

The Schrödinger atom

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Aspects of an electrodynamical interpretation of the wave function suggested by Schrödinger are described. According to this conception, electron charge is continuously distributed throughout the interior of the atomic system. A proof is given that classical electrodynamics holds within an atom. The Schrödinger atom is shown to be the only model in which electrons do not lose their energy through emission when they move around the nucleus. The value of the self-field of the distributed electron charge is estimated. Practical applications of this concept are noted, including a new trend in quantum electrodynamics. Experimental and theoretical confirmation of the model of an atom with continuously distributed charge is presented.

1. INTRODUCTION

The creation of nonrelativistic quantum mechanics was essentially complete by the end of 1927 and from then on belonged to history. The radical wave-probability concepts adopted at that time were described in detail and comprehensively in monographs and textbooks. This is in contrast to alternative ideas and solutions put forward at the same period, which have remained poorly known until now. Some of them, proposed by Einstein, Duane, Schrödinger, and Heisenberg, are probably deserving of attention.

In this article we consider the role of Schrödinger, which is usually by no means fully revealed. Schrödinger did not just derive the fundamental equation bearing his name. He proposed his own model of the atom and physical interpretation of the wave function, which he called the electrodynamic interpretation.

The Schrödinger model of the atom did not receive recognition at that time and was essentially not subjected to serious discussion. Interest in it revived only many decades later in connection with the development of a fruitful new direction in quantum electrodynamics (see Sec. 4).

Between January and June of 1926 Schrödinger submitted for publication four articles, jointly entitled "Quantization as an eigenvalue problem."¹

Schrödinger started from the ideas of de Broglie about matter waves, although he did not share the hypothesis regarding the dual corpuscular-wave nature of particles. He looked for a wavelike solution in order to describe the motion of electrons in an atom. And he succeeded in finding a partial differential equation describing the discrete stationary states of atomic systems. In his first communication he presented the equation for the hydrogen atom, and in the second the general equation for systems in a steady state:

$$\frac{\hbar^2}{2m} \Delta \psi + (E - U) \psi = 0; \quad (1)$$

here U is the electron potential energy in the external field and E is the energy of the system. As boundary conditions Schrödinger assumed the natural requirements that the function ψ be real, single-valued, bounded, and twice differentiable. The condition that it be real was soon relaxed. In time it became clear that it was almost always sufficient for ψ to be quadratically integrable. The fourth paper derived the equation for time-dependent systems:

$$\frac{\hbar^2}{2m} \Delta \psi - U \psi = i \hbar \frac{\partial \psi}{\partial t}. \quad (2)$$

In the interval between the second and third papers Schrödinger also published an article in which he showed the mathematical equivalence of his equation and Heisenberg's matrix mechanics,² thereby laying the foundations of the theory of transformations. It is impossible to disagree with the words of M. Jammer: "The new mechanics ... was created essentially entirely and over just a few months by none other than Schrödinger."³

The Schrödinger equation was met enthusiastically by physicists around the world, particularly by those who adhered to classical scientific principles: Einstein, Planck, Sommerfeld. The new method was much more convenient and more attractive than the totally abstract matrix mechanics and the Dirac algorithm. Contemporaries were struck by the fact that the "completeness" and discreteness of the energy spectrum were obtained as conditions in which there was no arbitrariness, in contrast to the incomprehensible and contradictory postulates of the Bohr planetary theory. A universal technique for calculating atomic systems developed which was extremely powerful, logically consistent, and quite transparent.

Schrödinger originally called his theory "micromechanics" and "undulatory mechanics," and the function ψ he called the "mechanical scalar field" and the "field scalar." Only in later papers did he switch to the terms "wave mechanics" and "wave function."

To complete the theory it was necessary to reveal the physical meaning of the wave function and to find an adequate atomic model.

Specifically, the goal originally set by Schrödinger, to describe matter waves, assumed a continuous distribution of electron material throughout the volume of an atomic system. But in addition, wavelike variations of the electron charge were assumed to exist in space and time. "It is quite natural to associate the function ψ with some vibrational process in the atom," writes Schrödinger in his first paper. As he initially assumed, in emitting light an atom vibrates simultaneously with two frequencies, which causes beating. Of course, this reflects the influence of the ideas of de Broglie. Moreover, Schrödinger did not accept them literally. He calls the atomic vibrational process "hypothetical," and the sequence of positions "arbitrary." In the second communication, retreating from his original views, he represents the electron in the form of a "wave group" (wave packet).

