On the charge density inside a conductor carrying a current

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The textbook literature on electromagnetism states that in an immobile homogeneous metal conductor with a constant current the space electric charge is equal to zero. (I. E. Tamm, *Foundations of the Theory of Electricity* (in Russian), Nauka, M., 1966; R. Feynman, R. Leighton, M. Sands, *Feynman Lectures on Physics*, Addison-Wesley, Reading, MA, 1963 [Russ. transl., Mir, M., 1977]; J. Orear Physics, MacMillan, N.Y., 1979 [Russ. transl., Mir, M., 1981]; A. A. Detlaf and B. M. Yavorsky, *A Course on Physics* (in Russian), Vysshaya shkola, M., 1989.

This follows from the continuity of current and Ohm's law

$$\mathrm{div}\mathbf{j} = \sigma \mathrm{div}\mathbf{E} = 0.$$

However, if one considers the effect of the Lorentz force on the electron:

$$\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v}\times\mathbf{B}/c)$$

(where $\mathbf{v}$ is the average current velocity of the electrons), then from

$$\mathrm{div}\mathbf{j} = 0$$

it follows that

$$\mathrm{div}\mathbf{E} = 4\pi\rho = \frac{\mathbf{v}}{c}\,\mathrm{rot}\mathbf{B} = \frac{4\pi}{c^2}\mathbf{v}\mathbf{j}$$

Hence

$$\rho = \rho_- v^2/c^2, \tag{1}$$

where ρ_- is the density of the space charge of free electrons.

This small relativistic correction contradicts the following lines of reasoning from the Feynman Lectures (Vol. 5).

Let us denote by S a system in which the conductor is at rest. Then in a system of coordinates (S') which is moving with the average drift velocity of the electrons *a space charge arises due to relativistic effects*.

In system S the positive charges are at rest, and in system (S')

$$\rho'_+ = \rho_+[1-(v^2/c^2)]^{-1/2}.$$

Electrons are at rest in system S', and their space charge in S is equal to

$$\rho_- = \rho'[1-(v^2/c^2)]^{-1/2}.$$

Since

$$\rho_- = -\rho_+,$$

then in S'

$$\rho' = \rho'_+ + \rho'_- = \rho_+(v^2/c^2)\,[1-(v^2/c^2)]^{-1/2}.$$

The magnetic field has no effect on an electron outside the conductor moving with the same velocity v in S, that is, at rest in S'. The force of attraction of the electron to the conductor, which corresponds to the attraction of parallel currents in S, is provided by the electric field of the positive space charge in system S'. This corresponds to Fig. 1 (*Feynman Lectures*, Vol. 5, p. 275 in Russ. transl.). We stress that we are talking about a space charge.

Let us now apply this same line of reasoning to electrons inside the conductor. In system S their movement creates a current

$$\mathbf{j} = \rho_-\mathbf{v}.$$

In system S' the magnetic field has no effect on them and there is no other force which could balance the effect of the electric field of the space charge. In contrast to the case discussed above we should require that

$$\rho' = \rho'_+ + \rho'_- = 0. \tag{j}$$

In this system the Lorentz force acts on ions, but its effect is balanced by forces which hold the ions in the crystalline "framework" of the conductor. If one uses the equations for ρ_+ and ρ_- in immobile and moving reference frames, it is easy to see that the volume of the immobile conductor with current is charged. The density of this charge is

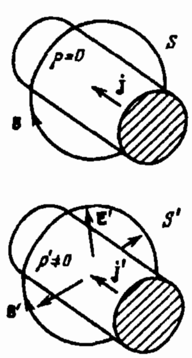

FIG. 1.

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$$\rho = \rho_+ + \rho_- = \rho'_+ [1 - (v^2/c^2)]^{1/2} + \rho'_- [1 - (v^2/c^2)]^{-1/2} = \rho_- v^2/c^2 = jv/c^2.$$

Of course we obtained the same result as in Eq. (1) because we used the same relativistically invariant condition, that the component transverse to the current of the Lorentz force that acts on the conductivity electrons is equal to zero.

This negative space charge is formed because some of the electrons move into the interior of the conductor until the effect of the magnetic field is balanced by their mutual repulsion. A positive charge Q_{surf} should remain on the surface, which has no effect on the electrons inside the conductor. It is this surface charge in system S' which will attract an external immobile electron. Figure 1 from the Feynman book should be changed as shown in Fig. 2. The statement that an attractive electric force acts on the electron outside the conductor remains in effect.

Where does the positive electric charge come from in the moving system S'? To answer this question one must consider a second conductor with current flowing in the opposite direction.

In the "forward" conductor

$$\rho' = 0, \quad Q'_{\text{surf}} = (Jv/c^2)\, [1 - (v^2/c^2)]^{-1/2}.$$

According to the Lorentz transformation formula

$$\rho = [1 - (v^2/c^2)]^{-1/2}\, [\rho - (jv/c^2)]$$

in the "reverse" conductor

$$\rho'' = -(2jv/c^2)\, [1 - (v^2/c^2)]^{-1/2},$$

the surface charge is

$$Q''_{\text{surf}} = (Jv/c^2)\, [1 - (v^2/c^2)]^{-1/2},$$

and the total charge per unit length is

$$Q''_{\text{surf}} + Q''_{\text{sp}} = -(2Jv/c^2)\, [1 - (v^2/c^2)]^{-1/2} + (Jv/c^2)\, [1 - (v^2/c^2)]^{-1/2} = -(Jv/c^2)\, [1 - (v^2/c^2)]^{-1/2}.$$

Thus, the total charges of the forward and reverse wires are equal in magnitude and opposite in sign. This also holds true for an arbitrary velocity of the reference frame, which follows from the general propositions of the theory of relativity.

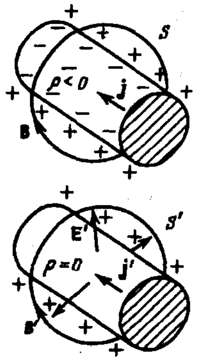

FIG. 2.

In view of the fact that the aforementioned books have deservedly acquired a position of authority and many young people are taught using them, we have found it expedient to refine the issue of the density of the space charge of a conductor carrying a current. We note that in Tamm's book it is repeatedly stressed that he limits himself only to quantities of the first order of smallness of v/c. Thus, it can be considered justified to ignore the space charge. In Purcell's book, *Electricity and Magnetism* (Berkeley course, Vol. 2, McGraw-Hill, N.Y., 1965 [Russ. transl. Nauka, M., 1971]), in a similar examination of the interaction of a moving charge with a current-carrying conductor, only the total linear charge of the conductor is involved. The distribution of this charge over the cross section is not examined.

We note that, in contrast to the space charge of an immobile conductor carrying a current, its surface charge will inevitably vary over its length (for a finite specific resistance of the conductor), and will also depend on many other factors, such as grounding at some point in the current circuit, the closeness of other conductors, etc.

(In the Russian translation of the illustration in *Feynman Lectures B* was erroneously indicated in the opposite direction.)

Translated by C. Gallant