Structural features, critical currents and current-voltage characteristics of high temperature superconducting ceramics

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1. INTRODUCTION

The use of high-T_c superconducting ceramics is the only possible way of making a variety of engineering applications of high temperature superconductivity practical. One of the basic obstacles on the path of realization of these possibilities are the small (even in a zero magnetic field) critical currents of HTSC ceramics which are by factors of 10–1000 (depending on the production technique) weaker than those of crystals and decrease rapidly with increasing magnetic field. It obviously follows from the fact that the only fundamental distinction between HTSC ceramics and HTSC crystals is the macrogranular structure inherent to the former, that the reason for "bad" superconducting characteristics of HTSC ceramics lies not so much in the properties of individual superconducting crystallites (granules, or grains) as in the contacts between them, or so-called intergranular (intergrain) boundaries (GBs). Hence the way to increasing the current-carrying abilities of HTSC ceramics runs through a better insight into and improvement (based on this better insight) of the properties of intergranular boundaries.

In this review we shall discuss problems related to the "arrangement" and properties of those elements of the granular structure of HTSC ceramics (with an emphasis on the ceramics of the $YBa_2Cu_3O_\delta$ composition) which define the critical currents and current-voltage characteristics of ceramic materials. As we have already mentioned, the basic element of this sort are intergranular boundaries which may, in principle, play a dual role: as pinning centers they may increase the critical current, while as "weak" sites (that is, areas with a low critical current density) they would lower it. In materials of practical interest the predominant one seems to be the second (negative) function of intergranular boundaries, so the task is to reduce it to a minimum.

The discussion of the role of GBs in determination of superconducting properties of HTSC ceramics should have been started with their classification. Unfortunately, there exists no single physical parameter of such boundaries which might be used as the basis of a nonambiguous classification scheme. Hence there are several different approaches, each of them proceeding from a concept of the decisive role of this or that property of a GB.[1] As an appropriate parameter one could use:

1) the misorientation angle of neighboring (separated only by a GB) grains;[2]

2) the GB plane orientation relatively to (001) and (100) planes in which the anisotropic correlation length takes on extreme values;[1]

3) the oxygen stoichiometry of near-to-boundary areas of grains;[5]

4) the probability of impurities segregation on a GB;

5) the tendency toward the formation of stacking faults and "foreign" or amorphous phases on a GB;[3]

No matter which of the above-listed (as well as unlisted) factors forms the basis of "bad" superconducting behavior of an HTSC ceramic, they all ultimately lead to the formation of weak superconducting links between its individual grains. That is why for modeling purposes a ceramic can be regarded as a Josephson medium, that is a set of superconducting grains interconnected by Josephson junctions. It provides a means of describing a good number of properties of superconducting ceramics without going into detail of specific genesis and "arrangement" of weak intergranular links. Although this kind of a model allows one to describe magnetic and transport properties of ceramics, it is expedient for working out concrete recommendations as to purposeful modification of these properties to take advantage of certain ideas of physical mechanisms responsible for a distinction between GB properties and bulk properties of superconducting crystallites of ceramics.

2. MORPHOLOGICAL AND STRUCTURAL FEATURES OF HTSC CERAMICS

2.1. Grains and intergrain contacts

From the point of view of morphology, an HTSC ceramic[1)] is a set of superconducting crystallites of various shapes, sizes and crystallographic orientations, which are in mechanical contact with each other. $YBa_2Cu_3O_\delta$- ceramics consist mainly of grains of an orthorhombic phase although the composition of a part of grains corresponds to a tetragonal phase; besides, it contains inclusions of other phases of different compositions (Y_2BaCuO_5, $BaCuO_2$, CuO, etc.).[6]

In early studies, when HTSC ceramics were produced using a "ceramic" technology, all the above-cited parameters of different crystallites were but weakly correlated, so microscopic-scale images of ceramics looked quite chaotic. Nevertheless even then it was noticed that the shape of an individual crystallite is not completely random: a good number of them were shaped as relatively thin lamellas (of irregular form, most often stretched out in one of the directions) whose plane is close to the (001) plane. (It is due to the fact that the preferred direction of crystallite growth is [100].) Characteristic crystallite dimensions are strongly dependent on the method of ceramics production and usually are in the 1–100 μm range. The internal region of

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crystallites is liable to strong twinning, it may contain microcracks (their number usually grows with the increase of crystallite dimensions); there are pores (voids) near the contacts as well as on the boundaries between and (more rarely) inside the grains. On the grain boundaries one can often see structure distortions and amorphization and changes in stoichiometry. A considerable part of intergrain contacts is realized through high-angle intergrain boundaries with a poor current-carrying ability (see below). Hence critical currents of such ceramics are not large ($1-10^3$ A/cm^2 in the absence of an applied field, $T=77$ K) and they rapidly diminish with increasing magnetic field (down to 1 A/cm^2 in the field of 0.1 T).

A substantial element of the $YBa_2Cu_3O_\delta$ ceramics structure is grain alignment which manifests itself in such kind of "ordering" of its crystallites that the directions of their *c*-axes ([001] directions) are in proximity to each other. A moderate degree of grain orientation in HTSC ceramics was observable already in the days of the initial ceramic technology. After it had been realized that the texture improves noticeably the current carrying ability production methods of almost completely textured $YBa_2Cu_3O_\delta$- ceramics were devised. The most promising of them are those utilizing liquid-phase processes (ensuring nearly equilibrium crystal growth): melt-textured growth,[7] quench and melt growth,[8] zone melting.[9]

The structure of liquid-phase produced materials is arranged in the following scheme: ceramics—macrograins—subgrains—crystallites—twin domains (with the corresponding hierarchy of scales: 1 cm—10^{-1} cm—10^{-2} cm—$10^{-(3-4)}$ cm—$10^{-(5-6)}$ cm) and is notable for a much higher degree of ordering as compared to ceramics. The existence of grain alignment manifests itself in the proximity of the [001] axes directions of all the elements of any level of this structure. The situation with the base *ab*-plane ((001)-plane) orientation, however, is somewhat different: the misorientation on macrograin boundaries is close to a random one, it is about 10° and 1° (Ref. 10) at the subgrain boundaries and crystallite boundaries respectively, and it is equal to 90° at twin boundaries. Critical currents in such materials are considerably higher than for $YBa_2Cu_3O_\delta$ ceramics (10^4-10^5 A/cm^2 in the magnetic field of the current, $T=77$ K) and are far less dependent on the applied magnetic field (10^3-10^4 A/cm^2 in a field of ~ 1 T).[11] The critical current anisotropy observable both when there is and there is no magnetic field is substantial: the critical current is higher when aligned with the *ab*-plane, and the magnetic field parallel to this plane produces a weaker effect on the critical current.

The major difference between dense textured $YBa_2Cu_3O_\delta$-materials and $YBa_2Cu_3O_\delta$ ceramics lies in the fact that a considerable part of intergranular contacts available in them are small-angle inter-crystallite boundaries whose planes are parallel to the [001] direction common to all the crystallites. (The paramount formation of small-angle boundaries is due to the high diffusion rate in the liquid phase and to the long lifetime of this phase, which contribute to the GB "tuning" to the low-energy small-angle configuration. The remaining high-angle boundaries acquire in this case a clearly seen cut and contain no amorphous layers.[12]) Hence it is natural to assume that it is this small misorientation of neighboring granules which is responsible for the increased current-carrying ability of textured materials. A great number of experimental studies have pursued the aim of checking and confirming this assumption (see below).

From all the foregoing it follows that the current-carrying ability of materials under consideration is related to their "weak sites," i.e., to intergrain (intercrystallite) contacts. The critical current density of a ceramic is determined by the number of such contacts and their critical currents. In order to understand by what, in its turn, the critical current of GBs in HTSC ceramics is determined, one should pay special attention to their structure.

First we note that a greater part of the analyzed ceramics is marked by the absence (or, to be exact, by very rare occurrence) of amorphous and foreign crystalline phases on the GBs.[13,14] In Ref. 14, for instance, it has been shown (using the method of *Z*-contrast electron microscopy) that the stoichiometric composition of the grains of the $YBa_2Cu_3O_\delta$-ceramics at distances of ~ 2 Å from GBs corresponds to the formula unit $Y_{0.87<x<1.13}\,Ba_{1.92<y<2.07}Cu_{2.79<z<3.21}O_{4.91<x<9.09}$, where the content of only one of the components may vary within the limits cited. In this way the existence of phases of types $Y_2Ba_4Cu_7O_\delta$, $YBa_2Cu_4O_\delta$, $BaCuO_2$ and others on the phase boundary is excluded. It means that the critical current decrease on the GB (as compared to that in the crystallite bulk) is due to the boundary microstructure rather than to its "microchemistry." The main elements of the GB microstructure are as follows: 1) crystallographic orientation of both crystallites in contact; 2) boundary dislocations; and 3) oxygen stoichiometry.

Boundary surfaces of crystallites of which HTSC ceramics are composed are, as a rule, facets, that is they are crystalline planes with small Miller indexes. In nontextured ceramics the contact of most of crystallites (about 90%) occurs in such a way that the contact boundary surface of at least one of them coincides with the (001) plane; in grain-aligned (textured) ceramics, possessing much higher critical currents, the majority of boundaries is perpendicular to this plane.[3,13] From this it follows that the boundary plane (001) is "bad," that is, associated with low critical current density.

There is a number of factors which could provide an explanation of this property of the ⟨001⟩ surface. Firstly, it is a planar defect frequently occurring on this surface, which is an excessive Cu–O layer creating a nonsuperconducting (metallic or even dielectric) interlayer in-between the contacting crystallites.[3,13] Secondly, it is an anisotropy of the correlation length of superconducting electrons which manifests itself in the correlation length ξ_c along the *c*-axis (i.e., in the direction normal to ⟨001⟩ planes) being significantly smaller than the correlation length ξ_{ab} in the ⟨001⟩ plane. Both these factors result in a small critical current of a weak link originating in an intergranular contact. Quantitative studies of such contacts, however, are yet to be carried out.

Much better investigated (both experimentally and theoretically) are intergranular contacts with the planes parallel to the c-axis, especially those of them which are associated with small-angle tilt boundaries and were studied in detail in a series of studies.[15-20] One can find indications of sufficiently high critical currents in small-angle twist boundaries as well,[18,21] though their properties have been insufficiently studied as yet.

An important factor determining the properties of intergranular contacts is oxygen stoichiometry in near-contact areas of superconducting grains.[5] The reason for a possible difference between the oxygen atom concentration in direct proximity to a boundary and that in the grain bulk lies in the arrangement of these areas occurring in order to lower the boundary energy. Energetically beneficial (and, consequently, predominantly formed) are the boundaries with a considerable number of lattice sites simultaneously belonging to both contacting crystals. The formation of such boundaries is favored by a corresponding "tuning" of lattice parameters which may take place in $YBa_2Cu_3O_\delta$-like crystals as a result of variations of oxygen stoichiometry. Experimental data[5] suggest that the oxygen content near the boundary (in a sub-boundary layer less than ~ 100 Å thick) depends on the misorientation angle of grains in contact: for small-angle boundaries $\delta \cong 7$, and for boundaries with large misorientation angles $\delta < 7$. A well-known interrelation of superconducting properties of $YBa_2Cu_3O_\delta$ compounds with the oxygen content (T_c drop with a decrease in δ) explains the fact that it is contacts corresponding to small-angle intergrain boundaries that are "good."

2.2. Twinning planes

Experiments with $YBa_2Cu_3O_\delta$ single crystals reveal that twinning planes available in them are (anisotropic) pinning centers and in such capacity they are capable of producing a considerable effect on the critical current.[104,105] Obviously, exactly the same pinning mechanism may appear to be equally vital for $YBa_2Cu_3O_\delta$ ceramics. The reason for $YBa_2Cu_3O_\delta$ microcrystal twinning lies in a structural phase transition occurring at $T(0 \leftrightarrow t) \cong$ 600–700 °C (the exact transition temperature value depends on the oxygen concentration in the environment). The origination of a twin structure is not related to the atom diffusion and may take place very promptly which is characteristic for a typical martensite transition. This kind of transition occurs on cooling a ceramic sample in an oxygen-containing atmosphere from the synthesis temperature (~ 900 °C) down to room temperature and is accompanied by a transformation of the initial tetragonal structure (stable at $T > T(0 \leftrightarrow t)$) into an orthogonal one (stable at $T < T(0 \leftrightarrow t)$). At the transition moment the dimension of a $YBa_2Cu_3O_\delta$ unit cell along the c-axis is almost unchanged while lattice constants a and b in the basal plane change discontinuously. The relative magnitude of this discontinuity amounts to $\Delta b/b \cong -\Delta a/a \cong 0.008$, which corresponds to crystal compression along the a axis and its stretching along the b axis. The changes of the microcrystal shape resulting from such kinds of deformations in ceramics are to a great extent restricted by neighboring microcrystals and this should lead to the appearance of strong mechanical stresses. The relaxation of these stresses takes place by means of formation (in the microcrystal bulk) of a twin domain structure which is an aggregate of alternating areas of an ortho-phase with parallel c-axes but mutually rotated with respect to each other basal ab-planes. The misorientation angle of the basal planes in neighboring domains equals $\phi = 2\,\mathrm{arctg}(b/a)$. In $YBa_2Cu_3O_\delta$ $a \cong b$ and hence $\phi \cong 90°$, i.e., the twinning plane must coincide with the $\langle 110 \rangle$ or $\langle 1\bar{1}0 \rangle$ planes. The resulting twin structure "helps" the microcrystal to preserve its shape (or reduce its change to a minimum). The areas of differently oriented twin domains in this case must be roughly equal.

Analyzing microcrystals of the $YBa_2Cu_3O_\delta$ ceramics with the use of optic (and electron) microscopes proves them to be in fact strongly liable to twinning, their twin domains usually having the shape of long ($\geqslant 1\ \mu$m) narrow "strips" parallel to the [110] or [1$\bar{1}$0] axis. The width of these domains, $\Delta = 50$–3000 Å (Ref. 106), depends on the ceramics manufacturing process and, in particular, on the size G of the microcrystalline grains constituting it. Ref. 107 suggests an empirical relation $\Delta[\mu\mathrm{m}] \cong 0.03\sqrt{G[\mu\mathrm{m}]}$, which is true for small-sized microcrystals (1–30 μm) and seemingly reflects that effect of intergranular boundaries which they produce in the process of elastic stress relaxation. On the other hand, Ref. 108 discusses a simple model providing a means of evaluating the width Δ of twin domains in sufficiently large microcrystals (where the role of boundaries becomes insignificant) proceeding from crystal chemistry ideas about the $YBa_2Cu_3O_\delta$ structure.

In contrast to a high temperature tetragonal structure of $YBa_2Cu_3O_\delta$ in which oxygen atoms lying in $\langle 001 \rangle$ basal planes are distributed in a random manner, in low temperature orthogonal structure these oxygen atoms in combination with copper atoms form a regular pattern of chains stretched along the b axis. The quantity of oxygen vacancies in such chains depends on the total oxygen content in $YBa_2Cu_3O_\delta$: they are completely absent at $\delta = 7$ and appear in ever greater quantity as $\delta \to 6$. The model in Ref. 108 assumes that within each of the twin domains $\delta = 7$, and the compositions with lower oxygen content are realized as the result of an increase in the number of the twinning boundaries (i.e., the narrowing of domain "strips") to which oxygen vacancies in oxygen-copper chains are "bound." (This model is supported by the EPR studies of the twin structure of superconducting $YBa_2Cu_3O_\delta$ single crystals.[109] For a domain width Δ the average number of unit cells between domain boundaries equals $N_\Delta \sim \Delta/a$, and the ratio of the number of oxygen vacancies concentrated on this boundary to the number of chain Cu atoms is $\sim 1/N_\Delta \sim a/\Delta$. On the other hand, this ratio is determined only by the stoichiometry of $YBa_2Cu_3O_\delta$ and equals $(7-\delta)/6$. Consequently, $\Delta \sim 6a/(7-\delta)$. For samples annealed in oxygen the typical value is $\delta = 6.98$ and the twin domain width must amount to ~ 1000 Å.

