# Channeling of neutral particles and photons in crystals 

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#### Abstract

A review is given of the principles and prospects of directed channeling of neutral particles and photons in perfect crystals and in crystals containing microchannels of arbitrarily small width. It is shown that, in all cases (including a homogeneous lattice), the channeling of x rays and $\gamma$ rays is accompanied by the modulation of the transverse structure of the wave and by a change in the longitudinal attenuation coefficient. The nontrivial special case of Mössbauer radiation channeling is investigated. A detailed discussion is given of the channeling of neutrons in crystals due to the 'optical' Fermi interaction with lattice nuclei, and the magnetic dipole-dipole and coherent Schwinger interactions in magnetic and nonmagnetic crystals. The effects of spin waves, elastic waves, and domain walls on channeling are examined. A discussion is presented of the possibility of channeling of neutral atoms with internal electromagnetic resonances by the induced-dispersive ordering interaction that takes place when the resonance frequency is equal to a harmonic of the bounce frequency associated with longitudinal motion in the periodic lattice. Experiments on neutron and $\gamma$-ray channeling are analyzed.


## 1. INTRODUCTION

Advances in the physics of charged-particle channeling in crystals, and the prospects for the future that they offer, have stimulated searches for mechanisms capable of ensuring similar directed ordered motion of neutral particles and $\gamma$ rays in single crystals.

Of the many different physical phenomena that determine particular interactions between photons, neutrons, and neutrals, on the one hand, and crystal atoms, on the other, the most important processes are those that 'tie' the motion of particles or photons to particular crystal planes, axes, and systems. For short-wave photons, the main (and probably the only) factor governing their motion is coherent scattering (followed by interference) by individual centers, disordered ensembles, and the periodic set of crystal planes and axes. This process is well known and is widely used in diffraction physics. All quantitative characteristics are then determined by analyzing the susceptibility or its Fourier transform. The interaction between neutrons and atoms is naturally divided into scattering by nuclei and by atomic electrons. The first category includes both $s$-wave scattering by the nucleus as a whole and (under special conditions and at high energies) by nuclear resonances.

We know that $s$-wave scattering is typically described in terms of the Fermi pseudopotential

$$
\begin{equation*}
V(\mathrm{r})=4 \pi \hbar^{2} b \delta(\mathrm{r}) / m, \tag{1.1}
\end{equation*}
$$

where $m$ is the neutron mass and $b$ is the scattering length. ${ }^{1,2}$
If we evaluate the macroscopic average over the volume of the unit cell, we find that the mean potential

$$
V \equiv n \int V(\mathbf{r}) \mathrm{d} v=4 \pi \hbar^{2} b n / m
$$

accounts for the macroscopic characteristics of the medium. The very considerable analogy between the Maxwell wave equation and the quantum-mechanical wave equation enables us to describe the motion of neutrons by introducing the effective susceptibility $\chi=V / 2 E$.

The other category of phenomena involves the interaction of neutrons with atomic electrons and the nuclear charge, and reduces to the Schwinger interaction ${ }^{3}$ due to the relativistic effect whereby the magnetic lattice gives rise to the magnetic field $\mathbf{H}=\mathbf{E} \times \mathbf{v}$ in the rest frame of the neutron as it crosses the electric field $\mathbf{E}$ of the atom. Since the neutron has an intrinsic magnetic moment, this field produces a sufficiently strong magnetic interaction that has a significant effect on the motion of the neutron.

Apart from the interaction with the effective magnetic field due to the relativistic transformation of the atomic electric field, there is the direct magnetic dipole-dipole interaction between the neutron and the uncompensated atomic magnetic moments, and this also affects the motion of the neutron. The latter effect is particularly significant in ordered magnetic structures.

All the processes accompanying the neutron-atom interaction can in principle accompany the interaction between a moving neutral atom and the lattice. However, the atomic electron shells give rise to a very significant change in the effectiveness of the process because the resonance levels available in these shells offer a fundamentally new mechanism for a coherent interaction with the periodically distributed lattice atoms.

The central question in the physics of channeling is whether or not these interaction mechanisms can ensure the directed channeling of neutrals and photons in crystals.

It is usually conjectured (see, for example, Refs. 4-7) that the phenomenon of channeling that occurs for charged particles does not occur for photons, which pass right through a perfect crystal. Moreover, it is conjectured that this type of radiation transfer does not take place even in channels whose width $a$ is much greater than the separation between the crystal planes, but is less than the threshold $\Delta x_{\text {min }}$ which amounts to a few Ångstroms. How is this concluson arrived at?

The basic argument is as follows. A wave localized
within an interval $\Delta x$ of the transverse coordinate $x$ (i.e., measured at right angles to the direction of the hypothetical channeling) has an unavoidable wave-number uncertainty $\Delta k_{x} \geqslant 2 \pi / \Delta x$. If it is obvious that channeling is possible only when $\Delta k_{x}$ has associated with it an uncertainty $\Delta \theta \approx \Delta k_{x} / k$ that is less than the angle of total internal reflection $\theta_{0}=(2|\chi|)^{1 / 2}$, so that

$$
\begin{equation*}
\Delta x_{\min } \approx \pi\left(2 / k^{2}|\chi|\right)^{1 / 2}, \tag{1.2}
\end{equation*}
$$

where $\chi$ is the susceptibility of the channel wall material.
In the case of nonresonant x rays, $\chi=-\omega_{e}^{2} / 2 \omega^{2}$ where $\omega_{e}=\left(4 \pi n_{e} e^{2} / m\right)^{1 / 2}$ is the plasma frequency and $n_{e}$ is the mean density of all the atomic electrons for which the transition frequency is less than the wave frequency. For this type of radiation, and for typical values $\omega_{e} \approx 3 \times 10^{16}-6 \times 10^{16}$ $s^{-1}$, we have $\Delta x_{\text {min }} \approx 300-600 \AA$ which is greater than the separation between the crystal planes by more than two orders of magnitude.

For resonant Mössbauer radiation with typical wavelength $\lambda \approx 1 \AA$, the magnitude of the resonant susceptibility ${ }^{8}$ $\left|\chi^{\prime}\right|_{\max } \approx \lambda^{3} n / 8 \pi^{2}$ in a medium with the concentration $n=10^{22} \mathrm{~cm}^{-3}$ of Mössbauer atoms is $\left|\chi^{\prime}\right|_{\max } \approx 10^{-4}$, which leads to $\Delta x_{\text {min }} \approx 100 \AA$.

