# Aurorae boreales and magnetic storms* 

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1. Aurorae boreales have long attracted the attention of observers, and scientists have been trying to puzzle out the nature of the phenomenon that has given rise to numerous legends and superstitions.

More than 200 years ago, Galileo drew attention to the similarity between the phenomena associated with the polar aurora and the glow observed during the outflow of electricity from highly electrified bodies. He also noted that, when the aurora takes the form of an arc, its apex lies in the magnetic meridian, whilst the angle of the rays or bands is very close to the angle of the magnetic needle.

Polar seafarers of the first half of the nineteenth century noted deviations of compass readings and disturbances of the magnetic needle during auroae boreales, and Franklin was among the first to perform systematic studies of this question.

Emission by rarified gases in electrical discharges provided a new analogy between electric phenomena and aurorae boreales. There were even theories in which aurorae boreales were ascribed to the quiet discharge of terrestrial electricity through the tenuous upper layers of the atmosphere.

When magnetic observatories were built on Gauss' initiative, Franklin's observations received systematic confirmation, and the connection between magnetic storms and aurorae boreales was reliably established.

Finally, observations of the solar surface and of sunspots, which were regularly performed since the early 1700 s, provided as early as the 1850 s the first indication of a connection between sunspots and aurorae boreales. It was found that years of developed and strong solar activity, with numerous sunspots, were accompanied by numerous aurorae, which reached their maximum strength and visibility-even at middle latitudes-during these years.

As far back as the 1790s, Dalton used the position of the apex of an auroral are relative to the fixed stars, observed from two points separated by 83 English miles ( 125 versts), to show that the apex lay at a height of 100 English miles, a little to the south of one of these points.
R. Potter, in a paper entitled 'Calculation of the heights of the aurorae boreales of 17th September and 12th October 1853', printed in Volume 8 of the Philosophical Transactions of the Cambridge Royal Society, reported a number of similar observations which also yielded heights of between 65 and $85^{\circ}$ English miles.

Spectroscopic observations then revealed the presence in the aurora borealis of a number of lines that were also found in terestrial bodies.
2. This was, roughly, the situation twenty years ago, i.e., the phenomenon was described and investigated externally, and mostly qualitatively, but had not been subjected to a systematic study by precise observations that could
then be compared with one another and could serve as a basis for an orderly theory.

And then, about twenty years ago, the Norwegian scientist, Professor Birkeland put together his first expedition to investigate the aurora borealis.

He took part in this expedition together with two of his students, Helland-Hansen and Laws, accompanied by Hett, an old Finnish postman.

The aim of the expedition was to reach a mountain peak 3,000 feet above sea level in northern Norway, not far from Hammerfest, and to use a log cabin near the top as their base for observations.

However, the expedition was unsuccessful: two versts from the log cabin, the expedition was caught in a snowstorm with strong northerly winds and temperatures of $-25^{\circ} \mathrm{C}$. Expedition members were hit by frostbite and were forced to jettison their baggage and instruments, and to turn back. It was only the experience of the old postman that helped them to find their way back to Gargia which they had left 31 hours earlier.

Skillful thawing out of frostbitten hands in icy water and timely medical help saved members of the expedition from gangrene and mutilation.
3. The failure of his first attempt did not weaken Birkland's resolve. On the contrary, it spurred him on to continue his project with greater care and prudence.

In the summer of 1897 and 1898, he visited northern Norway and climbed to the top of its highest mountain in an attempt to choose the most suitable position for an observatory.

He chose the top of Sukkertoppen and of Talviktoppen, separated by a distance of about $3-4 \mathrm{~km}$ at $70^{\circ} \mathrm{N}$ and $22^{\circ} 30^{\prime}$ E from Greenwich. Both points were at about 3,000 feet above sea level.

Solid stone-built observatories were put up at the top of both mountains and were linked by telephone with one another and with the rest of the country.

Magnetic and meteorological observations of aurorae boreales were made at these observatories in the winter of 1899/1900 under the direction of Professor Birkeland.

We shall not go into the details of the results obtained in this way, since they have now been superseded by other data, and we merely mention the severe gales and snowstorms which members of the expedition had to experience. The wind speed at these altitudes was about $46 \mathrm{~m} / \mathrm{s}$ and there were storms of up $38 \mathrm{~m} / \mathrm{s}$ at $-16^{\circ} \mathrm{C}$. Birkeland reported that it was 'difficult to imagine such storms and their effect on men'. An assistant who ventured out, returned after a few minutes later with a frostbitten hand because he did not put furlined gloves on top of woolen gloves when he handled anemometers.