As far as the boundary between two twin domains (differing in a,b-orientations) is concerned, the one-dimensional image of its crossing with a microcrystal boundary is usually a broken line consisting of a great number of straight line segments parallel to the [110] or [$1\bar{1}0$] axes. The sinuosity of this line can be characterized by its fractal dimension which, according to Ref. 110, may attain values of 1.2–1.4.

A considerable part of twin domains ends on the boundaries of ceramic microcrystalline grains. In numerous cases, however, they extend beyond these boundaries, especially in nearly "symmetrical" cases when the angles, made by domain "strips" in two neighboring grains with the boundary between them, differ but slightly from one another.[111]

Apart from the "mirror" twin boundaries considered above, "rotating" twin boundaries may originate in $YBa_2Cu_3O_\delta$ where an alteration of crystallographic axes in microcrystal neighboring domains takes place: the c axis transforms into the b axis (or, more rarely, into the a axis). The probability of forming such boundaries is great when the ratio $c/b \approx 3$, i.e. for $YBa_2Cu_3O_\delta$ compositions with a structure close to tetragonal and with relatively low critical temperature.[112,113]

3. PROPERTIES OF INDIVIDUAL INTERGRANULAR CONTACTS

In Refs.[16–20] properties of individual "artificial" GBs in epitaxial laser ablated $YBa_2Cu_3O_\delta$ films (where the film plane coincides with the basal ab-plane, the film thickness being $\sim 1\ \mu m$) on a bi-crystalline strontium titanate substrates have been discussed. The GB orientation in the film is determined by the bi-crystalline boundary of the substrate. Critical currents and current-voltage characteristics of GBs with different misorientation have been studied. Two universal relationships were brought to light: a) an average (over the area) density (j_c^{GB}) of critical current across the boundary is a function solely of the angle ϑ of the grain misorientation on this boundary; and b) the characteristic voltage $v_c = i_c r_N$, which is a product of the critical current i_c of an intergrain contact by its resistance r_N in the normal state depends only on the critical current density (j_c^{GB}) of this contact. The very existence of these universal relationships is an evidence of superconducting properties of GBs in $YBa_2Cu_3O_\delta$ being intrinsic properties of the material.

The first of the above-mentioned relationships is illustrated by Fig. 1, from which it follows that the critical current density j_c^{GB} rapidly falls off with an increase in ϑ, diminishing by an order of magnitude at $\vartheta \sim 10°$; during a further increase in ϑ, the j_c^{GB} fall-off slows down. In different studies where the $j_c^{GB}(\vartheta)$-dependence was used for calculations of the critical current of HTSC ceramics different approximations were suggested for it. Thus, in Ref. 22 it was shown that the experimental dependence $j_c^{GB}(\vartheta)$ is satisfactorily approximated with the expression

$$j_c^{GB}/j_c^{G} = \frac{\sin \delta}{\sin(\vartheta+\delta)}, \qquad (1)$$

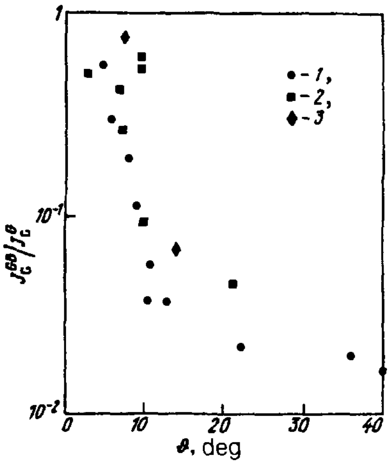

FIG. 1. Intergrain boundary critical current density j_c^{GB} as a function of the misorientation angle ϑ between contacting grains of a $YBa_2Cu_3O_\delta$-film at $T=5$ K (Ref. 18). Boundary type: ●—[001] tilt boundary; ■—[100] tilt boundary; ♦—[100] twist boundary.

where j_c^{G} is the critical current density in a grain, $\delta = 0.87 \cdot 10^{-2}$. In Ref. 23 a different approximation was utilized:

$$j_c^{GB}/j_c^{G} = 0.6\left[\frac{1}{1+\vartheta/\vartheta_0} + \frac{1}{1+(\pi/2-\vartheta)/\vartheta_0}\right] - 0.08, \qquad (2)$$

where $\vartheta_0 = 4°$. In Ref. 24, finally, experimental data are described by the exponential dependence

$$j_c^{GB}/j_c^{G} = \exp(-\vartheta/\vartheta_0), \quad \vartheta_0 = 5°. \qquad (3)$$

It should be borne in mind, however, that the aforementioned method of forming GBs in $YBa_2Cu_3O_\delta$ films based on the use of cubic substrates gives a possibility of producing boundaries with misorientation angle $0 \leqslant \vartheta \leqslant 45°$. Hence the assertion of monotonic GB critical current density decrease with the rise of the misorientation angle should be treated with caution. Thus, in Refs. 25, 26 it has been experimentally established that in $YBa_2Cu_3O_\delta$ the [010] twist boundary with $\vartheta = 90°$ does not display qualities typical of weak links. (A rotation of ab-planes by 90° around the common a or b axis occurs on a GB of this kind).

Magnetic field dependences $i_c(B)$ of the critical current of intergrain contacts testify to their being Josephson junctions:[27] in most cases periodic dependences (with periods correlating with the theory[28] and determined by geometric dimensions of the contacts) are observed. Depending on the critical current density of a contact the latter may be regarded as "narrow" or "wide";[19] the parameter determining the contact type is the ratio of its width L to the Josephson penetration depth

$$\lambda_J = \left(\frac{\hbar c^2}{8\pi e d j_c^{GB}}\right)^{1/2}, \qquad (4)$$

where $d \cong 2\lambda_L$ is an effective contact width equal to a double London penetration depth (λ_L). The contacts with $L/\lambda_J > 4$ are wide, otherwise they are narrow. For

$YBa_2Cu_3O_\delta$ $\lambda_L \sim 10^{-5}$ cm, hence at $j_c^{GB} \sim 10^5$ A/cm^2 (such current density values are typical for small-angle ($\vartheta \leqslant 10°$) GBs of the kind under analysis at $T=4.2$ K) $\lambda_J \sim 1$ μm, and taking into account that usually $L \sim 1$ μm, we arrive at the conclusion that at $T=4.2$ K small-angle boundaries correspond to wide Josephson junctions, and narrow junctions are correlated with high-angle ones. As for $T=77$ K, when $j_c^{GB} \sim 10^4$ A/cm^2 (for every value of ϑ), then practically all junctions are narrow.

Josephson junctions of a different sort in $YBa_2Cu_3O_\delta$ have different current-voltage characteristics: narrow junctions are well described by a resistive model where r_N is almost independent of the temperature (within the range of $4.2 < T < 77$ K), and at high voltages ($v \gg v_c = i_c r_N$) Ohm's law is valid: $v \cong r_N i$ (v, i are contact voltage and current, respectively); wide junctions are not described by the resistive model (their properties are defined by the so-called Josephson vortices[28]) and are characterized by the presence of "excess" current i_{exc} (comparable to the critical current i_c) violating Ohm's law at high voltages: $v \cong r_N(i - i_{exc})$. It should be noted, however, that in any case the magnitude of the critical voltage v_c is as small as 0.2–8 meV which is substantially lower than the theoretically predicted value equal to $\Delta/e \sim 15$–30 meV (here Δ is the energy gap width in $YBa_2Cu_3O_\delta$). This small magnitude of v_c is most probably due to a suppression of the order parameter in superconducting grains near the boundary for which (taking into account the small value of the correlation length) the small-scale inhomogeneities inherent to these boundaries are sufficient. These inhomogeneities are the result of structural disorder and/or mechanical stresses, stoichiometric deviations, impurities, etc. In any case it means that the Josephson junctions under consideration are not the classic tunnel SIS junctions (Superconductor-Insulator-Superconductor) and may be junctions of SS′IS′S- or SNINS-types (S′ is a superconductor with lower critical temperature, N is a normal metal). More definite conclusions as to the junction type can be reached proceeding from the temperature dependence of its critical current (see below).

The second of the above-mentioned universal relationships deals with narrow junctions and consists in the dependence of their characteristic voltage only on the critical current density: $v_c \propto (j_c^{GB})^{0.6}$ (Ref. 20). This relationship is illustrated by Fig. 2 pertaining to high-angle ($\vartheta \geqslant 15°$) tilt and twist boundaries in $YBa_2Cu_3O_\delta$. The intrinsic nature of this relationship has been confirmed by the fact that on artificial variation of the junction critical current (e.g., by means of annealing) the corresponding point "drifted" along the curve corresponding to this relationship. The correlation in question can be expressed in a different, equivalent, form:

$$j_c^{GB} \propto (\sigma_N)^{5/2}, \tag{5}$$

where $\sigma_N = (r_N \times \text{junction area})^{-1}$ is the normal conductivity of a junction of unit area.

In order to clarify the nature of Josephson junctions it is important to measure temperature dependences $i_c(T)$ of their critical current. The theory[28] predicts that near T_c

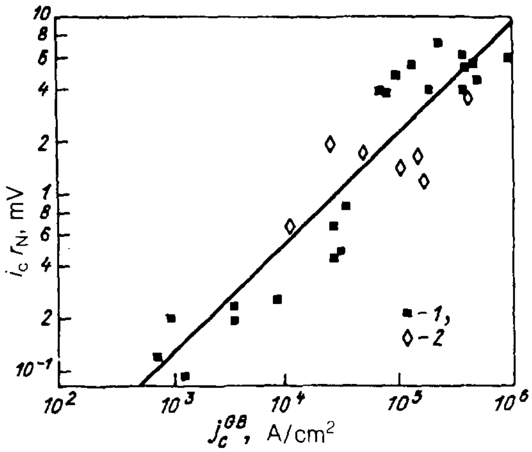

FIG. 2. Characteristic voltage $v_c = i_c r_N$ of the Josephson junction on the intergrain boundary in a $YBa_2Cu_3O_\delta$-film as a function of the critical current j_c^{GB} at $T=4.2$ K (Ref. 20). The data are valid for large-angle ($15 < \vartheta < 45°$) [001] tilt boundaries and [100] twist boundaries in films produced by laser ablation (■) and electron-beam evaporation (◇) processes. Straight line corresponds to the function $v_c \propto (j_c^{GB})^{0.6}$.

$$i_c(T) \propto (1 - T/T_c)^m, \tag{6}$$

where $m=1$ for a tunnel SIS-junction and $m=2$ for a SNS-transition. In the latter case the temperature dependence $i_c(T)$ has the form[29]

$$i_c(T) \propto \frac{\Delta_\infty^2(T)}{\xi_N(T)\,\mathrm{sh}[d_N/\xi_N(T)]}\,\zeta(T), \tag{7}$$

where $\Delta_\infty(T)$ is an order parameter in a superconductor at a large distance from the contact, d_N is the normal metal intergrain layer width,

$$\xi_N(T) \cong \begin{cases} 0.23\left[\dfrac{\hbar v_{FN} l_N}{kT}\right]^{1/2} & (l_N \ll \xi_N\text{—"dirty" limit}) \\ 0.16\,\dfrac{\hbar v_{FN}}{kT} & (l_N \gg \xi_N\text{—"clean" limit}) \end{cases} \tag{8}$$

is a coherence length in the normal layer material (v_{FN}, l_N are the Fermi velocity and the mean free path of charge carriers, respectively),[30] and

$$\zeta(T) = \mathrm{th}^2\left[\frac{x_0}{\sqrt{2}\xi_s(T)}\right] \tag{9}$$

is a factor accounting for an order parameter decrease at the "superconductor-normal metal" interface resulting from the proximity effect (ξ_S is the coherence length in a superconductor and x_0 is the characteristic length defining the suppression of the order parameter near the boundaries of an intergrain contact).

According to different estimates, $\xi_N = 10$–100 Å in $YBa_2Cu_3O_\delta$ (Ref. 27), in Refs. 31, 32 $\xi_N = 20$–80 Å, as obtained from an exponential dependence of the SNS-contact critical current on the thickness of the normal interlayer. From estimates made for the $x_0/\xi_S(0)$ ratio in Ref. 33 by means of comparison of the theoretical expression (7) with experimental temperature dependences of the ceramics critical current it follows that $x_0 \sim 50$ Å for intergranular contacts in $YBa_2Cu_3O_\delta$.

It should be kept in mind, however, that a direct utilization of Eqs. (6) and (7) for clarifying the nature of a junction is hindered by the presence of strong fluctuation effects near $T = T_c$ (Refs. 19, 27, 34). A comparison of the experimental dependences $i_c(T)$ with theory accounting for fluctuations reveals[19] that most of the GBs are Josephson junctions of the SNS- (or SNINS-) type and may be approximated with Eq. (7), which predicts an exponential dependence of the SNS-junction critical current on the contact width d_N in "thick" junctions:

$$i_c(T) \propto \frac{\Delta_\infty^2(T)}{\xi_N(T)} \zeta(T) \exp\left[-\frac{d_N}{\xi_N(T)}\right]. \tag{10}$$

An analysis of the magnetic field dependences $i_c(B)$ of the critical current of individual intergrain contacts not only confirms their Josephson nature but also allows one to draw conclusions about the spatial distribution $j_c^{GB}(x,y)$ of the critical current density over the contact area. The analysis is based on the well-known expression for a narrow ($L < 4\lambda_J$) Josephson junction:[28]

$$i_c(\Phi/\Phi_0) = \left| \iint j_c^{GB}(x,y) \exp[2\pi i(\Phi/\Phi_0)(x/L)] \mathrm{d}x\mathrm{d}y \right|, \tag{11}$$

which relates to the case when the magnetic field projection (B_y) on the junction plane (plane xy) is directed along the y-axis; $\Phi = B_y L d$ is the magnetic flux passing through the contact, L is the contact dimension perpendicular to the magnetic field, $d \cong d_N + 2\lambda_L$ is an effective contact thickness and $\Phi_0 = hc/2e \cong 2 \cdot 10^{-7}$ Gs$\cdot$cm^2 is the magnetic flux quantum.