For neutrons with energy $E$ in the crystal, the mean effective susceptibility is $\chi=V / 2 E$. We then have

$$
\begin{equation*}
\Delta x_{\min }=\pi \hbar /(m V)^{1 / 2}=2 \pi \hbar / m v_{0} \tag{1.3}
\end{equation*}
$$

where $V=m v_{0}^{2} / 2$ is the mean potential for a neutron in the crystal, which is a function of the nuclear scattering length $b$ and can be expressed in terms of the effective maximum to-tal-reflection velocity $v_{0}=\left(8 \pi \hbar^{2} n b / m^{2}\right)^{1 / 2}$ of ultracold neutrons. ${ }^{1-3}$ For most media, $v_{0}=3-6 \mathrm{~m} / \mathrm{s}$, which leads to $\Delta x_{\text {min }} \approx 300-600 \AA$ and this is of the same order as for nonresonant radiation.

Such estimates can lead to the not wholly correct conclusion that nondiffractive directed motion (at angle $\theta=0$ ) that is strongly coupled to the crystal is not possible for short-wavelength photons and neutrons in either natural perfect single crystals with lattice constant $d \approx 1-3$ $\AA<\Delta x_{\text {min }}$ or in superlattices with lattice constant $a>d$ and $a<\Delta x_{\text {min }}$.

Since the validity of this conclusion seems obvious, all research into the physics of channeling of neutrals and neutrons has been confined to studies of the possibility of channeling by complex structures with a large constant $a$ (Refs. 4-7 and 9-11), by structures with large $a$ and a graded refractive index (Refs. 5-7), and by systems with vacancies or voids having diameters $D>100 \AA$ (Refs. 12-15). For parameter values of this order, the transport of neutrons and photons is qualitatively similar to the large-scale neutron, $x$ ray and $\gamma$-ray optics of artificially produced waveguides (see, for example, Refs. 16-20) that have long been successfully used in ultracold-neutron transport and in focusing and monochromatization of $x$ rays, although they differ from the latter by the smaller number of modes.

On the other hand, and contrary to what has just been said, there are arguments that offer some evidence for the possibility of channeling even in channels (gaps) of width less than $\Delta x_{\text {min }}$. For example, channeling in crystals can be approached as the limiting case of an optical dielectric waveguide. It is well-known (see, for example, Ref. 21) that this
device has a rich mode spectrum, but only if the width of the middle layer whose susceptibility is higher than that of the surroundings satisfies the condition $a / \lambda \gg 1$. The spectrum becomes depleted as this ratio decreases, and only one mode can propagate in the waveguide when $a<\lambda$ ('mode without cutoff'). The corresponding propagation constant (i.e., the longitudinal wave number) is ${ }^{21}$

$$
\begin{equation*}
\beta=k\left[1+(\chi / 2)+\left(k^{2} a^{2} \chi^{2} / 8\right)\right], \quad k=\omega / c, \tag{1.4}
\end{equation*}
$$

and is found from the dispersion relation
$\tan \left[\alpha\left(k^{2}-\beta^{2}\right)^{1 / 21 / 2} / 2\right]=\left[\left(\beta^{2}-k^{2} \varepsilon\right) /\left(k^{2}-\beta^{2}\right)\right]^{1 / 2}, \varepsilon=1+\chi$,
which is a consequence of the compatibility of boundary conditions on the surface of the waveguiding layer. This value of $\beta$ differs from the value for continuous media without channels by the amount $\Delta \beta=k^{3} a^{2} \chi^{2} / 8$.

It is typical of this mode (contrary to the above qualitative argument) that it exists for arbitrarily small values of $a$. In a purely formal way, this type of mode should also continue to exist as $s$ tends to the crystal constant $d$. On the other hand, since the entire theory of optical waveguides is based on macroscopically averaged equations of electrodynamics, and takes as its basic quantity the volume-averaged susceptibility $\chi=\chi\langle(x)\rangle$ that ignores the atomic structure of the channel walls, the above 'direct conclusion' is undoubtedly incorrect.

A correct and convincing answer to the question of the possibility of, and mechanism for, the channeling of neutrals and photons in crystals must rely on the exact and not the average (over the crystal-plane separation) equations of quantum mechanics and electrodynamics. This requirement is also found to emerge from studies of the band structure of the energy spectrum of channeled motion of neutrals in the periodic field of crystal axes and planes.

Anticipating the results of numerical calculations, we note the following. Because the ordering mechanism that results in the directed motion of neutral particles is specific and relatively weak (for Coulomb channeling of charged particles, the typical barrier height or well depth associated with crystal axes and planes amounts to a few dozen eV, whereas for neutrals this figure is $\leqslant 1 \mathrm{eV}$ and often $\ll 1 \mathrm{eV}$ ), the very concept of channeling must be re-examined. With the possible exception of slightly relativistic electrons, charged particles can almost always be described with reasonable precision by the classical theory of channeling that involves the classical periodic trajectory with the oscillation amplitude limited by the channel width, but this is not the case for neutral particles. Even for the relatively massive neutrons, there is usually one energy level (or, more precisely, one energy band), and motion in the transverse direction is entirely quantum mechanical. Since the channel wall height is low, the sub-barrier 'tails' of the wave function extend to distances much greater than the channel width. Despite all this, the motion is undoubtedly channeled in the sense that it is directed, strongly bound (i.e., localized in the vicinity of the potential well), transversely quantized, and longitudinally free.

Because of major difficulties in the setting up of experiments, there are no systematic studies of the channeling of neutrals and photons. Apart from obvious problems such as the necessity for highly collimated, monochromatic, and in-
tense beams, the problem is complicated in no small measure by the lack of a generalizing analysis of channeling in the case of non-Coulomb interactions. This gap is partially filled by the review given below.

Our aim is to present a detailed examination of the leading mechanisms of interaction of neutrals and photons with ordered systems of atoms capable of producing directed motion in the lattice. We shall also be concerned with particular implementations of this type of motion and, ultimately, with possible experimental studies of the phenomenon.

Channeling can be analyzed in terms of four (at present) main interaction mechanisms.

A general formulation of the problem is given in the Introduction.

The 'optical' interaction of neutrons and photons with the lattice is discussed in Sec. 2, using the periodic susceptibility mechanism, traditional in optics. This allows a detailed study to be made of the possible channeled motion of neutrons, nonresonant x rays, and $\gamma$ rays, as well as Mössbauer radiation in perfect single crystals and in crystals containing microchannels of arbitrarily small width (including microchannels in zeolites and in asbestos fibers).