Nevertheless, once or twice a week, in all the snowstorms and gales, the short stocky Finn managed to deliver


FIG. 1.
mail to the observatory. On one occasion, when he arrived covered with frost so that he was hardly recognisable, Professor Birkeland asked him 'are you not afraid to walk about in such poor weather?' The Finn at first did not answer and sat quietly, until all the frost upon him melted, and he then said: 'I am too silly to be afraid'.
4. The main conclusion drawn from the observations made during the second expedition was that a determination of the origin of the aurora borealis and of magnetic storms would require simultaneous recordings with magnetic instruments and observations at different stations separated by about $1,000 \mathrm{~km}$, and also recordings made at a large enough number of stations distributed over the entire globe. Calculations showed that some of the extensive magnetic disturbances could be ascribed to an electric current flowing parallel to the Earth's surface in its polar areas at a height of a few hundred kilometers and amounting up to a million kiloamperes. The only assumption that had to be made was that the current of electrically charged particles could be determined-in the same way as for a galvanic current-from its magnetic effect. In polar countries, these currents were well identified and localized, and were occasionally found between the two stations at Jan Mayen and Bossekop.

The Norwegian government supported further studies with a contribution of 20,000 crowns. Five individuals gave 6,000 crowns each, and Professor Birkeland himself added to this sum a further 30,000 crowns and equipped his third expeditions that went out to investigate the aurora borealis in 1902-1903. The analysis of the data obtained is still continuing. Professor Birkeland has now published two huge books that constitute the first and second part of the first volume of his analysis of these results.

The stations were constructed at the following four points:
Kaafiord (northern Norway) ( $69^{\circ} 56^{\prime} \mathrm{N}, 22^{\circ} 58^{\prime} \mathrm{E}$ )
Dyrafiord (Iceland) $\left(66^{\circ} 15^{\prime} \mathrm{N}, 22^{\circ} 30^{\prime} \mathrm{W}\right)$
Axelöen (Spitsbergen) ( $77^{\circ} 41^{\prime} \mathrm{N}, 14^{\circ} 50^{\prime} \mathrm{E}$ )

## Matochkin Shar (Novaya Zemlya) ( $72^{\circ} 17^{\prime} \mathrm{N}$, $53^{\circ} 57^{\prime} \mathrm{E}$ )

All the stations were equipped with recording magnetic instruments, absolute magnetometers, inclinometers, a complete set of meteorological instruments, a theodolite for astronomical observations, and chronometers. Each station consisted of staff quarters, a magnetic observatory containing the automatic recording equipment, an observatory for absolute magnetic observations, a meteorological hut, and a hut for astronomical observations. Each station had a manager and two assistants, with Professor Birkeland himself managing Kaafiord and the administration of all the stations.

On Novaya Zemlya, the expedition had at its disposal a house that was originally built for the artist Borisov. Members of the expedition were brought to Motochkin Shar and back again on steamship 'Vladimir' by the then governor of Archangel, Vice-Admiral N. A. Rimskǐ̌Korsakov.

In addition to these special custom built and equipped stations, the principal magnetic observatories (a total of 23) across the globe took part at agreed times in a program of accelerated recordings, and communicated their results to Professor Birkeland.

This huge amount of material was then subjected by him to systematic analysis which I shall now try to summarize.
5. Birkeland adopted the following method of analysis, taking as his starting point the assumption that aurorae boreales and magnetic storms were not of terrestrial but of cosmic origin, and that their causes had to be sought in the motion of electrified particles (or particles of electricity) ejected by the Sun, whose spots could act sources of cathode rays.

For each of the observed magnetic disturbances, which since the time of Gauss were known to occur simultaneously over the entire globe, he calculated the magnitude and direction of the disturbing force at each of the points of


FIG. 2.
observation and thus obtained an idea of the perturbing magnetic field distribution over the Earth's surface. He performed this calculation for each disturbance at many instants of time in order to be able to follow the development of the disturbances in time. He adopted a special way of representing the above field. For each of the observing points he constructed a vector whose direction was the same as the direction of a horizontal electric current that could, by flowing above the given point, produce the observed horizontal disturbing magnetic force. The length of the vector was taken to be proportional to the current, i.e., to the horizontal component of the disturbing force.

The vertical component of the disturbing force was represented by a vector perpendicular to the first vector and pointing to the left of it when the force was downward and to the right when it was upward. In this representation, whenever the disturbance was actually due to horizontal currents flowing above the arc, the arrow representing the vertical force pointed toward the region of maximum current density.

The charts constructed in this way led Birkeland the following classification of magnetic disturbances or storms:
(1) equatorial positive
(2) equatorial negative
(3) polar positive
(4) polar negative
(5) cyclo-median

Positive equatorial disturbances are characterized by the following features:
(1) at middle and low latitudes, one observes the positive (i.e., one that increases the horizontal component) disturbing force that lies in the plane of the magnetic meridian, i.e., one that produces no changes in either declination or inclination, or only very small changes. The magnitude of the force is at its maximum at low latitudes, and decreases toward the pole.

The chart of Fig. 2 shows an example of disturbances of this type.