Applying Eq. (11) to the known function $i_c(B)$ it is in principle possible to deduce the distribution $j_c^{GB}(x,y)$, although this reverse problem is an incorrect one so it is common practice to limit oneself to "guessing" (proceeding from certain physical arguments) such a distribution $j_c^{GB}(x,y)$, which would provide an $i_c(B)$ close to the experimental one. The reference points here may be the known magnetic field dependences $i_c(B)$ for a rectangular contact (see Fig. 3) corresponding a) to a uniform distribution of the critical current density, $j_c^{GB}(x,y) = \text{const}$:

$$i_c(\Phi/\Phi_0)/i_c(0) = \left| \frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0} \right| \tag{12}$$

(the so-called "Fraunhofer diffraction dependence); b) to strongly nonuniform $1D$ U-like distribution $j_c^{GB}(x) \propto \mathrm{ch}[2\chi(x/L)]$ ($\chi \gg 1$) with increased critical current density at the contact edges, and c) to a randomly nonuniform $1D$ distribution $j_c^{GB}(x) = j_c^0 + j_c(x)$, where $j_c^0 = \text{const}$, and $j_c(x)$ is a random function with a zero average value $\overline{j_c(x)} = 0$, and a correlation radius r defined by the expression

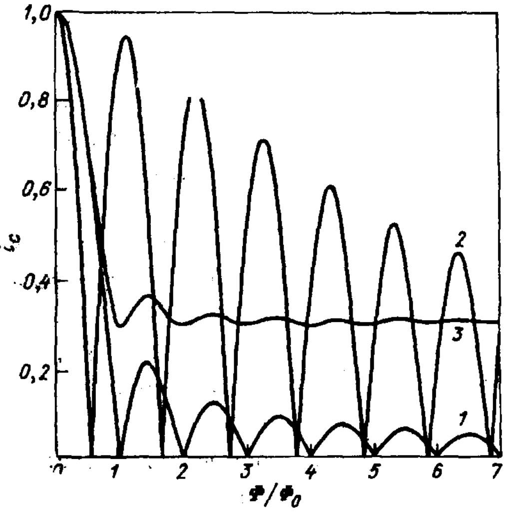

FIG. 3. Magnetic field dependences $i_c(B)$ of the critical current of a narrow Josephson junction for different distributions of the local critical current density $j_c^{GB}(x)$. *1*—uniform distribution [see Eq. (7)]. *2*—*U*-like distribution (see the text) with $\chi = 10$ [Ref. 28]. *3*—random distribution (see the text) with $2\pi r/L = 0.01$ and $\gamma^2 = 0.09$ [Ref. 96].

$$\overline{[j_c(x_1) - j_c^0][j_c(x_2) - j_c^0]} = \overline{[j_c - j_c^0]^2} \exp(-|x_1 - x_2|/r^{-1}),$$

and an effective variance $\gamma^2 = (2r/L)\overline{(j_c - j_c^0)^2}/(j_c^0)^2$. The following factors are characteristic of $i_c(B)$ functions: a) symmetric (in relation to the contact center) distributions $j_c^{GB}(x)$ give us $i_c(B) = 0$ at definite values of the magnetic field with the distance between the first zero minima (symmetric about $B = 0$) of the Fraunhofer dependence being twice as large as that between all the subsequent minima, while for a U-like distribution with $\chi \gg 1$ all these distances are equal; and b) for the random distribution $j_c^{GB}(x)$, i_c does not become zero for any value of B and the envelope of the dependence $i_c(B)$ may have a plateau.[2)]

Experiments show that although magnetic field dependences $i_c(B)$ of the critical current of separate contacts resemble those of Fraunhofer, they provide evidence of the inhomogeneous nature of the corresponding distributions $j_c^{GB}(x)$.[19] The wide intergrain Josephson junctions in $YBa_2Cu_3O_\delta$ are also inhomogeneous. An example of "guessing" the distribution $j_c^{GB}(x)$ in a Josephson junction corresponding to a large-angle intergrain tilt boundary in a polycrystalline $YBa_2Cu_3O_\delta$-film is given in Fig. 4.[38]

It has already been mentioned above that one of the basic parameters of GBs in $YBa_2Cu_3O_\delta$ defining the magnitude and the magnetic field dependence of their critical current is the angle ϑ of misorientation of contacting grains: as ϑ increases the superconducting properties of a contact worsen. Undoubtedly, one of the reasons for this phenomenon are the structural features of such a boundary. According to a dislocation model of an intergrain tilt boundary, it is a dislocation wall, that is a set of edge dislocations lying in the boundary plane and spaced at a distance from one another[39]

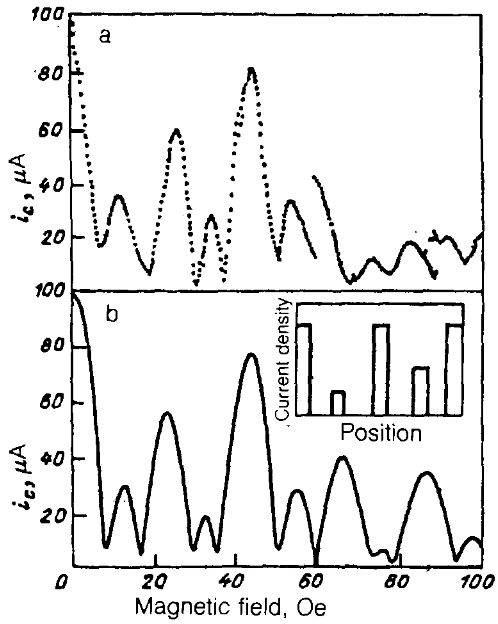

FIG. 4. Experimental (a) and model (b) magnetic field dependences $i_c(B)$ of the critical current of a narrow Josephson junction in a polycrystalline $YBa_2Cu_3O_\delta$-film at $T=4.2$ K [Ref. 38]. In the insert: model distribution $j_c^{GB}(x)$.

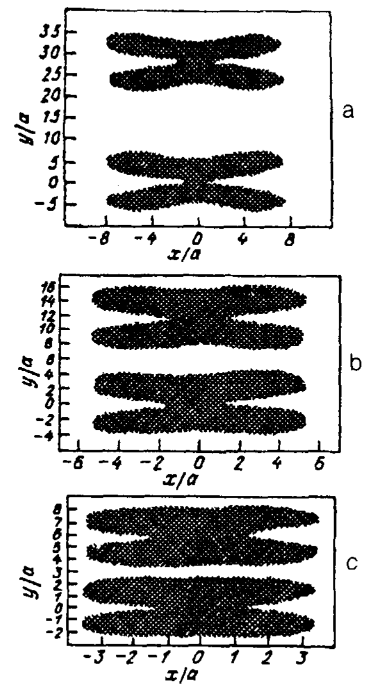

FIG. 5. Deformation field ε_{xx} (maximum deformation component) on a symmetrical intergrain tilt boundary ($x=0$) with an infinite dislocation wall [Ref. 14]. Within the darkened areas $\varepsilon_{xx} \geqslant 0.01$. Misorientation angle ϑ: a—2°, b—5°, c—10°.

$$D=\frac{a}{2\sin(\vartheta/2)} \tag{13}$$

Here a is the value of the Burgers vector which, for a boundary between grains with a common c-axis, practically coincides (because of a nearly tetragonal structure of $YBa_2Cu_3O_\delta$) with the lattice parameter in the basal plane. For small-angle boundaries between such grains $D \gg a$ and the lattices of neighboring grains are well adjoined everywhere except in the areas near dislocation nuclei; for large-angle boundaries these areas overlap, and the whole boundary becomes "bad."

Different reasons are possible for the deterioration of superconducting properties of a material due to the presence of dislocations on the tilt boundaries in question. One of them is the order parameter suppression in elastically deformed areas of the boundary.[14] Let ρ_m be the effective radius of the deformation area near a separate dislocation in the basal plane perpendicular to it (according to theory ρ_m is of the order of the lattice parameter in this plane[39]). Assuming the superconducting current across the boundary to flow only in its undeformed part we obtain $j_c^{GB}/j_c^G=(D-2\rho_m)/D$, where j_c^G is the critical current density in a grain. For small misorientation angles, when $D\cong a/\vartheta$, it brings us to

$$j_c^{GB}/j_c^G=1-\left(\frac{2\rho_m}{a}\right)\vartheta, \tag{14}$$

that is a linear (in ϑ) decrease in j_c^{GB}. The experimental data in Fig. 1 are in agreement with this dependence at $\vartheta \ll 1$ if we let $\rho_m \cong 2.9a$. Such a large value of ρ_m suggests that the superconducting properties of $YBa_2Cu_3O_\delta$ are very sensitive to the lattice deformation (this is indirectly confirmed by the fact that a deformation of $\sim 1\%$ is sufficient to prevent a tetra-ortho transition).

For further clarification of the model under consideration the authors of Ref. 14 calculated spatial contours of the constant magnitude (equal to 0.01) of one of the components of the relative deformation caused by a dislocation wall on a symmetric intergrain tilt boundary. Results of the calculation carried out within the framework of the isotropic theory of elasticity[39] are plotted in Fig. 5. It can be seen that with an increase in the angle ϑ the thickness of weakly deformed ($\varepsilon_{xx}<0.01$) near-to-boundary areas diminishes and at $\vartheta \cong 10°$ it becomes smaller than the unit cell dimension (in the ab-plane). This means that the structure required for the existence of superconductivity is destroyed along the complete interface. This "destroyed" near-to-boundary layer extends as deep into the contacting grains as $D/2\pi \cong a/2\pi\vartheta$ and is a normal metal (or insulator). It is in this manner that a Josephson junction of the SNS- (SNINS- or SIS) type originates on the boundary.

A somewhat different approach, which allows for the minimization of the tilt boundary energy by means of adjustment of its individual atoms has been undertaken in Ref. 40. It gives a possibility to estimate the thickness d_N of the transition layer on the boundary and by making use of Eq. (5) to find the critical current dependence on the misorientation angle. The result agrees qualitatively with the experiment.[16]

Nevertheless, the situation considered does not mean that any GB with a sufficiently large misorientation angle ϑ is necessarily a weak link. It has already been noticed above that in $YBa_2Cu_3O_\delta$ the [010] twist boundary with $\vartheta=90°$ exhibits no properties characteristic of weak links. Besides, from experiments of Ref. 25 it follows that no such properties are exhibited by the [001] twist boundary which corresponds to two contacting crystallites with a common c-axis, mutually rotated around the a- (or b-) axis by an angle $\vartheta=14°$. It means that the atomic structure of real GBs may (at least at certain misorientation angles of contacting grains) rearrange in such a way that it would not manifest properties of a weak link.[3)] The rearrangement of this kind becomes easier if the process of crystallites growth and intergrain boundaries formation goes in the presence of a liquid phase. That is why in the materials produced by the liquid phase technology the high angle intergrain boundaries often do not reveal the properties typical for the weak links.[102,103]

4. CALCULATIONS OF THE CRITICAL CURRENT OF HTSC CERAMICS (PERCOLATION MODEL)

4.1. Analytical methods

Calculation of the critical current density j_c of HTSC ceramics (regarded as an assembly of superconducting grains being connected by weak (Josephson) links) is a complex problem. When solving this problem in a general case one should take into account not only the spread of the coupling energy, ε_J, over Josephson contacts but also the correlation of the order parameter phases in different grains. The task may be significantly simplified if we neglect the latter: in this case currents in adjacent contacts may be regarded as mutually independent. Such a situation is realistic either at sufficiently high temperatures ($T\gg\varepsilon_J$), when temperature fluctuations of the order parameter are large, or in a sufficiently strong magnetic field $B\sim\Phi_0/a^2$ which results in strong "magnetic field" fluctuations of the order parameter. In this case the critical current calculation can be made on the basis of the percolation theory[42,43] or the effective medium theory.[44]

Any of these theories assumes the function $f(i_c)$ (normalized to unity) of the intergranular contact distribution in critical currents to be known. The form of this function may in principle be deduced on the basis of model concepts about the properties of HTSC ceramics intergranular contacts or experimentally. One of the schemes of an approximate calculation of the critical current density j_c of a ceramic has been suggested in Ref. 45 and consists in the following. The function

$$P(i)=\int_i^\infty f(i_c)\mathrm{d}i_c=1-\int_0^i f(i_c)\mathrm{d}i_c, \tag{15}$$

is introduced. It determines the portion of contacts with critical currents $i_c>i$. Let this portion be equal for $i=i^*$ to the so-called percolation limit P_c (i.e., $P(i^*)=P_c$). Then a set of contacts with $i_c\geqslant i^*$ forms an infinite cluster ensuring the superconducting current flow across the whole system. The critical current of such cluster, however, equals zero and in order to obtain a finite value of j_c it is also necessary to take into consideration contacts with $i<i^*$. We consider an infinite cluster consisting of contacts with critical currents $i_c>i^{**}$, where $i^{**}<i^*$. Its critical current density is finite and increases still more by an amount $\Delta j(i^{**})$, if contacts with critical currents within the limits of $i^{**}-\Delta i<i_c<i^{**}$ are added to the cluster. Then the function P increases by $\Delta P(i^{**})\equiv P(i^{**}-\Delta i)-P(i^{**})=f(i^{**})\Delta i$. If the cluster in question were a random net of identical (but broken with probability $1-P$) resistive bonds then, according to the effective medium theory, its conductivity would be a linear function of $P(i^{**})$ (Ref. 44). The basic (and dubious) supposition made in Ref. 45 comes down to an assertion that the contribution $\Delta j_c(i^{**})$ of the "added" contacts (with critical currents within $i^{**}-\Delta i<i_c<i^{**}$) to the critical current density of the whole cluster under consideration is proportional to $\Delta P(i^{**})$, and besides, to average critical current

$$\langle i_c\rangle=\frac{1}{P(i^{**})}\int_{i^{**}}^\infty i_c f(i_c)\mathrm{d}i_c$$

of the contacts of this cluster:

$$\Delta j_c(i^{**})\propto\langle i_c\rangle\Delta P(i^{**}). \tag{16}$$

Summing these contributions, we get the total critical current density of the ceramic

$$j_c=A\int_{P_c}^1\frac{\mathrm{d}P}{P}\int_0^P i(P)\mathrm{d}P. \tag{17}$$

where $i(P)$ is a function inverse to the function $P(i)$ [see Eq. (14)], A is a proportionality factor independent of the form of $f(i_c)$ which can be easily established by means of considering a cubic net whose bonds consist of identical contacts with critical current i_c. The critical current density in this sort of a system obviously equals $j_c=i_c/a^{D-1}$, where a is the spacing between contacts and D is the system dimensionality. The comparison of the result obtained with Eq. (17) brings us to $A=[a^{D-1}(1-P_c)]^{-1}$.

A somewhat different (and, to our mind, a more consistent) procedure of deriving a j_c value has been suggested in Ref. 46 and used for concrete calculations in Ref. 47. In essence it is as follows. Let us imagine that a cluster consisting of contacts with critical currents $i_c>i$ determines the critical current density j_c of the system (which means that for current density equal to j_c all the contacts outside this cluster are in a resistive state). It is clear that the magnitude of j_c is limited by the "worst" (i.e., with the smallest critical current $i_c=i$) contacts and, consequently, is directly proportional to i. Besides, the current flowing through the cluster under consideration is proportional to the density of percolation paths in this cluster. The latter density itself is proportional to the conductivity of an equivalent network of resistive bonds, where P is the fraction of unbroken couplings. This conductivity is, according to the percolation theory, directly proportional to $(P-P_c)^t$, where t is a critical conductivity factor depending on the form and dimensionality of the network. Thus,

$$j_c(P)\propto(P-P_c)^t i(P). \tag{18}$$

The actual critical current of the system is calculated by means of maximization of Eq. (18) relative to P and corresponds to some "critical" cluster with $P=P_{cr}$; at $P_c<P<P_{cr}$ the cluster, although being infinite, is very "sparse," while for $P>P_{cr}$ the cluster is crowded with "bad" contacts. (The unknown constant factor in Eq. (18) is found using the same procedure as was used above.)

In the calculation procedures under consideration the influence of any external parameter, ζ, (e.g, of magnetic field or pressure) on the magnitude of the critical current can be allowed for if we know the dependence $i_c=i_c(i_{c\zeta},\zeta)$ which interconnects the "old" critical current i_c of an individual contact with the "new" critical current $i_{c\zeta}$. A "new" distribution function, $f_\zeta(i_{c\zeta})$ of the contacts is calculated using the relationship

$$f_\zeta(i_{c\zeta})=f[i_c(i_{c\zeta},\zeta)]\left|\frac{\partial i_{c\zeta}}{\partial i_c}\right|^{-1}. \tag{19}$$

After that we may again utilize one of the previously mentioned procedures of the critical current calculation.