The motion of particles that have a magnetic moment in a magnetic lattice with a controlled ordered inhomogeneity such as a spin wave, an elastic wave, or a domain wall is discussed in Sec. 3.

Section 4 presents an analysis of magnetic channeling of particles with an intrinsic magnetic moment in nonmagnetic lattices, due to the coherent Schwinger interaction.

Directed motion of neutrals exhibiting an internal electromagnetic resonance whose frequency is equal to one of the spectral lines of a periodic perturbation applied to these particles as they travel along crystal axes or planes is discussed in Sec. 5.

The concluding Sec. 6 is devoted to a brief analysis of the results of the most convincing experiments on the channeling of fast neutrons and hard $\gamma$ rays in perfect germanium crystal crystals.

## 2. ‘OPTICAL’ CHANNELING OF NEUTRONS AND SHORTWAVE ELECTROMAGNETIC RADIATION IN PERFECT CRYSTALS AND CRYSTALS CONTAINING MICROCHANNELS

### 2.1. Properties and mode structure of channelled and quasichanneled wave motion of particles and radiation in crystals.

Following Ref. 22, we consider the general problem of channeled motion of photons and particles in crystals. To begin with, we assume that the crystal consists of two parts separated by a distance $a$. Each of these two parts consists of reflecting planes with plane separation $d$, all planes being parallel to the separation boundary. The $y, z$ plane lies in the space between the two parts, half way between them. This model can be used to examine the motion of either photons or neutrals in a wide channel with $a \gg d$ but, in contrast to 'ordinary' macroscopic optics, it takes into account the re-flecting-plane structure of the wall material. It also enables us to pass to the limit as $a \rightarrow d$ and to determine the motion in an ultrasonic channel or even the homogeneous lattice with $a=d$.

The assumption that there are continuous reflection planes, averaged along the Oz direction, is based on taking
into account the wave-like motion of photons and particles. Actually, the change $\Delta \beta$ in the longitudinal wave number on internal re-reflection, which is the necessary condition for channeling, is such that $\Delta \beta_{\text {max }}=k-\beta_{\max } \approx k \theta^{2} / 2$ where $\theta$ is the effective reflection angle. Since $\theta_{\max } \leqslant(2|\chi|)^{1 / 2}$, we find that the minimum coherence length along the scattering plane, within which the structure of the scatterer is fundamentaly unresolvable, is $\Delta z_{\text {min }} \geqslant 2 \pi / \Delta \beta_{\text {max }}$ which becomes

$$
\begin{equation*}
\Delta z_{\min } \geq 2 \pi / k|\chi| . \tag{2.1}
\end{equation*}
$$

This result yields the estimate $\Delta z_{\text {min }} \geqslant d_{z}$ for all possible wavelengths $\lambda$, susceptibility $\chi$, and longitudinal lattice constant $d_{z}$.

When a harmonic wave is incident at a grazing angle in the positive direction of the Oz axis, and the crystal planes extend to infinity in the $O y$ direction, there are no field variations in the latter direction, which is formally indicated by writing $\partial / \partial y=0$. Maxwell's equation then reduces to the wave equation

$$
\begin{equation*}
\frac{\partial^{2} E_{y}}{\partial x^{2}}+\frac{\partial^{2} E_{y}}{\partial z^{2}}+k^{2} \varepsilon(x) E_{y}=0, \quad \varepsilon(x)=1+\chi(x) \tag{2.2}
\end{equation*}
$$

It is important to note that this generally accepted structure of the wave equation, in which the properties of the medium are represented exclusively by the susceptibility $\chi(x)$ (but not its derivatives) is not wholly correct. This question will be examined in greater detail in Sec. 2.3 in which we analyze the channeling of Mössbauer photons.

For a set of homogeneous planes, the permittivity can be written in the form

$$
\begin{equation*}
\varepsilon(x) \equiv 1+\chi(x)=1+\chi d \sum_{n=0}^{\infty} \delta(|x|-(\alpha / 2)-n d) \tag{2.3}
\end{equation*}
$$

where $\chi=\langle\chi(x)\rangle$ is the macroscopic susceptibility averaged over the transverse crystal spacing $d$, which corresponds to the average permittivity $\epsilon=1+\chi$. Replacement of the true susceptibility for planes of thickness $2 R \approx 0.2-0.4$ $\AA$ with (2.3), which is valid for infinitesimally thin planes, is readily justified by the natural condition $\Delta x_{\min } \geqslant 2 R$ and by considerations similar to those introduced above that lead to $\Delta x_{\min } \approx \pi\left(2 / k^{2}|\chi|\right)^{1 / 2}$. Since $\Delta x_{\min } \gg 2 R$ for all crystals, the real spatial structure of $\chi(x)$ within the thickness $2 R$ [provided (2.2) is valid] is unimportant and can be approximately replaced with (2.3). We note that this replacement cannot be made for superlattices with $2 R \geqslant \Delta x_{\text {min }}$.

Let us write the field in the form $E_{y}=u(x) v(z)$. It then follows from (2.2) that $v(z)=\exp (i \beta z)$ where $\beta$ is the separation constant that can be looked upon as the longitudinal wave number. The equation for the transverse structure of the field $u(x)$ is

$$
\begin{align*}
& \frac{\mathrm{d}^{2} u}{\mathrm{~d} x^{2}}-G \sum_{n=0}^{\infty} \delta\left(|x|-\frac{a}{2}-n d\right) u+x^{2} u=0, \\
& G=-k^{2} \chi d, x^{2}=k^{2}-\beta^{2} \tag{2.4}
\end{align*}
$$

The eigenvalues $\kappa$ and the transverse-structure modes $u(x)$ can be determined from the boundary conditions. When the symmetry of the problem is taken into account, the transverse mode structure for $|x| \leqslant a / 2$ (i.e., within a channel) can be characterized by the even solution

$$
\begin{equation*}
u_{a}=A \cos (x x) \tag{2.5}
\end{equation*}
$$

or the odd solution

$$
\begin{equation*}
u_{a}=A \sin (x x) \tag{2.6}
\end{equation*}
$$

We shall show below that for small subthreshold widths $a<\Delta x_{\text {min }}$, channeling (i.e., subthreshold channeling) occurs only for the even mode (2.5).

For $a / 2 \leqslant|x| \leqslant(a / 2)+d$, the solution of (2.4) is

$$
\begin{equation*}
u=B \exp (i x x)+C \exp (-i x x) \tag{2.7}
\end{equation*}
$$

Because the susceptibility is periodic, i.e., $\chi(x)=\chi(x+d)$, the solution given by (2.7) must satisfy the condition $u(x+d)=f u(x)$ of Bloch's theorem, where $f=\exp (i v d)$ is the Bloch parameter and $v$ is the quasimomentum.