Negative equatorial disturbances are relatively rare. An example can be seen in the chart of Fig. 3 in which the


FIG. 3.
disposition of the arrows is similar to that of Fig. 2, but the directions are reversed.

Polar disturbances are characterized by a very considerable strength of the disturbing force in arctic countries, but are confined to a relatively small area whilst the magnitude of the force decreases rapidly with distance from this region at middle and low latitudes, and becomes very small.

This type of (negative) disturbance, i.e., a disturbance for which the horizontal component in the region of maximum disturbance is reduced by it, is shown in the chart of Fig. 4 in which the lengths of the vectors for Iceland and Spitsbergen are reduced by a factor of 5 as compared with the scale for Central Europe.

Positive polar disturbances differ from negative disturbances only by the direction of the forces.

Figure 5 shows an example of a vortical or cyclic disturbance in which the directions of the force arrows have a vortical character.
6. Before we present the explanations offered by Birkeland for his observations, we have to say a few words


FIG. 4.


FIG. 5.
about the mathematical theory of the aurora borealis, produced by the Norwegian mathematician Carl Störmer.

Störmer's theory was reviewed in volume 42 of the Journal of the Russian Physico-chemical Society, and I shall confine myself to presenting only its salient points.

Störmer investigated the conditions under which a charged particle ejected by the Sun enters the Earth's magnetic field and under its influence either reaches its surface or just misses it.

To solve this problem, he assumed that the Sun is a nonmagnetic body, i.e., that it does not generate a magnetic field, and that the entire motion of the particle is governed by the Earth's magnetic field. Störmer replaces the Earth with a small elementary magnet whose axis points along the magnetic axis of the Earth and whose magnetic moment is equal to the Earth's magnetic moment, i.e., $8.52 \times 10^{25} \mathrm{cgs}$. The differential equations of motion of a particle carrying an electric charge in a magnetic field can be written down if we assume that an element of the particle trajectory at a given time is similar to a current element, and if we use the rule describing the effect of a magnetic field on the current, the current being proportional to the velocity of the particle and to its charge.

Let $H_{x}, H_{y}, H_{z}$ be the magnetic field components along the coordinate axes, $m$ the particle mass, $e$ the particle charge, and $\alpha$ a constant. ${ }^{1)}$ The equations of motion then become

$$
\begin{align*}
& \frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}=\alpha \frac{e}{m}\left(H_{y} y_{z}-H_{z} v_{y}\right), \\
& \frac{\mathrm{d}^{2} y}{\mathrm{~d} t^{2}}=\alpha \frac{e}{m}\left(H_{z} v_{x}-H_{x} v_{z}\right),  \tag{1}\\
& \frac{\mathrm{d}^{2} z}{\mathrm{~d} t^{2}}=\alpha \frac{e}{m}\left(H_{x} v_{y}-H_{y} y_{x}\right),
\end{align*}
$$

where $v_{x}, v_{y}, v_{z}$ are the components of the velocity $v$ of the particle along the coordinate axes. Since the direction of the force acting on the particle is perpendicular to the direction of its velocity, the velocity $v$ remains constant,
and Störmer replaces the time $t$ in the equations with the variable $s$, i.e., the length of an arc of the trajectory, defined by $\mathrm{d} s=v \mathrm{~d} t$. He then directs the $z$ axis along the magnetic axis of the Earth and describes its potential by

$$
\begin{equation*}
V=M \frac{z}{r^{3}} \tag{2}
\end{equation*}
$$

in which $M$ is the magnetic moment of the Earth and $r^{2}=x^{2}+y^{2}+z^{2}$. Hence

$$
\begin{align*}
& H_{x}=\frac{\partial V}{\partial x}=-3 M \frac{x z}{r^{5}} \\
& H_{y}=\frac{\partial V}{\partial y}=-3 M \frac{y z}{r^{5}}  \tag{3}\\
& H_{z}=\frac{\partial V}{\partial z}=-M \frac{3 z^{2}-r^{2}}{r^{5}}
\end{align*}
$$

## Moreover

$$
\frac{\mathrm{d}^{2} x}{\mathrm{~d} t^{2}}=\frac{\mathrm{d}^{2} x}{\mathrm{~d} s^{2}} v^{2}, \quad \frac{\mathrm{~d}^{2} y}{\mathrm{~d} t^{2}}=\frac{\mathrm{d}^{2} y}{\mathrm{~d} s^{2}} v^{2}, \quad \frac{\mathrm{~d}^{2} z}{\mathrm{~d} t^{2}}=\frac{\mathrm{d}^{2} z}{\mathrm{~d} s^{2}} v^{2}
$$

and if we use primes to represent derivatives with respect to $s$, we have

$$
\begin{align*}
& x^{\prime \prime}=\frac{c_{1}^{2}}{r^{5}}\left[3 y z z^{\prime}-\left(3 z^{2}-r^{2}\right) y^{\prime}\right] \\
& y^{\prime \prime}=\frac{c_{1}^{2}}{r^{5}}\left[\left(3 z^{2}-r^{2}\right) x^{\prime}-3 x z z^{\prime}\right]  \tag{4}\\
& z^{\prime \prime}=\frac{c_{1}^{2}}{r^{5}}\left(3 x z y^{\prime}-3 y z x^{\prime}\right)
\end{align*}
$$