Thus, the problem comes down to the determination of the distribution function $f(i_c)$. To this end several methods have been suggested based on the results of different experiments: dependence of the contact critical current on the misorientation angle between contacting grains,[22] magnetic flux creep in a ceramic sample,[23] a current-voltage characteristic of HTSC ceramics in the resistive state.[47]

The first of them assumed that contacts are uniformly spread over disorientation angles ϑ and have equal areas. Then, if we let $\zeta\equiv\vartheta$ in Eq. (19), we shall easily obtain $f(i_c)\propto|\partial i_c/\partial\vartheta|^{-1}$. In Ref. 23 the angular dependence, $i_c(\vartheta)$, is described by relation (1). The form of this dependence seems to be rather unnatural and its applicability at $\vartheta>45°$ looks doubtful, especially if we recollect that experiments[18] underlying the presentation of $i_c(\vartheta)$ in the form (1)–(3) are limited in principle to angles $\vartheta<45°$; moreover, in other studies different forms of the $i_c(\vartheta)$ dependence are suggested [see, e.g. relations (2) and (14)]. Hence a simpler and more general two-parameter power-law dependence $i_c(\vartheta)\propto(\vartheta+\vartheta_0)^{-(n-1)}$, where ϑ_0 and n are fitting parameters, seems to be more appropriate for this approach. The distribution function in this case also turns out to be power-law: $f(i_c)\propto i_c^n$. Naturally, the last relationship may be valid only within some upper-bound-limited range of critical currents $i_c<i_0$ (i_0 is the maximum critical current of a contact), and the normalized distribution function $f(i_c)$ would be defined by the expression

$$f(i_c)=\begin{cases}\dfrac{n+1}{i_0}\left(\dfrac{i_c}{i_0}\right)^n, & i_c<i_0\\ 0, & i_c>i_0\end{cases}. \tag{20}$$

Such a distribution function has been put forward (but on quite different basis) in Ref. 47, where it was used for explanation of experimentally observed power-law current-voltage characteristics of HTSC ceramics (see below). Note that distribution (20) may be obtained for Josephson junctions of the SNS-type in which the critical current is described by Eq. (10) if we assume that their distribution $f_d(d_N)$ over the thickness d_N of the normal-metal intergranular layer is exponential: $f_d(d_N)\propto\exp(-d_N/\langle d\rangle)$.[47]

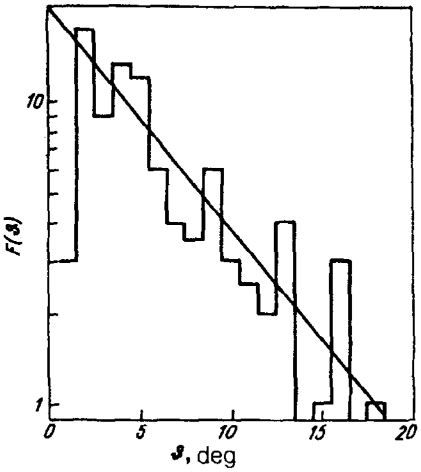

FIG. 6. The distribution $F(\vartheta)$ of the intergrain boundaries over misorientation angles ϑ in the $YBa_2Cu_3O_\delta$-ceramic [Ref. 24]. Straight line: $F(\vartheta)\propto\exp(-\vartheta/\langle\vartheta\rangle)$, $\langle\vartheta\rangle=6°$.

The magnitude and the temperature dependence of the parameter n near T_c here are governed by the relationship:

$$n=\frac{\xi_N(T)}{\langle d\rangle}-1, \tag{21}$$

where $\langle d\rangle$ is the average thickness of the intergranular layers.

A different procedure of determining $f(i_c)$ may be recommended for ceramics with a known $F(\vartheta)$ function of GB distribution over the misorientation angles ϑ. Then $f(i_c)=F[\vartheta(i_c)]|\partial i_c/\partial\vartheta|^{-1}$, where $\vartheta(i_c)$ is a function inverse to the (experimentally) known function $i_c(\vartheta)$. The distribution $F(\vartheta)$ may, in principle, be found from electron microscopy analysis of a large number of GBs. The result of such analysis for the melt-produced $YBa_2Cu_3O_\delta$-ceramics is given in Fig. 6.[18] It can be easily seen that this distribution is close to exponential:

$$F(\vartheta)\propto\exp(-\vartheta/\langle\vartheta\rangle). \tag{22}$$

In this case the average misorientation angle is $\langle\vartheta\rangle\cong6°$. But this value is typical only for the ceramics under analysis. Making use of the distribution found and the dependence $i_c(\vartheta)$ in the form (3), we arrive again at Eq. (20) where

$$n=\frac{\vartheta_0}{\langle\vartheta\rangle}-1. \tag{23}$$

Comparison of (23) with (21) allows us to determine the relation between the average thickness $\langle d\rangle$ of a SNS-junction and the average angle $\langle\vartheta\rangle$ of grain misorientation:

$$\langle d\rangle=\frac{\xi_N(T)}{\vartheta_0(T)}\langle\vartheta\rangle. \tag{24}$$

From this it follows that the temperature dependence of parameter ϑ_0, defining the angular dependence of the contact critical current [see Eq. (3)], should coincide with the temperature dependence $\xi_N(T)$.

One more method of determining $f(i_c)$ is based on the analysis of the magnetic flux creep in HTSC ceramics under conditions when it is due to thermal activation motion of separate vortices through potential barriers in Josephson contacts.[48] Processing of experimental results allows one to deduce a distribution function of contacts over heigts ε_J of such barriers. However, since $\varepsilon_J \propto i_c$, the same function would define the contact distribution over their critical currents. Although the function $f(i_c)$ obtained in Ref. 48 describes only the ceramic $(Y_{0.25}Ba_{0.75})CuO$ with $T_c \cong 80$ K utilized in this work that is "bad" (from the point of view of its current-carrying ability), it illustrates one important point, namely: a shift of the maximum of this function towards smaller critical currents of contacts in a weak (4 G) magnetic field. It is the allowance for this sort of magnetic-field-induced "deformation" of the distribution function that provides an explanation of the magnetic field dependence of the HTSC ceramics critical current (see below).

4.2. Numerical methods

For calcualtion of critical curents of HTSC ceramics various numerical methods have been used.[23,49,50] One of these is the critical surface method[23] based on the following concept. Let us imagine a high-temperature superconducting ceramic sample in the form of a collection of closely packed cubic superconducting grains (with equal density of the intragrain critical current). Their contacting faces are weak links with different (in value) critical currents i_{cl} (l is a link number), characterized by the distribution function $f(i_c)$. For a preset arrangement of current electrodes let us select an arbitrary (consisting of faces of cubic grains) surface γ, dividing the sample under consideration into two separate parts. Each of the parts contains one of these electrodes. Among these surfaces let us find the one for which the sum (Σi_{cl}) of critical currents of all $(l \in \gamma)$ the intergrain weak links would be maximal. This "critical" surface γ_c is exactly what defines the critical current of the sample equal to $I_{cr} = \Sigma i_{cl}$ $(l \in \gamma_c)$. The critical surface that is so determined is essentially a single large Josephson junction, neither flat nor spatially uniform.

Although in Ref. 23 numerical calculations were performed using this approach only for a $2D$ grain-aligned system with the basal planes of all the grains being parallel to each other (as is the case, e.g., in a grain-aligned polycrystalline $YBa_2Cu_3O_\delta$ film), the results quoted there make it possible to understand qualitatively the effect produced by the spread of critical current of intergrain contacts on the ceramics critical current. This spread is related to a different grain orientation determined by the angle ϕ between their a axis and the average direction of the current flow and described by distribuiton function $f_\phi(\phi) \propto \exp(-\phi^2/\phi_0^2)$. The angle of mutual misorientation of the grains is $\vartheta = |\phi - \phi'|$ (the angles ϕ and ϕ' pertain to two neighboring grains) and applying Eq. (2) we can easily find the distribution function $f(i_c)$ for any value of ϕ_0. The results of numerical calculations consist in the following. 1) The average critical current density j_{cr} of a nontextured material ($\phi_0 = \infty$) turns out to be by a factor of ~30 lower than in the case of an ideal texture ($\phi_0 = 0$). 2) The density j_{cr} of the ceramics critical current is determined by the critical current averaged over the misorientation angles ϑ $\langle i_c \rangle_\vartheta = \int i_c(\vartheta) f_\vartheta(\vartheta) d\vartheta$ of the intergrain contacts only in a strongly aligned system ($\phi_0 \leqslant 5°$): in this case we may assume $j_{cr} = \langle i_c \rangle_\vartheta / a^2$, where a is a grain size; otherwise the j_{cr} value turns out to be smaller by a factor of 2 or 3 than what might be expected. The latter conclusion is of crucial importance: it proves that a frequently used method of calculating the ceramics critical current by means of averaging the critical currents of its individual intergrain contacts (e.g., Ref. 51) is not always justified for it does not allow for the percolation nature of the phenomenon.

A basically different, variational, method of the numerical calculation of the critical current (as well as of a current-voltage characteristic of HTSC ceramics, see below) has been developed in Ref. 49. This method also makes use of the model of cubic superconducting grains with weak links which are a $3D$ network of Josephson junctions with different critical currents i_{cl}. This network is desribed by the nonlinear system of Kirchhoff's equations for currents i_l and voltages v_l, relating to separate Josephson junctions with individual current-voltage characteristics $v_l = v_l(i_{cl}, i_l)$. A solution of this system is obtained by means of iterations with the application of the variatonal principle which is a generalization of the principle of the dissipated power minimum[52] relevant only for linear systems. It has been shown that the actual distribution of currents and voltages in the system in question corresponds to the minimum of the functional

$$W = \sum_l \int_0^{i_l} v_l(i_{cl}, i_l') \mathrm{d}i_l', \qquad (25)$$

in which the summation is taken over all the system elements and which for a linear network ($v_l \propto i_l$) is equal to the power being dissipated in the system. Numerical calculations were made for the distribution function (20) with $n = 0, 2$.

The results obtained in the aforementioned studies are listed in Table I.

TABLE I. Results of numerical calculation of the critical current and CVC exponent for random lattices of elements with critical currents corresponding to a power-law distribution (20).

	Square lattice			Cubic lattice		
	$n=0$	1	2	0	1	2
I_{cr}/i_0 (per link*)	0,12 [47] 0,11 [50] 0,06 [23]	0,26 [47]	0,36 [47]	0,14 [47] 0,36 [49]	0,28 [47]	0,38 [47] 0,66 [49]
μ ($V \propto I^\mu$)	2,1 [47] 2 [50]	3,2 [47]	4,3 [47]	1,8 [47] 2,6 [49]	2,6 [47]	3,4 [47] 2,9 [49]

*) i_0 is the maximum critical current of a link [see (20)].

5. CALCULATION OF THE HTSC CERAMICS CRITICAL CURRENT (JOSEPHSON NETWORK MODEL)

Under conditions when the correlation of the order parameter phases in different granules of HTSC-ceramics (see above) cannot be neglected, its critical current and current-voltage characteristics become collective properties of a system which in this connection is called a "Josephson medium." The latter is usually analyzed using the Josephson network (or lattice) model. Randomly located sites of this network correspond to ceramics granules,[4)] and its couplings correlate with Josephson contacts with their inherent properties (critical current, current-voltage characteristic, etc.). The study of the properties of this sort of a network relies on the use of the Josephson equation

$$\frac{d}{dt}(\phi_k-\phi_l)=\frac{2e}{\hbar}(V_k-V_l) \tag{26}$$

(where ϕ_k is an order parameter phase in the kth granule and V_k is its potential), the equation defining the current i_{kl} of the weak link between the kth and the lth granules

$$i_{kl}=i^c_{kl}\sin(\phi_k-\phi_l)+(V_k-V_l)/R_{kl}, \tag{27}$$

(i^c_{kl} is the coupling critical current and R_{kl} is its resistance in the normal state), and of the Kirchhoff's equation

$$\sum_l i_{kl}=0, \tag{28}$$

provided with relevant boundary conditions. When there is no dissipation in the system (i.e., at currents below the critical one) $V_k=V_l$.

The calculation of the system state (for a given applied current) comes down to the determination of all the currents i_{kl}, the phases ϕ_{kl} and the potentials V_{kl}, for which this or some other numerical method is required.[53,54] As to the transport critical current, it can be calculated in different ways. Thus, in Ref. 53 the total system current as a function of the order parameter phase difference on the system boundaries (that is, electrodes) was determined and the maximum value of this current was taken as critical. With this in mind the contribution to the system free energy

$$\sum_{k,l}(\varepsilon_J)_{kl}[1-\cos(\phi_k-\phi_l)],\quad (\varepsilon_J)_{kl}=\hbar i^c_{kl}/2e, \tag{29}$$

resulting from passing a current[5)] through this system was calculated and a configuration (that is a set of the order parameter phases) was sought which corresponds to the absolute minimum of this energy. For a moderate spread of contact critical currents i^c_{kl} the total critical current of the sample made up of a large number of granules has turned out to coincide with the current-carrying ability averaged over its cross-sections. This current-carrying ability is regarded as the sum of critical currents of couplings penetrating the sample cross-section perpendicular to the current.

A numerical calculation of the critical current (and also of the current-voltage characteristic, see below) of a $2D$ system exhibiting an exponential dependence (3) of the critical current of individual Josephson junctions on the misorientation angle ϑ and a "cut-off" Gaussian distribution of these angles ($F(\vartheta<20°)\propto\exp(-\vartheta^2/2\langle\vartheta^2\rangle)$, $F(\vartheta>20°)=0$), has been carried out in Ref. 24. The critical current of this kind of system rapidly diminishes with the rise of the parameter $\langle\vartheta^2\rangle$ (that is, as the $F(\vartheta)$ distribution broadens): for $\langle\vartheta^2\rangle^{1/2}=5°$, 10° and ∞ the values obtained were $I_{cr}/I^0_{cr}\cong 0.2$, 0.1 and 0.06, respectively, where I^0_{cr} is the critical current at $\langle\vartheta^2\rangle=0$ (a single crystal).

In Ref. [54] the numerical solution of a system of nonlinear differential equations (26) was found for a $2D$ closely packed granular structure with regular hexagonal grains, their c-axis being perpendicular to the structure plane. The orientation of these hexagons in the basal plane has been assumed to be random, and the links connecting them (with similar normal resistances R_{kl}) were classified into two types according to the critical current magnitude: "good" links with a high critical current, $i^c_{kl}=i_1$, and "bad" links with critical current $i^c_{kl}=i_2\ll i_1$. The intergrain link was considered to be "good" if the intergrain misorientation angle was $\vartheta\leqslant\vartheta_1$ (in accordance with the results of Refs. 16–20, $\vartheta_1\approx 10°$), otherwise, the link was assumed to be "bad"; the magnitude of the ratio i_1/i_2 was taken equal to 25. Depending on the ratio of the portion of the "good" links $p=\vartheta_1/45°$ to the percolation threshold $p_c\approx 0.35$ two possible modes should be distinguished: a) $p<p_c$ when the system critical current is defined only by the "bad" links: $I_{cr}\propto i_2$; and b) $p_c<p<1$ when the critical current depends on the links of both types. In the latter case the numerical calculation results may be presented in the form

$$(i_{rh}-i_2)\propto(p-p_c)^v, \tag{30}$$

where $i_{cr}=I_{cr}/N$, N is a number of weak links in the system cross-section perpendicular to the current, $v\cong 1.23$.