Only in the fourth paper did Schrödinger finally do away with the concept of "fluctuating matter" and part with the vital concept of an atom as an "oscillator."

2. THE ELECTRODYNAMIC INTERPRETATION

According to Schrödinger's idea, formulated in his fourth paper, electron charges and the currents they produce are continuously distributed throughout the volume of an atom, molecule, or crystal with densities expressed in terms of the wave function ψ . In the steady state of an atomic system the spatial distribution of the electron charge and current is constant in time, and their electrical and magnetic fields are constant. "In a certain sense one can speak of returning to the electrostatic and magnetostatic model of the atom," writes Schrödinger.

Thus, electrons are not point charges and are not small in comparison with an atom. "According to this hypothesis the charge of an electron is actually confined to a region of several Ångströms," asserts Schrödinger.² The electron dimensions are of the same order as those of an atom.

For a time-independent state of a system it is necessary that the forces of the electric and magnetic fields be balanced by the nonelectrodynamic forces, represented in the Schrödinger equation by the term containing Planck's constant \hbar . Equilibrium is attained not for arbitrary distributions of electron charge and current, but only for certain chosen ones, determined by discrete solutions (eigenfunctions) of the Schrödinger equation with the corresponding energy levels E .

The density of the electron charge and current can be expressed most simply in terms of the wave function ψ in the case of the hydrogen atom and other single-electron systems:

$$\rho = -e\psi\psi^*, \quad (3)$$

$$\mathbf{J} = -\frac{ie\hbar}{2m} (\psi \text{ grad } \psi^* - \psi^* \text{ grad } \psi). \quad (4)$$

One of the objections advanced against the Schrödinger interpretation reduces to saying that it is unsuited

for multielectron atomic systems. In fact, when applied to an atom with Z electrons the function ψ depends not on three coordinates in physical three-dimensional space but on $3Z$ coordinates in a configuration space of $3Z$ dimensions.

However, Schrödinger succeeded in overcoming the difficulties that developed. In the fourth paper he represented the wave function in the form $\psi = \psi(x, x')$, where x is the set of coordinates of one particle and x' is the set of coordinates of all the other particles. This allowed the density of the distributed charge and current of the α -th particle to be expressed in terms of the three usual coordinates in the form of integrals:

$$\rho_\alpha(x) = q_\alpha \int \psi \psi^* dx', \quad (5)$$

$$\mathbf{J}_\alpha(x) = \frac{i\hbar q_\alpha}{2m_\alpha} \int (\psi \text{ grad } \psi^* - \psi^* \text{ grad } \psi) dx'. \quad (6)$$

Summation of expressions (5) and (6) over all particles yields the desired electron charge and current densities ρ and \mathbf{J} as functions of the three spatial variables. Schrödinger also derived the continuity equation

$$\frac{\partial}{\partial t} \rho_\alpha(x) + \text{div}_\alpha \mathbf{J}_\alpha(x) = 0. \quad (7)$$

In deriving integrals of the form (5) and (6), Schrödinger actually introduced for the first time the concept of the density matrix, but this achievement went unnoticed. Some time later density matrices were independently reintroduced into the theory and only then did they receive this appellation.

Note one surprising circumstance. Over the course of many successive decades the spatial distribution of the electron charge was characterized in the literature as inadmissible on the basis that it is appropriate only for one-electron systems. The generalization of this concept to multielectron systems, carried out by Schrödinger, went unremarked both by his contemporaries and by later generations of researchers. In the literature the opinion that the electrodynamic interpretation was inappropriate for complex systems was frequently asserted.^{3,5,6}

Schrödinger pointed out a number of advantages of his interpretation. "The hypothesis according to which the electron is transformed from one vibrational state to another in a quantum transition is considerably more satisfactory than the picture of an electron jumping around. The change in the vibrational form can always occur continuously in space and time; it can last a time equal to the experimentally determined time for the emission process," writes Schrödinger in the first paper.

And in the third paper he points out: "It is necessary to try to apply the ideas of Uhlenbeck and Goudsmit in wave mechanics ... since in it the electron is no longer a point charge, but fills space with a continuous flux, so that the unpleasant concept of a 'rotating point charge' is eliminated." In fact, the electron spin, which is equal to $\hbar/2$, is naturally explained in terms of its rotation about an internal axis with angular momentum $m\mathbf{v}\mathbf{R}$. The mean radius R

can be chosen to be equal roughly to the first Bohr radius $a_1 = \hbar^2 / me^2 = 0.529 \cdot 10^{-8}$ cm. Then it can easily be shown that it suffices to have rotational velocities of order $v = \hbar / 2ma_1 = \alpha c / 2$, which are small in comparison with the velocity of light ($\alpha = e^2 / \hbar c = 1/137$ is the fine-structure constant).

