Thermoelastic stresses as a mechanism of electromagnetic-acoustic transformation

A. N. Vasil'ev and F. M. Maallavi

M. V. Lomonosov Moscow State University, Moscow

M. I. Kaganov

P. L. Kapitsa Institute of Physics Problems, Russian Academy of Sciences, Moscow

(Submitted 29 July 1993)
Usp. Fiz. Nauk **163**, 81–93 (October 1993)

The excitation of thermal waves of frequency ω in an anisotropic conductor by an electromagnetic wave of the same frequency incident on its surface is investigated. The normal to the metal surface does not coincide with the axis of symmetry of the conductor. Due to the thermoelectric effect, oscillations of temperature appear in the metal which, in turn, excite ultrasonic waves as a result of thermoelastic stresses. The role played by the constant, relatively high, but nonquantizing magnetic field in the linear thermoelastic generation of sound has been investigated. The nonlinear (at a frequency of 2ω) thermoelastic generation of sound has been studied and an analysis is given of the experimental situation in this field.

1. INTRODUCTION

The traditional subject of investigation in problems of the electromagnetic-acoustic transformation (EMAT) are the mechanisms of the linear transformation responsible for the generation of ultrasound in a metal at the frequency of the electromagnetic wave incident on its surface. Although being significantly less effective than the piezoelectric and magnetostriction transformations, EMAT nevertheless is of interest both for the study of the intrinsic acoustic properties of conductors, and for the determination of the coupling parameters between the electron, ion and spin subsystems. Among the sources of linear transformation of waves that have been studied in greatest detail are the induction and deformation forces.[1] Moreover, the inertial force (the Stewart–Tolman force) can serve as a source of the linear EMAT,[1] as well as the inhomogeneous oscillations of the temperature due to the thermoelectric effect.[22] The manner in which the oscillations of temperature can serve as a source of acoustic oscillations under the action of electromagnetic waves is shown schematically as follows:

Transformation	Source
Electromagnetic wave	
↓	Thermoelectric effect
Temperature oscillations	
↓	Thermoelastic stresses
Ultrasound	

It is possible to realize the scheme of linear EMAT as a result of thermoelastic stresses only in a significantly anisotropic conductor or in a metal in which anisotropy of conductivity is produced by a magnetic field. In an isotropic metal the thermoelastic stresses serve as a source of generation of ultrasound at double the frequency of the electromagnetic wave.

Before going on to investigate the linear and the nonlinear thermoelastic generation of ultrasound in metals we note that in this article we use the notation for various physical quantities adopted in Ref. 1 and only the newly introduced quantities are defined here.

2. THERMAL WAVES IN A METAL

Temperature oscillations and electromagnetic oscillations (although obviously being different) have the following in common that the propagation of both types of oscillations in a metal is accompanied by a skin-effect. Indeed, the small temperature oscillations excited at a frequency ω by a heat source at the surface of a semi-infinite $(z>0)$ sample are described by the homogeneous equation of heat conductivity

$$C\partial\theta/\partial t-\varkappa\partial^2\theta/\partial z^2=0, \tag{1}$$

where $\theta=\mathrm{Re}\,\theta(z)e^{-i\omega t}$ is the variable (oscillating) part of the temperature. The total temperature of the body is equal to $T+\theta$ and it is assumed that $|\theta|\ll T$. This enables one to regard the heat capacity of a unit volume $C=C(T)$ and the heat conductivity coefficient $\varkappa=\varkappa(T)$ as being constant quantities independent of the coordinate z and of the time t. The solution of the equation (1) is the expression

$$\theta(z,t)=\mathrm{Re}(\theta_0 e^{i(k_\mathrm{T}z-\omega t)}),$$
$$k_\mathrm{T}^2=i\omega C/\varkappa,\quad \mathrm{Im}\,k_\mathrm{T}>0. \tag{2}$$

The value of the amplitude θ_0 is determined by the source (cf., below), while the depth of the thermal skin layer is

$$\delta_\mathrm{T}=(2\varkappa/\omega C)^{1/2}. \tag{3}$$

The depth of penetration of the temperature wave δ_T just as the electromagnetic depth of penetration

$$\delta_\mathrm{E}=c/(2\pi\sigma\omega)^{1/2}, \tag{4}$$

falls with the increase in the frequency ω inversely proportional to $\omega^{1/2}$. For making estimates it is convenient to use expressions valid under the simplest assumptions concerning the conduction electrons:

$$\sigma = ne^2 l/p_F,$$

$$\varkappa = \pi^2 T\sigma/3e^2 \sim Cl\, v_F. \tag{5}$$

We assume that the main transporters of heat are the electrons. Whether C coincides with the electron heat capacity, or with the heat capacity of the entire body is not too significant for the present, since formula (3) contains the coefficient of temperature conductivity $\varkappa/C$ which does not depend on the heat capacity.

Comparing the depths of penetration δ_T and δ_E one can see that their ratio is independent of the frequency:

$$\delta_T/\delta_E = l/\delta_0. \tag{6}$$

At room temperature when the mean free path of the carriers is less than the plasma depth of penetration ($l<\delta_0$), the depth of the thermal skin layer δ_T is less than the depth of the electromagnetic skin layer δ_E. At low temperatures in pure metals we have in contrast $l>\delta_0$ and $\delta_T>\delta_E$.

Apparently it was noted for the first time in Ref. 4 that in the case of propagation of temperature oscillations in a metal there is no phenomenon analogous to the anomalous skin-effect.[5,14] Formulas (3)–(6) and the estimates given above are based on a macroscopic description requiring the validity of a number of inequalities:

$$l \ll \delta_E, \quad l \ll \delta_T, \quad \omega\tau \ll 1. \tag{7}$$

If the last inequality does not hold (condition of a quasistatic situation), then naturally it is not possible to speak of the temperature oscillations (2), since the temperature is a characteristic of an equilibrium thermodynamic system. Throughout the entire article we shall assume that the inequality $\omega\tau \ll 1$ holds. We shall show first of all that in such a case the condition $\delta_T \gg l$ also necessarily holds. Indeed, in accordance with (3) and (5) we have

$$\delta_T \approx (l\, v_F/\omega)^{1/2} = l/(\omega\tau)^{1/2} \gg l. \tag{8}$$

On the other hand, since $\delta_E \sim \delta_0/(\omega\tau)^{1/2}$, then when $l \gg \delta_0$ (low temperature pure metal) there is a frequency range

$$(\delta_0/l)^2 < \omega\tau \ll 1, \tag{9}$$

when the condition for the skin effect to be anomalous ($\delta_E<l$) does not conflict with the condition for a quasistatic situation ($\omega\tau \ll 1$). We note in passing that the skin-effect being anomalous does not contradict the macroscopic nature of the phenomenon since although $\delta_E<l$, both these quantities definitely exceed the size of the crystalline cell of the metal.

In conclusion of this section we emphasize once more: the applicability of the macroscopic equation for heat transfer [of the type (1)] is restricted only by the condition for a quasistatic situation ($\omega\tau \ll 1$); consequently it can be used when the frequency of the source corresponds (for electromagnetic oscillations) to the anomalous skin-effect [of course, within the limits of the inequality (9)].

3. COUPLED ELECTROMAGNETIC-ACOUSTIC OSCILLATIONS

Taking into account thermoelectric phenomena in investigating the propagation of an electromagnetic wave in a conductor requires (in accordance with Ref. 6) the inclusion into the complete system of equations along with the Maxwell equations

$$\mathrm{rot}\,\mathbf{H} = 4\pi\mathbf{j}/c,$$

$$\mathrm{rot}\,\mathbf{E} = i\omega\mathbf{H}/c, \tag{10}$$

the equation for heat conductivity

$$C\theta + \mathrm{div}\,\mathbf{q} = 0; \tag{11}$$

here $\mathbf{q}$ is the heat flux density. Equations (10) and (11) must be supplemented by the material equations which express $\mathbf{j}$ and $\mathbf{q}$ in terms of $\mathbf{E}$ and θ:

$$E_i = \rho_{ik} j_k + \alpha_{ik}\partial\theta/\partial x_k, \tag{12}$$

$$q_i = T\alpha_{ki} j_k - \varkappa_{ik}\partial\theta/\partial x_k; \tag{13}$$

here we have used the generally accepted notation: $\rho_{ik} = \sigma_{ik}^{-1}$ is the tensor of specific resistances, σ_{ik} and $\varkappa_{ik}$ are the tensors for the specific electric and heat conductivity, α_{ik} is the tensor of the thermoelectric coefficients. The requirements of the principle of symmetry of kinetic coefficients have been taken into account (cf., Refs. 7,8).