In accordance with this requirement, and because the solution must be continuous on the $|x|=(a / 2)+d$ plane between the intervals $|x| \leqslant(a / 2)+2$ and $|x| \geqslant(a / 2)+d$, we have

$$
\begin{equation*}
u((a / 2)+d-0)=u((a / 2)+d+0), \quad u((a / 2)+d)=f u(a / 2) \tag{2.8}
\end{equation*}
$$

Let us now consider the conditions for the derivative. It follows from (2.8) that $u^{\prime}[(a / 2)+d]=f u^{\prime}(a / 2)$ and, simultaneously,

$$
u^{\prime}((a / 2)+d-0)=u^{\prime}((a / 2)+d+0)-G u((a / 2)+d)
$$

where the latter relation can be obtained directly from (2.4) after integration over the infinitesimal interval around $|x|=(a / 2)+d$ and contains an extra term that is typical for singular susceptibilities (and, in quantum mechanics, singular potentials). We then find from the last two equations that

$$
\begin{equation*}
u^{\prime}((a / 2)+d)=f u^{\prime}(a / 2)+G u((a / 2)+d) \tag{2.9}
\end{equation*}
$$

The solution of the homogeneous set of equations given by (2.8)-(2.9) gives nontrivial values for the amplitudes $B$ and $C$ only if its determinant vanishes, and this leads to the following equation for the Bloch parameter:

$$
\begin{equation*}
f^{2}-f[2 \cos (x d)+(2 G / x) \sin (x d)]+1=0 \tag{2.10}
\end{equation*}
$$

To find the spectrum of wave numbers $\kappa$, we use the boundary conditions for the solutions (2.5)-(2.7) on the plane $|x|=a / 2$, i.e., on the channel wall:
$u_{a}(a / 2)=u(a / 2), \quad u_{a}^{\prime}(a / 2)=u^{\prime}(a / 2)-G u_{a}(a / 2)$.
By eliminating the amplitude $C$ with the help of the relation

$$
C=B \exp (i x a)[f-\exp (i x d)] /[\exp (-i x d)-f]
$$

that follows from (2.8), we obtain the dispersion relation

$$
\begin{equation*}
\tan (x a / 2)=(G / x)+\{[\cos (x d)-f] / \sin (x d)\} \tag{2.12}
\end{equation*}
$$

for the even solution (2.5) and

$$
\begin{equation*}
\cot (x a / 2)=-(G / x)-\{[\cos (x d)-f] / \sin (x d)\} \tag{2.13}
\end{equation*}
$$

for the odd solution (2.6).
The expressions given by (2.10), (2.12), and (2.13) constitute the complete solution of the problem. We shall first show that, contrary to the direct analogy with the limit-
ing case $a=d$ of the microchannel in ordinary macroscopic dielectric waveguides with 'solid' walls (1.4), the homogeneous lattice does not produce channeling (in the sense of a mode localized in the transverse direction). Thus, eliminating $G / \varkappa$ from (2.10), (2.11) and also (2.10), (2.13), we obtain the following expression for even and odd modes, respectively:

$$
\begin{align*}
& f=[\cos (x d)+\tan (x a / 2) \cdot \sin (x d)]^{-1}  \tag{2.14}\\
& f=[\cos (x d)-\cot (x a / 2) \cdot \sin (x d)]^{-1}
\end{align*}
$$

In the limiting case $a=d$, the two equations in (2.14) reduce, respectively, to $f= \pm 1$ with real crystal momentum $v$. Taken together with (2.8), this conclusion demonstrates the strict periodicity (with period $d$ or $2 d$ ) of even and odd modes, and also the presence of totally delocalized modes. Moreover, we recall that the entire calculation was performed for $2 R \ll d, \Delta x_{\min }$ and is valid only for single crystals. For superlattices with thick barriers between channels, $2 R \geqslant \Delta x_{\text {min }}$, the entire conclusion is invalid and channeling is possible within the confines of an individual channel.

Let us now consider the solution for an arbitrary channel with $a>d$. Eliminating the parameter $f$ from (2.10), (2.12), and (2.13), we obtain the following dispersion relation for even and odd solutions, respectively:

$$
\begin{align*}
& 2 x / G=\sin (x a)+2 \cos ^{2}(x a / 2) \cot (x d),  \tag{2.15}\\
& 2 x / G=-\sin (x a)+2 \sin ^{2}(x a / 2) \tan (x d) . \tag{2.16}
\end{align*}
$$

We shall examine this for the most important and controversial case of a thin microchannel with $x a, x d \ll 1$.

Direct expansion of (2.15), subject to the last condition, leads to the following expression for the wave number:

$$
\begin{equation*}
x=\left\{(G / d)-G^{2}\left[1-3\left(1-a d^{-1}\right)^{2} / 12\right]\right\}^{1 / 2} \tag{2.17}
\end{equation*}
$$

which characterizes the single nonthreshold mode $u(x)$ that exists for all values of $a$ up to $a=d$. Consequently, the expression for $f \approx 1+i v d$ is then

$$
\begin{equation*}
f \approx 1-[G(a-d) / 2]=1+\left[k^{2} d(a-d) \chi / 2\right] \tag{2.18}
\end{equation*}
$$

The imaginary part of the crystal momentum $v^{\prime \prime}=-k^{2}(a-d) \chi^{\prime} / 2$ for the solution given by (2.17) describes the reduction in the mode amplitude with increasing distance from the channel in the transverse direction in the range $|x|>a / 2$. It is clear from the structure of the expression for $f$ that the total spatial width of a mode is approximately given by $\Delta \approx a+\left(2 /\left|v^{\prime \prime}\right|\right.$. The final overall structure of the nonthreshold channeled mode of short-wave radiation has the form

$$
\begin{align*}
& E_{y}((a / 2)+n d \leq|x| \leq(a / 2)+(n+1) d, z) \\
& =u(x) \exp [i(v n d+\beta z)] \\
& u(x)=A \cos (x u / 2)\langle f \sin [x(|x|-(a / 2)-n d)] \\
& \quad-\sin [x(|x|-(a / 2)-(n+1) d]\} / \sin (x d),  \tag{2.19}\\
& E_{y}(0 \leq|x| \leq a / 2, z)=A \cos (x x) \exp (i \beta z)
\end{align*}
$$

and is illustrated in Fig. 1.
For odd modes and $\varkappa a, \varkappa d \ll 1$, the dispersion relation given by (2.16) does not have a solution, so that there are no odd modes in a narrow channel.


