An outline of the development of the theory of the structure of the atomic nucleus (I,IV)*

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I. THEORY OF RADIOACTIVE DECAY

1. Everyone (or, at any rate, everyone who picks up this journal) knows that an atom consists of a heavy nucleus carrying a positive charge and a system of electrons rotating about it in analogy with a small planetary system. This is the so called Rutherford-Bohr atomic model.

During the last two decades the friendly cooperation of experimenters and theoreticians made it possible to study in detail and to explain the laws governing the electron system of an atom, and at the present moment the theory of atomic structure can be regarded as being practically completed. The study of the atom forced us to reexamine the applicability of the laws of classical mechanics which turned out to be true only approximately and led to a new system of quantum (or wave) mechanics.

In parallel with this the theory of the structure of the nucleus was being developed. Already the phenomenon of radioactivity discovered at the end of the last century indicated that the nucleus of an atom is not a simple entity but has a very complex structure. The α and β particles observed in the radioactive decay of the elements were interpreted by Rutherford as component parts of the nucleus ejected from the unstable nuclei of heavy atoms, and the quite hard radiation, the γ rays observed in this decay were interpreted as electromagnetic perturbations brought about by the restructuring of the nuclei subsequent to decay.

Rutherford's further experiments showed also the possibility of artificial breaking up of nuclei of usually stable elements under the influence of external energetic interactions.

The discovery of isotopes and Aston's investigations that showed that the atomic weights of the isotopes are expressed by numbers very close to integers made more than probable the supposition that the nuclei of all the elements are built up from protons and electrons where a very great role in the structure of the nucleus is played by formations consisting of four protons and two electrons (α particles) that are very stable.

A very accurate measurement of the atomic weights of the isotopes demonstrated deviations from integers (mass defect) which led to the possibility of determining the total energy binding the individual structural elements of a nucleus into one whole.

Detailed studies of the spectra of γ rays which showed their line structure—investigations for which we are obligated primarily to Ellis and Meitner, led to the conclusion that within the nucleus of the atom we are dealing with the existence of definite quantum energy levels completely analogous to those which we encounter in the electron system of the atom.

Finally in the most recent times the observation of hyperfine structure of lines of the optical spectrum indicated the existence of a definite magnetic moment of the nucleus and the possibility of determining it.[1)]

At the present time we have an exceedingly rich but, to be frank, quite disorderly experimental material concerning the atomic nucleus and the time has come for theoreticians armed with the powerful tool of modern quantum mechanics to tackle the problem of the structure of the nucleus and of explaining the observed facts and regularities.

2. In view of the very great difficulty to act on the atomic nucleus by the means available to us the material obtained in this way is quite sparse and it is naturally to be expected that the first theoretical conclusions concerning nuclear structure can be obtained from the study of the natural decay of heavy atoms (the phenomenon of radioactivity)—a field which at the present time has been studied in considerable detail.

The most surprising fact which we encounter in the theory of the spontaneous decay of nuclei are those frequently very long time intervals in the course of which the unstable nucleus remains in *statu quo* before ejecting an α or a β particle. The average lifetime of radioactive elements varies from a very small fraction of a second to unusually long periods of many millions of years, and for any given element is a quite definite quantity.

It appeared to be very difficult to find causes which delay the emergence of a particle for such long periods of time, if the particle has enough energy to leave the nucleus, and yet the α and β particles ejected from the nucleus carry very considerable amounts of energy.

Already for a long time the fact was known of the existence of a very definite relationship between the energy of the ejected particle and the average period of its existence within the nucleus in an unstable state (the period for the decay of the nucleus). In 1912 Geiger and Nuttall noticed that if for elements which undergo decay we plot along the abscissa the energy of the α particles and along the ordinate the logarithm of the corresponding decay constant then for a given radioactive family the points will lie approximately on a straight line. The three radioactive families known to us as uranium-radium, thorium and actinium families are represented by three parallel straight lines. The Geiger–Nuttall diagram is shown in Fig. 1 where we can note a number of deviations from the linear law. First of all, the values of $\log \lambda$ corresponding to very large or very small values of the energy of the particle turn out to be systematically smaller than required by the linear law (a fact first noted by Jacobsen and Gudden) indicating that in fact we are dealing with a curve concave downwards.

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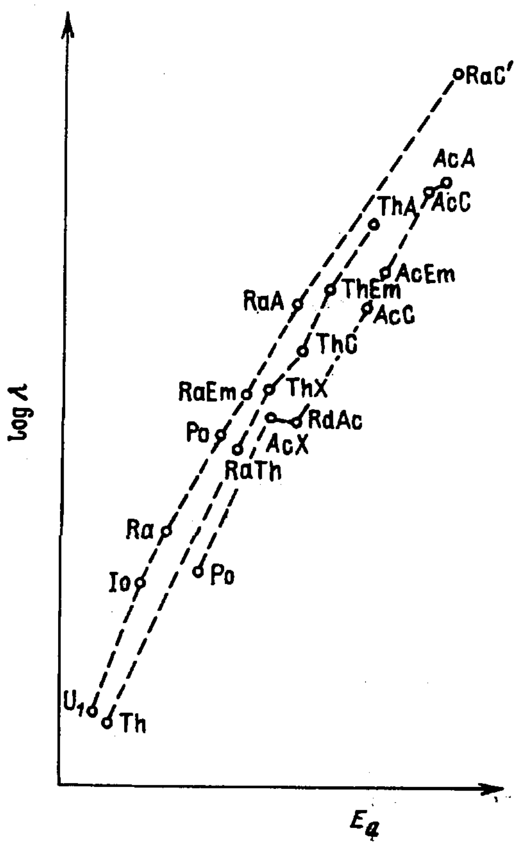

FIG. 1.

Secondly, we note (particularly sharply for *AcX*) cases where the experimental point deviates from the Geiger–Nuttall straight line indicating some kind of a sharp anomaly.

It is quite clear that before we try to explain the regularities associated with the emergence of α particles from the nucleus we must know something about the forces acting on an α particle close to and inside the nucleus itself.

The α particles carrying a positive charge will of course experience the Coulomb repulsion from the remainder of the nucleus. The potential energy of the Coulomb forces can be written in the form

$$U_C(r) = +\frac{2(Z-2)e^2}{r}, \tag{1}$$

where Z is the atomic number of the decaying nucleus, and e is the elementary charge. In order to explain the existence of α particles inside the nucleus it is necessary to assume the existence also of some attractive forces which act only at very close distances from the nucleus. We can formulate different hypotheses on the nature of these forces—they may be either polarization forces (decreasing with distance as $1/r^5$, or forces of quantum interaction (Austauschenergien) between the internal structures of the α particle and the remainder of the nucleus—these forces fall off with distance exponentially. The existence of such attractive forces can be noted in an experiment: in experiments on the scattering of α particles in different elements the particles approach to the nucleus to very close distances and can enter the region of action of those forces.