We now study the normal incidence of electromagnetic waves onto the half-space $z>0$ occupied by an anisotropic metal. For simplicity we assume that the metal is uniaxial. The crystal axis makes an angle φ with the normal to the surface (with the z axis). The incident wave is polarized so that $E_y = H_x = 0$, while $E_x, H_y \neq 0$. All the quantities depend only on the z coordinate, $j_y = j_z = 0$, while the nonvanishing functions ($E_x, H_y, q_z, \partial\theta/\partial z$) are related by the following expressions:

$$-\mathrm{d}H_y/\mathrm{d}z = 4\pi j_x/c,$$

$$\mathrm{d}E_x/\mathrm{d}z = i\omega H_y/c, \tag{14}$$

$$-i\omega C\theta + \mathrm{d}q_z/\mathrm{d}z = 0, \tag{15}$$

$$E_x = \rho_{xx} j_x + \alpha_{xz}\mathrm{d}\theta/\mathrm{d}z, \tag{16}$$

$$q_z = T\alpha_{xz} j_x - \varkappa_{zz}\partial\theta/\partial z. \tag{17}$$

It is convenient to rewrite the system of equations (14)–(17) as follows:

$$\partial^2 j_x/\partial z^2 + 4\pi i\omega j_x/c^2\rho_{xx} = -(\alpha_{xz}/\rho_{xx})\partial^3\theta/\partial z^3,$$

$$\partial^2\theta/\partial z^2 + i\omega C\theta/\varkappa_{zz} = T(\alpha_{xz}/\varkappa_{zz})\partial j_x/\partial z. \tag{18}$$

The characteristic equation of the system (18) for the wave vectors k can be rewritten as follows:

$$(k_E^2 k^{-2} - 1)(k_T^2 k^{-2} - 1) = T\alpha_{xz}^2/\rho_{xx}\varkappa_{zz},$$

$$k_E^2 = 4\pi i\omega/c^2\rho_{xx},$$

$$k_T^2 = i\omega C/\varkappa_{zz}. \tag{19}$$

From this it can be seen that the coupling between the electrodynamic and thermal oscillations is determined by the dimensionless parameter

$$a = T\alpha_{xz}^2/\rho_{xx}\varkappa_{zz}, \tag{20}$$

which for "good" metals satisfies $\sim (T/\varepsilon_F)^2 \ll 1$.

Before proceeding we note that k_E, k_T and a involve different tensor components (for example ρ_{xx} and $\varkappa_{zz}$). In a strongly anisotropic conductor their values can depend in an essential manner on the geometrical formulation of the problem. For example, in a layered conductor they may depend on the polarization of the electric field of the wave with respect to the layers. These problems go beyond the framework of our article and require a special examination.

Since $a \ll 1$ we can use the approximate values of the roots of the characteristic equation (19). When

$$k_T^2 \neq k_E^2,$$

$$k_1^2 \approx k_E^2[1 - ak_E^2/(k_T^2 - k_E^2)],$$

$$k_2^2 \approx k_T^2[1 - ak_T^2/(k_E^2 - k_T^2)]. \tag{21}$$

At high temperatures $|k_T^2| \gg |k_E^2|$ and

$$k_1^2 \approx k_E^2(1 - ak_E^2 k_T^{-2}),$$

$$k_2^2 \approx k_T^2(1 + a), \tag{21'}$$

at low temperatures $|k_T^2| \ll |k_E^2|$ and

$$k_1^2 \approx k_E^2(1 + a),$$

$$k_2^2 \approx k_T^2(1 - ak_T^2 k_E^{-2}). \tag{21''}$$

If $k_T^2 = k_E^2$ (this condition, analogous to the resonance condition, must hold at an intermediate temperature, when $l \sim \delta_0$) then

$$k_{2,1}^2 \approx k^2(1 \pm a^{1/2}),$$

$$k^2 = k_T^2 = k_E^2. \tag{22}$$

Naturally it is not difficult to write down the exact expressions for the roots of equation (19)

$$k_{1,2}^{-2} = \tfrac{1}{2}(k_E^{-2} + k_T^{-2}) \pm \tfrac{1}{2}[(k_E^{-2} - k_T^{-2})^2 + 4a(k_E k_T)^{-2}]^{1/2}. \tag{23}$$

We note for any value of the parameter $a (a > 0)$ and k_1^2, and k_2^2 are purely imaginary quantities [cf., the definition of k_E^2 and k_T^2 in (19)]; moreover, strict resonance between two types of oscillations is impossible: if $a \neq 0$ then $k_1^2 \neq k_2^2$.

The system of equations (14)–(17) [or (18)] for the solution of the problem concerning the propagation of coupled electrodynamic thermal oscillations requires, in addition to the natural electrodynamic boundary conditions, the formulation of conditions describing the thermal contact of the sample with the external medium. It is natural to assume that within the bulk of the sample the temperature is equal to the equilibrium temperature, i.e., $\theta = 0$ as $z \to \infty$.

The boundary condition at $z = 0$ (on the surface of the metal) depends on the particular conditions of heat removal. We consider two limiting cases:

isothermal boundary

$$\theta|_{z=0} = 0, \tag{24}$$

adiabatic boundary

$$q_z|_{z=0} = 0,$$

or

$$\varkappa_{zz} d\theta/dz|_{z=0} = T\alpha_{xz} j_x(0). \tag{25}$$

We have utilized equation (17).

The solution (dependence of all the quantities on the z coordinate) can be conveniently written down by introducing two amplitudes A_1 and A_2, the relationship between which is obtained from the boundary condition for the temperature

$$j_x = A_1 \exp(ik_1 z) + A_2 \exp(ik_2 z),$$

$$\theta(z) = iT\alpha_{xz}/\varkappa_{zz}[k_1 A_1 \exp(ik_1 z)(k_T^2 - k_1^2)^{-1} + k_2 A_2 \exp(ik_2 z)(k_T^2 - k_2^2)^{-1}],$$

$$E_x(z) = 4\pi i\omega/c^2[A_1 \exp(ik_1 z)k_1^{-2} + A_2 \exp(ik_2 z)k_2^{-2}],$$

$$H_y(z) = 4\pi i/c[A_1 \exp(ik_1 z)k_1^{-2} + A_2 \exp(ik_2 z)k_2^{-2}]; \tag{26}$$

here k_1 and k_2 are the roots of the characteristic equation (19) [cf., (23)] keeping in mind that in order to guarantee the damping of all waves within the bulk of the sample we have chosen Im $k_{1,2} > 0$.

Using the boundary conditions (24) and (25) we find in the case of an isothermal boundary

$$A_2/A_1 = -k_1(k_T^2 - k_2^2)[k_2(k_T^2 - k_1^2)]^{-1}, \tag{27}$$

and the case of an adiabatic boundary

$$A_2/A_1 = -(k_T^2 - k_2^2)/(k_T^2 - k_1^2). \tag{28}$$

On the basis of these expressions it is easy to write out the distribution of the temperature over the sample and to calculate the impedance of the metal

$$Z = E_x(0)/H_y(0). \tag{29}$$

For an isothermal boundary:

$$\theta(z) = \mathrm{Re}\{(cH_y(0)/4\pi)T\alpha_{xz}k_1^2 k_2^2[\exp(ik_1 z) - \exp(ik_2 z)][\varkappa_{zz}k_T^2(k_2^2 - k_1^2)]^{-1}\}, \tag{30}$$

$$Z = (\omega/ck_1)[k_T^2(k_1^2 + k_1 k_2 + k_2^2) - k_1^2 k_2^2] \times [(k_T^2(k_1 + k_2)k_2)]^{-1}. \tag{31}$$

For an adiabatic boundary:

$$\theta(z) = \mathrm{Re}\{(cH_y(0)/4\pi)T\alpha_{xz}k_1 k_2 \times [k_1 \exp(ik_1 z) - k_2 \exp(ik_2 z)] \times [\varkappa_{zz}(k_2 - k_1)(k_1 k_2 + k_T^2)]^{-1}\}, \tag{32}$$

$$Z=(\omega/ck_1)(k_1+k_2)k_{\rm T}^2[k_2(k_1k_2+k_{\rm T}^2)]^{-1}. \quad (33)$$

Substituting into formulas (30)–(33) the values of the roots of the characteristic equation (23) we obtain expressions for the temperature distribution $\theta=\theta(z)$ and for the impedance Z. For subsequent discussion we require the temperature distribution. As regards the impedance we restrict ourselves to a single assertion. As we have already stated $k_{1,2}^2$, are purely imaginary quantities, $\operatorname{Im} k_{1,2}>0$; moreover, both roots are proportional to $\omega^{1/2}$. Therefore in accordance with formulas (31) and (33) in both cases we have

$$Z=|Z|(1-i)/\sqrt{2}, \quad |Z|=K\omega^{1/2},$$

the coefficient of proportionality K depends on the components of the tensors of the kinetic coefficients and differs in the case of an isothermal and an adiabatic boundary.