FIG. 1. Transverse structure of a nonthreshold photon-channeling mode in a microchannel of width $a>d(1)$ and in the case of quasichanneling with $a==d$ (2).

It follows from (2.4) that the expression for the longitudinal wave number is

$$
\begin{align*}
& \beta \equiv \beta^{\prime}+\dot{\beta} \beta^{\prime \prime}=\left(k^{2}-x^{2}\right)^{1 / 2} \\
& \approx k-(G / 2 k d)\left\{1-(1 / 12) G d\left[1+3\left(1-a d^{-1}\right)^{2}\right]\right\},  \tag{2.20}\\
& \beta^{\prime} \approx k\left\{1+\left(\chi^{\prime} / 2\right)+(1 / 24) k^{2} d^{2}\right. \\
&\left.\times\left[\left(\chi^{\prime}\right)^{2}-\left(\chi^{\prime \prime}\right)^{2}\right]\left[1+3\left(1-a d^{-1}\right)^{2}\right]\right\}, \\
& \beta^{\prime \prime} \approx k \chi^{\prime \prime}\left\{1+(1 / 6) k^{2} d^{2} \chi^{\prime}\left[1+3\left(1-a d^{-1}\right)^{2}\right]\right\} / 2 .
\end{align*}
$$

Analysis of the above expressions for $\nu^{\prime \prime}$ and $\beta$ shows that true channeling (i.e., the existence of a transversely localized mode with decreasing amplitude on either side of the channel and longitudinal attenuation coefficient that is lower by $\delta \beta_{a}^{\prime \prime} \approx k^{3} \chi^{\prime \prime} \chi^{\prime}(a-d)^{2} / 4$ as compared with the homogeneous lattice), occurs for arbitrary $a>d$ subject to the condition $\chi^{\prime}<0$.

It is important to note that, although in a homogeneous lattice without a channel and with $a=d$ we have $v^{\prime \prime}=0$ and the localized mode is absent, the values of $\beta^{\prime}, \beta^{\prime \prime}$ and the wave structure (Fig. 1, curve 2) nevertheless differ by the following amounts from the values $\beta^{\prime}=k i\left[1+\left(\chi^{\prime} / 2\right)\right]$, $\beta^{\prime \prime}=k \chi^{\prime \prime} / 2$, deduced from the average macroscopic electrodynamic equations for a plane transversely unmodulated wave:

$$
\delta \beta_{d}^{\prime} \approx k^{3} d^{2}\left[\left(\chi^{\prime}\right)^{2}-\left(\chi^{\prime \prime}\right)^{2}\right] / 24, \quad \delta \beta_{d}^{\prime \prime}=k^{3} d^{2} \chi^{\prime} \chi^{\prime} / 12
$$

This type of directed wave propagation in the perfect lattice with $a=d$ corresponds to nonthreshold quasichanneling ${ }^{22}$ and exhibits a number of properties. In particular, in addition to the absolute change in the longitudinal attenuation coefficient by the amount $\delta \beta_{d}^{\prime \prime}\left(\chi^{\prime}, \chi^{\prime \prime}\right)$, the coefficient is asymmetric as a function of frequency on either side of the absorption-line center, which does not occur in the isotropic medium.

### 2.2.Excitation of an 'optical' channeling mode and possible experiments'

The efficiency of excitation of a nonthreshold channeled mode is characterized by an amplitude that can be determined from the condition that the wave

$$
E_{y 0}(r)=\exp \left[i\left(k_{x} x+k_{z} z\right] / L^{1 / 2}\right.
$$

incident on the front surface of the crystal $(z=0)$ must be continuous across it, where the incident wave is normalized to the transverse beam size or the linear dimension $L$ of the
crystal, and the channeled mode $E_{y}(x, z)$ given by (2.18). This leads to the following expression for the excitation amplitude when the orthogonality of the eigenmodes is taken into account:

$$
B=\int E_{y 0}(x, 0) E_{y}^{*}(x, 0) d x
$$

The 'mating' condition does not include the wave reflected by the surface, whose amplitude for x -ray and shorter-wavelength radiation is negligible.

Since the transverse mode size $\Delta \approx a+\left(2 /\left|v^{\prime \prime}\right|\right) \gg a, d$ is significantly greater than the channel width and the lattice constant, and the amplitude of the oscillations in the field $E_{y}(x, z)$ is very small, the approximate expression for the field is

$$
E_{y} \approx\left|v^{\prime \prime}\right|^{1 / 2} \exp \left(i \beta z-\left|v^{\prime \prime} x\right|\right)
$$

In this approximation, the total probability of excitation of a non-threshold mode is

$$
\left.P \equiv|B|^{2} \approx 4 /\left\{\left|v^{\prime \prime}\right| L \mid 1+\left(k_{x}| | v^{\prime \prime} \mid\right)^{2}\right]\right\},
$$

provided $L \geqslant 2 /\left|\nu^{\prime \prime}\right|$. In particular, when the incident radiation propagates exactly along the channel axis, i.e., when $k_{x}=0$, the excitation probability $P \approx 4 /\left|v^{\prime \prime}\right| L$ is actually determined by the ratio of the transverse mode size to the width of the crystal (beam width).

Let us now estimate the quantitative characteristics of channeling. For nonresonant radiation $\chi^{\prime}=-\omega_{e}^{2} / 2 \omega^{2}$, which leads to $v^{\prime \prime}=\omega_{e}^{2}(a-d) / 4 c^{2}$. The initial conditions $x a, x d \ll 1 \quad$ with $\quad x=k \chi^{1 / 2} \quad$ are satisfied for $a \lesssim a_{\max }=\sqrt{2} c / \omega_{e} \approx 150-300 \AA$. For a lattice with $d \approx 2.5$ $\AA$ and typical values $a \approx 5-30 \AA, \omega_{e} \approx(3-6) \times 10^{16} \mathrm{~s}^{-1}$, we find that $\nu^{\prime \prime} \approx(0.5-5) \times 10^{4} \mathrm{~cm}^{-1}$, which corresponds to the total mode width $\Delta \approx 4-0.4 \mu \mathrm{~m}$. The spatial mode width falls off rapidly in the neighborhood of resonance. Thus, near the $K$-absorption edge of the diamond crystal with $\hbar \omega_{0} \approx 284 \mathrm{eV}$, we have $\chi^{\prime} \approx-4 \times 10^{-3}$ (Ref. 23) and $\Delta \approx 0.8-0.08 \mu \mathrm{~m}$ for the same range of values of $a$. Still greater localization of the nonthreshold mode corresponds to Mössbauer transitions for which $\left|\chi^{\prime}\right|$ increases rapidly. For example, for the tantalum crystal containing in the natural state $99.9 \%$ of ${ }^{181} \mathrm{Ta}$, and if we take $\hbar \omega_{0} \approx 6.25 \mathrm{keV}$ and $k \approx 3 \times 10^{8} \mathrm{~cm}^{-1}$, we obtain $\chi^{\prime} \approx-10^{-4}$ at $\omega=\omega_{0}+\Gamma$, and the mode size for a microchannel width $a \approx 5-12 \AA$ falls to $120-40 \AA$.