3. NONRADIATING ELECTRONS AND CLASSICAL ELECTRODYNAMICS

Among the advantages of this interpretation Schrödinger points out another one. "The mechanical scalar field (which I denote by ψ) is completely suited to play the role of 'source' for the electromagnetic field vector even in the case when the latter satisfies the unaltered Maxwell-Lorentz equations." As can be seen, Schrödinger assumed that classical electrodynamics also applies inside an atom.

Subsequently this assumption of Schrödinger was fully corroborated. In the literature one can encounter several versions of a mathematically rigorous proof that the equations of classical electrodynamics can be derived from the special theory of relativity, the law of conservation of electric charge, and the Coulomb law (or the principle of least action).⁷⁻⁹ A fairly concise proof has been presented by M. Bowler (see Appendix). True, proofs of this sort are not intended for applications to the atom. But inside an atom and other atomic systems all the initial assumptions are satisfied. One must conclude that classical electrodynamics remains valid inside atomic systems, although it is clearly inadequate to describe the atom.¹⁰

"Furthermore, the absence of radiation in the normal state can at any rate be simply explained," writes Schrödinger in the fourth paper. In fact, it turns out to be possible to explain simply and naturally how electrons avoid losing energy to radiation while moving around atomic nuclei, despite the validity of Maxwell electrodynamics.

In the simplest case, that of the hydrogen atom, the time-independent spatial distribution of the electron charge and current has circular symmetry about an axis passing through the nucleus. The electric and magnetic field inside and outside of the atom are also axisymmetric and time-independent. The electric and magnetic field vectors \mathbf{E} , \mathbf{H} everywhere lie in planes passing through the axis of the atom. The electromagnetic energy flux, whose density is given by the Poynting vector $\mathbf{E} \times \mathbf{H} c / 4\pi$, is perpendicular everywhere to these axial planes, i.e., directed azimuthally and closed on itself. The electromagnetic energy, rather than being radiated, only circulates inside the atom and in its immediate vicinity. Such an atom can be called "rotational" in view of the circular symmetry of its electronic charges and currents, electric and magnetic fields, and finally, energy fluxes.

Obviously, the system does not have to be circularly asymmetric. There also exist distributions of electron charge and current about atomic nuclei which do not have circular asymmetry, do not radiate energy, and are constant in time. It is only necessary that the electromagnetic energy circulate inside and around them in closed contours, and not escape into the surrounding space. In the

presence of two or more atomic nuclei such electronic charge and current distributions form molecules or crystals.

Thus, according to the Schrödinger interpretation atoms and molecules are unique spatially self-restricted "resonators"—spatial distributions of electron charge and current about atomic nuclei having a discrete series of possible stationary configurations and corresponding energy levels. Of course, transitions from one stationary state to another are accompanied by radiation or absorption of photons with a frequency proportional to the difference in the energy levels in the initial and final states.

4. THE SELF-FIELD AND EXTERNAL PROBLEMS

According to Schrödinger, as already noted, the distributed electronic charges and currents are sources of the self-electromagnetic field.² In the Schrödinger equation (1) it is necessary to add to the quantity U the potential energy U_s of the electron charge in its self-field,

$$\frac{\hbar^2}{2m} \Delta \psi + (E - U - U_s) \psi = 0. \quad (8)$$

The quantity U_s is determined from the Poisson equation

$$\Delta U_s = -\frac{1}{2} e \Delta V_s = 2\pi e \rho, \quad (9)$$

where V_s is the potential of the self-electric field of the electron, or else is expressed in terms of the Green's function

$$U_s(x) = e \int D(x-y) \rho(y) dy.$$

With the addition of U_s the Schrödinger equation ceases to be linear, and solving it becomes more complicated. The quantity U_s is fairly small, so that near the nucleus the self-field of the electron is negligible in comparison with that of the nucleus, while at the periphery of the atom where these fields are of the same order, they are both quite weak. Consequently, the Schrödinger equation yields fairly accurate results even with the addition of U_s .

However, the presence of U_s significantly affects some properties of the atom. Thus, the electron charge density at the periphery of the atom falls off not exponentially but faster, and vanishes at a finite distance from the nucleus. The atom has clearly delineated boundaries, and the distributed electron charge does not extend to infinity.

