Hydrodynamic theory of multiple process and the physics of the quark-gluon plasma

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This paper is concerned with the refinement of the quantitative conclusions of the hydrodynamic theory of multiparticle processes which was put forward by Landau 40 years ago. The experimental data obtained by modern accelerators are compared with the predictions and conclusions of the theory. A comparison is made between the model proposed by Landau and the scaling model. The authors point out that in the latter model the initial temperature and energy density values are underestimated. The concepts of the hydrodynamic theory and the quark-gluon theory are compared. The common nature of the two approaches is noted and some discussion is presented regarding ways of developing the two directions. Collisions of heavy nuclei are analyzed.

By the end of this century the large heavy-ion accelerators RHIC (Brookhaven) and LNC (CERN) should be in operation. One of the principal objects of investigation on these accelerators will be the quark-gluon plasma, a hypothetical state consisting of quasifree quarks and gluons. At the present time the literature on this topic is extensive, and for this reason it is not possible to present a detailed review of the subject in this paper. We have put before ourselves the limited goal of summing up the progress in the development of the hydrodynamic theory of multiple processes formulated by L. D. Landau 40 years ago, to relate its basic ideas to the concept of a quark-gluon plasma, and to make a comparison with contemporary experimental data. Therefore we do not claim to present a comprehensive review of the literature on this topic.

1. SOME HISTORY

In the early 1930s a group of cosmic ray particles were detected that were genetically related.[1–3] The imperfect methods used at that time did not permit a distinction between two possibilities: cascade multiplication of particles or their formation in a single event. W. Heisenberg[4] adopted the second hypothesis, and on the basis of the existence of nonlinearities in some versions of the theory of β decay proposed a model for the formation of several electrons, positrons, and neutrinos in a single event. This phenomenon was later given the designation multiple processes.

Then it was found that multiple processes do not involve the formation of electrons and positrons, but mesons, and Heisenberg,[5,6] to describe multiple processes, used the nonlinear Lagrangian of Born and Infeld. In this version of the theory the nonlinearity (and hence the interaction between the particles) is so strong, that the average multiplicity $\langle N\rangle$ in secondary particles turned out to be the maximum allowed by the conservation laws

$$\langle N\rangle \approx s^{1/2}/m_\pi; \qquad (1)$$

where m_π is the mass of the pion and $s^{1/2}$ is the total energy of the colliding particles in the center-of-mass system.

The papers of Heisenberg contain two important ideas:

1. For the formation of many particles in a single event it is necessary that the equations describing this process be nonlinear.

2. To analyze the conversion of two primary particles into many secondary particles it is necessary to use the concepts of macroscopic physics—nonrelativistic hydrodynamics.

These two ideas were developed (partially independently) by Fermi.[7] His model was based on three postulates:

1. As a result of collision all secondary particles are contained in a Lorentz-contracted volume with transverse dimensions $R_\perp \approx 1/m_\pi$ and longitudinal dimensions $R_\parallel \approx 1/E_C$, where E_C is the energy of the primary particles in the center-of-mass system (the C-system).

2. In this volume the secondary particles are in statistical equilibrium.

3. All the energy of the primary particles is deposited into the equilibrium system.

From these postulates follows uniquely the formula for $\langle N\rangle(s)$

$$\langle N\rangle \approx (s/m_\pi^2)^{1/4}, \qquad (2)$$

which agrees qualitatively with experimental data.

The angular distribution of the secondary particles in the C system in the first version of the Fermi model must be isotropic. However, as Pomeranchuk noted at once,[8] the Fermi model contains an internal contradiction. It is not possible to put N real particles in the Lorentz-contracted volume when each of them has about that same volume. Therefore the Lorentz-contracted volume can be only the initial volume for the expansion of the system of virtual particles. These particles expand isotropically, and the expansion processes terminates when the final temperature is $T_f \approx m_\pi$ and the virtual particles are converted into real particles with a dimension $r \approx 1/m_\pi$ in the proper system of coordinates. In this case the principal contradiction of the Fermi model is removed. The final volume of the system

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can contain all the real secondary particles. The final results of the Pomeranchuk model are close to the conclusions of the theory of Heisenberg, Eq. (1).

Forty years ago Landau[9] synthesized the ideas of his predecessors by introducing a new and important factor: the expansion of the elements occupying the Lorentz disk are described by the relativistic hydrodynamics of an ideal fluid. During the expansion the temperature decreases, and the expansion stops when $T_f \approx m_\pi$. At this time real hadrons are formed. In the Landau theory the average multiplicity $\langle N \rangle$ is described by formula (2), and the angular distribution in the C system has a strong anisotropy. In conclusion, let us make one important comment. In the time of Heisenberg, Fermi, Pomeranchuk, and Landau there was no clear conception of the nature of the elements composing the expanding fluid. At that time they spoke of the excited vacuum or the "boiling operator" fluid, or even more indefinitely, of the "constituents." In modern language, one might try to identify these constituents in terms of their properties with quarks and gluons.

2. THE PRO AND CONTRA OF THE LANDAU HYDRODYNAMIC THEORY

In this section we recall qualitatively the fundamentals of the physical ideas of the hydrodynamic theory. Within the framework of this theory the collision of two nucleons can be described in the following way: in the initial stage of the process there is a hard collision of the valence and sea quarks and gluons making up the structural functions of the projectile nucleons. As a result of the strong interaction of mainly the gluon components, a quark-gluon plasma, which participates in the hydrodynamic process, is formed in the central region. The valence quarks interact more weakly and play an important role in the formation of the leading particles. Of course, in order to describe the hydrodynamic of the evolution, one must prescribe the boundary and initial conditions in addition to the equations of relativistic hydrodynamics

$$\frac{\partial T_{ik}}{\partial x_k}=0, \qquad (3)$$

$$T_{ik}=(p+\varepsilon)w_i u_k+p g_{ik} \qquad (4)$$

here T_{ik} is the relativistic energy-momentum density tensor of an element of an ideal fluid, x_k is the 4-coordinate of an element of the fluid, ε is its energy density, p is the pressure, u_i is the 4-velocity of this element, and g_{ij} is the metric tensor.

In the theory of Landau these conditions are given in the form of a disk with transverse dimensions $1/m_\pi$ and longitudinal dimensions 2 $\eta s^{-1/2}$. An important point is that in all the stages of the hydrodynamic process it is assumed that there is thermodynamic equilibrium characterized by a temperature T and an equation of state that, as a rule, is expressed in the form

$$p=\varepsilon/3, \qquad (5)$$

which is characteristic of an ideal relativistic gas.

As the quark-gluon fluid expands the temperature T of the fluid decreases, and when it reaches the value $T_f \approx m_\pi \approx$ 140–150 MeV, the hadron phase transforms into real hadrons that compose the region of pionization (the quasi-central plateau in the distribution $\mathrm{d}N/\mathrm{d}y$ or $\mathrm{d}N/\mathrm{d}\eta$, where y and η are, respectively, the rapidity and the pseudorapidity)

$$y=\tfrac{1}{2}\ln\frac{E_i^* + p_{\parallel\,\mathrm{C}}^*}{E_i^* - p_{\parallel\,\mathrm{C}}^*}=\operatorname{Arsh}\frac{p_{\parallel\,\mathrm{C}}^*}{m_{\perp\,\mathrm{C}}}, \qquad (5')$$

$$\eta^* = -\ln\left(\frac{1}{2}\operatorname{tg}\theta^*\right); \qquad (6)$$

E_{C}^*, $p_{\parallel\,\mathrm{C}}^*$ are the energy and the longitudinal momentum of the secondary particles in the C system, θ^* is the outgoing angle, and $m_\perp = (m^2+p_\perp^2)^{1/2}$.

A particularly strong point of the hydrodynamic theory is its unprecedented heuristic power. This circumstance is of particular significance, since the multiple processes are extremely complicated phenomena characterized by many free parameters that allow one to interpret practically any of its properties.

Let us recall the predictions made in the initial stage of the development of the theory and subsequently verified by experiment.

1. The functional dependence of the average multiplicity $\langle N \rangle (s)$ (Ref. 9).
2. The derivatives $\mathrm{d}N/\mathrm{d}y$ and $\mathrm{d}N/\mathrm{d}\eta$ (Ref. 9, see also Refs. 10 and 11).
3. The boundedness and the value of the transverse momentum $\langle p_\perp \rangle$ and its distribution. This property of high importance for high-energy physics was predicted on the basis of the hydrodynamic model.[12]
4. The dependence of the transverse momenta on the mass of the secondary particles.[13]
5. The extremely weak dependence $p_\perp (s)$ (Refs. 14 and 15). This point is of fundamental importance for the entire hydrodynamic concept and will be considered in greater detail below (see sections 4 and 5).
6. The formation of photons and lepton pairs in multiple processes.[16,17] The exceptional heuristic power of the hydrodynamic theory is a serious argument in its favor. Let us now consider some "contra" arguments.

The principle arguments refer to the formation and the characteristics of the initial state (the Lorentz volume). The main argument of the opponents of the theory reduces to the fact that the formation of the Lorentz-contracted volume contradicts the uncertainty principle.[18,19] The essence of this objection is that if the disk is divided into n layers in the transverse direction, then it would appear as if the inequality $n<1$ were satisfied, which naturally is in a conflict with the uncertainty principle. However, simple numerical estimates show that there is no contradiction. Actually, from the uncertainty principle we have in the present specific case the inequality

$$\varepsilon > \left(n m_\pi^2 \frac{s^{1/2}}{m_{\mathrm{p}}}\right)^2; \qquad (7)$$

where ε is the energy density in the initial state and m_p is the proton mass.

Using the obvious relations

$$\varepsilon = s^{1/2}/V_0, \quad V_0 = \tfrac{8}{3}\pi \frac{m_p}{m_\pi^3 s^{1/2}},$$

we obtain the condition

$$n^2 < \frac{m_p}{m_\pi} \approx 7, \tag{8}$$

which (although in the limit) does not contradict the uncertainty principle. However, in our opinion the principal remark is more important. The division of the disk into independent layers may be problematical because of the relatively strong interaction between the elements of the fluid. In particular, because of the interaction of the partons the uncertainty principle rather has the form $\Delta x \cdot \Delta p \sim \hbar/N$, where N is the number of colors.[20] Of course it is not possible to calculate this interaction from first principles (QCD). Therefore the assumption of the existence of the Lorentz-contracted disk may be regarded as a postulate of the theory.

Another objection amounts to the statement that in the process of deceleration and the formation of the disk the primary partons lose all their energy to electromagnetic radiation.[21] However, as is shown by the careful analysis in the paper of Feinberg,[22] the calculations in Ref. 21 were carried out under the assumption of point partons. If the finite dimensions of the primary particles are taken into account this inconsistency is removed. Moreover, most of the constituents—gluons—are electrically neutral.

