Scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences (24 February 1993)

Usp. Fiz. Nauk 163, 107–111 (June 1993)

A scientific session of the Division of General Physics and Astronomy of the Russian Academy of Sciences was held on February 24, 1993 at the P. L. Kapitsa Institute of Physics Problems. The following reports were presented at the session:

1. V. B. Braginskiĭ and F. Ya. Khalili. Interaction of electromagnetic and mechanical oscillations at the quantum-ground-state level.

2. A. S. Gadun and V. N. Karpinskiĭ. Problems of the structuration of the sun and stars.

A brief summary of the reports is given below.

V. B. Braginskiĭ and F. Ya. Khalili. *Interaction of electromagnetic and mechanical oscillations at the quantum-ground-state level.* It is well known that zero-point vibrations of the electromagnetic field are responsible for Casimir's effect[1]—the fact that a constant interaction force acts between close bodies. This is not the only demonstration of the existence of a fundamental relation between the electromagnetic vacuum and the mechanical world (see, for example, Refs. 2 and 3). In particular, it was shown in Ref. 3 that the electromagnetic pressure of zero-point vibrations on ideally conducting half-space should have a fluctuation component.

In this work a number of other effects, caused by the interaction of zero-point vibrations of the electromagnetic field with mechanical boundary conditions, are studied.

1. *Friction in free space.*[4] It can be shown with the help of the results obtained in Ref. 3 that the spectral density of fluctuations of the pressure force of zero-point electromagnetic vibrations acting on an ideally conducting plate is, if the area S is such that $S^{1/2} \gg c/\omega$,

$$F_\omega^2 = \frac{\hbar^2 \omega^5 S}{60\pi^2 c^6}, \tag{1}$$

where $\hbar$ is Plank's constant, ω is observation frequency, and c is the speed of light. These fluctuations pump energy from the electromagnetic into the mechanical degrees of freedom. In the equilibrium state energy should also flow in the opposite direction, i.e., mechanical friction should occur. The physical mechanism of this effect consists of parametric transfer of energy from mechanical to electromagnetic degrees of freedom. In accordance with the fluctuation-dissipation theorem the coefficient of friction for oscillations of the plate perpendicular to its surface is

$$H(\omega) = \frac{\hbar \omega^4 S}{120\pi^4 c^4}. \tag{2}$$

It is significant that $H(\omega)$ is proportional to ω^4 and, correspondingly, the friction force is proportional to the fifth derivative of the coordinate with respect to time. Thus there are no contradictions with the principle of relativity: Under the conditions of uniform motion there is no friction force.

This friction is very weak. For example, the Q of the lowest associated mode of transverse mechanical oscillations of the plate is

$$Q_m = \frac{m\omega_m}{2H(\omega_m)} = \frac{60\pi\rho c^4 a^4}{\hbar v^3}, \tag{3}$$

where v is the speed of sound, ρ is the density and a is the thickness of the plate, and ω_m is the frequency of the oscillations. For $a = 10^{-4}$ cm, $\rho = 4$ g/cm^3, and $v = 10^6$ cm/sec, we have $Q_m \approx 5 \cdot 10^7$. This estimate shows that under ordinary conditions this effect is virtually unobservable with the existing experimental techniques.

2. *Friction and fluctuations under resonance conditions.*[4] This friction and the associated fluctuations can be significantly intensified if the corresponding resonance conditions are satisfied. Consider the quantum ground state in a resonator, formed by lumped inductance L and capacitance C and weakly coupled with a thermostat, so that its $Q_e \gg 1$. The expression for the spectral density of the fluctuation component of the attractive force acting between the capacitor plates in this case has the form

$$F_\omega^2 = \frac{1}{\pi}\left(\frac{\hbar\omega_e^3}{Q_e d}\right)^2 \int_0^\infty \frac{\omega'(\omega-\omega')\,d\omega'}{|z(\omega)|^2 |z(\omega')|^2}, \tag{4}$$

where

$$z(\omega) = \omega_e^2 - \omega^2 - \frac{i\omega\omega_e}{Q_e}$$

(we note that this force is additional to Casimir's force and appears when the inductance is connected). Near the frequency $2\omega_e$ the spectral density is maximum and equal to

$$F_\omega^2 \approx \frac{\hbar^2 \omega_e Q_e}{4d^2}. \tag{5}$$

The correlation time of these fluctuations is Q_e/ω_e. It is obvious that for a resonator of the klystron type (with a well-localized capacitance) with frequency, for example, $\omega_e \approx 10^{10}$ sec^{-1}, $Q_e \approx 10^{10}$, and $d \approx 10^{-4}$ cm, this force is

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many orders of magnitude stronger than for a plate in free space [see Eq. (1)].

The appearance of these fluctuations can be explained qualitatively as follows. In the system "resonator +thermostat" the energy of each mode is exactly $\hbar\omega/2$, some of the energy residing in the resonator and the rest in the thermostat. The resonator "selects" from the entire continuum of modes only those modes whose frequencies lie in the range $\Delta\omega_e \approx \omega_e/Q_e$. The fluctuation component of the attractive force results from beats of modes from this range.

