Light scattering from electron-density fluctuations in mutlivalley semiconductors and metals

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The theoretical concepts of Raman scattering by free current carriers in semiconductors and metals are reviewed. Various elementary excitations are discussed as the sources of the light-scattering fluctuations: one-particle charge-density fluctuations and plasmons, spindensity fluctuations, and fluctuations of the density of electron energy and momentum. These elementary excitations are common to different solids: metals, semimetals, semiconductors, and superconductors. As an adequate mathematical apparatus that reflects the unitary nature of the Raman spectra of these elementary excitations in various solids, a macroscopic approach to describing their relaxation is proposed. In this approach one distinguishes two mechanisms of relaxation: the diffusion mechanism, in which relaxation occurs via diffusional fluxes of varying nature, and the Mandel'shtam-Leontovich mechanism, in which adiabatic relaxation of the light-scattering fluctuations occurs. In multivalley semiconductors these mechanisms coexist, giving an additive contribution to the reciprocal relaxation time of the fluctuations: in single-valley semiconductors, metals, and superconductors, one of these mechanisms is realized, depending on the details of the electron band structure. Here the correlation function that determines the Raman cross section satisfies the same kinetic or diffusional equation as the fluctuating quantity itself. As technical applications, the possibility is pointed out of contact-free determination of the parameters of the electronic spectrum of semiconductors, metals, semiconductor superlattices, and superconductors.

1. INTRODUCTION

Raman scattering by charge carriers in solids is one of the best developed branches of spectroscopy. There are a multitude of electronic excitations that are manifested in Raman scattering. They include fluctuations of charge density, fluctuations of electron energy and momentum, fluctuations of spin density, etc. A large number of parameters of the electronic spectrum of semiconductors and metals and their kinetic coefficients can be determined from scattering spectra.

The conservation laws for the elementary scattering event have the form

$$\mathbf{p}' - \mathbf{p} = \hbar \mathbf{q}, \quad \varepsilon_{\mathbf{p}'} - \varepsilon_{\mathbf{p}} = \hbar \omega;$$
 (1.1)

here

$$\omega = \omega^{\mathbf{I}} - \omega^{\mathbf{S}}, \quad \mathbf{q} = \mathbf{k}^{\mathbf{I}} - \mathbf{k}^{\mathbf{S}} \tag{1.2}$$

are the frequency and wave vector transmitted upon scattering, ω^{I} and \mathbf{k}^{I} in (1.2) are the frequency and wave vector of the incident light, while ω^{S} and \mathbf{k}^{S} are the same parameters of the scattered light, and ε_{p} is the energy of the elementary excitation having the quasimomentum **p**. Despite the considerable difference of electronic properties of different solids: metals, semimetals, semiconductors, and superconductors, deep relationships exist among their Raman spectra, since the one-particle excitations in them are controlled by the very same laws. The conservation laws in (1.1) give a good example of such laws.

For a degenerate electron gas at zero temperature T=0, Eqs. (1.1) and (1.2) imply that only the electrons that lie in a layer of depth $\hbar\omega/v_{\rm F}$ contribute to scattering, where $v_{\rm F}$ is the Fermi velocity below the Fermi surface. The number of such electrons depends on the geometry of the Fermi surface. For a spherical surface the scattering cross section increases linearly with increase of ω up to $\omega = qv_{\rm F}$, and then declines to zero at $\omega = qv_{\rm F} + (\hbar q^2/2m)$ (Fig. 1, curve I). At a finite temperature T an additional rounding-off will occur near $\omega = qv_{\rm F}$ (curve 2). Fluctuations of the bottom of the conduction band caused by fluctuations of the density of ionized donors lead to the same effect (curve 3). Curve 4 shows the experimental data (from the review, Ref. 3). Thus, for Raman scattering by current carriers, it is essential in principle to take account of spatial dispersion. Usually the Raman cross section is determined by the square of the classical electron radius

$$r_0 = \frac{e^2}{mc^2} = 2.82 \cdot 10^{-13} \text{ cm}$$
 (1.3)

(see Ref. 1, Sec. 78). However, near resonance it can be enhanced by a factor of up to $10^{10.2}$ This very substantial enhancement factor, being a resonance effect, depends very strongly on the details of the electronic band structure and on the scattering mechanism.

In line with this, the present review begins with deriving the Raman cross section (Sec. 2) for a model of a band structure applicable to semiconductors and semimetals. Only several energy bands are considered. They allow one



FIG. 1. Frequency dependence of the scattering cross section of oneparticle excitations illustrating the law of conservation of energy. Curves I-3—calculation for GaAs with $n=6.4 \times 10^{18}$ cm⁻³, $qv_{\rm F}=280$ cm⁻¹, curve 4—experiment from Ref. 3.

to take account of the major contribution to the Raman cross section. The degeneracy of the bands is taken into account both in the virtual states and in the initial and final electronic states. In the special case of semiconductors with a simple conduction band, such as n-InP and GaAs, the Raman cross section reduces to (1.3). However, at high concentrations of charge carriers easily attainable in the plasma of a solid, this very simple scattering mechanism, which is associated with charge fluctuations, is suppressed by screening. In this case, to explain the observed spectra requires other mechanisms of Raman scattering, which arise from features of the band structure of the materials.^{3,4} These scattering mechanisms are discussed in Secs. 3.2-3.7. The electronic energy bands in typical semiconductors are classified according to the quantum numbers of the angular momentum. The corresponding transitions lead to scattering with spin reversal, which in the absence of spin splittings degenerates into the ordinary Thompson scattering with the cross section of (1.3). In this case the spin subbands can be treated as two valleys of a multivalley semiconductor, with which light interacts in different ways. This is the so-called scattering by spin-density fluctuations. In the case of a real multivalley semiconductor (containing several valleys), singularities of scattering that occurs in an anisotropic multicomponent plasma of a solid are manifested.⁵ As more complex multicomponent systems, one can treat the electrons of metals having an arbitrary anisotropy of the Fermi surface.⁶ In the collision-free case the singularities of the Raman spectra are directly associated with the topology of the Fermi surface.⁶ Here the multicomponent character of the plasma is attained by the differing interaction of the light with different regions of the Fermi surface. In nonparabolic semiconductors the role of different plasma components is played by the carriers that do not lie at the Fermi surface.⁷ Under a number of conditions, the scattering by energy fluctuations that occurs in this case allows a macroscopic description. A special scattering mechanism, whose contribution predominates in the isotropic spectrum of the scattering particles,

exists in semiconductors with degenerate bands.⁸

Section 4 is devoted to calculating the form of the spectrum for each of the cited scattering mechanisms. To do this, it proved necessary to discuss the kinetics of the fluctuations that scatter light. The existing experiments on Raman scattering are also analyzed in this section.

Within the framework of the effective-mass approximation, the mechanisms of scattering in semiconductors having superlattices and quantum wells are the same as in volume materials. It is shown in Sec. 5 how to apply all of the material discussed in the review for the case of electrons trapped in superlattices and wells; Sec. 6 is devoted to metals and superconductors.

In all places where it seems convenient in this article, we discuss the theoretical and experimental results obtained in our research group.

2. THE SCATTERING CROSS SECTION

One can derive the most general expression for the light-scattering cross section of free current carriers on the basis of a quantum-mechanical description of both the electronic system and the radiation. If the energy spectrum of a semiconductor contains degenerate or closely spaced bands, then, in the absence of external agents, the Hamiltonian of the free carriers \hat{H}_0 of any crystal can be written in the form of a matrix in the indices of the degenerate or closely spaced bands.^{9,10} To find the matrix Hamiltonian of the interaction \hat{H}_{int} of such free carriers with the field of an electromagnetic wave, we must replace in \hat{H}_0 the kinematic momentum **p** with the generalized $\hat{\mathbf{p}} + (e/c)\mathbf{A}(\mathbf{r},t)$, where **A** is the vector potential of the electromagnetic wave, and must separate out the terms containing **A**. As a result the sought interaction Hamiltonian equals

$$\hat{H}_{\text{int}} = \frac{e}{c} \int d^3r \bigg(\hat{j}_i(r) A_i(\mathbf{r}) + \frac{e}{2mc} A_i(\mathbf{r}) \hat{\mu}_{lk}(r) A_k(\mathbf{r}) \bigg).$$
(2.1)

Here the operator for the carrier current equals

$$j(\mathbf{r}) = \sum_{a} \left(\hat{v}_{a} \delta(\mathbf{r} - \mathbf{r}_{a}) + \delta(\mathbf{r} - \mathbf{r}_{a}) \hat{v}_{a} \right), \qquad (2.2)$$

where the summation over a is taken over all particles, while the velocity operator has the form

$$\hat{v}_a = \frac{\partial \hat{H}_0}{\partial \hat{\mathbf{p}}_a}.$$
(2.3)

The tensor

$$\hat{\mu}_{ik}(\mathbf{r}) = m \sum_{a} \frac{\partial^2 \hat{H}_{0a}}{\partial \hat{p}_{al} \partial \hat{p}_{ak}} \delta(\mathbf{r} - \mathbf{r}_{a})$$
(2.4)

is defined as the operator for the reciprocal effective mass. The vector potential $\hat{\mathbf{A}}(\mathbf{r})$ can be written in terms of the operators for creation and annihilation of photons \hat{c}^+ and \hat{c} in the form

$$\hat{\mathbf{A}}(\mathbf{r}) = \sum_{n} (\hat{c}_{n} \mathbf{A}_{n} + \hat{c}_{n}^{\dagger} \mathbf{A}_{n}^{*}).$$
(2.5)

Here the summation over n is performed over the states of the electromagnetic field, and the A_n are the wave functions of the electromagnetic field. We note that the interaction in (2.1) contains effects of spatial dispersion and is suitable for describing scattering in inhomogeneous media for bounded crystals, their surfaces, superlattices, and quantum wells.

Inelastic scattering of light is a process in which the absorption of an incident electromagnetic wave with frequency ω^{I} , wave vector k^{I} and polarization e^{I} is accompanied by simultaneous emission of a wave with the parameters ω^{S} , k^{S} , and e^{S} . The probability of the corresponding quantum-mechanical transition has the form

$$W = \frac{V^2(\omega^{\rm S})^2}{2\pi\hbar c^4} |M_{21}|^2 \delta(E_2 - E_1 + \hbar(\omega^{\rm S}\omega^{\rm I}))$$
(2.6)

$$M_{21} = \frac{e^2}{mc^2} \int d^3r_1 d^3r_2 A_i^{I}(\mathbf{r}_1) A_k^{S*}(\mathbf{r}_2) \left[\mu_{21}^{ik}(\mathbf{r}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2) + m \sum_n \frac{j_{2n}^i(\mathbf{r}_1) j_{n1}^k(\mathbf{r}_2)}{E_1 - E_n + \hbar \omega^I + i0} + \frac{j_{2n}^k(\mathbf{r}_2) j_{n1}^i(\mathbf{r}_1)}{E_1 - E_n - \hbar \omega^S + i0} \right]$$
(2.7)

is the total matrix element of the transition calculated in first- and second-order perturbation theory in \hat{H}_{int} from (2.1) (see Ref. 11). In (2.6) V is a normalizing volume for photons. The probability of the transition (2.6) describes all the secondary emission of the crystal¹² and contains the contribution of luminescence and inelastic light scattering. If the light scattering occurs with a small frequency change (the so-called quasielastic scattering):

$$\omega^{\mathbf{I}} - \omega^{\mathbf{S}} \ll \omega^{\mathbf{I}}, \tag{2.8}$$

then in this case it proves possible to write the Raman cross section in terms of the electronic contribution to the polarizability of the crystal $\delta\chi(\mathbf{r},t)$. To do this it suffices to reverse the rule of passing around the pole in the second term within the summation sign in (2.7). If the scattering system exists in the quantum-mechanical ground state, then only "upward" quantum transitions in which $E_1 - E_n < 0$ are possible. Therefore the denominator in the second term does not vanish, which makes the replacement identical.¹¹ If (at $T \neq 0$) the crystal does not exist in the ground state, then, if we assume the frequency of the light ω^{I} to be large enough, in view of (2.8) we can write

$$E_1 - E_n \ll \hbar \omega^{\mathbf{I}} \approx \hbar \omega^{\mathbf{S}}.$$

This implies that the second denominator in (2.7) still does not vanish. As a result, using the relationship

$$\frac{1}{E+i0} = -\frac{i}{\hbar} \int_0^\infty \mathrm{d}t e^{iEt/\hbar}, \qquad (2.9)$$

we obtain

$$M_{21} = -\left(\frac{\omega^{\mathbf{I}}}{c}\right)^{2} \int d^{3}r_{1}d^{3}r_{2}A_{i}^{\mathbf{I}}(\mathbf{r}_{1})A_{k}^{\mathbf{S}_{*}}(\mathbf{r}_{2})\chi_{ik}^{21}(\mathbf{r}_{1},\mathbf{r}_{2}),$$
(2.10)

where

$$\hat{\chi}_{ik}(\mathbf{r}_{1},\mathbf{r}_{2}) = -\frac{e^{2}}{m(\omega^{1})^{2}} \left[\hat{\mu}_{ik}(\mathbf{r}_{1})\delta(\mathbf{r}_{1}-\mathbf{r}_{2}) + \frac{i}{\hbar} \int_{0}^{\infty} dt_{1} e^{i\omega^{I}t_{1}} (\hat{j}_{i}(\mathbf{r}_{1},t)\hat{j}_{k}(\mathbf{r}_{2},0) - \hat{j}_{k}(\mathbf{r}_{2},0)\hat{j}_{i}(\mathbf{r}_{1},t)) \right]$$
(2.11)

is the operator for the dielectric polarizability of the electrons.

Upon substituting (2.10) into (2.6) and using the relationship

$$\delta(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \mathrm{d}t e^{-iEt/\hbar}, \qquad (2.12)$$

as well as the normalization condition for photons (one photon in the volume V), we can derive the following expression for the scattering cross section:

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{\omega^{\mathbf{I}}(\omega^{\mathbf{S}})^{3}}{2\pi c^{4}} - \frac{1}{|A_{0}^{\mathbf{I}}A_{0}^{\mathbf{S}}|^{2}} \int \mathrm{d}^{3}\mathbf{r}_{1}$$

$$\times \mathrm{d}^{3}\mathbf{r}_{2}\mathrm{d}^{3}\mathbf{r}_{3}\mathrm{d}^{3}\mathbf{r}_{4} \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t}A_{i}^{\mathbf{I}}(\mathbf{r}_{1})A_{k}^{\mathbf{S}}(\mathbf{r}_{2})$$

$$\times \langle \chi_{ik}(\mathbf{r}_{1},\mathbf{r}_{2},t)\chi_{nm}(\mathbf{r}_{3},\mathbf{r}_{4},0)\rangle A_{n}^{\mathbf{I}}(\mathbf{r}_{3})A_{m}^{\mathbf{S}}(\mathbf{r}_{4});$$
(2.13)

Here the angle brackets $\langle ... \rangle$ denote statistical averaging over the initial states of the crystal, and A_0^1 and A_0^3 are the amplitudes of the waves outside the crystal. Equation (2.13) is valid for absorbing crystals, and also for films under the condition of no interference of the incident wave in the scattering volume. However, Eq. (2.13) is inapplicable for substantially inelastic, e.g., multiphonon, scattering¹³ in which the condition (2.8) fails, and one must not change the rule for passing around the poles in the denominators of (2.7). In this case the scattering cross section is not expressed in terms of the operator for the electronic polarizability.¹² The absorption length in typical semiconductors is rather large ($\delta' \sim 10^{-5}$ cm) even under conditions of resonance enhancement of the Raman cross section. This allows us to neglect the spatial dispersion of the dielectric susceptibility χ_{ik} from (2.11) and write the following expression to an accuracy up to q < 1, where a is the lattice constant:

$$\chi_{ik}(\mathbf{r}_1,\mathbf{r}_2) = \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta \chi_{ik}(\mathbf{r}), \qquad (2.14)$$

Here we have $\mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2)/2$, while the quantity $\delta \chi_{ik}(\mathbf{r})$ is defined as the fluctuation of the dielectric susceptibility of the system of electrons. Substituting (2.14) into (2.13), we obtain

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{\omega^{\mathrm{I}}(\omega^{\mathrm{S}})^{3}}{2\pi c^{4}} - \frac{1}{|A_{0}^{\mathrm{I}}A_{0}^{\mathrm{S}*}|^{2}} \int \mathrm{d}^{3}\mathrm{r}\mathrm{d}^{3}\mathrm{r}'A_{i}^{\mathrm{I}*}(\mathbf{r})A_{k}^{\mathrm{S}}(\mathbf{r})$$
$$\times \langle \delta\chi_{ik}(\mathbf{r})\delta\chi_{mn}(\mathbf{r}')\rangle_{\omega}A_{m}^{\mathrm{I}}(\mathbf{r}')A_{n}^{\mathrm{S}*}(\mathbf{r}'), \quad (2.15)$$

where the spectral correlation function equals

$$\langle \delta \chi_{ik}(\mathbf{r}) \delta \chi_{mn}(\mathbf{r}') \rangle_{\omega} = \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t} \langle \delta \chi_{ik}(\mathbf{r},t) \delta \chi_{mn}(\mathbf{r}',0) \rangle.$$
(2.16)

We note that the Raman cross section in the subsurface layer of a nontransparent medium was treated in Ref. 14 by constructing the Green's function of the wave equation outside the medium. The dependence of the correlator (2.16) on the boundary conditions at the surface of the crystal enables one to study various scattering processes that occur with participation of surface excitations of the crystal.¹⁵ For transparent media one can calculate the integrals over the volume in (2.15):

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{V\omega^{\mathrm{I}}(\omega^{\mathrm{S}})^{3}}{2\pi c^{4}} \left(e_{k}^{\mathrm{I}}e_{j}^{\mathrm{S}}\right) \ast e_{i}^{\mathrm{I}}e_{n}^{\mathrm{S}}\langle\delta\chi_{ij}\delta\chi_{kn}\rangle_{q,\omega}; \quad (2.17)$$

here $\langle \delta \chi_{ij} \delta \chi_{kn} \rangle_{q,\omega}$ is the completely Fourier-transformed correlation function (see Ref. 16). The substitution of the current operator of (2.2) and the effective-mass operator of (2.4) into Eq. (2.11) yields

$$\hat{\delta}\chi_{ik}(\mathbf{q}) = \left(\frac{e}{\omega^{\mathbf{I}}}\right)^2 \sum_{\mathbf{p},\xi,\xi'} \hat{a}_{\xi,\mathbf{p}}^+ + \frac{\hbar\mathbf{q}}{2} \hat{a}_{\xi',\mathbf{p}} - \frac{\hbar\mathbf{q}}{2} \gamma_{ik}^{\xi\xi'}(\mathbf{p}).$$
(2.18)

Here $\hat{a}_{\xi \mathbf{p}}^+$ and $\hat{a}_{\xi \mathbf{p}}$ are the operators for creation and annihilation of an electron in the band ξ with the quasimomentum **p**, while the matrix $\hat{\gamma}_{ik}$ equals

$$\gamma_{ik}^{\xi\xi'} = \left(\frac{\partial^2 H_0}{\partial \hat{p}_i \partial \hat{p}_k}\right)_{\xi\xi'} + \sum_m \left(\frac{(\hat{v}_i)_{\xi\mathbf{p},m\mathbf{p}}(\hat{v}_k)_{m\mathbf{p},\xi'\mathbf{p}}}{E_{\xi\mathbf{p}} - E_{m\mathbf{p}} + \hbar\omega^{\mathbf{\Gamma}}} + \frac{(\hat{v}_k)_{\xi\mathbf{p},m\mathbf{p}}(\hat{v}_i)_{m\mathbf{p},\xi'\mathbf{p}}}{E_{\xi\mathbf{p}} - E_{m\mathbf{p}} - \hbar\omega^{\mathbf{\Gamma}}}\right).$$
(2.19)

The diagonal components of the matrix $\hat{\gamma}_{ik}$ describe intraband scattering. The corresponding contribution to Eq. (2.18) describes the thermodynamic fluctuations of the dielectric susceptibility associated with the fluctuations of the electron distribution function:

$$\hat{\delta}f_{\xi\mathbf{p}} = \hat{a}_{\xi,\mathbf{p}}^{+} + \frac{\hbar\mathbf{q}}{2}\,\hat{a}_{\xi,\mathbf{p}} - \frac{\hbar\mathbf{q}}{2}\,. \tag{2.20}$$

All the information on the mechanism of interaction of the carriers with light is concentrated in the matrix $\hat{\gamma}_{ik}$. In the nonresonance situation we have $\hbar \omega^{I} \ll E_{g}$, where E_{g} is the width of the forbidden band of the crystal. As Abrikosov and Fal'kovskiĭ showed,¹⁷ the first term in the Hamiltonian of the interaction (2.1), which is linear in the field of the electromagnetic wave, yields a contribution to the cross section that is small (-v/c) in comparison with the quadratic term. Correspondingly, the contribution of the second and third terms in (2.19) is small⁵ in comparison with the first term, and we should neglect them. Here, for a simple nondegenerate band, the first term in (2.19) reduces to the reciprocal effective mass $(m^*)^{-1}$. Therefore, also in (1.4) the mass of a free electron should be replaced by its effective mass. However, if the resonance condition $|E_{\sigma} - \hbar \omega^{I}| \ll \hbar \omega$ is satisfied, the second and third terms make the fundamental contribution to the scattering. In

the case of complex bands all the terms $\hat{\gamma}_{ik}$ amount to matrices in the symbols of the subbands. The existence of nonscalar components in all of them that do not reduce to an effective mass leads to new scattering mechanisms, which are discussed in Sec. 3.7.

We can represent the fluctuation of the dielectric susceptibility in (2.18) in the form of a series in the small deviations of statistically independent fluctuating quantities from their equilibrium values. Both the magnitude of the cross section and the form of the scattering spectrum substantially depend on whether purely classical fluctuations of these quantities occur, as determined by the temperature, or quantum "zero-point oscillations" of the fluctuating quantities occur. The transition from the one scattering cross section to the other is carried out with the coefficient

$$F(\omega) = \frac{\hbar\omega}{1 - e^{-\hbar\omega/T}}.$$
 (2.21)

This coefficient gives rise to the well known¹⁶ ratio of the cross sections of Stokes and anti-Stokes processes

$$\frac{\mathrm{d}\Sigma^{\mathrm{Stokes}}}{\mathrm{d}\Sigma^{\mathrm{anti-Stokes}}} = e^{\hbar\omega/T}.$$
(2.22)

At low temperatures $T \ll \hbar \omega$ the elementary excitations of the crystal from which scattering occurs are mainly created by the light. Therefore there are no anti-Stokes processes. In the converse case $T \gg \hbar \omega$, scattering occurs from already existing thermal fluctuations. Therefore the cross sections of the two processes are the same.

The immediate information on the mechanisms of Raman scattering is contained in the so-called integral scattering cross section, which is obtained by integrating (2.17) over the transmitted frequency ω . The integral is calculated by using a representation in δ -functions of the type of (2.12). As a result the nonsimultaneous correlator $\langle \delta \chi_{ij}^{(t)} \delta \xi_{kn}^{(0)} \rangle$ from (2.16) becomes simultaneous, and the integral cross-section acquires the form

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \frac{V\omega^{\mathrm{I}}(\omega^{\mathrm{S}})^{3}}{2\pi c^{4}} e_{k}^{\mathrm{I}} e_{j}^{\mathrm{S}} (e_{i}^{\mathrm{I}} e_{n}^{\mathrm{S}})^{*} \langle \delta \chi_{i} \delta \chi_{kn} \rangle_{\mathrm{q}}.$$
(2.23)

This expression is simplified at high temperatures $T \gg \hbar \omega$, for which the fluctuations are classical.¹⁹ Apart from charge-density fluctuations, the classical simultaneous fluctuations are uncorrelated. Therefore we can set q=0 in (2.23). Then the scattering cross section in (2.23) breaks down into the sum of the mean-square fluctuations of fundamental thermodynamic quantities. The values of the integral cross sections for the fundamental scattering mechanisms are derived in Sec. 3.

The scattering cross section of the charge-density fluctuations substantially depends on \mathbf{q} owing to screening effects. Another example of dependence on \mathbf{q} of the simultaneous correlator of fluctuations $\delta \chi$ is given by spin-density fluctuations at low temperatures;³ see also Ref. 20.

3. SCATTERING MECHANISMS

3.1. Scattering of light by charge-density fluctuations

This type of scattering is caused by charge carriers created either by ionization of dopant admixtures²¹ or by optical pumping;^{22,23} see also Refs. 69 and 70. The essential question in treating light scattering by charged current carriers is the screening of the fluctuations that scatter light. At low carrier concentrations the screening radius r_s is so large that any excitation having the light wave vector **q** is not screened. Here we have

$$qr_s \gg 1.$$
 (3.1)

In this case the most essential fluctuations in the Raman spectra are the charge-density fluctuations. The corresponding contribution to the fluctuation of the dielectric-susceptibility tensor has the form^{16,17}

$$\delta\chi_{ij} = -\frac{e^2}{m(\omega^1)^2} \mu_{ij} \delta n.$$
(3.2)

Here the tensor for the reciprocal effective mass $\mu_{ij} = m\gamma_{ij}^{25}$ equals the diagonal matrix element of the matrix $\hat{\gamma}_{ij}$ from (2.19). This type of scattering has been studied in a gas plasma^{24,28} and in semiconductors.²¹ The spatial Fourier component of the correlator of the susceptibility fluctuations from (2.17) is reduced for (3.2) to the correlator of the density fluctuations:

$$\delta n_q = \sum_a e^{i\mathbf{q} \cdot \mathbf{r}_a}.$$
(3.3)

The main contribution to the Fourier integral comes from distances of the order of $r \sim q^{-1} \ll r_s$. Thus we can neglect the interference of the waves scattered from different Debye spheres of radius r_s . Yet within the limits of one sphere the motion of carriers is uncorrelated. As a result the differential Raman cross section acquires the form²⁵

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = V r_{0}^{2} |e^{\mathbf{I}} \mu_{ik} e_{k}^{\mathbf{S}*}|^{2} \operatorname{Re} \frac{1}{\pi} \int_{0}^{\infty} \mathrm{d}\tau e^{i\omega\tau} \langle e^{i\mathbf{q}\Delta\mathbf{r}(\tau)} \rangle.$$
(3.4)

Here the displacement $\Delta \mathbf{r}(\tau) = \mathbf{r}(t+\tau) - \mathbf{r}(t)$ is associated with the classical trajectory and is determined kinematically. This formula, derived under the assumption of absence of screening, is also valid for scattering by spindensity fluctuations. In this case the role of the tensor μ_{ij} from (3.2) is played by the corresponding "Raman tensor,"³ which determines the polarization dependence of the spectrum.

To trace in detail the transition from the differential cross section of (2.17) to the integral cross section of (2.23), let us integrate (3.4) over ω with the aid of (2.12):

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = Vnr_0^2 |e_i^{\mathrm{I}}\mu_{ik}e_k^{\mathrm{S}*}|^2. \tag{3.5}$$

Equation (3.5) differs from the standard formula [(78.5) from Ref. 1] in that it takes account of the anisotropy of the scattering medium. Here the different scattering mechanisms are manifested in different forms of μ_{ij} ; see Secs.

3.3-3.9. Moreover, the coefficient $e_i^{I} e_j^{S} \mu_{ij} = m e_i^{I} e_j^{S} \gamma_{ij}^{\xi\xi}$ according to (2.19) reflects the possibility of resonance enhancement of the scattering cross section.

Quite high concentrations of the current carriers, a condition, reverse to that of (3.1), is fulfilled. In the isotropic case it is the following

$$qr_s \ll 1. \tag{3.6}$$

At high enough concentrations Eqs. (3.4) and (3.5) lose validity, since the interference of waves scattered by various particles becomes substantial. This implies that oneparticle quasielastic scattering by charge-density fluctuations is transformed into substantially inelastic Raman scattering by plasmons. The general formula describing these changes in the scattering spectrum has the following form for an anisotropic medium:

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = VTr_0^2 |e_i^{\mathrm{I}} \mu_{ij} e_j^{\mathrm{S}*}|^2 \frac{q^2}{4\pi e^2} \left(1 - \frac{q^2}{q_{\mathcal{E}_{ik}}(0,\mathbf{q})q_k}\right). \quad (3.7)$$

Here $\varepsilon_{ik} = (0,\mathbf{q})$ is the dielectric permittivity tensor at zero frequency. In general, the cross section cannot be expressed in terms of the number of scattering particles and the screening radius, since the problem of screening in an anisotropic medium requires special treatment. If the medium is isotropic, then, using the known expression for $\varepsilon(0,\mathbf{q})$, we obtain

$$\frac{\mathrm{d}\boldsymbol{\Sigma}}{\mathrm{d}\boldsymbol{\Omega}} = VT\left(\frac{\partial n}{\partial \boldsymbol{\xi}}\right)_T r_0^{*2} |e^{\mathbf{I}}e^{\mathbf{S}_{*}}|^2 \frac{q^2}{q^2 + r_{\mathrm{s}}^{-2}},$$

where ζ is the chemical potential of the carriers. Thus a comparison of Eqs. (3.7) and (3.5) shows that, in view of the parameter (3.6), the scattering spectrum of plasmons is smaller by a factor of $(qr_s)^{-2}$ in intensity than the oneparticle spectrum. In Eq. (3.7), instead of the total concentration *n*, the coefficient $T(\partial n/\partial \zeta)_T$ enters, reflecting the Pauli principle and giving the fraction of the carriers in a layer of thickness *T* near the Fermi surface that participate in scattering. Crystals of high-temperature superconductors are anisotropic conductive media²⁷ (see Sec. 6), as well as doped superlattices; see Sec. 5.²⁶

3.2. Unscreened scattering mechanisms from the standpoint of symmetry

A distinguishing feature of plasmas of solids is that they are multicomponent plasmas. This leads to a number of unscreened mechanisms of light scattering that differ from the mechanism described in Sec. 3.1 of Raman scattering by charge-density fluctuations. The main difference consists in the fact that the corresponding contributions to the fluctuation of the susceptibility in (2.18) are nonscalar. Therefore they are not screened.