It is interesting to note that for radiation in the $\gamma$-ray range, whose frequency lies symmetrically on the other side of the nuclear resonance (e.g., for $\omega=\omega_{0}-\Gamma$ ), there is no channeling and instead of absorption we have amplification, which leads to a distortion of the emission spectrum.

The relative reduction in the absorption coefficient during channeling (as compared with unchanneled motion in an isotropic medium with the same optical density or in crystals without a channel) is $\delta \beta^{\prime \prime} / \beta^{\prime \prime} \approx 10^{-4}-10^{-2}$ for resonant x rays (in the above case of diamond). Correspondingly, near the Mössbauer $\gamma$-ray transition in the ${ }^{181} \mathrm{Ta}$ crystal, we have $\delta \beta^{\prime \prime} / \beta^{\prime \prime}=5 \cdot 10^{-3}-10^{-2}$. These effects can be detected, for example, by recording the angular dependence of the transmission coefficient. The relative contrast of the transmission maximum in the direction of channeling is then

$$
F=\left[\exp \left(\delta \beta^{\prime \prime} z\right)-1\right] /\left[\exp \left(\delta \beta^{\prime \prime} z\right)+1\right]
$$

It increases with the thickness of the medium and, despite the fact that $\delta \beta^{\prime \prime}$ is small, it can reach values in the range $1-$ $100 \%$ for $z \gg 1 / 1 \beta^{\prime \prime}\left|\beta^{\prime \prime}\right|$. For extended samples, the nonthreshold quasichanneling effect should be readily detectable and should lead to the appearance of a transmission maximum in the direction of quasichanneling, i.e., exactly along single-crystal planes.

The channeled propagation of short-wave radiation should lead to one further spatial effect that is significantly different from all other known diffraction phenomena. It follows from the general solution given by (2.19) and from the field structure illustrated in Fig. 1 that the wave propagating in perfect single crystals without microchannels has a transverse fine structure with period equal to the plane separation $d$. The probability of excitation of this type of mode at small angles of incidence, i.e., along the crystal planes, is close to unity, and the mode retains its transversely-modulated structure when it leaves the crystal. It is clear that the subsequent emission of this wave into the space behind the crystal will occur not only in the initial direction $\theta=0$, but also in a direction defined by the interference conditions

$$
d \sin \theta=n \lambda(n=1,2,3, \ldots)
$$

If we compare this condition with the condition for Bragg diffraction, we see that the quasichanneling directions and the Bragg directions $\theta_{B}$ are related by $\sin \theta=2 \sin \theta_{B}$, which can be used to identify the phenomenon of quasichanneling.

If we introduce the effective susceptibility $\chi=V / 2 E$ for neutrons, we find that all the above results on the channeling and quasichanneling of x rays and $\gamma$ rays remain valid for neutrons in crystals when their motion is described in terms of the 'optical' nuclear interaction, provided we introduce the obvious replacement $E \rightarrow \Psi$. For example, the above case of the ${ }^{181} \mathrm{Ta}$ Mössbauer isotope corresponds to the channeling of neutrons with velocity $3 \times 10^{4} \mathrm{~cm} / \mathrm{s}$ in a copper crystal.

We also note that $10-50 \AA$ voids in intercalated crystals can be used as microchannels for the more effective channeling of photons and neutrons.

### 2.3. Nonthreshold channeling of Mössbauer $\gamma$ rays in crystals

It was noted in connection with the wave equation given by (2.2) that its structure had to be generalized to the case of propagation of $\gamma$ rays in crystals. This wave equation is widely used in electrodynamics and in macroscopic wave theory. It is also valid in the microscopic theory of $x$-ray channeling, but can lead to errors in the case of $\gamma$ rays.

The complete set of equations for the microfield takes the form
$[\mathbf{\nabla}, \mathrm{H}]=\frac{1}{c} \frac{\partial \mathrm{D}}{\partial t},[\nabla, \mathrm{E}]=-\frac{1}{c} \frac{\partial \mathrm{H}}{\partial t}, \mathrm{D}=\varepsilon \mathrm{E}, \varepsilon=1+\chi(\mathrm{r})$.
The wave equation

$$
\begin{equation*}
\Delta \mathbf{E}+\nabla(\mathbf{E} \nabla \varepsilon / \varepsilon)+k^{2} \varepsilon \mathbf{E}=0 \tag{2.21}
\end{equation*}
$$

is obtained by eliminating the magnetic field $\mathbf{H}$ from (2.21). When $\varepsilon$ is a constant, (2.2) follows from (2.22). However, in our case, the change in susceptibility in one lattice period is significant and the magnitude of $\nabla \varepsilon$ is nonzero, so that the propagation of radiation in the crystal must be examined in
terms of the complete equation (2.22). For the one-dimensional function $\varepsilon$ we have
$\frac{\partial^{2} E_{y}}{\partial x^{2}}+\frac{\partial^{2} E_{y}}{\partial z^{2}}+\frac{\partial E_{y}}{\partial x}-\frac{\partial \varepsilon}{\partial x}+E_{y}\left[\frac{\partial^{2} \varepsilon}{\partial x^{2}}-\left(\frac{\partial \varepsilon}{\partial x}\right)^{2}\right]+k^{2} \varepsilon E_{y}=0$.