Finally, we advance the last contra argument. It has been suggested that because of the existence of asymptotic freedom, conditions arise that prevent the formation of the disk. The quarks and gluons will "slip past" without forming a quasistatic disk structure. However, because of confinement the validity of such assessments is very problematical. Moreover, it follows from rough estimates (no others are possible here) that the energy of the quarks and the gluons is ~ 1 GeV, and consequently in the present case a relatively strong interaction is quite probable.

Therefore the hypothesis that the initial state forms in the shape of a Lorentz-contracted disk does not encounter logical contradictions, but its basis has very much the nature of a model.

3. THE MODERN INTERPRETATION OF THE HYDRODYNAMIC THEORY

The first version of the hydrodynamic theory[9] contained some statements that underwent revision during the development of the theory. It should be emphasized that the subsequent modifications did not alter its remarkable predictions, but they did refine the theoretical values of the characteristics of the multiple processes. This remark refers particularly to one of the postulates of the first version—that *all* the energy of the particles is transferred into a statistical hydrodynamic system. As was shown by the first cosmic ray experiments and later by more precise experiments on accelerators, approximately a fraction $K = 1/2$ of the initial energy is deposited into the statistical system. The other half of the energy goes into leading particles that preserve their quantum numbers. In modern terminology one can say that two regions are formed in the distribution over rapidity y (or pseudorapidity η). The first of them, usually called the region of pionization or the region of the central plateau, includes the value $y \approx 0$, and the second—the region of fragmentation—includes particles with values $y \approx y_{\max}$ ($y_{\max}$ is defined by the kinematic limits). Semiqualitatively, these regions may be defined in the following way:[23]

$$\tfrac{1}{2} \ln \frac{s}{m_\perp^2} + l \leqslant y \leqslant y_{\max} \tag{9}$$

is the region of fragmentation of the projectile particles

$$y_{\min} \leqslant y \leqslant \tfrac{1}{2} \ln \frac{s}{m_\perp^2} - l \tag{10}$$

is the region of fragmentation of the target particles, and

$$\tfrac{1}{2} \ln \frac{s}{m_\perp^2} - l \leqslant y \leqslant \tfrac{1}{2} \ln \frac{s}{m_\perp^2} + l \tag{11}$$

is the region of pionization.

In these relations

$$y_{\max} \approx \ln \frac{s}{m_b m_\perp}, \quad y_{\min} \approx \ln \frac{m_\perp}{m_b},$$

the constant $l \approx 1$, $m_\perp = (m^2 + p_\perp^2)^{1/2}$, m is the mass of a secondary particle, and m_b is the mass of a target particle.

At high energies $s^{1/2} \geqslant 100$ GeV more than 90% of these particles are concentrated in the region of pionization. This is the region that is the main topic of the hydrodynamic theory and of our paper. This region forms mainly as a result of the interaction mainly of the gluon component and it forms a single statistical system (a quasi-fireball). For a more detailed account see the discussion in the collection, Ref. 24. A similar idea has also been expressed in Ref. 25.

Landau divided the hydrodynamic process into two stages. In the first stage he took into account one-dimensional expansion, which was then "merged" with a three-dimensional solution, described very approximately on the basis of conical (inertial) expansion. In both stages thermal motion of the particles of the relativistic stage was not taken into account. This approach gave a substantial overestimate of the transverse momentum $p_\perp$.

In Ref. 14 another approach was marked out: thermal motion was assumed to be responsible for the lateral expansion and the longitudinal expansion was described by the one-dimensional solution (the quasi-one-dimensional approximation). On the basis of this approximation it was possible to predict the correct distribution $dN/dp_\perp$.

Then in Ref. 26 the quasi-one-dimensional approximation was used as the basis for a numerical estimate of the three-dimensional stage and of the dependence $\langle p_\perp \rangle (s)$.

However, in the calculations a noninvariant expression was used for the one-particle distributions, which resulted in some overestimate of the value of $\langle p_\perp \rangle$.

It is necessary to emphasize the extreme importance of the estimation of the transverse characteristics in the assessment of any model of multiple processes, and especially for the hydrodynamic theory. The point is that these characteristics are nearly independent of the initial conditions of the problem and are the most vulnerable to criticism. In the framework of the other constructs based on quantum field concepts it has not been found possible to interpret uniquely the experimental data on the transverse momenta. In view of these facts, we once again recalculated the characteristics of the multiple processes[27,28] (in particular, the transverse characteristics).

These calculations were based on the following ideas:

1. Most of the hydrodynamic expansion is one-dimensional motion.
2. The motion in the transverse direction is mainly governed by thermal motion.
3. The quasi-one-dimensional solution is valid as long as the transverse path of an element of the fluid is less than the transverse dimension of the system, $r_0 = 1/m_\pi$. The surface at which the transition is made from the one-dimensional to the three-dimensional stage is determined by the relativistically invariant condition

$$\tau^2 = t^2 - x^2 = r_0^2; \tag{12}$$

where τ is the proper time of the element.

It is possible to show that if $\tau > r_0$ the motion is inertial, which means, in essence, the termination of the interaction (freezeout). Each element in the fluid moves with a constant velocity. Therefore the rapidity distribution is determined by relation (12) and does not depend on the freezout temperature, T_f.

4. CALCULATION OF TRANSVERSE MOMENTA

We shall be concerned principally with a more thorough calculation of the dependence $\langle p_\perp \rangle (s)$. This choice is dictated by the fact that the transverse characteristics of multiple processes are only weakly dependent on the initial conditions, and moreover, the exceptionally weak dependence of $\langle p_\perp \rangle$ on the energy of the primary particles probably has no precedent, although it may be verified experimentally.

Using the exact solution of the one-dimensional equations of relativistic hydrodynamics[29] with our choice of initial conditions, relation (12) can be written in the form

$$\left(\frac{\partial \varkappa}{\partial \tau_1}\right)^2 - \left(\frac{\partial \varkappa}{\partial y_1}\right)^2 = r_0^2 e^{2\tau_1}; \tag{13}$$

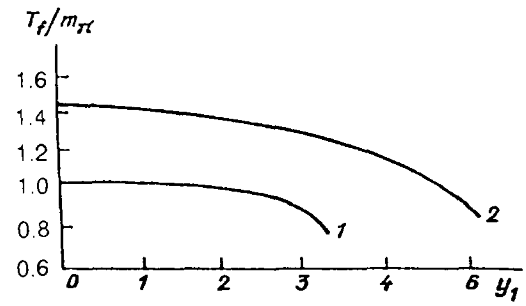

FIG. 1. Profile of the freezout temperature $T_f(y)$. Curve *1* corresponds to $s^{1/2} \simeq 53$ GeV, Curve *2* to $s^{1/2} = 540$ GeV. The inelasticity coefficient is $K = 1/2$.

where $\tau_1 = \ln(T/T_0)$, $y_1 = \tanh^{-1} V_C$, T_0 is the initial temperature, V_C is the velocity corresponding to the end of the one-dimensional stage, and

$$\varkappa = \frac{\Delta}{2c_s}(\exp \tau_1) \int_{c_s y_1}^{\tau_1} \exp\left(-\frac{1+c_s^2}{2c_s^2}\right) I_0\left(\frac{1-c_s^2}{2c_s^2}\right)\left[\langle \alpha' \rangle^2 - c_s^2 y_1^2\right]^{1/2} d\alpha'. \tag{14}$$

Here c_s is the velocity of sound, which is assumed to be constant and equal to $1/\sqrt{3}$, $\Delta = 4m_p r_0/s^{1/2}$ is the longitudinal dimension of the initial volume with allowance for the leading effect, and I_0 is the Bessel function. In Eqs. (13) and (14) we do not take into account thermal motion, whose effect we shall consider later. From conditions (13) and (14) we can obtain the distribution of the entropy s over the rapidity

$$\frac{ds}{dy_1} \sim e^{-\tau_1} \Phi(\tau_1, y_1), \tag{15}$$

$$\Phi(\tau_1, y_1) = \left[\left(\frac{\partial \psi}{\partial y_1}\right)^2 - c_s^2\left(\frac{\partial \psi}{\partial \tau_1}\right)^2\right]\left(\frac{\partial \varkappa}{\partial y_1}\frac{\partial \psi}{\partial y_1} - \frac{\partial \varkappa}{\partial \tau_1}\frac{\partial \psi}{\partial \tau_1}\right)^{-1} \tag{16}$$

where $\psi = \partial\varkappa/\partial\tau_1 - \varkappa$.

We note that in his seminal paper[9] Landau obtained an asymptotic solution valid for $s^{1/2}/m_p \gg 1$ and $\tau_1 \gg y_1$. From relations (14) and (15) it is possible, by expanding the Bessel function, to obtain the solution to essentially any degree of accuracy. In fact, expanding

$$I_0(Z) = 1 + Z^2/4 + Z^4/64 + \ldots,$$

$$Z = \frac{1-c_s^2}{4c_s^2}(\tau_1^2 - c_s^2 y_1^2)^{1/2},$$

one can retain only the first three terms in the expansion of I_0 out to energies $s^{1/2} \sim 1000$ GeV with an accuracy or 5–7%. The rather complicated explicit expression for the functions $\partial\varkappa/\partial\tau_1$ and $\partial\varkappa/\partial y_1$ are given in Ref. 27. We shall only remark that these functions depend on the parameters c_s and y_1. Using the explicit expressions for the functions $\partial\varkappa/\partial\tau_1$ and $\partial\varkappa/\partial y_1$, and relation (12) we can determine the temperature T_f at which the one-dimensional and the three-dimensional stages merge. The function

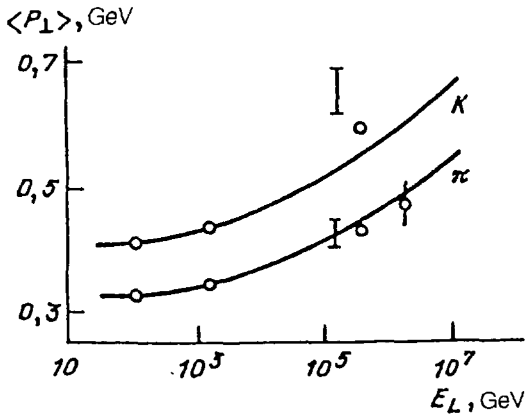

FIG. 2. Average transverse momentum $\langle p_\perp \rangle$ vs the energy $E_\perp$ of the primary particles in the laboratory system for pions (π) and kaons (K).

$$\frac{T_f}{m_\pi}(y_1)$$

is plotted in Fig. 1 for values $s^{1/2}=53$ and 540 GeV. Using these expressions we can show that after the end of the one-dimensional stage (condition (12)) the motion of the elements of the fluid is practically inertial. For a calculation of the transverse momenta it is necessary to take into account the thermal motion that determines their values.[14] The idea of the calculations is to take into account the decay of the elements of the fluid at the temperature $T_f(y_1)$ and then integrate over curves such as are shown in Fig. 1. The one-particle distribution can be written[30]

$$E\frac{dN}{d^3p}=\int f(x,p)p^\mu d\sigma_\mu; \tag{16'}$$

where p_μ are the momenta of the particles and $d\sigma_\mu$ is an element of the 3-volume in a 4-hypersurface. For bosons

$$f(x,p)=\frac{g}{(2\pi)^3}\left(\exp\frac{p^\mu u_\mu}{T}-1\right)^{-1}. \tag{17}$$

The quantity $\langle p_\perp \rangle$ is defined by the relation

$$\langle p_\perp \rangle=\int p_\perp\, dN\Big/\int dN, \tag{18}$$

where dN is the number of particles in an element of phase volume:

$$dN=Ap_\perp\, dp_\perp\, dy_1 dy\left[\exp\frac{m_\perp\ \mathrm{ch}(y-y_1)}{T_f(y_1)}-1\right]^{-1} p^\mu d\sigma_\mu; \tag{19}$$

where $m_\perp=(m^2+p_\perp^2)^{1/2}$ is the transverse mass.