Generally speaking, the symmetry analysis of the quasielastic Raman cross section must be conducted with respect to the group of the wave vector **q** transmitted in scattering, i.e., to G_q . The expansion of the tensor $\hat{\gamma}_{ik}$ from (2.19) or $\delta \chi_{ik}$ from (2.18) into irreducible tensors must also be performed with respect to this group. The symmetry analysis of the integral cross section is often simplified

by the fact that it is determined by the simultaneous correlation function, in which we may neglect spatial dispersion, assuming that $\mathbf{q}=0$. This allows us to use the pointgroup symmetry. For example, for the cubic groups T_d , O, and O_h there are four irreducible representations that are contained in the representation formed by the arbitrary second-order tensor $\hat{\gamma}_{ik}$. Correspondingly, $\delta \chi_{ik}$ can be represented in the form

$$\delta \chi_{ik} = \delta \chi^{(\Gamma_1)} \delta_{ik} + \delta \chi^{(\Gamma_{12})}_{ik} + \delta \chi^{(\Gamma'_{23})}_{ik} + \delta \chi^{(\Gamma_{15})}_{ik}.$$
 (3.8)

Here $\delta \chi^{(\Gamma_i)}$ is the scalar contribution containing the trace of the tensor $\delta \gamma_{ik}$. It contains the charge fluctuations from (3.2). The rest of the terms have higher symmetry and are described by traceless matrices. After substituting $\delta \chi_{ik}$ from (3.8) into (2.23) and averaging over the symmetry directions the separate terms prove to be statistically independent. Therefore only the first term of scalar symmetry, which contains the charge fluctuation, is screened, while the remaining terms of higher symmetry are not screened. For less symmetrical crystals several terms having the same symmetry appear in the expansion (3.8). Here the cross term remains after statistical averaging. This is the situation, e.g., in crystals of classes T and T_h, where there are seven independent types of scattering instead of the four types following (3.8). The cross term can also appear in transforming to the group of the wave vector G_{a} , which is lower than the point group symmetry of the crystal. In all these cases the selection rules in Raman scattering cannot be derived by analyzing the second-order tensors γ_{ik} or $\delta \chi_{ik}$. The cross terms describe the overlap of wave functions having the same symmetry but differing energy. The theory based on the second-order Raman tensor is applicable only for narrow lines when the overlap integrals between them are insignificant. In describing deformation effects in Raman scattering in n-Si (see Sec. 3.8) and in calculating the differential section for Raman scattering by holes (see Sec. 3.6), we encounter broad lines, for which one must use a fourth-order tensor.³²

3.3. Light scattering by intervalley fluctuations in multivalley semiconductors

In a multivalley semiconductor the current carriers occupy several energy minima (valleys) in the Brillouin zone. The dependence of the energy on the quasimomentum for an electron lying in the α th minimum is described by the formula

$$\varepsilon_{\alpha}(\mathbf{p}) = \frac{1}{2m} p_{\sharp} \mu_{ik}^{(\alpha)} p_k.$$
(3.9)

Here $\mu_{ik}^{(\alpha)}$ is the tensor for the reciprocal effective mass in valley α . In the semiconductors n-Si and n-Ge, the application of the symmetry operations transforms the valleys into one another (Figs. 2 and 3). Therefore the valleys are equivalent. In deformed crystals the valleys cease to be equivalent. For example, upon hydrostatic compression of GaAs its side valleys drop lower in energy, approaching the Γ -valley, so that a system arises of several nonequivalent valleys. An analogous situation is realized in hydro-



FIG. 2. Orientation of the valleys in n-Si. The arrows indicate the directions of the vectors e^{I} , e^{S} , and q, which correspond to the Γ_{12} scattering geometry used in the study.

static compression of n-Ge, with the difference that here the side valleys are initially lower. Raman scattering in hydrostatically³ and also in uniaxially compressed²⁹ crystals is being intensively studied.

The contribution of the current carriers of a multivalley semiconductor to the dielectric susceptibility is determined by a generalized formula of the type of (3.2):

$$\delta \chi_{ik} = -\frac{e^2}{m(\omega^1)^2} R_{12} \sum_{\alpha} \mu_{ik}^{(\alpha)} \delta n_{\alpha}. \qquad (3.10)$$

Here R_{12} is a resonance factor having the standard form³⁰

$$R_{12} = \frac{E_{g}^{2}}{E_{g}^{2} - (\hbar\omega^{I})^{2}}.$$
 (3.11)

If the α th valley has axial symmetry with respect to the direction $v^{(\alpha)}$, then the tensor $\mu_{ik}^{(\alpha)}$ can be written in the form

$$\mu_{ik}^{(\alpha)} = \frac{1}{3} (\mu_{\parallel} + 2\mu_{\perp}) \delta_{ik} + (\mu_{\parallel} - \mu_{\perp}) (\nu_{i}^{(\alpha)} \nu_{k}^{(\alpha)} - \frac{1}{3} \delta_{ik}),$$
(3.12)

where μ_{\parallel} and μ_{\perp} are the principal values of the tensor $\mu_{ik}^{(\alpha)}$. This representation according to Sec. 3.2 is convenient, since the terms of differing symmetry are separated in it. The corresponding expression for $\delta \chi_{ik}$ has the form



FIG. 3. Orientation of the valleys in n-Ge and the unit vectors used in the present article. The directions of all the unit vectors pass through the coordinate origin—the center of the cube.

$$\delta\chi_{ik} = -\frac{e^2}{m(\omega^{\rm I})^2} \left[\frac{1}{3} \left(\mu_{\parallel} + 2\mu_{\perp} \right) \delta_{ik} \delta n + \frac{1}{2S} \left(\mu_{\parallel} - \mu_{\perp} \right) \right] \times \sum_{\alpha,\beta} \left(\nu_i^{(\alpha)} \nu_k^{(\alpha)} - \nu_i^{(\beta)} \nu_k^{(\beta)} \right) \left(\delta n_\alpha - \delta n_\beta \right) \left[(3.13) \right]$$

Here S is the number of valleys, δn_{α} is the fluctuation of the concentration of electrons in the valley α , and

$$\delta n = \sum_{\alpha} \delta n_{\alpha}$$

is the fluctuation of the total concentration. Substituting (3.13) into (2.23) leads to the cross section for scattering by neutral fluctuations. The correlation function for neutral fluctuations is well known:¹⁹

$$\langle \delta n_{\alpha} \delta n_{\beta} \rangle_{q \to 0} = T \delta_{\alpha\beta} \left(\frac{\partial n_{\alpha}}{\partial \zeta} \right)_{T}.$$
 (3.14)

The final answer depends on the number and orientation of the valleys in the Brillouin zone. For the six valleys of n-Si the unit vectors $v^{(\alpha)}$ are oriented along the fundamental fourfold crystallographic axes. The integral scattering cross section equals

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \frac{1}{27} V(R_{12}r_0)^2 T\left(\frac{\partial n}{\partial \xi}\right)_T (\mu_{\perp} - \mu_{\parallel})^2 \\ \times \sum_i \left|\sum_k \left(e_i^{\mathrm{I}} e_i^{\mathrm{S}*} - e_k^{\mathrm{I}} e_k^{\mathrm{S}*}\right)\right|^2.$$
(3.15)

The combination of polarization vectors obtained in (3.15) corresponds to the so-called Γ_{12} scattering geometry. In pure form it exists, e.g., for $e^{I} = \langle 110 \rangle / \sqrt{2}$, $e^{S} = \langle 1\overline{10} \rangle / \sqrt{2}$. An analogous result was obtained in Ref. 30. For n-Ge, where the valleys are oriented along the threefold axes (see Fig. 3), an analogous procedure yields the following result for the integral cross-section:³³

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \frac{1}{9} V(R_{1,2}r_0)^2 \left(\frac{\partial n}{\partial \zeta}\right)_T (\mu_1 - \mu_{\parallel})^2 \\ \times \sum_{i>k} |e_i^{\mathrm{I}} e_k^{\mathrm{S}*} + e_k^{\mathrm{I}} e_i^{\mathrm{S}*}|^2.$$
(3.16)

As is known,³³ this Γ'_{25} geometry is realized, e.g., when $e^{S} = \langle 100 \rangle$, $e^{I} = \langle 010 \rangle$. Thus one can determine the orientation of the valleys from the electronic Raman scattering in multivalley semiconductors. On the whole, multivalley semiconductors give a graphic example of how a complex structure of the spectrum of carriers creates conditions for the appearance of excitations active in light scattering.

3.4. Light scattering in metals

Light scattering in metals occurs in the subsurface layer (the skin layer), whose thickness δ is determined by the depth of penetration into the crystal of the field of the incident light. In (2.15) one must use the Fresnel formulas for the coupling of the incident and scattered electromagnetic waves inside and outside the crystal.¹⁶

The unscreened scattering of light in a metal is caused by the fluctuations of the electron density distribution along the complex Fermi surface. Upon taking account of the strong degeneracy of the statistics, the fluctuation of the electronic polarizability can be represented in the form^{34,35}

$$\delta\chi_{ik} = -\frac{e^2}{(\omega^1)^2} \int \frac{2dS_F}{(2\pi\hbar)^3 v_F} \varphi_P \frac{\partial^2 \varepsilon}{\partial p_i \partial p_k}.$$
 (3.17)

Here φ_p is the fluctuation of the electron distribution function at the Fermi surface, which is associated with the total distribution function δf_p according to

$$\delta f_{\mathbf{p}} = \varphi_{\mathbf{p}} \left(\frac{\partial f_0}{\partial \zeta} \right)_T. \tag{3.18}$$

Here f_0 is the Fermi distribution function of the electrons. The role of the reciprocal effective mass is played in (3.17) by the curvature of the Fermi surface $\partial^2 \varepsilon / \partial p_i \partial p_k$, which depends on the quasimomentum of the electron. Since the wave vector of the light in the metal has the considerable uncertainty $\Delta q \sim 1/q > q$, the role of the condition of strong screening in (3.6) is played by the stricter condition

$$r_{\rm s} \ll \delta. \tag{3.19}$$

Instead of the summation over the valleys, an integral over all the Fermi surface figures in the condition of neutrality for a metal:

$$\delta n = \int \frac{2d^3 p}{(2\pi\hbar)^3} \, \delta f_{\mathbf{p}} = \int \frac{2dS_{\mathbf{F}}}{(2\pi\hbar)^3 v_{\mathbf{F}}} \, \varphi_{\mathbf{p}} = 0. \tag{3.20}$$

This condition implies that the fluctuations of the electron distribution function are not accompanied by charge fluctuations. It replaces the Poisson equation in the case of (3.19) (see Ref. 36). Since metals are opaque materials, one employs the "backscattering geometry" in them (see Ref. 3). Upon substituting (3.17) into (2.15) and integrating over the frequency with the aid of (2.12), we obtain

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \frac{16}{\left[(N+1)^2 + \alpha^2\right]^2} r_0^2 S \delta T \left(\frac{\partial n}{\partial \zeta}\right)_T m^2 \\ \times \left\langle \left| e_i^{\mathrm{I}} \frac{\partial^2 \varepsilon}{\partial p_i \partial p_k} - \left\langle \frac{\partial^2 \varepsilon}{\partial p_i \partial p_k} \right\rangle e_k^{S*} \right|^2 \right\rangle.$$
(3.21)

Here the first factor corresponds to the square of the transparency coefficient T at normal incidence of the electromagnetic wave on the boundary of the metal:

$$T = \frac{4}{(N+1)^2 + \alpha^2},$$
 (3.22)

Here N and α are the refractive and absorption coefficients of the metal, S the area of the scattering surface, and the angle brackets denote averaging over the Fermi surface. The second-order tensor

$$\delta\mu_{ik}(\mathbf{p}) = m \left(\frac{\partial^2 \varepsilon}{\partial p_i \partial p_k} - \left\langle \frac{\partial^2 \varepsilon}{\partial p_i \partial p_k} \right\rangle \right)$$
(3.23)

in (3.21) vanishes for a parabolic dependence of the energy of the electron on the quasimomentum, when the fluctuations δn are fully screened (see Sec. 3.1). Thus the separation of the trace of the tensor $\partial^2 \varepsilon / \partial p_i \partial p_k$ effected by averaging implies taking account of screening.³¹ On the other hand, this operation leads to numerical smallness of the cross section of (3.21).

3.5. Mechanisms of light scattering in semiconductors with a nonparabolic dispersion law

An important group of semiconductors is the A³B⁵ compounds. The band structure of these materials is well described by the Kane model,³⁷ in which the dependence of the energy of the electrons on the quasimomentum proves to be isotropic, but substantially nonparabolic. As in the case of metals, nonparabolicity of the electron spectrum of $A^{3}B^{5}$ compounds implies a dependence of the curvature of the isoenergy surface on the quasimomentum. Therefore the principal features of unscreened mechanisms of light scattering that we discussed in Sec. 3.4 for metals are maintained also in these semiconductors. The difference consists in the fact that a nondegenerate statistics can be realized in semiconductors, in which all free carriers are essential, rather than only those lying at the Fermi surface, as in metals. In this regard the fluctuation of the electronic polarizability is determined by the total fluctuation of the distribution function δf_n :

$$\delta\chi_{ik} = -\frac{e^2}{(\omega^1)^2} \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \delta f_p \frac{\partial^2 \varepsilon}{\partial p_i \partial p_k}.$$
 (3.24)

One can obtain the integral cross-section by substituting (3.24) into (2.23). The symmetry considerations presented in Sec. 3.2 enable us to separate this cross section into two contributions corresponding to the separation of the tensor $\delta \chi_{ik}$ into the scalar $\delta \chi_{ii}/3$ and traceless symmetrical components. The latter equals

$$\delta \chi_{ik}^{(\mathbf{S})} = \frac{1}{2} (\delta \chi_{ik} + \delta \chi_{ki'} - \frac{2}{3} \delta \chi_{el} \delta \chi_{ik}). \qquad (3.25)$$

The correlation function $\langle \delta \chi_{ij} \delta \chi_{kn} \rangle_q$ from (2.23) is reduced for the case of an isotropic medium to the following equation [see Eq. (117.13 in the book, Ref. 16)]:

$$\langle \delta \chi_{ij} \delta \chi_{kn} \rangle_{\mathbf{q} \to 0} = \langle (\frac{1}{3} \delta \chi_{ii})^2 \rangle_V \delta_{ij} \delta_{kn} + \frac{1}{10} \langle (\delta \chi^{(\mathbf{S})})^2 \rangle_V$$
$$\times (\delta_{ik} \delta_{jn} + \delta_{in} \delta_{kj} - \frac{2}{3} \delta_{ij} \delta_{kn}).$$
(3.26)

The two terms in (3.26) correspond to the two independent mechanisms of light scattering by energy fluctuations and momentum fluctuations of electrons. The physical differences between these mechanisms of Raman scattering are manifested most sharply for weak nonparabolicity, when the dependence of the energy on the quasimomentum can be represented in the form⁷

$$\varepsilon_p = \frac{p^2}{2m^*} - \frac{1}{E_g} \left(\frac{p^2}{2m^*}\right)^2.$$
 (3.27)

Here m^* is the value of the effective mass at the bottom of the conduction band. In this case the curvature of the isoenergy surface that enters into (3.24) proves to be a linear function of the energy ε . Then the scalar contribution to (3.26) has the form

$$\delta\chi^{(\varepsilon)} = \frac{1}{3} \delta\chi_{ii}$$
$$= -\frac{e^2}{m(\omega^I)^2} \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \left(1 - \frac{4p^2}{3m^* E_g}\right) \delta f_{\mathbf{p}}^{(1)}.$$
(3.28)

Here the first term does not contribute to the integral owing to (3.20), while the second term differs from zero only for spherically symmetric fluctuations of the electronic distribution function $\delta f_p^{(1)}$ and reduces to the energy fluctuation

$$\delta E = \int \frac{2\mathrm{d}^3}{(2\pi\hbar)^3} \frac{p^2}{2m^*} \,\delta f_p^{(1)}. \tag{3.29}$$

Raman scattering from energy fluctuations was first studied by Wolf,⁷ observed by Mooradian,²¹ and identified in Ref. 38 at T = 10 K from the strong resonance dependence of the scattering cross section, which in this review is contained in $\gamma_{ik}^{\xi\xi}$ from (2.19). However, for simplicity we assume here that $\gamma = 1/m^*(\varepsilon)$. Below we identify the energy fluctuations from the temperature dependence of the integral scattering cross section. Exact formulas for the resonance factors can be found in the review, Ref. 30. The cross section for scattering by energy fluctuations is derived by substituting (3.28) into (2.23) and has the form

$$\frac{\mathrm{d}\boldsymbol{\Sigma}^{(\varepsilon)}}{\mathrm{d}\Omega} = \left(\frac{e^2}{m^*c^2}\right)^2 \left(\frac{8}{3}\right)^2 |\mathbf{e}^{\mathbf{I}}\mathbf{e}^{*\mathbf{S}*}|^2 C_v \left(\frac{T}{E_g}\right)^2. \tag{3.30}$$

Here C_v is the electronic heat capacity at constant volume.¹⁹ We see from (3.30) that the integral scattering cross section of energy fluctuations has a strong temperature dependence. For nondegenerate statistics we have $C_v = 3nV/2$, and the cross section is proportional to T^{2} .^{7,30} When degenerate statistics sets in we have $C_v \sim nT/\zeta$, so that the cross section is proportional to the cube of the temperature. This gives rise to a considerable decrease in the cross section. When we take account of the higher powers in the expansion of the energy in (3.27), the temperature dependence of the cross section varies. In particular, in going to degenerate statistics, according to Ref. 30, a factor $\hbar q/mv_F$ arises, rather than C_v ; see Eq. (4.100) from Ref. 30. On the whole, for strong nonparabolicity the role of the discussed mechanism of Raman scattering is more significant.

The second term in (3.26) arises from the symmetric contribution to $\delta \chi_{ik}$ from (3.24), which we can represent with the aid of (3.27) in the form

$$\delta\chi_{ik}^{(S)} = -\frac{e^2}{m^*(\omega^1)^2} \cdot \frac{16}{E_g} \times \int \frac{2d^3p}{(2\pi\hbar)^3} \frac{p_i p_k - (p^2 \delta_{ik}/3)}{2m^*} \,\delta f_p^{(2)}.$$
 (3.31)

We see from (3.31) that $\delta \chi_{ik}^{(S)}$ differs from zero only for those fluctuations of the electron distribution function

 $\delta f_p^{(2)}$ that have the symmetry of the second spherical harmonic. The corresponding integral cross section has the form

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \left(\frac{e^2}{m^*c^2}\right)^2 \cdot \frac{8}{15} TV \left(\frac{\partial n}{\partial \zeta}\right)_T \left\langle \left(\frac{\varepsilon_p}{E_g}\right)^2 \right\rangle \\ \times \left(1 + \left|\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}}\right|^2 - \frac{2}{3} \left|\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}*}\right|^2\right).$$
(3.32)

Here the angle brackets denote an averaging performed while taking account of the Pauli principle, i.e., with the distribution function $(\partial f_0/\partial n)_T$. At T=0 this is an averaging over the Fermi surface as in a metal [see (3.21)]. Wolf⁷ also predicted theoretically the unscreened scattering of (3.32). Despite the relative smallness of the cross section that was obtained, caused by the numerical factors 8/15 in (3.32) and 64/9 in (3.30), its contribution must be taken into account at low temperatures in the scalar geometry $e^I || e^S$ The calculation performed in Ref. 39 with account taken of the resonance enhancement of both mechanisms yields the following for the frequency of an Nd:YAG laser with $\hbar \omega^I = 1.17$ eV:

$$\frac{\mathrm{d}\Sigma^{(\varepsilon)}}{\mathrm{d}\Sigma^{(p)}} = \left(\frac{10T}{\zeta}\right)^2. \tag{3.33}$$

Equation (3.33) implies a possibility of observing scattering by momentum fluctuations of (3.32) in materials with degenerate statistics. For example, for the semiconductors n-InP and GaAs at T=300 K, a change of Raman scattering mechanisms from (3.30) to (3.32) must occur at $n \approx 10^{18}$ cm⁻³.

3.6. Light scattering by spin-density fluctuations

The interaction of charge carriers with electromagnetic radiation also can serve as a source of scattering. The most effective mechanism arises from the spin-orbital interaction.⁴ Scattering by spin-density fluctuations was the first one-particle scattering by a solid-state plasma that could be observed experimentally.^{3,21} In the nonrelativistic theory⁴⁰ the spin arises as a result of taking account of the quantum-mechanical degeneracy of states. In the tensor $\hat{\gamma}_{ik}$ from (2.19), nondiagonal matrix elements appear here among the states of the spin subbands described by the values s=1/2 for electrons and J=3/2 for holes.

In the zero order of perturbation theory the spinorbital interaction is taken into account by a "choice" of the "correct" linear combinations of spinor wave functions.⁴⁰ Here the axis of quantization of the angular momentum is associated with the direction of the quasimomentum.¹⁰

The electronic Hamiltonian of the spin-orbital interaction has the form

$$\hat{H}_{\rm S0} = -\frac{i\hbar^2}{4m^2c^2} \ \vec{\sigma} \ [\nabla u, \nabla]. \tag{3.34}$$

Here $\vec{\sigma}$ is a vector composed of Pauli matrices as projections. When we take account of the Hamiltonian of (3.34),

the light is scattered by the fluctuations δn_{\uparrow} and δn_{\downarrow} associated with the individual spin subbands. Here the condition of neutrality can be fulfilled:

$$\delta n = \delta n_1 + \delta n_1 = 0. \tag{3.35}$$

The cross section for Raman scattering by spin-density fluctuations was calculated by Hamilton and McWhorter.^{4,30} It proved to be antisymmetric in e^{I} and e^{S} Therefore we should represent the susceptibility $\delta \chi_{ik}$ in the form of a sum of scalar, traceless symmetric, and antisymmetric contributions by analogy with Eq. (117.11) from Ref. 16:

$$\delta \chi_{ik} = \frac{1}{3} \delta \chi_{ll} \delta_{ik} + \frac{1}{2} [(\delta \chi_{ik} + \delta \chi_{ki}) - \frac{2}{3} \delta \chi_{ll} \delta_{ik}]$$

+ $\frac{1}{2} (\delta \chi_{ik} - \delta \chi_{ki}).$ (3.36)

The scalar term in $\delta \chi_{ik}$ makes no contribution to the Raman cross section, since this contribution is screened. The expressions for the second and third terms in (3.36) can be written by associating the matrix $\hat{\gamma}_{ik}$ from (2.19) with the operators for the angular momentum of the carriers.

3.6.1. Electrons In A³B⁵ semiconductors

Let us examine first the conduction band of symmetry Γ_6 . Here the role of the angular momentum is played by the spin s=1/2. Therefore the matrix $\hat{\gamma}_{ik}$ from (2.19) depends on the Pauli matrices $\hat{\sigma}_i$. It was shown in Ref. 40, p. 250, that an arbitrary function of Pauli matrices is reduced to a linear function. The most general form of a nonscalar linear function of Pauli matrices is reduced to the following:

$$\hat{\gamma}_{ii} = B_{\sigma} \delta_{iik} \hat{\sigma}_k, \qquad (3.37)$$

where δ_{ijk} is a unitary antisymmetric tensor, and B_{σ} is a phenomenological coefficient. Substituting (3.37) into (2.18), we obtain the following expression for the spin contribution to the susceptibility fluctuations:

$$\hat{\delta}\chi_{ij}^{(\alpha)}(\mathbf{q}) = \frac{e^2}{m(\omega^{\mathbf{I}})^2} B_{\sigma} \delta_{ijk} \hat{\sigma}_k(\mathbf{q}). \qquad (3.38)$$

Here $\hat{\sigma}_k(q)$ is the spin-density operator:

$$\hat{\sigma}_{k}(q) = \sum_{p} \sum_{\xi\xi'} \hat{a}_{\xi,p}^{+} + \frac{\hbar q}{2} \hat{a}_{\xi',p} - \frac{\hbar q}{2} \sigma_{k}^{\xi\xi'}, \qquad (3.39)$$

where $\xi = \pm 1/2$ is the index that numbers the degenerate spin subbands, and $\sigma_k^{\xi\xi'}$ is the corresponding element of the *k*th Pauli matrix. The microscopical expression for B_{σ} when p=0 has the form⁴

$$B_{\sigma} = \hbar \omega^{\rm I} \frac{2P_{cv}^2}{3m} \frac{\Delta(\Delta + 2E_{\rm g})}{[E_{\rm g}^2 - (\hbar \omega^{\rm I})^2][(E_{\rm g} + \Delta)^2 - (\hbar \omega^{\rm I})^2]}.$$
(3.40)

Here $P = \hbar \langle S | \nabla_x | x \rangle$ and Δ are the parameters of the Kane modulus.³⁷ The resonance dependence of B_{σ} on ω^{I} can be used to amplify the spectrum by choosing ω^{I} close to $E_{g} + \Delta$; see the reviews, Refs. 3, 30, and 31.

The convolution of the tensor $\delta \chi_{ik}$ with the polarization vectors of the incident and scattered light $e_i^I \delta \chi_{ik} e_k^S$



FIG. 4. Temperature dependences of the integral scattering cross section. Solid and dashed curves—contribution of fluctuations of density of energy and momentum, dot-dash curve—contribution of spin-density fluctuations. The corresponding selection rules are indicated in brackets. The experimental points were obtained at the frequency $\omega^{I} = 1.17$ eV.

enters into the cross sections of (2.15), (2.17), and (2.23). In it the antisymmetric Kronecker symbol δ_{ijk} converts the polarization vectors into a vector product. This yields

$$e_{i}^{\mathbf{I}}\delta\chi_{ij}e_{j}^{\mathbf{S}} = \frac{e^{2}}{m(\omega^{\mathbf{I}})^{2}} B_{\sigma} \sum_{p,\xi,\xi'} \hat{a}_{\xi,\mathbf{p}}^{+} \frac{\hbar\mathbf{q}}{2} \hat{a}_{\xi',\mathbf{p}} -\frac{\hbar\mathbf{q}}{2} \left([\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}\mathbf{*}}]\sigma^{\xi\xi'} \right).$$
(3.41)

Although in semiconductors described by the Kane model the nonparabolicity can be substantial, at small electron concentrations we can neglect it and use the expression (3.40) for B_{σ} . Under these conditions the axis of quantization of the spin can be chosen arbitrarily.⁴¹ It is convenient to make use of this arbitrariness by choosing the axis of quantization of the spin parallel to the vector product $[e^{I}e^{S}]$, i.e., having oriented the axis Oz in this direction. Then only the Pauli matrix σ_z remains in (3.41), reducing the convolution $e_i^{I}\delta\chi_{ik}e_j^{S}$ to the difference of populations of the spin subbands:

$$e_{i}^{\mathbf{I}}\delta\chi_{ij}e_{j}^{\mathbf{S}} = \frac{e^{2}}{m(\omega^{\mathbf{I}})^{2}} B_{\sigma}|[\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}}]|(\delta n_{\uparrow} - \delta n_{\downarrow}). \qquad (3.42)$$

Upon substituting (3.42) into (2.23), we obtain the integral cross section for scattering by spin-density fluctuations

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = (B_o r_0^2)^2 V T \left(\frac{\partial n}{\partial \zeta}\right)_T |[\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}*}]|^2.$$
(3.43)

The temperature dependences of the integral intensities of scattering by fluctuations of energy or momentum $I_{\varepsilon,p}$ and by spin-density fluctuations I_{σ} obtained by Eqs. (3.30), (3.32), and (3.43) are shown in Fig. 4. The solid and the dashed lines give the cross sections $I_{\varepsilon,p}$ calculated with and without account taken of the temperature dependence of the electronic heat capacity $C_v(T)$. The solid dots show the corresponding experimental results obtained by Baĭramov and Toporov for a specimen of n-InP with $n=1.1\times10^{18}$ cm⁻³ for parallel polarizations of the inci-

dent and the scattered light $\mathbf{e}^{\mathbf{I}} \parallel \mathbf{e}^{\mathbf{S}}$. The best agreement of the theory of (3.30)-(3.32) with experiment is attained upon taking account of the $C_v(T)$ relationship. The dotdash curve and the open experimental points give the cross section I_{σ} for crossed polarization $\mathbf{e}^{\mathbf{I}} \perp \mathbf{e}^{\mathbf{S}}$. The nonlinearity of the theoretical curve (3.43) is due to the increase in *n* with increasing temperature, which can be explained by the linear temperature narrowing of the width of the forbidden band. By fitting, it was possible to determine the values of the chemical potential ζ and the temperature coefficient of narrowing α :

$$\alpha = -\left(\frac{\partial E_g}{\partial T}\right)_p = 3.45 \frac{\text{eV}}{\text{K}}, \quad \zeta = 99 \text{ MeV}.$$

3.6.2. Multivalley semiconductor

In a multivalley semiconductor there is no possibility of free choice of the axis of quantization of the spin, since the valleys have the defined axis $v^{(\alpha)}$. Here the carriers of one valley are characterized by a common axis of quantization of the spin parallel to $v^{(\alpha)}$, while the different valleys correspond to different axes of quantization. The contribution of the fluctuations of spin density of the separate valleys corresponds to Eq. (3.42), while the final formula for $\delta \chi_{ik}$ is represented in the form of the summation

$$\delta\chi_{ij} = \delta_{ijk} \frac{e^2}{m(\omega^{\mathrm{I}})^2} B_{\sigma}^{(1)} \sum_{\alpha} \nu_k^{(\alpha)} (\delta n_{\uparrow}^{(\alpha)} - \delta n_{\downarrow}^{(\alpha)}). \quad (3.44)$$

The expression for the coefficient $B_{\sigma}^{(1)}$ that enters here is obtained from the corresponding expression for B_{σ} (3.40) by replacing E_{g} and Δ taken at the point Γ (p=0) with the corresponding values E_{g1} and Δ_{1} pertaining to the edge of the Brillouin zone where the valleys lie. The integral scattering cross section also differs from the expression (3.43) obtained above in an inessential numerical factor. To analyze the resonance dependence of the cross section of (3.44) on ω^{I} , it is important that the relative spin-orbital splitting at the edge of the Brillouin zone be considerably smaller than at the center. The expansion (3.40) in the small parameter $\Delta/E_{g} \ll 1$ yields

$$B_{\sigma}^{(1)} = \hbar \omega^{\mathrm{I}} \cdot \frac{4P_{cv}^{2}}{m} \frac{\Delta_{1}E_{\mathrm{g}1}}{[E_{\mathrm{g}1}^{2} - (\hbar \omega^{\mathrm{I}})^{2}]^{2}}.$$
 (3.45)

Comparison of $B_{\sigma}^{(1)}$ from (3.45) with the resonance enhancement factor R_{12} from (3.11) shows that the resonance dependence of the cross section on ω^{I} in the case of scattering by spin-density fluctuations is sharper than in the case of intervalley fluctuations.