Let us now estimate the realtive importance of each of the additional terms. First, we introduce explicitly the spatial structure of susceptibility within each interchannel 'wall' (crystal plane), assuming that the nuclei responsible for the interaction with the Mössbauer $\gamma$ rays take part in the thermal motion that is taken into account by introducing their spatial probability density

$$
\chi(x)=\langle\chi) d \exp \left(-x^{2} / 2 u^{2}\right) / \sqrt{2 \pi} u
$$

where $\langle\chi\rangle$ is the susceptibility averaged over the lattice period (i.e., the macroscopic susceptibility) and $u$ is the root mean square amplitude of the displacement of the nuclei. If we use the solution given by (2.19), the first of the additional terms assumes the form
$\left(\partial E_{y} / \partial x\right) \partial \varepsilon / \partial x \approx E_{y} k((\chi))^{3 / 2} x d \exp \left(-x^{2} / 2 u^{2}\right) / u^{3} \sqrt{2 \pi}$.
If we compare it with the existing term in the analogous structure $k^{2} \varepsilon E_{y}$, we find that the ratio is exceedingly small ( $\sim 10^{-3}$ ), so that the term can be neglected.

The two other additional terms that contain susceptibility derivatives can be combined with the previous term $k^{2} \varepsilon E_{y}$ by introducing the effective susceptibility

$$
\chi_{0}=\chi(x)-\left(\frac{\partial \chi}{\partial x}\right)^{2} k^{-2}+\frac{\partial^{2} \chi}{\partial x^{2}} k^{-2}
$$

which finally enables us to employ the structure of the simplified equation (2.2).

It was shown above that the weak wave interaction of short-wave radiation with each individual crystal plane ensures that it is expedient to replace the true susceptibility with its layer average. In our case, we find that

$$
\begin{equation*}
\left\langle\chi_{0}\right\rangle=\langle\chi\rangle-\left\langle\left(\partial \chi^{\prime} \partial x\right)^{2}\right\rangle / k^{2}=\langle\chi\rangle-(\langle\chi\rangle)^{2} d / 4 u^{3} k^{2} \sqrt{\pi} \tag{2.24}
\end{equation*}
$$

It is clear that when the spatial dependence of susceptibility is taken into account, this results in the additional nonzero term $-(\langle\chi\rangle)^{2} d / 4 u^{3} k^{2} \sqrt{\pi}$ in the expression for the effective susceptibility $\left\langle\chi_{0}\right\rangle$. Let us estimate the magnitude of this term. If we take $u \approx 2 \times 10^{-10} \mathrm{~cm}, k \approx 3 \times 10^{8} \mathrm{~cm}^{-1}$, and $d \approx 2.5 \AA$ (which is close to the properties of ${ }^{181} \mathrm{Ta}$ ), we obtain $\left\langle\chi_{0}\right\rangle=\langle\chi\rangle-5 \times 10^{3}(\langle\chi\rangle)^{2}$. For the estimated resonance value $\langle\chi\rangle=-10^{-4}$, the additional term produces a $50 \%$ change in the average susceptibility, which gives rise to a significant change in the quantitative characteristics of channeling. Moreover, it is clear that the additional term is very small for nonresonant $x$ rays because, in the case of scattering by atomic electrons, the magnitude of $u$ cannot be smaller than the root mean square localization radius of atomic electrons, i.e., $\sim 10^{-9}-3 \times 10^{-9} \mathrm{~cm}$. We thus obtain $\langle\chi\rangle \approx-10^{-7}-10^{-9}$.

We also note that such terms do not appear in the case of neutron channeling because of the absence of gradient terms from the Schrödinger equation.

### 2.4.Possibility of channeling in natural microchannels in zeolites and asbestos

Among the analogous systems of crystal structure elements that are suitable for channeling of short-wave radiation and neutrons, the most interesting is the system of natural rectilinear microchannels with radii $R \approx 4-10 \AA$ in zeolites. ${ }^{24,25}$

The utility of these microchannels has frequently been discussed in the literature in connection with attempts to develop x-ray resonators with distributed feedback, ${ }^{26}$ and also in direct modelling of x -ray lasers with active medium in the form of a beam of relativistic positrons. ${ }^{8,27-29}$ Let us therefore consider the propagation of short-wave radiation in an ultranarrow channel. The solution of the wave equation

$$
\begin{aligned}
& \left(\frac{\partial^{2}}{\partial r^{2}}+\frac{1}{r} \frac{\partial}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2}}{\partial \varphi^{2}}+\frac{\partial^{2}}{\partial z^{2}}\right) E_{z}+k^{2} \varepsilon(r) E_{z}=0, \\
& \varepsilon(r)=1+\chi(r),
\end{aligned}
$$

for a cylindrical hollow channel $R$ surrounded by a medium with susceptibility $\chi$ is

$$
E_{z}=\exp \left[i(\beta z-m \varphi) \times\left\{\begin{array}{l}
C_{1} J_{|m|}(x r), r \leq 2, \\
C_{2} K_{|m|}\left(\left(-k^{2} \chi-x^{2}\right)^{1 / 2} r\right), r \geq R,
\end{array}\right.\right.
$$

where $J_{|m|}(x)$ is a Bessel function and $K_{|m|}(y)$ is a Kelvin function.

Since the tangential components of the electric field and its derivative must be continuous, we find that the dispersion relation is

$$
\begin{aligned}
& x J^{\prime}{ }_{|m|}(x R) K_{|m|}\left(\left(-k^{2} \chi-x^{2}\right)^{1 / 2} R\right) \\
& \quad=\left(-k^{2} \chi-x^{2}\right)^{1 / 2} J_{|m|}(x R) K_{|m|}^{\prime}\left(\left(-k^{2} \chi-x^{2}\right)^{1 / 2} R\right) .
\end{aligned}
$$

For a channel of small radius, e.g., a microchannel in zeolite, we can expand these functions and their derivatives in the dispersion relation and obtain the following equation for the wave number:

$$
\left(x^{2} R^{2} / 2\right) \ln \left[2 /\left(-k^{2} \chi-x^{2}\right)^{1 / 2} R\right]=1 .
$$

It is readily verified that this equation has no solutions for nonzero azimuthal wave numbers. The transverse wavenumber structure is obtained by simplifying the last equation and putting $m=0$ :

$$
x=2 / R\left[\left|\ln \left(-4 / k^{2} R^{2} \chi\right)\right|^{1 / 2} .\right.
$$

If we use the relation between the longitudinal and transverse wave numbers $\beta=\left(k^{2}-\chi^{2}\right)^{1 / 2}$, we readily find the expression for the longitudinal wave number (propagation constant)

$$
\beta \approx k-\left[4 / k R^{2} \ln \left(-k^{2} R^{2} \chi / 4\right)\right]
$$

and the longitudinal attenuation coefficient

$$
\beta^{\prime \prime} \approx-4 \chi^{\prime \prime} / R^{2} k \chi^{\prime} \ln ^{2}\left(-k^{2} R^{2} \chi / 4\right)
$$

that follows from it. Starting with the mode structure $E_{z} \sim K_{0}\left(\left(-k^{2} \chi-\varkappa^{2}\right)^{1 / 2} r\right)$, outside a channel, we find the condition for the confinement of the propagating wave to a single channel with a small adjacent region $l_{R} \approx\left(-k^{2} \chi-\varkappa^{2}\right)^{-1 / 2}$. This case corresponds to the neglect of wave tunneling into other channels, i.e., true channeling. The size of the near-channel mode localization region

$$
l_{K} \approx R\left\{-k^{2} R^{2} \chi-\left[4 / \ln \left(-4 / k^{2} R^{2} \chi\right)\right]\right\}^{-1 / 2}
$$

depends on the channel size, the wavelength $\lambda$, and the value of $\chi$.