The final expression for $\langle p_\perp \rangle$ is obtained after integration over $p_\perp$ and y. The integration is carried out partly analytically and partly numerically. Figure 2 shows the theoretical (from formula (19)) and experimental values of $\langle p_\perp \rangle$ for various values of the energy of the primary protons (antiprotons) in the laboratory system. The agreement between the theoretical and experimental results is quite satisfactory.

5. ALLOWANCE FOR RESONANCES

Formula (19) takes into account only the direct particles (pions and kaons) that are formed as a result of the multiple processes. However, as is known, a considerable fraction of the light mesons are formed as a result of the decay of resonances (see, e.g., Ref. 31). A natural question arises: does the presence of resonances change the conclusions of the preceding section? This question was analyzed thoroughly in Ref. 28, in which the treatment took into account the decay of ρ, ω, η, φ, A, B, A_α, f, K^*_{890} resonances into two or three particles with the proper weighting factors. Here we shall only describe the method that made it possible to take into account two-particle decays. Let us consider a particle of mass m_0 that decays into two particles of masses m_1 and m_2. Then, using standard kinematics (see, e.g., Ref. 32) and transforming from the rest system of the resonance to the C system of the colliding particles, we can calculate the spectrum of the particles with mass m_1:

$$\frac{1}{N}\frac{dN}{dE_1}=\frac{m_0}{2p_1^*}\int_{E_0^-}^{E_0^+}\frac{W(E_0)dE_0}{p_0}, \tag{20}$$

where

$$E_0^\pm=m_0\left(\frac{E_1}{m_1}\frac{E_1^*}{m_1}\pm\frac{p_1p_1^*}{m_1^2}\right);$$

$$E_1^*=\frac{m_0^2+m_1^2-m_2^2}{2m_0}.$$

We consider only relatively heavy particles, where it is possible to use the Boltzmann distribution. Then we have

$$\frac{dN}{dp_1}=\frac{Ap_1}{E_1}\int_0^{(y_1)_{\max}}dy_1\Bigg\{\Bigg[\left(\frac{E_0^-}{T_f}+1\right)\times\exp\left(-\frac{E_0^-}{T_f}\right)-\left(\frac{E_0^+}{T_f}+1\right)\times\exp\left(-\frac{E_0^+}{T}\right)\Bigg]\exp(2\tau_1)\frac{\partial x}{\partial\tau_1}\Phi(\tau_1,\ y_1)\Bigg\}. \tag{21}$$

The function $\Phi(\tau,y_1)$ was defined previously in Eq. 16. Using Eqs. (16) and (21) we can estimate the effect of two-particle decay on the value of $\langle p_\perp \rangle$. As a result of this estimate (also including three-particle decays) it was found that although the transverse momenta due to the decay of several resonances are quite different from the values of $\langle p_\perp \rangle$ corresponding to direct mesons, the net contribution of resonance pions and kaons to the value of $\langle p_\perp \rangle$ is small. The average value of $\langle p_\perp \rangle$ changes by at most 2–3%, which is within the range of the statistical error of the measurements.

6. DISTRIBUTION OVER THE RAPIDITY AND PSEUDORAPIDITY

The number of particles in an element of 4-volume is given by formula (19). Integrating this expression over y_1 and $p_\perp$ we can obtain the distribution of particles over the rapidity.

However, the distribution over the pseudorapidity η usually changes. In order to convert from the rapidity y to the quantities $p_\perp$ and η we must use the formula

$$y = \tanh \frac{p_\perp \operatorname{th} \eta}{[p_\perp^2 + (m_\pi/\operatorname{ch} \eta)^2]^{1/2}}. \tag{22}$$

The results of the transformation and the integration (partly numerical) are given in Fig. 3.

This figure shows that there is agreement between the calculated and the experimental data in the distributions of dN/dy or $dN/d\eta$. One circumstance should be noted. Although for relatively small values of $\eta(y) \leqslant 2$ one might speak of the existence of a plateau, for large values of y there is a relatively slow decline in the function $dN/d\eta$. As a result, one can speak only of the existence of a quasiplateau in this distribution, which is in agreement with the first calculations[9,15] of the hydrodynamic theory, where the distributions of $dN/d\eta$ and dN/dy are Gaussian functions.

Also calculated in Ref. 33 were the correlations between the transverse and longitudinal momenta of the secondary particles, which showed good agreement with experiment for $s^{1/2} \approx 53$ GeV.

7. AVERAGE MULTIPLICITY

In accord with the fundamental ideas of Fermi and Landau, the average multiplicity is determined by the statistical weight (entropy) of the initial state. Since, as noted, the problem of the initial state is the bottleneck of the theory, it might appear necessary to ignore this experimental test. However, such a comparison was made up to energies $s^{1/2} \approx 500$ GeV by the following method. Up to an energy $s^{1/2} \approx 100$ GeV the exact values of the statistical weights were used, and at higher energies, the thermodynamic relations. The comparison between the theoretical and experimental data on the dependence $\langle N \rangle (s)$ demonstrated satisfactory agreement,[34] which indicates that in order of magnitude the choice of initial conditions in the form proposed by Fermi-Landau has a certain justification.

8. HYDRODYNAMIC THEORY WITH SCALE-INVARIANT INITIAL CONDITIONS (SCALING MODEL)

The requirement that the characteristics of the fluid be independent of the form of the inertial system[35–37] (frame-independence symmetry) has been suggested as an alternative to the initial conditions in the form of a Lorentz-contracted disk. This model became particularly popular after the work of Bjorken,[38] in which he used this model to analyze the collisions of fast nuclei (see below, Sec. 12). The characteristics of a one-dimensional relativistic fluid are given by the 4-vectors $x^\mu(t,x)$ and the scalar $\tau = (t^2 - x^2)^{1/2}$. The condition of Lorentz invariance will be satisfied if all the vector characteristics of the fluid are proportional to x^μ and the scalars are determined by τ. Essentially, this requirement is equivalent to the postulate that there exists some instant of time τ when all the scalar quantities are constants. Then the two-dimensional equations of hydrodynamics reduce to the relation[39]

$$\frac{ds}{d\tau} + \frac{s}{\tau} = 0; \tag{23}$$

where s is the specific entropy.

The solution of Eq. (23) has the form

$$s = \frac{s_f \tau_f}{\tau}, \tag{24}$$

where the quantities s_f and τ_f correspond to the transition from the quark-gluon phase to the hadron phase.

Using standard thermodynamic relations, we obtain

$$\varepsilon = \varepsilon_f (\tau_f/\tau)^{1+c_s^2}, \tag{25}$$

$$T = T_f (\tau_f/\tau)^{c_s^2}; \tag{26}$$

where ε is the energy density.

To evaluate the distribution over the rapidity, we must use the relation $dN \propto ds$. Then

$$\frac{dN}{dy} = \frac{\pi}{m_\pi^2} s_f \tau_f, \quad y_{\min} < y < y_{\max}. \tag{27}$$

In other words, within the framework of this model $dN/dy = \text{constant}$, which is in poor agreement with experimental data (Fig. 3). Strictly speaking, the plateau in the distribution of dN/dy is absent. The condition on dN/dy leads to the dependence $\langle N \rangle (s) \propto \ln s$, which is also in poor agreement with the experimental data. It is possible that also taking into account some distributions (for example, of the inelasticity coefficients) might improve the agreement between the conclusions of this model and the experiment. However, in the presently available data this detailed comparison is lacking.

Yet another (theoretical) feature of the model has been pointed out by Gorenshtein *et al.*[39,40] In the initial formulation of the model[35–37] only the initial conditions for the hydrodynamic equations were given. However, in addition to the initial conditions, it is necessary to formulate the boundary conditions on the fluid-vacuum interface.

In Ref. 39 it was proposed that the boundary conditions be given in an invariant form as a step function, which enters as an additional term in the hydrodynamic equations. Then the requirement of the conservation of energy-momentum leads to the necessity of introducing special states that are interpreted as leading particles. It must be pointed out that in this procedure the particle-like states are essentially not in contact with the relativistic fluid, which results in a certain simplification.

In reviewing the situation with the scale-invariant solution, we may point out the absence of any comparison of it with experimental data relating to inclusive reactions in pp or *nn* collisions at high energies.

9. INTERACTIONS BETWEEN NUCLEI (EXPERIMENTAL DATA)

In the description of the interaction between relativistic nuclei one can use two limiting models.

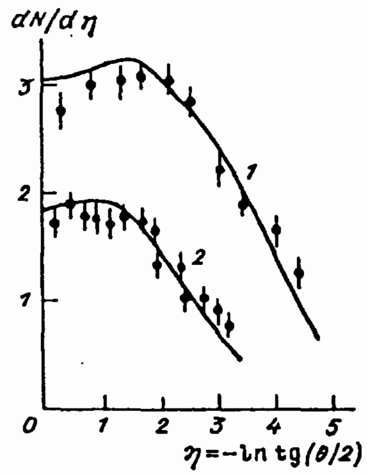

FIG. 3. Distribution $dN/d\eta$ for energies $s^{1/2}=540$ GeV (curve *1*) and 53 GeV (curve *2*). Solid curves are the results of calculations using the hydrodynamic theory.

1. *The "independent-interaction" model.* In this model each nucleon of the projectile particle interacts only with one nucleon of the target nucleus, and the secondary particles do not interact at all. Consequently the net interaction reflects all the properties of the nucleon-nucleon interactions.

2. *Cascade model.* Each nucleon of the projectile nucleus interacts with all the nucleons of the target nucleus. In this model the dependences $\langle N(A)\rangle$ and $\langle p_\perp\rangle(A)$ are highly nonlinear.