Next, the concept of a spatially distributed electron charge called into existence a new direction in quantum electrodynamics. One of its problems is the calculation of the so-called radiative corrections caused by the self-field of electrons. As is well known, previously this area of theoretical physics was unsatisfactory in its logical structure.¹¹ The Schrödinger interpretation permitted the problem to be reduced to that of including the self-potential energy U_s . The shift in the energy levels of atoms and in the spectral lines (the Lamb shift), the anomalous magnetic moment of the electron, and other radiative effects are now calculated by more efficacious, accurate, and physically

transparent methods without divergences, renormalization and second quantization.¹²⁻¹⁶ Of course, the inclusion of relativistic factors is desirable.¹⁴

Among the problems of quantum mechanics it is worthwhile to distinguish between internal and external ones. Internal problems include the study of the nature of the electron charge and current spatial distributions in an atomic system—atom, molecule, crystal, etc. (e.g., a potential well). These problems are solved using the Schrödinger equation. It was just for this purpose that it was devised by its author when he advanced the electrodynamic interpretation of the function ψ .

External problems include the investigation of atomic systems in which particles move through relatively extended regions of space, free of electron charge. Examples are particle collision and scattering processes, particle diffraction by crystals, etc. In solving external problems use is made of energy and momentum quantization, the relation between the photon frequency and momentum, and other rules.

The Schrödinger equation was not designed to solve external problems directly. Nevertheless, it is widely used in external problems also, with the probabilistic interpretation of the ψ function, and it yields results that are confirmed by experiment. However, on examination these results may turn out to be wholly illusory. Thus, for the process of scattering of α particles by an atom the Schrödinger equation leads to the same Rutherford formula as does the classical treatment. This formula does not contain the Planck constant \hbar at all; the Schrödinger equation drops out, so to speak, in the algebra. The correction term obtained in next order, which contains \hbar , yields only an apparent improvement, since the calculation that is done does not take into account the self-field of the distributed electron charge.

Thus, the applicability of the Schrödinger equation to the solution of external problems requires careful study, and it cannot be taken for granted at all. For internal and external problems Barut¹⁴ has introduced two different notations for the wave function, ψ and Ψ .

In the Thomson atom (1901) positive electrical charge filled the whole volume of the atom, and electrons were interspersed throughout it like raisins in a bun. In 1911 Rutherford showed that the positive charge of the atom is concentrated in a very small nuclei, but the electrons were still regarded as small. The Schrödinger interpretation proposed in 1926 finally forced us to change our view of electrons. It turned out that electrons are not small, and their charge fills the whole volume of the atom. According to Schrödinger, real atoms are the reverse of the Thomson atom.

As already noted, Schrödinger did not arrive at his interpretation directly. It ripened gradually and in the beginning was interwoven with erroneous ideas about oscillations of the electron charge. Viewed against the background of the triumphant success of the Schrödinger equation his exegesis of the meaning of the ψ function appeared tangled and unconvincing. For a long time it was aimed only at the hydrogen atom. Only in the fourth paper

did Schrödinger extend his interpretation to multielectron systems. But by this time, of course, readers had simply gotten tired of following Schrödinger's constantly changing ideas. Instead of publishing just one final result, he revealed the whole process of his search, the picture of his long wandering in the darkness. And this had quite undesired consequences. Furthermore, in the middle of 1926 the Schrödinger atom already had to compete with the probability interpretation of the wave function put forward by Max Born.

After encountering staunch objections from Bohr, Heisenberg, and Dirac, the Schrödinger interpretation was rejected.

The events unfolded dramatically. Schrödinger's fourth paper reached the journal *Annalen der Physik* on June 21, 1926. On June 25 the editors of another journal, *Zeitschrift für Physik*, received a short note from Born, and on July 21 his detailed paper.¹⁷ Born apparently had not been acquainted with Schrödinger's fourth paper prior to that.

According to Born, particles are material points, and the square of the absolute value of the wave function, $\psi\psi^*$, is the probability density for finding a particle at a given point of space. According to this interpretation, which became widely accepted, quantum mechanics cannot predict events, but only the probability for them to occur.

5. REALITY OF THE SCHRÖDINGER CONCEPT

The Schrödinger interpretation of atomic structure, as already noted above, has served as the basis for a fruitful new direction in quantum electrodynamics. This is an important, but by no means the only, practical application of the Schrödinger concept.