This kind of a scaling dependence (but with a different index v) also describes the current of a more complex $3D$ system analyzed in Ref. 56. The critical current numerical calculation of a random $3D$ cubic lattice, consisting of identical Josephson links, gives $I_{cr}\propto(p-p_c)^v$, where $v\cong 1.7$ and $p_c\cong 0.37$.[6)]

6. EFFECT OF EXTERNAL FACTORS ON THE CRITICAL CURRENT OF HTSC CERAMICS

6.1. Magnetic field dependence of the critical current

As has been previously mentioned, allowance for the magnetic field effect on the current-carrying ability of HTSC ceramics within the framework of analytical calculations of their critical current requires knowledge of the distribution function $f_B(i_{cB})$ of intergranular contacts over their critical currents i_{cB} "reorganized" by the magnetic field. The latter may be calculated using relation (19) if the function $i_{cB}(i_c,B)$, describing variations in the critical current of a single contact in the magnetic field, is known. This function depends on four factors, namely, 1) the size, 2) the orientation, 3) the shape and 4) the spatial distribution of local density $j^{GB}_c(x,y)$ of the critical current over the contact area. For calculations the shape and distribution $j^{GB}_c(x,y)$ are usually taken identical for all the

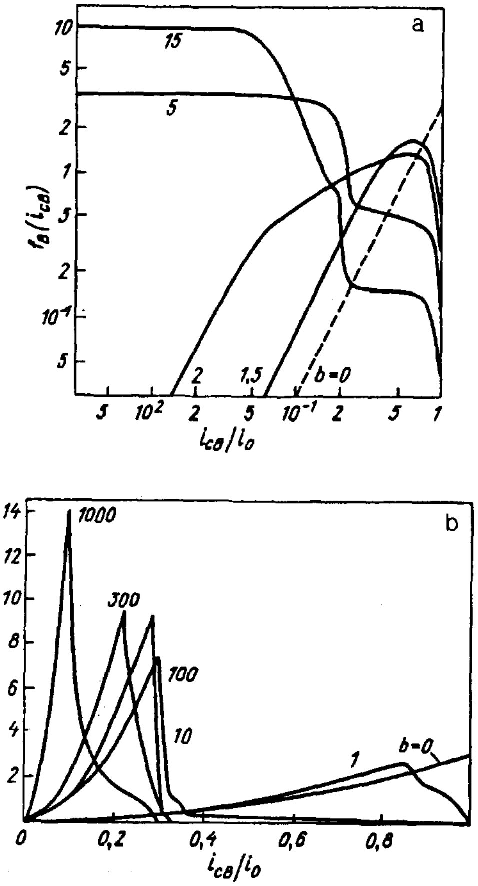

FIG. 7. Evolution of the function $f_B(i_{cB})$ of contact distribution over critical currents i_{cB} under the action of a magnetic field. a—Uniform distribution of the critical current density [Ref. 47]. b—Randomly nonuniform $1D$ distribution [Ref. 96]; correlation radius $r=0.01(L/2\pi)$, variance $\gamma^2=0.09$. Initial $(b=0)$ distribution function $f(i_c)$ is described by relation (20) with $n=2$.

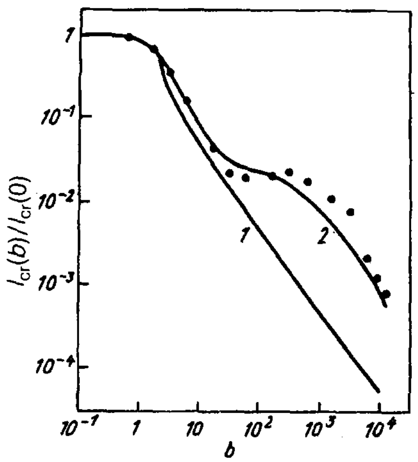

FIG. 8. Magnetic field dependences of the ceramics critical current. I_{cr} 1—Uniform distribution of the local critical current density;[47] dots: experimental data for the "bad" $YBa_2Cu_3O_\delta$-ceramic ($T_c \cong 80$ K, $T=77$ K). 2—Randomly nonuniform $1D\ j_c^{GB}(x)$- distribution;[96] correlation radius $2\pi r/L=1/300$, variance $\gamma^2=4.10^{-4}$, initial $(b=0)$ distribution function $f(i_c)$ is described by relation (20) with $n=2$. Dots: experimental data for the $YBa_2Cu_3O_\delta$-ceramic ($T_c \cong 90$ K, $T=77$ K) [Ref. 59].

contacts. As far as the contact sizes and orientations are concerned, they are averaged assuming for simplicity that corresponding distributions are uniform (within certain limits of parametric variations).[7)]

In Refs. 47, 96 this problem has been solved for contacts of a square form with the initial distribution function (20) for a uniform distribution of the critical current density, $j_c^{GB}(x,y)=\text{const}$ [see Eq. (12)] and for a randomly nonuniform one-dimensional distribution $j_c^{GB}(x)$, discussed earlier. The results demonstrating the evolution of the distribution function $f_B(i_{cB})$ in the magnetic field are plotted in Figs. 7a, b. The magnetic field magnitude is determined by the dimensionless parameter $b=2\pi\lambda_G aB/\Phi_0$, where a is the average contact size; the initial $(b=0)$ power-law distribution function $f(i_c)$ is described by Eq. (20) with $n=2$. In both cases the magnetic field shifts the distribution function $f_B(i_{cB})$ towards smaller critical currents, although in a strong magnetic field $(b\gg 1)$ the form of this function is quite different: for a uniform j_c^{GB}-distribution the function $f_B(i_{cB})$ varies with the magnetic field increase gradually and at small critical currents $f_B(i_{cB})$ = const, while for a randomly nonuniform j_c^{GB}-distribution there is a magnetic field region (in Fig. 7a these are fields $3\leqslant b\leqslant 300$), where the variation of this function is negligible (as compared to fields below and above these limits) and at small critical currents $f_B(i_{cB}) \propto i_{cB}^2$. Such differences result in different behavior of the magnetic field dependences $I_{cr}(b)$ of the ceramics critical current in both situations under consideration: in the first case $I_{cr}(b\gg 1)\propto 1/b$, and in the second case the function $I_{cr}(b)$ has a plateau (see Fig. 8a, b).[8)]

Experiments[59,60] actually show that the magnetic field dependences $I_{cr}(b)$ for HTSC ceramics of different composition have a plateau in the region of sufficiently strong magnetic fields.[9)] Is this an argument for spatial nonuniformity of Josephson contacts of HTSC ceramics causing a random distribution of the local density of the critical current over the contact area? In Refs. 59, 60 a different interpretation is suggested: every (or nearly every) Josephson contact is spatially nonuniform and contains a region of a "strong" link. It is these regions that provide the current transfer in strong magnetic fields lessening the local density of the critical current in the rest ("weak") sections of each of the contacts.

The evidence in favor of this interpretation, in the authors' opinion,[60] is provided by the investigation results for magnetic field dependences $I_{cr}(b)$ in grain-aligned $YBa_2Cu_3O_\delta$-ceramics which are aggregates of crystallites with nearly parallel ab-planes (misalignment of their c-axes is $\sim\pm 10°$). In these experiments the transport current was passed along the ab-plane, and the magnetic field perpendicular to the current was directed either along or across this plane. In both cases a plateau in $I_{cr}(b)$ could be observed; however, the field B^*, corresponding to its high-field boundary on the side of strong fields is much stronger in the first case: $B^*(\parallel ab)\cong 30$ T and $B^*(\parallel c)\cong 7$ T at $T=76$ K. Assuming these values of the field to be close to the values $B_{c2}(\parallel ab)$ and $B_{c2}(\parallel c)$ of the upper critical field for $YBa_2Cu_3O_\delta$, the authors of Ref. 60 come to the conclusion that the critical current of a textured material in a strong magnetic field is limited by the material properties close to those of a single crystal rather than to those of weak links. In this connection two observations are appropriate here. Firstly, the known values of $B_{c2}(\parallel ab) \cong 130$ T and $B_{c2}(\parallel c) \cong 25$ T (for $T=76$ K)[62] are noticeably higher than the measured values of B^*. Secondly, when making an estimate of the magnetic field within crystallites and intercrystallite "layers" (which in this case have the form of comparatively thin lamellas parallel to the ab-plane) it is necessary to account for demagnetization factors which are different for different field directions. Then the difference of the measured boundary fields, $B^*(\parallel c)$ and $B^*(\parallel ab)$ may be due to the difference of these factors. Thus, the interpretation suggested in Ref. 60 is not well-grounded.

Both hypotheses considered above (randomly nonuniform contacts and contacts with a "strong" coupling) may be made consistent if we assume that in non-textured ceramics "strong" intergrain links are absent and its behavior in the region of the plateau of I_{cr} as a function of the magnetic field is defined by randomly nonuniform weak links while in textured ceramics "strong" links appear owing to peculiarities of its structure. One of the possible reasons for this phenomenon consists in favorable structural conditions for the formation of the above-mentioned intercrystallite [001] twist boundaries (a mutual rotation of contacting crystallites with respect to each other around the common c-axis) with the misorientation angles near $\vartheta = 14°$ (and also, possibly, with other favorable misorientation angles), exhibiting the properties of a "strong" link.[25] A portion of such boundaries may be sufficient ($\geqslant 15\%$) for the formation of continuous current paths in strong magnetic fields, when all the remaining (weak) links turn out to be "destroyed."[10)] These considerations might explain the results obtained in Ref. 60.

The analysis of experimental data suggests that the plateau on the $I_{cr}(b)$ dependences is observed only in ceramics with a sufficiently high critical current density and is absent in the case with low current densities. The meaning of this correlation is discussed in the Section dealing with ceramics current-voltage characteristics.

Experiments show that a magnetic field not only reduces the critical current of HTSC ceramics but also promotes the appearance of its anisotropy (even in nontextured ceramics): the critical current densities along $j_{cr}(I \parallel B)$ and across $j_{cr}(I \perp B)$ the magnetic field differ substantially (by a factor of 2–3) from each other.[61–64] This effect is due to a selective action of the magnetic field on different contacts: an effective decrease of the critical current occurs only for contacts whose planes form a small angle with the magnetic field direction; the contacts whose planes are almost perpendicular to this direction preserve their current-carrying ability. Hence current trajectories in HTSC ceramics in a strong magnetic field for a current I nearing the critical one are of completely different forms for various B and I orientations: when $I \parallel B$ almost all the sections of the trajectories are directed roughly along the

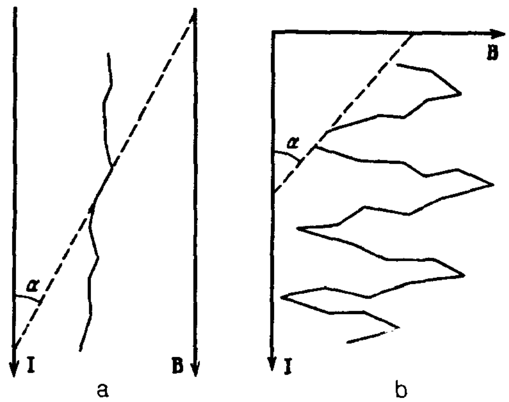

FIG. 9. Current paths in an HTSC-ceramic in a strong magnetic field for the current nearing the critical one. The magnetic field is parallel (a) and perpendicular (b) to the average current I direction. Sinuosity $\langle\alpha\rangle$ of the current path is characterized by the average values of path declination angle α off the average current direction.

current, while for $I \perp B$ they are perpendicular to the current (see Fig. 9). By virtue of the difference of these trajectories, the averaging defining the j_{cr} values gives different results.[65]

Apart from the analytical calculations cited above of magnetic field dependences $I_{cr}(B)$ of the ceramics critical current within the framework of the percolation model calculations of these dependences are known based on the use of the Josephson lattice model.[56,66–68] Analytical results obtained using this model are related only to $2D$ systems made up of identical Josephson contacts. The influence of the spread of contact properties in a $2D$ system as well as a $3D$ system (composed of identical contacts) has been investigated only by using numerical methods.

Thus, in Ref. 67 a $2D$ square lattice has been analyzed with the links formed by Josephson contacts with critical currents i^c_{kl}, dependent exponentially on the magnetic field:

$$i^c_{kl}(B)/i^c(0) = \begin{cases} 1 & , \ B < B_{kl} \\ [i_c(\infty)/i_c(0)]^{1-B_{kl}/B} & , \ B > B_{kl} \end{cases}, \tag{31}$$

where $i_c(0)$ and $i_c(\infty)$ are the critical currents that are the same for every contact in small ($B < B_{kl}$) and large ($B \gg B_{kl}$) fields, B_{kl} is an individual "critical" field for each contact.[11)] Calculations of $I_{cr}(B)$ (based on the minimization of the system energy (29)) have been made for two types of contact distribution $f(B_{kl})$ over the "critical" fields B_{kl}: a uniform distribution ($f(B_{kl}) = $const within a certain range $1 < B_{kl}/B_* < b_*$, outside of which $f(B_{kl}) = 0$; the parameter satisfies $b_* \gg 1$), and a power-law distribution ($f(B_{kl}) \propto B_{kl}^{-\beta}$ with the exception of a certain region of small fields $0 < B_{kl} < B_*$, where $f(B_{kl}) = 0$; the index is $\beta = 1.8$–2.3). In all the cases analyzed it appeared that in the strong field region ($B \gg B_*$) the critical current of the system decreases exponentially with the magnetic field increase:

$$\ln[I_{cr}(B)] \propto \text{const} + \frac{1}{B^a}. \tag{32}$$

The index a depends on the distribution of the contact parameters and in the cases analyzed varies within 0.8–1.0.

The exponential dependences $I_{cr}(B)$ of the form (32) have in fact been observed for the $YBa_2Cu_3O_\delta$ ceramics;[69] the experimental value of the parameter $a \cong 0.5$, however, was noticeably different from the calculated one. This difference is likely to result from the $3D$ nature of the real ceramics.

The numerical analysis of the magnetic field dependence of the critical current in a $3D$ lattice of Josephson contacts is limited by the available calculating facilities and hence has been carried out only for small-size lattices. Thus, in Ref. 56 a $(7\times7\times7)$ $3D$ lattice of superconducting grains interconnected with weak links with identical critical currents was investigated.[12)] The disorder inherent in actual HTSC ceramics was imitated in this system by random shifts of constituent granules. The fundamental property of such a system is the inherent frustration in the magnetic field, that is the existence of a multitude of metastable states corresponding to local energy minima and differing in the magnetic flux distribution $\Phi_\alpha = (\text{an integer} + \delta_\alpha)\Phi_0$ $(0 < \delta_\alpha < 1)$ over various lattice cells (α is the cell number). The authors of Ref. 56 provide qualitative considerations in favor of the argument that in a strong magnetic field the parameters δ_α are uniformly distributed between 0 and 1 and this distribution remains almost unchanged as the magnetic field increases. The field of transition to this situation is estimated to be $B_\delta \sim \Phi_0/\Delta S$, where ΔS is the r.m.s. fluctuation of the area of the cells. As the result a plateau is expected to appear on the magnetic field dependence $I_{cr}(B)$.

Unfortunately, it is difficult to check experimentally the correctness of the model suggested. The problem is that the underlying element of the model under investigation is disorder and frustration (rather than weak links) which, in the authors' opinion, should cause the appearance of a plateau on the $I_{cr}(B)$ dependence also for a (sufficiently "holey") lattice of "strong" links. Nevertheless, further development of this course look promising.

6.2. Temperature dependence of the critical current

The temperature dependence of the critical current, $I_{cr}(T)$, of HTSC ceramics is determined by two factors, namely, 1) by temperature dependence $i_c(T)$ of the critical current of individual intergrain Josephson contacts of the ceramics and 2) by the increase in the number of effective (i.e. participating in the near-critical current transfer) contacts as the temperature decreases. As has been previously emphasized, the majority of intergranular contacts are of the SNS (or of the SNINS) type, hence the temperature dependence of their critical current near T_c obeys Eq. (5) with $m=2$. As far as the other of the two factors in question is concerned, its role can be assessed with the help of the analytical method of the ceramics critical current calculation discussed in the previous Section. As has been demonstrated in Ref. 47, for a distribution function $f(i_c)$ of the form (20) and for the temperature dependence $i_c(T) \propto (1-T/T_c)^m \exp(-d/\xi_N)$ the magnitude of I_{cr} can be deduced from the expression:

$$I_{cr}(T) \propto K(T)\left(1-\frac{T}{T_c}\right)^m, \tag{33}$$

$$K(T) = [t(n+1)]^t [t(n+1)+1]^{-[1/(n+1)+t]} \times (1-P_c)^{1/(n+1)},$$

where $t \cong 1.5$ and $P_c = 0.25$ are percolation parameters for a $3D$ medium, $n+1 = \xi_N(T)/\langle d\rangle$ [see Eq. (21)]. For an intergrain layer made of "clean" material $n(T)+1 = [\xi_N(T_c)/\langle d\rangle](T_c/T)$. Application of (33) reveals that for $1-T/T_c \sim 0.1$ the temperature dependence of the coefficient $K(T)$, related to the distribution of contacts over the critical currents, is negligible and practically does not alter the power-law temperature dependence $I_{cr}(T) \propto (1-T/T_c)^m$, typical for individual intergrain contacts. It provides the grounds for the frequently used method of determination of the nature of the ceramic contacts from the asymptotic behavior of its critical current near T_c.[13)]

At temperatures noticeably different from T_c the temperature dependence of the distribution function, $f(i_c)$ might have turned out to be substantial. However, $n \to \infty$ as $T \to 0$, i.e. $K(T) \to 1$; hence such dependence becomes irrelevant and the temperature behavior of $I_{cr}(T)$ is again dependent on the properties of individual contacts.