Of course, since there is no theory of nuclear interactions, the choice between the two models can be made only on the basis of experimental data. Let us consider first the question of the average multiplicity $\langle N\rangle$. For example, in Ref. 41 Simic *et al.* studied the interaction of p, d, He, and C with the carbon nucleus at an energy of 4.2 GeV/nucleon. Figure 4 shows the dependence of the average multiplicity of π^+ mesons as a function of the number Q of protons participating in the interaction. As can be seen from this figure, the average multiplicity of secondary particles per secondary proton does not depend on the atomic number A.

Figure 5 shows the ratio $\langle N_\pi\rangle/N_p$ for collisions of various nuclei (N_p is the number of nucleons participating in the collision).[42] The solid line corresponds to a fitting of the results of pp collisions. This figure shows that the number of secondary pions per nucleon of the colliding nuclei is also independent of A. Bartke[43] has derived the following empirical formula for central collisions:

$$\langle N\rangle_{AB}=A\langle N^-\rangle_{nB}; \tag{28}$$

where N^- is the number of negatively charged particles. This empirical formula is confirmed by data obtained in the comparison of the multiplicities in p-Au and ^{16}O-Au collisions at 200 GeV nucleon.[44–46] In the latter collisions $\langle N\rangle_{\mathrm{OAu}}=124\pm0.2$. For p-Au collisions $\langle N\rangle_{\mathrm{pAu}}=7.42\pm0.21$. Thus, $16\langle N\rangle_{\mathrm{pAu}}=119\pm4$, which is in excellent agreement with the results of collisions of ^{16}O nuclei with Au. In Ref. 47 Satz obtained the value for the ratio $(dN/dy)_{AA}/(dN/dy)_{pp}=A^\alpha$, where $\alpha=1.1$. Going over to the distribution over $p_\perp$ (Ref. 43) it must be noted that in the collisions with nuclei this distribution is somewhat distorted relative to the case of pp collisions, but the distortion is small. For example, in central ^{16}O-Au collisions at 200 GeV/nucleon the value of $\langle p_\perp\rangle$ in the range $2<y<3$ changes by about 10–20%. Thus the empirical data indicate that the reality is close the model of collisions of independent nucleons.

This result has a simple physical interpretation. A nucleon after an interaction at a distance $\approx E/m^2$ (the formation length) loses its ability to undergo a secondary interactions, which leads to the so-called "transparency" of the nuclei. The first investigations for the determination of the formation length data to the 1950s.[48,49] Of course, the model of independent collisions is only the first approximation. Theoretical interpretations are possible that take into account cascade processes with the introduction of the formation length. The existence of collective interactions is also highly likely. This is indicated, for example, by the observation of a cumulative effect[50] and nuclear scaling,[51] where the secondary particles in $A+A$ collisions have momenta that exceed the kinematic limit for pp collisions. However the cross section for these processes is only a small fraction of the total cross section.

In summarizing this section we may say that the model of independent collisions is close to reality, but in such a complicated physical phenomenon as AA-collisions collective processes as well as cascade processes also exist, which, although relatively small, do have an influence on such basic characteristics as $\langle N\rangle(s)$ and $\langle p_\perp\rangle(s)$. Later, we shall consider phenomena that are comparatively rare but very important for the understanding of physics.

10. THE QUARK-GLUON PLASMA (MAIN CONCEPTS)

The term "quark-gluon plasma" was introduced at the end of the 1970s.[52] Essentially, this concept means the existence of an ensemble of weakly interacting quarks and gluons at a temperature T in a spatial region comparable to the dimensions of a hadron. The confinement, that is, the nonescape of free quarks and gluons, in the popular bag model[53] is due to the pressure B of the QCD vacuum on the "surface" of the hadrons. In fact, this is the confinement of color. It is proposed that the quark-gluon plasma can be formed as the result of the collision of relativistic hadrons, for which even before the collision the gluon component has about half of the energy of the hadrons. In the expansion and cooling of the quark-gluon plasma the interaction between the quarks and the gluons increases (as the distance between them increases) and the quark pairs and the gluons are converted into hadrons. This process is interpreted as a phase transition from a quark-gluon plasma to hadrons.

An estimate of the temperature T_c of the phase transition can be made most simply in the bag model if the interaction between the quarks and gluons is ignored and it is assumed that only pions are formed (a detailed discus-

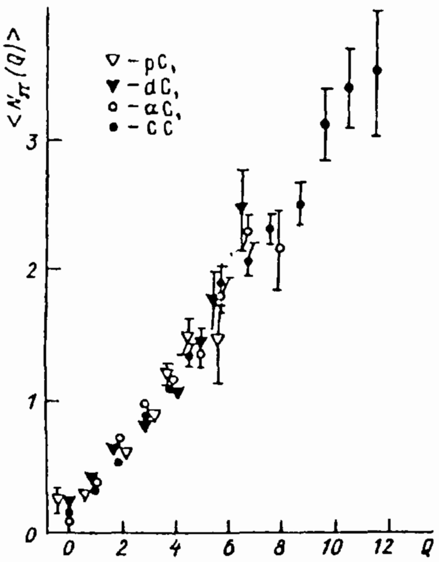

FIG. 4. Average multiplicity of secondary π^+ mesons as a function of the number of protons participating in the interaction.[41]

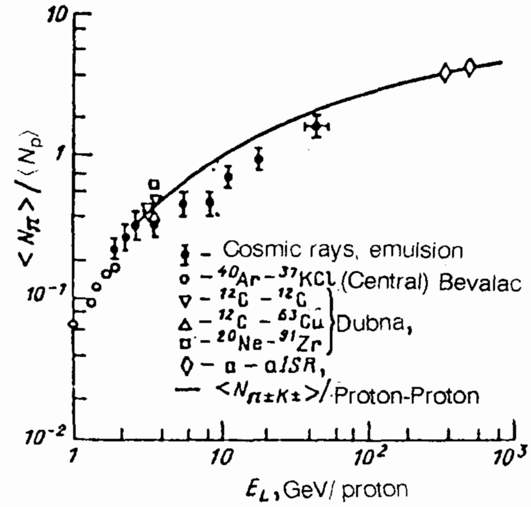

FIG. 5. Average multiplicity $\langle N_{\pi}\rangle$ for collisions of nuclei having various atomic numbers.[42]

sion of phase transitions in the quark-gluon material has been given in Ref. 54). Equating the pressure of the pion gas

$$P_{\mathrm{h}}=a_{\mathrm{h}}\frac{\pi^2}{90}T^4$$

to the pressure of the quark-gluon gas

$$P_{\mathrm{qg}}=a_{\mathrm{qg}}\frac{\pi^2}{90}T^4-B,$$

we can determine the critical temperature T_{c}:

$$T_{\mathrm{c}}=\frac{90B}{(a_{\mathrm{qg}}-a_{\mathrm{h}})\pi^2}. \qquad (29)$$

For pions,

$$a_{\mathrm{h}}=3, \quad a_{\mathrm{qg}}=16+\tfrac{21}{2}N_{\mathrm{f}},$$

where N_{f} is the number of flavors of the quarks. Setting $B\sim250$ MeV and $N_{\mathrm{f}}=3$, we obtain $T_{\mathrm{c}}\approx160$ MeV. However, the indeterminacy of the parameters is such that it is usually assumed that

$$T_{\mathrm{c}}\sim150-200 \text{ MeV}$$

(see, e.g., Refs. 47 and 55). At sufficiently high energies the initial temperature T_0 can exceed T_{c}; that is, the formation of a quark-gluon plasma is possible. We note that as the number a_h of degrees of freedom in the hadron phase increases (for example, if resonances are taken into account) the temperature T_{c} increases.

In the expansion and cooling of the quark-gluon system the phase transition into hadrons occurs and then the hadron phase expands and cools to free expansion at the temperature T_{f}.

We note that if only pions are taken into account, the great difference in the numbers of degrees of freedom a_{qg} and a_{h} results in a very strong first-order phase transition with a large jump in the entropy $\Delta s=\Delta\varepsilon/T$ and energy $\Delta\varepsilon=4B$. This transition follows from Monte-Carlo lattice calculations for SU(3) gluon dynamics (see, e.g., Ref. 97 and the references cited therein). There exists a relatively long-lived mixed phase, where the hadrons, quarks, and gluons coexist at $T=T_{\mathrm{c}}$. This means that at $T=T_{\mathrm{c}}$ thermodynamic and chemical equilibrium are attained between the hadrons and the quarks. For this to happen it is necessary for energy to be evolved that exactly compensates its diminution due to the expansion. Although in principle this situation is possible, it appears probable that the state of the mixed phase will be metastable, or possibly unstable.[56]

It should be mentioned that if other particles and resonances are taken into account besides pions, the mixed phase may be essentially absent, and the phase transition becomes close to second-order.[58]

It is interesting to note how close are the properties of the quark-gluon plasma to those of the "fluid" in the hydrodynamic theory of Landau. It turns out that Landau's assumption of an ideal fluid (without viscosity or heat conductivity) with the equation of state $p=\varepsilon/2$ gives the best agreement between the rapidity distributions of the secondary hadrons and modern data for the proton energies $s^{1/2}=63$ and 540 GeV (this was shown in Ref. 98).

It is also interesting to note that the smooth transition between the expanding "fluid" and the final hadrons as analyzed by Landau is close to a possible second-order phase transition of the quark-gluon plasma into hadrons as it expands. Thus the hydrodynamic theory in terms of its properties is close to the concept of a quark-gluon plasma.

In conclusion we present Fig. 6, showing a space-time diagram of colliding hadrons and nuclei at high energies.[52] This process can provisionally be divided into the following stages:

1. A pre-equilibrium period, during which a thermalized system is formed as a result of the interaction of the

partons of the projectile particles; this is the quark-gluon plasma (the fluid of Landau).

2. During the next stage the quark-gluon plasma expands and cools, and when it reaches a temperature of T_c (in a time t_c) the plasma begins to transform into hadrons.

3. If the hadronization is of the nature of a first-order phase transition (a thorough study of the various types of first-order phase transition has been reported in Ref. 99), then the system at the temperature T_c can exist for a time t_h in a mixed phase, which then transforms into an equilibrium pion gas. During this process there is a considerable increase in the volume of the gas and the energy and entropy densities undergo discontinuities due to the large difference in the number of degrees of freedom in the plasma and pion phases. During the period of the mixed phase (if it exists) the interaction of the gluons ensures conservation of the total entropy by the formation of the additional partons that are required for the filling of the large volume by pions.[57]

However, if the possibility of the formation of not only pions, but other stable particles and resonances is taken into account, there will not be a sharp difference between the number of degrees of freedom of the plasma phase and the hadron phase; that is, there will be no sharp jumps in the energy and entropy. This leads to the possibility of a phase transition that is close to second-order at a higher temperature T_c than for the first-order transition.[58] The mixed phase then is essentially absent or becomes very short-lived (this has also been noted recently by Chakrabarty *et al.*[100]

4. In the next stage the hadron gas is already expanding, and then after a time t_f the "freezout" occurs, with the free escape of hadrons. The freezout temperature T_f is close to m_π (Ref. 9), but may increase slowly with the initial energy, which causes a slow growth in $\langle p_\perp \rangle$ with energy pp collisions.[27]

11. SOME COMMENTS ON THE SCALING MODEL

After the appearance of the paper by Bjorken[38] the scaling model was widely used for the analysis of $A+A$ collisions and searches for signals of the quark-gluon plasma. However, as we have mentioned above (Section 8), this version of the theory leads to an equilibrium rapidity distribution of the particles ($dN/dy=$constant), which does not agree with the experimental data and can lead to underestimated values for the most important parameter for the identification of the quark-gluon plasma—the initial temperature T_0. Below, we shall make some estimates of this parameter.