It can be seen that the discovery of the role played by the thermoelectric effect in the surface impedance is possible only in the case that its absolute value can be measured and compared with the classical value

$$Z=(1-i)(\omega\rho_{xx}/8\pi)^{1/2}.$$

It is true, perhaps, that it is possible to discover the role played by the heat flux based on the violation of the relation $(\operatorname{Re} Z)^2=(\operatorname{Im} Z)^2=Q\rho_{xx}$ with the factor Q being independent both of the temperature and of the characteristics of the metal.

Using the fact that the parameter satisfies $a\ll 1$, we write the approximate expressions for the temperature.

For an isothermal boundary ($a\ll 1$):

$$\theta(z)\approx\operatorname{Re}\{(cH_y(0)/4\pi)(T\alpha_{xz}/\varkappa_{zz})[k_E^2/(k_{\rm T}^2-k_{\rm E}^2)] \times(e^{ik_{\rm E}z}-e^{ik_{\rm T}z})\}. \quad (34)$$

For an adiabatic boundary ($a\ll 1$):

$$\theta(z)\approx\operatorname{Re}\{(cH_y(0)/4\pi)(T\alpha_{xz}/\varkappa_{zz})[k_{\rm E}/(k_{\rm T}^2-k_{\rm E}^2)] \times(k_{\rm E}e^{ik_{\rm E}z}-k_{\rm T}e^{ik_{\rm T}z})\}. \quad (35)$$

The smallness of the parameter a which indicates the weak coupling between the electrodynamic and thermal oscillations enables one to calculate the temperature distribution approximately by using the equation for the heat conductivity [the second equation of the system (18)] in which the term $T\alpha_{xz}\partial j_x(z)/\partial z$ serves as the source, while $j_x(z) = j_x(0)e^{ik_{\rm E}z}$ is the current density unperturbed by the thermoelectric forces. Taking into account the fact that in such a case [in accordance with (14) $j_x(0)=-ik_{\rm E}cH_y(0)/4\pi$] we obtain formulas (34) and (35). In conclusion of this section we call attention to the fact that at low temperatures ($|k_{\rm T}^2|\ll|k_{\rm E}^2|$) the temperature distribution at relatively large distances from the surface is determined by the thermal characteristics of the metals and not by the electrodynamical ones: the heat wave is damped over a distance $\delta_{\rm T}$, which significantly exceeds the depth of the skin layer $\delta_{\rm E}$. Moreover, the electromagnetic field and the current density $j_x(z)$ are also pulled by the thermal wave to a depth $\delta_{\rm T}\gg\delta_{\rm E}$ [cf., formula (26)]. Naturally the smaller is the coupling between the oscillations the smaller is the amplitude of the pulled wave [$A_2\to 0$ as $a\to 0$, cf., formulas (27) and (28), if $a=0$, then $k_2=k_{\rm T}$].

4. THERMAL OSCILLATIONS IN THE CASE OF A SURFACE SOURCE

At a low temperature when the depth of the thermal skin-layer $\delta_{\rm T}$ exceeds considerably the electromagnetic depth $\delta_{\rm E}$ the temperature distribution at a distance significantly in excess of the depth of the skin-layer $\delta_{\rm E}$ can be determined without specifying the specific dependence of the heat source on the z coordinate. In accordance with the second equation of the system (18)

$$d^2\theta/dz^2+k_{\rm T}^2\theta=dQ(z)/dz; \quad (36)$$

here

$$Q(z)=(T\alpha_{xz}/\varkappa_{zz})j_x(z)\equiv q_e(z)/\varkappa_{zz}, \quad (37)$$

where $q_e(z)$ is a part of the heat flux density due to the thermoelectric effect. In the case of an isothermal boundary $\theta(z)$ satisfies condition (24), while in the case of an adiabatic boundary it satisfies condition (25) which in the new notation assumes the following form:

$$d\theta/dz|_{z=0}=Q(0). \quad (38)$$

With the aid of Green's functions satisfying the zero boundary conditions one can easily obtain expressions for $\theta(z)$.

For an isothermal boundary:

$$\theta(z)=\frac{e^{ik_{\rm T}z}}{2}\int_0^z(e^{ik_{\rm T}z'}+e^{-ik_{\rm T}z'})Q(z')dz' +\tfrac{1}{2}(e^{ik_{\rm T}z}-e^{-ik_{\rm T}z})\int_z^\infty e^{ik_{\rm T}z'}Q(z')dz',$$

$$\operatorname{Im} k_{\rm T}>0. \quad (39)$$

For an adiabatic boundary:

$$\theta(z)=-\frac{e^{ik_{\rm T}z}}{2}\int_0^z(e^{ik_{\rm T}z'}-e^{-ik_{\rm T}z'})Q(z')dz' -\tfrac{1}{2}(e^{ik_{\rm T}z}+e^{-ik_{\rm T}z})\int_z^\infty e^{ik_{\rm T}z'}Q(z')dz',$$

$$\operatorname{Im} k_{\rm T}>0. \quad (40)$$

In the above expressions for z close to the boundary both terms are important, but the asymptotic behavior for $z\gg\delta$ where δ is the depth of damping of the source $Q=Q(z)$, is determined in both cases only by the first term in which one must make the limit of integration tend to infinity, while the expression in brackets under the integral sign must be expanded in powers of $k_{\rm T}z'$ and restrict oneself to the first nonvanishing term. As a result we shall have:

For an isothermal boundary:

$$\theta(z)\approx\operatorname{Re} e^{ik_{\rm T}z}\int_0^\infty Q(z')dz',$$

$$|k_{\rm T}|\delta_{\rm E}\ll 1. \quad (41)$$

For an adiabatic boundary:

$$\theta(z)\approx\mathrm{Re}\left(-ik_{\mathrm{T}}e^{ik_{\mathrm{T}}z}\int_0^\infty z'Q(z')\mathrm{d}z'\right),$$

$$|k_{\mathrm{T}}|\delta_{\mathrm{E}}\ll 1. \tag{42}$$

We note that the temperature distribution in the entire body (with the exception of a narrow layer near the surface) is determined by the integral characteristic of the source and not by its detailed structure, and the boundary condition (adiabatic or isothermal boundary) "selects" the form of the integral characteristic [cf., formulas (41) and (42)]. And in addition: it might appear strange that in the case of the adiabatic boundary condition when heat is not at all removed from the surface the amplitude of the thermal wave is smaller than in the case of the isothermal boundary condition when there is a definite heat flux across the boundary. The seeming paradox, apparently, can be explained in the following manner: due to the value of the derivative $\mathrm{d}\theta/\mathrm{d}z|_{z=0}$ fixed by the adiabatic boundary condition the greater part of the energy of the source remains in the electromagnetic wave [cf., formula (35)] and is dissipated over a distance $\sim\delta_{\mathrm{E}}$.

If in formulas (41) and (42) we substitute

$$Q(z)=-\frac{iT\alpha_{xz}}{\varkappa_{zz}}\frac{k_{\mathrm{E}}c}{4\pi}H_y(0)e^{ik_{\mathrm{E}}z}, \tag{43}$$

then from (41) and (42) we obtain in accordance with (34) and (35) expressions for θ.

For an isothermal boundary:

$$\theta\approx\frac{cH_y(0)}{4\pi}\frac{T\alpha_{xz}}{\varkappa_{zz}}\mathrm{Re}\,e^{ik_{\mathrm{T}}z},$$

$$|k_{\mathrm{T}}|\ll|k_{\mathrm{E}}|, \tag{44}$$

for an adiabatic boundary:

$$\theta\approx\frac{cH_y(0)}{4\pi}\frac{T\alpha_{xz}}{\varkappa_{zz}}\frac{k_{\mathrm{T}}}{k_{\mathrm{E}}}\mathrm{Re}\,e^{ik_{\mathrm{T}}z},$$

$$|k_{\mathrm{T}}|\ll|k_{\mathrm{E}}|. \tag{45}$$

Formulas (41) and (42) together with formula (37) can be used to calculate the temperature field under the conditions of anomalous electrodynamical skin-effect when the mean free path is $l\gg\delta_{\mathrm{E}}$ while the heat flux density $q_e(z)$ is damped nonexponentially as the z coordinate increases.