Mestres *et al.*⁴² studied light scattering by spin-density fluctuations in n-Ge. The scattering mechanism was identified by using polarization measurements with the symmetry Γ_{12} , for which the scattering by intervalley fluctuations is forbidden by symmetry; see Eq. (3.16). The resonance dependence of $B_{\sigma}^{(1)}$ was well described by Eq. (3.45). Contreras *et al.*⁴³ studied scattering by intervalley fluc-

Contreras *et al.*⁴³ studied scattering by intervalley fluctuations in n-Ge, while using the application of pressure to the specimen to identify it. It is assumed rather often that Raman scattering by spin-density fluctuations is not sensitive to the presence of uniaxial deformation. However, the selection rules for Raman scattering that follow from (3.44) prove sensitive to deformations. If one applies to an n-Ge specimen pressure along the $\langle 111 \rangle$ axis, then a singlet valley is formed in this direction that lies lower in energy than the triplet valleys. Here, for the "backscattering" geometry used in Ref. 43 with **q** parallel to $\langle 1\overline{10} \rangle$, the convolution

$$e_{i}^{\mathbf{I}}\delta\chi_{ij}e_{j}^{\mathbf{S}} = \frac{e^{2}}{(m\omega^{\mathbf{I}})^{2}} B_{\sigma}^{(1)} \sum_{\alpha} \nu^{(\alpha)} [\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}}] (\delta n_{\uparrow}^{(\alpha)} - \delta n_{\downarrow}^{(\alpha)})$$
(3.46)

vanishes. Actually we have $\delta n_{\gamma}^{(\alpha)} - \delta n_{\gamma}^{(\alpha)} \neq 0$ only for the single filled singlet valley oriented along $\langle 111 \rangle$. At the same time the vector product $[e^{I}e^{S}]$ is directed along the perpendicular direction $\langle 1\bar{1}0 \rangle$. Therefore the scalar product in (3.46) equals zero: $v^{(\alpha)}$ $[e^{I}e^{S}]=0$. Thus, by separating out the singlet valley with the aid of external pressure, one can quench the scattering by spin-density fluctuations. The situation on the whole recalls the quenching of scattering by fluctuations of electron density in the presence of external pressure.⁴³ The resonance properties of the cross section from Ref. 43 also rather correspond to (3.45) than to Eq. (3.11) for intervalley fluctuations. On this basis we can conclude that in the spectra of n-Ge from Ref. 43 a more significant contribution of spin-density fluctuations exists than of intervalley fluctuations.

For completeness we should mention the study by Aronov and Ivchenko,⁴⁴ which took account of the small spin splitting of the electronic band that arises upon taking account of the spin-orbital interaction. This splitting exists only in semiconductors having no center of inversion. For the case of A^3B^5 semiconductors this splitting, as well as the spin relaxation arising from it, were described in the review, Ref. 41. The Hamiltonian of the spin-orbital splitting used in Ref. 41 has the form

$$\hat{H}_{s0} = \alpha [\hat{\sigma}_x \hat{p}_x (\hat{p}_y^2 - \hat{p}_z^2) + \hat{\sigma}_y \hat{p}_y (\hat{p}_z^2 - \hat{p}_x^2) + \hat{\sigma}_z \hat{p}_z (\hat{p}_x^2 - \hat{p}_y^2)].$$
(3.47)

Here α is a phenomenological constant. The substitution into (3.47) of $\hat{p} \rightarrow \hat{p} + (e/c)A$ and the separation of the terms quadratic in A allows one to obtain a Hamiltonian of the interaction H_{int} of electrons with photons equivalent to the first term in (2.19). The intersubband transitions described by this Hamiltonian determine by Eq. (2.6) the electronic scattering cross section. However, this cross section should be compared with the Raman cross section for spin-density fluctuations that was discussed above. This comparison shows that the former Raman scattering can be observed at low temperatures, small ω^{I} , and high concentrations n of electrons, at which the effects of nonparabolicity described by the Hamiltonian of (3.47) become substantial. The fundamental difference of the Aronov-Ivchenko scattering mechanism from the scattering by spin-density fluctuations discussed above consists in the fact that the former can be observed both when $e^{I} \perp e^{S}$ and when $e^{I} \parallel e^{S}$. Finally, we should note the recent observation of Raman scattering with spin reversal in a binary heterostructure.45



FIG. 5. Structure of the subbands of the valence band of p-Si.⁴⁶ The intersubband transitions corresponding to the boundaries of the Raman spectrum are indicated by arrows. Cross-hatched region—quantity of transitions near the threshold.

3.7. Mechanisms of light scattering in semiconductors having a degenerate energy spectrum

The hole bands in cubic (Ge,Si) and tetrahedral semiconductors (InP,GaAs) prove to be degenerate at the point Γ of the Brillouin zone and are described by the spinor representation Γ_8 of the cubic or tetrahedral groups. These zones at the point Γ have fourfold degeneracy, which is characterized by the four projections of the total angular momentum of a hole J=3/2. A displacement to $p \neq 0$ leads to splitting into bands of light $(J_z = \pm 1/2)$ and heavy $(J_z = \pm 3/2)$ holes. Thus the hole plasma of semiconductors proves to be of multicomponent type, which leads to a number of light-scattering mechanisms that do not exist for electrons. The earliest study on light scattering by holes in semiconductors is the already discussed study by Aronov and Ivchenko,⁴⁴ who discussed scattering associated with transitions between the subbands of light and heavy holes. The corrugation of the hole isoenergy surfaces is not taken into account in this study. The actual band structure of the hole subbands $\varepsilon_1(p)$ and $\varepsilon_p(p)$ for p-Si is shown in Fig. 5.46 The arrows in Fig. 5 show the boundaries ω_{\min} and ω_{\max} of the possible intersubband transitions. At frequencies smaller than ω_{\min} intersubband scattering is absent, while between ω_{\min} and ω_{\max} the cross section reaches a maximum. The study of Balkanskiĭ *et al.*⁴⁷ performed a numerical calculation of the cross section of the intersubband electronic scattering in p-Si that took account of the corrugation of the subbands and the nonparabolicity of the hole spectrum. This calculation showed that the presence of corrugation appreciably blurs the boundaries of the spectrum. The experimental study of scattering in p-Si,48 which was performed at room temperature, which also enhances the blurring, did not allow detecting either boundaries or a maximum. Lowering the temperature to T=2 K also did not lead to detecting them.⁴⁷ A maximum could be detected only in uniaxially deformed crystals of p-Si⁴⁸ in which the bands of light and heavy holes are split at p=0. However, even here a tail is

preserved in the low-frequency region $\omega < \omega_{\min}$, whose form recalls the analogous spectrum of n-Si.⁴⁸

All of this involves the fact that another unscreened scattering mechanism operates in the low-frequency region, associated with intraband fluctuations of the total angular momentum of the holes. To clarify the essence of this mechanism, we can take a simplified isotropic description of the hole bands within the framework of the isotropic Hamiltonian of Luttinger¹⁰

$$\hat{H}_{0} = \frac{1}{2m} \left[\gamma_{1} \hat{p}^{2} - \gamma(\hat{p} \hat{J})^{2} \right].$$
(3.48)

Here

$$\gamma_1 = \frac{1}{2} m \left(\frac{1}{m_1} + \frac{1}{m_h} \right)$$

and

$$\gamma = \frac{1}{4} m \left(\frac{1}{m_1} - \frac{1}{m_h} \right)$$

are the parameters of the Hamiltonian of Luttinger associated with the masses of the light and heavy holes m_1 and m_h . The wave functions of the Hamiltonian in (3.48) are four-component columns owing to the fourfold degeneracy of the hole bands at the point Γ of the Brillouin zone. Correspondingly the magnitudes of J here are four-row matrices in the quantum numbers of the hole bands. These matrices are four-component analogs of the Pauli matrices used above for the total angular momentum J=3/2. One can find them in explicit form, e.g., in the book, Ref. 9.

The interaction of the holes with the field of the electromagnetic wave is obtained by the standard substitution $\hat{p} \rightarrow \hat{p} + (e/c)A$ in the Luttinger Hamiltonian with subsequent separation of the terms linear and quadratic in A. The matrix $\gamma_{ik}^{\xi\xi}$ obtained as a result is associated in the standard way (see Sec. 3.1) with the projections J_i of the total angular momentum:

$$\gamma_{ik}^{\xi\xi} = \gamma_1 + \delta_{ik}\delta_{\xi\xi} + \gamma \left[\sum_{\eta} \left(J_i^{\xi\eta} J_k^{\eta\xi} + J_k^{\xi\eta} J_i^{\eta\xi} \right) - \frac{2}{3} J(J+1)\delta_{ik}\delta_{\xi\xi} \right] + B_j \delta_{ikj} J_j^{\xi\xi}.$$
(3.49)

The first term in γ_{ik}^{k} is a scalar. It does not contribute to the scattering because of screening. The second term, which has the symmetry of a traceless second-order tensor, in contrast to the case with spin s=1/2 (see Sec. 3.6), does not vanish, and contributes to the scattering cross section. The third term is the antisymmetric contribution to γ_{ik}^{\sharp} , which is analogous to the spin term from (3.37). The microscopical expressions for the coefficients γ_1 , γ , and B_i can be obtained from (2.19). In particular, the expression for the coefficient B_J is obtained by separating out the antisymmetric term in the square brackets in (2.19) (see Ref. 11). Since the denominators in (2.19) contain ω^{I} with opposite signs, in forming the antisymmetric contribution a factor arises of the order of ω^{I}/E_{g} . When $\omega^{I} \ll E_{g}$, the coefficient B_J becomes small, since it contains the small factor $\omega^{I}/E_{g} \ll 1$. In this sense B_{J} proves to be analogous to B_{σ} from Sec. 3.6. Since scattering by spin-density fluctuations has been treated in detail in Sec. 3.6, we shall focus attention below on the second, symmetric contribution, which is absent in the case of spin fluctuations. The symmetric traceless contribution to $\gamma_{ik}^{\xi\xi}$ has the form

$$\mathcal{Q}_{ik}^{\xi\xi} = \gamma \bigg[\sum_{\eta} \left(J_i^{\xi\eta} J_k^{\eta\xi} + J_k^{\xi\eta} J_i^{\eta\xi} \right) \bigg(-\frac{2}{3} J(J+1) \delta_{ik} \delta_{\xi\xi} \bigg].$$
(3.50)

We recall that ξ takes on the values $\pm 1/2$ and $\pm 3/2$. Equation (3.50) contains both diagonal and off-diagonal matrix elements in the stated indices of the hole subbands. The nondiagonal matrix elements of $Q_{ik}^{\xi\xi}$ describe the intersubband scattering in light-to-heavy-hole transitions discussed in Refs. 44, 47, and 48. The corresponding diagonal matrix elements describe the quasielastic intrasubband scattering by fluctuations of the total angular momentum density. We can obtain its contribution to $\delta\chi_{ik}$ by restricting the summation in (2.19) only to the terms diagonal in the symbols ξ . Here we obtain

$$\delta\chi_{ik} = \frac{e^2}{m(\omega^1)^2} \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \mathcal{Q}_{ik}^{\xi\xi} \delta f_{\xi,p}. \qquad (3.51)$$

Here $\delta f_{\xi,p}$ is the fluctuation of the distribution function in the subband with the projection of the angular momentum ξ . Substituting (3.51) into (2.23), we find the corresponding integral scattering cross section:

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = r_0^2 \sum_{\xi} \int \frac{2V \mathrm{d}^3 p}{(2\pi\hbar)^3} |Q_{ik}^{\xi\xi} e_i^1 e_k^{\mathbf{S} *}|^2 f_0(\xi, p) \times (1 - f_0(\xi, p)).$$
(3.52)

We see from (3.52) that the Raman cross section equals the sum of the additive contributions from the light (ξ =1/2) and heavy (ξ =3/2) holes. They are related as the densities of the corresponding states: $(m_1/m_h)^{3/2} \leq 1$. Therefore in the summation over ξ in (3.52) only the one term with $\xi = 3/2$ is essential. We find terms of two types under the sign of the squared modulus in (3.52). First are the terms of the form $J_i J_k$ with $i \neq k$, which have no diagonal matrix elements and hence do not contribute to the cross section of (3.52). Second are the perfect squares $\langle J_x^2 \rangle = \langle J_y^2 \rangle$ and $\langle J_z^2 \rangle$, which reduce simply to half-integral numbers. The dependence of the matrix element being discussed on the direction of p arises from the fact that the axis of quantization of the total angular momentum of the holes coincides with the direction of p. This dependence has the form

$$Q_{ik}^{3/2,3/2} e_i^{\mathbf{I}} e_k^{\mathbf{S}} = 2(e_x^{\mathbf{I}} e_x^{\mathbf{S}} + e_y^{\mathbf{I}} e_y^{\mathbf{S}}) \langle J_x^2 \rangle + e_z^{\mathbf{I}} e_z^{\mathbf{S}} \langle J_z^2 \rangle - \frac{5}{2} \mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}}$$
$$= 3 \frac{(\mathbf{p} \mathbf{e}^{\mathbf{I}}) (\mathbf{p} \mathbf{e}^{\mathbf{S}})}{p^2} - \mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}}.$$
(3.53)

Substituting (3.53) into (3.52) and calculating the angular integral, we obtain the polarization dependence characteristic of symmetric scattering:



FIG. 6. Structure of the conduction band of n-Si. The intersubband transitions corresponding to the boundaries of the Raman spectrum are indicated by arrows.

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \frac{3}{10} (\gamma r_0)^2 T V \left(\frac{\partial n}{\partial \zeta}\right)_T \left(1 + \left|\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}}\right|^2 - \frac{2}{3} \left|\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S} \star}\right|^2\right).$$
(3.54)

Equation (3.54) implies that the Thompson cross section of the scattering that we have discussed is determined by the effective mass of the light holes $m/\gamma \approx 4m_1$, while the number of scattering particles coincides with the number of heavy holes, which depends on the statistics of the carriers. This situation favorable to experiment is explained by the fact that scattering occurs by virtual transitions of holes between the heavy and light subbands, which are taken into account exactly by the matrix of (3.50)

The discussed mechanism of Raman scattering is the sole cause of scattering at low frequencies ω at low temperatures in semiconductors having a parabolic and isotropic spectrum in the presence of quantum-mechanical degeneracy of the bands. The best material satisfying these requirements is p-Ge.⁵⁰

3.8. Features of Raman scattering by electrons lying near the *x*-point of the conduction band of n-Si

The theory developed in the previous section is also applicable to n-type silicon, whose six valleys lie at a distance of 0.19 K_0 from the x-point of the Brillouin zone, where quantum-mechanical degeneracy⁵¹ of the bands of symmetry Δ_1 and Δ'_2 exists (Fig. 6). Here K_0 is the boundary wave vector of the Brillouin zone at the x-point. This degeneracy is substantial at a concentration *n* so high that the Fermi energy ε_F is comparable to the intersubband spacing ε_{Δ} at the point of the minimum of the valleys. The Hamiltonian \hat{H}_0 describing the electronic spectrum of silicon near the x-point has the form^{9,51}

$$\hat{H}_{0} = \frac{1}{m} \begin{bmatrix} \frac{1}{2} (\hat{p}_{\perp}^{2} \mu_{\perp} + \hat{p}_{z}^{2} \mu_{\parallel}), & \hat{p}_{x} \hat{p}_{y} \mu' \\ \hat{p}_{x} \hat{p}_{y} \mu', & \frac{1}{2} (\hat{p}_{\perp}^{2} \mu_{\perp} + \hat{p}_{z}^{2} \mu_{\parallel}) + P_{\hat{p}_{z}} \end{bmatrix}.$$
(3.55)

Here $P = \hbar \langle x_1 | \nabla_z | x_1 \rangle$, and μ' is the reciprocal effective mass, which equals

$$\mu' = \frac{2}{m} \sum_{l} \frac{\langle \Delta_1 | \hat{p}_y | \Delta_l \rangle \langle \Delta_l | \hat{p}_z | \Delta_2' \rangle}{E_{\Delta_l} - E_{\Delta_1}}, \qquad (3.56)$$

where the summation is performed over the bands of symmetry Δ_5 .^{9,51} The width of the optical forbidden band $E_g=4.3$ eV for n-Si substantially exceeds $\hbar\omega^{I}$ for the visible-light lasers used for Raman scattering. Therefore we can neglect the first term in (2.1).⁵ Substituting $\hat{p} \rightarrow \hat{p} + (e/c)A$ and separating out the terms quadratic in A, we obtain the Hamiltonian of the interaction

$$\hat{H}_{\rm int} = \frac{1}{m} \begin{bmatrix} \frac{1}{2} (A_{\perp}^2 \mu_{\perp} + A_{z}^2 \mu_{\parallel}), & A_{x} A_{y} \mu' \\ A_{x} A_{y} \mu', & \frac{1}{2} (A_{\perp}^2 \mu_{\perp} + A_{z}^2 \mu_{\parallel}) \end{bmatrix}.$$
(3.57)

The diagonal matrix elements of \hat{H}_{int} describe the Raman scattering by intervalley fluctuations discussed in Sec. 3.3. The nondiagnoal part of \hat{H}_{int} from (3.57), which has the symmetry Γ'_{25} , also yields an unscreened scattering mechanism to which the theory from Sec. 3.7 is applicable. We can obtain the corresponding Raman cross section by taking the diagonal matrix element of \hat{H}_{int} over the state corresponding to the Δ_1 subband and averaging the result while allowing for the Pauli principle. In the linear approximation in $(\varepsilon_F/\varepsilon_{\Delta})^2$ we obtain

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = \frac{r_0^2}{4\pi} TV \sum_{\alpha} \left(\frac{\partial n_{\alpha}}{\partial \xi}\right)_T v_k^{(\alpha)} |\delta_{ijk}| |e_i^{\mathrm{I}} e_j^{\mathrm{S}*} + e_j^{\mathrm{I}} e_i^{\mathrm{S}*}|^2 \left\langle \left(\frac{p_\perp^2}{2\pi k_0 P}\right)^2 \right\rangle.$$
(3.58)

Here p_1 is the component of the quasimomentum perpendicular to the axis of the valley; the averaging is performed with the distribution function $(\partial f_{0\alpha}/\partial n_{\alpha})_T$, where $f_{0\alpha}$ is the Fermi function of the α th valley. According to (3.58), in order that the α th valley contribute to the cross section, the polarization vectors e^I and e^S must have nonzero projections on the plane perpendicular to the axis of the valley. This property is reflected by the symbol $|\delta_{iik}|$.

Equation (3.58) enables us to explain certain deformation effects in Raman scattering in uniaxially compressed n-Si from Ref. 49. Table I enumerates the contributions to the cross section of (3.58) from the different valleys with different scattering geometries. The nonzero contributions are denoted with the word "Yes" in the fourth column.

If external uniaxial pressure shifts any of the valleys downward in energy, then the corresponding concentration increases and the scattering cross section of (3.58) also increases. This increase in the intensity of Raman scattering in n-Si under uniaxial deformations has been observed in Ref. 49.

A better geometry for detecting the deformational flare-up of Raman scattering might be the Z(Y',Z)X ge-

TABLE I. Γ'_{25} component of the spectrum of one-particle quasielastic light scattering in n-Si.

Polarization of spectrum	Valley	Polarization factor $ e_i^{I}e_j^{S} + e_j^{T}e_i^{S} ^2$	Are contributions of valleys present?	Relative value of cross section
<u>(Y',Z)</u>	100	$(e_{v}^{I}e_{z}^{S}+e_{z}^{I}e_{v}^{S})^{2}=1/2$	Yes	1
	010	$(e_x^{\mathbf{f}}e_x^{\mathbf{S}} + e_x^{\mathbf{f}}e_x^{\mathbf{S}})^2 = 1/2$	Yes	
	001	$(e_{x}^{I}e_{y}^{S}+e_{y}^{I}e_{x}^{S})^{2}=0$	No	
(Y',Y')	100	$2e_{v}^{I}e_{z}^{S}=0$	No	1
	010	$2e_{x}^{i}e_{z}^{s}=0$	No	
	001	$(2e_{x}^{\hat{I}}e_{y}^{\hat{S}})^{2}=0$	Yes	
(5,5)	100	$(e_{y}^{I}e_{z}^{S}+e_{z}^{I}e_{y}^{S})^{2}=1/18$	Yes	1/3
	010	$(e_{x}^{\mathbf{I}}e_{z}^{\mathbf{S}}+e_{z}^{\mathbf{I}}e_{x}^{\mathbf{S}})^{2}=1/18$	Yes	
	001	$(e_x^{I}e_y^{S}+e_y^{I}e_x^{S})^2=2/9$	Yes	
(5,5)	100	$(2e_{v}^{I}e_{z}^{S})^{2} = 2/3$	Yes	5/3
	010	$(2e_x^{f}e_z^{S})^2 = 1/3$	Yes	
	001	$(2e_{y}^{\hat{I}}e_{x}^{\hat{S}})^{2}=1/3$	Yes	
(ζ,ζ)	All valleys	$(2e^{i}e^{s})^{2}=4/9$	Yes	4/3

ometry with compression along $\langle 010 \rangle$. As we see from (3.15), in this geometry the scattering by intervalley fluctuations is forbidden. Therefore the cross section is completely determined by the second row of Table I.

There is a sole experimental geometry, the (ξ,ζ) geometry, in which the deformational effect is absent. It is shown below that in this geometry the different deformational effects have opposite signs and therefore compensate one another. Let us examine the Raman scattering by intervalley fluctuations in n-Si upon deformation along $\langle 111 \rangle$, which leaves the valleys of the semiconductor equivalent. In this case a deformation-induced anisotropy of the transverse mass of the valleys appears in the spectra. This is accompanied by complication of the polarization dependence of the cross section as compared with (3.15). This dependence is determined by the direct product of the symmetric tensors composed of the polarization vectors e^{I} and e^{S} (see Ref. 32). To find its most general form in a deformed crystal, let us expand the representation of the

group G_{σ} of the deformed crystal performed with the tensor $e_i^{I}e_k^{S} + e_k^{I}e_i^{S}$ into irreducible representations, and construct the corresponding basis functions φ_l^{xs} . They are given in Table II. In agreement with the general method of invariants^{9,32} we have

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = F(\omega) V \mathbf{r}_{0}^{2} \sum_{\varkappa, s, s'} \alpha_{ss'}^{\varkappa}(\omega) \sum_{l} \varphi_{l}^{\varkappa s}(\mathbf{e}^{\mathbf{I}}\mathbf{*}, \mathbf{e}^{\mathbf{S}}) \times \varphi_{l}^{\varkappa, s'}(\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}}\mathbf{*}).$$
(3.59)

Here κ gives the numbers of the different irreducible representations, and s and s' are representations of the same type. For generality we have presented in Table II the functions that allow one to calculate not only the Raman cross section in n-Si upon deformation along $\langle 111 \rangle$ (lefthand side of the table), but also in n-Ge under deformation along $\langle 100 \rangle$. However, we present the results of calculation only for n-Si, since a corresponding experiment

TABLE II. Basis functions of irreducible representations formed by the tensor $e_i^1 e_k^S + e_k^1 e_i^S$.

$G_{q,\sigma} = C_{3v}; q, \vec{\sigma} \parallel (111)$		$G_q = 0, \ \sigma = 0, \ q = 0$			$G_{q,\sigma} = C_{4v}; q, \vec{\sigma} \parallel (001)$		
$\varphi_l^{\Lambda_{\Gamma^s}}$	Λ,	Г	$\varphi_i^{\Gamma_i}$	Γι	Δ,	$\varphi_l^{\Delta_{\Gamma^{I}}}$	
e ¹ e ^s		r,	e ^l e ^s	r, —		e ^l e ^s	
$2e_{\xi}^{\dagger}e_{\xi}^{S}-e_{\xi}^{\dagger}e_{\xi}^{S}-e_{\eta}^{\dagger}e_{\eta}^{S}$			$\frac{1}{\sqrt{3}} \left(2e_z^{\dagger}e_z^{S} - e_z^{\dagger}e_z^{S} - e_y^{\dagger}e_y^{S} \right)$	/		$2e_z^{\dagger}e_z^{\rm S} - e_z^{\dagger}e_x^{\rm S} - e_y^{\dagger}e_y^{\rm S}$	
$e_{\xi}^{i}e_{\eta}^{s}+e_{\eta}^{1}e_{\xi}^{s}$		Γ ₁₂	$e_x^1 e_x^S - e_y^1 e_y^S$	Γ ₁₂	> A ₂	$e_x^1 e_x^S - e_y^1 e_y^S$	
$e_{\eta}^{1}e_{\zeta}^{1}+e_{\zeta}^{1}e_{\eta}^{S}$	A3 4	<u> </u>			A 4'2	$e_x^{I}e_y^{S} + e_y^{I}e_x^{S}$	
$e_{r}^{1}e_{r}^{S}-e_{r}^{T}e_{r}^{S}$		\sum_{r}	$e_x^{\dagger}e_y^{\rm S} + e_y^{\dagger}e_x^{\rm S}$			$e_x^{S}e_z^{I} + e_z^{S}e_x^{I}$	
$e_{\xi}^{1}e_{\zeta}^{S}+e_{\xi}^{1}e_{\xi}^{S}$		1 25	$e_{z}^{l}e_{y}^{S} + e_{z}^{l}e_{z}^{S}$ $e_{z}^{l}e_{y}^{S} + e_{y}^{l}e_{z}^{S}$	1 25	→ ^Δ 5	$e_y^{\rm S}e_z^{\rm I}+e_z^{\rm S}e_y^{\rm I}$	
The unit vectors of the c	oordinate svs	tems QXYZ	and OEnč are indicated in	Fig. 3. OE =	= (112)/√6.	1	

exists.⁴⁹ The functions $a_{11}^{\Lambda_1} = a_{12}^{\Lambda_1} = 0$ pertaining to the scalar representation Λ_1 equal zero owing to screening.

Since the correspondence rules shown in Table II with arrows associate the nonscalar unitary representation Λ_1 with the representation Γ'_{25} of the original cubic crystal, the corresponding function is proportional to the square of the deformation tensor. Consequently, to take full account of the lowering of symmetry upon deformation, we must write also the remaining functions $a_{ss'}^{\varkappa}$ to the same accuracy. They have the form

$$a_{22}^{\Lambda_1} = \frac{16}{81} \left(\frac{\mu' C' \sigma S_{44}}{\varepsilon_{\Delta}} \right)^2 \frac{1}{\pi} \left(\frac{\partial n}{\partial \zeta} \right)_T L(\omega), \qquad (3.60)$$

$$a_{11}^{\Lambda_3} = \frac{1}{9} \left(\mu_{\perp} - \mu_{\parallel} - \frac{4}{3} \frac{\mu' C' \sigma S_{44}}{\varepsilon_{\Delta}} \right)^2 \frac{1}{\pi} \left(\frac{\partial n}{\partial \zeta} \right)_T L(\omega),$$
(3.61)

$$a_{22}^{\Lambda_3} = \frac{1}{18} \left(\mu_{\perp} - \mu_{\parallel} + \frac{8}{3} \frac{\mu' C' \sigma S_{44}}{\varepsilon_{\Delta}} \right)^2 \frac{1}{\pi} \left(\frac{\partial n}{\partial \zeta} \right)_T L(\omega),$$
(3.62)

$$a_{12}^{\Lambda_3} = \frac{\sqrt{2}}{9} \left[-\frac{1}{2} \left(\mu_{\parallel} - \mu_{\perp} \right)^2 + \frac{2}{3} \mu' \frac{C' \sigma S_{44}}{\varepsilon_{\Delta}} \right] \times \left(\frac{8}{3} \frac{\mu' C' \sigma S_{44}}{\varepsilon_{\Delta}} + \mu_{\parallel} - \mu_{\perp} \right) \frac{1}{\pi} \left(\frac{\partial n}{\partial \zeta} \right)_T L(\omega).$$

$$(3.63)$$

Here C' is the intersubband matrix element of the operator for the deformation potential, S_{44} is the component of the deformation tensor, σ is the stress, and

$$L(\omega) = \frac{6/\tau}{\omega^2 + (6/\tau)^2}$$
(3.64)

is the Lorentzian factor determining the frequency dependence of the differential cross section (see Sec. 4.2.2). Substituting the polarization functions from Table II into (3.62) and neglecting the terms quadratic in the deformation, we can find the linear deformational increment to the cross section:

$$\frac{\mathrm{d}^{2}(\Sigma^{(6)}-\Sigma^{(0)})}{\mathrm{d}\omega\mathrm{d}\Omega} = r_{0}^{2}VF(\omega)\left(\frac{\partial n}{\partial\xi}\right)_{T}L(\omega)$$

$$\times \frac{(\mu_{1}-\mu_{\parallel})^{2}}{3\pi}\frac{\mu'C'\sigma S_{44}}{\varepsilon_{\Delta}}I(\mathbf{e}^{\mathrm{I}},\mathbf{e}^{\mathrm{S}}),$$
(3.65)

where the polarization factor $I(e^{I}, e^{S})$ equals

$$I(\mathbf{e}^{\mathbf{I}}, \mathbf{e}^{\mathbf{S}}) = \sum_{i \neq j \neq k} \left[e_i^{\mathbf{I}} e_i^{\mathbf{S}*} - \frac{1}{3} \left(\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}} \right) \right] \left(e_k^{\mathbf{I}} e_j^{*\mathbf{S}} + e_j^{\mathbf{I}} e_k^{\mathbf{S}} \right) + \text{complex conjugate.}$$
(3.66)

As Table II and Eq. (3.60) imply, in the (ξ,ζ) geometry the cross section is proportional to $a_{11}^{\Lambda_3}$. According to (3.60) the function $sa_{11}^{\Lambda_3}$ decreases with increasing pressure, and therefore the intensity of the spectrum must decline. However, this does not happen, and up to pressures $\sigma = 16$ kbar the spectrum remains unchanged. The most probable reason for this is that the corrections (3.65) and (3.58) compensate one another. Upon equating these two cross sections in order of magnitude, we can obtain

$$\frac{\mu' C' \sigma S_{44}}{\varepsilon_{\Delta}} = \left(\frac{\varepsilon_{\rm F}}{\varepsilon_{\Delta}}\right)^2. \tag{3.67}$$

Using the known parameters: $\varepsilon_{\Delta} = 0.5 \text{ eV}$, $\varepsilon_{\rm F} \approx \varepsilon_x = 0.1 \text{ eV}$, $C'/\varepsilon_{\Delta} \approx 8.3$, $S_{44} = 1.47 \times 10^{-11} \text{ M}^2/\text{H}$, and $\sigma = 16 \text{ kbar}$,⁵¹ we can obtain from (3.67) the magnitude of the reciprocal effective mass μ' , which is determined according to (3.56) as: $\mu' = 0.20$. The obtained value is smaller than the corresponding theoretical value $\mu' \approx \mu_{\perp}$ found in Ref. 51. This disagreement involves the sharp elevation of the Fermi level, which is expelled into the gap between the Δ_1 and Δ'_2 subbands that is formed upon deformation.