where $c_{1}$ is a constant that depends on the ratio of the charge of the particle to its mass, its velocity $v$, the particle species, and the magnetic moment $M$. It is clear from (4) that $c_{1}$ is a length whose magnitude for the above value of $M$ is as follows:
for cathode rays $c_{1}$ is in the range $4.0-8.5$ million km for the radium beta particles it is the range 1.4-2.2 million km
for radium alpha particles it is $150,000-170,000 \mathrm{~km}$
It is clear that any length can be adopted as the unit of length because the equations in (4) are homogeneous.

The equations can be simplified by taking $c_{1}=1$ which then defines the scale of the trajectories. We shall assume henceforth that $c_{1}=1$.

It is found that (4) has an integral similar to the integral of areas. In point of fact, we find that

$$
x y^{\prime \prime}-y x^{\prime \prime}=\frac{1}{r^{5}}\left[\left(3 z^{2}-r^{2}\right)\left(x x^{\prime}+y y^{\prime}\right)-3\left(x^{2}+y^{2}\right) z z^{\prime}\right]
$$

or, putting $x^{2}+y^{2}=R^{2}$, we have

$$
\begin{equation*}
x y^{\prime \prime}-y x^{\prime \prime}=\frac{3 z^{2}-r^{2}}{r^{5}} R R^{\prime}-\frac{3 R^{2}}{r^{5}} z z^{\prime} \tag{5}
\end{equation*}
$$

The expression on the right hand side of (5) is the total derivative with respect to $s$ of the quantity $R^{2} / R^{3}$, since

$$
\frac{\partial}{\partial R} \frac{R^{2}}{r^{3}}=\frac{3 z^{2}-r^{2}}{r^{3}} R, \frac{\partial}{\partial z} \frac{R^{2}}{r^{3}}=-\frac{3 R^{2}}{r^{5}} z
$$

We now introduce the cylindrical coordinates

$$
x=R \cos \varphi, \quad y=R \sin \varphi
$$

and keep the $z$ axis the same as before. Instead of (5) we now have

$$
\frac{\mathrm{d}}{\mathrm{~d} s}\left(R^{2} \frac{\mathrm{~d} \varphi}{\mathrm{~d} s}\right)=\frac{\mathrm{d}}{\mathrm{~d} s} \frac{R^{2}}{r^{5}}
$$

and the other two equations in the system are

$$
\begin{align*}
& \frac{\mathrm{d}^{2} R}{\mathrm{~d} s^{2}}=R\left(\frac{\mathrm{~d} \varphi}{\mathrm{~d} s}\right)^{2}+\frac{r^{2}-3 z^{2}}{r^{5}} R \frac{\mathrm{~d} \varphi}{\mathrm{~d} s}, \\
& \frac{\mathrm{~d}^{2} z}{\mathrm{~d} s^{2}}=\frac{3 z}{r^{5}} R^{2} \frac{\mathrm{~d} \varphi}{\mathrm{~d} s}, \tag{6}
\end{align*}
$$

whereas instead of ( $5^{\prime}$ ) we take the corresponding integral

$$
\begin{equation*}
R^{2} \frac{\mathrm{~d} \varphi}{\mathrm{~d} s}=2 \gamma+\frac{R^{2}}{r^{3}} \tag{7}
\end{equation*}
$$

where $2 \gamma$ represents an arbitrary constant.
Moreover, since $s$ is the arc length on the trajectory, we have

$$
\begin{equation*}
R^{2}\left(\frac{\mathrm{~d} \varphi}{\mathrm{~d} s}\right)^{2}+\left(\frac{\mathrm{d} R}{\mathrm{~d} s}\right)^{2}+\left(\frac{\mathrm{d} z}{\mathrm{~d} s}\right)^{2}=1 \tag{8}
\end{equation*}
$$

Equation (7) corresponds to an integral of areas and is very important in the analysis given below.

If we write this equation in the form

$$
\begin{equation*}
R \frac{\mathrm{~d} \varphi}{\mathrm{~d} s}=\frac{2 \gamma}{R}+\frac{R}{r^{3}} \tag{9}
\end{equation*}
$$

and note that $R \mathrm{~d} \varphi / \mathrm{d} s$ is the sine of the angle $\vartheta$ between the tangent to the trajectory and the plane drawn through the point of contact and the $z$ axis, we obtain the inequality

$$
\begin{equation*}
-1 \leqslant \frac{2 \gamma}{R}+\frac{R}{r^{3}} \leqslant 1 \tag{10}
\end{equation*}
$$

which separates regions of space that contain trajectories for given arbitrary value of $2 \gamma$, and any initial conditions, from regions in which there are no such trajectories. It is clear that the bounding surfaces are obtained by taking the equality signs in (10). Figures $6-9$ show the meridians of these surfaces, which take the form of surfaces of revolution around the $z$ axis whilst the black areas are the regions from which the trajectories are excluded.