5. EXCITATION OF ULTRASOUND

In an isotropic crystal three waves can be propagated along any arbitrary direction, the three modes are analogs of transverse and longitudinal oscillations in elastically isotropic media—with three (in the general case) different velocities S_1,S_2,S_3. The mutually orthogonal polarization vectors $\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3$ of these oscillations depend on the direction of the wave vector of sound $\mathbf{k}_{\mathrm{S}}$ with respect to the crystallographic axes of the crystal. Even a cubic crystal is elastically anisotropic, and in it the directions of the polarization vectors depend in a complicated manner on the direction of the $\mathbf{k}_{\mathrm{S}}$ vector. The oscillations of the three modes are independent and are described by three wave equations:

$$\frac{\mathrm{d}^2U_i}{\mathrm{d}z^2}+k_{\mathrm{S}}^2U_i=F_i,$$

$$F_i=\frac{(\mathbf{e}_i\mathbf{f}(z))}{\rho S_i^2}, \tag{46}$$

$U_i=(\mathbf{e}_i\mathbf{U})$ is the component of the displacement vector U along the ith polarization vector, $\mathbf{f}(z)$ is the density of the force exciting the sound (the factor $e^{-i\omega t}$ has been omitted). In the case of the thermoelastic mechanism of EMAT

$$f_i(z)=B_{iz}\partial\theta/\partial z, \tag{47}$$

where B_{ik} is the tensor determining the thermoelastic part σ_{ik}^{T} of the stress tensor σ_{ik}:

$$\sigma_{ik}=\sigma_{ik}^{\mathrm{el}}+\sigma_{ik}^{T},$$

$$\sigma_{ik}=-B_{ik}\theta(z,t), \tag{48}$$

$\sigma_{ik}^{\mathrm{el}}$ is the elastic part of the stress tensor,[9] while $B_{ik}=\rho S^2\beta_{ik}$, where β_{ik} in order of magnitude is the tensor of the coefficients of thermal expansion ($[\beta_{ik}]=1/\mathrm{deg}$). Therefore in our case we have

$$F_i=\beta_{iz}\partial\theta/\partial z. \tag{49}$$

The boundary conditions for equations (46) depend on the particular formulation of the problem. In the theory of EMAT it is customary to distinguish two limiting cases:

$$\text{fixed boundary } \mathbf{U}|_{z=0}=0, \tag{50}$$

$$\text{free boundary } \sigma_{iz}|_{z=0}=0. \tag{51}$$

In the case of a fixed boundary condition (50) denotes that each of the three functions U_i vanishes at the boundary of the sample

$$U_i|_{z=0}=0. \tag{50'}$$

The condition at the free boundary

$$\sigma_{iz}^{\mathrm{el}}|_{z=0}=-\sigma_{iz}^{\mathrm{T}}|_{z=0} \tag{51'}$$

can be treated as the appearance of a surface source of an exciting force

$$f_i^{\mathrm{surf}}=B_{iz}\theta(z)|_{z=0}. \tag{52}$$

The surface force in the EMAT theory is usually regarded as an independent source of sound waves.[10] The existence of a surface force, as a rule, is related to the nonspecular nature of the reflection of electrons from the surface. It may be seen that in the case of a thermoelastic mechanism of EMAT if $\theta(z=0)\neq 0$ (a nonisothermal boundary, cf., Sec. 3), the surface force differs from zero independently of the nature of reflection of electrons by the boundary.

Formulas (46)–(52) supplemented by the expressions obtained in Sec. 3 for the temperature distribution $\theta=\theta(z)$ [cf., (30), (32) or (34), (35) and (44), (45)], enable one to calculate the amplitude of sound in the metal.

In what follows in order to avoid additional complication we shall assume the metal to be elastically isotropic. It is true that even in this case one cannot neglect the anisotropy of the electron properties: the amplitude of the temperature oscillations is proportional to α_{xz}—the *nondiagonal* component of the tensor of the thermoelectric coefficients, which is equal to zero in an isotropic conductor.

The neglect of elastic anisotropy, it would appear to us, will not lead to the loss of any interesting features of EMAT. Perhaps, one should only note that in an elastically anisotropic body the boundary condition (51) can lead to mixing of oscillations with different polarizations (cf., with the theory of Rayleigh surface waves[9]).

One can easily convince oneself that due to the thermoelectric effect only the longitudinal wave is excited in an elastically isotropic conductor. We denote its amplitude by the letter U (without subscripts). The problem concerning the excitation of sound waves assumes the following form:

$$d^2U/dz^2+k_S^2U=\beta d\theta/dz, \tag{53}$$

where β in order of magnitude coincides with the volume coefficient of thermal expansion $k_S=\omega/S$, while S is the velocity of longitudinal sound. The boundary conditions for the problem can be rewritten in the form:

$$\text{fixed boundary } U|_{z=0}=0, \tag{54}$$

$$\text{free boundary } dU/dz|_{z=0}=\beta\theta|_{z=0}. \tag{55}$$

It is of interest to note that when the latter boundary condition is satisfied the surface force is equal to the volume force acting on the lattice with a reversed sign. This property holds also when a deformation force is acting on the lattice, and the reflection of electrons is not specular.[1,10]

We shall calculate the temperature distribution in two cases: for an isothermal boundary [formulas (30), (34) and (44)] and for an adiabatic one [formulas (32), (35) and (45)]. Therefore we have to calculate the sound amplitude in four cases. In order to regulate the results we shall separately consider the case of a fixed boundary, and also separately that of a free boundary.

There are many different parameters in the problem. In particular, there are the three wave vectors: the electrodynamic one k_E, the thermal one k_T and the sound one $k_S=2\pi/\lambda$. Usually the EMAT is realized under conditions when the wavelength of sound λ considerably exceeds the layer near the surface of the metal where the exciting force is concentrated. Therefore in subsequent discussion we shall quote results valid for

$$k_S\ll|k_E|,\quad |k_T|. \tag{56}$$

This means that the frequency ω is not too small and satisfies the following conditions easily fulfilled in practice:

$$\omega\gg 2\pi\sigma S^2/c^2,\quad \nu S^2/v_F^2,$$

$$\nu=1/\tau. \tag{57}$$

Fixed boundary. By using the Green's function method it is easy to solve equation (53) with the boundary condition (54) with an arbitrary dependence $\theta=\theta(z)$. At large distances from the surface the asymptotic equality

$$U(z)\sim U_\infty^{\text{fix}}e^{ikz}, \tag{58}$$

holds, where

$$U_\infty^{\text{fix}}=\beta\int_0^\infty \theta(z')dz'. \tag{59}$$

In the case of an adiabatic boundary [cf., (32) and (35)]

$$\int_0^\infty \theta(z)dz=0$$

and one should use the quadratic term of the expansion in terms of k_Sz. Thus, in the case of an adiabatic boundary we have

$$U^{\text{fix}}=\frac{\beta k_S^2}{2}\int_0^\infty z^2\theta(z)dz. \tag{60}$$

Substituting expression (34) into formula (59) and the expression (35) into (60) we obtain ($k_S\ll|k_E|$, $|k_T|$):

isothermal boundary

$$|U_\infty^{\text{fix}}|=\frac{cH_y(0)}{4\pi}\beta\frac{T\alpha_{xz}}{\varkappa_{zz}}\left|\frac{k_E}{k_T(k_T+k_E)}\right|,$$

adiabatic boundary

$$|U_\infty^{\text{fix}}|=\frac{cH_y(0)}{4\pi}\beta\frac{T\alpha_{xz}}{\varkappa_{zz}}\frac{k_S^2}{|k_T^2k_E|}. \tag{61}$$

Free boundary. When the mechanically free boundary is under isothermal conditions, then the boundary condition (55) is simplified

$$\left.\frac{\partial U}{\partial z}\right|_{z=0}=0-\text{isothermal boundary}. \tag{55'}$$

Independently of thermal conditions in the case of a (mechanically) free boundary

$$U(z)\sim U_\infty^{\text{free}}e^{ikz}, \tag{62}$$

$$U_\infty^{\text{free}}=-ik_S\beta\int_0^\infty z\theta(z)dz.$$

Using formulas (34) and (35) we obtain

isothermal boundary

$$|U_\infty^{\text{free}}|=\frac{cH_y(0)}{4\pi}\beta\frac{T\alpha_{xz}}{\varkappa_{zz}}\frac{k_S}{|k_T^2|},$$

adiabatic boundary

$$|U_\infty^{\text{free}}|=\frac{cH_y(0)}{4\pi}\beta\frac{T\alpha_{xz}}{\varkappa_{zz}}\frac{k_S}{|k_T(k_E+k_T)|}. \tag{63}$$

Leaving to later discussion the absolute evaluation of the EMAT mechanism under discussion here we compare the effect of the boundary conditions (mechanical and thermal) on EMAT. If the boundary is fixed then it can be seen from formula (60) that the isothermal nature of the boundary favors the transformation, in the case of a free boundary everything depends on the ratio $|k_T/k_E|$ [cf., formula (63)]. In concluding this section we note: formulas (60) and (62) should be regarded as examples. In a comparison of theory with experiment (and/or in planning an experiment) with the aid of the formulas available here for $\theta=\theta(z)$ it is not difficult to calculate the amplitude of the excited sound wave for any ratio between the wavelength of sound λ and the depths of the skin-layers (the electromagnetic $\delta_{\rm E}$ and the thermal $\delta_{\rm T}$) and to select a convenient frequency range, temperature range etc.