The experiments of Rutherford and Chadwick showed that in the case of very close collisions of α particles with nuclei of light elements deviations observed of the number of scattered particles from the formula derived on the assumption of a Coulomb interaction. The observed deflections can be explained by the assumption of the existence of the attractive forces mentioned above—we can in this way form an idea of the range of action and the laws governing these forces. Unfortunately at the present time we do not yet have a sufficiently detailed investigation of the anomalous scattering of α particles, and the theoretical conclusions amount roughly to the following. For light elements (Mg,Al) the anomalous attractive forces begin to become *felt at distances of the order of* 10^{-12} *cm varying approximately* inversely proportional to the fourth or fifth power of the distance, and overcome the Coulomb repulsion at a distance of approximately $3 \cdot 10^{-13}$ cm from the center of the nucleus—at closer distances the α particle is evidently already under the influence of the total attractive forces. For the nuclei of the heavy radioactive elements in which we are interested in view of their large charge the α particles available to us cannot approach to such close distances and reach the region of the anomalous forces. Rutherford and Chadwick in experiments on the scattering of α particles by uranium could attain (using the fastest α particles) only distances of $3 \cdot 10^{-12}$ cm, and no deviations from the normal scattering were noted—the region of the attractive forces evidently lies here much closer to the nucleus than $3 \cdot 10^{-12}$ cm.

It might appear that the results of these experiments with uranium could be of very little help to us, since the region of attractive forces cannot be reached by us; but it is just in these experiments that the key lay to deciphering the phenomenon of α decay.

In comparing the data on the decay of the uranium nuclei themselves these experiments lead to the paradox totally inexplicable from the point of view of classical mechanics. Indeed: the nuclei of uranium atoms are unstable and eject α particles with energies of approximately $6.8 \cdot 10^{-6}$ erg. In accordance with our hypothesis on the existence of attractive forces near the nucleus, the α particle situated within the nucleus of a radioactive element is surrounded by a kind of potential barrier as shown in Fig. 2. The fact that even at distances of $3 \cdot 10^{-12}$ cm we have only the Coulomb forces indicates that the maximum height of the barrier is in any case greater than

$$\frac{2(Z-2)e^2}{3 \cdot 10^{-12}} = 14 \cdot 10^{-6} \text{ erg (for uranium } Z=92).$$

How can an α particle of uranium with an energy of only $6.8 \cdot 10^{-6}$ erg "roll over" such a barrier? In other words: if the α particles of *RaC′* used in experiments on scattering by uranium "rolling up" the outer slope of the barrier were far from being able to reach its peak, how can the uranium α particles having a considerably lower energy roll over the barrier and emerge outside? From the point of view of

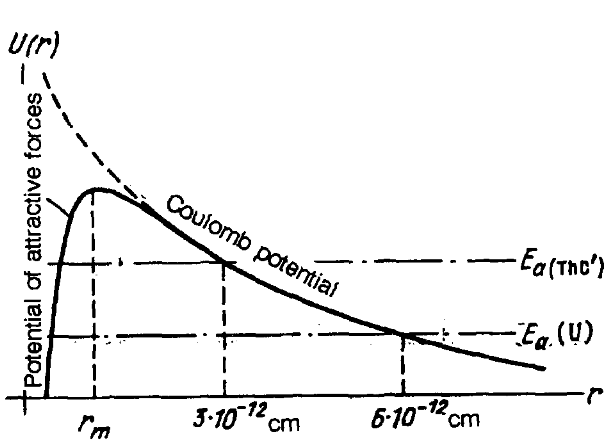

FIG. 2.

classical mechanics an α particle in passing over such a barrier that is higher than its total energy should have inside the barrier a "negative kinetic energy" and, consequently, an "imaginary velocity." However the possibility of such a phenomenon that is in sharp contradiction to classical mechanics is a direct consequence of the modern wave mechanics. In analogy with how in wave optics light falling on the interface of two media at an angle greater than the angle of total internal reflection, partially penetrates into the second medium, so in exactly the same way in wave mechanics the de Broglie–Schrödinger waves may partially penetrate into the region of "imaginary velocity" thereby giving the possibility for particles to "roll over" the barrier.

We now shall discuss the simplest case of a rectangular barrier and shall derive the formulas for its "penetrability." We specify the distribution of the potential by the conditions

$$U(x)=0, \quad x<0,$$

$$U(x)=U_0, \quad 0<x<l, \tag{2}$$

$$U(x)=0, \quad l<x.$$

The Schrödinger equation can be written in the form

$$\frac{\partial^2\Psi}{\partial x^2}-\frac{4\pi i}{h}\frac{\partial\Psi}{\partial t}+\frac{8\pi^2 m}{h^2}U(x)=0. \tag{3}$$

Taking

$$\varphi(x,t)=\Psi(x)\exp\left(\frac{2\pi i}{h}Et\right), \tag{4}$$

where E is an arbitrary constant specifying the energy of the system we have for the determination of Ψ

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{8\pi^2 m}{h^2}(E-U(x))\varphi=0. \tag{5}$$

We examine the case of penetration through the barrier which is classically impenetrable, and therefore $E<U_0$ (Fig. 3). The solutions of equation (5) in regions I, II, and III will be given, respectively, by

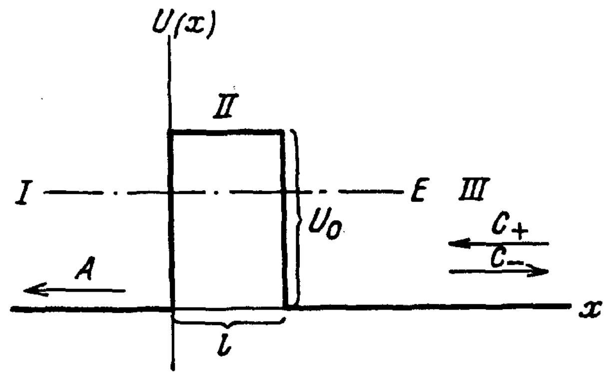

FIG. 3.

$$\Psi_{\rm I}(x)=A_+e^{ikx}+A_-e^{-ikx}, \tag{6}$$

$$\Psi_{\rm II}(x)=B_+e^{k'x}+B_-e^{-k'x}, \tag{6'}$$

$$\Psi_{\rm III}(x)=C_+e^{ikx}+C_-e^{-ikx}, \tag{6''}$$

where

$$k=\frac{2\pi}{h}\sqrt{2mE}, \quad k'=\frac{2\pi}{h}\sqrt{2m(U_0-E)}. \tag{7}$$

These solutions must at the boundaries of the discontinuity of the potential ($x=0$, $x_1=l$) satisfy the conditions of continuity of the function itself and of its first derivative.

Substituting the values of (6) into (4) we see that the expressions (6) and (6″) each represent two waves propagating in opposite directions with amplitudes A_+ and A_- and correspondingly C_+ and C_-. According to the physical sense of the solution being sought we must have two waves (incident and reflected) in region III but only one wave transmitted through the barrier) in region I.