The values of the constant $\gamma$ for which these figures are constructed are as follows:

Fig. 6a: $\gamma=0.03$
Fig. 6b: $\gamma=0.2$
Fig. 7a: $\gamma=-0.05$
Fig. 7b: $\gamma=-0.05$
Fig. 8a: $\gamma=-0.97$
Fig. 8b: $\gamma=-1.016$
We note that each of these surfaces, and especially the one corresponding to $\gamma$ close to -1 , approaches the origin of coordinates within a very narrow sector. This means


FIG. 6.
that all the trajectories, whatever the corresponding initial conditions, must approach the origin for the particular value of $\gamma$ inside a very narrow region.

Inspection of these surfaces shows that they are simply connected, i.e., they consist of a single piece and extend to infinity. They enclose the origin when $-1 \leqslant \gamma \leqslant 0$.

The trajectories do not reach the origin for very small positive values of $\gamma$, but when $\Delta$ is small and $\gamma \leqslant\left(2 c_{1} / \Delta\right)^{3}$, the trajectories approach the origin to a distance less than $\Delta$.

Finally, by taking $c_{1}=5.2 \times 10^{6} \mathrm{~km}$ for cathode rays, so that the distance between the Earth and the Sun is $28.8 c_{1}$, or simply 28.8 when $c_{1}$ is taken equal to unity, Störmer obtained several other inequalities that restricted the possible values of $\gamma$, and the initial conditions, for which a particle leaving the Sun can reach the Earth.

Analysis of the shape of regions containing trajectories adjacent to the origin for the above values of $2 \gamma$ led Störmer to the following conclusions.

All the trajectories reach the atmosphere in belts that run around the magnetic poles and are constrained within the following ranges:
for cathode rays, between $2.3^{\circ}$ and $3.4^{\circ}$
for radium beta particles, between $4.6^{\circ}$ and $5.8^{\circ}$
for radium alpha particles, between $16.6^{\circ}$ and $18.1^{\circ}$.


FIG. 7.

As can be seen, these belts are narrower than the auroral zone, but we must remember that simplifying assumptions were made in these calculations and that the 'rigidity' of the particles emitted by sunspots, i.e., under conditions that are quite inaccessible to our experiments, may be different from the 'rigidity' of the more familiar particles.

From (9)

$$
\varphi=\int_{s_{0}}^{s}\left(2 \frac{\gamma}{R^{2}}+\frac{1}{r^{3}}\right) \mathrm{d} s+\varphi_{0}
$$

which is found by quadrature after which $x$ and $y$ are determined, and the trajectory can be constructed.

The whole procedure is thus reduced to integration, i.e., to the setting up of a table of values of $R$ and $z$ for given $s$.

Let us suppose that we take a series of successive equidistant values of the independent variable $s$ with an interval $\Delta s=h$ that is sufficiently small to enable us to put

$$
s_{\lambda}=s_{0}+\lambda h(\lambda=1,2, \ldots, \quad n-2, \quad n-1, \quad n, \quad n+1, \ldots)
$$

and let us denote the corresponding values of $R$ and $z$ by

$$
\boldsymbol{R}_{\lambda}=\boldsymbol{R}\left(s_{\lambda}\right), \quad z_{\lambda}=\boldsymbol{z}\left(s_{\lambda}\right)
$$

Moreover we introduce the quantities $\rho_{\lambda}$ and $\xi_{\lambda}$, defined by


FIG. 8.

$$
\rho_{\lambda}=R^{\prime \prime}\left(s_{\lambda}\right) h^{2}, \quad \zeta=z^{\prime \prime}\left(s_{\lambda}\right) h^{2}
$$

The calculation consists of successively adding a row to tables of $R_{n}, \rho_{n}, z_{n} \zeta_{n}$ and their differences, where $\rho_{n}$ and $z_{n}$ for known $R_{n}$ and $Z_{n}$ are found from the first two differential equations in the above system. ${ }^{2}$

Thus, suppose that the table is filled as indicated and that similar tables are obtained for $z$ and $\zeta$. It is required to calculate $R_{n+1}$ and $z_{n+1}$.