6. THE ROLE PLAYED BY THE MAGNETIC FIELD

We have repeatedly emphasized that the linear thermoelastic mechanism of EMAT is possible only in an anisotropic conductor. However, anisotropy can be created artificially by placing the conductor in a constant magnetic field $\mathbf{H}_0$. Since the electrons move differently along the magnetic field and in the plane perpendicular to the magnetic field an anisotropy of the kinetic coefficients appears which is necessary for the excitation of a temperature wave.

Equations (12) and (13) for $H_0\neq 0$ take on the following form:

$$E_i=\rho_{ik}(\mathbf{H}_0)j_k+\alpha_{ik}(\mathbf{H}_0)\partial\theta/\partial x_k, \qquad (12')$$

$$q_i=T\alpha_{ki}(-\mathbf{H}_0)j_k-\varkappa_{ik}(\mathbf{H}_0)\partial\theta/\partial x_k, \qquad (13')$$

where $\rho_{ik}(\mathbf{H}_0)=\rho_{ki}(-\mathbf{H}_0)$ and $\varkappa_{ik}(\mathbf{H}_0)=\varkappa_{ki}(-\mathbf{H}_0)$; the coefficient in front of j in (13′) is written down observing the principle of symmetry of kinetic coefficients. The principle of symmetry of kinetic coefficients does not require the symmetry of the tensor α_{ik} for $\mathbf{H}_0=0$, but in the majority of metals $\alpha_{ik}(\mathbf{H}_0=0)=\alpha_{ki}(\mathbf{H}_0=0)$ due to the relatively high symmetry of the crystalline lattice.

If we neglect the quantum oscillations of the kinetic coefficients their dependence on the magnetic field $\mathbf{H}_0$ can be determined on the basis of Boltzmann's kinetic equation, naturally, taking into account the structure of the energy spectrum of the electrons. Without using simplified models one can determine only the asymptotic behavior of the components of the tensors ρ_{ik}, $\varkappa_{ik}$ and α_{ik} in a strong magnetic field.[8] Here we wish to demonstrate the effect of the structure of the energy spectrum of the electrons on the dependence on the magnetic field of the amplitude of the thermal wave. Therefore we shall greatly simplify the formulation of the problem. First of all we assume that the conductor is isotropic in the absence of a magnetic field. Then equations (12′) and (13′) are simplified significantly. We shall require the components transverse with respect to $\mathbf{H}_0$ of the vectors $\mathbf{E},j$ and others. For them (in the notation adopted in Ref. 7) we have

$$\begin{aligned}&\mathbf{E}=\rho_\perp\mathbf{j}+\alpha\nabla T+RH_0[\vec\eta\mathbf{j}]+NH_0[\vec\eta\nabla T],\\&\mathbf{q}=\alpha T\mathbf{j}-\varkappa_\perp\nabla T+NH_0T[\vec\eta\mathbf{j}]+LH_0[\vec\eta\nabla T],\\&\vec\eta=\mathbf{H}/H.\end{aligned} \qquad (64)$$

We assume (again for the sake of simplicity) that the magnetic field is parallel to the metal surface (it is specifically because of this that we shall not need the longitudinal components of the vectors $\mathbf{E}$ and $\mathbf{q}$). All the coefficients ($\rho_\perp$, α, R, N, $\varkappa_\perp$ and L) appearing in formulas (64) are functions of the magnetic field. Since the component normal to the metal surface of the current density $j_z=0$ the electromagnetic field in the wave has its previous form with the replacement of ρ_{xx} and $\rho_\perp=\rho_\perp(H_0)$ [cf., (19)]

$$\begin{aligned}&j_x(z)=j_x(0)e^{ik_{\rm E}z},\\&j_x(0)=-ik_{\rm E}cH_y(0)/4\pi,\\&k_{\rm E}^2=4\pi i\omega/c^2\rho_\perp(H_0).\end{aligned} \qquad (65)$$

In the equation for heat conductivity the source in the present case is the dirivative of the term in the flux density of heat q that describes the Ettingshausen effect. Therefore

$$\frac{d^2\theta}{dz^2}+k_{\rm T}^2\theta=-\frac{iNH_0T\omega cH_y(0)}{c^2\rho_\perp\varkappa_\perp}e^{ik_{\rm E}z}. \qquad (66)$$

In order to determine $\theta(z)$ we need to know three kinetic coefficients: $\rho_\perp$, $\varkappa_\perp$ and N. Regarding the specific resistance $\rho_\perp$ and the coefficient of thermal conductivity $\varkappa_\perp$, their behavior in a magnetic field is known: $\rho_\perp$ increases with increasing magnetic field (significantly if the metal is compensated, i.e., the number of electrons $n_{\rm e}$ is equal to the number of holes $n_{\rm h}$ and insignificantly if $n_{\rm e}\neq n_{\rm h}$) $\varkappa_\perp$ decreases with increasing magnetic field tending in the limit to the value of the phonon heat conductivity of the given crystal, which, as a rule, is lower by a factor of several fold than the electron coefficient of heat conductivity.

The Nernst–Ettingshausen N, as has been noted in Ref. 3, has a quite complicated nature: for example $N=0$ in conductors with one type of carriers without dispersion, i.e., if their relaxation time τ does not depend on the energy. In order to determine the order of magnitude of the quantity $N=N(H_0)$ with an arbitrary (but not a quantizing) magnetic field one can use the linearized Boltzmann kinetic equation in the τ—approximation (however, assuming that τ is a function of the energy). After some quite tedious calculations we obtain

$$NH_0=e\rho_\perp\left(\frac{\left\langle\dfrac{\omega_{\rm c}\tau^2}{1+\omega_{\rm c}^2\tau^2}\right\rangle}{\left\langle\dfrac{\tau}{1+\omega_{\rm c}^2\tau^2}\right\rangle}\left\langle\frac{\varepsilon-\xi}{T}\frac{\tau}{1+\omega_{\rm c}^2\tau^2}\right\rangle-\left\langle\frac{\varepsilon-\xi}{T}\frac{\omega_{\rm c}\tau^2}{1+\omega_{\rm c}^2\tau^2}\right\rangle\right); \qquad (67)$$

here ξ is the chemical potential of the electrons, which in subsequent discussion does not differ from the Fermi energy, $\omega_{\rm c}=eH_0/m^*c$ is the cyclotron frequency, m^* is the effective mass ($m^*>0$ of the electrons, $m^*<0$ for holes), the angle brackets denote averaging over the energy:

$$\langle\psi(\varepsilon)\rangle=\frac{2}{3(2\pi\hbar)^3}\int v^2\left(-\frac{\partial F}{\partial\varepsilon}\right)\psi(\varepsilon)d^3p, \qquad (68)$$

$v=\partial\varepsilon/\partial p$, F is the equilibrium Fermi function; formulas (67) and (68) assume that the dispersion law of both electrons and holes is isotropic. Formula (68) can be written in a somewhat different form taking into account that

$v=|m^*|^{-1}p$ (if one measures the quasimomentum p from the center of the corresponding constant energy sphere):

$$\langle\psi(\varepsilon)\rangle=\frac{1}{|m^*|}\int\left(-\frac{\partial F}{\partial\varepsilon}\right)n(\varepsilon)\psi(\varepsilon)\mathrm{d}\varepsilon, \qquad (69)$$

$n(\varepsilon)$ is the number of occupied electron states of energy smaller than ε and $m^*>0$, or the number of free states with an energy greater than ε and $m^*<0$. The number of states $n(\varepsilon)$ is naturally positive, while the sign of $\mathrm{d}n/\mathrm{d}\varepsilon$ agrees with the sign of m^*. If several groups of electrons and holes exist one should carry out a summation over the groups.

