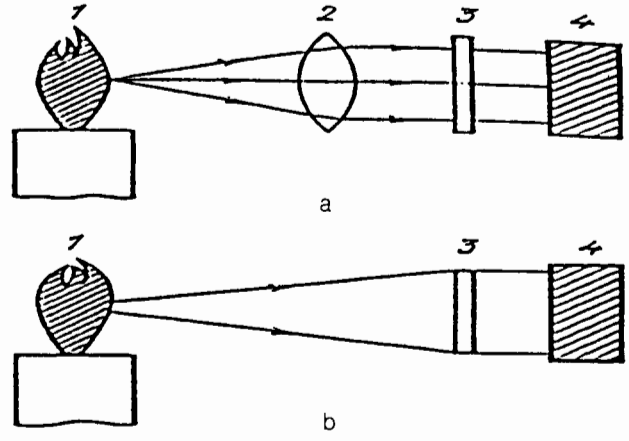

FIG. 1. A schematic representation of the measurement of spectral parameters of the flame of a candle a—Measurement of the spectral density of the radiation flux. b—Measurement of the spectral radiated power of the flame. *1*—flame of the candle, *2*—lens the focus of which is at the point being investigated in the flame, *3*—selector of radiation according to wavelength, *4*—collector of radiation.

$$P_\omega=\int Np_\omega d\mathbf{r}=Np_\omega(T_0)\int\exp\left(-\frac{\hbar\omega}{T_0^2}\alpha r^2\right)d\mathbf{r}$$

$$=Np_\omega(T_0)\left(\frac{\pi T_0^2}{\hbar\omega\alpha}\right)^{3/2}, \tag{17}$$

where N is the density of the soot particles. The spectral flux of radiation amounts to

$$I_\omega=\int\frac{Np_\omega d\mathbf{r}}{4\pi r^2}=\frac{Np_\omega(T_0)}{4}\left(\frac{\pi T_0^2}{\hbar\omega\alpha}\right)^{1/2}. \tag{18}$$

The effective area of the flame S_ω is equal to

$$S_\omega=\frac{P_\omega}{I_\omega}=4\pi\frac{T_0^2}{\hbar\omega\alpha}. \tag{19}$$

Thus, in accordance with formulas (3), (15), and (17) we have for the spectral radiated power of the flame

$$P_\omega=Np_\omega\left(\frac{S_\omega}{4}\right)^{3/2}=Nr^3\frac{\pi f_\omega}{\lambda}j_\omega\left(\frac{S_\omega}{4}\right)^{3/2}. \tag{20}$$

The principal parameters determined from experiment are the spectral radiation flux I_ω and the spectral radiated power P_ω. In order to measure the spectral flux of radiation the point in the flame being investigated is placed at the focus of the optical system of Fig. 1, while for the measurement of the spectral radiated power the detector is placed at a great distance from the flame and into is collected the radiation from a known solid angle. The assumption concerning the isotropy of the radiation (which is not difficult to check by moving the detector) enables one to find from this the spectral radiated power. The spectral technique created at the Petrozavodsk State University enabled the absolute values of these parameters to be determined. Further, on the basis of the frequency dependence

of these parameters (for example, $I_\omega \sim \omega^{7/2} \exp(-\hbar\omega/T)$) one can determine the temperature of the hottest point of the flame.

An additional method for determining the temperature of the flame was used along with the ones previously indicated. Sodium chloride was added to it and as a result of its decomposition sodium atoms entered the flame producing blackbody radiation near the centers of the lines of resonance transitions (wavelength $\lambda = 589.592$ and 588.995 nm). Since the maximum fluxes of radiation corresponded to an absolutely black body one could determine from this the temperature of the hot region. The set of measurements indicated above gave for the maximum temperature of a flame the value[45,46]

$$T_0 = 1800 \pm 50 \text{ K}. \tag{21}$$

Moreover, from these measurements the following values of several parameters were obtained: the effective area of the surface of the flame in the green portion of the optical spectrum $S_\omega = 2.6 \pm 0.5$ cm^2, the parameter $\alpha = 600 \pm 150$ K/cm^2, the parameter in formula (9)

$$Nr_0^3 = 6 \cdot 10^{-5 \pm 0.3}. \tag{22}$$

To the optical parameters of the flame one should add the chemical ones which are associated with the nature of the combustion process occurring according to the scheme

$$C_{18}H_{36}O_2 + 26O_2 \rightarrow 18CO_2 + 18H_2O + 39.7 \text{ kJ/g}. \tag{23}$$