In Refs. 38 and 59 the $A+A$ collisions were treated as a collection of individual independent nucleon collisions. As a result, the initial conditions were formulated for a single hydrodynamic system. These initial conditions are in fact given by the parameter τ_0 (the time for formation of the plasma), which is related to the non-hydrodynamic state of the process, depending on quantum effects. In Ref. 38 the values $\tau_0=1$ fm and the initial volume $V_0=\pi R_A^2\tau_0$ were used, where $R_A\approx 1.2A^{1/3}$ fm is the radius of the nucleus. The initial energy density is estimated from the formula

$$\varepsilon_0=\frac{1}{V_0}\, m_\perp \left.\frac{dN}{dy}\right|_{y=0}, \tag{30}$$

where $m_\perp=(m^2+p_\perp^2)^{1/2}\approx 400$ MeV.

However, it should be noted that this estimate does not take into account the longitudinal energy of the generated pions for $y>0$, which also is included in the initial volume. What does this estimate (30) give, for instance, for pp collisions at an energy $s^{1/2}=540$ GeV? Using the experimental estimate

$$\left.\frac{dN}{dy}\right|_{y=0}\approx 3.2, \quad m_\perp\approx 430 \text{ MeV}$$

we obtain for $\pi^{\pm,0}$ $dE_0/dy\approx 3.2\cdot 1.5\cdot 0.43\approx 2$ GeV, $\varepsilon_0=2$ GeV$/V_0\approx 2$ GeV$/\pi r^2\tau_0\approx 0.64$ GeV/fm^3(!) (for $r\approx\tau_0\approx 1$ fm).

In the region of the quasiplateau, $-3\leqslant y\leqslant 3$ for the equilibrium energy distribution we would obtain the value of the total of secondary particles $E_0\approx 2\cdot 6=12$ GeV, $\varepsilon_0\approx 3.8$ GeV/fm.

However, the value of E_0 in the range of rapidities $-y_0\leqslant y\leqslant 3$ must be calculated by the formula

$$E_0=\int_{-y_0}^{y_0} m_\perp \frac{dN}{dy}\,\mathrm{ch}\, y dy \tag{31}$$

(see, e.g., Ref. 60).

Using the distribution dN/dy taken from the experimental results of Ref. 61, we obtain for $y_0=3$ $E_0=42$ GeV and $\varepsilon_0\approx 13$ GeV/fm^3. In the interval of rapidity $-5\leqslant y\leqslant 5$, $E_0\approx 260$ GeV; that is, the secondary particles have almost half the initial energy, a conclusion that corresponds to a coefficient of inelasticity $K\approx 1/2$ (that is, $\varepsilon_0\approx 86$ GeV/fm^3). A similar estimate for the energy of the ISR ($s^{1/2}=53$ GeV) for $K\approx 1/2$ is $\varepsilon_0\approx s^{1/2}K/V_0\approx 8.4$ GeV/fm^3. However, the estimate by Bjorken[38] gives (taking $dN/dy|_{y=0}\approx 2$) $\varepsilon_0\approx 2\cdot 1.5\cdot 0.4/\pi r^2\tau_0\approx 0.38$ GeV/fm^3. In our opinion this estimate does not correspond with the initial energy density in the hydrodynamic model.

Thus at energies of the ISR the initial energy density ε_0 even for a longitudinal dimension $\tau_0\sim 1$ fm is sufficient for the formation of a quark-gluon plasma ($\varepsilon_0\gg 1$ GeV/fm^3). In the Lorentz-contracted initial volume this energy density will be even higher. We suppose that a similar situation occurs in $A+A$ collisions.

12. USE OF THE HYDRODYNAMIC MODEL TO INTERPRET THE COLLISIONS OF RELATIVISTIC NUCLEI

With the appearance of beams of relativistic heavy ions the hydrodynamic approach has begun to find wide use also for the study of collisions of heavy nuclei. The number of secondary particles increases manyfold, and includes kaons, photons, and lepton pairs, which improves the conditions for the diagnostics of the quark-gluon plasma. For the heavy nuclei the stopping of the initial nucleons is enhanced, the coefficient of inelasticity increases, and it would appear that the possibility exists of the formation of

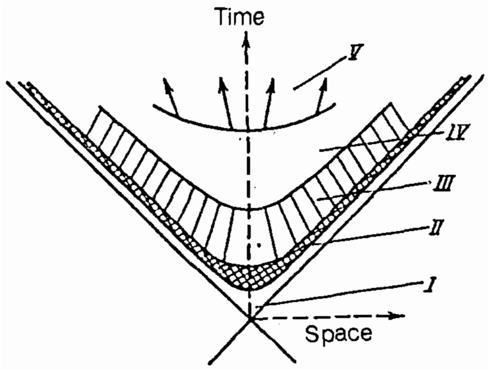

FIG. 6. Evolution of collisions of relativistic nuclei. *I*—Preliminary state; *II*—quark-gluon plasma; *III*—mixed phase; *IV*—hadron gas; *V*—free expansion.

a plasma enriched in baryons. What initial energy densities and temperatures might one expect for the collisions? Table I shows the parameters for planned and already existing accelerators of heavy ions for a lead target, and gives estimates of the initial energy density and temperature[47] according to Bjorken.[38] However, as we have noted, these values are probably underestimates.

Let us make some estimates for the planned accelerators RHIC and SPS. To estimate dN/dy in $A+A$ collisions we use an extrapolation of the data from $p+A$ collisions:

$$\left(\frac{dN}{dy}\right)_{AA} \Big/ \left(\frac{dN}{dy}\right)_{pp} = A^{\alpha},$$

where $\alpha \approx 1.1$ (Ref. 47). Using the empirical formula for pp collisions $dN/dy|_{y=0} \approx 0.8 \ln s^{1/2}$ (Ref. 62) we obtain for the RHIC accelerator ($s^{1/2}=200$ GeV)

$$\left.\frac{dN}{dy}\right|_{pp} \approx 4.24,$$

$$\left.\frac{dN}{dy}\right|_{AA} \approx 208^{1.1} \cdot 4.24 \approx 1500;$$

while the estimate according to Bjorken gives $E_0 \approx m_\perp \cdot 1500 \approx 750$ GeV and $\varepsilon_0 \approx 750/\pi(1.2)^2 A^{2/3}\tau_0$ ≈ 4.7 GeV/fm^3 (for $\tau_0 = 1$ fm). If we assume that in the initial volume V_0 an ideal quark-gluon plasma is formed consisting of gluons and three kinds of quarks, u, d, and s, and if we know the value of ε_0, then we can find the initial temperature from the formula

$$\varepsilon_0 = \frac{47.5\pi^2}{30} T_0^4.$$

From this we find $T_0 = 220$ MeV.

Let us repeat these estimates using formula (31). For the minimum coefficient of inelasticity, $K = 1/2$, an amount of energy $\sim s^{1/2} A K \approx 100$ GeV/A goes into secondary particles. If this energy were enclosed in the initial volume $\pi R_A^2 \tau_0$, then we find

$$\varepsilon_0 = K s^{1/2} A^{1/3} \cdot 0.22,$$

$$\varepsilon_0 \approx \frac{100 \text{ GeV}/A}{\pi(1.2)^2 A^{2/3} \tau_0} \approx 22 A^{1/3} \text{GeV/fm}^3 \approx 127 \text{ GeV/fm}^3.$$

This corresponds to an initial temperature $T_0 \approx 500$ MeV. In the initial Lorentz-contracted volume $V_0 \approx \pi R_A^2 \cdot R_A (2M_p/s^{1/2})$ the values of ε_0 and T_0 will be still larger: $\varepsilon_0 \approx 890$ GeV/fm^3 and $T_0 \approx 800$ MeV. We note that strictly speaking, the initial temperature T_0 should be calculated with allowance for the interaction of the quarks and gluons according to the formula of Ref. 58 (for three kinds of quarks)

$$\varepsilon_0 = T_0^4 (15.75 - 14.25\alpha_s + 6.8\alpha_s^2) + B,$$

where $\alpha_s = 2\pi/9 \ln(3T_0/\Lambda)$ is the coupling constant in QCD and $\Lambda \approx 150$ MeV. This results in a small increase in T_0 (< 10%).

As we shall see below, at a temperature $T_0 \geqslant 500$ MeV the yield of thermal lepton pairs of large mass M exceeds the yield of pairs in the Drell–Yan mechanism[63] (we recall that the Drell–Yan mechanism is the annihilation $q\bar{q} \to l\bar{l}$ (l is a lepton) in the region of the deep inelastic interaction. The characteristics of the target quark are determined by the structural functions of the target hadron). For the SPS accelerator ($s^{1/2} = 17$ GeV/A) the estimate according to Bjorken gives $\varepsilon_0 \approx 2.5$ GeV/fm^3 and $E_0 \approx 190$ MeV.

Our estimate for the Landau model with $K = 1/2$ gives $\varepsilon_0 \approx 27$ GeV/fm^3 and $T_0 \approx 330$ MeV.

For heavy nuclei and low energies the Lorentz-contracted initial dimensions in order of magnitude are close to $\tau_0 = 1$ fm.

Thus we suggest that the values of the initial energy density ε_0 and temperature T_0 listed in the Table I are underestimated.

An item of interest is an estimate of the loss of energy of the initial nucleons in $A+A$ collisions. Estimates were made in Ref. 64 for the loss of the initial energy in the central region for the reaction $S^{32}+S^{32}$ for an energy $s^{1/2} = 20$ GeV/A. The average number of negative particles per pair of interacting nucleons increases by $\sim 10\%$ from the value 3.2 in pp collisions to 3.5 in the reaction $S^{32}+S^{32}$. In the central region there are 54 nucleons, and each loses an energy $E \approx 5.81 \pm 3$ GeV. The average loss is 313 ± 38 GeV (58% of the initial energy of these nucleons; that is, the inelasticity coefficient is $K \approx 60\%$). It can be shown that in this case the nucleons lose on the average ~ 0.9 units of rapidity.