Substituting expression (67) into the right hand part of equation (66) and integrating it we obtain

$$\frac{\theta(z)}{T}=\frac{eH_y(0)}{cC[1-(k_{\mathrm{E}}^2/k_{\mathrm{T}}^2)]}\left[\frac{\left\langle\dfrac{\omega_c\tau^2}{1+\omega_c^2\tau^2}\right\rangle}{\left\langle\dfrac{\tau}{1+\omega_c^2\tau^2}\right\rangle}\right.$$

$$\left.\times\left\langle\frac{\varepsilon-\xi}{T}\frac{\tau}{1+\omega_c^2\tau^2}\right\rangle-\left\langle\frac{\varepsilon-\xi}{T}\frac{\omega_c\tau^2}{1+\omega_c^2\tau^2}\right\rangle\right]$$

$$\times(e^{ik_{\mathrm{T}}z}-e^{ik_{\mathrm{E}}z}). \qquad (70)$$

In order not to complicate the presentation we restrict ourselves to the case of an isothermal boundary [cf., (24)] and, moreover, we use the value of k_{T}^2 according to one of the formulas (19). We note that in terms of the notation adopted by us the heat capacity C has the dimensionality of cm^{-3}, and in order of magnitude is equal to $C_e\sim nT/\varepsilon_{\mathrm{F}}$ if the electrons play the principal role in heat conductivity, and $C_{\mathrm{ph}}\sim nT^3/\theta_{\mathrm{D}}^3$ if the heat conductivity is due to phonons ($T\ll\theta_{\mathrm{D}}$, θ_{D} is the Debye temperature, n in order of magnitude is the number of cells of the crystal per unit volume). Formula (70) enables us to make estimates in the case of low ($\omega_c\tau\ll1$) and high ($\omega_c\tau\gg1$) magnetic fields. Naturally we shall assume that the electron gas is highly degenerate and use an expansion in powers of $T/\varepsilon_{\mathrm{F}}$, retaining the first nonvanishing term of the expansion, i.e., we take

$$\langle\psi(\varepsilon)\rangle=\frac{n\psi(\varepsilon_{\mathrm{F}})}{|m^*|}, \qquad (71)$$

and

$$\left\langle\frac{\varepsilon-\xi}{T}\psi(\varepsilon)\right\rangle=\frac{\pi^2T}{3|m^*|}\frac{\mathrm{d}n\psi}{\mathrm{d}\varepsilon}\bigg|_{\varepsilon=\varepsilon_{\mathrm{F}}}. \qquad (72)$$

We begin with the expression in angle brackets. For $\omega_c\tau\ll1$ its order of magnitude certainly does not depend on whether the metal is compensated ($n_e=n_h$) or not, and

$$\langle...\rangle\approx\frac{\pi^2Tn}{3|m^*|}\omega_c\tau\frac{\mathrm{d}\tau}{\mathrm{d}\varepsilon}\bigg|_{\varepsilon=\varepsilon_{\mathrm{F}}}. \qquad (73)$$

For $\omega_c\tau\gg1$ one must distinguish between the cases $n_e\neq n_h$ and $n_e=n_h$. In the former case neglecting in all the denominators unity compared with $(\omega_c\tau)^2$ we obtain

$$\langle...\rangle\approx-\frac{\pi^2Tn}{3|m^*|}\frac{1}{\omega_c\tau}\frac{\mathrm{d}\tau}{\mathrm{d}\varepsilon}\bigg|_{\varepsilon=\varepsilon_{\mathrm{F}}},$$

$$n_e\neq n_h. \qquad (74)$$

In the case of a compensated metal the first term in the figure brackets is equal to zero (as a result of the first factor) and the second one differs from zero, and

$$\langle...\rangle\approx\frac{9\pi^2Tn}{4\omega_c|m^*|}\left(\frac{1}{\varepsilon_{\mathrm{F}}^{\mathrm{e}}}+\frac{1}{\varepsilon_{\mathrm{F}}^{\mathrm{h}}}\right),$$

$$n_e=n_h=n, \qquad (75)$$

$\varepsilon_{\mathrm{F}}^{\mathrm{e(h)}}$ is the Fermi energy of the electrons and holes measured from the bottom (top) of the corresponding band.

We can see that the value of $\langle...\rangle$ for a compensated metal differs from that for an uncompensated one in a not very significant manner.

We now evaluate the ratio $k_{\mathrm{E}}^2/k_{\mathrm{T}}^2$ taking into account the fact that $\rho_\perp$ and $\varkappa_\perp$ are functions of the magnetic field. For $\omega_c\tau\ll1$ the previously obtained estimate [cf., (6)] is retained. According to it at low temperatures $|k_{\mathrm{T}}|\ll|k_{\mathrm{E}}|$, and at relatively high temperatures $|k_{\mathrm{T}}|\gg|k_{\mathrm{E}}|$ in both cases we have

$$\frac{k_{\mathrm{E}}^2}{k_{\mathrm{T}}^2}=\frac{l^2(T)}{\delta_0^2}, \quad \omega_c\tau\ll1. \qquad (76)$$

For $\omega_c\tau\gg1$ it is convenient to rewrite the ratio $k_{\mathrm{E}}^2/k_{\mathrm{T}}^2$ in the form which enables us to take into account its dependence on the value of $\omega_c\tau$. For compensated and uncompensated metals we have

$$\frac{k_{\mathrm{E}}^2}{k_{\mathrm{T}}^2}=\frac{l_\varkappa l_\rho}{\delta_0^2},$$

where $l_\varkappa$ "arises" from the heat conductivity $\varkappa_\perp$ ($l_\varkappa\sim l/(\omega_c\tau)^2$) and l_ρ "arises" from the resistance $\rho_\perp$ ($l_\rho\sim l$) for $n_e\neq n_h$ and $l_\rho\sim l/(\omega_c\tau)^2$ for $n_e=n_h$. Thus, for $\omega_c\tau\gg1$ we have

$$\frac{k_{\mathrm{E}}^2}{k_{\mathrm{T}}^2}\sim\frac{l^2}{\delta_0^2}\frac{1}{(\omega_c\tau)^2}\sim\left(\frac{r_H}{\delta_0}\right)^2, \quad n_e\neq n_h,$$

$$\sim\frac{l^2}{\delta_0^2}, \quad n_e=n_h,$$

$$r_H\sim v_{\mathrm{F}}/\omega_c. \qquad (77)$$

The results (70), (73)–(77) show that in order to make an estimate of the amplitude of the temperature oscillations $\theta(z)$ one can use the following expression

$$\frac{\theta(z)}{T}\approx\frac{eH_y(0)\tau}{c|m^*|}\frac{1}{1-(k_{\mathrm{E}}^2/k_{\mathrm{T}}^2)}F(\omega_c\tau)(e^{ik_{\mathrm{T}}z}-e^{ik_{\mathrm{E}}z}). \qquad (78)$$

The function $F(\omega_c\tau)$ has a maximum at $\omega_c\tau\sim1$,

$$F(\omega_c\tau)\sim\omega_c\tau \quad \text{for } \omega_c\tau\ll1,$$

$$\sim(\omega_c\tau)^{-1} \quad \text{for } \omega_c\tau \gg 1. \tag{79}$$

We have assumed that C is the heat capacity due to the electrons [cf., (70)]. We shall not write out the amplitude of the sound wave excited by thermoelastic stresses. This is not difficult to do using the results of Sec. 5. We note only that the distinctive dependence of the amplitude of the thermal wave on the magnetic field H_0 is particularly characteristic of a compensated metal when the ratio k_E^2/k_T^2 depends only weakly on H_0 [cf., (76) and (77)] may enable us to determine the existence of a thermoelastic linear mechanism of EMAT. In fact the dependence of $\theta(z)$ on H_0 is reproduced by the amplitude of the sound wave. Consult Ref. 3 for more details concerning the thermoelastic mechanism of the excitation of sound in a magnetic field (the case of $\omega_c\tau \ll 1$).

7. NONLINEAR GENERATION OF LONGITUDINAL ULTRASOUND

The smallness of the coupling between the electromagnetic and the elastic subsystems characteristic for problems of electromagnetic excitation of ultrasound that has been repeatedly emphasized by us, forces us, it might seem, to restrict ourselves only to linear interactions, when the frequencies of the incident and transformed waves coincide. A more careful analysis shows, however, that such a restriction is justified essentially only for the inertial mechanism of EMAT. For the inductive and deformational interaction the level of manifestation of nonlinear effects in the transformation is determined by the degree of action of the electromagnetic field on the dynamics of conduction electrons in the layer near the surface. Regarding the thermal mechanisms of EMAT the basic one (at any rate in an isotropic conductor or when the axis of symmetry of a single crystal is perpendicular to the surface) appears to be specifically the nonlinear interaction that is due to the appearance of thermoelastic stresses in the skin-layer as Joule heat $Q=\mathbf{Ej}$ is liberated within it. In this case the frequency of the ultrasound being excited is equal to the doubled frequency of the incident electromagnetic wave. To avoid misunderstanding we shall at once emphasize that we are speaking of sources of nonlinearity concealed specifically in the transformation mechanisms. Other sources of nonlinearity contained in particular in the equations of the theory of elasticity or in the kinetic equation (cf., Ref. 11 on this subject) are not discussed here. Examination of different mechanisms of electromagnetic excitation of ultrasound independent of one another that is due to the low effectiveness of linear EMAT is all the more justified in the case of a nonlinear transformation. Quantitative estimates of the amplitude of the excited ultrasound given below confirm this.