Thus, almost within the whole temperature range of $T < T_c$ the temperature dependence of the ceramics critical current is close to that of the critical current of its separate intergrain contacts.

6.3. Critical current dependence on pressure (Uniform and axial compression)

6.3.1. Uniform compression. Numerous experiments prove that the critical current of HTSC ceramics increases under hydrostatic pressure.[70,77] The relative variation of the critical current at $P \sim 10$ kbar goes as high as $\sim 100\%$ and depends on its magnitude at zero pressure: for "bad" ceramics (with low critical current) relative variations are, as a rule, greater than for "good" ones. For interpretation of these results it is essential that the conclusion made in the previous Section (concerning the temperature dependence of I_{cr}) is equally valid for the dependence of the critical current of a ceramic on pressure: the latter is determined mainly by the variation of the critical current of individual intergrain contacts.[14)]

Thus, the mechanism of pressure action on the properties of a single Josephson SNS contact should be analyzed.[78] Local density $j_c^{GB}(x,y)$ of the critical current in such a contact is defined by the relationship [see Eq. (9)]

$$j_c^{GB}(x,y) \propto \exp\left[-\frac{d(x,y)}{\xi_N}\right], \tag{34}$$

where $d(x,y)$ is the local thickness of a normal layer. For analyzing the "responses" of an intergranular contact to external pressure it is convenient to use a very simple model: two spherical granules of radius R compressed by an external force F. In this case the contact radius r and the local pressure, $p(x,y)$, in its plane obey the relations[79]

$$r=\alpha[(F+F_0)R]^{1/3},\quad p(x,y)=\frac{3(F+F_0)}{2\pi r^2}\left[1-\frac{x^2}{r^2}-\frac{y^2}{r^2}\right]^{1/2}, \tag{35}$$

where $\alpha=[3(1-\nu^2)/8E]^{1/3}$, E and ν are Young's modulus and Poisson ratio of the granule material, respectively. The "internal" force F_0 takes into account the fact that even when there is no external force F the area of their contact is characterized by the initial radius $r_0=\alpha(F_0R)^{1/3}$ (e.g., owing to thermal tensions originating in the course of the ceramic synthesis). The "external," F, and the "internal," F_0, forces are correlated with "external" (P) and "internal" (P_0) pressure by simple relationships: $P\sim F/R^2$, $P_0\sim F_0/R^2\alpha(r_0/R)^3$. Let $r_0/R\sim 0.1$ and $E\sim 10^3$ kbar, then $P_0\sim 1$ kbar$\ll E$, that is, the initial deformation is of the elastic type. From Eq. (35) it follows that the contact radius r_P increases with pressure according to:

$$r_P=r_0\left(1+\frac{P}{P_0}\right)^{1/3}. \tag{36}$$

A nonuniform pressure distribution over the contact area [see (35)] leads to a change of its thickness $d(x,y)=d_0[1-p(x,y)/E]$ and, in accordance with Eq. (10), to a nonuniform distribution of the local current density

$$j_c^{GB}(x,y)\propto\exp\left[\frac{A}{r_0}(r_P^2-x^2-y^2)^{1/2}\right],\quad r_P=r_0(1+P/P_0)^{1/3}, \tag{37}$$

where $A=(4/\pi)(r_0/R)(d_0/\xi_N)/(1-\nu^2)$, d_0 is the normal layer thickness at the contact edge. For relatively small pressures when the ceramic deformation is reversible, the distribution $j_c^{GB}(x,y)$ may be substantially nonuniform only for "bad" ceramics for which $d_0\gg\xi_N$. In "good" ceramics ($d\sim\xi_N$) this distribution is always close to a uniform one. [Here we, naturally, do not take into account any other reasons which are not connected with the nonuniformity of the local pressure on a contact but which cause a nonuniform distribution $j_c^{GB}(x,y)$].

The total critical current i_c of a contact in the presence of a magnetic field B, lying in its plane and directed along the y-axis is determined by Eq. (11). For "good" ceramics the pressure action is confined to a change of areas of intergranular contacts.[15] In this case

$$i_c(P,B)\propto\frac{J_1(kr_P)}{kr_P}r_P^2, \tag{38}$$

where J_1 is a Bessel function. Oscillations of i_c disappear after averaging over all the contacts of the ceramics and then the expressions for its critical current density acquire the following form: in a zero magnetic field

$$j_{cr}(P,0)=(r_P/r_0)^2j_{cr}(0,0)=j_{cr}(0,0)\left(1+\frac{P}{P_0}\right)^{2/3}, \tag{39}$$

which correlates well with the experimentally observed dependences; and in a strong magnetic field ($kr_P\gg 1$)

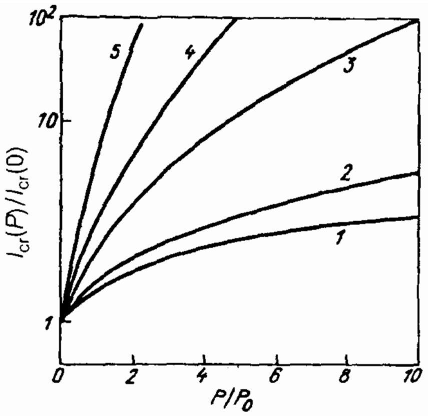

FIG. 10. Critical current dependences of different types of ceramics on pressure for $B=0$, calculated using Eqs. (39), (41) [Ref. 78]. The ceramics "quality" is defined by the parameter A: ($A\ll 1$ is a "good" ceramic, $A>1$ is a "bad" ceramic). *1*—$A\ll 1$, *2*—$A=1$, *3*—$A=3$, *4*—$A=5$, *5*—$A=9$.

$$j_{cr}(P,B)=j_{cr}(0,B)\left(1+\frac{P}{P_0}\right)^{1/6}, \tag{40}$$

which corresponds to a much weaker dependence of the ceramics critical current on pressure. It results from the fact that the contact area increase, potentially favorable from the point of view of raising its critical current, simultaneously brings about the rise of the magnetic flux through a contact lateral surface ($\Phi\propto r_P\lambda_G B$), which in its turn assists the critical current decrease [Eq. (12)].

For "bad" ceramics an approximate (asymptotically exact at $A\gg 1$) expression for the critical current of an individual contact has the form:

$$i_c(P,B)\propto(1+P/P_0)^{1/2}\Theta(P,B), \tag{41}$$

where

$$\Theta(P,B)=\int_0^{\pi/2}\exp[(\mathrm{A}r_P/r_0)\sin\theta]\times\cos[kr_P\cos\theta]\sin^{3/2}\theta\,d\theta.$$

The ceramics critical current dependences on pressure in the absence of a magnetic field ($B=0$) calculated using Eqs. (39) and (41) are plotted in Fig. 10. They provide an illustration of the idea put forward in this connection by the authors of Ref. 75: "He who has little has more to gain!" For our case: the lower is the initial ($P=0$) critical current the greater is its rise under pressure. At $A\gg 1$ (very "bad" ceramics) the critical current may increase by several orders of magnitude.

A different situation is expected to arise in a strong magnetic field ($kr_P\gg 1$). Here the critical current rise due to pressure turns out to be comparatively small even for "bad" ceramics (see Fig. 11). The reason for this is the same as for "good" ceramics (see above).

6.3.2. Uniaxial compression. Up till this point we were considering the critical current variations under uniform pressure. New interesting effects are brought about by uniaxial compression.[65] In this case the deformation $e(\phi)$

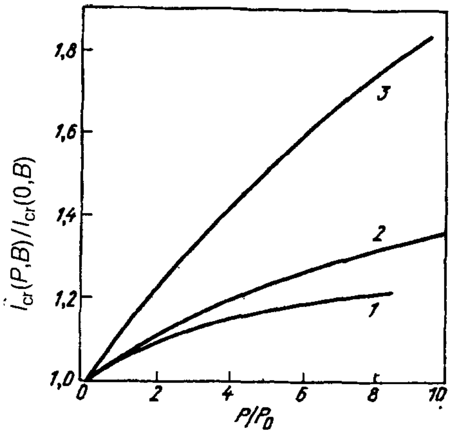

FIG. 11. Critical current dependences of different types of ceramics on pressure in a strong magnetic field ($kr_P=10$), calculated using Eqs. (40) and (41) [Ref. 78]. The ceramics "quality" is defined by parameter A: ($A\ll1$ is a "good" ceramic, $A\gtrsim1$ is a "bad" ceramic). *1*—$A\ll1$, *2*—$A=1$, *3*—$A\gg5$.

of the medium is anisotropic. Both its value and sign depend on the angle ϕ between the direction of compression and that of deformation:[80]

$$e(\phi)=\frac{P_\phi}{E}, \quad P_\phi=P[(1+\nu)\cos^2\phi-\nu]. \tag{42}$$

In accordance with relation (42), uniaxial compression is accompanied with expansion in directions for which $|\pi/2-\phi|<\arccos[\nu/(1+\nu)]$. It means that all the intergranular contacts may be divided into two types: critical currents of the contacts of the first type (their planes form an angle with the compression direction which does not exceed $\arccos[\nu/(1+\nu)]$), decrease, and those of the second type, on the contrary, increase. The total effect of the uniaxial compression effect on the ceramics critical current in this situation depends on the orientation of an average direction of the transport current relatively to the compression direction as well as of the "sinuosity" of current-conducting paths along which the near-to-critical current transfer is performed (see Fig. 9). If this sinuosity characterized by the average deviation $\langle\alpha\rangle$ of current paths from the transport current direction is not large [if, to be exact, $\langle\alpha\rangle\leqslant(6\nu)^{1/2}$ (Ref. 65)], then for the current flowing (on the average) along the compression direction it is the compressed contacts that are critical and for the current perpendicular (in general) to the compression direction the leading role belong to expanded contacts. Correspondingly, the critical current density should increase under uniaxial pressure in the former case and decrease in the latter one. Calculations reveal the ratio of variations of the densities of these currents (for small sinuosity of current paths) to be equal to $\Delta j^{\perp}_{cr}/\Delta j^{\parallel}_{cr}\cong-\nu$ (for the $YBa_2Cu_3O_\delta$ ceramics $\nu\cong0.2$). Experiments[73] carried out with the $YBa_2Cu_3O_\delta$ ceramics (with the critical current j_{cr} (77 K) $\sim10^2$ A/cm^2) confirm this conclusion fully. From this, in particular, it follows that the sinuosity of current paths in this ceramics is not large: $\langle\alpha\rangle\leqslant30°$.

Nontrivial results can be obtained with ceramics uniaxial compression in a magnetic field.[65] Pressure raises the critical current of contacts with planes normal to the compression direction, and the magnetic field reduces the critical current of contacts with planes parallel to the magnetic field direction. Hence the uniaxial compression effect is to a great extent dependent on the mutual orientation of the directions of uniaxial compression, magnetic field and current. Ref. 65 scrutinizes the situation when all the three directions are parallel (this situation corresponds to experimental conditions in Ref. 74). It is shown that in a strong magnetic field ($b\gg1$) the influence of uniaxial compression on the ceramics critical current is described by the relation[16)]

$$I_{cr}(P,B)\propto\frac{1+P/P_1}{B^{3/2}}, \quad P_1\cong\tfrac{1}{3}P_0(E/P_0)^{1/3} \tag{43}$$

and rapidly declines with the field increase: $I_{cr}(P,B)-I_{cr}(B,0)\propto P/B^{3/2}$. Exactly this kind of a relationship has been established experimentally.[74]

7. CALCULATION OF VOLTAGE-CURRENT CHARACTERISTICS OF HTSC CERAMICS (PERCOLATION MODEL)

7.1. Analytical techniques

Calculation of current-voltage characteristics (CVC) of superconducting ceramics at currents I exceeding the critical current I_{cr} is complicated by the fact that the resistive elements of such a system (intergrain contacts) are essentially nonlinear: the voltage drop v across a SNS contact with critical current i_c and normal resistance r_N is described by the function[81]

$$v(i,i_c)=\begin{cases}0 & , i<i_c\\ r_N(i^2-i_c^2)^{1/2} & , i>i_c\end{cases} \tag{44}$$

where i is the current flowing through the contact. Since for nonlinear systems the superposition principle is invalid the utilization of well-developed percolation and effective medium methods is inapplicable for the calculation of the conductance of such a system. Particularly, for a considerable spread of the parameters i_c and r_N determining the nonlinear conductance of separate elements of the system the CVC of the system may differ strongly from that of the constituent elements.

It can be easily seen from a simple and frequently used model which regards an HTSC ceramic as a set of $1D$ "threads" connected in parallel and composed of a large number of weak links connected in series.[82] In this case the voltage V at the ends of a "thread" with a current i is

$$V(i)\propto\int_0^i v(i,i_c)f(i_c)di_c. \tag{45}$$

The normal resistance r_N of a contact is proportional to its thickness d_N and for "thick" contacts [see Eq. (10)] depends only weakly (logarithmically) on i_c. If we neglect this dependence and use the distribution function (20) we shall obtain from Eqs. (44) and (45):[47]

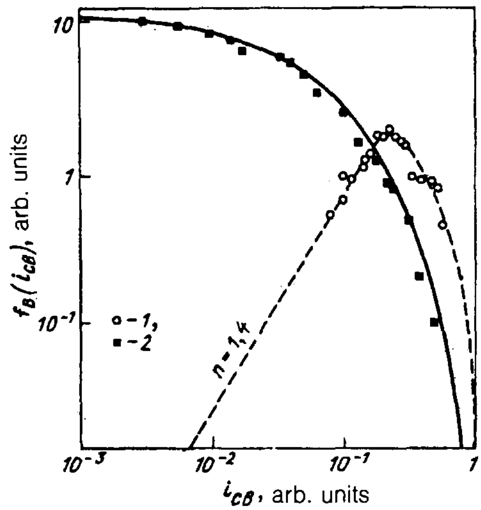

FIG. 12. Distribution function of intergrain Josephson junctions over critical currents in the $YBa_2Cu_3O_\delta$-ceramics in the absence (O) and presence (■) of a magnetic field $B=250$ G [Ref. 47].

$$V(i)\propto\int_0^i (i^2-i_c^2)^{1/2}f(i_c)\,\mathrm{d}i_c\propto\begin{cases} i^{n+2} & , \ i<i_0 \\ i & , \ i\gg i_0 \end{cases}, \tag{46}$$

from which it follows that a $1D$ chain of Josephson links possesses a power-law CVC at moderate currents, is characteristic becoming linear for large currents. The natural current scale here is the maximum critical current of couplings, i_0.