Corresponding to this we must in formula (6) set $A_-=0$ (and $A_+=A$). The conditions of continuity at the boundaries yield

$$B_+=\frac{1}{2}A\left(1+i\frac{k}{k_1}\right), \quad B_-=\frac{1}{2}A\left(1-i\frac{k}{k_1}\right), \tag{8}$$

$$C_+=A(\cosh k'l+iD\sinh k'l),$$

$$C_-=iAS(\sinh k'l)e^{ikl}, \tag{9}$$

where

$$S=\frac{1}{2}\left(\frac{k}{k'}+\frac{k'}{k}\right), \quad D=\frac{1}{2}\left(\frac{k}{k'}-\frac{k'}{k}\right). \tag{9'}$$

From (9) we obtain:

$$|A|^2=|C_+|^2-|C_-|^2, \tag{10}$$

which gives us the law of conservation of the flux of particles. The coefficient of penetrability of the barrier given by the ratio of the squares of the amplitudes of the transmitted and incident waves turns out to be equal to

$$\varkappa=\frac{|A|^2}{|B|^2}=\frac{1}{\cosh^2 k'l+D^2\sinh^2 k'l} \tag{11}$$

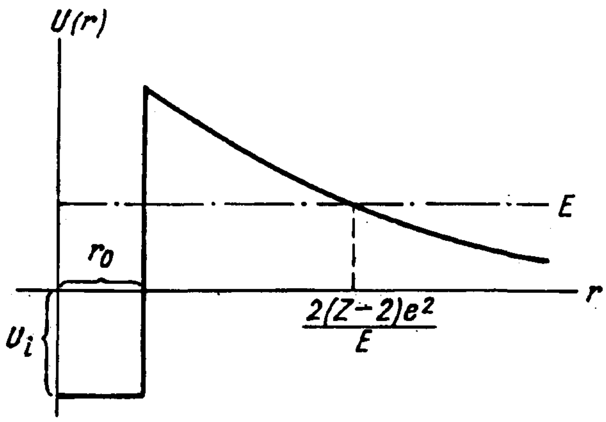

FIG. 4.

in the case $k'l \gg 1$ which always holds for barriers encountered in α-decay we can replace the hyperbolic functions by $e^{k'l}/2$ and obtain for the coefficient of transparency

$$\varkappa = \frac{4}{(1+D)^2} \exp\left(-\frac{4\pi\sqrt{2m}}{h} \sqrt{U_0 - E}\, l \right). \qquad (11')$$

From (11) we see that here the principal role is played by the exponential factor

$$\exp\left(-\frac{4\pi\sqrt{2m}}{h} \sqrt{U_0 - E}\, l \right), \qquad (12)$$

which for a sufficient height and width of the barrier can be exceedingly small; for radioactive nuclei this factor turns out to be of the order of 10^{-30} and this explains the very long periods of radioactive decay.

We have examined the case of a rectangular barrier, however it can be shown that quite an analogous formula will hold for a barrier of any shape if the penetrability of that barrier is small. In such a case the factor (12) should be replaced by

$$\exp\left(-\frac{4\pi}{h} \sqrt{2m} \int_{r_1}^{r_2} \sqrt{U(r) - E}\, \mathrm{d}r \right), \qquad (13)$$

where the integration is taken over the entire region of the imaginary velocity (i.e., where $U(r) > E$).

3. Going over to the question of the emergence of an α-particle from the nucleus surrounded by a certain potential barrier (cf., Sec. 2). We first of all have to know the shape of this barrier. We have seen already that the variation of the potential of the anomalous attractive forces both close to and inside the nucleus (the internal slope) is not known exactly; on the other hand, it is easy to see that the exact variation of the potential on the inside steep slope of the barrier has but little effect on its penetrability. In such a case the most rational procedure is to make the simplest assumptions concerning its shape; for subsequent calculations we shall take the model of the barrier given by formulas (14) (Fig. 4):

$$U(r) = \frac{2(Z-2)e^2}{r} \quad \text{for } r > r_0,$$
$$U(r) = U_i = \text{const} \quad \text{for } r < r_0. \qquad (14)$$

This model is characterized by two unknown quantities: the nuclear radius r_0 and the internal potential U_i.

The question of the emergence of an α-particle from the space surrounded by the potential barrier reduces to the solution of the wave equation that gives outside the nucleus an outgoing spherical wave. This problem leads to a number of discrete (quantum) energies of the α-particle situated within the barrier, and to a number of corresponding probabilities of emergence.

However in the present outline we shall not dwell on the exact solution of the problem and will be satisfied by an approximate derivation which, however, is quite sufficient for a comparison with experimental data. In view of the great height of the barrier we can in the first approximation regard the motion of the particle within the nucleus as being confined between infinitely high walls forgetting the fact that after a couple of million years the particle nevertheless will emerge. We shall be interested only in the state of lowest energy (the principal orbit) since at present one can regard it as being more than probable that all the α-particles in the nucleus have the quantum number—unity.

In this case,[2)] as is well known, the kinetic energy of the particle will be expressed by the following formula:

$$K = \frac{h^2}{8\pi m r_0}. \qquad (15)$$

Taking into account the fact that the bottom of our potential well is at the level U_i we have for the total energy with which the α-particle can emerge outside the value

$$E = U_i + \frac{h^2}{8\pi r_0^2}. \qquad (15')$$

The probability of emergence can be calculated approximately as the product of a "number of collisions of the α-particle with the barrier" by its penetrability, i.e.,

$$\lambda = \frac{\sqrt{E - U_i}}{\sqrt{2m}\, r_0} \times \exp\left[-\frac{4\pi}{h} \sqrt{2m} \int_{r_0}^{2(Z-2)e^2/E} \sqrt{\frac{2(Z-2)e^2}{r} - E}\, \mathrm{d}r \right]. \qquad (16)$$

Taking into account the fact that within the region of integration we have

$$\frac{2(Z-2)e^2}{r} \gg E,$$

and integrating we obtain

$$\lambda=\frac{\sqrt{E-U_i}}{\sqrt{2m}r_0}\exp\left[-\frac{4\pi^2e^2\sqrt{2m}}{h}\frac{Z-2}{\sqrt{E}}\right.$$
$$\left.+\frac{16\pi e\sqrt{m}}{h}\sqrt{(Z-2)r_0}\right], \qquad (16')$$

or, introducing the velocity v of the α-particle,

$$\lambda=\frac{\sqrt{v^2-(2U_i/m)}}{2r_0}\exp\left[-\frac{8\pi^2e^2}{h}\frac{Z-2}{v}\right.$$
$$\left.-\frac{16\pi e\sqrt{m}}{h}\sqrt{(Z-2)r_0}\right]. \qquad (16'')$$

The formulas (15′) and (16′) are sufficient for the calculation of the energy and the decay constant for the given model of the nucleus, and also for the inverse calculation of the constants of the model r_0 and U_i for the known radioactive elements.

Here we must emphasize the essential difference between the applicability of both formulas to real cases. In the formula which determines λ the primary role is played by the exponential factor which depends in addition to known quantities only on the nuclear radius r_0. The quantity U_i which determines some average potential within the nucleus and essentially depends on the form of the model appears only in the first factor which plays a negligibly small role.[3)]

As a result of this the formula (16′) (and others similar to it) can be used for a quite accurate calculation of the nuclear radius of radioactive elements r_0. The formula for E on the contrary very strongly depends on the model adopted for the interior of the nucleus and in view of this the values of U_i obtained in this manner can give at present only a very general idea of the internal potential.