In the optimum regime of combustion which corresponds to the optimum ratio between the fuel and the oxidant: 14 grams of air correspond to 1 gram of stearin. The maximum temperature of the products of the process corresponds to the optimum composition of the burning mixture and the assumption that the entire chemical energy of the fuel is spent on heating the products. It amounts to 2500 K. It follows from this and from formula (21) that not more than 30% of the chemical energy of the candle can be spent on radiation.

We shall check the validity of the assumption made concerning the transparency of the flame. We have that the ratio of the measured spectral flux of the radiation J_ω to the spectral flux of an absolutely black body at the maximum flame temperature j_ω amounts to

$$\tau = J_\omega / j_\omega = (\lambda_0/\lambda)^{1/2}, \tag{24}$$

where in accordance with the measured values of the parameters $\lambda_0 = 20 \text{ nm} \cdot 10^{\pm 0.6}$. In particular, for the green region of the spectrum this corresponds to an optical thickness of the flame of $\tau = 0.2 \cdot 10^{\pm 0.3}$, and this confirms the assumption used concerning the transparency of the flame. The maximum temperature of the flame (21) and the measured values of the parameters lead under optimal conditions of combustion to the following value of the density of carbon in soot, if it consists of particles:

$$\rho_c = (1.4 \pm 0.1) \cdot 10^{-5} \text{ g/cm}^3. \tag{25}$$

Considering that the density of carbon in soot is the same as in graphite (2.1 g/cm^3) we obtain from this

$$Nr^3 = (1.6 \pm 0.1) \cdot 10^{-6} \xi. \tag{26}$$

Comparing this with the value of (22) which follows from the values of the measured parameters we obtain the fraction of carbon contained in soot:

$$\xi = 4 \cdot 10^{\pm 0.4}. \tag{27}$$

As may be seen we have arrived at a contradiction and the only method of overcoming it corresponds to the assumption that the soot is not in the state of individual particles, but in a state of fractal aggregates. Such a situation was observed in a flame[47] and below we shall make estimates using the parameters of fractal aggregates obtained in Ref. 26 as a result of exploding carbon-containing compounds. These parameters are as follows: $r_0 = 3$ nm, $R = 20$ nm, $D = 1.9$. As follows from formula (14) the formation of fractal aggregates leads to an increase in the radiated power per unit mass by a factor of $(R/r_0)^{3-D}$. In the present case this factor is equal to 9, so that the fraction of soot in the carbon of the flame taking into account the process of formation of fractal aggregates and under optimum conditions of combustion amounts to

$$\xi = 40\% \cdot 10^{\pm 0.4}, \tag{28}$$

which no longer contains a contradiction. Thus, the fractal concept of a flame leads to a correspondence between the measured values of the parameters and the calculated ones and requires a more detailed analysis of the composition of the flame.

3. FRACTAL DISCHARGES

3.1. Special features of fractal discharges

A gas discharge in which fractal clusters (or fractal aggregates) are formed shall be called fractal discharge for brevity. Such discharges are characterized by a relatively high yield of radiation. We note a special feature of fractal discharges. On the one hand they require high currents to the electrode, and this leads to an effective destruction of the electrode and ejection of its material in the form of an expanding weakly ionized vapor into the region of the discharge. Further particles-clusters are formed from this vapor, and their uniting produces fractal aggregates or fractal clusters. On the other hand, the temperature in the region of the discharge must not exceed the melting temperature of the particles, otherwise fractal aggregates will not be able to exist. These, in some sense contradictory conditions, determine the parameters for the existence of fractal discharges.

Evidently the indicated conditions can be realized in a pulsed arc discharge in which the instant of evaporation of the material and the instant of formation of fractal aggregates are distinct. The duration of the existence of a discharge must exceed the time for the formation of fractal aggregates, i.e., the duration of fractal discharges must exceed some milliseconds.