An attractive feature of the model in question is, apart from its simplicity, the feasibility of reconstructing the form of the contact distribution function $f(i_c)$ over the critical currents from the shape of the CVC (Ref. 47). For an approximate solution of this problem let us approximate the CVC of a single contact (44) with the step function

$$v(i,i_c)=\begin{cases} 0 & , \ i<i_c \\ r_N i & , \ i>i_c \end{cases}. \tag{47}$$

Then instead of (46) we obtain

$$r(i)\equiv\frac{V(i)}{i}\propto\int_0^i f(i_c)\,\mathrm{d}i_c, \tag{48}$$

for the nonlinear "thread" resistance, from which it follows that

$$f(I/N)\propto\frac{\partial R}{\partial I}, \tag{49}$$

where $R=Nr$ and $I=Ni$ are the resistance and the current of a sample consisting of N parallel "threads." An example of the application of Eq. (49) is given in Fig. 12, where the distribution functions $f(i_c)$ and $f_B(i_{cB})$ (in magnetic fields $B=0$ and 250 G, respectively) as calculated using this equation are plotted for the $YBa_2Cu_3O_\delta$ ceramics with a low critical current j_{cr} (78K) $\cong 10$ A/cm^2. Comparison with Fig. 7 reveals that in this case the majority of the contacts of HTSC ceramics are contacts with a uniform distribution of the critical current density.

This simple model seems to provide an adequate description of HTSC ceramics only in the direct proximity to T_c, when the network of conducting paths is sufficiently sparse and may be roughly represented as a set of "threads" connected in parallel. On lowering the temperature more and more intergranular contacts get involved in this network, its structure becomes still more complex and CVC calculations require a more general approach. The first approximation consists in letting the contact resistance in the normal state be identical while retaining the contact spread over critical currents.[47] Then an analogy with a percolation problem on the conductance in a random system made of resistive and superconducting bonds can be utilized.[83]

If the portion P_s of superconducting bonds of such a random lattice is smaller than the threshold value, P_c, then its conductance Σ is finite and equals $\Sigma\propto(P_c-P_s)^{-s}$. (For a $3D$ cubic lattice $P_c=0.25$ and $s=0.7$–0.9; for a $2D$ square lattice $P_c=0.5$ and $s=1.1$–1.15). The "distance" to the superconducting state transition point is determined in this problem by the difference (P_c-P_s). Its counterpart in CVC calculations is the portion of broken links in the above deduced (see Sec. 4) critical cluster, increasing with the $I>I_{cr}$ growth, i.e., the difference $(1-P')$, where $P'=P'(I)$ is the portion of contacts remaining in the superconducting state for $I>I_{cr}$. For a power-law distribution function (20) the dependence $P'(I)$ is also expected to be a power-law one, besides, taking into consideration that $P'(I_{cr})=1$, we find[17)] $P'(I)-1\propto(I-I_{cr})^{n+1}$. Then for $I>I_{cr}$ we obtain the following expression for the conductance of the system under analysis:

$$\Sigma(I)\propto\langle r_N\rangle^{-1}[1-P'(I)]^{-s}=\langle r_N\rangle^{-1}(I-I_{cr})^{-s(n+1)},$$

where $\langle r_N\rangle$ is the average contact resistance in the normal state. Thus, the CVC of the system in question $V(I)\propto I/\Sigma(I)$ is described by the relation

$$V(I)\propto\langle r_N\rangle I(I-I_{cr})^{(n+1)s} \tag{50}$$

and at $I\gg I_{cr}$ it acquires the power-law form:

$$V\propto I^{\mu},\quad \mu=1+(n+1)s. \tag{51}$$

Taking into account that $s\cong 1$ we find that, in conformity with the above deduced expression for CVC of $1D$ "threads," $V\propto I^{n+2}$ [see Eq. (46)].

7.2. Numerical techniques

The complexity of an analytical solution of the problem concerning the CVC of a random system of nonlinear elements which is the case with HTSC ceramics has initiated attempts of its numerical solution[49,50,84] and experimental investigations on studying CVC of artificial nonlinear systems.[85] In the course of these studies an interesting circumstance was discovered, namely: a system made of threshold elements with section-linear individual CVC exhibits a power-law CVC under random spread of threshold values. An example of this kind of system is given in Ref. 50. It is a square $(L\times L)$ lattice where every bond is a nonlinear threshold element (of the Zener diode type) with the following interconnection between current i and voltage v:

$$i=\begin{cases}0 & , v<v_g \\ \sigma(v-v_g) & , v>v_g\end{cases}. \tag{52}$$

If all the elements of the lattice are identical, its CVC has a similar form: $I\propto(V-V_g)$, where $V_g=Lv_g$. If the elements have equal conductance σ but random thresholds v_g (uniformly distributed over the range from 0 up to v_s), then a numerical calculation shows that its CVC (in the range of intermediate voltages where not all the elements are involved in the current transfer) becomes nonlinear:

$$I\propto(V-V_g)^{\gamma}, \tag{53}$$

with the threshold $V_g=(0.22\pm0.02)Lv_s$ and exponent $\gamma=2\pm0.08$. For $L\gg1$ the voltage interval in which the CVC is described by Eq. (53) is sufficiently broad (for $L=100$ $1<V/V_g<10^3$).

To get a better insight into the origin of this result is possible using the following reasoning. For a small increase (δV) of the voltage across the lattice the voltage drop on each of its elements is proportional to δV. For a uniform distribution of their thresholds v_g the number of the "activated" (i.e., starting to transfer current) elements, δn, is also proportional to δV. Neglecting the correlation between conducting elements we may use the classical result of the effective medium theory[44] according to which the variation of the total conductance of the lattice is $\delta\Sigma\propto\delta n\propto\delta V$. Taking into account that $\delta I=\Sigma\delta V$ we obtain $I\propto(V-V_g)^2$, which is in conformity with the numerical result (53).

The principle of the dual nature of electric circuits[86] enables one to translate the result obtained to a system imitating HTSC ceramics and composed of nonlinear elements with a current threshold rather than a voltage threshold (an analog of a weak intergrain link possessing a critical current). Then for a square lattice of superconducting elements with randomly distributed critical currents we obtain, instead of Eq. (53), a power-law CVC of the form:

$$V\propto(I-I_{cr})^{\mu}, \quad \mu=2\pm0.08. \tag{54}$$

with the exponent practically coinciding with that in Eq. (51) since the latter equals $\mu=2.1$ for a $2D$ square lattice (if we assume $t=1.1$ and take into account that for a uniform distribution of elements over the critical currents $n=0$).

A $2D$ lattice model consisting of "Josephson" elements whose critical currents may take on two strongly different values has already been discussed (see Section 5).[54] Apart from a numerical calculation of the critical current of such a system, a study of its CVC evolution due to a variation of the portion p of its "high-current" elements was conducted. When $p\ll p_c\cong0.35$ or $p\cong1$, the CVC of the system coincides with those of its constituent elements ("weak-current" elements in the former case and "strong-current" ones in the latter). However, within the limits of $p_c<p<1$ the randomness in the arrangement of elements of different current-carrying ability becomes essential and it is not difficult to notice (although this circumstance has not been commented upon by the authors of Ref. 54) that near I_{cr} the calculated CVC of the system exhibits symptoms of power-law behavior.

A more realistic model (discussed above in Section 5) was used as the basis of CVC calculations for a $2D$ square lattice of "Josephson" elements in Ref. 24. The distribution $f(i_c)$ of the critical currents over the lattice elements, as distinct from the previous work, has been assumed to be continuous and was determined by a combination of the exponential dependence $i_c(\vartheta)$ [see Eq. (3)] and the "cut-off" (for misorientation angles $\vartheta>20°$) Gauss distribution function $F(\vartheta)\propto\exp(-\vartheta^2/\langle\vartheta^2\rangle)$. The analysis of CVC adduced in Ref. 24 suggests that they are described by power-law dependences of the form $V\propto(I-I_{cr})^{\mu}$, with the exponent diminishing with the broadening of the distribution $F(\vartheta)$. Thus, for $\langle\vartheta^2\rangle^{1/2}=5°$, 10° and ∞ we may find $\mu\cong4.4$, 2.4 and 1.4, respectively. It is easy to confirm that the distributions $f(i_c)$ in these three cases are such that their "weak-current" part (essentially the one responsible for the CVC form) is close to the power-law function $f(i_c)\propto i_c^n$ with $n\cong2$, 0.6 and -1, respectively. If we calculate the exponent μ substituting these values of n into Eq. (51), it will give us $\mu\cong4.3$, 2.7 and 1, which is in good agreement with the analysis of CVC cited in Ref. 24.

The numerical CVC calculation of a $3D$ network of "Josephson" contacts requires far more calculating time than for a $2D$ case and it was performed in Ref. 49 with the help of the aforementioned (see Section 5) variational principle. The results relating to different means of disordering the parameters of system elements prove that in all the cases when such disordering is crucial (that is, the spread of the parameters of "Josephson" elements is large) the CVC of the system has a considerable portion of the exponential type. At the same time in Ref. 56 it has been shown that a percolation $3D$ system consisting of "Josephson" elements with equal critical currents also has power-law sections of the CVC. The values of the exponents μ in this case are small ($1<\mu<2$) however.

Thus, all the investigations reviewed in this Section prompt us to the conclusion that the power-law nature of the CVC is a universal property of a system composed of a large number of threshold nonlinear elements with a broad spread of parameters. Nevertheless it is not clear now if this conclusion is universal and it is impossible to set requirements for individual elements of the system which are necessary for the appearance of a power-law CVC.

In conclusion to this Section we note that there is a considerable number of studies whose authors attribute the power-law nature of the CVC of HTSC ceramics to critical fluctuations of the order parameter phase near the transition of the system consisting of superconducting grains (interconnected with "Josephson" intergranular contacts) into a coherent state.[87–89] This model (a so-called "superconducting glass model") also predicts CVC of the form (54) with $\mu\cong2$, but the range of its applicability is limited by the proximity to the transition temperature, and the predicted value of $\mu\sim1$ with weak temperature and magnetic field dependences raise doubts about the effectiveness of such a mechanism over the whole wide range of fields and temperatures where the power-law CVC of HTSC ceramics has been observed.

8. EFFECTS OF EXTERNAL FACTORS ON THE CURRENT-VOLTAGE CHARACTERISTICS OF HTSC CERAMICS

8.1. Magnetic field and temperature dependences of CVC

Within the framework of the model considered in Sec. 7.1. the CVC dependence on temperature and magnetic field is derived from the corresponding dependences of the exponent μ, the critical current I_{cr} and the average resistance $\langle r_N \rangle$ in the normal state [see Eq. (50)]. Of special interest and importance are, undoubtedly, the functions $\mu(T)$ and $\mu(B)$.

The temperature dependence of the exponent μ may be found from relations (21), (51):

$$\mu(T)-1=s\frac{\xi_N(T)}{\langle d\rangle}$$

$$=s\frac{\xi_N(T_c)}{\langle d\rangle}\times\begin{cases}(T/T_c)^{-1/2} & (\text{"dirty" limit})\\ (T/T_c)^{-1} & (\text{"clean" limit})\end{cases}. \tag{55}$$

According to (55), the parameter μ should grow monotonically with the temperature increase. As to the magnetic field dependence of the CVC exponent $\mu(B)$ it depends on the nature of the evolution of the distribution function $f_B(i_{cB})$ in the magnetic field (see Fig. 7). In the case of contacts with randomly nonuniform distribution of local critical current density $j_c^{GB}(x)$ the magnetic field produces a scarcely noticeable effect on the form of the distribution function and this correlates with a weak dependence of $\mu(B)$ on B. For contacts with a uniform local current density ($j_c^{GB}(x)=\text{const}$) the part of the distribution function $f_B(i_{cB})$, which determines the ceramics critical current I_{cr} corresponds (in a sufficiently strong magnetic field) to the value of $n=0$. It means [see Eq. (51)] that with the magnetic field increase μ should decrease down to $\mu=1+s$.

Power-law CVCs of the form (51) have been observed more than once in experiments with HTSC ceramics of different composition.[90-95] Analysis proves $\mu(B=0)$ to be always ~ 1 for ceramics with low current carrying ability (j_{cr} (77 K) ~ 10 A/cm^2 at $T_c=80$–90 K),[91,92,94] and $\mu(B=0)\gg 1$ for ceramics with sufficiently high critical current density (j_{cr} (77 K) $\sim 10^2$ A/cm^2).[93,95] Such behavior is naturally explained with the help of Eq. (55): the former case ($\mu\sim 1$) corresponds to "thick" contacts where $\langle d\rangle/\xi_N \gg 1$ and, consequently, critical currents are small [see Eq. (10)] and the latter case corresponds to "thin" contacts ($\langle d\rangle/\xi_N \leqslant 1$) with large critical currents.

This approach also explains the difference in the form of magnetic field dependences $\mu(B)$ in ceramics with both low and high current-carrying ability. It only requires taking into consideration the fact that a randomly nonuniform distribution of the local critical current density, $j_c^{GB}(x)$, can exist only in "thin" contacts sensitive to structural inhomogeneities of near-contact regions of ceramic super-

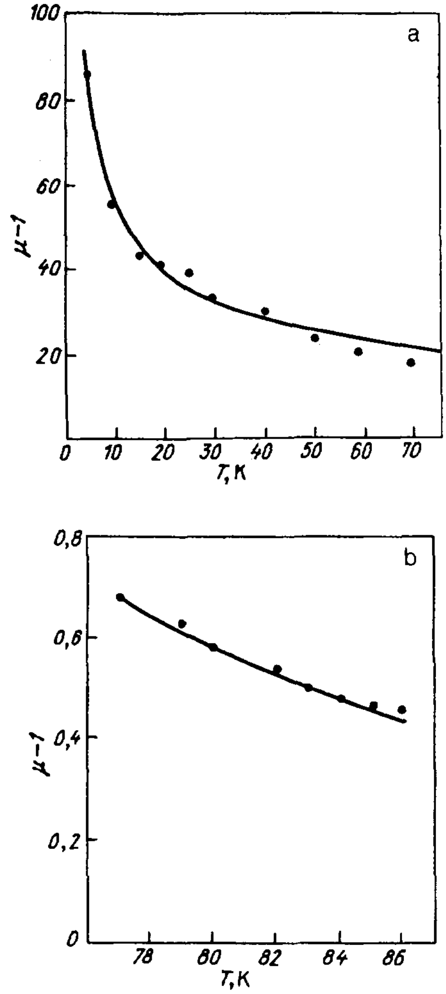

FIG. 13. Temperature dependences of the exponent μ of a power-law current- voltage characteristic. a) Bi-ceramics (phase 2212) with j_{cr} (77 K) $\sim 10^3$ A/cm^2, $B=0$ [Ref. 95]; solid curve—dependence $\mu-1\propto T^{-1}$ ("dirty" limit). b) $GdBa_2Cu_3O_\delta$-ceramics with j_{cr} (77 K) ~ 1 A/cm^2, $B=0$ [Ref. 91]; solid curve—fitting dependence $\mu-1\propto T^{-4}$.

conducting granules. For "thick" contacts, on the contrary, the spatial distribution $j_c^{GB}(x)$ of the local critical current density is practically uniform. It has already been discussed above why it should lead to different magnetic field dependences $\mu(B)$.

In Fig. 13a, b temperature dependences $\mu(T)$ are plotted for Bi-ceramics (phase 2212) with j_{cr} (77K) $\sim 10^3$ A/cm^2 (Ref. 95) and $GdBa_2Cu_3O_\delta$-ceramics with j_{cr} (77K) ~ 1 A/cm^2 (Ref. 91) at $B=0$. It can be easily seen that in the first case, in fact, $\mu \gg 1$, while in the second case $\mu\sim 1$. Estimates made using Eq. (55) give us $\langle d\rangle/\xi_N \sim 0.1$ for the Bi-ceramics with a considerable critical current density and $\langle d\rangle/\xi_N \sim 1$ for the $GdBa_2Cu_3O_\delta$-ceramics with a low critical current density.