The formula (16′) which gives an exponential dependence of the decay constant on the energy of the α particle represents a mathematical expression of the Geiger–Nuttall law. The expression (16″) shows that $\log\lambda$ is not a linear function of E and can be taken as such only for small changes of E; in actual fact the graph of ($\log\lambda$, E) is a curved line concave towards the E axis which agrees well with the experimental data (cf., Sec. 2). The second important conclusion following from the theory consists of the fact that $\log\lambda$ depends not only on E but also on the atomic number of the element Z and the graph of ($\log\lambda$, E) is in fact not realistic. However, due to the fact that in a number of radioactive elements the energy of the emitted α-particle varies usually in parallel with the atomic number Z, the Geiger–Nuttall graph gives a more or less smooth curve. At those points where the parallel variation of the energy of the α-particle and the atomic number of the element is violated (for example, for AcX), one should expect anomalies in the variation of the Geiger–Nuttall curve. This is the explanation of the long known deviations from this law; the observed angularities in the graph of ($\log\lambda$, E) completely coincide with the predictions of the theory.

We have already pointed out that formula (16″) can serve for a quite accurate determination of the radius of the nucleus. The obtained values of the radius r_0 of our model for the uranium-radium family are given in Fig. 5. We see that the radius decreases fairly regularly with the decrease in the atomic weight of the nucleus. (Should the nuclei along the abscissa of Fig. 5 be listed in reverse order? Transl.) The decrease in the radius is approximately inversely (Should "inversely" be deleted? Transl.) proportional to the cube root of the atomic weight (this regularity extends also into the region of light elements for which the radius can be determined from the anomalous scattering of α-particles), which leads to the conclusion that the density of the nucleus always remains constant.

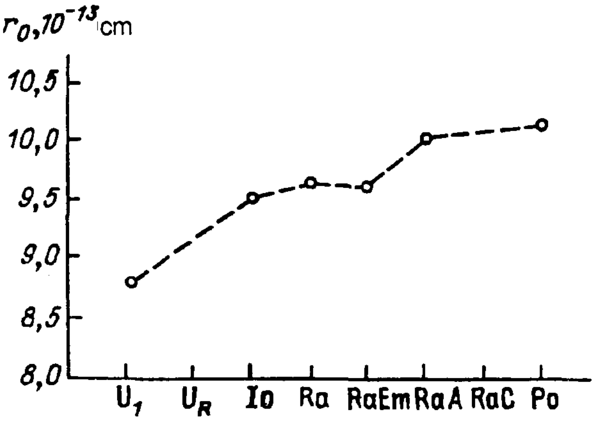

FIG. 5.

4. It might seem that the phenomenon of β-decay should be easily explainable on the same general considerations as α-decay.

In actual fact the phenomenon of ejecting a nuclear electron is in many respects analogous to the emission of an α-particle. We encounter here the same very long periods and with a quantitatively the same dependence between the energy and the period of decay: the slower β-particles correspond to the longer lifetimes of the nucleus.

An essential difference, however, is the fact that the spectrum of the β-particles is smeared out.

The investigations of Ellis have quite reliably established that the β-particles leave the nucleus with velocities varying within wide limits; on the other hand there does not exist any process which might compensate this smearing out of energies and produce a balance of the total energy of the nucleus. According to the law of conservation of energy, nuclei resulting from β-decay ought to have a widely varying amount of energy, and yet the discrete nature of the velocities of the α-particles and the line nature of the γ-spectra points to a quite definite discrete energy of the nuclei. We thus arrive at the conclusion that for the electrons existing within a nucleus and emerging from it the law of conservation of energy turns out to be inapplicable.

This and a number of other difficulties associated with the question of the motion of electrons inside the nucleus indicate that here we have encountered something entirely new, which cannot be explained on the basis of the modern theoretical concepts. Undoubtedly that all these difficulties of quantizing particles moving with a velocity very close to the velocity of light are in a direct relationship with those fundamental contradictions which modern theoretical physics has encountered in its efforts of generalizing wave mechanics to the case of relativistic motion. The investigation of the properties of electrons within the nucleus is at the present time the only field which might provide experimental material for a further development of the basic principles of theoretical physics.

[1]Cf., the article by O. Frisch in the same issue of Usp. Fiz. Nauk 10(4), 570 (1930).

[2]The problem reduces to finding the fundamental frequency of a spherical resonator (acoustics).

[3]This is supported for example by the fact that the five methods proposed until now for obtaining λ give five different expressions for this coefficient, which, however, does not in any way affect the numerical results.

IV. THE OVERALL STRUCTURE OF THE NUCLEUS[1]

1. In the preceding articles of this outline we examined in detail a number of nuclear processes such as: natural and artificial transformations of nuclei and the excitations of the nucleus associated with these transformations which lead to the emission of γ-rays. We now go over to the general question of the components of the nucleus and of the forces binding them into a single whole. According to modern concepts every nucleus is composed of two kinds of elementary particles—protons and electrons. The number of the former is directly given by the value of the atomic weight M, while the number of the latter is given by the difference between the atomic weight of the nucleus and its atomic number Z. It is well known that the mass of any nucleus is not equal to the sum of the masses of the protons and electrons of which it is composed, but is less than the latter by a small amount ΔM which is called the total mass defect and is associated with the total internal binding energy of the nucleus by the relativistic relationship

$$E=\Delta M\cdot c^2, \qquad (1)$$

where c is the velocity of light. Exact measurements of atomic weights of different isotopes for which we are indebted primarily to the work of Aston give us the possibility of calculating these binding energies for a number of nuclei. The material available at the present moment on this subject is presented graphically in Fig. 1, where the internal binding energy is plotted as a function of the atomic weight. We see that in the first approximation we can regard the total binding energy proportional to the number of the component parts of the nucleus. However the assumption suggests itself that in complex nuclei its elementary component parts (protons and electrons) are associated into some stable formations which play an independent role in complex nuclei. Such second order units can, for example, be the recently discovered simplest nuclei—neutrons (proton+electron), the nuclei of a hydrogen isotope (two protons plus electron) and finally, the known for a long time extremely stable helium nuclei, or α-particles (four protons+two electrons). Making some

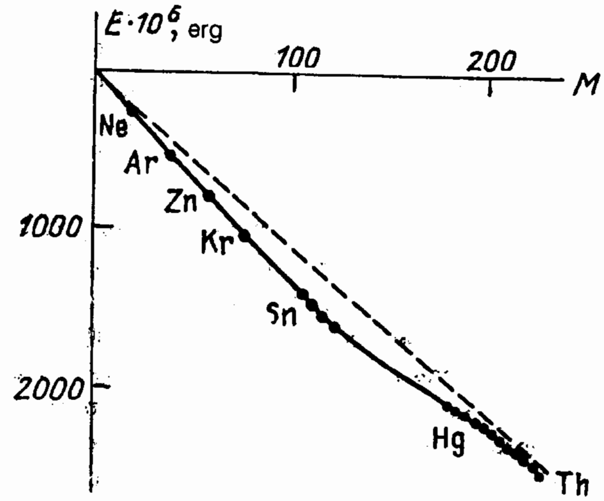

FIG. 1.

definite hypotheses on the composition of the nucleus we can obtain the energy binding these component parts together, by subtracting from the total energy of the nucleus the internal energy of these formations.