This is done by using the equation

$$
\begin{aligned}
& R^{\prime \prime}=\left(\frac{2 \gamma}{R}+\frac{R}{r^{3}}\right)\left(\frac{2 \gamma}{R^{2}}+\frac{3 R^{2}}{r^{5}}-\frac{1}{r^{3}}\right) \\
& z^{\prime \prime}=\left(\frac{2 \gamma}{R}+\frac{R}{r^{3}}\right) \cdot \frac{3 R z}{r^{5}}
\end{aligned}
$$

TABLE I.

| $s_{n-3}$ | $\dot{R}_{n-3}$ | $4 R_{n-3}$ | ( $\Delta^{2} R_{n-4}{ }^{2}$ | $\\|_{\dot{\rho}_{n-3}}$ | $\square$ | $\left\|\dot{\Delta}^{2} \rho_{n-4}\right\|$ | $\Delta^{3} \rho_{n} \because$ | $\int^{4} \dot{\rho}_{n-5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{n-2}$ | $R_{n-2}$ | $\Delta R_{n-3}$ | $\mid \Delta^{2} R_{n-3}$ | $\rho_{n-2}$ | $\Delta \rho_{n-2}$ | $\dot{\Delta}^{2} \rho_{n-3}$ | $\dot{\lambda}^{3} \rho_{n-3}$ | $\Delta_{2}^{4} \rho_{n-4}$ |
| $s_{r-1}$ | $\boldsymbol{R}_{n-1}$ | $\Delta R_{n-3}$ | $\Delta^{2} R_{2-2}$ | $\left\|\rho_{n-1}^{n}\right\rangle$ | $\Delta \begin{array}{ll}  & \\ \Delta \phi_{n-1} \end{array}$ | $\Delta^{2} \rho-2$ | $\cdots$ |  |
| $S_{n}$ | $R_{n}$ |  |  | $\rho_{n}$ |  |  |  |  |



FIG. 9.
to calculate directly the quantities

$$
R^{\prime \prime}(n) \quad \text { and } z^{\prime \prime} n
$$

and hence

$$
\rho_{1}=R^{\prime \prime}{ }_{n} h^{2} \quad \text { and } z_{n}=z_{n}^{\prime \prime} h^{2},
$$

after which we add to the table of values of $\rho_{n}$ the numbers

$$
\rho_{n}, \Delta \rho_{n-1}, \quad \Delta^{2} \rho_{n-2}, \quad \Delta^{3} \rho_{n-3}, \quad \Delta^{4} \rho_{n-4}
$$

and, in exactly the same way, we add to the table of values of $\zeta_{n}$ the numbers

$$
\zeta_{n}, \Delta \zeta_{n-1}, \quad \Delta^{2} \zeta_{n-2}, \quad \Delta^{3} \zeta_{n-3}, \quad \Delta^{4} \zeta_{n-4} .
$$

Using the Taylor expansion, we set up the following relation, neglecting terms containing $h^{5}$ :

$$
\Delta^{2} R_{n-1}=\rho_{n}+\frac{1}{12}\left(\Delta^{2} \rho_{n-2}+\Delta^{3} \rho_{n-3}+\Delta^{4} \rho-\frac{1}{20} \Delta^{4} \rho_{n-4}\right)
$$

and hence find $\Delta^{2} R_{n-1}$ and then, knowing $\Delta R_{n-1}$, we obtain

$$
\begin{aligned}
& \Delta R_{n}=\Delta R_{n-1}+\Delta^{2} R_{n-1} \\
& R_{n+1}=R_{n}+\Delta R_{n}
\end{aligned}
$$

The quantity $z_{n+1}$ is found in precisely the same way. Störmer used this process, helped by his assistant and two calculators, to evaluate about 120 trajectories for the following 27 values of $\gamma_{1}=-\gamma$ :

$$
\begin{aligned}
\gamma_{1}= & 0.1 ; 0.2 ; 0.3 ; 0.4 ; 0.5 ; 0.6 ; 0.7 ; 0.8 \\
& 0.85 ; 0.90 ; 0.92 ; 0.926 ; 0.9285 ; 0.93 \\
& 0.9335 ; 0.939 ; 0.94 ; 0.95 ; 0.9456 ; 0.957 \\
& 0.97 ; 0.999 ; 1 ; 1.2 ; 1.5 ; 2 ; 5 .
\end{aligned}
$$

About 100-120 points were calculated on each trajectory, which involved more than 5,000 hours of work.

For all these trajectories, the initial conditions were chosen so that each trajectory crossed the origin of coordinates and departed to infinity. Störmer used his analysis of (4) to show that for each value of $\gamma$ between 0 and -1
there were two and only two such trajectories. He also showed how the initial values of the variables could be found for these trajectories.

These calculations were used by Störmer to construct the models shown in the photographs reproduced in Figs. 10-12.

In addition to trajectories that depart to infinity, there are closed or periodic trajectories that require roughly the same amount of work as those departing to infinity.

These periodic trajectories sometimes take the form of very complicated spirals such as those shown in Fig. 12, and it is not difficult to imagine the amount of labor necessary to calculate a sufficient number of points to construct such spirals.
7. Störmer's investigations undoubtedly show that particles of electricity similar to cathode rays or the radium beta or alpha rays can under the above assumptions enter the Earth's magnetic field and reach the Earth in a very restricted region close to the Pole, in which their trajectories are confined to an exceedingly narrow sector as they approach the Earth. The diameter of this sector at about 100 km above the Earth's surface is between a few meters


FIG. 10.