Additional arguments in favor of the above considerations are given in Fig. 14 where magnetic field dependences $\mu(B)$ are plotted for the Bi-ceramics (phase 2212) with j_{cr} (77K) $\sim 10^3$ A/cm^2 (Ref. 93) (upper curve) and the $YBa_2Cu_3O_\delta$-ceramics with j_{cr} (77K) ~ 1 A/cm^2 (Ref. 92) (lower curve) at $T=50$ and 77 K, respectively. Here, as in the case, $\mu\sim 1$ (and is essentially dependent on the

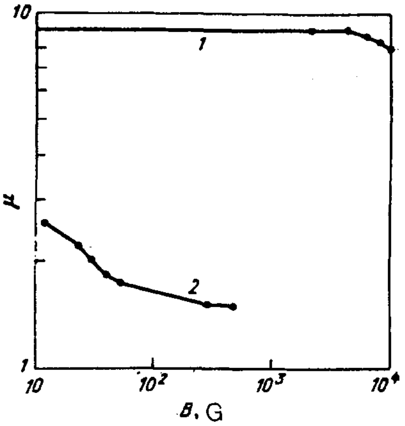

FIG. 14. Magnetic field dependences $\mu(B)$ of the exponent of a power-law current-voltage characteristic. Top curve—Bi-ceramics (phase 2212) with j_{cr} (77 K)$\sim 10^3$ A/cm^2, $T=50$ K [Ref. 93]; bottom curve—$YBa_2Cu_3O_\delta$-ceramic with j_{cr} (77 K)~ 1 A/cm^2, $T=77$ K [Ref. 92].

magnetic field) and as in the second case, $\mu \gg 1$ (and is slightly dependent on the magnetic field up to $B \sim 1$ T).

One of the interesting and frequently noted properties of HTSC ceramics CVC's is the existence of a peculiar scaling: on extrapolating the power-law portions of a CVC which correspond to different temperatures (or to different magnetic fields) they are all found to intersect in a single point (or, at least all their intersection points are very close to each other).[92–94] This peculiarity of the CVC's also turns out to be "built-in" in the model in question. Fig. 15 presents a family of current-voltage characteristics obtained using Eq. (50). The critical current here was calculated using Eq. (33) and, besides, it was taken into account that $\langle r_N \rangle \propto T$ (a typical temperature dependence of the normal resistance of superconducting metaloxides). Fig. 15 demonstrates clearly the above property of the CVC.

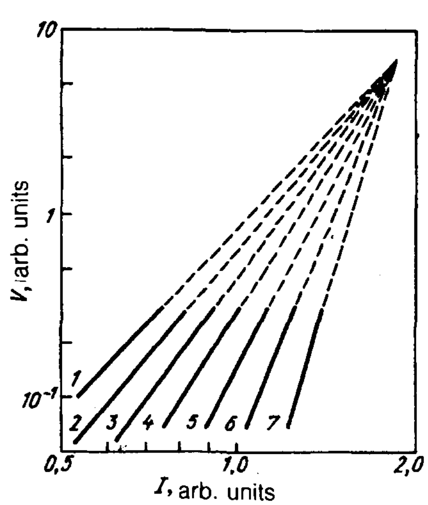

FIG. 15. Current-voltage characteristics of an HTSC ceramic calculated using Eq. (50) for the following parameter values: $t=1.5$; $s=0.9$; $P_c=0.25$ (a $3D$ system); $n[T=(77/91)T_c]=2$; $m=2$ (SNS-contacts). t: *1*—0.9; *2*—0.8; *3*—0.7; *4*—0.6; *5*—0.5; *6*—0.4; *7*—0.3.

8.2. Current-voltage characteristic dependence on pressure (Uniform and uniaxial compression)

A study of uniform pressure or uniaxial compression effects on the CVC of HTSC ceramics has been a key issue of the not too numerous experimental studies.[71–74] Uniform compression has been found to cause the CVC to shift to larger currents[71–33] while qualitatively retaining its form. Analysis of the CVC reveals that within the complete pressure range investigated ($P \leqslant 9$ kbar) CVCs of the $YBa_2Cu_3O_\delta$-ceramics with critical currents j_{cr} (77K)~ 20 A/cm^2 retain their power-law behavior, the exponent μ being virtually independent of pressure ($\mu \cong 1.5$–2). In the light of the previously discussed model, according to which $\mu = \xi_N/\langle d \rangle - 1$, this signifies that pressure produces no changes in the normal interlayer thickness (in the ceramics analyzed) in SNS-type intergranular contacts and affects solely the contact area (see Sec. 6.3).

Uniaxial compression influence on a CVC of the $YBa_2Cu_3O_\delta$-ceramics is not so unambiguous:[73,74] the CVC corresponding to a current perpendicular to the compression direction moves to the high current side while for a current parallel with the compression direction the CVC goes to the opposite side. It corresponds to the previously discussed ambiguity of the uniaxial compression effect on the ceramics critical current (see Sec. 6.3.).

On the whole we may affirm that the compression effect on the CVC of HTSC ceramics consists in the main in critical current renormalization while retaining the general form of CVC which is described by the relation

$$V \propto (I - I_{cr})^{\mu}. \tag{56}$$

9. CONCLUSION

The problem of increasing the critical current density of bulk high temperature superconductors is related to the solution of two problems, namely: elimination (or minimization) of "bad" intergrain weak links and creation of effective pinning centers in the bulk of superconducting grains. In this review we passed over the questions connected with a solution of the second problem (of vital importance!) and just tried to trace the interconnection between the properties of individual intergrain contacts of HTSC ceramics with its macroscopic transport properties such as the critical current and the current-voltage characteristic. Revelation of this interconnection provides a better understanding of the physical essence of previously described methods of raising the current carrying capacity of HTSC ceramics and points to determination of new promising approaches of achieving this goal. Among these methods let us dwell on the following.

1) *Predominant formation of intergrain contacts with a favorable "geometry."* Typical contacts of this type are small angle tilt boundaries with the planes parallel to the c-axis. It is these contacts that have the highest (characteristic of a single-crystal film) critical current density. It should be specially noted that well-known methods of ceramics texturing[98,99] result in meeting only one of these requirements, for ceramics of this type usually consist of

grains with (nearly) parallel c-axes but randomly oriented in the basal ab-planes. As the result, the majority of intergrain contacts are either tilt boundaries (parallel to the c-axis) with a broad distribution of misorientation angles ϑ, or twist boundaries with planes perpendicular to the c-axis with a reduced critical current density. Hence any further improvement of the properties of textured HTSC ceramics requires either a development of a production process which would ensure a more complete ordering of individual grains (including their ordering in the basal plane) or, else, producing ceramics with "architectural" features which would provide a possibility to achieve sufficiently high critical current densities even with relatively "poor" intergrain twist boundaries. The former of these directions has not yet yielded any tangible results, while the latter is seemingly implemented in a number of liquid-phase methods of HTSC ceramics production.

2) *Creation of randomly nonuniform intergrain contacts.* From the practical point of view, it is of primary importance (apart from raising the absolute value of the critical current density) to preserve high values of j_{cr} in sufficiently strong magnetic fields. The only effective mechanism of limiting the rapid drop of the Josephson contact critical current with the rise of the magnetic field is the inherent (or induced in one way or another) random nonuniformity of the local critical current density $j_c^{GB}(x,y)$ which is apparently exactly the reason for the appearance of a plateau observable on magnetic field dependences $j_{cr}(B)$ for HTSC ceramics of different composition.[96,101] The ratio γ of critical current densities in magnetic fields corresponding to this plateau and in zero magnetic field is $\gamma \sim (r_0/L)\delta^2$ for a $1D$ random function, $j_c^{GB}(x)$ (Ref. 96) and $\gamma \sim [r_0^2/S]^{1/2}\delta$ for a $2D$ random function $j_c^{GB}(x,y)$ (Refs. 28, 100) (S is the contact area). Here r_0 and δ are a correlation radius and a r.m.s. relative fluctuation of this function, respectively. Hence one should aim for an increase of the δ and r_0 values which are, naturally limited to values $\delta \cong 1$ and $r_0 \ll L$ ($1D$ nonuniformity) and $r_0^2 \ll S$ ($2D$ nonuniformity). On the other hand, a magnetic field corresponding to the upper limit of the plateau of the magnetic field dependence $j_{cr}(B)$ is inversely proportional to r_0.[96] This signifies the necessity of a certain compromise in the choice of the value of r_0 (if, of course, we assume that there are technological possibilities for such a choice).

3) *Manufacturing of textured ceramics with favorable "architecture."* An example of this kind of architecture is a ceramic with the grains in the form of comparatively thin platelets (parallel to the ab-plane) arranged similarly to bricks in the wall. In this case the total contact area between the "bricks" (grains) belonging to different rows (contacts of the A type) exceeds noticeably the total contact area between the grains forming a row (type B contacts). The ratio of these areas equals $L/D \gg 1$, where $2L$ is the grain size (along the row), D is its thickness. Hence the lines of the current directed (on the average) along the rows will have the form of sinusoidal lines passing through type A contacts of larger area and by-passing type B contacts of smaller area. As the result the critical current density of the ceramic increases and becomes $j_{cr} \sim \langle j_c^{GB} \rangle (L/D)$, where $\langle j_c^{GB} \rangle$ is the average (over the contact area) density of the critical current across the twist boundary (type A contacts). Thus, raising the L/D ratio one can increase substantially the current-carrying ability of the ceramic.[99]

It should be borne in mind, however, that the efficiency of this method is limited by the $\langle j_c^{GB} \rangle$ decrease as the contact length L rises because of the magnetic field of the current: $\langle j_c^{GB} \rangle \propto \lambda_J/L$ at $L \gg \lambda_J$.[28] Hence any further improvements of j_{cr} will require a decrease of the grain thickness D.

And lastly, one more possibility of raising the critical current density consists in the purposeful variation of the intergrain boundary "chemistry." An example of this approach is the investigation of Ag-doped ceramics (see Ref. 72 and references therein). Small (several percent) Ag admixtures increase substantially (by several times) the critical current density of the $YBa_2Cu_3O_\delta$ ceramic without changes in its critical temperature. The mechanism of the phenomenon is still vague and needs further investigation.

Finally we formulate several conclusions that follow from the above discussion of the properties of HTSC ceramics: 1) the distribution function of the critical currents of intergranular contacts is close to a power law; 2) the random structural inhomogeneity of "good" intergranular contacts is the reason for the plateau on the magnetic-field dependence of the critical current of HTSC ceramics; 3) there is a correlation between the index of the CVC and the critical current density; 4) the power-law CVC of HTSC ceramics is a universal property of systems that represent a set of nonlinear elements with a large spread of parameters.

[1)] Hereafter the term "HTSC ceramics" will apply first of all to ceramics of the $YBa_2Cu_3O_\delta$ composition. At present it is this material which has been most thoroughly investigated from the point of view of the problems discussed in our article. However, a good deal of models considered and results obtained may be applied with equal success to HTSC ceramics of a different composition.

[2)] The nonuniformity of the local critical current density $j_c^{GB}(x)$ in the contact may result not only from its intrinsic structural inhomogeneity but also from some "external" factor, e.g., a magnetic field. Thus, if the superconducting boundaries of a contact are in the mixed state (which is possible when the magnetic field B exceeds the lower critical magnetic field B_{c1}^G of the superconducting granule material) the nonuniform magnetic field of the Abrikosov vortices close to the contact plane results in inhomogeneity of $j_c^{GB}(x)$. This effect is discussed in detail in Refs. 35 (randomly situated pinned vortices) and 36 (regularly spaced vortices). Similarly, the reason for the inhomogeneity of $j_c^{GB}(x)$ in a wide contact may lie in the field of Josephson vortices.[37]

[3)] It has long been known of the existence of "special" large-angle GBs along which the impurity diffusion proceeds with the greatest rate. Such boundaries exist for orientations corresponding to good adjustment of lattices of neighboring grains.[41] It is quite probable that it is these boundaries in $YBa_2Cu_3O_\delta$ that do not exhibit properties of a weak link.

[4)] It is equivalent to an assumption that the order parameter phase inside each granule suffers no changes.

[5)] Eq. (29) does not allow for terms proportional to i_{kl}^2 corresponding to the magnetic field energy of weak links and to the kinetic energy of electrons.

[6)] The above value of p_c differs from the one known for an infinite cubic lattice ($p_c=0.312$, Ref. 42). The difference results most likely from the small number of elements in the analyzed $7\times7\times7$ network.

[7)] As has been shown in Ref. 51, the actual form of the contact distribution over sizes and orientations affects the results of averaging only insignificantly.

[8)] The form of dependence $I_{cr}(b)$ in strong magnetic fields ($b \gg 1$) for a uniform j_c^{GB}-distribution depends on the contact form: $I_{cr}(b \gg 1) \propto 1/b$ for square contacts, but $I_{cr}(b \gg 1) \propto 1/b^{3/2}$ for round contacts.[57] The allowance for contact anisotropy resulting from anisotropy of the London penetration depth produces only an insignificant effect on this result: the calculation in Ref. 58 gives $1 < |\partial \ln I_{cr}/\partial \ln b| < 2$.

9)Initial dependences $I_{cr}(b)$ are meant which correspond to a monotonic variation of the magnetic field from zero up to a preset value. Otherwise hysteresis phenomena may be observed resulting from magnetic flux capture inside superconducting grains.[61]

10)Since there are no systematic studies of the boundaries of this kind it is highly possible that $\vartheta = 14°$ is not the only misorientation angle ensuring the properties of a "strong" link. Besides, if we suppose, by analogy with the results of Ref. 18 (relating to small-angle tilt boundaries), that the properties of a "strong" link are preserved for all boundaries of the type under consideration with misorientation angles $\vartheta = 14° \pm \Delta\vartheta$, where $\Delta\vartheta \sim 5°$, then the portion of the boundaries with "strong" links can be estimated as being equal to $(2\Delta\vartheta/45°) \cong 0.2$ (for this estimate no difference is made between axes a and b, i.e., the properties of contacts with misorientation angles $\vartheta = 14$ and $76°$ are considered identical).

11)In strong magnetic fields ($B > B_{kl}$) Eq. (31) is reduced to an asymptotic magnetic field dependence of the critical current of a wide Josephson contact where the nonuniformity of the $j_c^{GB}(x)$-distribution results from the field of Josephson vortices penetrating it.[37]

12)A system of equations was investigated that differed from Eqs. (26)–(28) by the fact that the phase difference appearing in expression (27) was augmented by the phase factor that appears in a magnetic field

$$A_{kl} = (2\pi/\Phi_0) \int_{(k)}^{(l)} \Phi \mathrm{d}l$$

which is obtained by means of integration of the vector potential along the line connecting the centers of the kth and the lth grains.

13)It should be borne in mind, however, that in close proximity to T_c the power-law dependence (6) may be distorted because of fluctuations.[19,27,34]

14)Nevertheless, the fact that a ceramic is a set of differently oriented contacts with various critical currents appears to be important in a number of cases (see below). This fact should be taken into consideration if there is a specified direction in the system, e.g., a direction of axial compression or of a magnetic field (when the critical current of a ceramic becomes anisotropic[78]), or in a textured ceramic.

15)Experiments reveal a weak dependence of intrinsic superconducting parameters proper of high temperature superconductors on pressure (for $YBa_2Cu_3O_\delta$, for instance, $\partial[T/T_c]/\partial P \sim 10^{-2}$–$10^{-3}$ kbar^{-1}).

16)For the $YBa_2Cu_3O_\delta$ ceramics $P_1 \sim 10$ kbar.[65]

17)The increasing of exponent in Equation for $P'(I)$ by one as compared with the exponent in the distribution (20) is related to the integral form of the dependence $P'(I) \propto \int f(i_c) \mathrm{d}i_c$.

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