Until very recently the hypothesis that was considered to be the most probable one consists of the fact that within the nucleus there is formed a maximum possible number of α-particles with the remainder containing always not more than three protons and a number of electrons both of which have not entered within an α-particle structure. This hypothesis was based primarily on the relatively huge mass defect of an α-particle which is equal, as is well known, to $42.3\cdot 10^{-6}$ erg. On the basis of this hypothesis we can calculate the binding energy between the α-particles and the protons and electrons which are not incorporated into their structure. This energy obtained simply as the difference between the curve of Fig. 1 and the straight line (represented by dashes) with an angle coefficient equal to the binding energy of one α-particle, is shown in Fig. 2. We see that approximately to the middle of the graph the curve falls rather smoothly, but beyond that it begins to rise in a very unusual manner: the experimental points give sections of a curve which is still moving downward from left to right, but these sections are separated by great jumps. Such a variation of the curve is very strange and gives rise to a suspicion as to whether this might be the consequence of the incorrectness of the hypothesis concerning the formation of a maximum number of α-particles in the compound nucleus. Indeed, the curve of Fig. 2 can be smoothed out if we assume that in heavy nuclei a part of the α-particles is dissociated and that the jumps occurring on the curve are due to this fact not having been taken into account. Such a supposition is also confirmed by a number of indications from other fields; for example, having in the nucleus always not more than three protons it would be very difficult to explain the large angular momenta observed in a number of heavy nuclei.

An entirely new hypothesis respecting the composite

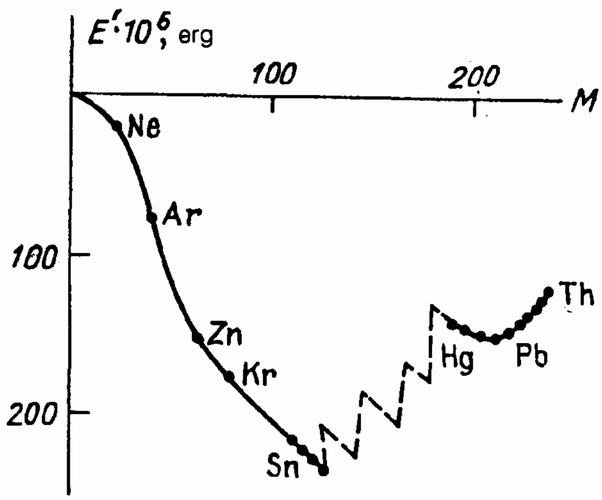

FIG. 2.

parts of the nucleus is the assumption which was a direct consequence of the discovery of neutrons according to which each nuclear electron is associated in the first instance with one of the nuclear protons by forming a neutron. Thus, we have in the nucleus Z protons and A-Z neutrons which in their turn combining into groups with two pairs each form α-particles. Thus, we obtain the following composition of the nucleus; for an even atomic number $Z/2$ α-particles and A-$2Z$ neutrons, for an odd atomic number $(Z-1)/2$ α-particles, A-$2Z+1$ neutrons and 1 proton. We see that on such an assumption the number of α-particles in heavy nuclei will be somewhat smaller than in the case of the previous assumption (for example for mercury $Z=80$ $A=200$ the number of α-particles according to the new hypothesis is equal to only 40 instead of 50). The curve of the internal binding energy of the nucleus calculated according to this latter hypothesis on the structure is shown in Fig. 3 in which, as can be seen,

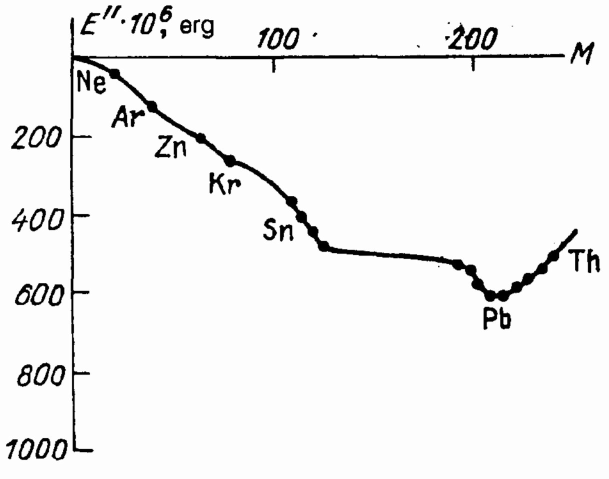

FIG. 3.

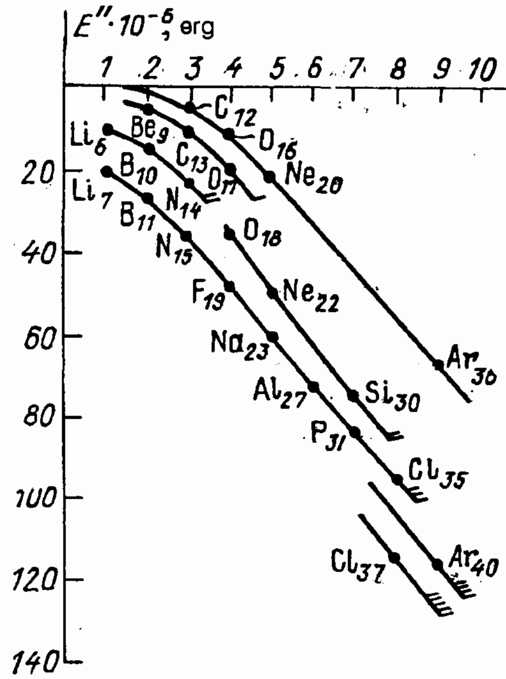

FIG. 4.

the curve proceeds quite smoothly beginning to rise only in the range of the radioactive elements which supports the correctness of the assumption that had been made.

Unfortunately, in spite of Aston's heroic perseverance the data on the mass defects until now are far from complete and not very accurate, which does not give us the possibility of carrying out a more detailed analysis of the experimental curves that is necessary for obtaining information on the distribution of binding energy between α-particles, neutrons, and protons.

It is only in the region of the light elements using both the results of a direct measurement of the mass defect and also the data on the energy balance in artificial transformation of elements (the latter gives us the difference between the internal energy of the initial nucleus and the product nucleus of the transformation), can one construct a curve of the energy in a more or less satisfactory manner. Such a curve shown in Fig. 4 can be very valuable for predicting the energy balance of a nuclear reaction.