The main feature of fractal discharges is associated with a high yield of radiation from the discharge. In contrast to ordinary discharges in which the radiation is determined by the excitations of atoms and molecules in a

plasma created in collisions with electrons, or is determined by other processes of collisions with participation of electrons, the afterglow of fractal discharges is created by a medium in which there are no electrons. Therefore the time of radiation by such discharges significantly exceeds the recombination time of electrons and ions, i.e., is much greater than the time of afterglow of ordinary discharges.

As may be seen the set of processes needed for realization of a fractal discharge includes evaporation of the material of the electrodes and their penetration into the hot zone of the discharge. Then after the electrical current of the discharge has been switched off fractal aggregates are formed from this vapor which are responsible for the high yield of the radiation from the hot zone of the discharge while it has not yet cooled off. These general requirements can be satisfied in the case of different methods of exciting a discharge. This refers to an ultrahigh-frequency discharge,[48,49] to an electrical breakdown of air and a gas,[50–53] to a powerful arc discharge,[54–62] to a radiofrequency discharge[63,64] to an underwater electrical breakdown,[65–67] a corona discharge, etc. In all these cases of excitation of a gas discharge the conditions for the formation of fractal aggregates are fulfilled in the afterglow of the discharges, and this guarantees a high yield of radiation from the hot region of the discharge.

We note the special feature of experiments associated with fractal discharges. Not in a single one of them had there been carried out an analysis concerning the presence in the discharge of fractal aggregates. Moreover, in the majority of them the experimental analysis of the discharge is onesided, which limits the possibilities of analyzing them. Usually in virtue of the high yield of radiation and the absence of an external source of energy fractal discharges are utilized as an analogy of ball lightning. However, the lifetime of ball lightning significantly exceeds the time of emission by fractal discharges. Thereby the indicated analogy is restricted, and fractal discharges should be examined without reference to ball lightning.

3.2. The Powell–Finkelstein discharge

Among the investigations of fractal discharges a particular place is occupied by the experiment of Powell and Finkelstein[48] in view of the many sided and detailed analysis carried out in this experiment. Although this experiment was performed long before the appearance of the concept of a fractal aggregate, in view of the large amount of information obtained in it, the Powell and Finkelstein experiment will help us to check the validity of this concept by indirect methods. Below we shall examine the experiment of Powell and Finkelstein.[48]

In this experiment in order to excite a gas discharge they used a generator of electromagnetic field at a frequency of 75 MHz. The discharge was excited in a pyrex tube of 15 cm diameter at an air pressure from 0.5 to 3 atm. At lower pressures the duration of light emission decreased sharply with decreasing pressure. If the discharge tube was connected to a large volume which enabled one to maintain constant pressure as the discharge was switched on and off, light emission continued up to 1 sec after the discharge

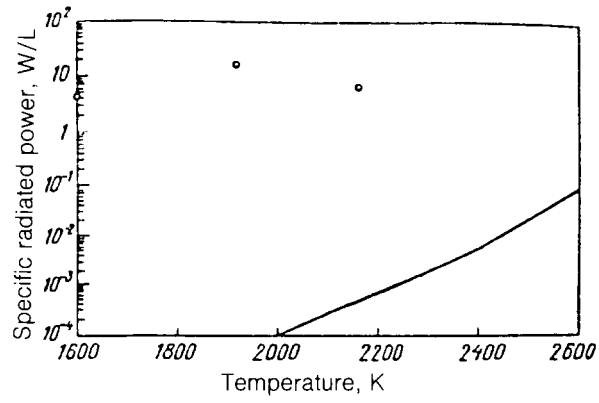

FIG. 2. The temperature dependence of the specific radiated power of atmospheric air (solid curve) and of air undergoing cooling in the afterglow of the Powell–Finkelstein[48] discharge (circles).

stopped. If the discharge was ignited in open air it lasted only half as long. Stable light emission was observed both in air and in nitrogen, oxygen, and nitrous oxide.

An important result is associated with the role played by the electrodes which were made of different metals, with stable light emission corresponding to the high melting point metals Pt, Ag, Au, Cu, Zn, Cd, C, Sn, W, Al. It was established that in using as electrodes such easily vaporized materials as Hg and Pb the duration of light emission does not exceed 50 msec. Since the effect of the electrodes persists after the discharge is switched off it follows that it is specifically the material of the electrode that creates this light emission.