FIG. 11.
and a few kilometers, depending on the 'rigidity' of the rays.

It is clear that the flux of particles ejected by the Sun simultaneously from very different points on its surface travel along such trajectories and are responsible for the aurora borealis with its draperies and rays.
8. Birkeland approached this question in a somewhat different way. He undertook an experimental investigation of the the motion of electrical particles in a magnetic field. He did this by constructing, in effect, a model of the globe, which he called the terrella.

Figure 13 shows the first of his models and Fig. 14 the second, with a larger diameter.

The principle of both devices is the same: a glass vessel or a box with glass walls, evacuated to a pressure of a few thousandths of a millimeter of mercury, contains an electromagnet in the form of a sphere suspended at the center


FIG. 13.
of the box. The spherical electromagnet is surrounded by a thin brass shell, coated with barium platinocyanide. A source of cathode rays is placed on one of the walls of the box. The spherical electromagnet-the terella-can be mounted so that its magnetic axis assumes any required position relative to the line joining its center to the cathode. Moreover, it can be made to rotate so as to simulate the diurnal rotation of the Earth.

By varying the magnetizing current, it is possible to vary the magnetic moment of the spherical magnet, i.e., the constant $c_{1}$ in Störmer's equations, whose value for the Earth depends on the type of rays. A given set of cathode rays can thus be used to study the behavior of rays with other 'rigidities'.

Cathode rays produce a glow in the low-presure gas and bright spots on the barium platinacyanide layer at points at which they strike the surface of the terrella.

Birkeland also equipped his terrella with different screens, coated with the same material, so that he was able to follow more closely the paths of these rays. He used slotted screens to define a narrow beam of rays travelling in a particular direction.

Figures 14-16 shows some of Birkeland's results. These clearly demonstrate the presence of the equatorial ring and of an emission localized in a narrow region at a distance of about $20^{\circ}$ from the pole.

The shape of these curves and the points at which they


FIG. 14.


FIG. 15.
touch the sphere are generally consistent with Störmer's mathematical theory.
9. For certain initial conditions, the trajectories found by Störmer include those that seem to have a vertical branch approaching the Earth, a short horizontal segment near the Earth's surface, and again a vertical branch departing from the Earth. Such trajectories come close to the Earth in regions near the Pole.

Similar paths were also obtained by Birkeland for certain particular magnetizations of the terrella and for certain positions of its magnetic axis relative to the direction of the incident rays.

This fact, and the relatively simple field structure obtained for polar magnetic storms, led him to consider the following problem: what is the length of the horizontal segment, its height above the Earth's surface, and the current that would produce a disturbing field similar to that found by observation.

He found that the height had to be about 200 km , the length about 1500 km , and the current about $10^{6} \mathrm{~A}$, in which case the resulting field was close to one of those found by observation. Values of the same order were obtained for other observations.


FIG. 16.
10. Störmer's trajectories approach the Earth's surface mostly on the side that is not illuminated by the Sun. The same result was revealed by Birkeland's experiments with the terrella. He therefore decided to investigate the distribution of magnetic storms with the time of day at his four polar stations. A simple count of the disturbances, or allowance for their duration at different times of day did not suffice, and the strength of the disturbance itself had to be taken into account.

Suppose, for example, that $P_{\mathrm{h}}$ is the horizontal component of the disturbing force in the plane of the magnetic meridian. The integral of the absolute magnitude of this component, i.e.,

$$
\left|S_{\mathrm{H}}\right|=S_{\mathrm{H}}^{\mathrm{a}}=\frac{1}{T} \int_{0}^{T}\left|P_{\mathrm{h}}\right| \mathrm{d} t
$$

then gives the absolute average of the strength of the horizontal disturbance over the meridian. Similarly, if $P \mathrm{f}$ and $P_{\mathrm{h}}^{\mathrm{n}}$ are the positive and negative values, the integrals

$$
\begin{aligned}
& S_{\mathrm{H}}^{\mathrm{p}}=\frac{1}{T} \int_{0}^{T} P_{\mathrm{H}}^{\mathrm{p}} \mathrm{~d} t, \\
& S_{\mathrm{H}}^{\mathrm{n}}=\frac{1}{T} \int_{0}^{T} P_{\mathrm{h}}^{\mathrm{n}} \mathrm{~d} t
\end{aligned}
$$

give the mean values of the positive and negative disturbances associated with the above-mentioned components.

It is clear that similar quantities can be constructed for the transverse component (disturbances of inclination) and the vertical component.