2. Another very essential factor for understanding the internal structure of the atomic nucleus is the knowledge of its rotational and magnetic moments. The moment of an atomic nucleus can be observed as a result of its action on the energy levels of the external atomic electrons which under the influence of its action are split into several closely lying sublevels, the number of which is related in a definite manner to the angular momentum of the nucleus, while the value of the splitting is determined by the magnetic moment. (Another method of determining the angular momentum of a nucleus is based on the investigation of the distribution of intensities of the band spectra of molecules but appears to be less convenient.) Figure 5 shows the values of the angular momentum of different nuclei

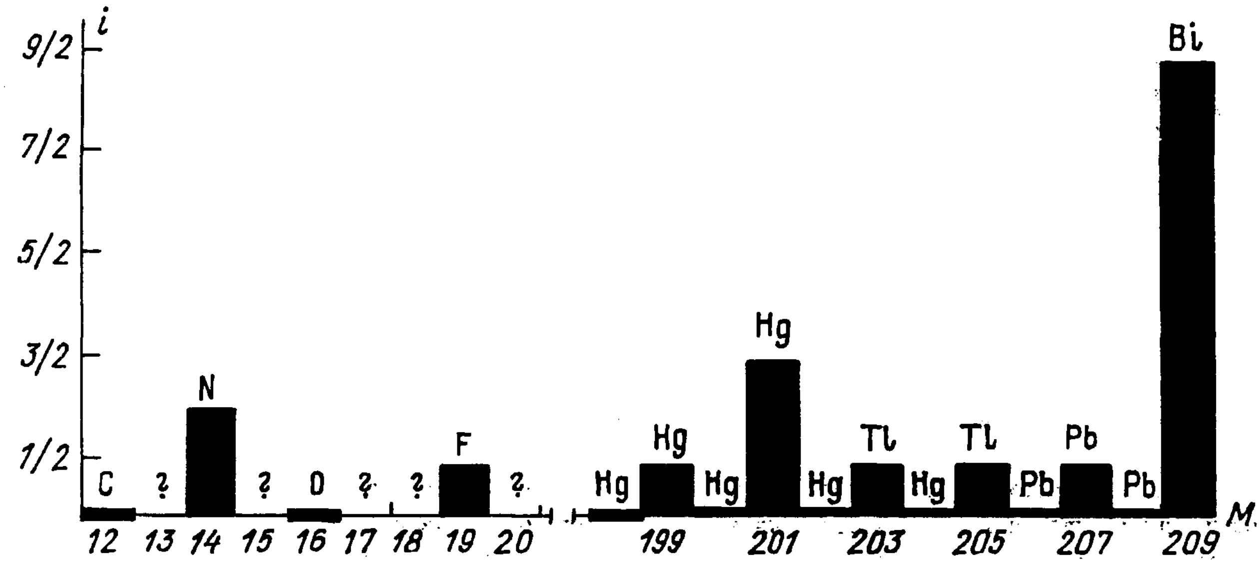

FIG. 5.

expressed in units of the rotational quantum $h/2\pi$. One immediately notices that nuclei with an even atomic weight (with the exception of nitrogen) usually have no angular momentum at all, while in the case of an odd atomic weight the angular momentum always differs from unity being for light elements usually equal to one half, and for heavy nuclei taking on sometimes quite large values. What can the angular momentum tell us about the structure of the nucleus?

First of all we must take into account that an α-particle (as experimental data show) has no angular momentum at all. Since, moreover, all the α-particles of a nucleus in its normal state are in the ground energy state which also has no angular momentum we arrive at the conclusion that the angular momentum of a nucleus is due entirely to protons and neutrons which do not enter into the composition of the nuclear α-particles. The angular momentum of the proton is equal to, as is well known, $\pm 1/2$; apparently the same is also true for the neutron.[2)] Besides that, since the number of neutrons in heavy nuclei can be as large as fifty-four, and the Pauli principle forbids more than two neutrons to be in the same orbit, in the formation of the angular momentum of a nucleus angular momenta of different neutron orbits may play a role.

The angular momentum of a nucleus observed by us is, of course, only the total result of the intrinsic and orbital angular momenta of the neutrons and the protons (for odd Z) in the nucleus, but the knowledge of it is essential for checking any hypothesis concerning the distribution of neutrons among the different quantum levels within the nucleus.

Unfortunately a very large number of attempts being made at the present time with the aim of explaining the observed value of nuclear moments on the basis of different assumptions on the distribution of nuclear particles among different quantum levels have not so far led to an unambiguous result.

3. We now proceed to examine the question of the stability of an atomic nucleus with respect to different transformations. For this it is necessary first of all to make different assumptions concerning the nature of the interaction between the different component parts of the nucleus. For the interaction of two protons which we can here regard as point charges (since the proton radius

$$r_p = \frac{e^2}{m_p c^2} = 2 \cdot 10^{-16} \text{ cm}$$

is much smaller than the radius of the nucleus) we can with confidence assume the Coulomb repulsive forces described by a potential.

The interaction between a proton and a neutron or between two neutrons will, evidently, be manifested only at distances comparable with the dimensions of the neutron (i.e., a few 10^{-13} cm) and will fall off very rapidly as the particles are separated.

Using an analogy borrowed from the field of the interaction of atoms and ions we can assume that in both cases attractive forces will exist with the mutual potential energy $-I(r)$ in the interaction of a proton with a neutron will be considerably greater than the energy $-K(r)$ corresponding to the interaction of two neutrons. Here it is necessary to point out that another additional assumption must be made with respect to the potentials $-I(r)$ and $-K(r)$, specifically that in the case of a too close approach of the particles these potentials must begin to grow giving rise to repulsive forces, for in the opposite case the nuclear model would not be stable tending to draw together to a point.

As regards the interaction between the α-particles, it will evidently be made up of a Coulomb repulsion and an average force of cross interaction between the protons and the neutrons of which they are composed. This last statement leads, as can be shown, to an attraction with a potential energy close to the interaction between the neutrons (the forces associated with the potential $-I(r)$ mutually cancel out), so that we can write for the potential energy of two α-particles

$$+\frac{4e^2}{r}-L(r), \qquad (2)$$

where $L(r)=K(r)$ and also very rapidly decreases with distance.

The exact expressions for the potentials $-I(R)$, $-K(r)$, and $-L(r)$ are at the present time unknown. Their theoretical derivations is impossible without a relativistic quantum theory, while experimentally they can be deduced from data on the scattering of α-particles in helium and in hydrogen, of neutrons in hydrogen, etc. However, in view of the mathematical complexities of such a calculation, and partially in view of the lack of exact experimental data, such a calculation so far has not been carried out.

We now examine what will be the behavior of a collection of such particles with masses of approximately the same order of magnitude attracted to one another by forces that fall off very rapidly with distance (one can in the first approximation neglect the Coulomb repulsive forces inside the nucleus). The state of such a system must be very analogous to the one that we have in a small liquid drop inside which the forces acting on any particle are in equilibrium (since the range of forces is less than the radius of nucleus), while near the surface strong forces appear which prevent the particle from leaving the drop (surface tension). Although we do not have until now an exact solution of the problem concerning such a collection we can make a number of interesting conclusions concerning the properties of such a model. First of all we must suppose that the volume of such a model will be approximately proportional to the number of particles so that the radius will vary approximately as the cubic root of the atomic weight.[3)] The potential energy for any given particle within such a model must be more or less constant and increase sharply at the boundaries forming thereby a kind of a "potential well."