The deformation effects in light scattering in n-Si in the $(\xi\xi)$ - and $(\xi\xi)$ -geometries were calculated in Ref. 49, where results were obtained that coincide with (3.62) and (3.63), respectively. In the $(\xi\xi)$ geometry in Ref. 49 an increase in the cross section resulting from an applied deformation was experimentally detected, which was especially sharp at $n=10^{20}$ cm⁻³. Such an increase is explained by the fact that in this geometry the different deformational increments to the cross section add up. The large value of the cross section in deformed n-Si enables one to recommend the $(\xi\xi)$ -geometry and uniaxial compression in the $\langle 111 \rangle$ direction as means for detecting the signals of single-particle Raman scattering in specimens strongly doped by the ion-implantation method.⁵²

Up to now the function $a_{12}^{\Lambda_3}$ has not been studied experimentally, as is explained, apparently, by the fact that this function does not fit within the framework of the standard analysis using second-order scattering tensors.¹⁴

3.9. Taking account of corrugation of the hole subbands in the integral scattering cross section

In the presence of corrugation of the hole isoenergy surfaces, the symmetry of the Hamiltonian of (3.48) is lowered to cubic. The representation formed by the tensor Q_{ik} from (3.50) becomes reducible. In agreement with (3.8) the tensor of (3.50) breaks down into two independent tensors of Γ_{12} and Γ'_{25} symmetry. The tensor basis functions of the cited irreducible representations are found, e.g., in Ref. 9. Using them, we obtain the following expression for the diagonal part of the tensor $Q_{ii}^{\xi\xi'}$, which has the symmetry Γ_{12} :

$$Q_{ii}^{\xi\xi'} = \frac{2}{3} \gamma_2 [3(\hat{J}_i^2)^{\xi\xi'} - J(J+1)\partial_{\xi\xi'}].$$
(3.68)

The nondiagonal part having symmetry Γ'_{25} equals

$$\mathcal{Q}_{ik}^{\xi\xi'} = \gamma_3 \sum_{\eta} \left(J_i^{\xi\eta} J_k^{\eta\xi'} + J_k^{\xi\eta} J_i^{\eta\xi'} \right), \quad i \neq k.$$
(3.69)

As was shown in Sec. 3.7, to calculate the integral cross section one must take the diagonal matrix elements from (3.68) and (3.69) over the states ξ and ξ' in the subbands of light and heavy holes, then calculate their convolutions

and average the square of the convolutions over the corresponding distribution functions $(\partial f_0/\partial n)_T$ of the light and heavy holes. These calculations are more complicated than the calculation of (3.53), which leads from (3.52) to (3.54). The columns of the hole wave functions, which depend on the direction of **p**, can be found in Refs. 9 and 10. Retaining only the one contribution of the heavy holes, which is most essential according to the parameter $m_l/m_h \ll 1$, we obtain

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = 9r_0^2 VT \left(\frac{\partial n}{\partial \xi}\right)_T \left[\gamma_2^4 I_{\Gamma_{12}}(\mathbf{e}^{\mathrm{I}},\mathbf{e}^{\mathrm{S}}) \left\langle \frac{(p_x^2 - p_y^2)^2}{(\varepsilon_1(\mathbf{p}) - \varepsilon_h(\mathbf{p}))^2} \right\rangle + \gamma_3^4 I_{\Gamma_{25}}(\mathbf{e}^{\mathrm{I}},\mathbf{e}^{\mathrm{S}}) \left\langle \frac{4p_x^2 p_y^2}{(\varepsilon_1(\mathbf{p}) - \varepsilon_h(\mathbf{p}))^2} \right\rangle \right]. \quad (3.70)$$

In (3.70) $I_{\Gamma_{12}}$ and $I_{\Gamma'_{25}}$ denote the polarization factors from (3.15) and (3.16), respectively. They equal

$$I_{\Gamma_{12}}(\mathbf{e}^{\mathbf{I}},\mathbf{e}^{\mathbf{S}}) = \frac{2}{9} \sum_{i} \left| \sum_{k} (e_{i}^{\mathbf{I}} e_{i}^{\mathbf{S}*} - e_{k}^{\mathbf{I}} e_{k}^{\mathbf{S}*}) \right|^{2}, \qquad (3.71)$$
$$I_{\Gamma'_{25}}(\mathbf{e}^{\mathbf{I}},\mathbf{e}^{\mathbf{S}}) = \sum_{i>j} |e_{i}^{\mathbf{I}} e_{j}^{\mathbf{S}*} + e_{i}^{\mathbf{I}} e_{j}^{\mathbf{S}*}|^{2}.$$

Summing the expressions (3.71), we can obtain a linear combination of the squares entering into (3.32) and (3.54) for the polarization dependence of the cross section of symmetrical scattering:

$$I_{\Gamma_{12}}(\mathbf{e}^{\mathrm{I}},\mathbf{e}^{\mathrm{S}}) + I_{\Gamma'_{25}}(\mathbf{e}^{\mathrm{I}},\mathbf{e}^{\mathrm{S}}) = 1 + |\mathbf{e}^{\mathrm{I}}\mathbf{e}^{\mathrm{S}}|^{2} - \frac{2}{3} |\mathbf{e}^{\mathrm{I}}\mathbf{e}^{\mathrm{S}}|^{2}.$$
(3.72)

The cross section of (3.70) in the case of an isotropic hole spectrum goes over into (3.54). Therefore Eq. (3.72) implies that the averages entering into (3.70) must coincide for weak corrugation. The standard method (see, e.g., Ref. 53) of finding the corrections, which is applicable particularly to the integral cross section, is based on the approximate expansion of the radical in the parabolic dispersion law of holes

$$\varepsilon(\mathbf{p}) = \frac{1}{2m} \{ \gamma_1 p^2 \pm [4\gamma_2^2 p^4 + 12(\gamma_3^2 - \gamma_2^2) \\ \times (p_x^2 p_y^2 + p_x^2 p_z^2 + p_y^2 p_z^2)]^{1/2} \}$$
(3.73)

as a series in the parameter

$$\eta = \frac{\gamma_3^2 - \gamma_2^2}{\gamma_3^2 + \gamma_2^2}.$$
(3.74)

Here γ_2 and γ_3 are the parameters of the cubic Hamiltonian of Luttinger.⁹ The concrete values of the given parameter are $\eta = 0.284$ for Ge and $\eta = 0.863$ for Si. Restricting the treatment to the linear terms of the expansion (3.73), we shall write the averages entering into (3.70) in the form

$$\left\langle \frac{(p_x^2 - p_y^2)^2}{(\varepsilon_{\rm h}(p) - \varepsilon_{\rm l}(p))^2} \right\rangle = \frac{m^2}{15\bar{\gamma}^2} \left(1 + \frac{\eta}{7} \frac{\gamma_1 - \frac{7}{2}\bar{\gamma}}{\gamma_1 - 2\bar{\gamma}} \right), \quad (3.75)$$

$$\frac{4p_{x}^{2}p_{y}^{2}}{(\varepsilon_{h}(p)-\varepsilon_{l}(p))^{2}} = \frac{m^{2}}{15\bar{\gamma}^{2}} \left(1 - \frac{3}{7}\eta \frac{\gamma_{l} - \frac{1}{2}\bar{\gamma}}{\gamma_{l} - 2\bar{\gamma}}\right), \quad (3.76)$$

where $\bar{\gamma} = (\gamma_2^2 + \gamma_3^2)^{1/2}$. Since in (3.75) and (3.76) the small parameter η of (3.74) is multiplied further by the small numerical coefficient 1/7, the averages entering into (3.70) practically always coincide. Therefore, in the zero-order approximation in the parameter of (3.74), taking account of corrugation is reduced to introducing the coefficients γ_2^4 and γ_3^4 for the polarization factors $I_{\Gamma_{12}}$ and $I_{\Gamma'_{25}}$ in Eq. (3.70). The ratio of the Raman cross sections in the Γ'_{25} and Γ_{12} geometries equals

$$\frac{\mathrm{d}\Sigma^{\Gamma_{25}'}}{\mathrm{d}\Sigma^{\Gamma_{12}}} = \frac{\gamma_3^4}{\gamma_2^4} \gg 1. \tag{3.77}$$

For p-Si, in particular, the ratio γ_3/γ_2 is close to 10, while the parameter of (3.77) is as much as 10⁴. This means that the practically observable Raman spectrum has the symmetry of the selection rules of Γ'_{25} . In the polarization corresponding to Γ_{12} symmetry scattering is absent. This result enables explaining the selection rules found in the spectra of bulk p-Si⁴⁸ and in superlattices (see Sec. 5.3).

It is of interest to compare the cross section of (3.70) with the cross section obtained in Ref. 48 after substituting the hole dispersion law (3.73) into (3.24) and (2.23). From this substitution we obtain a formula that is a special case of Eq. (3.21). Calculation of the ratio $d\Sigma^{\Gamma'_{25}}/d\Sigma^{\Gamma_{12}}$ by the described method yields

$$\frac{\mathrm{d}\Sigma^{\Gamma_{25}'}}{\mathrm{d}\Sigma^{\Gamma_{12}}} = \left(\frac{\gamma_3^2}{\gamma_2^2} - 1\right)^2. \tag{3.78}$$

We note that one must not use Eq. (3.78) in the case of weak anisotropy of the hole spectrum. For strong anisotropy Eqs. (3.77) and (3.78) practically coincide.

Corrugation was not taken into account in the cross section for intersubband scattering from the study of Aronov and Ivchenko.⁴⁴ When it is taken into account the cross section is determined by the sum of the squares of the moduli of the convolutions of the nondiagonal matrix elements of (3.68) and (3.69) between states in the subbands of light and heavy holes. After statistical averaging with allowance for the Pauli principle we obtain

$$\frac{\mathrm{d}\Sigma}{\mathrm{d}\Omega} = 3r_0^2 \int \frac{2Vd^3p}{(2\pi\hbar)^3} f_{0\mathrm{h}}(\mathbf{p}) [1 - f_{0\mathrm{l}}(\mathbf{p})] \\ \times \left[\gamma_2^2 \left[1 - \frac{3\gamma_2^2(p_x^2 - p_y^2)}{m^2(\varepsilon_1(\mathbf{p}) - \varepsilon_\mathrm{h}(\mathbf{p}))^2} \right] I_{\Gamma_{12}}(\mathbf{e}^{\mathrm{I}}, \mathbf{e}^{\mathrm{S}}) \right. \\ \left. + \gamma_3^2 \left[1 - \frac{12\gamma_3^2 p_x^2 p_y^2}{m^2(\varepsilon_1(\mathbf{p}) - \varepsilon_\mathrm{h}(\mathbf{p}))^2} \right] I_{\Gamma_{25}'}(\mathbf{e}^{\mathrm{I}}, \mathbf{e}^{\mathrm{S}}) \right].$$

$$(3.79)$$

The expressions entering into (3.79) in square brackets prove after integration to be of the same order of magnitude. Therefore we can estimate from (3.79) the ratio of the cross sections Γ'_{25} and Γ_{12} for intersubband scattering in the following way:

$$\frac{\mathrm{d}\Sigma^{\Gamma_{25}}}{\mathrm{d}\Sigma^{\Gamma_{12}}} \sim \frac{\gamma_3^2}{\gamma_2^2} \,. \tag{3.80}$$

The numerical calculation of the spectrum of intersubband scattering in p-Si from Ref. 47 gives a ratio of cross sections close to (3.80) in order of magnitude.

Thus the corrugation of the isoenergy surfaces of holes leads to the appearance of a substantial difference in the spectra of Γ'_{25} and Γ_{12} scattering. The ratio of these cross sections for intrasubband scattering is two orders of magnitude larger than for intersubband scattering. Both types of intersubband scattering are observed in superlattices. Comparison of the temperature dependences of the cross sections of intra- and intersubband Raman scattering in (3.70) and (3.79) shows that decreasing temperature favors the observation of the latter. Within the framework of the isotropic model of the valence band, the ratio of cross sections is estimated to be

$$\frac{\mathrm{d}\Sigma^{\mathrm{inter}}}{\mathrm{d}\Sigma^{\mathrm{intra}}} \geq 8,$$

where the equality sign corresponds to Maxwellian statistics.

4. FLUCTUATIONAL AND KINETIC PARAMETERS OF SEMICONDUCTORS MANIFESTED IN RAMAN SPECTRA

4.1. Form of the spectrum of scattering by charge-density fluctuations at low electron concentrations

The frequency dependence of the differential scattering cross section contains information on the relaxation kinetics of the fluctuations that scatter light. Naturally the form of the spectrum here depends on the type of fluctuations at which scattering occurs.

At low concentrations the condition (3.1) is satisfied, which means that screening effects are absent in the spectra. Here the scattering occurs at one-particle excitations,^{30,31} while collective plasma excitations are absent.

On the other hand, the spectra are usually taken at room temperature, at which there is a sufficient number nof electrons in the conduction band to fulfill the condition of strong screening of ionized impurities:

$$nr_{\rm s}^3 \gg 1. \tag{4.1}$$

The region of electron concentrations and temperatures in which the conditions (3.1) and (4.1) are compatible is discussed below (see below, Fig. 10). Shklovskiĭ and Éfros⁵⁴ showed that, when the condition (4.1) is fulfilled in crystals, only the large-scale impurity potential remains, having a characteristic dimension of the order of r_s and the rms value

$$\gamma = U(r_{\rm s}) (Nr_{\rm s}^3)^{1/2}.$$
 (4.2)

Here $U(r) = e^2/\varepsilon r$ is the potential of an isolated impurity, and N is the concentration of impurities. The one-particle electronic excitations created upon light scattering are scattered by the large-scale potential. Since the thermal momentum of the carriers $p_t = (2mT)^{1/2}$ far exceeds the light momentum $\hbar g$ at room temperature, we have

$$\frac{p_t r_s}{\hbar} \ge 1. \tag{4.3}$$

The parameter of (4.3) implies that the potential of the impurities deflects the carriers mainly at small angles $\theta \sim \hbar/pr_s < 1$. In view of the condition (4.3), the de Broglie wavelength of an electron $\lambda_{dB} = p_t/\hbar < r_s$ is far smaller than the characteristic scale of the impurity potential wells r_s . Therefore we can consider the impurity potential to be quasiclassical. The motion in this potential occurs under the action of a constant random force, ⁵⁵ and the classical displacement of an electron $\Delta r(\tau)$ in the time τ has the form

$$\Delta \mathbf{r}(\tau) = \mathbf{v}\tau + \frac{\mathbf{F}\tau^2}{2m},\tag{4.4}$$

where F is the force acting on the electron. The presence of impurities in the crystal also slightly alters the chemical potential of the electron ζ . Thus, in the expression for the scattering cross section (3.4) we should average over the set of three random quantities: ζ , F, and the electron velocity v. For a degenerate statistics, the fluctuations of the chemical potential are the most substantial. Their contribution is shown by curve 3 in Fig. 1. For nondegenerate statistics, the overall averaging is performed by using the following formula:

$$\langle e^{iq\Delta r(\tau)} \rangle = \int d\zeta \int \frac{2d^3p}{(2\pi\hbar)^3} \int d^3F f_0(p) \Phi(\zeta,F)$$
$$\times \exp\left[iq\left(v\tau + \frac{F}{2m^*}\tau^2\right)\right]. \tag{4.5}$$

Here

$$f_0(p) = \frac{n}{(2\pi m^* T)^{3/2}} \exp\left(\frac{\zeta - \varepsilon_p}{T}\right)$$

is the Maxwell-Boltzmann distribution function, and $\Phi(\zeta, F)$ is the distribution function of the random quantities ζ and F. Under the condition (4.1) the fluctuations of ζ and F are small. Therefore they are described by the Gaussian distribution function

$$\Phi(\zeta,F) = \frac{1}{2\pi^2} \frac{1}{\gamma \rho_0^{3/2}} \exp\left[-\frac{1}{2} \left(\frac{\zeta^2}{\gamma^2} + \frac{F^2}{3\rho_0}\right)\right], \quad (4.6)$$

where γ^2 and

$$\rho_0 = N \int \mathrm{d}^3 r F^2(r) \tag{4.7}$$

are the corresponding rms fluctuations. Substitution of Φ from (4.6) into (4.5) leads, after integration and calculation of the imaginary component of (3.4) to a spectrum with the form

$$\frac{\mathrm{d}^2 \Sigma}{\mathrm{d}\omega \mathrm{d}\Omega} = V n \left(\frac{e^2}{m^* c^2}\right)^2 (\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}})^2 \int \frac{\mathrm{d}t}{2\pi} \cos(\omega t)$$



FIG. 7. Theoretical spectra of one-particle scattering by charge-density fluctuations obtained by numerical integration of Eq. (4.8). The numbers on the curves indicate the values of the dimensionless concentration of impurities *a* from (4.10).

$$\times \exp\left\{-\left[\left(\frac{qv_Tt}{2}\right)^2 + \rho_0 \frac{q^2 t^4}{24m^2}\right]\right], \qquad (4.8)$$

where v_T^2 . If in (4.8) we neglect the term containing ρ_0 in the exponential, then the integral over *t* is taken in analytic form and yields a Gaussian form of the scattering spectrum with the half-width $\Gamma_G = qv_T$:

$$\frac{\mathrm{d}^2 \Sigma}{\mathrm{d}\omega \mathrm{d}\Omega} = Vn \left(\frac{e^2}{m^* c^2}\right)^2 (\mathbf{e}^{\mathrm{I}} \mathbf{e}^{\mathrm{S}})^2 \frac{1}{q v_T} \exp\left(-\frac{\omega^2}{q^2 v_T^2}\right). \quad (4.9)$$

This Raman cross section is known for a classical atomic plasma.⁵⁶ Its broadening reflects the velocity distribution of the atoms of the gas. The spectra obtained by numerical integration of Eq. (4.8) by the formula of Ermitt⁵⁷ are shown in Fig. 7. We see that the half-width Γ of the spectra increases with increase in the dimensionless concentration

$$a = N \frac{\pi \int (r \nabla U)^2 \mathrm{d}^3 r}{24 (T^2 q)}.$$
 (4.10)

Using a Gaussian approximation of the spectra in Fig. 7, we obtained the $\Gamma(N)$ relationship shown in Fig. 8a by the line of points. The crosses show the experimental values of the half-widths determined from the spectra shown in Fig. 9, taken from Ref. 58. The more detailed studies⁵⁸ showed that the experimental points better fit a logarithmic $\Gamma(N)$ dependence than a linear one. This is reflected by the dots in Fig. 8b.

If we extrapolate in Figs. 8a, b to small concentrations, then the straight line cuts off on the axis of ordinates a constant contribution to Γ , which, according to (4.9) should coincide with the broadening of the Gaussian curve



FIG. 8. a—The theoretical concentration dependence of the half-width $\Gamma_{\rm G}$ of the spectra in Fig. 7 at the 1/e level obtained by Gaussian approximation of Eq. (4.8). The experimental points are indicated by crosses. (From Ref. 25). b—The experimental concentration dependence of the half-widths $\Gamma_{\rm G}$ determined in the same way as in Fig. 7.



FIG. 9. Experimental spectra of n-InP for the range of low electron concentrations, illustrating the dependence of the form of the spectral line on the electron concentration $n \approx N$ (solid lines). The lines of dots obtained by subtracting the difference overtone of two-phonon scattering represent the pure electronic spectra.



FIG. 10. Region of electron concentrations and temperatures (crosshatched) in which a Gaussian contour of a collision-free plasma in strongly doped semiconductors is realized.

for free electrons $\Gamma = qv_T$. However, the numerical estimates of $\Gamma = qv_T$ for InP show that the experimental value $\Gamma = 30$ cm⁻¹ differs from the theoretical value $\Gamma = 50$ cm⁻¹. This difference can be associated with the increase in the mass of an electron owing to nonparabolicity up to 0.1 m. Usually in InP one uses the value $m^* = 0.07 m$. The graphs in Figs. 8a, b can serve for determining N (from the slope angle of the straight line). This method of measuring the concentration of impurities N is necessary in cases in which this concentration does not coincide with the concentration n of electrons. This is the situation in commercial specimens in which a low concentration of conduction electrons is attained by introducing compensating impurities.

We note that, although isotropy of the electronic spectrum was assumed above, the obtained results can be generalized to the case of ellipsoidal isoenergy surfaces. To do this we should make the following substitutions in Eqs. (4.5) and (4.6):

$$q \to \mu_{\parallel}^{1/6} \mu_{\perp}^{1/3} (q_{\sharp} \mu_{ik} q_{k})^{1/2},$$

$$n \to n/s m^{\ddagger} \to \mu_{\parallel}^{1/3} \mu_{\perp}^{2/3}.$$
 (4.11)

Now let us discuss the boundaries of applicability of the obtained results. The regime that we have studied of one-particle scattering is realized in the cross-hatched region of concentrations and temperatures in Fig. 10. The bound on the side of low concentrations is set by the inequality (4.1). In InP, e.g., at T = 300 K and $n = 10^{14}$ cm⁻³, the parameter of (4.1) equals $nr_s^3 = 7$. On the highconcentration side the bound is determined by the inequality (3.1). According to Sec. 3.1, breakdown of the condition (3.1) is accompanied by the appearance in the polarized scattering spectra of a plasmon peak. This process is discussed in Sec. 4.5. At the limiting concentration $n = 10^{16}$ cm⁻³ in n-InP the experimental scattering spectrum for parallel polarizations of the incident and scattered light contains a broad plateau that extends from $\omega = 0$ to the plasma frequency $\omega_{\rm p}$ (see below, Fig. 18). The experimental points in Figs. 8a, b corresponding to the highest concentrations $n = 10^{16}$ cm⁻³ were taken with crossed polarization $e^{I} \perp e^{S}$, in which plasmons are forbidden by the selection rules (3.7).

The lower temperature bound in Fig. 10 is determined by the effect of freezing-out of carriers T_f , while the upper bound Θ_D is determined by the influence of electronphonon interaction. Interaction with acoustic phonons leads only to small-angle scattering of carriers; therefore taking account of them in Eqs. (4.6)-(4.9) is reduced to renormalizing the constants γ and ρ_0 , as in the case of impurities.

Light scattering at temperatures above Θ_D is governed by diffusional processes of relaxation of electron-density fluctuations. The condition for realizing this regime is

$$ql \leqslant 1, \tag{4.12}$$

where l is the mean free path of an electron with respect to interaction with optical phonons. In this case the mean of (4.5) equals

$$\langle \exp(i\mathbf{q}\Delta r)\rangle = \exp\left(-q^2 \frac{\langle\Delta x^2(\tau)\rangle}{2}\right),$$
 (4.13)

where $\Delta x^2(\tau) = 2D\tau$, and D is the electron diffusion coefficient. Substituting (4.13) into (3.4), we obtain the scattering cross section in the form of a Lorentzian contour

$$\frac{\mathrm{d}^2\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = nV \left(\frac{e^2}{m^*c^2}\right)^2 (\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}})^2 \frac{q^2D}{\omega^2 + (q^2D)^2}.$$
 (4.14)

Broadening of spectral lines of Lorentzian type, which reflects the diffusional behavior of the carriers, was first obtained by Sobel'man (see, e.g., Ref. 59) for a classical atomic plasma. The reason for conversion of the Gaussian contour of Raman scattering of (4.9) into the Lorentzian contour of (4.14) is the frequent collisions of electrons with optical phonons, which realize the condition (4.12).

Quasielastic scattering was recently observed in the superionic glasses $(AgI)_x(AgPO_3)_{1-x}$. The scattering spectrum recalled in its behavior the spectrum of electronic Raman scattering. A considerable increase was observed in its intensity and a broadening with increased concentration x of the superionic component.⁶⁰

4.2. The form of the light-scattering spectrum of oneparticle excitations in a multicomponent system of current carriers.

4.2.1. A photoexcited electron-hole plasma

We noted above that the cross section for electronic excitation is rather small; therefore most experiments have been performed at high concentrations of current carriers; see the reviews.^{3,30} Under the conditions of strong screening of (3.6), scattering can occur only from fluctuations that satisfy the condition of neutrality (3.20). In a number of cases the satisfaction of condition (3.20) involves the existence of a multicomponent system of current carriers.

A very simple system of electrons and holes with an isotropic spectrum is the good model of a photoexcited plasma in semiconductors.^{22,23,69,70} The fluctuation of the dielectric susceptibility from (3.2) includes the contributions of electrons and holes:

$$\delta \chi = -\left(\frac{e^2}{m_{\rm e}(\omega^{\rm I})^2} \,\delta n_{\rm e} + \frac{e^2}{m_{\rm h}(\omega^{\rm I})^2} \,\delta n_{\rm h}\right). \tag{4.15}$$

Since in photoexcitation the current carriers are created in pairs, the local neutrality condition holds that

$$\delta n_{\rm e} = \delta n_{\rm h} = \delta n. \tag{4.16}$$

The condition (3.6) implies that a multitude of impurity potential wells is packed within the wavelength of the electronic excitation q^{-1} . This means that, to calculate the spectrum, one must take account of the scattering of carriers at arbitrary angles. Here the condition (4.12) is usually satisfied, under which the regime of motion of the carriers is diffusional. Therefore, to describe the relaxation of the light-scattering fluctuations, one can use the macroscopic transport equations. In a multicomponent system, as is known (Ref. 61, Sec. 25), the diffusion of the two types of charged particles—electrons and holes—occurs under the action of the electric field *E* that arises in the diffusion process itself. Therefore the system of equations of continuity and diffusion has the following form [see Ref. 61, p. 135, Eqs. (25.1) and (25.2)]:

$$\frac{\partial}{\partial} t \delta n_{e} + \frac{1}{e} \operatorname{div} \delta \mathbf{j}_{e} = 0, \quad \frac{\partial}{\partial t} \delta n_{h} + \frac{1}{e} \operatorname{div} \delta \mathbf{j}_{h} = 0,$$

$$\delta \mathbf{j}_{e} = -\sigma_{e} \mathbf{E} - e D_{e} \operatorname{grad} \delta n_{e}, \qquad (4.17)$$

$$\delta \mathbf{j}_{h} = \sigma_{h} \mathbf{E} - e D_{h} \operatorname{grad} \delta n_{h}.$$

Here $\delta \mathbf{j}_{e}$ and $\delta \mathbf{j}_{h}$ are the fluctuational currents created by the fluctuations of the concentrations of electrons and holes. The initial conditions for the equations that we have written are the values of the simultaneous correlators determined by the external conditions under which the semiconductor exists. These simultaneous correlators are calculated in a purely thermodynamic way. It is precisely they which we discussed in Sec. 3 in calculating the integral scattering cross sections. In the case of electrons and holes forming two equilibrium subsystems, the simultaneous correlator has the following known value¹⁹ [Sec. 113, Eq. (113.2)]:

$$\langle \delta n_{\rm e}^2 \rangle_{q \to 0} = \langle \delta n_{\rm h}^2 \rangle_{q \to 0} = \langle \delta n_{\rm e} \delta n_{\rm h} \rangle_{q \to 0} = T \left(\frac{\partial n}{\partial \zeta} \right)_T.$$
(4.18)

Here $n_e = n_h = n$ is the common value of the concentrations of electrons and holes. Upon performing a one-sided Fourier transform in time and a complete Fourier transform in the coordinates in (4.17) and taking account of the initial condition (4.18) as well as the condition of neutrality (4.16), we obtain

$$(-i\omega + q^{2}D_{e})(\delta n^{+})_{q,\omega}^{+} - \frac{i}{e}\sigma_{e}\mathbf{q}(\mathbf{E}\delta n)_{q,\omega}^{+} = T\left(\frac{\partial n}{\partial\xi}\right)_{T},$$

$$(4.19)$$

$$(-i\omega + q^{2}D_{h})(\delta n^{2})_{q,\omega} + \frac{i}{e}\sigma_{h}\mathbf{q}(\mathbf{E}\delta n)_{q,\omega}^{+} = T\left(\frac{\partial n}{\partial\xi}\right)_{T}.$$

$$(4.20)$$

Here the symbol $(\delta n^2)_{q,\omega}^+$ denotes the one-sided time Fourier transform of the correlator of the density fluctuations.

Solving the system (4.19), (4.20) for this correlator and calculating its real component Re $(\delta n^2)_{q,\omega}^+$, we obtain for the scattering cross section

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = (\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}})^{2} \frac{e^{4}}{2\pi c^{4}} \left(\frac{1}{m_{\mathrm{e}}} + \frac{1}{m_{\mathrm{h}}}\right)^{2} \left(\frac{\partial n}{\partial \zeta}\right)_{T} F(\omega) \frac{q^{2}D_{\mathrm{a}}}{\omega^{2} + (q^{2}D_{\mathrm{a}})^{2}}.$$
 (4.21)

Here D_a is the ambipolar diffusion coefficient, which equals (see Ref. 51, Chap. 5, Sec. 2)

$$D_{\rm a} = 2 \frac{n}{(\partial n/\partial \zeta)_T} \frac{b_{\rm e} b_{\rm h}}{e(b_{\rm e} + b_{\rm h})}, \qquad (4.22)$$

where

$$b_{\rm e} = \sigma_{\rm e(h)} / en_{\rm e(h)}$$

is the mobility of electrons (or holes). In Ref. 69 (Fig. 2) a result equivalent to (4.21) and (4.22) was obtained by a complicated numerical integration.