However, these losses increase for the heavy nuclei. From the studies of nucleon-nucleus collisions[65] it is known that a nucleon in passing through heavy nuclei lose on the average two units of rapidity (i.e., the nucleons lose more than 80% of their energy). However, for the Pb+Pb reaction the loss of rapidity may be even higher: ~ 3.5 (Ref. 66), and the regime free of baryons in the central region appears at $s^{1/2}/\Lambda \approx 1000$ GeV, i.e., higher than the energy of the RHIC. Figure 7 shows the rapidity distribution of baryons for peripheral and central collisions in the reaction $S^{32}+S^{32}$ for $s^{1/2} = 20$ GeV, and also data for the reaction p+Au extrapolated to Pb+Pb collisions.[64]

TABLE I.

Year of Startup	Name of Accelerator	Beam	$s^{1/2}$, GeV/A	$\left.\frac{dN}{dy}\right\|_{y=0}$	ε_0, GeV/fm^3	E_0, MeV
1986	AGS	Si^{28}	5	110	1,3	160
	SPS	S^{32}	20	220	2,4	190
1993	AGS	Au^{197}	4	390	1,2	160
	SPS	Pb^{208}	17	800	2,5	190
1998	RHIC	Au^{197}	200	1500	4,7	220
	LHC	Pb^{208}	6300	2500	7,8	250

AGS — Alternate Gradient Synchrotron, RHIC — Relativistic Heavy Ion Collider, LHC — Large Hadron Collider, SPS — Super Proton Synchrotron.

Thus at the energies presently available, the central region of rapidity in $A+A$ collisions is largely filled with baryons. However it is still not entirely clear whether these baryons form a statistical system along with the secondary hadrons.

For a rough estimate of the loss of initial energy in nuclear collisions one may use the formula for the inelasticity coefficient:[67] $K(AA) \approx K(\mathrm{pp}) A^{0.09 \pm 0.03}$. For collisions of heavy nuclei an increase in K can lead to an increase in the initial energy density ε_0 (for Pb+Pb collisions almost double that for pp collisions).

The distributions of rapidity and $p_\perp$ were studied in Ref. 68 for $A+A$ collisions for a beam of oxygen nuclei and various targets—Cu, Ag, and Au. The hydrodynamic model of Landau was used as well as the equation of state for an ideal gas and an energy of expansion of $T_f \approx 0.15$ GeV. The cross section $d\sigma/dE_\perp$ was calculated as a function of $E_\perp$ for an energy per nucleon of 10 GeV. The experimental data for $d\sigma/dE_\perp$ and for the correlation of the total energy and $E_\perp$, as well as for the distribution over N_{ch} were in good agreement with the model if it is assumed that there is almost complete stopping of the nucleons, $\sim 95\%$. At higher energies, ~ 50 and 200/GeV nucleon the experiment can be described for heavy targets if it is assumed that $\sim 85\%$ of the initial energy of the nucleons is expended in the collision.

We should briefly mention the interferometric measurements of the radius of "freezout." These measurements[69,70] indicate that the radius increases with energy for a fixed value of A, a result that evidently indicates that the freezout volume expands. This expansion, may perhaps be a consequence of hydrodynamic evolution.

An interesting effect observed in $A+A$ collisions is the increase in the yield of soft pions compared to the case of pp collisions.[71–72] The spectra for $p_\perp$ for the reactions O+Au and S+S, unlike the pp reaction, do not fit the simple thermal distribution. A number of investigations have been devoted to this effect. In the work of Kataja and Ruuskanen[73] the observed distribution was interpreted by introducing a positive chemical potential μ for the pions (that is, an increase over chemical equilibrium).

In another investigation, Sollfrank *et al.* studied two- and three-particle decay of resonances. However the results for the reactions S+S at 200 GeV/nucleon agreed with experiment at too high a temperature $T \sim 200$ MeV.

Bell *et al.*[74] explained the observed excess in soft pions at small $p_\perp$ for heavy ions at energies of 200 GeV/nucleon in the three-dimensional hydrodynamic approximation within the framework of the Landau theory. It was shown that in this interpretation the role of resonances is an important consideration. In the calculations the equation of state of the mixed phase was used, with a freezout temperature $T_f = 130$ MeV and baryon number $B = 35$.

However, it must be pointed out that an analogous (although smaller) excess was found also for pp collisions ($s^{1/2} = 63$ GeV) at large multiplicities $N_{ch} > \langle N_{ch} \rangle$ in the range of rapidity $|y| < 2$ (Ref. 75). However, in pp collisions the chemical potential is $\mu = 0$. Therefore the reason for the excess in not entirely clear. We note that here it might be necessary to take into account the finite width of the resonances.

13. DIAGNOSTICS OF THE QUARK-GLUON PLASMA

The issue of the diagnostics of the quark-gluon plasma in collisions of nuclei has been addressed by many investigations. The experiments indicate that there is an increase in the relative yield of strange particles in $A+A$ collisions compared with pp and pA collisions.

This effect has been regarded[76] as a signal of the formation of the quark-gluon plasma. A specific feature of the collisions of heavy nuclei, unlike in the case of the collision of nucleons, is the suppression of light quarks because of the nonzero baryon chemical potential $\mu_B \neq 0$. This results in a relative increase in the yield of strange s- and $\bar{s}$-quarks.

It was found, however, that the increase in the yield of strangeness (the ratio K^+/π^+) is also observed for an equilibrium hadron gas.[77] For the interpretation of this effect those investigators took into account the contributions of all the observed non-strange and strange meson and baryon resonances to mass ~ 2 GeV and invoked the repulsion of baryons at small distances.

The excess in strangeness in nuclear collisions is here related to the nonzero baryon number $n_B \neq 0$ and to the equilibrium of baryons in the hadron gas (a more thorough discussion of the role of strangeness in the diagnostics of the quark-gluon plasma can be found in Refs. 78 and 79).

The most direct information on the quark-gluon plasma can be provided by dileptons and the direct photons created within it. Unlike the secondary hadrons, which are mainly created in the freezout process, the lepton pairs can be created in the hottest and densest zone and can carry information about this zone. The creation of leptons and electromagnetic radiation was studied for the first time in the work of Feinberg[16,17] within the framework of the hydrodynamic theory. Later, with the appearance of quantum chromodynamics it became possible to calculate these phenomena on the basis of the kinetics in the quark-gluon plasma (for more detail, see Refs. 55 and 79).

An important assumption is that of local equilibrium in the quark-gluon plasma. Thermal dileptons of high mass M can serve as a "thermometer" for the initial temperature.

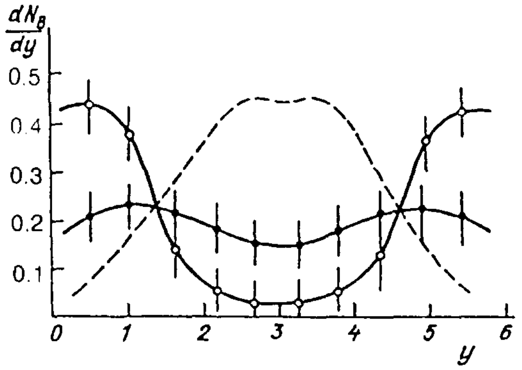

FIG. 7. Rapidity distribution of baryons for peripheral (○) and central (●) S+S collisions with $s^{1/2}=20$ GeV. The dashed line shows the extrapolation of data of p—Au collisions to Pb+Pb collisions.

In order to illustrate the order of magnitude of the yield of dileptons and the degree of indeterminacy in the calculations, we present a formula taken from Ref. 55:

$$\frac{d^3N}{dM^2dydp_T^2}=1.6\cdot 10^{-7}\frac{\tau_0}{\tau_p}A^{\delta+\lambda+(2/3)}\left(\frac{1\ \text{GeV}}{M_t}\right)^6\text{GeV}^{-4}. \tag{32}$$

The formation length τ_p is included in the interval $1/3<\tau_p<\tau_0\approx 1$ fm, $1/2<\lambda<1$. $0<\delta<1/3$, and M and M_t are the mass and transverse mass of the lepton. For dilepton masses below 1.5 GeV the spectrum is determined by the tail of the vector mesons, and for very high mass $M>5$ GeV the spectrum is determined by the pre-equilibrium creation of Drell–Yan lepton pairs[63] in hard collisions of quarks. A diagram of the dilepton mass spectrum is shown in Fig. 8.

The cross section for the creation of Drell–Yan pairs is determined by the hadron structure function and the parton interaction cross section. The structure functions are measured in deep inelastic processes (the scattering of leptons by hadrons). The elementary cross sections are calculated by perturbation theory taking into account the higher approximations.[80]

The calculated yield of Drell–Yan pairs in pp collisions reproduce very well the continuous part of the observed dilepton spectrum in the region above the mass of the J/ψ-particles. However, below the J/ψ mass the results of perturbation theory are less reliable (the structure functions are not well known for small x, where x is the fraction of the energy of a quark). For high initial temperatures $T_0>500$ MeV the behavior of the dilepton spectra is similar to the behavior in the Drell–Yan process; it goes as $\propto M^4$. At lower temperatures the mass spectrum is exponential and intersects the Drell–Yan curve (Fig. 9). The dependence of the yield of Drell–Yan pairs on the atomic weight A of the nucleus has the form $\propto A^{4/3}$. If the multiplicity is proportional to A, then the thermal dileptons also have the dependence $\propto A^{4/3}$ (Ref. 81). If the multiplicity increases faster than proportional to A, then the yield of thermal pairs will exceed the yield of Drell–Yan pairs. With an increase in the energy $s^{1/2}$ the fraction x of momentum of the annihilating q and $\bar{q}$ quarks becomes small: $x=M/s^{1/2}\to 0$ and corrections $\propto \ln x$ are possible (for Drell–Yan pairs).

The yield of thermal pairs depends strongly on the temperature. Figure 9 shows a comparison of the quantity[81] $d^2N^{\mu^+\mu^-}/dy$ for Drell–Yan pairs and thermal dileptons for initial temperatures $T_0=0.2$, 0.3, 0.4, and 0.5 GeV. For $T_0\geqslant 350$ MeV thermal pairs appear with high masses, and for $T_0<250$ MeV the resonance tails intersect the Drell–Yan curve. Therefore the initial temperature T_0 plays an important role in the observation of thermal dileptons of high mass.

Similar conclusions were also drawn previously by Kajantie *et al.* In this paper the investigators used the one-dimensional hydrodynamic scaling model and calculated the yield of leptons from the quark-gluon plasma, the pion gas, and the mixed phase. The results, as these investigators pointed out, depend only weakly on the nature of the phase transition. The calculation in the hydrodynamic model of Landau shows that in pp collisions at the energies of the ISR ($s^{1/2}=53$ GeV) in the initial state a temperature $T_0\geqslant 500$ MeV is reached, which is sufficient to study the creation of thermal pairs of high mass. Therefore it is of interest to study the creation of lepton pairs in the framework of the hydrodynamic model of Landau for $A+A$ collisions for various scenarios of the phase transition.