The temperature in the region of light emission amounts to 2000–2500 K as measured by the electrical resistance of a thin tungsten wire placed in the region of light emission. It should be noted that at these temperatures the fractal aggregates can exist only in the case of platinum and tungsten. The melting temperature of other materials is considerably below these temperatures. However one should take into account that the melting temperature of oxides of the majority of materials that have been used is higher than that indicated above, i.e., the oxides of these materials can form fractal aggregates.

Yet another special feature of the light emission under discussion is associated with the kind of gas in which the process is taking place. For the CO_2 or Ar buffer gases the light emission is similar to the light emission in an arc and lasts for a time of the order of a second after the discharge is switched off. Light emission in nitrogen is weak and blue. As oxygen is being added the intensity of light emission increases and in air it is of moderate intensity and is of a yellow-white color. In pure oxygen it becomes bright and white. The most intense light emission occurred in using N_2O as the buffer gas. Then the light was of weakly orange color, had a maximum intensity and lasted up to 2 sec.

In Fig. 2 the intensity of radiation emerging from 1 L of air in the wave length range of 0.4–1.1 μm is comparable

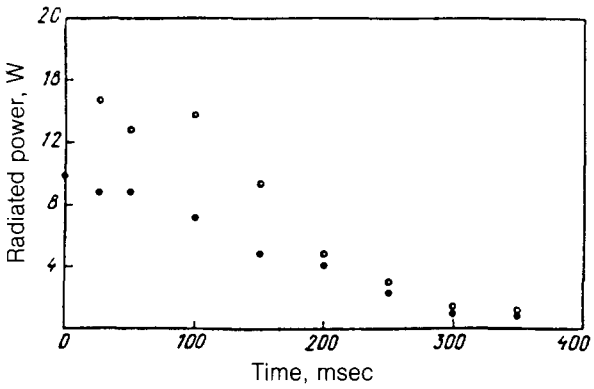

FIG. 3. Temperature dependence of the radiated power from a Powell–Finkelstein[48] discharge in the afterglow for the visible (0.4–0.7 μm) (bright circles) and for the infrared (0.7–1.1 μm) (dark circles) regions of the spectrum of the radiation. The measurements refer to platinum electrodes and air under atmospheric pressure as the discharge gas.

with the equilibrium radiation in air at the temperature in question. As can be seen the equilibrium radiation of oxygen does not contribute to the observed light. In Fig. 3 the dependence on time is shown for the visible (wavelength of 0.4–0.7 μm) and the infrared (wavelength of 0.7–1.1 μm) radiation. We note that according to Wien's law the maximum of blackbody radiation at a temperature of 2000 K corresponds to a wavelength of 1.5 μm. The spectral properties of the system under discussion imply a shift of this maximum towards the blue side.

We now analyze the results obtained by Powell and Finkelstein. We shall assume that the radiation is produced by particles or by structures which are formed in the hot zone from atoms of the material evaporated from the electrodes. We cannot choose between these two possibilities on the basis of the information being utilized. We shall make estimates for both cases assuming that the radiation is produced by individual particles-clusters or by fractal aggregates composed of them utilizing for the particles the radiation parameters of soot $f(\omega)=0.9$.

We shall first estimate the contribution of the radiation to the energy balance of the system under consideration. We assume that cooling of the air is associated with the emergence from the volume of the radiation produced by small particles-clusters. We make an estimate for the time of cooling of the volume. From the heat balance equation (12) we have $\tau_{\text{rad}}=C_pT/4J_{\text{rad}}$. Assuming in accordance with experiment $J_{\text{rad}}\sim 10$ W/L, $T=2000$ K we obtain $\tau\sim 10$ sec. According to experiment the curves for the time dependence of the radiated power are approximated by exponential curves with a time constant of 0.2–0.3 sec. In that case only a small part of the heat energy of the air is expended on radiation. The estimate that we have carried out shows that in principle it is possible to produce a construction of a discharge with low heat losses and with time constants up to 10 sec.