4.2.2. Electrons in multivalley semiconductors

Another example of a multicomponent plasma is the carriers in multivalley semiconductors. The mechanisms of the observable scattering are identified by applying external pressure (see Secs. 3.6.2 and 3.8). Therefore we must take account here of the nonequivalence of the valleys. On the other hand, with the model of nonequivalent valleys we can describe "pockets" on the Fermi surface of certain semimetals such as Bi, Sb, and As. In describing the Raman spectrum of electrons in the cited semimetals and multivalley semiconductors we should write the diffusion equations with account taken of intervalley transitions of electrons that occur between nonequivalent anisotropic valleys.⁶² These transitions are described by the matrix of the intervalley collisions $\hat{I}_{\alpha\beta}$. Otherwise the equations of continuity and diffusion correspond to the system of (4.17):

$$\frac{\partial}{\partial t}\delta n_{\alpha} + \frac{1}{e}\operatorname{div}\delta j_{\alpha} = \sum_{\beta} I_{\alpha\beta}\delta n_{\beta}, \qquad (4.23)$$

$$\delta f_{i\alpha} = -e D_{ik}^{(\alpha)} \operatorname{grad}_k \delta n_\alpha + \sigma_{ik}^\alpha E_k.$$
(4.24)

Here $D_{ik}^{(\alpha)}$ and $\sigma_{ik}^{(\alpha)}$ are the tensors of the diffusion and conductivity coefficients of the electrons of valley α . Sometimes it is convenient to transform the column of variables $\{\delta n_{\alpha}\}_{\alpha=1}^{s}$ in the system (4.23) and (4.24) by separating out the density of states at the Fermi level by analogy with (3.18):

$$\delta n_{\alpha} = \left(\frac{\partial n_{a}}{\partial \zeta}\right)_{T} \Psi_{\alpha}. \tag{4.25}$$

The system (4.23) and (4.24), which is written with respect to Ψ_{α} , contains the collision matrix $I_{\alpha\beta}^{c}$, which is symmetric with respect to α and β . In scattering by impurities this matrix has the form

$$\hat{I}_{inter}^{c} = \frac{\mathbf{S}}{\tau_{inter}} \left(\frac{\partial n_{a}}{\partial \zeta} \right)_{T} \left[\delta_{\alpha\beta} - \frac{(\partial n_{\beta}/\partial \zeta)_{T}}{(\partial n/\partial \zeta)_{T}} \right], \quad (4.26)$$

where τ_{inter} is the characteristic time of intervalley transitions. We note that this matrix has the form of the difference of departure and arrival terms and satisfies the law of conservation of the number of particles in intervalley transitions:

$$\sum_{\alpha} I^{c}_{\alpha\beta} = \sum_{\beta} I^{c}_{\alpha\beta} = 0.$$

The equilibrium concentration of electrons n_{α} is determined in the standard way (see, e.g., Ref. 43) by their chemical potential ζ . The equations that we have written must be supplemented by the initial condition for the simultaneous correlator $(\delta n_{\alpha}\phi n_{\beta})_{q\to 0}$, which, upon taking account of the condition of neutrality

$$\sum_{\alpha} \delta n_{\alpha} = 0 \tag{4.27}$$

acquires the form

$$\langle \delta n_{\alpha} \delta n_{\beta} \rangle_{\mathbf{q} \to 0} = T \left(\frac{\partial n_{\alpha}}{\partial \zeta} \right)_{T} \left[\delta_{\alpha\beta} - \frac{(\partial n_{\beta}/\partial \zeta)_{T}}{(\partial n/\partial \zeta)_{T}} \right].$$
 (4.28)

We note that the transition from Eq. (3.14) to (4.28) is discussed in the book, Ref. 61 (Sec. 20, p. 115).

4.2.2-1. The differential Raman cross section in semiconductors with equivalent valleys. In the semiconductors n-Si and n-Ge under deformations along the directions $\langle 111 \rangle$ and $\langle 100 \rangle$, respectively, the valleys remain equivalent (see Sec. 3.8). They are the same also in the absence of deformation. In this case the collision integral (4.26) in the system (4.23), (4.24), and (4.27) acquires the form of an approximation of the relaxation time, while we can replace the initial condition with a simpler one, reverting from (4.28) to (3.14). To calculate the differential section it is convenient to introduce the function

$$G_{\alpha}^{+}(\mathbf{q}\omega) = \sum_{\beta} \left(\delta n_{\alpha} \delta n_{\beta} \right)_{q,\omega}^{+} \mu^{(\beta)}, \qquad (4.29)$$

where $\mu^{(\alpha)} = e_{ik}^{I} \mu_{ik}^{(\alpha)} e_{k}^{S}$. Performing a one-sided Fourier transformation of (4.23) and (4.24) over the time, with account taken of (4.27) and (4.28), we find

$$G_{\alpha}^{+}(\mathbf{q},\omega) = T\left(\frac{\partial n}{\partial \zeta}\right)_{T} \left[\frac{\mu^{(\alpha)}}{-i\omega + q^{2}D_{\alpha} + (S/\tau_{\text{inter}})} - \frac{\sigma_{\alpha}}{-i\omega + q^{2}D_{\alpha} + (S/\tau_{\text{inter}})} \right]$$
$$\times \frac{\sum_{\beta} \frac{\mu^{(\beta)}}{-i\omega + q^{2}D_{\beta} + (S/\tau_{\text{inter}})}}{\sum_{\beta} \frac{\sigma_{\beta}}{-i\omega + q^{2}D_{\beta} + (S/\tau_{\text{inter}})}}\right]. \quad (4.30)$$

Here $q^2 D_{\alpha} = q_i D_{ik}^{(\alpha)} q_k$ and $\sigma_{\alpha} = q_i \sigma_{ik}^{(\alpha)} q_k / q^2$ are the longitudinal components of the diffusion and conductivity tensors, and the summation is performed over the ellipsoids occupied by carriers. Multiplying (4.30) by $\mu^{(\alpha)}$ from (3.12) and summing over the valleys, we obtain the following expression for the scattering cross section in the undeformed crystal:

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi T}F(\omega)\operatorname{Re}\sum_{\alpha}\mu_{\alpha}G_{\alpha}^{+}(\mathbf{q},\omega),$$

or

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{r_{0}^{2}}{\pi} V \frac{\hbar\omega}{1 - e^{-\hbar\omega/T}} \frac{(\mu_{\parallel} - \mu_{\perp})^{2}}{S} \left(\frac{\partial n}{\partial \zeta}\right)_{T}$$

$$\times \operatorname{Re}\left\{\sum_{\alpha} \frac{\left[(\mathbf{e}^{\mathbf{I}\vec{v}^{(\alpha)}})(\mathbf{e}^{\mathbf{S}\vec{v}^{(\alpha)}}) - (\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}/3})\right]^{2}}{-i\omega + q^{2}D_{\alpha} + (S/\tau_{\mathrm{inter}})}\right\}$$

$$+ \left[\sum_{\alpha} \frac{(\mathbf{e}^{\mathbf{I}\vec{v}^{(\alpha)}})(\mathbf{e}^{\mathbf{S}\mathbf{I}\vec{v}^{(\alpha)}}) - (\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}/3})}{-i\omega + q^{2}D_{\alpha} + (S/\tau_{\mathrm{inter}})}\right]^{2}$$

$$\times \frac{-i\omega + (S/\tau_{\mathrm{inter}})}{\sum_{\alpha} \frac{q^{2}D_{\alpha}}{-i\omega + q^{2}D_{\alpha} + (S/\tau_{\mathrm{inter}})}}\right\}.$$
(4.31)

We note that the dependence of the scattering cross section in (4.31) on the intervalley relaxation time τ_{inter} is reduced to a displacement of the diffusion pole from the imaginary axis into the complex plane. To ensure convergence of the integral cross section from (4.31) at T=0, the diffusion coefficient should be treated as a high-frequency coefficient, i.e., with account taken of frequency dispersion. The first term in (4.31) describes scattering by opposite-phase fluctuations of the concentration of electrons in different valleys. The second term arises from the inhomogeneous selfconsistent electric field E that arises in the diffusion process (see Ref. 61, Sec. 25), which splits apart equivalent valleys. The existence of this scattering mechanism, which is associated with fluctuations of the self-consistent electric field, was noted in Ref. 6-for the case of metals with complex Fermi surfaces, and in Ref. 8-for semiconductors with a complex valence band. In Eq. (4.31) this scattering mechanism is manifested for a diffusional, but not a free type of the motion of carriers, as in Refs. 6 and 8. When $T > \hbar \omega$, in line with Secs. 4.1 and 4.2.1, the scattering spectrum has the form of a Lorentzian contour determined by the diffusion coefficient D. To calculate the corresponding coefficients of the tensor of diffusion coefficients, we must separate Eq. (4.31) into simple parts. The form of this decomposition depends on the chosen directions of q and of the polarizations e^I and e^S. Experiments on Raman scattering are usually designed so that q is directed along one of the three different symmetry axes of a cubic semiconductor. 42,43,49,63 The parameters of the Lorentzians Γ and the diffusion coefficients measured from the spectra of n-Ge and n-Si for these directions of q and the corresponding experimental geometry are given in Table III. Only a few experiments (noted in Table III in a footnote) have been set up thus far. Nevertheless, in Ref. 63, with the aid of the temperature, concentration, and frequency (on

TABLE III.	Independent	experiments	on	Raman	scattering	in	multival	lley	semicono	iuctors
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Direction of q (group Gq)	Scattering geometry	Half-width of Lorentzians			
Germanium					
$v_q = \langle 001 \rangle$	$Z(X,Y)\overline{Z}^{*})$	$\Gamma = q^2 D + \frac{S}{\tau_{\text{inter}}}$			
$v_{q} = \frac{1}{\sqrt{2}} \langle 1\overline{1}0 \rangle$	$\begin{bmatrix} X'(Z,Y')\overline{X}'\\ X'(\zeta,\xi)\overline{X}' - \frac{2}{9}X(Y,X)Y \end{bmatrix}$	$\Gamma_i = q^2 D_\perp + \frac{S}{\tau_{\text{inter}}}$			
(D ₂)	$\frac{2}{9} \Big[X(YX)Y + X(YZ)Y \Big] - X'(\zeta,\xi)\overline{X}'$	$\Gamma_2 = q^2 \frac{D_\perp + 2D_{\parallel}}{3} + \frac{S}{\tau_{inter}}$			
	$\begin{bmatrix} X(Y,X)Y\\ X'(\zeta',\xi)\overline{X}' - \frac{1}{9}X'(Z,Y')\overline{X} \end{bmatrix}$	$\Gamma_3 = 3q^2 \frac{D_1(D_1 + 2D_{\parallel})}{D} + \frac{S}{\tau_{inter}}$			
$\nu_{q} = \frac{1}{\sqrt{3}} \langle 111 \rangle$	$\zeta \left(X',X' \right) \overline{\zeta} - \zeta \left(X',\xi \right) \overline{\zeta}$	$\Gamma_{\rm I} = q^2 \frac{D_{\rm II} (8D_{\perp} + D_{\rm II})}{9D} + \frac{S}{\tau_{\rm inter}}$			
(C _{3v})	ζ (Χ',ξ) ξ	$\Gamma_2 = q^2 \frac{8D_\perp + D_{\parallel}}{9} + \frac{S}{\tau_{\text{inter}}}$			
Silicon					
$\nu_q = \langle 001 \rangle$	$Z(X',Y')\overline{Z}^{*})$	$\Gamma_{\rm I} = q^2 D_{\perp} + \frac{S}{\tau_{\rm inter}}$			
(C _{4v})	$Z(Y',Y')\overline{Z}^{\bullet})$	$\Gamma_2 = q^2 D_\perp D_\parallel / D^{-1} + S \tau_{\text{inter}}^{-1}$			
$\nu_q = \frac{1}{\sqrt{2}} \langle 1\overline{1}0 \rangle$	$X'(\zeta,\xi)\overline{X'}^*)$	$\Gamma_{\downarrow} = \frac{1}{2} q^2 \frac{D_{\perp} (\overline{D_{\perp} + D_{\parallel}})}{D} + \frac{S}{\tau_{\text{inter}}}$			
(D ₂)	ζ'(Υ',ξ')ζ''	$\Gamma_2 = \frac{1}{2} q^2 \left(D_{\parallel} + D_{\perp} \right) + \frac{S}{\tau_{\text{inter}}}$			
$\nu_{q} = \frac{1}{\sqrt{3}} \langle 111 \rangle$	ζ(ξ, <i>Χ'</i>) ζ *)	$\Gamma = q^2 D + \frac{S}{\tau_{inter}}$			
(0 _h)					
*)Experiments set up in Refs. 4	3, 49, and 63.				
The symbols of the scattering get (113) $c' = 1/2 \overline{6}(132)$. The brack	cometry are given to agree with Ref. 30. The	unit vectors are indicated in Fig. 3, $\xi = (1/\sqrt{6})$			

 ω^{I}) dependences of the widths of the Lorentzian contours, it was possible to separate the diffusional and relaxational (associated with au_{inter}) contributions to the width of the Lorentzians. In n-Si precisely the latter contribution proved to be fundamental. Figure 11 shows the spectra of several specimens of n-Si from Ref. 63, where the curves marked with the numbers I-3 correspond to close-lying values of the concentration n of electrons, denoted in Fig. 11 in standard format. The impurity contribution to the frequency τ_{inter}^{-1} for specimens 1 and 3, which were doped with phosphorus, is approximately half as large as for specimen 2, which was doped with arsenic. The scattering of an electron by an impurity that leads to an intervalley transition can be treated as resulting from the mismatch of the pseudopotentials of the impurity and the main material. The pseudopotential for arsenic is closer to the pseudopotential of silicon than that of phosphorus, while the frequency τ_{inter}^{-1} for the crystal doped with arsenic is larger than for that doped with phosphorus. We can explain this

contradiction by assuming that the magnitude of τ_{inter} is controlled by processes of capture of electrons by an intermediate state bound to the impurity. The radius of the wave function of this state, and hence also the capture cross section, are inversely proportional to the binding energy⁶⁴ determined by the pseudopotential. Consequently for arsenic the frequency $1/\tau_{inter}$ must be larger than for phosphorus, in agreement with experiment.⁶³ Thus the spectra of one-particle electronic scattering can be used to identify the chemical nature of an impurity in multivalley semiconductors.

The second, diffusional, contribution to the width Γ contains (see Table III) longitudinal and transverse diffusion coefficients D_{\parallel} and D_{\perp} with respect to the axis of the valley. They are expressed in terms of the corresponding intravalley relaxation times τ_{\parallel} and τ_{\perp} (see Refs. 10 and 53). For example, we have the following expression for the longitudinal coefficient D_{\parallel} for a Fermi statistics:



FIG. 11. Raman spectra of several specimens of n-Si obtained⁶³ at T=80 K. Curves 1-3, which illustrate the dependence of $1/\tau_{inter}$ on the type of doping impurity, are discussed in the text.

$$D_{\parallel} = \frac{2}{3} \frac{\varepsilon_{\rm F}}{m^*} \mu_{\parallel} \tau_{\parallel} \tag{4.32}$$

and analogously for D_{\perp} , with $\tau_{\parallel} \neq \tau_{\perp}$. It is evident from Table III and (4.32) that Raman spectra enable one to determine an ill-studied parameter of multivalley semiconductors—the coefficient of anisotropy of relaxation times $K_{\tau} = \tau_{\perp} / \tau_{\parallel}$. Usually this parameter is determined by using deformation effects in the conductivity of specimens.^{9,53,65} To illustrate this we shall make use of the fact that the coefficient K_{τ} enters into the Einstein relationship that associates the transverse diffusion coefficient D_{\perp} measured in Ref. 63 with the mobility b obtained from electrical measurements in Ref. 66:

$$D_{1} = \frac{3K}{2K+1} b \frac{T}{e} \frac{d \ln n}{d(\zeta/T)},$$
 (4.33)

where $K = K_{\tau}K_{\mu}$, $K_{\mu} = \mu_{\perp} / \mu_{\parallel}$. Figure 12 gives the experimental values of the ratio D/b as functions of the electron concentration *n*. The crosses, curve 4, show the values in a crystal in which collisions of electrons with acoustic pho-



FIG. 12. Comparison of the values of the mobility *b* obtained from the electronic Raman spectra in Refs. 43 and 63 with the results of electrical measurements in Ref. 66. Values of the points: *1*—n-Ge; 2–4 correspond to n-Si; 4—acoustic scattering; 3—impurity scattering; the points with number 2 were obtained by using the fitting value of the parameter $\tau_{\parallel} / \tau_{\perp} = 20$. (From Ref. 62.)

nons predominate, the solid squares, curve 3, are for collisions of electrons with impurities; the lower solid curve is drawn according to (4.33) for n-Si with the fitted value of $\tau_{\parallel}/\tau_{\perp}$, while the upper solid curve is drawn for n-Ge by the ordinary Einstein relationship. We see from Fig. 12 that the experimental points for both types of scattering correspond to the theoretical curve. Indeed, there is a systematic shift of the experimental points for n-Si, which can be associated with the influence of the contacts and inhomogeneities in the electrical measurements in Ref. 66.

4.2.2-2. The influence of external pressure on light scattering in multivalley semiconductors. The method developed in the previous section for solving the diffusionkinetic equations (4.23) and (4.24) allows one in a number of cases to find analytically the differential Raman cross section in a deformed crystal. In their original form, these equations describe the relaxation of the electronic system of a multivalley semiconductor toward local equilibrium, the parameters of which vary from point to point in space.⁶¹ Let us write Eqs. (4.23) and (4.24) in such a way that they describe relaxation to complete equilibrium. To do this, after a one-sided Fourier transformation and elimination of the fluctuation of the electric current δj in them, we must introduce the following new unknown function instead of $G_a^+(\mathbf{q},\omega)$:

$$F_{\alpha}(\mathbf{q},\omega) = \frac{i\omega}{T} G_{\alpha}^{+}(\mathbf{q},\omega) - (\mu^{(\alpha)} - \langle \mu \rangle) \left(\frac{\partial n_{\alpha}}{\partial \zeta}\right)_{T},$$
(4.32')

which differs from the old function G_{α}^{+} in an increment associated with the fluctuations from point to point of the local equilibrium value of the chemical potential. In (4.32) $\langle \mu \rangle$ is defined as an average that takes account of the Pauli principle (see Secs. 3.5, 3.7, and 3.8):

$$\langle \mu \rangle = \sum_{\beta} \mu^{(\beta)} \frac{(\partial n_{\beta} / \partial \zeta)_T}{(\partial n / \partial \zeta)_T}.$$
(4.33')

We note that the function F_{α} describes the response of the system of electrons to a harmonic external perturbation:

$$u(\mathbf{r},t) = u_{\alpha}(\mathbf{q},\omega) \exp[i(\mathbf{q}\mathbf{r}-\omega t)]. \qquad (4.33'')$$

In the case q=0 the equation for this response was derived in Ref. 67. Generalization to the case $q\neq 0$ yields the following equation:

$$(-i\omega + q^2 D_{\alpha}(\omega))\psi_{\alpha} + q^2 D_{\alpha}(\omega)(\mu^{(\alpha)} - ieEq^{-1})$$

= $-\left(\frac{\partial \zeta}{\partial n_{\alpha}}\right)_T \sum_{\beta} I^C_{\alpha\beta}\left(\psi_{\beta} - \mu^{(\beta)} + \frac{ieE}{q}\right),$ (4.34)

where we have used the substitution $F_{\alpha} = \psi_{\alpha} (\partial n_{\alpha} / \partial \zeta)_T$ analogous to (4.25). We recall that the intensity of the self-consistent electric field *E* in a crystal with equivalent valleys was found in the previous section from the neutrality condition (4.27). In the case of nonequivalent valleys of a deformed crystal, the neutrality condition for certain valleys depleted of carriers may not be fulfilled. Therefore, in the given case, we should find the electric field from the Poisson equation

div
$$\mathbf{D} = -4\pi e \sum_{\alpha} \left(\frac{\partial n_{\alpha}}{\partial \zeta}\right)_T \psi_{\alpha},$$
 (4.35)

where **D** is the electrostatic induction vector.

Equations (4.32) and (4.33) imply that both the Poisson equation (4.35) and the condition of neutrality (4.27) are equally applicable both for the fluctuations with respect to local equilibrium, as described by the correlator $G_{\alpha}^{+}(q,\omega)$, and for the fluctuations with respect to complete equilibrium described by the response function $F_{\alpha}(q,\omega)$. In view of this circumstance, not the relative value $\mu^{(\alpha)} - \mu$, but the absolute value of the "perturbing force" $\mu^{(\alpha)}$ enters into (4.34). A more detailed discussion of the kinetics of the fluctuations of a multicomponent plasma can be found in Ref. 61 (Sec. 51). Expressing ψ_{α} from (4.34) and calculating *E* from (4.35), we obtain

$$eE = iq \frac{\Phi_q \sum_{\alpha\beta} (\partial n_{\alpha} / \partial \zeta)_T (\delta_{\alpha\beta} + i\omega B_{\alpha\beta}^{-1}) \mu^{(\beta)}}{1 + \Phi_q \sum_{\alpha\beta} (\partial n_{\alpha} / \partial \zeta)_T (\delta_{\alpha\beta} + i\omega B_{\alpha\beta}^{-1})}.$$
 (4.36)

Here $\Phi_q = 4\pi e^2 / \varepsilon_0 q^2$, $\varepsilon_0 = D/E$ is the lattice dielectric permittivity, and $B_{\alpha\beta}^{-1}$ is the matrix reciprocal to

$$B_{\alpha\beta} = (-i\omega + q^2 D_{\alpha})\delta_{\alpha\beta} + \left(\frac{\partial\xi}{\partial n_{\alpha}}\right)_T I^c_{\alpha\beta}.$$
 (4.37)

To find the cross section we must substitute (4.36) into (4.34) and solve the equation that is obtained for ψ_{α} . The scattering cross section is expressed in terms of $F_{\alpha}(\mathbf{q},\omega)$ according to

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{r_{0}^{2}}{2\pi} \frac{V\hbar}{1 - e^{-\hbar\omega/T}} \operatorname{Im} \Pi(\mathbf{q},\omega), \qquad (4.38)$$



FIG. 13. Graphs for the polarization operator $\Pi(\mathbf{q},\omega)$.

where the electronic polarization operator of the multivalley semiconductor

$$\Pi(\mathbf{q},\omega) = \sum_{\alpha} \mu^{(\alpha)} F_{\alpha}(\mathbf{q},\omega)$$

is represented by the diagram series in Fig. 13. It equals

$$\Pi(\mathbf{q},\omega) = -\sum_{\alpha,\beta} \left(\frac{\partial n_{\alpha}}{\partial \zeta}\right)_{T} \mu^{(\alpha)} (\delta_{\alpha\beta} + i\omega B_{\alpha\beta}^{-1}) \mu^{(\beta)} - \left[\Phi_{q} \sum_{\alpha,\beta} \left(\frac{\partial n_{\alpha}}{\partial \zeta}\right)_{T} (\delta_{\alpha\beta} + i\omega B_{\alpha\beta}^{-1}) \mu^{(\beta)} \times \sum_{\gamma,\varepsilon} \mu^{(\gamma)} (\delta_{\gamma\varepsilon} + i\omega B_{\gamma\varepsilon}^{-1}) \left(\frac{\partial n_{\gamma}}{\partial \zeta}\right)_{T}\right] \times \left[1 + \Phi_{q} \sum_{\alpha,\beta} \left(\frac{\partial n_{\alpha}}{\partial \zeta}\right)_{T} (\delta_{\alpha\beta} + i\omega B_{\alpha\beta}^{-1})\right]^{-1}.$$

$$(4.39)$$

The expressions (4.38) and (4.39) generalize Eq. (4.31) to the case of a deformed crystal with nonequivalent valleys. They are applicable for any value of the parameter qr_s and any frequencies. In particular, they can be used to analyze Raman spectra of plasmons in deformed materials.²⁹ We shall apply Eq. (4.39) to the case of n-Ge, Si, or GaAs subjected to strong hydrostatic pressure. Such a pressure shifts the central Γ -valley downward or upward in energy so that it reaches the energy of the side valleys. For scalar scattering with the wave vector **q** directed along axes of high symmetry as defined in Sec. 3.8, the side valleys are equivalent and can be combined into one isotropic valley. For simplicity we shall assume that the combined side valley and the central valley are described by the same coefficient of intravalley diffusion $D_{\rm L} = D_{\Gamma} = D$. Then the matrix that enters into (4.39)

$$\left(\frac{\partial n_{\alpha}}{\partial \zeta}\right)_{T} (\delta_{\alpha\beta} + i\omega \hat{B}_{\alpha\beta}^{-1})$$

acquires the form

$$\frac{1}{\Omega\left(-i\Omega+\frac{S}{\tau_{\text{inter}}}\right)} \times \begin{bmatrix} \left(\frac{\partial n_1}{\partial \xi}\right)_T [\Omega(V_1+\Gamma)+iV_2\Gamma], & -\omega V_1\left(\frac{\partial n_1}{\partial \xi}\right)_T \\ -\omega V_1\left(\frac{\partial n_1}{\partial \xi}\right)_T, & \left(\frac{\partial n_2}{\partial \xi}\right)_T [\Omega(V_2+\Gamma)+iV_1\Gamma] \end{bmatrix}.$$
(4.40)

Here the matrix I^{C} was taken from (4.26), while the quantities Γ , $V_{1,2}$, and Ω equal

$$\Gamma = q^2 D, \quad V_{1,2} = \frac{(\partial n_{2,1}/\partial \zeta)_T}{(\partial n/\partial \zeta)_T},$$

$$\Omega = \omega + i\Gamma.$$
(4.41)

Equation (4.40) implies that the process of formation of the plasma oscillations and the electronic screening are not altered in the presence of intervalley transitions, since, upon substituting (4.41) into (4.40) in the polarization operator Π from (4.39), the intervalley time drops out of the high-frequency dielectric function:

$$\varepsilon(q,\omega) = 1 + \frac{4\pi e^2}{\varepsilon_0 q^2} \left(\frac{\partial n}{\partial \zeta}\right)_T \frac{q^2 D(\omega)}{-i\omega + q^2 D(\omega)}.$$
 (4.42)

In rare intervalley transitions with $q^2 D\tau_{\text{inter}} > 1$, the scattering cross section of (4.38) and (4.39) with $\varepsilon(q,\omega)$ from (4.42) describes the plasmon peak in the scattering spectra. It is distinctly observed in n-Ge.²⁹ In n-Si the opposite limiting case is realized of frequent intervalley transitions with $q^2 D\tau_{\text{inter}} < 1.^{63}$ Owing to the vanishing of the numerator of the second term in (4.39), scattering by plasmons is absent here. Instead of it, (4.39) yields one-particle scattering by intervalley fluctuations having the cross section

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi} (\mu_{\Gamma} - \mu_{L})^{2} \frac{(\partial n_{1}/\partial \zeta)_{T} (\partial n_{2}/\partial \zeta)_{T}}{(\partial n/\partial \zeta)_{T}} \times F(\omega) \frac{S/\tau_{\mathrm{inter}}}{\omega^{2} + (S/\tau_{\mathrm{inter}})^{2}}.$$
(4.43)

An indirect confirmation of these ideas is the absence of plasmon peaks in the allowed geometry in n-Si,⁶⁸ where plasmons are observed only in the forbidden geometry.

The cross section for one-particle scattering of (4.43), which has not been experimentally detected up to now, is governed by a Lorentzian with a half-width equal to the frequency of the intervalley transitions between the side and central valleys. This frequency is an important parameter of devices having intervalley transport of electrons. The intensity of the scattering being discussed is determined by the square of the difference of the mean reciprocal masses of the valleys μ_G and $\mu_L = (1/3) \times (2\mu_{\perp} + \mu_{\parallel})$. The reduced density of states at the Fermi level that enters into the integral intensity in (4.43) is very sensitive to the presence of external pressure.

4.3. Form of the spectrum in scattering by spin-density fluctuations

The physical cause of appearance of light scattering by spin-density fluctuations involves the presence in the crystals of spin-orbital interaction. It was shown in Sec. 3.6 that the nature of this interaction is the same for a large class of substances. This is reflected in the similarity of the corresponding formulas for the integral cross section of light scattering. Thus the cross section of (3.43) pertains both to multivalley semiconductors and to direct-band A_3B_5 semiconductors.

Conversely, the differential scattering cross section of spin-density fluctuations depends on the details of the band structure, which dictate a certain choice of directions of the quantization axis of the spin. Thus, upon taking account of the Hamiltonian of spin-orbital interaction (3.47), which is cubic in the quasimomentum, the quantization axis of the spin must be directed along the axial vector \varkappa whose projection

$$\kappa_{x}(p) = \frac{p_{x}(p_{y}^{2} - p_{z}^{2})}{p^{3}}, \quad \kappa_{y}(p) = \frac{p_{y}(p_{z}^{2} - p_{x}^{2})}{p^{3}}$$
(4.44)

enters into the Hamiltonian of (3.47).

In p-type semiconductors, as is implied by the Luttinger Hamiltonian (3.48), the quantization axis of the spin proves to be rigidly bound to the direction of the momentum **p** even in the parabolic approximation.

We recall that in multivalley semiconductors the quantization axis of the spin is codirected with the axis of the valley $v^{(\alpha)}$, as we see from (3.44).