Finally,

$$
S^{\mathrm{T}}=\sqrt{\left|S_{\mathrm{H}}\right|^{2}+\left|S_{\mathrm{D}}\right|^{2}+\left|S_{\mathrm{V}}\right|^{2}}
$$

is the average total strength where $\left|S_{\mathrm{D}}\right|$ and $\left|S_{\mathrm{v}}\right|$ represent the average values of the transverse and vertical components.


FIG. 17.


FIG. 18.

Birkeland performed these calculations for two-hour intervals during each day, and also for the same hours over five-day periods, then over monthly periods, and, finally, for the entire time of observations.

He found that the average strength of the disturbances near the auroral belt follow the diurnal motion of the Sun and, when these disturbances are represented by the current direction vectors, two principal systems are noted at each station, namely, (1) the first system, which has a maximum at about 6 pm local time, with the arrow pointing east along the auroral belt and (2) the second system with a maximum near midnight, with arrow pointing west.

Magnetic calm, i.e., the absence of disturbances, occurs at about 9-10 am local time.

These two principal systems correspond to disturbances that were earlier referred to as positive and negative polar disturbances.

Examination of the diagrams themselves demonstrated to Birkeland that the center of these disturbances was located between Axelöen and Kaafiord, but there were also some local storms of lower strength near the Earth's magnetic pole and north of the auroral belt.

The diurnal period, the position of the zone of polar disturbances, and the time of day at which they attain their highest intensity are consistent with both Störmer's theory and Birkeland's terrella experiments in which he found that by varying the rigidity of the rays it was possible to obtain two types of electrical precipitation on the evening side of the Earth. In one of them the motion of the particles occurs eastward and in the other westwards (Birkeland


FIG. 19.
found this by placing his screens near the poles of the terrella and noting their bright and dark sides). These two types of precipitation can provide as an explanation of the positive and negative polar disturbances.

On the other hand, the source of the electrified radiation is the Sun.
11. Birkeland also performed interesting experiments in which the electromagnet itself, or the terrella, was the source cathode rays.

He observed phenomena similar to the solar corona and the rings of Saturn, as can be seen in Fig. 17.
12. It is difficult to convey, in the limited time at our disposal, an adequate idea of the two huge volumes produced by Birkeland. They are full of observational, experimental, and theoretical material, of which I have given only a brief review.

I now turn to Störmer's recent researches in which he reveals his talent as an observer and experimentalist; in his previous work he demonstrated his mathematical skills.

I noted at the beginning the calculations made by $\mathrm{Dal}-$ ton, in which he determined the position of the apex of an auroral arc, and Potter's work of 1833.

Neither photography nor telegraphy was available at the time, and the telephone was not even thought of. It is therefore clear that two-point observations of a variable event such as the aurora borealis, carried out simply by eye or by goniometers, with a view to determining the position of particular points on the aurora relative to fixed stars, were not distinguished by high precision, mostly because they were difficult to perform.

Störmer decided to employ modern scientific equipment in a systematic study of this question. He built two observatories in northern Norway, one at Bossekop and the other at Stor Korsnes (the former $70^{\circ} \mathrm{N}, 24^{\circ} \mathrm{E}$ of Greenwich and the latter on the same meridian, but 27 km further north).

Each observatory was equipped with identical, specially adapted photographic cameras. The two stations were linked by telephone so that simultaneous photographs could readily be taken. The cameras were pointed at a sky region containing the aurora borealis and a bright star or


FIG. 20.
planet which could be used as a reference for images of the same point on the aurora.

It is clear that such simultaneous photographs readily yield the position of any chosen feature recorded on film, including its height and azimuth. The position can then be marked on a chart with an indication of the height of the feature above ground level. Störmer was mostly concerned with the lower edge of the aurorae boreales, where they take the form of draperies, and examined several hundred photographs at several thousand points on the aurorae.

The photographs of Figs. 18 and 19 clearly show the images of aurorae and of Venus.

Figure 20 presents a summary of these observations. It shows the heights of all the calculated points. It is clear that they tend to lie in a layer at about 100 km above ground.

Störmer is continuing his work. He is carrying out magnetic observations, using recording variometers in parallel with his photogrammetry.

It is clear that analysis of simultaneous observations will yield accurate data for calculations, similar to those initiated by Birkeland, i.e., calculations of the current of charged particles which, as they enter the atmosphere, generate both the aurora borealis and magnetic storms. However, we can already declare that the work of Birkeland and Störmer has ellucidated the essence of the situation and that subsequent work will be mainly concerned with detail.
${ }^{\text {*) }}$ (Paper read to a meeting of the Russian Physico-Chemical Society, held in January 1917.) First published in UFN in April 1918.
${ }^{1)}$ The constant $\alpha$ is the ratio of the electrostatic unit of electric mass to the electromagnetic unit. It is equal to $1 / c$ where $c$ is the velocity of light.
${ }^{2)}$ A detailed account of this method can be found in the following article: A. N. Krylov, Arkhiv. Fiz. Nauk, Nos. 1-2, 68 (1918).

Translated by S. Chomet