Let us estimate the amount of evaporated material which provides for the observed radiated power $J_{\text{rad}}\sim 10$ W/L at a temperature of $T=2000$ K. If the emitting substance (platinum) is in the form of particles-clusters we shall obtain that the concentration of platinum in the form of individual particles which provides for the observed amount of emission must constitute $\sim 10^{-4}$ g/g (gram of platinum per gram of air), i.e., in 1 liter of air there must be contained approximately 30 μg of platinum. We note that the radiating volume is optically transparent. Indeed, the observed radiated power of 10 W per liter of air is emitted from the surface of an absolutely black body at a temperature of 2000 K whose area is ≈ 0.1 cm^2.

Let us assume now that the radiating particles are united into fractal aggregates. Then the radiated power per unit mass of radiating material increases. To make an estimate we assume that the parameters of the fractal aggregate are the same as the ones that had been observed in the experiment of Ref. 17 and are shown in Table I, i.e., the size of the particles in the aggregate amounts to $r=3$ nm, and the fractal dimensionality is equal to $d=1.6$ and the radius of the fractal aggregate is $R_0=1$ μm. Then, as follows from formulas (5) and (13), the radiated power per unit mass increases by a factor of $(R_0/r)^{3-D}\sim 3000$ which corresponds to a concentration of platinum in the discharge of $\sim 10^{-8}$ g/L.

The formation of fractal aggregates can be of fundamental significance for the radiative processes under discussion since it leads to a sharp increase in the efficiency of using the evaporated material, while its concentration is relatively low. According to the estimates that have been made the observed radiation can be provided by a concentration of the atoms of evaporated material in air of ~ 0.01 ppm (10^{-8} molecules of the material per molecule of air), if it exists in the form of fractal aggregates.

Let us discuss the possibility and the nature of formation of fractal aggregates in the discharge under consideration. Earlier it had been noted that of the 10 materials of the electrodes for which prolonged light emission was observed after the discharge was switched off only in two cases the temperature of the light-emitting zone (~ 2000 K) is below the melting temperature of the material, i.e., fractal aggregates may be formed from the given material. However, this does not reject the possibility of formation of fractal structures in other cases. For example, in the case of Al the fractal aggregate may consist of Al_2O_3 and the same situation can occur in other cases.

The physical picture of the formation of fractal aggregates in the system under consideration can be described in the following manner. As long as the discharge is taking place the temperature of the air in the discharge zone is relatively high, and the fractal aggregates are destroyed under the action of the electrical current or processes associated with it. As a result the evaporated material is in the gas in the form of atoms or particles-clusters, so that the radiation from such a system is relatively low. After the source of the discharge has been switched off the possibility arises of forming fractal aggregates. As the fractal aggregates are being formed the radiated power increases, and subsequently it falls as the gas cools down.

Assuming the cluster-cluster mechanism of the formation of fractal aggregates[68,69] one can estimate the time of their formation in air which under the parameters being considered amounts to 0.01–0.1 sec. This time can be noticeable on the scale of times being used, which will give a nonmonotonic dependence of the radiated power on the time. Specifically after the discharge has been switched off the radiated power at first increases, while the evaporated material has not yet been transformed into clusters, and subsequently it falls due to the cooling of the gas. Such dependences have been repeatedly observed in the experiment of Powell and Finkelstein[48] and are and indirect confirmation of the presence of fractal aggregates in the afterglow.

Let us analyze the role played by chemical processes in the phenomenon under consideration. We note that the chemical energy liberated when the evaporated material enters into a chemical bond is relatively not very great. Under the conditions being considered in the case when the radiation is produced by small particles (i.e., the concentration of the evaporated material is relatively high), this energy does not exceed 0.1 J and is lower by one or two orders of magnitude than the energy emitted in the form of radiation. Only in the case of using N_2O as a buffer gas the structures formed from the evaporated material can operate as a catalyst in the decomposition of this compound. In the case of all other buffer gases there is no such possibility, i.e., the chemical energy liberated in chemical processes does not affect the processes of radiation in the afterglow.

However the chemical processes themselves determine to a high degree the spectral characteristics and the radiated power through the parameters of the fractal structures being formed. The rate of formation of fractal aggregates and also their mechanical and optical parameters depend on the chemical composition of the compounds formed by the evaporated material. Therefore the buffer gas affects the color and the radiated power. For example, in pure oxygen the radiated power is several times higher than in air. If in the buffer gas being used there is no compound of the evaporated material from which fractal aggregates can be formed, then in such a buffer gas prolonged radiation will not be observed in the afterglow. The experiment of Powell and Finkelstein indicates that different versions can exist.