The quantization axis of the spin can be fixed in explicit form in the Hamiltonian of interaction of the carriers with light, as was done in the review, Ref. 3 [Sec. 2.3.1, Eqs. (2.88) and (2.91)]. For the case in which the quantization axis is correlated with the quasimomentum, Eq. (2.88) from Ref. 3 can be generalized as follows:

$$H_{\rm int} = \frac{e^2}{4mc^2} \sum_p A_i^{\rm I} A_k^{\rm S} B_{ik}(p) \left(\delta f_{\uparrow p} - \delta f_{\downarrow p}\right), \qquad (4.45)$$

where the tensor coefficient $B_{ik}(p)$ depends on the scattering mechanism. It equals

$$B_{ik}(p) = \delta_{ikj} \varkappa_j(p) B_{\sigma},$$

= $\delta_{ikj} \frac{p_j}{p} B_J,$
= $\delta_{ikj} \nu_j^{(\alpha)} B_{\sigma}^{(1)}$ (4.46)

respectively, for n- and p-type A_3B_5 semiconductors and multivalley semiconductors, while δf_{1p} is the fluctuation of the distribution function of electrons having a certain spin. Depending on the type of material, the relative population of the spin subbands $\delta f_{1p} - \delta f_{1p}$ pertains in the Hamiltonian of (4.45) either to one point of the Brillouin zone, or to one valley. The fluctuation corresponding to (4.45) of the dielectric susceptibility $\delta \chi_{ij}$ equals

$$\delta\chi_{ij} = \frac{e^2}{m(\omega \mathbf{I})^2} \sum_{p} B_{ij}(p) (\delta f_{\uparrow p} - \delta f_{\downarrow p}). \qquad (4.47)$$

Substituting (4.47) into (2.18), we obtain the differential scattering cross section in the form

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi T} F(\omega) \sum_{\mathbf{p},\mathbf{p}'} R_{\mathbf{p}} \langle (\delta f_{\uparrow \mathbf{p}} - \delta f_{\downarrow \mathbf{p}}) \rangle \\ \times \langle \delta f_{\uparrow \mathbf{p}'} - \delta f_{\downarrow \mathbf{p}'} \rangle \rangle_{\mathbf{q},\omega} R_{\mathbf{p}'}; \qquad (4.48)$$

Here we have $R_{\mathbf{p}} = e_i^{\mathbf{I}} B_{ik}(\mathbf{p}) e_k^{\mathbf{S}}$, and the function $f(\omega)$ is defined in (2.21). To calculate the correlator of the relative populations of the spin subbands from (4.48), we shall use the kinetic equation (Ref. 61, p. 126). By analogy with (4.29), we shall define the function

$$G_{\mathbf{p}}^{+}(\mathbf{q},\omega) = \sum_{\mathbf{p}'} \left\langle (\delta f_{\uparrow \mathbf{p}} - \delta f_{\downarrow \mathbf{p}}) (\delta f_{\uparrow \mathbf{p}'} - \delta f_{\downarrow \mathbf{p}'}) \right\rangle_{\mathbf{q},\omega}.$$
(4.49)

It was shown in Ref. 61, p. 126 that G_p^+ satisfies the same kinetic equation as $\delta f_{\uparrow p} - \delta f_{\downarrow p}$. This equation, with account taken of the initial condition for the simultaneous correlator (see, e.g., Ref. 19, Sec. 117),

$$\langle (\delta f_{\uparrow \mathbf{p}} - \delta f_{\downarrow \mathbf{p}}) (\delta f_{\uparrow \mathbf{p}'} - \delta f_{\downarrow \mathbf{p}'}) \rangle_{\mathbf{q} \to 0} = f_0 (1 - f_0) \delta_{\mathbf{p}\mathbf{p}'}$$
(4.50)

acquires the form

$$-i(\omega - \mathbf{q}\mathbf{v})G_{\mathbf{p}}^{+}(q,\omega) = T\left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T}R_{\mathbf{p}} + \sum_{\mathbf{p}'}I_{\mathbf{p}\mathbf{p}'}G_{\mathbf{p}'}^{+}(\mathbf{q},\omega).$$
(4.51)

A solution of an equation of this type, which describes the relaxation to local equilibrium, was obtained in Ref. 71 (Sec. 12.1, Eq. 12.6). We shall also solve Eq. (4.51) for the case of elastic collisions of carriers, to which the collisions with acoustic phonons and impurities are reduced. The kernel of the collision integral equals

$$I_{pp'} = W_{pp'} - \delta_{pp'} \sum_{p'} W_{pp'}, \qquad (4.52)$$

where

$$W_{\mathbf{p}\mathbf{p}'} = \frac{2\pi}{\hbar} \sum_{\sigma,\sigma'} |V_{\mathbf{p}\sigma;\mathbf{p}'\sigma'}|^2 \delta(\varepsilon_{\mathbf{p}} - \varepsilon_{\mathbf{p}'})$$
(4.53)

is the probability of electronic transition per unit time from the state $\mathbf{p}\sigma$ to the state $\mathbf{p}'\sigma'$ when acted on by the Hamiltonian V associated with impurities or phonons.

For n-type A_3B_5 semiconductors, owing to the simple band structure, at low temperatures $T \ll \zeta$, the collision integral with the kernel (4.52) is brought to the form (see, e.g., Refs. 59, 72)

$$\sum_{\mathbf{p}'} I_{\mathbf{p}\mathbf{p}'} G_{\mathbf{p}'}^+(\mathbf{q}, \omega) = \frac{1}{\tau} \left[\left(\frac{\partial f_0}{\partial n} \right)_T \sum_{\mathbf{p}'} G_{\mathbf{p}'}^+(\mathbf{q}, \omega) - G_{\mathbf{p}}^+(\mathbf{q}, \omega) \right], \qquad (4.54)$$

where τ is the intravalley relaxation time. In the case of quantum-mechanical degeneracy of the bands in (4.53), an additional angular dependence of the following form arises:

$$W(\mathbf{p}-\mathbf{p}') = \sum_{\xi\xi'} |V_{\mathbf{p}\xi;\mathbf{p}'\xi'}|^2 = \frac{1}{2} [1+3\cos^2(\hat{\mathbf{p}} \ \mathbf{p}')] |V|^2,$$
(4.55)

where $|V|^2$ is a coefficient having no angular dependence. Even in the case of elastic scattering, this dependence does not allow us to use the approximation of the relaxation time of (4.54).

After substituting (4.54) into (4.51), we obtain

$$G_{\mathbf{p}}^{+}(\mathbf{q},\omega) = \frac{1/\tau}{-i(\omega - \mathbf{q}\mathbf{v}) + (1/\tau)} \left(\frac{\partial f_{0}}{\partial n}\right)_{T}$$

$$\times \sum_{\mathbf{p}} T\left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T} \frac{R_{\mathbf{p}}}{-i(\omega - \mathbf{q}\mathbf{v}) + (1/\tau)}$$

$$\times \left[1 - \frac{1}{\tau} \sum_{\mathbf{p}} \left(\frac{\partial f_{0}}{\partial n}\right)_{T} \frac{1}{-i(\omega - \mathbf{q}\mathbf{v}) + (1/\tau)}\right]^{-1}$$

$$+ T\left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T} \frac{R_{\mathbf{p}}}{-i(\omega - \mathbf{q}\mathbf{v}) + (1/\tau)}.$$
(4.56)

The scattering cross section is expressed in terms of the function of (4.56) by analogy with (4.31):

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi T}F(\omega)\mathrm{Re} \sum_{\mathbf{p}}R_{\mathbf{p}}G_{\mathbf{p}}^{+}(\mathbf{q},\omega).$$
(4.57)

Substituting (4.56) into (4.57), we obtain

 Vr_0^2

 $d^2\Sigma$

$$\overline{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{2\pi}{2\pi} F(\omega)$$

$$\times \operatorname{Re} \left\{ \sum_{\mathbf{p}} \left(\frac{\partial f_0}{\partial \xi} \right)_T \frac{R_{\mathbf{p}}^2}{-i(\omega - q\mathbf{v}) + (1/\tau)} + \left(\frac{\partial \xi}{\partial n} \right)_T \left[\sum_{\mathbf{p}} \left(\frac{\partial f_0}{\partial \xi} \right)_T \frac{R_p}{-i(\omega - q\mathbf{v}) + (1/\tau)} \right]^2 \times \left[\tau - \left(\frac{\partial \xi}{\partial n} \right)_T + \left[\frac{1}{2\pi} \left(\frac{\partial f_0}{\partial r} \right)_T \frac{1}{-i(\omega - q\mathbf{v}) + (1/\tau)} \right]^2 \right] \right\}.$$

$$\times \sum_{\mathbf{p}} \left(\frac{\partial f_0}{\partial n} \right)_T \frac{1}{-i(\omega - q\mathbf{v}) + (1/\tau)} = 1. \quad (4.58)$$

The scattering cross section of (4.58) is a generalization of Eqs. (2.35) and (2.92) from Ref. 3, based on the study of Mermin.⁷² The generalization allows the possibility of a fixed axis of quantization of the spin, as, e.g., in the case of electrons in semiconductors described by the Hamiltonian of (3.47). We note that the cross section of (4.58) that is found, in contrast to the result of Mermin from Ref. 3, is not expressed in terms of the dielectric susceptibility of the carriers, although it transforms into the result of Ref. 3 for the case of a free quantization axis of the spin. Although the conclusion (4.58) directly pertains only to n-type A_3B_5 semiconductors, it can be used in the case of rare collisions also for p-materials.

There is also a possibility of applying (4.58) to the case of multivalley semiconductors under conditions of fast in-

travalley relaxation. In this case the kinetic equation (4.51) is reduced by the method developed in Ref. 61 (Sec. 6) to a system of hydrodynamic equations of the type of (4.23) and (4.24). In calculating the relative population of the electronic spin subbands

$$\delta n_{\uparrow\alpha} - \delta n_{\downarrow\alpha} = \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \left(\delta f^{(\alpha)}_{\uparrow p} - \delta f^{(\alpha)}_{\downarrow p}\right)$$

the self-consistent electric field E, which generally leads to screening effects, drops out of the treatment. The mixed diffusion-kinetic equation has the form (4.34) with E=0. Comparison of (3.34) and (4.51) shows that the role of the term with qv is played by the term containing the diffusion coefficient $q^2 D_{\alpha}$, while the role of the collision integral is played by the matrix of intervalley transitions. The corresponding scattering cross section is obtained from (4.58) by the replacement of qv by $q^2 D$, and of the intravalley time τ by the intervalley time τ_{inter} .

4.4. Scattering spectra of spin-density fluctuations in certain materials

4.4.1. n-Type A₃B₅ semiconductors

In n-type A_3B_5 semiconductors in the parabolic approximation for the electronic dispersion law, the quantization axis of the spin is not fixed (see Sec. 3.6.1). Therefore the coefficient R_p from (4.58) does not depend on p, and to analyze the scattering spectra one can use the expression for the Raman cross section from Ref. 3 (Sec. 2.2.5). Neglecting the dependence of R_p on p in (4.58), we obtain

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi} B_{\sigma}^{2} F(\omega) [\mathbf{e}^{\mathrm{I}} \times \mathbf{e}^{*\mathrm{S}}]^{2} \\ \times \mathrm{Im} \left\{ \int \mathrm{d}^{3} p \, \frac{(\partial f_{0}/\partial \zeta)_{T}}{\omega - \mathbf{q}\mathbf{v} + (i/\tau)} \right. \\ \left. \times \left[\int \mathrm{d}^{3} p \, \frac{\mathbf{q}\mathbf{v} - \omega}{\omega - \mathbf{q}\mathbf{v} + (i/\tau)} \left(\frac{\partial f_{0}}{\partial n} \right)_{T} \right]^{-1} \right].$$

$$(4.59)$$

A relationship equivalent to (4.59) from the review, Ref. 3, is widely employed⁶⁹⁻⁷⁰ to determine the electron concentration n and τ by a contact-free method. When T=0, the cross section in (4.59) is reduced to that obtained by Fal'kovskii⁷³ for the case of normal metals.

Recently two groups of authors—Baĭramov et al.^{58,74-76} and Tsen et al.^{77,78}—have performed measurements of scattering spectra in n-type A_3B_5 semiconductors with controlled introduction of doping impurities. In these studies the concentration of electrons was varied over a broad range—from 10^{12} to $n \sim 10^{19}$ cm⁻³. Correspondingly, also the kinetics of the fluctuations scattering light varied over a broad range. The case of low concentrations was discussed in Sec. 4.1.

Figure 14 shows the spectra of electronic scattering by spin-density fluctuations ($e^{I} \perp e^{S}$) in n-InP at three electron concentrations, $n=1\times10^{16}$ cm⁻³, $n=1.1\times10^{17}$ cm⁻³, and $n=5\times10^{17}$ cm⁻³, from Refs. 74 and 75. The



FIG. 14. Spectra of electronic scattering by fluctuations of spin density $(e^{I} \perp e^{S})$ in n-InP for the region of electron concentrations from 1×10^{16} to 5×10^{17} cm⁻³ illustrating the narrowing of the line with increasing concentration N of impurities. (From Refs. 74 and 75.)

width of the spectra successively decreases with increasing impurity concentration $N \approx n$. In form the first spectrum is close to Gaussian, while the second and third are Lorentzian. The earlier results of Bray⁷⁸ for GaAs are shown by dots in Fig. 15 for two electron concentrations $n=7\times10^{16}$ cm⁻³ and $n=7\times10^{17}$ cm⁻³. Both spectra are well described by Eq. (4.59) (solid line). The corresponding τ from (4.59) equals $\tau = 2 \times 10^{-13}$ s. Comparison of the spectra in Figs. 14 and 15 shows that the range of concentrations n in which the described transformation of the spectra from Gaussian to Lorentzian occurs amounts to 10^{16} - 10^{17} cm⁻³. The existence of this qualitative reorganization of the spectra depending on the value of the concentration N of impurities enables one to use the spectra to estimate the quality of specimens and to determine the relaxation time τ .

To trace the transformation from a Gaussian to a Lorentzian spectrum, it suffices to take from (4.59) the



FIG. 15. Spectra of electronic scattering by spin-density fluctuations in n-GaAs for $n=7\times10^{16}$ cm⁻³ and 7×10^{17} cm⁻³. (From Ref. 78.)



FIG. 16. Stokes spectra of electronic scattering by spin-density fluctuations in the solid solution $n-Ga_xIn_{1-x}P$ and the binary semiconductor n-InP. (From Ref. 76.)

different limiting cases in the parameter of (4.12). When $ql \ll 1$ the denominator of (4.59) equals unity, while the numerator, according to the formula

$$\frac{1}{\omega - qv + (i/\tau)} = -\pi \delta(\omega - \mathbf{qv})$$

yields for nondegenerate statistics the Gaussian contour discussed above with the half-width

$$\Gamma_{\rm G} = q v_T. \tag{4.60}$$

For a degenerate statistics Eq. (4.59) gives rise to the triangular contour shown in Fig. 1.

When $ql \ll 1$ the cross section is determined by the diffusion pole that arises in the denominator of (4.59) owing to its series expansion in the small parameter:⁷⁹

$$z = \frac{\mathbf{q}\mathbf{v}}{-i\omega + (1/\tau)} \leqslant 1. \tag{4.61}$$

Here the cross section acquires the form

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi} B_{\sigma}^{2} \left(\frac{\partial n}{\partial \zeta}\right)_{T} F(\omega) \operatorname{Re} \frac{1}{-i\omega + q^{2} \widetilde{D}(\omega)}.$$
(4.62)

Here, in view of $q^2 D\tau \approx (ql)^2 \leq 1$, we can replace

$$\widetilde{D}(\omega) = \frac{D(\omega)}{1 - \frac{q^2 D(\omega)}{-i\omega + (1/\tau)}} \approx D(\omega), \qquad (4.63)$$

where $D(\omega)$ is the high-frequency diffusion coefficient associated with the high-frequency conductivity by the Einstein relationship (4.33). Using the Drude value for the electron conductivity, we can obtain the high-frequency asymptotics of the cross section

$$\frac{\mathrm{d}^2\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = V B_{\sigma}^2 r_0^2 \hbar \left(\frac{\partial n}{\partial \zeta}\right)_T \frac{q^2 v_{\mathrm{F}}^3}{l\omega^3}.$$

TABLE IV. Comparison of the experimental values of the mobility b obtained from electrical measurements and from Raman spectra.

Material	Optical measurements (from Raman spectra)	Electrical measurements	Concentration n	
nInP	1700	2000	$\frac{1.1\times10^{18} \text{ cm}^{-3}}{3\times10^{17} \text{ cm}^{-3}}$	
nGa _x In _{1-x} P	1470	1540		

For the case of normal metals here we should replace $q \rightarrow \delta^{-1}$ ⁷³ (see Sec. 3.4). Thus convergence of the integral cross section, i.e., of the integral over the frequency ω from (4.62), generally occurs only upon taking account of the frequency dispersion $D(\omega)$.

The narrowing of the contour in Fig. 14 from curve 1 to curve 2 is explained by the fact that $\Gamma_L = q^2 D = qv \cdot ql \ll \Gamma_G$ from (4.60). The further narrowing (curve 3) involves the decrease in the diffusion coefficient with increasing concentration of impurities in the specimen.

Figure 16 shows the spectra of the solid solution $n-Ga_xIn_{1-x}P$ and the binary semiconductor InP.⁷⁶ In the solid solution the width of the Lorentzian contour is smaller. Apparently this involves the decrease in the diffusion coefficient described above, caused by the additional scattering of carriers by fluctuations of the composition of the solid solution. Table IV compares the data of the electrical measurements of the mobility and the values of the diffusion coefficient obtained from the spectra of the solid solution $Ga_rIn_{1-r}P$ and the binary compound InP. The agreement is relatively good. However, with further increase in the concentration of electrons (and impurities) the spectra, while maintaining their Lorentzian form, substantially broaden. We can see this from Fig. 17, which shows the experimental results on light scattering by spindensity fluctuations in InP^{75} in the concentration range from 5×10^{17} cm⁻³ to 10^{19} cm⁻³. The magnitude of Γ for the spectrum corresponding to the highest concentration amounted to $\Gamma = 100$ cm⁻¹. This behavior of Γ is not de-



FIG. 17. Spectra of electronic scattering by spin-density fluctuations $(e^{I} \perp e^{S})$ in n-InP for the region of electron concentrations from 5×10^{17} to 9.44×10^{18} cm⁻³ illustrating the increase in the line width with increasing concentration N of impurities. (From Refs. 74 and 75.)

scribed by Eq. (4.63), since the diffusion coefficient $D \sim v_F^2 \tau \sim N^{-1/3}$ does not increase with increasing concentration of impurities.

At high electron concentrations *n* the nonparabolicity of the electronic energy band becomes substantial. According to (3.47) nonparabolicity leads to spin-orbital splitting of the band. The mean value of the Hamiltonian of (3.47) for the typical values $\alpha = 20$ eV Å³ (see Ref. 41) and $n = 10^{19}$ cm⁻³ amounts to $\langle H_{SO} \rangle = 2$ meV. This is comparable to the minimal half-width of the Lorentzian in Figs. 14-16.

The spin splitting manifests itself in the intrasubband scattering by spin-density fluctuations, which leads to a dependence of R_p from (4.58) on **p**. Moreover, it leads to the separation out of scattering with spin reversal, which was observed in Ref. 45. The wave functions have a symmetry such that scattering by spin-density fluctuations proceeds via virtual states in the subband of heavy holes. Conversely, intersubband scattering occurs only with participation of the subband of light holes and the spinorbital split-off subband. The use of an Nd:YAG laser, the frequency of which is closest to the fundamental threshold, implies separation of the Raman scattering by spin-density fluctuations.

When the condition (4.12) is satisfied, in all parts of Eq. (4.58) we can neglect the term qv in comparison with $1/\tau$. Here the cross section acquires a relaxational form:⁸⁰

$$\frac{\mathrm{d}^2 \Sigma}{\mathrm{d}\omega \mathrm{d}\Omega} \approx \frac{V r_0^2}{2\pi} B_\sigma^2 \left(\frac{\partial n}{\partial \zeta}\right)_T \frac{F(\omega)\tau}{1+(\omega\tau)^2}.$$
(4.64)

The half-width of the Lorentzian contour of (4.64) increases with increasing concentration of impurities, which agrees with experiment (see Fig. 17).

In contrast to (4.62), Eq. (4.64) does not yield a convergent integral cross section since, when $\tau = \text{const}$, the integral over the frequency of (4.64) diverges at high frequencies. According to Ref. 81, when analyzing the spectra of high-temperature superconductors (HTSCs), one should use specifically Eq. (4.64) rather than (4.62), as was done in Ref. 82. Actually the standard vertex coefficient from the first formula of Ref. 82 corresponds to R_p , which depends on **p**, which leads to Eq. (4.64). According to Ref. 81, this formula contains an explanation of the giant electronic scattering observed in HTSC compounds, which stems from the fact that the integral cross section, first, diverges, and second, is proportional to the large frequency of electron collisions $1/\tau$, which is large because of the imperfection of HTSC structures.

4.4.2. Multivalley semiconductors

For multivalley semiconductors Eq. (4.58) must be modified in line with what we have said at the end of Sec. 4.3. Its most important difference from its one-valley analog (4.62) is that it contains a dependence on the intervalley relaxation time τ_{inter} . For example, for backward scattering with q parallel to $\langle 111 \rangle / \sqrt{3}$, one can obtain for the case of n-Ge

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{8}{27} \left| \left[\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}*} \right] \right|^{2} \frac{F(\omega)}{\pi S} \left(\frac{\partial n}{\partial \zeta} \right)_{T} \left\{ 2 \frac{q^{2} D_{1} + (S/\tau_{\mathrm{inter}})}{\omega^{2} + \left[q^{2} D_{1} + (S/\tau_{\mathrm{inter}}) \right]^{2}} \right. \\ \left. + \operatorname{Re} \frac{-7i\omega + q^{2}(6D_{1} + D_{\parallel}) + (3S/\tau_{\mathrm{inter}})}{(-i\omega + q^{2} D_{1}) + \frac{S}{\tau_{\mathrm{inter}}} (-i\omega + 3q^{2} D)} \right\},$$

$$(4.65)$$

where $D_1 = (8D_1 + D_{\parallel})/9$. We see that the dependence on τ_{inter} , in contrast to the case of Raman scattering of intervalley fluctuations from Sec. 4.2.2-1, is not reduced simply to a displacement of the diffusion pole from the imaginary axis into the complex plane by the amount S/τ_{inter} . For directions of **q** such as (001) in n-Ge with regard to which all valleys are visible at the same angles, the denominators of the simple fractions in (4.65) for all valleys prove to be the same. Owing to this, the time S/τ_{inter} drops out of the Raman cross section, and it acquires the "one-valley" form of (4.62). However, for other directions of **q**, as we see from (4.65), the spectrum is represented as a superposition of two or several Lorentzians with width Γ depending on τ_{inter} , so that the method adopted in Ref. 42 of describing the spectra is generally inapplicable. As an example, Table V presents the results⁸⁰ for Γ for one of the directions distinguished by symmetry $\mathbf{q} \parallel \langle 1\overline{10} \rangle / \sqrt{2}$.

An indirect experimental confirmation of the dependence of Γ on the intervalley frequency $1/\tau_{inter}$ may be the unsuccessful attempts to detect scattering by one-particle excitations in specimens of n-Ge doped by the ionimplantation method.⁵² It was noted in Ref. 61 that the most probable reason for the absence of one-particle Raman spectra in these specimens, despite the large electron concentration $n \approx 10^{21}$ cm⁻³, is the large value of $1/\tau_{inter}$ arising from defects in these materials.

To observe electronic scattering in ion-implanted specimens of n-Ge, a Γ_{12} experimental geometry is needed, e.g., $Z(X',Y')\overline{Z}$ in the notation of Fig. 3. This geometry singles

TABLE V. Irreducible representations of the group of the wave vector $q \parallel \langle 110 \rangle G_q = D_2$ and its corresponding basis functions, geometries of independent experiments on Raman scattering by spin-density fluctuations in n-Ge and parameters determined in them.

Basis functions of group G_q	Geometry of experiments	Half-width of Lorentzian
[e ¹ e ⁵]	<i>X</i> '(ξ, ζ) <i>X</i> ' *)	$\Gamma = q^2 \frac{2D_{ } + D_{\perp}}{3} + \frac{S}{\tau_{\text{inter}}}$
[e ^{'e^s]₁}	$X(YZ)Y - \frac{3}{8}X'(\xi,\zeta)\overline{X}'$	$\Gamma = q^2 D_{\perp} + \frac{S}{\tau_{\text{inter}}}$
[e ^l e ^s] ₁₂	X(YX)Y	$\Gamma_{1,2} = 3q^2D + \frac{S}{2\tau_{inter}} \pm \left[q^4 \left(\frac{D_{\perp} - D_{\parallel}}{3}\right)^2 + \left(\frac{S}{2\tau_{inter}}\right)^2\right]^{1/2}$
	Basis functions of group G_q $[e^l e^s]_{ }$ $[e^l e^s]_{\perp 1}$ $[e^l e^s]_{\perp 2}$	Basis functions of group G_q Geometry of experiments $[e^1e^S]_{ }$ $X'(\xi, \zeta)\overline{X'}^{*}$ $[e^te^S]_{\perp 1}$ $X(YZ)Y - \frac{3}{8}X'(\xi, \zeta)\overline{X'}$ $[e^1e^S]_{\perp 2}$ $X(YX)Y$

riments set up in Ref. 43.

For the symbols of the scattering geometry, see, e.g., Ref. 30. The unit vectors of the Cartesian axes used in the Table are defined in Fig. 3. out the scattering by spin-density fluctuations, which does not depend on the intervalley relaxation time.

4.5. Light scattering by energy-density fluctuations

Light scattering by energy fluctuations is studied in semiconductors over a broad range of electron temperatures and concentrations (see Sec. 3.5).^{21,22,69} When scattering spectra are excited with a visible-light laser, the energy fluctuations are manifested only under conditions of exact resonance $|E_{\rm g} + \varepsilon_{\rm F} - \hbar \omega^{\rm I}| < \hbar \omega^{\rm I}$, including degeneracy of statistics (see the reviews, Refs. 3 and 30). At room temperature, to observe scattering by energy fluctuations requires an infrared laser (see Fig. 4). For a photoexcited plasma with a high electron temperature, ^{22,69} special mechanisms of interaction of the light with the energy fluctuations are possible, e.g., via the dependence of the electronic relaxation frequency on the electron temperature.⁸³

Often scattering by energy fluctuations is understood in the broad sense—as any scattering process caused solely by the nonparabolicity of the electronic spectrum. This is precisely how the one-particle electronic scattering in A15 compounds of the transition metals with charge-density waves was treated.⁸⁴ The advances in the technology of synthesis of perfect HTSC crystals and the observation in them of giant electronic scattering have compelled a search for a new explanation of this phenomenon that would not involve frequent electron collisions. In Ref. 85 an attempt was made to explain giant electronic scattering in HTSC crystals by a specific form of nonparabolicity in which the Fermi surface has parallel regions.

As a general approach for describing Raman spectra involving the nonparabolicity of the electronic spectrum, one can propose expanding the function $\gamma_{ik}^{\xi\xi}$ from (2.19) in a series in the eigenfunctions of the collision integral.⁸⁶ Such an expansion for the carrier-density matrix was performed in Ref. 87. A unified formula for the cross section, suitable for all variants of the electronic spectrum and values of the parameter $ql \ge 1$, assumes the calculation of inverse kinetic operators similar to the matrix $B_{\alpha\beta}$ from (4.37). It was shown in Sec. 3.5 that, in the case of weak nonparabolicity, the principal term in the expansion of the matrix $\gamma_{12}^{f_2}$ is the scalar diagonal term, which reduces simply to the energy ε_p . Since the collision integral (4.54) does not generate a corresponding basis function proportional to ε_p , one must not use the Mermin procedure⁷² described in Sec. 4.3 to solve the problem being discussed.