The yield of lepton pairs from the hadron material has also been studied.[82] The principal reaction channel is determined in this case by the processes $\pi^++\pi^-\to\rho$, $\rho'\to e^+e^-+X$. The pion form factor $F_\pi(M^2)$ in the Breit–Wigner form $F_\pi(M^2)=m_\rho^2/(m_\rho^2-M^2-im_\rho\Gamma$ was used in these calculations.

It should be mentioned that for small mass $M\leqslant 2m_\pi$ the principal channel for the yield of dileptons is the emission of virtual photons, created in the $\pi\pi$ interaction.

Allowance for single pions is inadequate for a quantitative estimate of the yield of lepton pairs (there are φ and other particles). Estimates[82] show that in the mass region $0.4<M<2$ GeV, hadron annihilation dominates quark annihilation.

Let us note another characteristic of the spectra—in the region of the J/ψ mass the thermal lepton pairs will have a considerably broader distribution over $p_\perp$ than Drell-Yan pairs or J/ψ particles (Ref. 81). This may be due to the elastic scattering of partons within the nuclei.

The direct photons in the diagnostics of the plasma in $A+A$ collisions play a role similar to that of the dileptons,[83] but experimentally they are hard to distinguish from the π^0 and η background.[84]

Another phenomenon that may be an indication of the formation of the quark-gluon plasma in $A+A$ collisions is the suppression of the yield of J/ψ particles[85] below that for pp collisions. This suppression has been observed experimentally.[86] The reason for the suppression may be that $c\bar{c}$ pairs, which form J/ψ or ψ', are screened in the plasma, and this screening prevents the coupling of quarks. In Fig. 10 we see that the ratio of the number of J/ψ particles to the continuum of Drell–Yan dileptons falls off by a factor of two in going from lower to higher values of

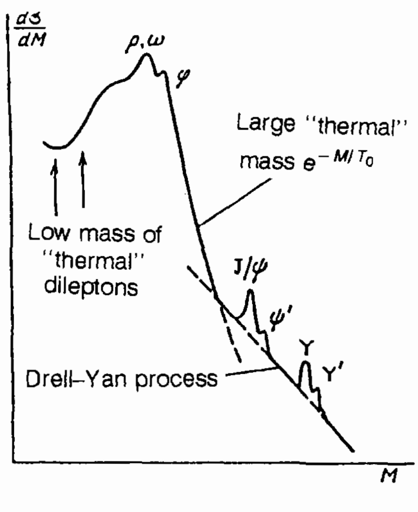

FIG. 8. Behavior of the "ideal" mass spectrum of dileptons.

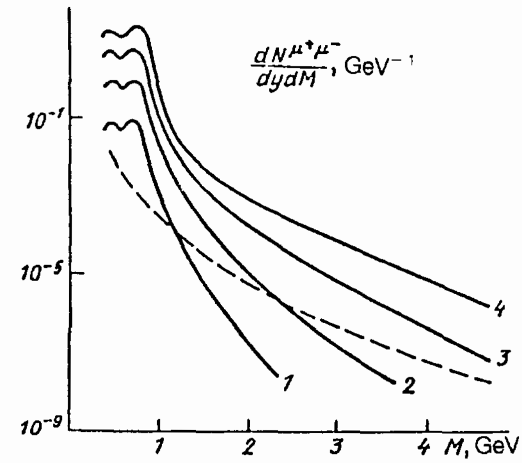

FIG. 9. Comparison of Drell–Yan dileptons (dashed line) and thermal dileptons for a system with a first-order phase transition between a quark-gluon plasma and a hadron gas with the initial temperature *1*) 0.2 GeV; *2*) 0.3 GeV; *3*) 0.4 GeV; *4*) 0.5 GeV.

E_T. It was also noted that with an increase in $p_\perp$ of the J/ψ particles (for O+U collisions at the energy of the SPS) there is no suppression. A possible reason may be that the fast $c\bar{c}$ pairs escape from the plasma without experiencing the screening.[87]

However, it should be noted that the suppression is also possible in the hadron medium by the dissociation $J/\psi+X\rightarrow D+\bar{D}$, where X is one of the constituents of the medium[88] (for example, nucleons in the nuclear material). The softening of the parton spectra in the nuclei may also lead to the suppression. If this softening is larger for gluons than for quarks, then suppression of the J/ψ is possible.[89]

It remains unclear, however, whether these effects can explain the observed suppression. Estimates[90,91] show that an extrapolation of the data for the $J/\psi+X\rightarrow D+\bar{D}$ reaction cross sections in pA collisions to the region of the nuclear collisions does not appear to explain the observed suppression. If it is assumed that in the nuclear collisions material is formed that is denser than nuclear material, then the observed suppression can be explained by using sufficiently large cross sections for the collisions with hadrons in the reaction $J/\psi+X\rightarrow D+\bar{D}$.[92] In order to distinguish the global screening from local dissociation in the suppression of the J/ψ particles a complete spectral analysis is needed, that is, a comparison of the suppression for various states of the charmonium and bottomonium,[93] with allowance for the possible "contamination" of the J/ψ particles from B-meson decays.[94] The most characteristic feature is the presence of a energy density threshold for the screening as compared to the continuous nature of dissociation. Therefore it would be of interest to study the suppression of the J/ψ for various colliding particles as a function of the energy density ε_0.

It should also be pointed out that since 40% of the J/ψ particles are formed in χ decays ($m_\chi \approx 3500$ MeV) the question arises as to whether the suppression of the J/ψ particles is a consequence of the suppression of the χ.

Van Hove[95] has studied the correlation between $\langle p_\perp \rangle$ and the multiplicity in the central region of rapidity as a signal of the plasma-hadron first-order phase transition. The flattening out of the curve of $\langle p_\perp \rangle (N_{ch}/\Delta y)$ for increased N_{ch} is regarded as a possible signal of the formation of the mixed phase, during which the temperature T_c remains practically constant and $p_\perp \propto T_c$. However, it has been shown[96] that this behavior in the pp collisions is determined by the flattening out of the quantity $\langle \bar{p}_\perp \rangle$ for increased inelasticity coefficient (and hence multiplicity N_{ch}) above the average value $K\approx 1/2$ and is not related to the phase transition.

From a review of even a limited number of investigations we have arrived at the conclusion that the observation of the quark-gluon plasma is a complex matter. Theoretical recommendations, as we have seen, are not always unambiguous. The most direct information on the formation of the quark-gluon plasma may be provided by a study of the yield of lepton pairs and hard photons if it is possible to separate reliably the background from the hadron phase and the Drell–Yan leptons.

14. CONCLUSIONS

The hydrodynamic theory of multiple processes proposed by Landau provides a good description of almost all the present-day experimental data on the inclusive reactions in pp(p$\bar{\text{p}}$) collisions. Here it is necessary to stipulate: the issue concerns soft processes. For a description of hard processes it is necessary to use QCD.

Although the theory in the form proposed by Landau does not contain any internal contradictions, nonetheless, the scale-invariant initial conditions appear to be esthetically more attractive. Unfortunately, however, no one has yet carried out a complete comparison of this model with *all* the mass of existing experimental data. This comparison is necessary in order to differentiate experimentally the two versions of the model. It would appear that the most promising tactic in this direction is to study high-energy secondary photons, moving a large angles.

The quark-gluon plasma is very close in spirit and letter to the hydrodynamic concept.

We can say with confidence that the high-energy, heavy-ion accelerators to be started up in the near future will play an important role in the verification of this concept.

For further investigations we suggest that the following factors will be important in the future:

1. A more rigorous choice of theoretical model for its heuristic power.
2. Further theoretical investigations of the problem of confinement and the quark-gluon plasma.
3. The experimental investigation of photons and high-energy lepton pairs at large angles in pp, pA, and AA collisions.

As yet it is probable that the most persuasive indication of the existence of a new state—the quark-gluon plasma—is the good agreement of the hydrodynamic theory with the experimental data. It is because of this good agreement that we consider it important to make a combined and comprehensive study of multiple processes on the basis of hydrodynamics and the relatively infrequent appearance of leptons, photons, and other "exotic" particles on the basis of the kinetics.

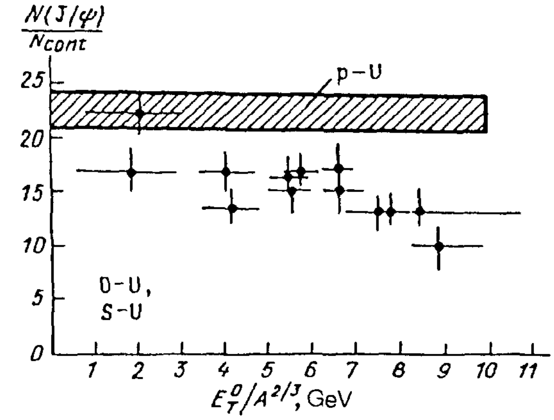

FIG. 10. Ratio of the yield of J/ψ particles to the continuum in A+A and pU interactions for the SPS accelerator as a function of the transverse energy E_T normalized to the area of the incident nucleus $A^{2/3}$ (in GeV/nucleon).