To compare the effectiveness of the linear and the nonlinear induction mechanisms of EMAT is easy. The ratio of the amplitude of generation at the first harmonic U_ω to the amplitude of generation at the second harmonic $U_{2\omega}$ is equal to the ratio of the constant H_0 and the variable H magnetic fields. The part that is quadratic in the amplitude of the wave incident on the metal of the induction force acting on the lattice is directed into the bulk of the metal, and this leads to the excitation in it of compression waves propagating along the normal to the surface. For the case $k_S\delta_E \ll 1$ the amplitude of the ultrasound of frequency 2ω excited by the Lorentz force is determined by the expression:

$$|U^{\text{ind}}| = \lambda \frac{H^2}{64\pi^2\rho S^2} = \frac{\lambda\Sigma}{8\pi}, \tag{80}$$

where λ is the wavelength of the ultrasound being excited, and Σ is the ratio of the energy density of the variable magnetic field $H^2/8\pi$ to the elastic modulus ρS^2. The value of $|U^{\text{ind}}|$ depends on the temperature only weakly, and this enables one to use it as a scale in representing the dependence on the temperature of the amplitude of ultrasound excited as a result of other mechanisms of transformation. At the frequency of $f=\omega/2\pi=1$ MHz in the variable field $H=100$ Oe the characteristic value is $|U^{\text{ind}}|\sim 10^{-13}$ cm.

If the nonlinear induction interaction reduces essentially only to the appearance of a source of variable pressure at the boundary of the metal, then the nonlinear deformation interaction (as generally also in the linear case) represents a significantly more subtle effect. The theoretical analysis of the deformation mechanism of EMAT covers the regimes of weak[11] and strong[12,13] nonlinearities. These two cases differ in the degree of action of the variable magnetic field on the dynamics of the electrons in the skin-layer and give different asymptotic behavior of the amplitude of the ultrasound being excited depending on the amplitude of the incident wave and on the mean free path of electrons in the metal. Within the regime of the normal skin-effect the deformation mechanism of EMAT is ineffective while in the regime of the anomalous skin-effect in the case of a well-developed electrodynamic nonlinearity which is characterized by the parameter

$$b=(Hel^2/8cp_F\delta)^{1/2},$$

$$\delta=\delta_A[1-\exp(-1/b)]^{1/3} \tag{81}$$

(where δ_A is the thickness of the skin-layer in the regime of the anomalous skin effect) the amplitude of ultrasound is described by the interpolation formula (cf., Ref. 13)

$$|U^{\text{def}}| = \frac{\lambda\Sigma}{6\pi^4}\frac{\tilde{m}}{m}(ql)^2\left(\ln\frac{1}{q\delta_A}\right)\left[1-\exp\left(-\frac{1}{b}\right)\right]^2. \tag{82}$$

Except for the deformation mass $\tilde{m}$ all the parameters needed for a quantitative calculation of $|U^{\text{def}}|$ using formula (82) can be obtained from independent measurements. Due to the presence in this formula of the nonlinearity parameter the dependence of $|U^{\text{def}}|$ on H deviates from the quadratic dependence as the amplitude of the variable field increases.

We now consider the thermoelastic mechanism of the nonlinear generation in the regimes of normal and anomalous skin-effects. In the regime of the normal skin-effect the source of thermoelastic stresses is the part of the Joule heat that depends on the time

$$Q=-\mathrm{Re}\left\{\frac{i\omega H^2}{8\pi}\exp[2i(k_Ez-\omega t)]\right\}, \quad (83)$$

and the boundary condition reduces to the requirement of the absence of a heat flux across the surface

$$\varkappa\frac{d\theta(z)}{dz}\bigg|_{z=0}=0. \quad (84)$$

In this case the expression for the oscillating increment to the temperature can be written in the form

$$\theta(z)=\mathrm{Re}\left[\frac{i\omega H^2}{8\pi\varkappa}\frac{\exp(2ik_Ez)-(2k/k_T)\exp(ik_Tz)}{k_T^2-4k_E^2}\right]. \quad (85)$$

The substitution of this expression into equation (53) in which one should replace k_S by $2\omega/S$, enables one to calculate the amplitude of the longitudinal ultrasound excited in the metal as a result of the thermoelastic stresses in the regime of the normal skin-effect. For $|k_S|\ll|k_E|$, $|k_T|$

$$|U^{\text{therm}}|=\lambda\frac{\beta H^2}{64\pi^2C}\frac{k_T}{k_T+2k_S}=\frac{\lambda\Sigma}{8\pi}\frac{\beta\rho S^2}{C}\frac{k_T}{k_T+2k_S}. \quad (86)$$

It can be seen that when the condition $K_E\ll k_T$ is satisfied which is valid at high temperatures the ratio of the amplitude of the thermoelastic and Lorentz generation is equal to the Grüneisen parameter $\gamma=\beta\rho S^2/C$.

In the regime of the anomalous skin-effect the heat in the sample is liberated in a layer of thickness $\sim\delta_A\ll l$. In the macroscopic approach the source of heat in this case cannot be regarded as a distributed one. The correct approach in this case is as follows: in formula (83) one should set $Q=0$, and for the boundary condition one should specify the flux of heat across the boundary:

$$\varkappa\frac{d\theta(z)}{dz}\bigg|_{z=0}=\mathrm{Re}\int_0^\infty E(z)j(z)dz. \quad (87)$$

Using the expressions for the distributions of the field and of the current for $l>\delta_A$ (Ref. 14), the wave part of the solution of the equation of heat conductivity can be written in the form:[4]

$$\theta(z,t)\approx-\mathrm{Re}\left\{(0.35+0.2i)\frac{\omega\delta_AH^2}{4\pi^2\varkappa k_T}\times\exp[i(k_Tz-2\omega t)]\right\}. \quad (88)$$

Then the amplitude of the longitudinal ultrasound excited in the metal as a result of thermoelastic stresses in the regime of the anomalous skin-effect is equal to

$$|U^{\text{therm}}|=\frac{\lambda\Sigma}{4\pi^2}\gamma k_S\delta_AG(|k_T|/k_S), \quad (89)$$

$$G(x)=\frac{x^3}{1+x^4}[(a_1+a_2x)^2+(a_1x-a_2)^2]^{1/2},$$

$$a_1\approx0.24,\quad a_2\approx0.067. \quad (90)$$

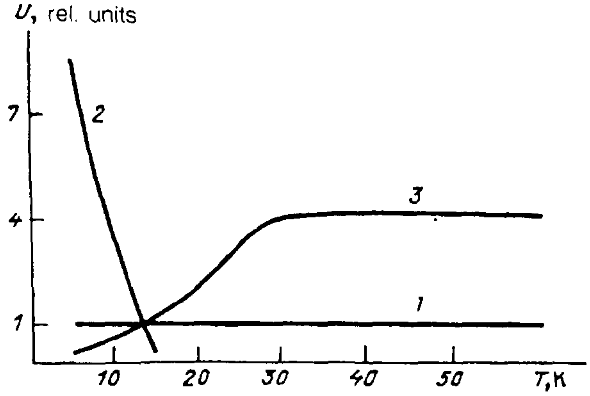

FIG. 1. Calculation of the temperature dependences of the amplitudes of the nonlinear generation of longitudinal ultrasound U in Zn at the frequency of the electromagnetic wave of 9.5 MHz in a variable magnetic field of 35 Oe under the action of the induction (curve *1*), deformation (curve *2*) and thermoelastic (curve *3*) mechanisms of transformation.

According to formulas (80), (82), (86), and (89) the temperature dependences of the amplitude of the longitudinal ultrasound excited under the action of the nonlinear induction, deformation and thermoelastic interactions can be represented by the curves shown in Fig. 1. Having in mind the discussion of the results of the experiment of Ref. 4 designed to elucidate the role of each of the mechanisms of nonlinear EMAT the curves in Fig. 1 were calculated for a specific metal—zinc. The value of $|U^{\text{ind}}|$ has been taken as the unit along the coordinate axis. It can be seen that at high temperatures the determining role in the processes of transformation is played by the thermal elastic force. As the temperature is decreased $|U^{\text{therm}}|$ decreases and this is determined by the temperature dependences of the electrodynamic $|k_E|^{-1}$ and of the thermal $|k_T|^{-1}$ penetration depths. Under the conditions of the anomalous skin-effect $|U^{\text{therm}}|$ is negligibly small. An estimate of the magnitude of displacements due to thermoelastic stresses was carried out utilizing data on the coefficients of thermal expansion, heat conductivity and heat capacity of zinc. The generation of ultrasound due to the deformation force is manifested only at low temperatures under conditions of the anomalous skin-effect (at $T\leqslant15$ K).