We shall develop here an approach based on an iterational solution of the kinetic equation (4.51) in the parameter of (4.61).⁸⁸ This approach, which is a variant of the Chapman-Enskog method,⁸⁹ under conditions of nonsteady-state external action, allows one to describe spectra over a broad range of transmitted frequencies ω , including the frequencies of the plasmon-phonon modes. Here a need arises in connection with the fact that the fluctuations of energy and charge density have the same-scalarsymmetry and therefore are not statistically independent. Their spectra cannot be separated by using the selection rules.^{3,30} The value of the function R_p from (4.51) arises from the expression for the fluctuation of the dielectric susceptibility (3.28):

$$R_{\mathbf{p}} = \left(1 - \frac{8}{3} \frac{\varepsilon_{\mathbf{p}}}{E_{\mathbf{g}}}\right) (\mathbf{e}^{\mathbf{I}} \mathbf{e}^{\mathbf{S}}). \tag{4.66}$$

In (4.51) one must take account of the electric field E that arises for fluctuations of charge density. This field involves the fluctuations with respect to total equilibrium. At the same time, the kinetic equation in the form of (4.51) describes the relaxation of the system only toward local equilibrium. Therefore, to take account of E in (4.51), this equation must be transformed in such a way that it describes relaxation toward total equilibrium. The nonequilibrium deviation of F_{n} from the distribution function f_{0} for total equilibrium as a result of an external force proportional to $R_{\rm p}$ from (4.66) (Incomplete sentence; please check.) In view of the fluctuation-dissipation theorem, ^{19,20} $F_{\mathbf{p}}$ is expressed linearly in terms of the correlator $G_{\mathbf{p}}^+$ by analogy with (4.32):

$$F_{\mathbf{p}}(\mathbf{q},\omega) = \frac{i\omega}{T} G_{\mathbf{p}}^{+}(\mathbf{q},\omega) - R_{\mathbf{p}} \left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T}.$$
(4.67)

After substituting G_p^+ from (4.67) into the kinetic equation (4.51), the inhomogeneous term of this equation, which is written with respect to $F_{\mathbf{p}}(\mathbf{q},\omega)$, acquires the standard form of a field term.⁶¹ This transformation is discussed in Sec. 4.2.2-2 and in Ref. 71. As is known, the self-consistent electric field E should be taken into account as a corresponding increment to the field term.⁶¹ As a result the equation for $F_{\rm p}(q,\omega)$ acquires the form

$$i(\omega - \mathbf{q}\mathbf{v})F_{\mathbf{p}}(\mathbf{q},\omega) - i\mathbf{v}(e\mathbf{E} + \mathbf{q}R_{\mathbf{p}})\left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T}$$
$$= -\sum_{\mathbf{p}'} I_{\mathbf{p}\mathbf{p}'}\left(F_{\mathbf{p}'}(\mathbf{q},\omega) + R_{\mathbf{p}'}\left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T}\right).$$
(4.68)

Here we should seek the electric field E from the Poisson equation

$$i\mathbf{q}\mathbf{E} = -\frac{4\pi e}{\varepsilon_0} \sum_{\mathbf{p}} F_{\mathbf{p}}(\mathbf{q},\omega). \tag{4.69}$$

To write the Raman cross section in terms of $F_p(q,\omega)$, we must express G_p^+ from (4.67) and substitute it into (4.57). As a result we obtain

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{r_{0}^{2}}{2\pi} \frac{\hbar}{1 - e^{-\hbar\omega/T}} \operatorname{Im} \Pi(\mathbf{q}, \omega).$$
(4.70)

Here $\Pi(\mathbf{q},\omega)$ is the electronic polarization operator, which equals

$$\Pi(\mathbf{q},\omega) = \sum_{\mathbf{p}} R_{\mathbf{p}} F_{\mathbf{p}}(\mathbf{q},\omega). \tag{4.71}$$

A graphic equation for $\Pi(\mathbf{q},\omega)$, which is equivalent to the system (4.68) and (4.69) and can serve to substantiate it, is shown in Fig. 13. Instead of the Mermin procedure,⁷² we must take account in the collision integral $I_{pp'}$ of inelastic processes, for which a single relaxation time cannot be introduced. Therefore we shall introduce a set of eigenvalues v_{α} of the collision integral according to⁸⁶⁻⁸⁸

$$\sum_{\mathbf{p}'} I_{\mathbf{p}\mathbf{p}'} \Psi_{\alpha}(\mathbf{p}') = v_{\alpha} \Psi_{\alpha}(\mathbf{p}), \qquad (4.72)$$

where the $\Psi_{\alpha}(\mathbf{p})$ are the corresponding eigenfunctions of the collision integral. Let us expand the functions of the momentum *p* that enter into (4.68), (4.69), and (4.71) in generalized Fourier series in $\Psi_{\alpha}(\mathbf{p})$:

$$F_{\mathbf{p}}(\mathbf{q},\omega) = \left(\frac{\partial f_0}{\partial \zeta}\right)_T \sum_{\beta} a_{\beta}^{q,\omega} \Psi_{\beta}(\mathbf{p}), \qquad (4.73)$$

$$R_{\mathbf{p}} = \sum_{\beta} r_{\beta} \Psi_{\beta}(p). \tag{4.74}$$

Since we already know R_p from (4.66), we can also consider its coefficients in the Fourier series (4.74) to be known. It was shown in Sec. 4.1 that the collisions of electrons with phonons exert no substantial influence on the light-scattering spectrum. Therefore we can assume that the "inelastic part" of the collision integral $I_{pp'}$ includes primarily collisions of electrons with one another. In view of the law of conservation of energy that is satisfied in these collisions, the function R_p from (4.66) and (4.74) proves to be one of the eigenfunctions of the collision integral, corresponding to the eigenvalue of zero, v=0. Therefore its expansion (4.74) contains only two orthogonal contributions that correspond to the two laws of con-

servation of energy and of number of particles. Therefore the polarization operator $\Pi(\mathbf{q}\omega)$ from (4.71) also contains a restricted number of terms. We can obtain them by substituting (4.74) and (4.73) into (4.71):

$$\Pi(\mathbf{q},\omega) = r_0^2 \Pi_{00} + 2r_0 r_1 \Pi_{01} + r_1^2 \Pi_{11}.$$
(4.75)

Before we calculate the matrix elements of the polarization operator $\Pi_{\alpha\beta}$, we must first eliminate from (4.68) the electric field E by using the Poisson equation (4.69). The result is expressed in terms of the operator $\hat{B}_{pp_1}^{-1}$, which is reciprocal to

$$\hat{B}_{\mathbf{p}\mathbf{p}_1} = (-i\omega + i\mathbf{q}\mathbf{v})\delta_{\mathbf{p}\mathbf{p}_1} + \hat{I}_{\mathbf{p}\mathbf{p}_1}.$$
(4.76)

The polarization operator $\Pi_{\alpha\beta}$ is associated with the matrix $\widetilde{B}_{\alpha\beta}^{-1}$ by⁹⁰

$$\Pi_{\alpha\beta} = i\omega(\hat{B}^{-1})_{\alpha\beta} - \frac{[\delta_{\alpha0} + i\omega(\hat{B}^{-1})_{\alpha0}][\delta_{\beta0} + i\omega(\hat{B}^{-1})_{0\beta}]}{1 + (qr_s)^2 + i\omega(\hat{B}^{-1})_{00}}.$$
(4.77)

Here the indices α and β take on two values 0 and 1, with the index 0 denoting the only eigenfunction $\Psi_0=1$, which arises from the law of conservation of number of particles in collisions, while the index 1 denotes the function defined by Eq. (37) from Ref. 88, which arises from the law of conservation of energy. The procedure of finding the inverse operator \hat{B}_{pp}^{-1} in the first approximation in the parameter of (4.61) assumes the solution of the kinetic equation in the hydrodynamic approximation. This solution was obtained in Ref. 61 in the usual, "coordinate" representation and in Refs. 88 and 90 in the matrix representation. Following these methods, we shall derive an equation for the inverse operator $\hat{B}_{pp'}^{-1}$ in the matrix representation, which generalizes Eq. (20) of Ref. 86 to the case of energydensity fluctuations:

$$\hat{B}^{-1}$$

$$= \left[-i\omega + q^2 D(\omega), \left[\frac{1}{TC_v} \left(\frac{\partial \zeta}{\partial n} \right)_T \right]^{1/2} q^2 D_T(\omega) \right] \times \left[\frac{1}{TC_v} \left(\frac{\partial \zeta}{\partial n} \right)_T \right]^{1/2} q^2 D_T(\omega),$$
$$-i\omega + q^2 \left[\chi(\omega) + \frac{1}{C_v} \left(\frac{\partial \zeta}{\partial n} \right)_T \frac{D_T^2(\omega)}{TD(\omega)} \right] \right]. \quad (4.78)$$

Here $D(\omega)$, $D_T(\omega)$, and $\chi(\omega)$ denote the components of the diffusion, thermodiffusion, and temperature conductivity tensors longitudinal with respect to **q**. One can find their frequency dependences in Ref. 88. Substituting (4.78) into (4.77) and calculating the summation in (4.75), we obtain the differential scattering cross section in a form that also takes account of the contribution of charge-density fluctuations, i.e., plasmons, and the contribution of energy-density fluctuations:

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi} \frac{F(\omega)}{\omega} \left(\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}}\right)^{2} \left(\frac{\partial n}{\partial \xi}\right)_{T} \operatorname{Im}\left\{q^{2}D(\omega)\left[-i\omega+q^{2}D(\omega)+\frac{4\pi\sigma(\omega)}{\varepsilon}-i\omega q^{2}\frac{D_{T}^{2}(\omega)}{TD(\omega)C_{v}}\left(\frac{\partial \zeta}{\partial n}\right)_{T}\right]^{-1}(-i\omega)\right\}$$

$$+q^{2}\chi(\omega)^{-1}\left[-i\omega+q^{2}D(\omega)+\frac{4\pi\sigma(\omega)}{\varepsilon}\right]\left[-i\omega+q^{2}\chi(\omega)\right]$$

$$-i\omega q^{2}\frac{D_{T}^{2}(\omega)}{TD(\omega)C_{v}}\left(\frac{\partial \zeta}{\partial n}\right)_{T}\right]^{-1}+r_{1}^{2}i\omega\left[-i\omega+q^{2}\chi(\omega)-i\omega q^{2}\frac{D_{T}^{2}(\omega)}{TD(\omega)C_{v}}\left(\frac{\partial \zeta}{\partial n}\right)_{T}\left(-i\omega+q^{2}D(\omega)\right)\right]$$

$$+\frac{4\pi\sigma(\omega)}{\varepsilon}\left[-i\omega\right]$$

$$+\frac{4\pi\sigma(\omega)}{\varepsilon}\left[-i\omega\right]$$

$$(4.79)$$

The Fourier coefficient r_1 , which plays simultaneously the role of the nonparabolicity parameter, can be found by expanding Eq. (4.66):

$$r_{1} = \frac{8}{3E_{g}} \int \frac{2d^{3}p}{(2\pi\hbar)^{3}} \varepsilon_{p} \Psi_{1} \left(\frac{\partial f_{0}}{\partial n}\right)_{T}$$
$$= \frac{8}{3E_{g}} \left[\left(\frac{\partial \zeta}{\partial n}\right)_{T} TC_{v} \right]^{1/2}.$$
(4.80)

In calculating this coefficient in the case of resonance enhancement of scattering, in (4.80) the quantity E_g is replaced by $E_g - \hbar \omega^I$. Here the possibility arises of resonance enhancement of r_1 . At low frequencies $\omega \ll \omega_p$, where ω_p is the plasma frequency, by dropping the terms in (4.79) that are small in the parameter of (3.6), we obtain the cross section for scattering by energy fluctuations in the form

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = (r_{0}r_{1})^{2}F(\omega)(\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}})^{2}C_{v}T^{2}$$
$$\times \operatorname{Re}\frac{1}{-i\omega+q^{2}\chi(\omega)}.$$
(4.81)

We repeat that one must take account of the frequency dispersion of the temperature conductivity coefficient γ in (4.81), since it ensures convergence of the integral cross section. At the same time, the cross section of (4.81) has a maximum at $\omega \tau < 1$. In this frequency region we can neglect the dispersion of χ . Therefore, when $T > \hbar/\tau$, the spectrum of (4.81) acquires the form of a Lorentzian contour with the half-width $\Gamma = q^2 \chi_0$, which is determined by the static electronic temperature conductivity χ_0 . With increasing concentration of impurities, which play in the semiconductor plasma the role of a second, heavy component, thermodiffusion is enhanced. Here, as was shown in Ref. 88, χ_0 declines. Thus the narrowing of the Raman-scattering contour noted in Secs. 4.3 and 4.1 acquires a macroscopic treatment for the scattering mechanism being discussed. We note that the narrowing of spectral lines is known in atomic physics as the Dicke effect.⁹¹

The contribution of the electron-phonon interaction to the width of the spectrum being discussed can be taken into account by introducing the energy relaxation time τ_{ε} . The total width of the Lorentzian of (4.81) has the form

$$\Gamma_l = q^2 \chi_0 + \frac{1}{\tau_{\varepsilon}}.$$
(4.82)

The first term of Eq. (4.79) omitted above proves essential at high frequencies $\omega \sim \omega_p$. It describes the scattering of light by plasmons. At medium concentrations *n*, at which $\omega_p \sim q^2 \chi_0$, we must keep all three terms in (4.79). In this case, for a complete description of the Raman spectrum we must take account of frequency dispersion in all three kinetic coefficients D, D_T , and χ .⁸⁸ In the case of elastic collisions of electrons, the calculation of the frequency dispersion is simplified, since the velocity vector v of an electron proves to be another eigenfunction of the collision integral that corresponds to an eigenvalue reciprocal to the transport relaxation time τ_{tr} . Here the frequency dependences of the kinetic coefficients are given by known relationships obtained in the relaxation-time approximation:⁶¹

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau_{\rm tr}}, \quad D(\omega) = \frac{D_0}{1 - i\omega\tau_{\rm tr}}$$
(4.83)

etc., where σ_0 is the static conductivity and D_0 is the corresponding diffusion coefficient. Substituting Eq. (4.83) into (4.79), we obtain

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{Vr_{0}^{2}}{2\pi} (\mathbf{e}^{\mathbf{I}}\mathbf{e}^{\mathbf{S}})^{2} \left(\frac{\partial n}{\partial \zeta}\right)_{T} \tau F(\omega) [NA - MB - (2N) (\omega\tau)^{2} + M(\omega\tau)^{4}] \{[(\omega\tau)^{4} - (A+1) (\omega\tau)^{2} + B]^{2} + (\omega\tau)^{2}[A - 2(\omega\tau)^{2}]^{2}\}^{-1}.$$

$$(4.84)$$

Here we have

 $\tau = \tau_{\rm tr}$,

$$A = (\omega_{\mathbf{p}}\tau)^2 + (\Gamma_1 + \Gamma_2 + \Gamma_c)\tau, \qquad (4.85)$$

$$B = \Gamma_1 \Gamma_2 \tau + (\omega_p \tau)^2 \Gamma_2 \tau, \qquad (4.86)$$

$$M = \Gamma_1 \tau + 2\Gamma_1 (\Gamma_1 \Gamma_c)^{1/2} \tau + r_1^2 (\Gamma_2 + \Gamma_c) \tau, \qquad (4.87)$$

$$N = \Gamma_1 \Gamma_2 \tau^2 + r_1^2 B, \tag{4.88}$$



FIG. 18. Theoretical spectra of electronic scattering with $e^{I}|| e^{S}$ in semiconductors with a nonparabolic dispersion law, illustrating the mixing of fluctuations of energy and charge.

$$\Gamma_{c} = q^{2} \frac{D_{T_{0}}^{2}}{T D_{0} C_{v}} \left(\frac{\partial \zeta}{\partial n}\right)_{T},$$

$$\Gamma_{1} = q^{2} D_{0}, \quad \Gamma_{2} = q^{2} \chi_{0}.$$
(4.89)

Figure 18 shows the results of an elementary calculation performed with Eqs. (4.84)–(4.89) for $r_1=0.2$. The parameters for which the calculation was performed are indicated in Table VI. Curves 1-3 correspond to electron concentrations from 5×10^{17} to 10^{16} cm⁻³. The values of the diffusion widths $\Gamma_1 - \Gamma_c$ from (4.89) were taken as common for all three curves. These values were calculated from the typical values of the electric conductivity and the mobility (see Table IV). The results of the calculation demonstrate an asymmetry of the spectrum characteristic of a regime of interference of two Raman-scattering mechanisms. The suppression of the plasma maximum on curve 3, which is drawn on the larger scale at the right, is due to interference of plasmons with energy fluctuations, and does not involve Landau damping.

For the experiments of Refs. 58 and 74–78, which were performed in a region of transparency of the crystals at $q \approx 10^5$ cm⁻¹, the interference of plasmons with energy fluctuations discussed here is more essential than Landau damping. In these experiments the plasmon peak was observed at low temperature $T \approx 10$ K and was not observed

TABLE VI. Values of the electron concentrations n, plasma frequencies ω_p , and relaxation frequencies $1/\tau$ for the spectra in Fig. 18.

$\frac{1}{n, \text{ cm}^{-3}}$	2×10 ¹⁷	=== 1×10 ¹⁷	3.3×10 ¹⁶
$\omega_{\rm n} {\rm cm}^{-1}$	150	100	60
$1/\tau$, cm ⁻¹	100	62	100

at room temperature: see also the book of Platzman and Wolf.⁹² This also involves the influence of energy fluctuations.

The generalization of Eq. (4.79) to the case of plasmon-phonon mixing can be obtained by taking account of the frequency dispersion of the lattice dielectric function:³

$$\varepsilon(\omega) = \varepsilon_{\infty} \frac{\omega_{\rm L}^2 - \omega^2}{\omega_T^2 - \omega^2 - i\omega\Gamma}.$$

This frequency dispersion, which was not taken into account in Fig. 18, is most essential at high electron concentrations $n > 5 \times 10^{17}$ cm⁻³.

4.6. Form of the intrasubband scattering spectrum in materials with degenerate bands

As was shown in Sec. 3.7, in materials with degenerate or close-lying bands, two unscreened mechanisms of Raman scattering exist, described by the symmetric and antisymmetric components of the matrix γ_{ik}^{kk} from (3.49). The spectrum corresponding to the antisymmetric contribution associated with fluctuations of spin density was calculated in Sec. 4.3. To calculate the spectrum of the symmetric scattering, we must substitute the tensor Q_{ik}^{kk} from (3.50) instead of B_{ik} in the expression for $R_p = e_i^{l} e_k^{S} B_{ik}$. This yields

$$R_{\mathbf{p}} = Q_{ik}^{\xi\xi} e_i^{\mathbf{l}} e_k^{\mathbf{S}}. \tag{4.90}$$

The fundamental difference of this spectrum from that calculated in Sec. 4.3 arises from the orbital nature of the matrix $Q_{ik}^{\xi\xi}$, which vanishes, being composed solely of Pauli matrices. Owing to this, in the kinetic equation (4.51) with R_p from (4.90), we must take account of the self-consistent electric field. This kinetic equation with R_p from (4.90) looks exactly like Eq. (4.68) with R_p from (4.66). Therefore the differential Raman cross section for fluctuations of the total angular momentum is given by solving Eq. (4.68), i.e., by Eqs. (4.70), (4.75), and (4.77).

4.6.1. The collision-free case

In single heterostructures and superlattices a condition is often satisfied that is the opposite of (4.12), since the experiments are performed at low temperature $T \ll \Theta_p$. Such a collision-free regime is characterized by the high mobility $10^6 \text{ cm}^2/\text{V} \cdot \text{s}$. Structures with a broad potential well and free from dimensional quantization were studied in Refs. 69 and 70. Times were attained of $\tau = 0.4 \times 10^{-10}$ s and values of the parameter ql = 10. By analogy with the case of intervalley fluctuations (see Sec. 4.2.2-1), it is convenient in (4.77) to single out two independent contributions to the scattering cross section. The first of them is fully determined by fluctuations of the angular momentum, while the second involves fluctuations of the screening electrostatic potential $\varphi = -i\mathbf{q}\mathbf{E}$. Calculation of the imaginary component of the polarization operator from (4.77) in this case yields

$$\frac{\mathrm{d}^2\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = V r_0^2 F(\omega) \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \left(\frac{\partial f_0}{\partial \zeta}\right)_T |R_p$$

$$-\varphi(\mathbf{q},\omega)|^{2}\delta(\omega-\mathbf{q}\mathbf{v}). \tag{4.91}$$

Here, to an accuracy of $qr_s \ll 1$, the screening potential equals

$$\varphi = \int d^3 p \, \frac{\mathbf{q} \mathbf{v}_p (\partial f_0 / \partial \zeta)_T}{\omega - \mathbf{q} \mathbf{v}_p + (i/\tau)} \, R_p \int d^3 p \, \frac{\mathbf{q} \mathbf{v}_p (\partial f_0 / \partial \zeta)_T}{\omega - \mathbf{q} \mathbf{v}_p + (i/\tau)} \,.$$
(4.92)

A possibility exists of calculating the integrals in (4.91) and (4.92) in analytic form in the case of an isotropic approximation for the spectrum of the scattering particles. There are five independent frequency functions a_{ss}^{\times} from (3.60) in this case, which should be obtained for a complete description of the spectra at all scattering angles. However, in superlattices and monolayers, as a rule, one uses the geometry "in reflection", which is convenient in combination with resonance enhancement of scattering. For this geometry only two frequency coefficients a_{ss}^{\times} remain. They correspond to the two scattering mechanisms cited above. As a result, with a degenerate statistics of the carriers, i.e., when $n \ge (m_T T)^{3/2}/\hbar^3$, the scattering cross section equals

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = V[F_{1}(\mathbf{q},\omega) + (\mathbf{e}^{\mathrm{I}}\mathbf{e}^{\mathrm{S}})^{2}F_{2}(\mathbf{q},\omega)].$$
(4.93)

Here we have

$$F_{1}(\mathbf{q},\omega) = \left(\frac{e^{2}\gamma}{mc^{2}}\right)^{2} \frac{\hbar\omega/qv_{\mathrm{F}}}{1 - e^{-\hbar\omega/T}} \frac{27}{32} \frac{n}{\varepsilon_{\mathrm{F}}} \Theta\left(1 - \frac{w}{qv_{\mathrm{F}}}\right) \\ \times \left[1 - \left(\frac{\omega}{qv_{\mathrm{F}}}\right)^{2}\right]^{2}, \qquad (4.94)$$

$$F_{2}(\mathbf{q},\omega) = \left(\frac{e^{2}\gamma}{mc^{2}}\right)^{2} \frac{\hbar\omega/qv_{\mathrm{F}}}{1 - e^{-\hbar\omega/T}} \frac{3n}{4\varepsilon_{\mathrm{F}}} \Theta\left(1 - \frac{\omega}{qv_{\mathrm{F}}}\right) \\ \times \left[\left[2 - \frac{\omega}{qv_{\mathrm{F}}} \ln \frac{1 + (\omega/qv_{\mathrm{F}})}{1 - (\omega/qv_{\mathrm{F}})}\right]^{2} + \pi^{2} \left(\frac{\omega}{qv_{\mathrm{F}}}\right)^{2}\right]^{-1}.$$

$$(4.95)$$

We see from (4.93) that, in the case $e^{I} \perp e^{S}$, the cross section is determined by the quantity $F_1(\mathbf{q},\omega)$. In the case $\mathbf{e}^{\mathbf{I}} \| \mathbf{e}^{\mathbf{S}}$, both $F_1(\mathbf{q},\omega)$ and $F_2(\mathbf{q},\omega)$ contribute to the cross section, while near $\omega = qv_F$ we have $F_2 \gg F_1$. The contributions of the individual terms in (4.93) are shown in Fig. 19 for an isotropic model of p-Ge with $p=1.5\times10^{19}$ cm⁻³ and T=2K (curves 1 and 3), T=300 K (curves 2 and 4). The total scattering cross section for crossed and parallel polarizations is shown in Fig. 20. We see from Figs. 19 and 20 that the singularity in the cross section that arises from the conservation laws (1.1) differs from that discussed in the Introduction. We might say that the singularity proves to be partially screened by the self-consistent electric field E. Equation (4.91) implies that only the electrons having a projection of the velocity $v_r = \omega/q$ and occupying a belt on the Fermi surface contribute to the scattering cross section. In the spherical case being discussed, this belt near the scattering threshold



FIG. 19. 1, 2—The function $F_1(\mathbf{q},\omega)$, 3, 4—the function $F_2(\mathbf{q},\omega)$ for an isotropic model of p-Ge with $n=1.5\times10^{19}$ cm⁻³. 1, 3—T=2 K, 2, 4—T=300 K. (From Ref. 8.)

$$\omega_{\rm max} = q v_{\rm F} + \frac{\hbar q^2}{2m^*}$$

(Fig. 1, curve 1) contracts to a point: $v_z = v_F$. However, in contrast to the situation shown in Fig. 1, the contribution of this single point of the Brillouin zone to the spectrum is completely screened. This screening of a feature in the cross section arises whenever the generation of a belt on the Fermi surface or a change in its topology is supported by a sole singular point (see Ref. 6).

A real Fermi surface of heavy holes is corrugated. It contains flat regions along planes of the type $\langle 100 \rangle$, and also indentations along directions perpendicular to them. The local flat regions enhance the singularities, while the presence of the indentations has the result that the belt on the Fermi surface that arises for $v_z = \omega_{\text{max}}/q$ proves to be multiply connected. That is, it has several points of support. The different points of support distinguish different



FIG. 20. Theoretical spectra of scattering by free holes at T = 300 K for the same model of p-Ge as in Fig. 19.



FIG. 21. Trace of the heavy-hole Fermi surface on a $\langle 010 \rangle$ -type plane. The arrows indicate the directions of the polarization vector $\mathbf{e}^{(S)}$ and the wave vector \mathbf{q} for which the calculation in Fig. 22 was performed. The numbers show the matching to the spectra. The polarization vector \mathbf{e}^{I} in both cases is perpendicular to the plane of the diagram.

groups of carriers that are in a state to scatter light without screening. They recall the valleys of a multivalley semiconductor that were discussed in Secs. 3.3 and 4.2.2. If the polarization directions e^{I} and e^{S} are chosen such that the points of support being discussed scatter light in different ways, then the character of the singularity discussed in the Introduction is restored. Figure 21 corresponds to one of these situations.

To offer a numerical illustration of the just discussed behavior of the spectra, we present a calculation of the Raman cross section of heavy holes with an orientation of the wave vector **q** along the direction of the (100) depression. In this case the function R_p that enters into (4.91) and (4.92) is divided into two irreducible contributions corresponding to Eqs. (3.68) and (3.69). Here the differential cross section breaks up into several terms, which we can write with the aid of Table II in the form of (3.60). Thus, calculating the diagonal matrix elements from (3.68) and (3.69) along the subband of heavy holes and substituting them into the expression for R_p from (4.91), we obtain the following expression for the functions a_{ss}^{x} from (3.60);

$$a_{22}^{\Delta_1} = \frac{9}{4} \gamma_2^4 \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \left(\frac{\partial f_0}{\partial \zeta}\right)_T |g^{-1}(\Omega)n_1^2$$
$$-\bar{\varphi}(\mathbf{q},\omega)|^2 \delta(\omega - \mathbf{q}\mathbf{v}_{\mathbf{p}}), \qquad (4.96)$$

$$a^{\Delta_2} = \frac{9}{4} \gamma_2^4 \int \frac{2\mathrm{d}^3 p}{(2\pi\hbar)^3} \left(\frac{\partial f_0}{\partial \xi}\right)_T \frac{(n_x^2 - n_y^2)^2}{g^2(\Omega)} \delta(\omega - \mathbf{q}\mathbf{v}_p),$$
(4.97)

$$a^{\Delta_{2}'}=9\gamma_{3}^{4}\int \frac{2\mathrm{d}^{3}p}{(2\pi\hbar)^{3}} \left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T} \frac{n_{x}^{2}n_{y}^{2}}{g^{2}(\Omega)} \,\delta(\omega-\mathbf{q}\mathbf{v}_{p}),$$
(4.98)

$$a^{\Delta_{5}} = 9\gamma_{3}^{4} \int \frac{2\mathrm{d}^{3}p}{(2\pi\hbar)^{3}} \left(\frac{\partial f_{0}}{\partial \zeta}\right)_{T} \frac{n_{x}^{2}n_{z}^{2}}{g^{2}(\Omega)} \,\delta(\omega - \mathbf{q}\mathbf{v_{p}}), \tag{4.99}$$

$$\varphi = \int \mathrm{d}^3 p \, \frac{\mathbf{q} \mathbf{v}_{\mathbf{p}}}{\omega - \mathbf{q} \mathbf{v}_{\mathbf{p}} + (i/\tau)} \frac{n_1^2}{g(\Omega)} \left(\frac{\partial f_0}{\partial \xi}\right)_T$$



FIG. 22. Differential light-scattering cross section of holes with account taken of the corrugation of the subbands of heavy holes. *I*—spectrum in the geometry $Y(ZX)\overline{Y}$; 2—in the geometry X'(ZX')Y'. The directions of the polarization vectors and the wave vector are indicated in Fig. 21.

$$\times \left[\int d^3 p \, \frac{\mathbf{q} \mathbf{v}_{\mathbf{p}}}{\omega - \mathbf{q} \mathbf{v}_{\mathbf{p}} + (i/\tau)} \left(\frac{\partial f_0}{\partial \zeta} \right)_T \right]^{-1}, \quad (4.100)$$

$$g(\Omega) = \left[\gamma_2^2 + 3\gamma_3^2 (n_x^2 n_y^2 + n_x^2 n_z^2 + n_y^2 n_z^2)\right]^{1/2}, \qquad (4.101)$$

where $n_{\perp}^2 = n_x^2 + n_y^2$. Fortunately, to illustrate what we need, it suffices to calculate the integrals of (4.96)-(4.100) in the first approximation in the parameter of (3.74). We shall present here only those functions a_{ss}^x from (3.60) that correspond to Γ'_{25} scattering according to Table II:

$$\begin{aligned} a^{\Delta_{2}^{\prime}} &= \frac{9\gamma_{3}^{4}}{4\bar{\gamma}^{2}} \cdot \frac{3n}{2\varepsilon_{\mathrm{F}}} \frac{1}{qv_{\mathrm{F}}} \left\{ \frac{(1-x^{2})^{2}}{4} \left[1 + \frac{3}{4} \eta \right] \\ &\times \left(\frac{13}{4} x^{4} - 5x^{2} - \frac{1}{6} \right) + 3\xi(1-x^{2}) \\ &\times \left(\frac{35}{4} x^{4} - \frac{39}{8} x^{6} - \frac{97}{24} x^{2} + \frac{1}{3} \right) \right] \Theta(1-x), \quad (4.102) \\ a^{\Delta_{5}} &= \frac{3\gamma_{3}^{4}}{4\bar{\gamma}^{2}} \cdot \frac{3n}{2\varepsilon_{\mathrm{F}}} \frac{1}{qv_{\mathrm{F}}} \left[\left[x^{2}(1-x^{2}) \left[1 + \frac{3}{2} \eta \right] \\ &\times \left(\frac{7}{2} x^{4} - 3x^{2} + \frac{1}{6} \right) \right] - 3\xi x^{2} \left(21x^{6} - \frac{173}{4} x^{4} \right) \\ &+ \frac{157}{6} x^{2} - \frac{17}{4} \right) \right] \Theta(1-x) \\ &+ (2 + \xi - 2x) \Theta(x-1) \Theta \left(1 + \frac{\xi}{2} - x \right) \right], \quad (4.103) \end{aligned}$$

where $x = \omega/qv_F$, $\xi = \eta \overline{\gamma}/(\gamma_1 - 2\overline{\gamma})$. The spectra of n-Ge calculated by these formulas are shown in Fig. 22. Curve 1 shows the spectrum in the geometry $Y(Z,X)\overline{Y}$, for which all the points of support are equivalent. Curve 2 shows the spectrum in the $X'(Z\overline{X'})Y'$ geometry, for which non-equivalent belts arise (see Fig. 21). We see that in the latter case there is an unscreened singularity in the scattering cross section near the threshold $\omega = qv_F$, which corresponds to the discussion just carried out.

In closing this section we note that in superlattices and monolayers both nonparabolicity and corrugation of the isoenergy surfaces of the holes are considerably enhanced in comparison with bulk materials (see, e.g., Ref. 93). The detection of the singularities pointed out above in the cross section might serve as a guarantee of the quality of a superlattice. Conversely, the absence of such singularities in the electronic spectra of HTSC crystals²⁷ renders unconvincing the description of the form of these spectra by using a concrete model of the Fermi surface.⁸⁵ This question is discussed in Sec. 6.

4.6.2. The case of frequent collisions qi ≪1

There are several factors that hinder the observation of the singularities of scattering spectra associated with the structure of the Fermi surface. Besides those discussed in the previous section, temperature and fluctuational blurring of these singularities can occur, as reflected in curves 2 and 3 in Fig. 1. The influence is described in Sec. 4.4 of electron collisions on scattering by spin-density fluctuations. In principle, collisions of holes exert an analogous influence on the spectra shown in Figs. 19, 20, and 22. Under the condition of frequent collisions (4.12) the second term of the polarization operator in (4.77) vanishes. We recall that the same situation arose in going from a regime of plasma scattering to the single-particle regime of (4.43) in the expression for the polarization operator of a multivalley crystal (4.39). As a result the scattering spectrum acquires a relaxational Lorentzian form. When we take account of corrugation, instead of one spherical function, infinite sets of eigenfunctions of the collision integral $I_{pp'}$ arise, having a definite and identical symmetry Γ_n . Another difference from the case of energy fluctuations discussed in Sec. 4.5 is that the function (4.90) itself is not generally an eigenfunction of the collision integral. It is represented in the form of an infinite series in the functions Ψ_{α} similar to (4.73). As a result the spectra of Γ_{12} and Γ'_{25} scattering will consist of a superposition of Lorentzian contours of the form

$$a^{\Gamma'_{25}} = \frac{1}{\pi} \left(\frac{\partial n}{\partial \zeta} \right)_T \sum_{\alpha} \gamma^2_{\alpha \Gamma'_{25}} \frac{v_{\alpha \Gamma'_{25}}}{\omega^2 + v^2_{\alpha \Gamma'_{25}}}.$$
 (4.104)

Here the $\gamma_{\alpha\Gamma'_{25}}$ are the coefficients of the expansion of the matrix elements from (3.69) in a generalized Fourier series in the functions Ψ_{α} . If we number the functions Ψ_{α} in order of increasing eigenvalues ν_{α} , then the number of zeros of these functions rapidly increases with increasing α . Correspondingly, the Fourier coefficients $\gamma_{\alpha\Gamma'_{25}}$ and $\gamma_{\alpha\Gamma_{12}}$ rapidly decline with increasing α . Their sums are normalized to the corresponding integral scattering cross sections of (3.70):

$$\sum_{\alpha} \gamma_{\alpha\Gamma_{12}}^{2} = \left(\frac{3\gamma_{2}^{2}}{m}\right)^{2} \int \frac{2\mathrm{d}^{3}p}{(2\pi\hbar)^{3}} \left(\frac{\partial f_{0}}{\partial n}\right)_{T} \times \frac{(p_{x}^{2} - p_{y}^{2})^{2}}{[\varepsilon_{1}(p) - \varepsilon_{b}(p)]^{2}}, \qquad (4.105)$$

Therefore only the first several terms in Eq. (4.104) are essential.