[1] D. V. Scobeltzyn, Z. Phys. **54**, 686 (1929).
[2] B. Rossi, Phys. Z. **33**, 304 (1932).
[3] P. M. S. Blackett and G. P. S. Occhialini, Proc. R. Soc. London Ser. A **139**, 699 (1933).
[4] W. Heisenberg, Z. Phys. **101**, 533 (1936).
[5] W. Heisenberg, Z. Phys. **126**, 569 (1949).
[6] W. Heisenberg, Z. Phys. **133**, 65 (1952).
[7] E. Fermi, Prog. Theor. Phys. A **5**, 570 (1950).
[8] I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **78**, 889 (1951).
[9] L. D. Landau, Izv. Akad. Nauk SSSR Ser. Fiz. **17**, (1953).
[10] P. Carruthers and M. D. Van, Phys. Lett. B **41**, 597 (1972).
[11] F. Cooper and E. Shonberg, Phys. Rev. Lett. **30**, 880 (1973).
[12] G. A. Milekhin and I. L. Rozental', Zh. Eksp. Teor. Fiz. **33**, 197 (1957) [Sov. Phys. JETP **6**, 154 (1957)].
[13] E. L. Feinberg, Usp. Fiz. Nauk **104**, 539 (1971) [Sov. Phys. Usp. **14**, 455 (1971)].
[14] I. L. Rozantal', Zh. Eksp. Teor. Fiz. **31**, 278 (1956) [Sov. Phys. JETP **4**, 217 (1956)].
[15] G. A. Milekhin, Zh. Eksp. Teor. Fiz. **35**, 1185 (1958) [Sov. Phys. JETP **8**, 829 (1958)].
[16] E. L. Feinberg, Izv. Akad. Nauk SSSR Ser. Fiz. **26**, 622 (1962).
[17] E. L. Feinberg, *Nuovo Cimento*, A **34**, 391 (1976).
[18] D. I. Blokhiptsev, Zh. Eksp. Teor. Fiz. **32**, 350 (1957) [Sov. Phys. JETP **5**, 286 (1957)].
[19] A. A. Tyapkin, Fiz. Elem. Chastits At. Yadra **8**, 544 (1977) [Sov. J. Part. Nucl. **8**, 222 (1977)].
[20] E. M. Levin and M. G. Ryskin, Usp. Fiz. Nauk **158**, 178 (1989) [Sov. Phys. Usp. **32**, 479 (1989)].
[21] M. J. Moravcsik and M. Teper, Phys. Rev. D **16**, 1693 (1977).
[22] E. L. Feinberg, Proc. First International Workshop "Local Equilibrium in Strong Interaction Physics," D. Scott and R. Weiner (eds.), World Scientific, Singapore p. 15 (1985).
[23] Yu. P. Nikitin and I. L. Rozental', *Theory of Multiple Processes* [in Russian], Nauka, M. p. 29 (1976).
[24] D. Scott and R. Weiner (eds.), Proc. First International Workshop "Local Equilibrium in Strong Interaction Physics," World Scientific, Singapore pp. 415–446 (1985).
[25] L. Van Hove and S. Pokorski, Nucl. Phys. B **86**, 243 (1975).
[26] G. A. Milekhin, Zh. Eksp. Teor. Fiz. **35**, 1185 (1958) [Sov. Phys. JETP **8**, 829 (1958)].
[27] I. L. Rozental' and Yu. A. Tarasov, Zh. Eksp. Teor. Fiz. **85**, 1535 (1983) [Sov. Phys. JETP **58**, 892 (1983)].
[28] Yu. A. Tarasov, Yadernaya Fiz. **42**, 411 (1985) [Sov. J. Nucl. Phys. **42**, 260 (1985)].
[29] I. M. Khalotnikov, Zh. Eksp. Teor. Fiz. **27**, 529 (1954).
[30] F. Cooper and G. Frye, Phys. Rev. D **10**, 186 (1974).
[31] V. S. Murzin and L. I. Sarycheva, *Interactions of High-Energy Hadrons* [in Russian], Nauka, M. Ch. 6 (1983).
[32] A. M. Baldin, V. I. Gol'danskii, V. M. Maksimenko, and I. L. Rozental' *Kinematics of Nuclear Reactions* [in Russian], Atomizdat, M. Ch. 3 (1968).
[33] Yu. A. Tarasov, Yadernaya Fiz. **45**, 1446 (1987) [Sov. J. Nucl. Phys. **45**, 898 (1978)].
[34] I. L. Rozental' and Yu. A. Tarasov, Pis'ma Zh. Eksp. Teor. Fiz. **35**, 349 (1982) [JETP Lett. **35**, 430 (1982)].
[35] C. B. Chiu, E. C. Sudarshan, and Kuo-Hsiang Wang, Phys. Rev. D **12**, 902 (1975).
[36] R. C. Hwa, Phys. Rev. D **10**, 2260 (1974).
[37] F. Cooper, G. Frye, and E. Schonberg, Phys. Rev. D **11**, 192 (1975).
[38] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).
[39] M. I. Gorenshtein, V. I. Zhdanov, and Yu. M. Sinyukov, Zh. Eksp. Teor. Fiz. **74**, 833 (1978) [Sov. Phys. JETP **47**, 535 (1978)].
[40] M. I. Gorenshtein, G. M. Zinov'ev, and Yu. M. Sinyukov, Pis'ma Zh. Eksp. Teor. Fiz. **28**, 371 (1978) [JETP Lett. **28**, 692 (1978)].
[41] L. Simic *et al.*, Phys. Rev. D **34**, 392 (1986).
[42] T. W. Atwater and P. S. Freier, Phys. Rev. Lett. **56**, 1350 (1986).
[43] J. Bartke, Intern. J. Mod. Phys. **4**, 1319 (1989).
[44] A. Bamberger *et al.*, Phys. Lett. B **184**, 271 (1987).
[45] A. Bamberger *et al.*, Phys. Lett. B **205**, 583 (1988).
[46] C. DeMarzo *et al.*, Phys. Rev. D **26**, 1019 (1982).
[47] H. Satz, Preprint CERN-TH 6216/91. October 1991.
[48] I. Ya. Pomeranchuk and E. L. Feinberg, Dokl. Akad. Nauk SSSR **95**, 439 (1953).
[49] E. L. Feinberg, Zh. Eksp. Teor. Fiz. **28**, 242 (1955) [Sov. Phys. JETP **1**, 176 (1955)].
[50] A. M. Baldin *et al.*, Yadernaya Fiz. **18**, 79 (1973) [Sov. J. Nucl. Phys. **18**, 41 (1973)].
[51] G. A. Leksin, "Nuclear Scaling. Elementary Particles," Fourth School of the Institute of Theoretical and Experimental Physics [in Russian], Atomizdat, M. (1977), No. 2, Ch. 6.
[52] E. V. Shuryak, Yadernaya Fiz. **28**, 796 (1978) [Sov. J. Nucl. Phys. **28**, 408 (1978)].
[53] A. Chodos *et al.*, Phys. Rev. D **9**, 3471 (1974).
[54] O. D. Chernavskaya and D. S. Chernavakii, Usp. Fiz. Nauk **154**, 497 (1988) [Sov. Phys. Usp. **31**, 263 (1988)].
[55] S. Raha and B. Sinha, Intern. J. Mod. Phys. **6**, 517 (1991).
[56] M. I. Gorenshtein and V. I. Zhdanov, Yadernaya Fiz. **45**, 1610 (1987) [Sov. J. Nucl. Phys. **45**, 999 (1987)].
[57] P. Koch. B. Muller and J. Rafelski, Phys. Rep. **142**, No. 4 (1986).
[58] Yu. A. Tarasov, Yadernaya Fiz. **48**, 820 (1988) [Sov. J. Nucl. Phys. **48**, 523 (1988)].

[59] K. Kajantie and L. McLerran, Phys. Lett. B **119**, 203 (1982).
[60] Yu. P. Nikitin and I. L. Rozental', *High-Energy Nuclear Physics* [in Russian], Atomizdat, M. (1980).
[61] G. Arnison *et al.*, Phys. Lett. B **107**, 320 (1981).
[62] F. Abe *et al.*, Phys. Rev. D **11**, 1719 (1990).
[63] T. M. Yan and S. D. Drell, Phys. Rev. D **1**, 2402 (1970).
[64] H. Ströbele *et al.*, Nucl. Phys. A **525**, 59 (1991).
[65] W. Busza and R. Ledoux, Ann. Rev. Nucl. Part. Sci. **38**, 119 (1988).
[66] A. von Keitz *et al.*, Phys. Lett. B **263**, 353 (1991).
[67] Yu. P. Nikitin, I. L. Rozental', and F. M. Sergeev, Usp. Fiz. Nauk **121**, 3 (1977) [Sov. Phys. Usp. **20**, 1 (1977)].
[68] J. Stachel and P. Braun-Munziger, Phys. Lett. B **216**, 1 (1989).
[69] J. Burtke, Phys. Lett. B **174**, 32 (1986).
[70] R. Stock, Preprint IKF 90-3, Frankfurt (1990).
[71] J. Sollfrank. P. Koch, and U. Heinz, Phys. Lett. B **252**, 256 (1990).
[72] S. Gavin and P. V. Ruuskanen, Phys. Lett. B **262**, 326 (1991).
[73] M. Kataja and P. Ruuskanen, Phys. Lett. **243**, 181 (1990).
[74] U. Ornik, and R. H. Weiner, Phys. Lett. B **263**, 503 (1991).
[75] W. Bell *et al.*, Z. Phys. C **27**, 191 (1985).
[76] J. Rafelski and B. Muller, Phys. Rev. Lett. **48**, 1066 (1982).
[77] J. Cleymans *et al.*, Phys. Lett. B **242**, 111 (1990).
[78] H. C. Eggers and J. Rafelski, Preprint G. S. 1.-90-37. July 1990.
[79] K. Kajantie, M. Kataja, L. McLerran, and P. V. Ruuskanen, Phys. Rev. D **34**, 811 (1986); L. McLerran, Proc. 18th Symposium "Multiparticle Dynamics 1987," I. Dremin and K. Gulamov (eds.), World Scientific, Singapore, p. 439 (1987).
[80] R. Baier, B. Pire, and D. Schiff, Phys. Rev. D **38**, 2814 (1988).
[81] P. V. Ruuskanen, Nucl. Phys. A **525**, 255 (1991).
[82] K. Kajantie *et al.*, Phys. Rev. D **34**, 2746 (1986).
[83] M. Neubert, Z. Phys. C **42**, 231 (1989).
[84] R. Albrecht *et al.*, Z. Phys. C **51**, 1 (1991).
[85] T. Matsui and H. Satz, Phys. Lett. B **178**, 416 (1986).
[86] C. Baglin *et al.*, (NA38) Phys. Lett. B **220**, 471 (1989); **251**, 465, 472 (1990).
[87] F. Karsch and R. Petronzio, Phys. Lett. B **193**, 105 (1987).
[88] D. M. Alde *et al.*, (E772) Phys. Rev. Lett. **66**, 133 (1991).
[89] R. Gavai, S. Gupta, and K. Sridhar, Phys. Lett. B **227**, 161 (1989).
[90] C. Gershel and J. Hufner, Phys. Lett. B **207**, 253 (1988).
[91] E. A. Capella *et al.*, Phys. Lett. B **206**, 354 (1988).
[92] S. Gavin, M. Gyulassy, and A. Jackson, Phys. Lett. B **207**, 257 (1988).
[93] F. Karsch and H. Satz, Z. Phys. C **51**, 209 (1991).
[94] L. Bergström, R. W. Robinett, and L. Weinkauf, Phys. Rev. D **42**, 825 (1990).
[95] L. Van Hove, Phys. Lett. B **118**, 138 (1982).
[96] Yu. A. Tarasov, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 562 (1986) [JETP Lett. **43**, 726 (1986)].
[97] J. Cleymans, R. Gavai, and E. Suhonen, Phys. Rep. **130**, 217 (1986).
[98] K. Wehrberger and R. U. Weiner, Phys. Rev. D **31**, 222 (1985).
[99] S. P. Baranov and L. V. Fil'kov, Yadernaya Fiz. **47**, 1423 (1988); **48**, 985 (1988) [Sov. J. Nucl. Phys. **47**, 905 (1988); **48**, 627 (1988)].
[100] Chadrabarty *et al.*, Phys. Rev. D **46**, 3802 (1992).

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