In the case when one plate of the resonator can move, the attractive force between the capacitor plates will contain a component proportional to the coordinate of the mobile plate. It can be shown that the corresponding generalized susceptibility is

$$\chi(\omega) = \frac{\hbar\omega_e^5}{2\pi Q_e d^2} \times \int_0^\infty \left[\frac{1}{z^*(\omega-\omega')} + \frac{1}{z^*(\omega+\omega')}\right] \frac{\omega d\omega'}{|z(\omega')|^2}. \tag{6}$$

The relations (5) and (6) imply that $F_\omega^2 = 2\hbar|\mathrm{Im}\chi(\omega)|$, in complete agreement with the fluctuation-dissipation theorem.

If the mobile capacitor plate is connected to a mechanical oscillator with mass m, frequency $\omega_m = 2\omega_e$, and intrinsic Q such that $(Q_m)_{\text{intrinsic}} \gg Q_e$, then its Q, owing to the susceptibility (6), will be

$$(Q_m)_{\text{loaded}} = \frac{m\omega_m^2}{2|\mathrm{Im}\,\chi(\omega_m)|} \approx \frac{16 m\omega_m d^2}{\hbar Q_e}$$

(if $(Q_m)_{\text{loaded}} \gg Q_e$). For an oscillator formed by the lowest mode of the transverse oscillations of a plate of thickness d and area S

$$(Q_m)_{\text{loaded}} \approx \frac{4\pi^2 YV}{\hbar\omega_e Q_e},$$

where Y is Young's modulus of the plate material and $V=Sd$ is the volume of the plate. For $S=10^{-3}$ cm^2, $d=10^{-4}$ cm, $\omega_e=10^{10}$ sec^{-1}, $Q_e=10^{11}$, and $Y=4\cdot 10^{12}$ g/cm$\cdot$sec^2 we obtain $(Q_m)_{\text{loaded}}=1.6\cdot 10^{13}$, which is not too far from the actually obtained values of the mechanical Q.

At frequencies close but not equal to $2\omega_e$, the susceptibility $\chi(\omega)$ has a nonzero real part, i.e., stiffness $k_{\text{e.m.}}$ is introduced into the mechanical motion. For the mechanical oscillator under consideration, this will lead to a frequency shift

$$(\Delta\omega_m/\omega_m) = \frac{k_{\text{e.m}}}{2m\omega^2} = \frac{\hbar Q_e}{64 m d^2 \omega_m} = \frac{\hbar\omega_e Q_e}{16\pi^2 YV} \approx 1.5\cdot 10^{-14}$$

if $|\omega_m - 2\omega_e| = \omega_e/Q_e$, which corresponds to maximum frequency shift.

3. *Redistribution of the energy of zero-point vibrations.*[5] The energy density of the zero-point electromagnetic vibrations can be changed by varying the boundary conditions. This effect can be observed experimentally. Let a capacitor with capacitance C and plate separation d be connected to a transmission line of length l and wave impedance ρ. The transmission band of the line must not have a lower limit (for example, the line can be coaxial). Then there appears in addition to Casimir's force an attractive force between the capacitor plates which is caused by the zero-point vibrations in the modes of the coupled system "capacitor +waveguide." The structure of these modes and therefore the energy density of the zero-point vibrations in the capacitor depend significantly on the boundary conditions at the second end of the line.

Calculations show that the attractive force is maximum if the second end of the line is short-circuited and minimum if the second end is open. This is easily explained by the fact that the lowest mode of the oscillations of a short-circuited line corresponds to oscillations of an LC circuit where the line plays the role of the inductance. For this mode a significant part of the energy (approximately one-half) is stored in the capacitor. For the other modes this fraction is significantly smaller. The difference of the attractive forces for the cases of open and closed ends depends on the dimensionless parameter $l/v\rho C$, where v is the velocity of an electromagnetic wave in the line. The maximum value

$$\Delta F \approx 0.07 \frac{\hbar v}{2dL} \tag{7}$$

is reached for $l/v\rho C \approx 0.42$. If $v \approx c$ (speed of light in vacuum), $d=10^{-6}$ cm, and $l=10^2$ cm, then $\Delta F \approx 10^{-14}$ dynes. Modern recording methods make it possible to observe such a force.

It follows from what we have said above that by changing the boundary conditions at the end of a transmission line information can be transferred along an electromagnetic channel without photon emission.

4. *Energy dispersion of the zeroth quantum state.*[5] The effect considered in Sec. 3 is static. At the same time, it is obvious that any mechanical fluctuations influencing the boundary conditions, including also zero-point mechanical vibrations, will cause the energy of the zeroth state of the electromagnetic modes to be indefinite.

Consider an electromagnetic resonator coupled to a mechanical oscillator by the ponderomotive effect, so that the resonance frequency ω_e depends on the coordinate x of the oscillator: $\omega_e = \omega_{e0}[1-(x/d)]$. Calculation shows that in the state when the total energy of this system is minimum (and, naturally, precisely defined), the uncertainty of the energy of its mechanical and electromagnetic parts separately are

$$\Delta E_e = \Delta E_m \approx \frac{\hbar\omega_e}{2}\frac{x_0}{d}, \tag{8}$$

where x_0 is the uncertainty of the coordinate of the mechanical oscillator in its ground state. This result is easily explained qualitatively by the presence of uncertainty in the frequency of the loop due to the dependence of this frequency on x.

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[5]V. B. Braginsky and F. Ya. Khalili, Phys. Lett. A **167**, 1 (1992).