In the isotropic approximation Q_{ik} , as we see from (3.53), is reduced to a second spherical harmonic, which is an eigenfunction of the collision integral. Consequently (in the notation of Ref. 87), we have

$$v_1(\Gamma'_{25}) = v_1(\Gamma_{12}) = \tau_1^{-1}(2),$$
 (4.107)

where $\tau_1(2)$ is the relaxation time of the second Legendre polynomial. Here the cross section of symmetrical scattering is reduced to a single Lorentzian contour with the half-width $1/\tau_1(2)$. It was shown in Ref. 87 that the relaxation of the irreducible components of the distribution function with orbital angular momentum $l \leq 2$ occurs independently of the other terms of the expansion. This opens up the possibility of calculating the half-width of the Lorentzian from (4.107) outside the limits of the Born approximation. Insofar as we know, there are as yet no data on the measurement of the differential Raman cross section of the type being discussed in gapless semiconductors. Therefore we have not performed such calculations. We shall restrict the treatment here to discussing crystals of p-Ge and p-Si, for which the Born approximation is applicable. Applying the collision integral of (4.52) to the second Legendre polynomial, we obtain

$$\frac{1}{\tau_1(2)} = \frac{3n}{2\varepsilon_{\rm F}} \left(\frac{4}{5} W_0 - \frac{2}{7} W_2 - \frac{18}{35} W_4 \right). \tag{4.108}$$

Here we have

$$W_{l} = \int_{0}^{\pi} W(\mathbf{p}_{\mathbf{F}} - \mathbf{p}_{\mathbf{F}}') p_{l}(\cos \theta) \sin \theta \mathrm{d}\theta, \qquad (4.109)$$

where the function $W(\mathbf{p}-\mathbf{p}')$ is defined in (4.55). If we take account of only the ionized impurities in calculating the Born amplitude, then in the Brooks-Herring approximation we obtain

$$\frac{1}{\tau_1(2)} = \frac{4e^4 m^*}{\pi \varepsilon_0^2 \hbar^3} a \bigg[(3a^2 - 1) \ln \frac{1 + a}{a - 1} - 6a \bigg], \qquad (4.110)$$

where $a=1+(\hbar^2/2r_s^2p_F^2)$. In Ref. 86 the author erroneously omitted the coefficient *a* in Eq. (35). Figure 23 shows several spectra of light scattering polarized with $e^I \parallel e^S$ in n- and p-type InP from Ref. 95. The corresponding half-widths are shown in Fig. 24, in which the left-hand vertical scale is graduated for electrons, and the right-hand for holes. The need for a double scale arises from the fact that the electronic half-widths prove to be severalfold smaller than the hole half-widths. The experimental spectra of n-InP (see the curves for specimens No. 2 and 5 in Fig. 23) are narrow Lorentzians whose half-widths are shown in Fig. 24 by crosses (referred to the left-hand scale). The narrowing of the electronic half-width is a direct proof of its diffusional nature; see Sec. 4.4.1. The the-



FIG. 23. Experimental Stokes spectra of electronic scattering in n- and p-type InP crystals with a high concentration of charge carriers.

oretical curve of the diffusional half-width drawn by Eq. (4.82) is shown in Fig. 24 by the dashed line. There is a qualitative agreement between the theoretical and experimental results for electron concentrations $n \leq 10^{18}$ cm⁻³. The increase of the electronic half-width for concentrations $n \geq 10^{18}$ cm⁻³ indicates a change in the scattering mechanism. An attempt to calculate this half-width using a broadening mechanism of relaxational type (see Ref. 39) leads to values severalfold larger than the corresponding experimental values.

The experimental spectrum of a p-InP crystal shown by the curve for specimen No. 6 in Fig. 23 is a very broad Lorentzian with a half-width $\Gamma = 150 \text{ cm}^{-1}$. For comparison this experimental point is marked in Fig. 24 by the open square referred to the right-hand scale. The theoretical curve for this half-width calculated with Eq. (4.110) is shown in Fig. 24 by the solid line. The only existing experimental point agrees not poorly with the theoretical curve.

5. LIGHT SCATTERING BY CURRENT CARRIERS IN SUPERLATTICES

5.1. Light scattering by plasmons in superlattices

In superlattices and monolayers one observes in the Raman spectra various elementary excitations, which are



FIG. 24. Dependence of the half-width Γ of the scattering spectrum on the concentration $N \approx n.^5$ Right-hand curve—relaxation mechanism of (4.110), left-hand curve—diffusion mechanism of (4.82). The crosses and squares show the experimental points for crystals of n- and p-types, respectively.

described in a special book.⁹⁴ Many of them involve transitions between subbands of dimensional quantization, while more rarely one finds spectra of intrasubband scattering. In particular, intrasubband scattering with spin reversal was recently detected in Ref. 45. There is a review article by Pinczuk and Abstreiter devoted to electronic scattering in superlattices (see Chap. 4 in the book, Ref. 94).

The macroscopic approach to the description of the relaxation kinetics of light-scattering fluctuations developed in Secs. 4.2, 4.5, and 4.7.2 is useful in the analysis of the known mechanisms of light scattering in superlattices. We shall start with describing Raman scattering by chargedensity fluctuations.⁹⁶ The fundamental feature that gives rise to the difference of superlattices from bulk materials is the dimensional quantization of the current carriers. Because of it, dimensionally quantized structures are characterized by a lowered dimensionality. This leads to a strong dispersion of the plasma frequency $\omega_{p}^{(2D)}$, even at small *a*—to the so-called directional dispersion (see Ref. 26). Our formula (4.79) allows a simple generalization in two limiting cases. If the period d of the superlattice is much smaller than the reciprocal of the wave vector q^{-1} , i.e., qd < 1, then the approximation of an effective medium is. applicable.²⁶ Directional dispersion arises in this case in finding the zeros of the longitudinal dielectric permittivity of the effective medium:

$$q_i \varepsilon_{ik} q_k = 0. \tag{5.1}$$

Equation (5.1) implies that the plasma frequency equals

$$\omega_{\mathbf{p}}^{(2\mathrm{D})} = \omega_{\mathbf{p}}^{(3\mathrm{D})} \cos\theta, \tag{5.2}$$

where $\omega_{\mathbf{p}}^{(3D)} = (4\pi e^2 n/\epsilon_0 m^*)^{1/2}$ is the ordinary threedimensional plasma frequency, and θ is the angle between the wave vector q and the plane of the superlattice. Experiments in which a dispersion law of the plasma frequency in (5.2) was observed have been the object of numerous publications (see Figs. 2.53 and 2.54 from Ref. 3). In these studies a method was developed of almost specular Raman scattering with a fixed projection of the wave vector on the plane of the superlattice q_{\parallel} .

In the opposite limiting case $dq_{\parallel} > 1$ the superlattice is described by the model of an isolated monolayer. The generalization of Eq. (4.79) to the case of a superlattice with a large period is obtained after taking adequate account of processes of Maxwellian relaxation. To do this, instead of the frequency of Maxwellian relaxation $\tau_M^{-1} = 4\pi\sigma/\varepsilon$ in Eq. (4.79), the velocity $V = 2\pi\sigma/\varepsilon$ of spread of charge over the monolayer should enter. With account taken of the frequency dispersion of the conductivity, this means that in (4.79) we must replace the three-dimensional conductivity σ with the product $\sigma^{(2D)}q_{\parallel}$, where

$$\sigma^{(2D)} = \frac{n_{\rm s} e^2 \tau}{m^*} \frac{1}{-i\omega + (1/\tau)}$$
(5.3)

is the two-dimensional high-frequency conductivity, and n_s is the two-dimensional concentration of electrons. In the case of rare collisions the scattering cross section obtained in this way has a pole at the two-dimensional plasmon frequency

$$\omega_{\mathbf{p}}^{(2\mathrm{D})} = \left(\frac{2\pi e^2 n_{\mathrm{s}} q_{\parallel}}{\varepsilon_0 m^{*}}\right)^{1/2} \sim q_{\parallel}^{1/2}.$$
 (5.4)

In the case of frequent collisions, when their frequency is much larger than the two-dimensional plasmon frequency of (5.4), the first term in (4.79) acquires the form

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = \frac{r_{0}^{*2}}{\pi} F(\omega) (\mathbf{e}^{\mathrm{I}}\mathbf{e}^{\mathrm{S}})^{2} \frac{q_{\parallel}^{2} n_{\mathrm{s}}}{m^{*}} \frac{\tau}{\omega^{2} + (Vq_{\parallel})^{2}}.$$
 (5.5)

Several spectra calculated by (5.5) for different values of q_{\parallel} are shown in Fig. 25a. In the inset to Fig. 25a the dots indicate the experimentally measured⁹⁸ frequencies of the plasmon peaks. The spectrum of (5.5) in its acoustic dispersion law corresponds to the lowest-frequency branch in the inset to Fig. 25a.

Figure 25b shows the spectra of collective intersubband excitations in a photoexcited superlattice of GaAs-Al_{0.3}Ga_{0.7}As obtained in the study of Klein *et al.*⁹⁷ The structures were grown by the method of molecularbeam epitaxy on a GaAs substrate grown in the $\langle 100 \rangle$ direction. They consisted of 30 periods, each of which contained a quantum well of width 215 Å and barriers of width 100 Å. Curves *1-4*, which were obtained at the temperature T=5 K, differ from one another in the delay time between the exciting and probe pulses. These times amounted to 37.5, 162.5, 412.5, and 662.5 ns for spectra I-4, respectively. The frequency of the incident light was $\omega^{I}=1.9$ eV. The decrease in the plasmon frequencies with increasing delay time was caused by interband recombination of current carriers. The most interesting feature of the



FIG. 25. a—Spectra of scattering by charge-density fluctuations in a two-dimensional electron plasma calculated by Eq. (5.5). The numbers on the curves indicate the values of the wave vector in units of 10^4 cm⁻¹. The inset shown the experimental dispersion laws measured in Ref. 98 (dots) and the theoretical values (solid lines) calculated by Eq. (5.4). The vertical cross-hatching shows the region of Landau damping. b—Raman spectra with time resolution of the intersubband plasmon-phonon modes I_+ and I_- of a GaAs–Al_xGa_{1-x}As superlattice with a width of quantum wells d=215 Å. (From Refs. 97 and 119.)

spectra is the narrowing with time of the high-frequency plasmon-phonon mode I_+ , whereas the width of the lowfrequency mode I_- remains practically invariant. One can explain this unusual behavior of the plasmon-phonon modes by starting with their dispersion curves. The dispersion curves of the collective intersubband excitations calculated in Ref. 100 for a binary heterostructure with the parameters $n_s = 5 \times 10^{11}$ cm⁻², d = 200 Å, are shown in Fig. 26 together with the cross-hatched region of intersubband one-particle excitations. The changes in the dispersion curves that arise upon decreasing the electron concen-



FIG. 26. Dispersion curves of collective intersubband excitations calculated in Ref. 100 for a binary heterostructure with parameters $n_s = 5 \times 10^{11}$ cm⁻², d = 200 Å.

tration n_s are shown in Fig. 26 by the dashed lines. We see from Fig. 26 that the decrease in the intersubband plasma frequency with time must be accompanied by narrowing of the region of one-particle excitations in which Landau damping exists. A corresponding increase in the lifetime initially occurs for the high-frequency mode I_+ , and only then for the low-frequency mode I_- . To a certain degree this pattern explains why only the high-frequency mode I_+ succeeds in exiting the region of Landau damping and decreases in width in the course of time, while the lowfrequency mode cannot do this and therefore does not change in width.

5.2. Light scattering by one-particle excitations in superlattices

The Lindhard-Mermin scattering cross section of (4.59) is most often used in describing the Raman spectra of spin-density fluctuations in superlattices.^{45,98} From such measurements one reconstructs the relaxation time τ that gives the best agreement of the experimental and theoretical curves (see Sec. 4.4.1). The fitted values of the relaxation times thus obtained prove, however, to be about three times smaller than those obtained from electrical measurements of spectral broadening in superlattices and monolayers. The additional cause of broadening might be large-scale fluctuations of the impurity potential (see Fig. 1 and Refs. 25 and 99).

Light scattering by energy fluctuations has also been found in superlattices. It has been observed in the polarized spectra shown in Fig. 4.12 from the book, Ref. 94. It was shown in Secs. 3.5 and 4.5 of this review that the corresponding scattering cross section is weakly sensitive to the temperature in the case of strong nonparabolicity. This explains why in superlattices, where the nonparabolicity is stronger than in bulk materials (see Ref. 93), one can detect the one-particle spectrum with parallel polarizations of the incident and scattered light $\mathbf{e}^{\mathbf{I}} \| \mathbf{e}^{\mathbf{S}}$. Another explanation is based on Fig. 25a and (5.5).

Finally we note that the spin splitting of the subbands of dimensional quantization greatly exceeds the magnitudes discussed in Sec. 4.4.1 for bulk materials. In addition to the Hamiltonian of the spin-orbital interaction of (3.47), which is cubic in the quasimomentum, also a linear term is present in dimensionally quantized structures (see Ref. 101). It has the result that processes that occur with spin reversal and with spin-density fluctuations become independent. Since they are practically the same in intensity, one can easily distinguish them experimentally. In Ref. 98 in Raman spectra of electrons in a well 500-A wide, the spin splitting was still insignificant, and therefore the processes being discussed could not be distinguished. However, in experiments with a narrower-180-Å widequantum well,⁴⁵ a doublet structure of one-particle electronic spectra is distinctly manifested. Here the highfrequency component of the doublet should be ascribed to scattering with spin reversal, whereas the low-frequency component involves fluctuations of spin density. We note that the authors of Ref. 45 attributed both components of the doublet to processes occurring with spin reversal. This leads to a number of contradictions between theory and experiment: the observed spin splitting proves to be twofold smaller than the calculated value, while the calculated intensities of the components of the doublet do not match those experimentally observed. All these contradictions can be eliminated if one identifies the low-frequency peak with the Raman scattering by spin-density fluctuations.

5.3. Light scattering in superlattices with quantummechanical degeneracy of states

Comparison of the light-scattering spectra of n- and p-type superlattices reveals a gigantic difference between them. In n-superlattices one observes a substantial frequency difference of the intersubband transitions obtained in crossed and parallel polarizations of the incident and scattered light. The frequency difference proves to be exactly equal to the frequency of an intersubband plasmon, and this effect is known in the literature as the effect of the "depolarizing field."³ although it is sometimes called an exciton wave.¹⁰² Its explanation arises from the selection rules discussed in Secs. 3.1 and 3.6, according to which, with parallel polarizations $e^{I} || e^{S}$ one observes plasmons, while with $e^{I} \perp e^{S}$ one observes one-particle excitations, e.g., spin-density fluctuations.

The Raman spectra of free holes obtained by Abstreiter *et al.* (see Chap. 4 in Ref. 94 and Ref. 103) demonstrate almost complete absence of this "effect of the depolarizing field." The theoretical ideas developed in Secs. 3.7, 3.9, and 4.6 allow us to give this the following treatment. The point is that, in both the parallel (Y'Y') and the crossed (Y'Z') experimental geometry, intersubband scattering actually takes place, which is described by the matrices of (3.69) and (3.68), respectively, and is not accompanied by an electric field. Another interesting feature of the Raman spectra of superlattices is the low-frequency peak manifested for parallel polarizations (Y'Y'). According to Chap. 4 in Ref. 94, its characteristic features consist in the following. It lies in energy below all the imaginable intersubband transitions. The corresponding abbreviation LEEX in Fig. 4.42 of Ref. 94 simply means "low-energy excitation". The LEEX peak has rigorous selection rules—it is observed in (Y'Y') geometry and is absent in (Y'Z'). Upon lowering the temperature to 2-3 K, this peak acquires a multiplet structure. In Ref. 94 a definite identification of the LEEX peak is absent, although in an earlier study¹⁰³ it was attributed to intrasubband collective excitations of holes.

The theory constructed in Secs. 3.7, 3.9, and 4.6 allows us to give an unequivocal treatment of the LEEX peak that explains all of its features. We note that, in the spectra in (Y'Y') geometry in the cited experiments from Refs. 94 and 103, not only the scalar excitations are allowed, but also the Γ'_{25} excitations (those of Γ'_{25} symmetry). It was shown in Sec. 3.9 that precisely the Γ'_{25} geometry, according to (3.77), is a test for one-particle scattering by free holes, which involves fluctuations of the density of total angular momentum. Therefore the absence of a LEEX peak in the (Y'Z') geometry, which coincides with Γ_{12} , and its presence in Γ'_{25} geometry are evidence in favor of its one-particle nature. This hypothesis is confirmed by the temperature dependence of the form of the LEEX peak, which acquires a multiplet structure with decreasing temperature, while with increasing temperature it is converted into a Lorentzian bell shape similar to (4.67) or (4.104). Apparently, with increasing temperature the decisive contribution to the LEEX peak arises from the spin-density fluctuations of (4.64) or the fluctuations of total angular momentum of (4.104). The condition corresponding to (4.12) of frequent collisions, which is necessary for applicability of these formulas, has the following form in a twodimensional electron gas:

$$q_{\parallel} \ l \ll 1. \tag{5.6}$$

This condition is milder than the condition (4.12) for bulk materials, since the component of the wave vector q_{\parallel} is small in comparison with its modulus q in back-scattering experiments.³ Therefore the condition (4.6) can be fulfilled by phonons even at moderately low temperatures. The multiplet structure of the LEEX peak at T=2-3 K arises from intrasubband hole transitions with spin reversal. On the whole its nature is the same as for the doublet structure of the one-particle spectrum of Ref. 45, which was discussed in Sec. 5.2. In particular, at the lowest temperatures discussed in this connection in Chap. 4 of Ref. 94, the condition (5.6) ceases to be fulfilled, and the multiplet structure must acquire a dependence on the wave vector.

6. FREQUENCY DEPENDENCE OF THE ELECTRONIC RAMAN CROSS SECTION IN METALS AND SUPERCONDUCTORS

6.1. Normal metals

Electronic Raman scattering in metals proves to be rather hard to observe because the resonance in $\hat{\gamma}_{ik}$ from (2.19) lies in the region of x-ray frequencies, while the nonresonance scattering by charge-density fluctuations is strongly screened owing to the high concentration of electrons. Success in detecting electronic Raman scattering in metals possibly will be attained after the invention of x-ray lasers. The fragmentary information existing today is carefully collected and analyzed in detail in the review, Ref. 104, where also the results on electronic Raman scattering in HTSC crystals are presented.

A structureless background of inelastic scattering was recently measured in certain rare-earth metals: Y, Dy, and Er.^{105,106} On the other hand, certain highly limited information on one-particle electronic excitations active in Raman scattering can be obtained by measuring the temperature dependence of the half-widths of the phonon lines.^{3,90} The contribution to this width associated with anharmonic decays of phonons increases with increasing temperature. An anomalous decrease in the width of the phonon lines that was observed in osmium¹⁰⁷ can be interpreted as the result of suppression of the high-frequency electronic excitations by frequent electron collisions. The corresponding contribution to the half-width of the phonon lines is determined by the imaginary component of the electronic polarization operator $\Pi(q,\omega)$ from Fig. 13:

Im
$$\Pi(q,\omega) \sim \Delta \Gamma_{\text{background}} \sim \frac{\Gamma \omega_0}{\Gamma^2 + \omega_0^2}$$
, (6.1)

where ω_0 is the frequency of the phonon, and $\Gamma = q^2 D + (1/\tau)$ is the half-width of the corresponding line of one-particle quasielastic scattering. One can obtain a decrease of $\Delta\Gamma$ with temperature by assuming that $\tau^{-1} > Dq^2$ and that $\tau^{-1} < \omega_0$ at low temperatures, but $\tau^{-1} \approx \omega_0$ at high temperatures. One can attain a remarkable agreement between the theory of (6.1) and experiment by calculating $\tau(T)$ from *dc* conductivity data.¹⁰⁷

6.2. Electronic Raman scattering in HTSC crystals

The data on the magnitude of the superconductive gap 2Δ in high-temperature superconductor compounds disagree very strongly among the various authors at present. The measurements have been conducted by various methods, ¹⁰⁸ and values of $2\Delta/kT_c$ were obtained, where T_c is the temperature of the superconductive transition, that lie in the range from 2 to 8. The Bardeen-Cooper-Schrieffer theory yields a value $2\Delta/kT_c \approx 3.52$, which lies exactly in the middle of the cited range of experimental values. In the case of the classical orthorhombic materials YBa₂Cu₃O₇, or briefly YBCO123, there are phonon lines that lie in the region of the broad maximum of the one-particle electronic background. These are phonons of A_g symmetry, which are observed with polarization vectors $\mathbf{e}^{\mathbf{I}} \parallel \mathbf{e}^{\mathbf{S}}$ lying in the plane of the layers. For example, there

is a Raman-active phonon of frequency $\omega_0 = 340 \text{ cm}^{-1}$ at $T < T_c$. These phonon lines are highly sensitive to rearrangement of the one-particle electronic excitations, which occurs at $T = T_c$ with opening of the superconductive gap.

The density of states of the one-particle electronic excitations through the superconductive gap within the framework of the Bardeen–Cooper–Schrieffer model equals

$$N_{\Sigma}(\omega) \sim (\omega - 2\Delta)^{-1/2}.$$
 (6.2)

These excitations lead to a rearrangement of the phonon lines and to one-particle electronic scattering. They are not screened, since they are accompanied by breakdown of the two half-filled Cooper pairs $\langle -p \uparrow, +p \downarrow |$ and $\langle -p \downarrow, +p \uparrow |$ with formation of one completely filled and one completely empty pair. This process is accompanied by reversal of the two spins of the components of the Cooper pairs, and hence can occur in the absence of charge fluctuations, i.e., screening. The theory of this process and its generalization to the case of finite q, finite T, and complex energy bands that allow unscreened scattering in the normal phase can be found in several studies.^{17,82,84,109,110} These theoretical studies do not make use of the specifics of high-temperature superconductors.

A recent study by Abrikosov¹¹¹ examined electronic scattering for a model containing the specifics of HTSC compounds. This model consists of superconductive layers with pairwise attraction between the electrons and the layers of the normal phase.¹¹¹ This model is applicable to YBCO123 compounds, which contain planes and chains, and also to compounds based on bismuth $(Bi_2Sr_2CaCu_2O_8)$, or briefly BISCO2212). The interaction between the two types of layers is taken into account in this model by using the hopping integral t. The described model predicts a peak in the scattering cross section at the frequency $\omega \approx 2\Delta$ for $\mathbf{e}^{\mathbf{I}} \parallel \mathbf{e}^{\mathbf{S}} \parallel a$ lying in the plane of the layers, and $\omega \approx \Delta$ for $\mathbf{e}^{\mathbf{I}} \parallel \mathbf{e}^{\mathbf{S}} \parallel c$ oriented perpendicular to the layers. This prediction was experimentally confirmed for BISCO2212 in Ref. 112 (Fig. 27) and for YBCO123 in Ref. 113. The anisotropy of the superconductive gap¹⁰⁸ was also established in $YBa_2Cu_4O_8$ (YBCO124), which compels us to assume a different magnitude of the gap in the CuO₂ planes (325 cm^{-1}) and in the double CuO chains (113 cm^{-1}) .¹¹⁴ In closing we note that the observation of a superconductive gap in spectra of HTSC compounds was first reported in Refs. 115 and 116.

As was mentioned in Sec. 4.4.1, an intense, broad, and almost structureless line extending to 1 eV is present in all compounds of high-temperature superconductors in the normal phase. Since no evidence of a plasmon peak was established, we should assume that this band involves unscreened scattering that occurs in the absence of chargedensity fluctuations. The most characteristic feature of this form of scattering is the practically complete independence of its cross section of the temperature. This behavior of the cross section disagrees with the coefficient of $F(\omega)$ in (2.21), according to which the cross section should be proportional to the temperature when $T/\hbar\omega > 1$.



FIG. 27. Stokes spectra of a crystal of BISC02212 in the superconductive phase for polarizations of the incident and scattered light parallel to the a(x) and c(z) axes, illustrating the anisotropy of the superconductive gap predicted in Ref. 111. (From Refs. 112 and 104.)

The observation of a frequency- and temperatureindependent Raman cross section enabled Varma *et al.*¹¹⁷ to propose that the coefficient $F(\omega)/\omega$ is compensated by the inverse dependence of the electronic polarization operator $\Pi(\omega)$ (see Fig. 13) on T and ω . Thus they proposed that the part of $\Pi(\omega)$ independent of q equals

Im
$$\Pi(\omega) \sim \omega/T$$
 for $|\omega| < T$,
~Sgn ω for $|\omega| > T$. (6.3)

Evidently the first of the equations of (6.3) allows compensation of the influence of the coefficient of $F(\omega)$ on the scattering cross section. The equations of (6.3) cannot be derived within the framework of any microscopical model of a Fermi liquid. Therefore (6.3) actually postulates that the electrons in the normal phase of HTSC compounds form a "magic Fermi liquid." In an ordinary Fermi liquid the reciprocal lifetime of electrons in states separated by ω from the Fermi surface is proportional to ω^2 . In a "magic" Fermi liquid according to (6.3) this reciprocal time should be proportional to $|\omega|$.

The magnitude of the scattering cross section in the normal phase is determined according to Ref. 82 by the square of the convolution of the tensor of (3.23) with the polarization vectors e^{I} and e^{S} . According to Secs. 4.4.1 and 4.7.2, the component of the Raman spectrum independent of the wave vector should be described by using a relaxation contour of the type of (4.64):

$$\frac{\mathrm{d}^{2}\Sigma}{\mathrm{d}\omega\mathrm{d}\Omega} = F(\omega)B\frac{\tau}{1+(\omega\tau)^{2}},\qquad(6.4)$$

Here τ is the characteristic relaxation time of the lightscattering fluctuations, and *B* is a parameter characterizing the intensity of scattering. Although both parameters τ and *B* depend on the mechanism of scattering, Eq. (6.4) itself is universal. It corresponds to the universal macroscopical mechanism of dissipation of the energy of a nonscalar external field of the type of (4.33') first discovered by Mandel'shtam and Leontovich.¹¹⁸ This problem was discussed in detail by one of the authors in Ref. 86.

Equation (6.4) enabled Virosztek and Ruvalds⁸⁵ to propose a mechanism that explains the appearance of the magic properties (6.3) of Fermi fluids. According to Ref. 85, the existence at the Fermi surface of HTSC-compounds of parallel regions leads to the following expression for $1/\tau$:

$$\tau^{-1} \approx \alpha (\omega^2 + \beta^2 T^2)^{1/2} \approx \alpha' (|\omega| + \beta T), \qquad (6.5)$$

where α , α' , and β are parameters of the order of unity. The theoretical value of the parameter β equals 4, whereas Raman-scattering data yield $\alpha \approx 0.5$. Upon substituting τ^{-1} from (6.5) into (4.64), we obtain for $T > \omega$

$$\frac{\mathrm{d}^2 \Sigma}{\mathrm{d}\omega \mathrm{d}\Omega} \approx \frac{B}{\alpha \beta} \approx B = \mathrm{const}$$
(6.6)

independently of ω and T, in agreement with the experimental observations. In the opposite limiting case $T \boldsymbol{<} \omega$, Eqs. (4.64) and (6.5) yield

$$\frac{\mathrm{d}^2 \Sigma}{\mathrm{d}\omega \mathrm{d}\Omega} = B \frac{\alpha}{1+\alpha^2} \approx \alpha \beta \tag{6.7}$$

which is also independent of ω and T, in agreement with experiment. Nevertheless, in the region

$$\alpha\beta T < \omega < \beta T \tag{6.8}$$

the frequency of (6.5) behaves like $\alpha\beta T$, so that the frequency dependence of the cross section is determined by the coefficient $F(\omega)$ from (2.21):

$$\frac{\mathrm{d}^2 \Sigma}{\mathrm{d}\omega \mathrm{d}\Omega} \approx \alpha \beta B \frac{T}{\omega} \,. \tag{6.9}$$

The increase of the cross section with decreasing ω predicted by Eq. (6.9) has not been observed experimentally. This increase might be diminished if we assume that α is larger than 0.5. Indeed, such an assumption impairs the agreement in the other regions of the spectrum. This is reflected in Fig. 28, which shows the electronic scattering spectrum measured in the normal phase of Bi₂Sr₂CaCu₂O₈ and its theoretical approximation by Eq. (6.4) with τ^{-1} from (6.5) from Ref. 85.

On the whole the theoretical description of the giant electronic scattering in HTSC crystals is still far from perfection. The results presented in this review show that a complete description is impossible without taking detailed account of the electronic band structure of these compounds. The unit cell of HTSC crystals contains several molecules. Owing to this, their space-group symmetry includes nontrivial translations and hence is not symmor-



FIG. 28. Electronic scattering spectrum measured in the normal phase of $Bi_2Sr_2CaCu_2O_8$ and its theoretical approximation by Eq. (6.4) with τ^{-1} from (6.3). (From Refs. 85 and 104.)

phic. As was shown in Ref. 51 with the example of n-Si (see Sec. 3.8), the presence of nontrivial translations leads to a specific degeneracy of the band states at certain points at the edge of the Brillouin zone. In n-Si this degeneracy occurs at the x-points. In HTSC crystals having a low enough symmetry group, degeneracy occurs along entire lines along the boundaries of the Brillouin zone. Therefore the reduced density of Raman-active interband excitations in HTSC crystals must be considerably larger than in n-Si, where electronic Raman scattering is reliably observed with insignificant resonance enhancement. As we see it, this circumstance must be taken into account in describing electronic Raman scattering in HTSC crystals.

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