In quantum chemistry the density matrices first introduced by Schrödinger are widely used. Density matrices provide a simple and clear picture of the role of each electron and atom in a complicated atomic system. They were found to be much better suited to the structural properties of complicated atoms, and also of molecules and crystals, than the excessively detailed wave functions.¹⁸

The density matrix multiplied by the total electron charge is called the electron distribution. The idea of the electron distribution is also used in treating collisions between particles and atoms,¹⁹ to provide a schematic picture of the structure of molecules and crystals, etc.

In calculations of multielectron systems the concept of the self-consistent field is introduced. This allows a many-body problem to be reduced approximately to the problem of the motion of a single particle in the average field of the remaining particles of the system. In connection with calculations of complicated atoms this method was proposed by D. Hartree in 1927 and improved by V. A. Fok in 1930.

The Schrödinger interpretation also finds support through its ability to explain in a simple and natural way a number of well-known phenomena which cannot be explained otherwise.

The most important of these phenomena is the extended stable existence of atoms in stationary states, i.e., the absence of energy losses through radiation by electrons

moving around atomic nuclei. For the Schrödinger atom with its spatially distributed electron charge this is easy to explain (see Sec. 3). But if we regard electrons as points or small in size compared with an atom, then according to the laws of classical electrodynamics it is impossible to explain reasonably the absence of radiation from moving electrons.

With the Schrödinger idea of a "large" electron, having dimensions comparable with those of an atom, the electron spin is easily explained through its rotation about its own axis (Sec. 2). But if we ascribe the "classical" radius r_e to an electron, the linear velocity of its rotation would have to be hundreds of times the speed of light.

A natural explanation of the tunneling effect is also possible. We can view the passage of a particle over a potential barrier without having the necessary energy as follows. When a particle has finite and sufficiently large dimensions its charge need not at any time be located entirely at the top of the barrier. A substantial part of its charge is at all times on the slopes or at the foot of the barrier, and the amount of energy needed to overcome it is thereby substantially reduced.

Thus, in the pole vault an athlete does not pass over the bar all at once; first his feet, then his body, and finally his head and hands go over. His center of gravity is always below the bar.

There are additional analogies to the tunneling effect in classical mechanics. For example a train of railroad cars (without a locomotive), coasting down from a switching hump of some height, may then climb over a second hill, even one that is higher than the first, provided that the train is sufficiently longer than the second hill.

The idea of vanishingly small electron dimensions, as is well known, leads to the so-called divergences. The simplest example of a divergence is that the energy of the external electric field of an electron, which is equal to e^2/r , blows up at $r=0$. If we equate the energy of the external field of an electron to its rest mass mc^2 , we find the "classical" electron radius $r_e = e^2/mc^2 = 2.8 \cdot 10^{-13}$ cm. Since the laws of classical electrodynamics apply (see Sec. 3), we must conclude that the electron dimensions are finite and in any case are not smaller than r_e .

From the expressions for the classical radius, the Compton wavelength, and the first Bohr radius,

$$r = \frac{e^2}{mc^2}, \quad \lambda_c = \frac{\hbar}{mc}, \quad a_1 = \frac{\hbar^2}{me^2}$$

we see that all the quantities with dimensions of length go to zero at the rest mass increases. The electron mass is smaller than that of the proton, and so the electron dimensions are naturally larger than those of a proton. If the electron radius were inversely proportional to the mass, it would be equal to

$$R_e = R_p m_p / m_e = 1836 R_p = 2.75 \cdot 10^{-10} \text{ cm},$$

but since the internal structures of these two particles are different the electron radius is larger, of order 10^{-8} cm.

The relatively large dimensions of the electron are also confirmed by the resolving power that has been achieved with electron microscopes. In present-day microscopes an

accelerating potential of up to 100 kV is used, which corresponds to de Broglie waves of length $\lambda = h/p = 3.7 \cdot 10^{-10}$ cm. However, the resolution of electron microscopes, regardless of their degree of technological refinement, is no better than 10^{-8} cm. Usually this is explained by distortion introduced by the magnetic lenses. But it also follows from the finite dimensions of the electron if these are of order 10^{-8} cm. Electron dimensions of this order have been confirmed by a new type of electron microscope, the scanning tunnel microscope of Binnig and Rohrer,²⁰ which appeared in the 1980s.

6. CONCLUSION

The Schrödinger model of the atom looks strange, but appears to be well founded.