Thus, a detailed investigation of the afterglow of a high-frequency discharge carried out by Powell and Finkelstein indicates that prolonged light emission after the discharge has been switched off is produced by material evaporated from the electrodes which forms chemical compounds with the buffer gas or its impurities. An analysis of the processes occurring in such a case shows that the most probable state of these compounds are fractal aggregates. Then, under the experimental conditions being considered, the concentration of the evaporated material in the buffer gas is of the order of 10^{-8} ppm or 10^{-8} g/L.

We note that the special feature of a fractal discharge is associated with the high efficiency and duration of light emission in the afterglow of the discharge. Therefore such discharges were made use of in the creation of a laboratory analog of ball lightning as a phenomenon in atmospheric air associated with the existence of a prolonged afterglow without an external source of energy. The paper by Powell and Finkelstein[48] examined above was devoted to this, and also other methods of creating a prolonged afterglow in air after the discharge has been switched off. Let us analyze the possible nature of the formation of fractal aggregates in some of them.

One of such methods refers to the arc discharge initiated in water.[65–67] After such a discharge has been initiated in the water between the electrodes a cavity is formed containing the discharge plasma. After the discharge has been switched off this cavity behaves as an autonomous object. The special feature of this type of a discharge is that the water surrounding the discharge delimits the zone occupied by plasma and the evaporated material. This accelerates the processes of formation of fractal aggregates as the system cools off.

Yet another type of afterglow of a discharge where apparently fractal aggregates are being formed was worked on by Tesla in the closing years of the last century and recently repeated by the brothers Corum.[63,64] The basis of the discharge is a radio-frequency spark gap (the frequency utilized was 67 kHz). After a multiple repetition of pulses a weakly conducting channel is formed in air. Such a discharge enables one to obtain a spark in air with a relatively small input of energy.

Sometimes at certain points of the conducting channel of a given discharge balls of diameter of several cm and with a lifetime up to several seconds were formed. The authors of Refs. 63, 64 assert that these are fractal structures and that the Tesla discharge is a laboratory analog of ball lightning.

However, the experimental data are insufficient for confirming the fractal structure of the light emitting formations. Nevertheless, since this effect is sensitive to the electrode material one can assert that the light emission is produced by the material of the electrode. The evaporated material is conduced into the zone of light emission along the conducting channel as a result of the process of cataphoresis. The presence of evaporated material in the conducting channel affects the parameters of the spark in the process of repetition of pulses.

Thus, an analysis of the fractal discharges shows that the presence in them of fractal aggregates is confirmed both by the high yield of radiation for these discharges, and also by the nature of the processes occurring in them. Nevertheless, additional detailed investigations of fractal discharges are required with a detailed study of the processes occurring in them and of the structures being formed.

4. THE FRACTAL CONCEPT OF BALL LIGHTNING

4.1. Properties of ball lightning as a luminous object

Let us examine the properties of ball lightning which follow from the analysis of observational data of Refs. 71–73 and which will be useful for the description of the fractal model of ball lightning. The average diameter of ball lightning amounts to 23 ± 5 cm, the average lifetime of

ball lightning is equal to $8 \cdot 10^{\pm 0.3}$ sec and the average brightness of ball lightning corresponds to the brightness of a 100–150 W electrical lamp. To these numerical parameters we add that, in any event in the majority of cases the light emission from ball lightning is not associated with an external source of energy and it has a spotted structure, and the temperature of the light emitting spots—hot zones inside the ball lightning—turns out to be ~ 2000 K, which follows from an analysis of the brightness. The average temperature of the air within the ball lightning is approximately by 100 K higher than the temperature of surrounding air, which follows from an analysis of the gas dynamics of the average ball lightning and the heat exchange with the surrounding air. Thus, the temperature field of ball lightning includes small highly heated regions which produce the light emission from ball lightning and which are inside the basic weakly heated region. We note that this conclusion follows from an analysis of observational data.

The above information can be utilized for an analysis of laboratory analogs of ball lightning and its models. In particular, since the time of light emission from fractal discharges (0.1–1 sec) is considerably shorter than the lifetime of ball lightning, they will not serve as a model for ball lightning, although with respect to other parameters (the absence of an external energy source, the brightness of light emission) they are an analog of ball lightning.