From what has been said about the nature of the interaction forces between different particles in the nucleus it follows that the "bottom" of this "well" for a proton will lie considerably lower than for neutrons on α-particles (Fig. 6). The total energy of such a model must be approximately proportional to the number of particles. We must not, however, forget about the existence of forces of Coulomb repulsion. These forces cannot significantly change the distribution of the potential within the nucleus, where the principal role is played by the attractive forces. However, these forces will decrease the values of the potential at large distances and will lead to the formation around the nucleus of a potential barrier playing such an important role in the theory of nuclear transformations. This raising of the potential well with respect to the values of the potential at infinity will evidently be quite absent for neutrons which do not have a charge, and for the proton will be by a factor of two lower than for an α-particle. The distribution of the potential within the nucleus taking the Coulomb

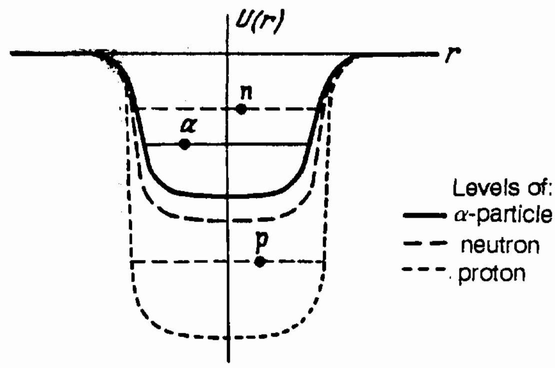

FIG. 6.

forces into account is shown in Fig. 7 where the case of a heavy nucleus is considered in which the level of an α-particle has already been raised above the zero level thereby making it possible to have a spontaneous α-decay.

The proton level even for the heaviest nuclei still remains in the negative region because even without taking the Coulomb forces into account the proton level lies considerably deeper than the level of an α-particle and, moreover, the raising of the level by the repulsive forces for a proton is less by a factor of two. For a neutron that has no charge the raising of the level by the Coulomb forces will not take place at all.

All that has been said above explains both the appearance of α-decay in the case of heavy elements, and also the absence of the phenomena of spontaneous emission of a proton or a neutron.

4. Until now we have been examining the neutrons existing within the nucleus as indivisible units and therefore could construct a model of the nucleus on the basis of the usual mechanics. We now turn to the decay of a nuclear neutron into a proton and an electron and ejection of the latter beyond the confines of the atom, i.e., to the so puzzling phenomenon of β-decay.

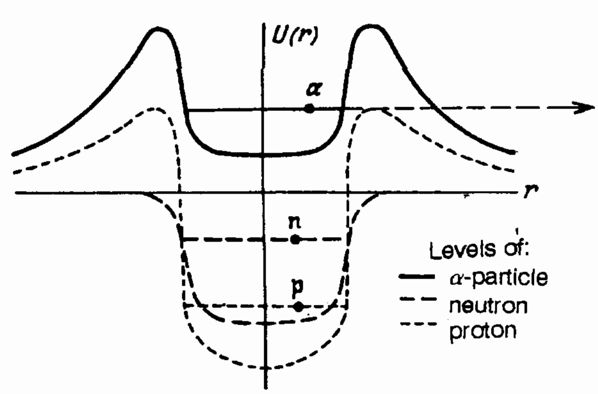

FIG. 7.

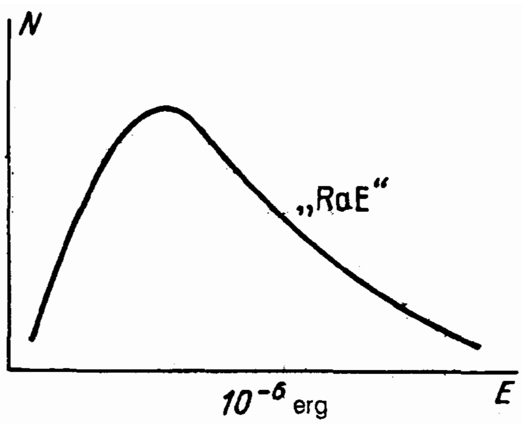

FIG. 8.

As is well known, β-decay represents one of the more sharp examples of the electron not obeying all the principles of the modern theory. While in nuclear reactions with the participation of heavy particles we always deal with sharply expressed quantum levels and strict observance of the energy balance, in the case of β-transformations neither the one nor the other occurs. As the experiments of Ellis have shown "the electrons ejected in the decay by different atoms of the same substance have very different values of the energy that vary continuously between zero and arbitrarily large values with the distribution curve being of the form very similar to the error curve (Fig. 8). Any other radiation that might compensate the difference in the energy created in this manner between different nuclei is totally absent, and yet all the properties and the subsequent behavior of the nuclei before and after the decay are completely identical." From a purely experimental point of view the situation here appears as if we are dealing with the violation of the law of conservation of energy. In addition to this basic fact there is also a number of no less fundamental arguments saying that the situation is bad with nuclear electrons; here for example are inconsistencies in the statistics of nuclei and the values of their angular momentum. The reasons for all these inconsistencies lie in the fact that as Bohr has shown we here already go beyond the boundaries of the region where one can apply the classical concept of an electron. Indeed, for the radius of the electron we have according to the classical theory the value

$$r_e = \frac{e^2}{m_e c^2} = 3 \cdot 10^{-13} \text{ cm},$$

i.e., a quantity comparable with the dimensions of the region where the electron has to move and under such conditions such a rough concept of an electron as a charged sphere is of course inapplicable.

In connection with this is the fact that evaluating the possible speed of the electron in the nucleus in accordance with the bases of the quantum theory we arrive at a value so close to the velocity of light (0.9998 c) that there is no possibility that one can neglect the theory of relativity and yet until now we do not have a relativistic quantum theory.

Until such a general theory which is an organic synthesis of the modern nonrelativistic quantum theory (wave mechanics) and an unquantized relativistic theory will be constructed there is no possibility of having a true understanding of the process of β-decay. However, even now we can attempt to construct working theories of β-decay using the old concepts. The main hypothesis of the theory of β-stability and β-decay proposed recently by Heisenberg consists of disregarding the indefiniteness of the energy of the β-particles to accept as the necessary and sufficient condition of the possibility of decay is the positive nature of the corresponding energy balance.

We consider a nucleus consisting exclusively of n "stuck together" neutrons. Since between neutrons there exist only attractive forces such a nucleus will of course be stable with respect to neutrons, i.e., in extracting a neutron from the nucleus we shall perform a certain amount of work which evidently will be of the order of $K(r)$, where r is the average distance between the particles within the nucleus. We shall decompose the extracted neutron into a proton and an electron which will require an amount of work determined by the internal binding energy of the neutron D (this quantity is quite insignificant and is equal to, according to Chadwick's measurements, to only one or two MeV, while the energies $K(r)$ and $I(r)$ amount to tens of MeV). We now return the proton so obtained to the nucleus obtaining in doing so an energy of the order of $+I(r)$; since $|I(r)| \gg |K(r)|$ then in such a process we shall have a positive energy balance. However, it is not difficult to see that the resulting reaction is equivalent simply to taking out of the nucleus one electron and, since the energy balance is positive we should expect the existence of spontaneous β-decay. Thus, the initially neutral nucleus will begin to emit sequentially a number of β-particles, the total number n_1 of neutrons entering into its composition will start to decrease, giving rise to an ever greater number n_2 of protons. However this process will not be carried through to the end; in view of the increase of the positive charge of the nucleus the introduction into it of new protons will be resisted by the Coulomb repulsive forces, and finally, "the replacement of a neutron by a proton" will become an energetically unfavorable replacement. To find the condition of equilibrium Heisenberg has to make a certain hypothesis concerning the dependence of the work needed to extract from the nucleus one neutron or one proton on the total number of neutrons in the nucleus. Such a hypothesis is made in two steps: first, it is assumed that this work in both cases is a function only of the relative number of neutrons and protons [$f(n_1/n_2)$ and $g(n_1/n_2)$]; secondly it assumes that these functions are linear.[4)] Since the work expended against the forces of Coulomb repulsion in introducing a proton into the nucleus (of charge $n_2 e$ and radius r_0), is equal to $n_2 e^2/r_0$ we can determine the region of instability with respect to β-decay by the inequality