The Schrödinger atom with its spatially distributed electron charge is the only atomic model in which the electrons moving around the atomic nucleus do not emit energy in the steady states of the atom, although the laws of classical electrodynamics do remain valid there.¹⁰

The idea of a continuous electron charge distribution in the volume of an atomic system has already been used in practical applications for a long time in calculating complicated atoms (the Hartree-Fock method) and in quantum chemistry (the "electron distribution"), and recently has been used as the basis for developing a fruitful new approach in quantum electrodynamics.¹²⁻¹⁶

Of course, more extensive information about effective and physically clear methods for calculating radiative effects is necessary. An adequate estimate of the Schrödinger atom would aid also in the broader discussion of the applicability of Maxwellian electrodynamics inside the atom. If classical electrodynamics applies we must acknowledge that the idea of point electrons is inconsistent: no matter how point electrons move in an atom, they would inevitably lose their energy to radiation, and the atom would be destroyed.

Moreover, in treating so-called external problems, and in particular in studying the processes of collision and scattering, it is entirely natural to take electrons and other microscopic particles as material points. An analogous simplification is used, e.g., in celestial mechanics in connection with the planets and the sun. The high accuracy achieved in such calculations, of course, in no way implies that celestial bodies have vanishingly small dimensions. In the same way, success in solving external problems in no way should be regarded as proof that the electron and other particles have vanishingly small dimensions.

The Schrödinger electrodynamic interpretation of the wave function, propounded in a classical spirit and consistent with the scientific principles that phenomena are deterministic and knowable (perspicuous) has been rejected in its time. The probabilistic interpretation, based on the idea of a point electron and contradicting the causational origin of individual events and phenomena in the microscopic world, has come to predominate. As it happens, Max Born arrived at the probabilistic interpretation precisely as a result of studying collision and scattering processes, and not the internal structure of the atom.¹⁷

Nearly all of the creators of quantum mechanics—Planck, Einstein, de Broglie, Schrödinger, Heisenberg, and Dirac—at one time or another and in various ways objected to abandoning classical principles. In connection with the negation of classical determinism Dirac wrote, “I feel very strongly that the stage physics has reached at the present day is not the final stage.”²¹

During the years in which the theory of atomic phenomena was being developed, along with the probabilistic-wave ideas, which have subsequently become generally accepted, alternative ideas and solutions lying in the classical mainstream were advanced. The Schrödinger model of the atom is only one of these solutions. The history of quantum mechanics is undoubtedly in need of more complete illumination and study.

APPENDIX

M. Bowler has presented a derivation of the Maxwell equations based on special relativity, the Coulomb law, and the law of electric charge conservation.⁸

First, the Coulomb law directly yields the laws of electrostatics, including the Poisson equation

$$\Delta\varphi = -4\pi\rho,$$

which related the potential φ and the electric charge density ρ . Of course, this equation does not satisfy the requirement of covariance under Lorentz transformations: it contains only the three spatial coordinates x, y, z . It is necessary to augment it by introducing the time t as a fourth coordinate, i.e., replacing the Laplacian Δ by the D'Alembertian \square :

$$\Delta\varphi - \frac{1}{c^2} \frac{\partial^2 \varphi}{\partial t^2} = \square\varphi = -4\pi\rho.$$

In this equation φ and ρ must have the same transformation properties. If we take into account charge conservation, this is only possible if φ and ρ are corresponding components of two 4-vectors. These 4-vectors can be written in the form

$$A_\nu = (A, i\varphi), \quad J_\nu = (\mathbf{J}, ic\rho).$$

It is natural to call \mathbf{A} the vector potential and \mathbf{J} the current density. It is clear that other pairs of corresponding components must also be related in the same way as φ and ρ :

$$\square A_\nu = -\frac{4\pi}{c} J_\nu \quad (\nu = 1, 2, 3, 4).$$

But this is the system of classical electrodynamic equations in the four-dimensional notation. The equation for φ is obtained from this for $\nu=4$. The transition from the potentials \mathbf{A}, φ to the field vectors \mathbf{E}, \mathbf{H} yields the Maxwell equation.

Finally, Bowler has derived the covariant equation for the rate at which the momentum of a charged particle changes in an electromagnetic field and has obtained the well-known expression for the Lorentz force on the right-hand side of this equation:

$$\frac{dp}{dt} = q \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}\mathbf{H}] \right).$$

This derivation of the equations of classical electrodynamics from the special theory of relativity is a fundamental result which could have been obtained, however, already in the period 1905–1907.

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