4.2. Fractal models of ball lightning

Fractal models have important significance in understanding ball lightning. The investigations of ball lightning using fractal concepts have shown the existence of new fractal structures with specific properties which are formed both under natural conditions, and also in laboratory systems. The study of such physical objects provides a qualitative understanding of new physical systems.

The fractal concept of ball lightning is based on its having a rigid core and the models describing it have undergone change as new laboratory analogs were adduced as models for the core of the ball lightning. At the first stage this model was constructed as a system of filament-like aerosols,[74] with this model being based on experiments on the electrical explosion of metallic wires.[75] The first fractal model of ball lightning[76] utilized the concept of fractal aggregates and the information associated with them. This was replaced by the aerogel model of ball lightning.[77] An aerogel consists of associated fractal aggregates and is a macroscopic system of laboratory size, in contrast to fractal aggregates whose dimensions do not exceed tens of microns. The basis for the next model—the model of a fractal tangle[78]—were experiments on the formation of fractal filaments as a result of laser irradiation of a metallic surface.[79] The fractal filaments[79,80] are formed as a result of rapid condensation of weakly ionized metallic vapor in an external electric field,[81,82] while a fractal tangle is a system of intertwined fractal filaments. In such a case the average density of the material of the fractal filaments is by 2–3 orders of magnitude lower than the density of a solid mass of a given material, and the filaments themselves occupy an ~ 0.01 fraction of the volume in which they are situated. As a result the average density of the material in the core of the ball lightning is by 4–5 orders of magnitude lower than the density of the material of which it is composed, i.e., the core of ball lightning has a relatively low weight.

As may be seen the development of the fractal concept of ball lightning occurred in accordance with one of the principles of the development of science—the principle of succession. This means that each subsequent model follows from the preceeding one and utilizes its principal elements. At the same time it includes within itself new elements based on additional physical information. As a result each succeeding model becomes more and more complicated. Correspondingly the last fractal model of ball lightning which includes a large number of factors and describes a large number of observed properties of ball lightning is at the same time too complex that it could have been proposed at the first stage of study of ball lightning.

In view of its rarified structure the core of ball lightning possesses simultaneously properties of a solid, a liquid and a gas; moreover the rarified structure of the core corresponds to its small specific gravity and its small surface tension. This explains a number of mechanical and gasdynamic properties of ball lightning: its ability to fly, to penetrate through small openings and narrow gaps, to be reflected from a solid surface, its small resistance to air in moving through it, etc.

An important property of a fractal tangle which represents a system of intertwined fractal filaments is the possibility of a phase transition tangle–globule, similar to the phase transition for a polymer filament with self-intersections.[83–85] This means that at low temperatures the fractal tangle acquires a spherical shape under the interaction between individual filaments in the zone of their contact. At high temperatures the order in the system of interacting filaments is destroyed and this leads to a loss of its spherical shape. The estimates of Ref. 78 show that for ball lightning with average parameters this transition is expected at a temperature of 700 ± 200 K. We note that approximately 90% of observed ball lightnings has a spherical shape. Usually the ball lightning retains its size and shape in the course of its existence but approximately in 1% cases of observations of ball lightning a change in its shape is recorded, as a rule, ribbon (rod)—sphere and vice versa.[86] We must add to the above that simple models of ball lightning are incapable of describing these transitions.

The fundamental property of ball lightning is associated with preservation and transformation of energy in ball lightning. Since elements of the structure of ball lightning are particles of nanometer size which unite into fractal aggregates and they in turn into macroscopic fractal structures, the core of ball lightning has a high specific surface energy attaining ~ 1 kJ/g. The transformation of internal energy into heat energy which is accompanied by the particles of the structure becoming larger can lead to heating of the elements of the structure to temperatures up to ~ 2000 K. Such a process is realized in the form of heat waves which propagate along fractal filaments of the structure.[87,88] At the same time many independent heat

waves can exist propagating along the individual fractal filaments of the structure.