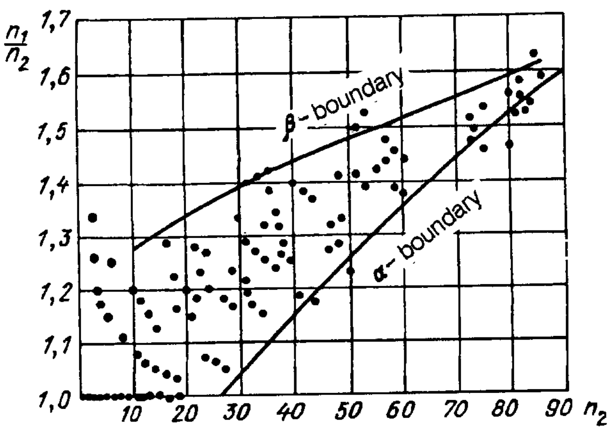

FIG. 9.

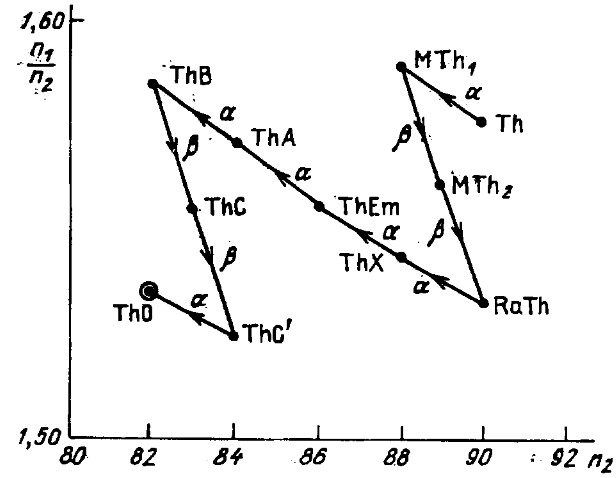

FIG. 10.

$$-f\left(\frac{n_1}{n_2}\right)-D+g\left(\frac{n_1}{n_2}\right)-\frac{e^2n_2}{r_0}\geqslant 0, \tag{3}$$

or, assuming linearity of f and g and assuming $r_0 \sim \sqrt[3]{n_1+n_2}$ (which approximately corresponds to the real situation),

$$\frac{n_1}{n_2}\geqslant C_1+C_2\frac{n_2}{\sqrt[3]{n_1+n_2}}. \tag{3'}$$

Considering now the condition for the possibility of α-decay (as has been shown in the preceding sections, emission of α-particles must begin much earlier than emission of protons), Heisenberg wrote down for the boundary of the region of instability with respect to the emission of an α-particle[5)]

$$\frac{n_1}{n_2}\leqslant C_1'+C_2'\frac{n_2}{\sqrt[3]{n_1+n_2}}. \tag{4}$$

For a comparison of the considerations presented above with experiment we can use the graph of Fig. 9 where along the horizontal axis we have plotted the total number of particles in the nucleus (i.e., $n=n_1+n_2$), and along the vertical axis the ratio of the number of neutrons to the number of protons for the different nuclei known to us. The two boundary lines have been drawn in accordance with equations (3) and (4) with the coefficients having been chosen in such a way that the curves would best include the experimental points (C_1 = 1.173, C_2 =0.0225, C_1'=0.47, C_2'=0.077).

In the region of the radioactive elements the two curves approach each other closely (Fig. 10) giving rise to the fact that a nucleus which is in the β-unstable region and emits two electrons (the fact that we always have two successive β-decays can also be obtained from the theory presented here) jumps over the region of general stability and lands into the α-unstable region. In the following series of α-decays the point representing a nucleus on the diagram gradually rises finally falling again into the β-unstable region. Such a process of alternating α- and β-decay will continue until in view of the gradual decrease of the number n the width of the region of general stability will become sufficiently broad so as not to permit transition through it in one move. Here lie the stable products of radioactive families.

Thus, the arguments of Heisenberg describe quite well the phenomenon of stability and instability with respect to β-decay, although all the principal difficulties associated with the continuity of the β-spectra are not touched upon by them. Also the existence of such long periods in the case of β-decay and the complete definiteness in the lifetime of the nuclei of a given element irrespective of the different values of the energy of β-decay remain at present completely unexplained.

*First published in "UFN" in June 1930 and in April 1993 (Usp. Fiz. Nauk **10**(4), 531–544 (1930); **13**(1), 46–57 (1933)).

[1)]cf.: Usp. Fiz. Nauk **10**(4), 531 (1930) (I [reproduced in this issue of the journal]); **12**(1), 31 (1932) (IT), **12** (4) 389 (1932) (III); **14**(4), 389 (1934) (V).

[2)]The investigation of the angular momenta of nuclei has already shown a long time ago that an electron being in the nucleus loses its angular momentum. This fact is understandable from the point of view of the modern theory and for its explanation one should wait for the appearance of the as yet nonexistent quantum theory of relativistic motion which will be able to explain all the puzzles associated with nuclear electrons.

[3)]That such a dependence, holds, it is true, quite roughly, for atomic nuclei—is well known.

[4)]Indeed, assuming that the interaction of a neutron with a nucleus is primarily due to its attraction to the nuclear protons, and the interaction of a proton with a nucleus is primarily due to its attraction to the nuclear neutrons (as is done by Heisenberg) and that the sphere of action of these attractions is small in comparison with the dimensions of the nucleus, we may expect that the work of pulling out from the nucleus a particle of one kind (neutron or a proton) will be a monotonically increasing function of the concentration within the nucleus of particles of the other kind. Therefore for the work in the two cases we should write

$$f'\left(\frac{n_1}{n_1+n_2}\right) \text{ and } g'\left(\frac{n_1}{n_1+n_2}\right)$$

and assume that both functions increase with an increase in the argument. Since the form of the functions f' and g' is not known we can go from this to the possibly less successful expressions of Heisenberg with the conclusion that $f(n_1/n_2)$ will decrease and $g(n_1/n_2)$ will increase with increasing argument. As to the hypothesis concerning the linearity of the functions f and g this hypothesis of course is a much more dangerous one and is justified only by the fact that the actual shape of these functions is totally unknown to us.

[5] Indeed, α-decay will begin at sufficiently large values of the specific charge of the nucleus $n_2/(n_1+n_2)$ when the forces of Coulomb repulsion acting on an α-particle will overcome the forces of intranuclear attraction, or for sufficiently low values of n_1/n_2.

Translated by G. Volkoff