The details of the experiment are fully described in Ref. 4. We note only that the measurements were conducted on single crystalplates of zinc, the normal to the plane of which coincided with the axis of symmetry of the sixth order in the temperature range 4–40 K, at three frequencies of the incident electromagnetic wave of 3, 5 and 9.5 MHz with the intensity of the variable magnetic field at the surface of the crystal of $\sim$35 Oe. The reception of ultrasound was carried out using lithium niobate transducers, the resonance frequencies of which corresponded to double the frequency of the incident signal, i.e., 6, 10, and 19 MHz.

The experimental data on the nonlinear generation of longitudinal ultrasound in zinc are shown in Figs. 2–4, the amplitude of the ultrasound excited by the induction force

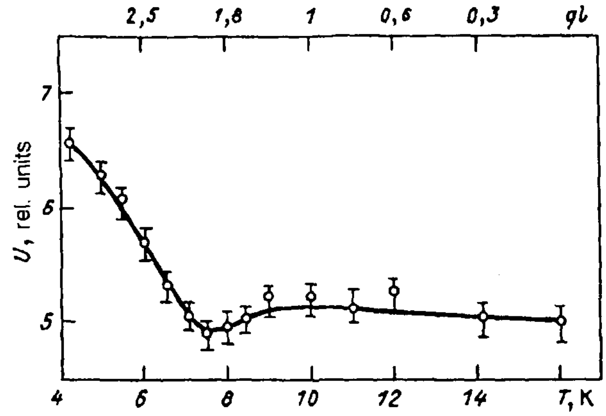

FIG. 2. The temperature dependence of the amplitude of the nonlinear generation of longitudinal ultrasound U in Zn at the frequency of electromagnetic wave of 3 MHz. The intensity of the variable magnetic field was 10 Oe. Along the vertical axis the unit is the amplitude of the nonlinear generation as a result of the induction interaction. Along the upper horizontal axis values are given of the dimensionless parameter $k_S l$.

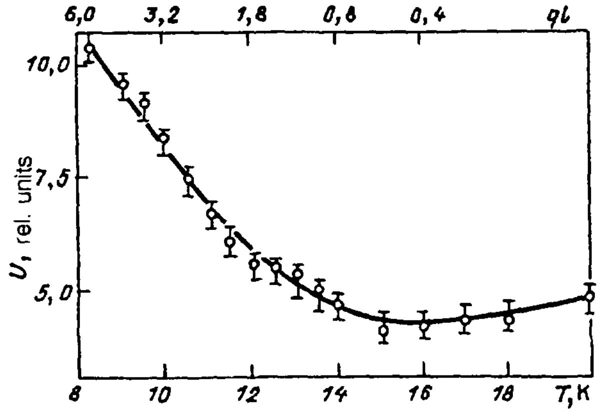

FIG. 4. The temperature dependence of the amplitude of the nonlinear generation of longitudinal ultrasound U in Zn at the frequency of the electromagnetic wave of 9.5 MHz. The intensity of the variable magnetic field is 35 Oe.

at the appropriate frequency was chosen as a scale along the ordinate axis. In the temperature range shown in the these figures the experimental dependences of $U_{2\omega}(T)$ are nonmonotonic. At all the frequencies that were investigated the amplitude of the generation decreased as the temperature was lowered, passed through a minimum and increased again. As the frequency increased the position of the minimum shifted into the region of higher temperatures, and the increase in the amplitude of the generation at low temperatures becomes more pronounced. The experimentally observed dependences qualitatively agree with theoretical concepts. A decrease in the amplitude of ultrasound as the temperature is lowered is due to the switching off of the thermoelastic mechanism the effectiveness of which in this temperature range decreases. An increase in the amplitude of generation with a further lowering of the temperature can be explained by the manifestation of the deformation mechanism of EMAT.

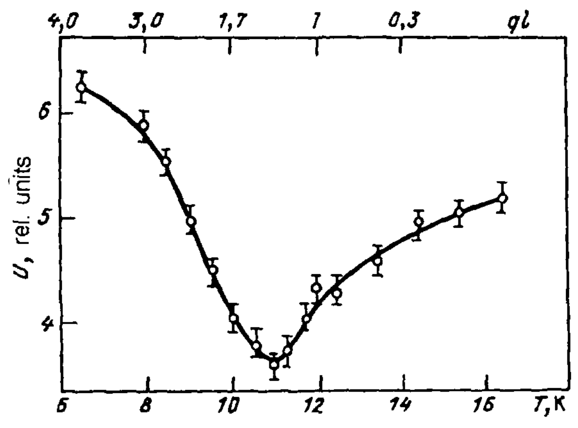

FIG. 3. The temperature dependence of the amplitude of the nonlinear generation of the longitudinal ultrasound U in Zn at the frequency of the electromagnetic wave of 5 MHz. The intensity of the variable magnetic field was 35 Oe.

An analysis of the expression for the amplitude of ultrasound excited as a result of the deformation force (89) shows that the role of this mechanism increases with an increase in the parameter ql which is shown along the upper horizontal axis in Figs. 2–4. As the frequency is increased this mechanism of EMAT begins to be manifested at ever higher temperatures. At the same time the temperature behavior of the amplitude of the thermoelastic generation (89) does not depend on the frequency. This explains the shift of the position of the minimum on the curves of $U(T)$ as the frequency is increased. In accordance with (82), $|U^{\text{def}}| \sim l^2$, which agrees well with the experimental data. If one uses the simplest ideas concerning the structure of the deformation tensor (see above) the numerical comparison of the results of theory and experiment can be carried out by varying the single parameter unknown in the problem—the deformation mass $\tilde{m}$. A satisfactory agreement is reached at the value of $\tilde{m}$, which exceeds by an order of magnitude the mass of the free electron m. As has been noted in Ref. 1, a similar situation exists also for the linear deformation interaction.

8. CONCLUSION

The induction, inertial and deformation mechanisms for the linear EMAT have been examined in detail in Ref. 1. In this paper the principal attention is devoted to the linear thermoelastic mechanism of transformation, and also the nonlinear generation of ultrasound has been examined which is due to the induction, deformation and thermoelastic interactions. In our opinion this achieves a qualitatively complete picture of the physical mechanisms responsible for the electromagnetic excitation of ultrasound both in the linear and in the nonlinear cases. It should be emphasized, however, that individual details in this picture so far have been obtained only theoretically (the linear thermoelastic and inertial mechanisms) and await their experimental confirmation.

[1] M. I. Kaganov and A. N. Vasil'ev, Usp. Fiz. Nauk **163**, 67 (1993) (in this issue of the Journal) [Phys. Usp. **36**, 1993)].
[2] M. I. Kaganov, Zh. Eksp. Teor. Fiz. **98**, 1828 (1990) [Sov. Phys. JETP **71**, 1028 (1990)].
[3] M. I. Kaganov and F. M. Maallavi, Fiz. Nizk. Temp. **18**, 737 (1992) [Sov. J. Low Temp. Phys. **18**, 521 (1992)].
[4] A. N. Vasil'ev, Yu. P. Gaĭdukov, and E. G. Kruglikov, Zh. Eksp. Teor. Fiz. **101**, 671 (1992) [Sov. Phys. JETP **74**, 357 (1992)].
[5] A. A. Abrikosov, *Fundamentals of the Theory of Metals*, North-Holland, Elsevier, Groningen, 1988 [Russ. original, Nauka, Moscow, 1987].
[6] M. I. Kaganov and V. M. Tsukemik, Zh. Eksp. Teor. Fiz. **35**, 474 (1958) [Sov. Phys. JETP **8**, 327 (1958)].
[7] L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon Press, Oxford, 1960 [Russ. original, Nauka, Moscow, 1957 and 1982].
[8] I. M. Lifshitz, M. Ya. Azbel', and L. D Landau, *Electron Theory of Metals*, Consultants Bureau, N.Y., 1973 [Russ. original, Nauka, Moscow, 1971].
[9] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity*, Pergamon Press, Oxford, 1959 [Russ. original, Nauka, Moscow, 1954 and 1987].
[10] M. I. Kaganov and V. B. Fiks, Fiz. Met. Metalloyed. **19**, 489 (1965) [Phys. Met. Metallogr. (USSR) **19**(4), 8 (1965)].
[11] A. N. Vasil'ev, M. A. Gulyanskiĭ, and M. I. Kaganov, Zh. Eksp. Teor. Fiz. **91**, 202 (1986) [Sov. Phys. JETP **64**, 117 (1986)].
[12] N. M. Makarov, F. P. Rodriguez, and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **94**, 368 (1988) [Sov. Phys. JETP **67**, 1943 (1988)].
[13] N. M. Makarov, F. P. Rodriguez, and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **96**, 2149 (1989) [Sov. Phys. JETP **69**, 1216 (1989)].
[14] G. E. Reuter and E. H. Sondheimer, Proc. R. Soc. London **A195**, 336 (1948).

Translated by G. M. Volkoff