This process explains the spotted structure of light emission of ball lightning[71–73] since in the region of the front of thermal waves hot zones arise with a temperature of ~ 2000 K. Moreover, the mechanism of light emission from ball lightning removes the apparent contradiction between the characteristic temperature of the light emitting zones (~ 2000 K) which corresponds to the observed brightness of ball lightning, and the average temperature of the air inside the ball lightning (~ 400 K) which follows from an analysis of the heat liberation and gas-dynamics of ball lightning; the color of ball lightning is determined by light emission of impurities situated in the hot zones.

The fractal concept of ball lightning enables one to bring together its various parameters. For example, the lifetime of ball lightning within the framework of this concept can be regarded as a characteristic time during which the material of its core can be worked over. The greater is the size of ball lightning the longer time is required for this. In this way one can explain the observed correlation between the size and the lifetime of ball lightning.[71,89] The fractal concept describes also the nature of the formation of the core of ball lightning as a set of processes in an expanding and cooling weakly ionized vapor existing in an external electric field.[81,82] This vapor is formed as a result of evaporation of the surface under the action of high energy fluxes. We note that the time of formation of fractal filaments and tangles is sufficiently great and amounts to minutes in the case of characteristic parameters of processes in ball lightning, while the stage of the process which ends by the formation of fractal aggregates lasts tens of milliseconds.

We call attention to another fundamental property of a fractal tangle which constitutes the core of ball lightning. In view of the high rarefaction of the structure individual bonds within it can be broken and new ones can subsequently be formed. This leads to the structure becoming denser, and the core of ball lightning is not stable, but ages in time. This property is similar to the corresponding property of fractal aggregates formed in solutions which was observed and modeled for example in Refs. 90–92.

In order to understand the possibility and the defects of the modern fractal concept of ball lightning let us compare the concept of the fractal tangle with the preceeding aerogel concept of the core of ball lightning.[77] An aerogel is a porous rarefied material consisting of oxides or organic compounds. The most frequently occurring material of an aerogel SiO_2 is the same as in the case of glass and quartz. The minimum specific gravity of samples of aerogel being prepared amounts to 3 g/L,[93] the specific area of the internal surface of the aerogel attains values of 1600 m^2/g.[94] These values can explain the observed gas-dynamic and energetic properties of ball lightning. As regards the nature of transformation of surface energy into heat energy, in an aerogel, as in an isotropic system, in the case of large dimensions only one heat wave can be propagated, while in a fractal tangle many heat waves can coexist simultaneously which propagate along individual fractal filaments. Thus, the model of a fractal tangle describes the spotted structure of ball lightning while the aerogel model does not include this specific feature of ball lightning. At the same time the model of a fractal tangle does not provide the possibility of estimating within the model itself the number of simultaneously present heat waves. This means that the model of a fractal tangle is an intermediate version of the fractal concept of ball lightning and requires subsequent modification. Apparently investigations of the fractal polymer model of ball lightning[35,95] are useful for this purpose.

Thus, the fractal concept of ball lightning enables us to explain the principal fundamental properties of ball lightning. We note that we here have not seen as an aim to describe different properties of ball lightning, some of which can be understood without bringing in fractal concepts. The aim of the analysis was to show that step by step, leaning on laboratory analogs one can arrive at a realistic fractal concept of ball lightning. The history of the establishment of this concept shows that it consists of a certain set of elements which can be included in it sequentially. Although at the present time the fractal concept of ball lightning has not yet been perfected, it enables one to explain many amazing properties of ball lightning which at first sight seem to be unreal. An understanding of the details of the nature of ball lightning prepares the path for detailed investigations of the fractal concept of ball lightning which by now has been partially traversed. As a result of these investigations a qualitatively new physical object is becoming outlined which has a relation to processes both in nature and in laboratory systems. This object is of interest without reference to ball lightning, and manifests simultaneously properties of a gas, a liquid, and a solid, and also other specific properties and leads us to qualitatively new positions in physics.

5. CONCLUSION

The examined examples show the role played by general fractal concepts[1,2] in the study of different physical processes and phenomena. Although fractal properties are not the basic parameters of objects and processes the existence of fractal structure may fundamentally change their properties. The examples provided above confirm this. Another result of the analysis that has been carried out refers to the incomplete set of experimental methods utilized in the investigation of the objects and phenomena being considered. With the exception of a small number of special investigations the arsenal of experimental methods usually does not include the analysis of fractal parameters of objects which greatly lowers the information about them.

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