It follows from the general structure of $l_{R}$ that the size of the near-channel mode localization region is greater than the 'thickness' of the skin layer near a plane surface $l_{\infty}$, as $R \rightarrow \infty$, where $l_{\infty}=1 / k(-\chi)^{1 / 2}$. For example, for x rays with $\lambda=2 \AA$ and $\chi=-10^{-6}$, we have $l_{\infty} \sim 300 \AA$, whereas for Mössbauer radiation with the same parameters as for ${ }^{181} \mathrm{Ta}$, we have $l_{\infty} \approx 30 \AA$. Since the channel separation in zeolite crystals is $d_{1} \approx 30 \AA$, and is comparable with or significantly smaller than $l_{\infty} \approx 30-300 \AA$, the concept of localized propagation in each individual microchannel is incorrect. The propagation of photons in a medium with periodically distributed microchannels must then be analyzed with allowance for the two-dimensional Bloch conditions.

For an isolated channel, the reduction in absorption during channeling as compared with absorption in a continuous medium $\beta_{0}^{\prime \prime}=k \chi^{\prime \prime} / 2$ in the optimum case of a small but negative real part of susceptibility, $\chi^{\prime}<0$, is described by

$$
\beta^{\prime \prime} / \beta_{0}^{\prime \prime}=-8 / k^{2} R^{2} \chi^{\prime} \ln ^{2}\left(-k^{2} R^{2} \chi / 4\right),
$$

which predicts a reduction in absorption with increasing channel size (radius).

For channels in asbestos fibers, the internal radius $R \approx 40 \AA$ satisfies the criterion for single-mode propagation $\varkappa R \ll 1$ and all the conclusions and estimates made above.

Since the effective 'optical' susceptibility for thermal and faster neutrons $\chi=\langle V\rangle / 2 E$ is characterized by the same range of values $|\chi| \leqslant 10^{-5}$ as for x -ray photons, all the conclusions about the channeling of photons in zeolites and asbestos apply fully to neutrons.

## 3. MAGNETIC CHANNELING OF NEUTRONS IN MAGNETIC CRYSTALS

In magnetic crystals, the effective neutron-crystal interaction can be due to both the purely nuclear mechanism of neutron channeling, which occurs within the framework of the 'optical' approximation (see Sec. 2), and the purely magnetic mechanism. Neutral particles with a magnetic momment participate in the same process. They are confined to a channel by the dipole-dipole interaction between their magnetic moment and the ordered but inhomogeneous distribution of magnetic moments in the lattice. ${ }^{30}$

To explain the origin and strength of the interaction, consider the motion a particle in a rectangular channel with sides $a$ and $b$ in the $x, y$ plane formed by the intersection of crystal planes consisting of ordered atomic magnetic moments forming a simple ferromagnetic (or oriented paramagnetic) dielectric structure with lattice constants $a_{0}, b_{0}$, $d_{0}$. To simplify the solution, we shall confine our attention to the motion of particles with magnetic moments $\mu_{0}$ aligned in the positive or negative direction of the $O z$ axis.

The secular part of the Hamiltonian describing the interaction of the particle magnetic moment $\vec{\mu}_{0}$ with all the magnetic moments $\vec{\mu}_{l}$ is

$$
\begin{equation*}
W=\sum_{\mathrm{n}}\left(A_{\mathrm{n}}+B_{\mathrm{n}}\right)\left(1-3 \cos ^{2} \theta_{\mathrm{n}}\right) r_{\mathrm{n}}^{-3} \tag{3.1}
\end{equation*}
$$

$$
\begin{aligned}
& A_{\mathrm{n}}=\mu_{0 z} \mu_{1 z \mathrm{n}}, \\
& B_{\mathrm{n}}=\frac{\mu_{0 z} \mu_{1 z \mathrm{n}}-\vec{\mu}_{0} \vec{\mu}_{1 \mathrm{n}}}{2} ;
\end{aligned}
$$

where $\mathrm{n}=\left\{n_{x}, n_{y}, n_{z}\right\}$ is the three-dimensional site number in the magnetic lattice. For the system under consideration, the term $B_{\mathrm{n}}$ that describes the magnetic dipole-dipole interaction with mutual overturning of the magnetic moments must be zero. We note that, for $a=a_{0}, b=b_{0}$, the problem corresponds to motion in the elementary crystal channel formed by the intersection of four nearest planes, and for $a \gg a_{0}, b_{0}$ or $b \gg a_{0}, b_{0}$ it corresponds to motion in the gap between two plane layers of a ferromagnet.

### 3.1. Magnetic channeling of particles interacting with a spin wave

We assume that all the magnetic moments of atoms in their static positions are aligned along the channel axis (the Oz axis). We shall consider the case where a transverse spin wave of amplitude $\delta_{s} \mu_{1}$ propagates in the magnet in the $O z$ direction:

$$
\begin{equation*}
\mu_{1 \perp n_{z}}=\delta_{s} \mu_{1} \cos \left(k z_{n_{z}}-\omega t+\varphi\right), \mu_{1 z n_{z}}=\left(\mu^{2}-\mu_{1 \perp n_{z}}^{2}\right)^{1 / 2} \tag{3.2}
\end{equation*}
$$

In the coordinate frame moving with the particle velocity $v_{z}$, equal to the spin-wave velocity, we have (Fig. 2)

$$
z_{n_{z}}^{\prime}=z_{n_{z}}-v_{z} t, \quad v_{z}=v_{\mathrm{sw}}=\omega / k
$$

If we now introduce the linear magnetic-moment density $\mu_{l} / d_{0}$ along the $O z$ axis, we can replace summation over $n_{z}$ with integration with respect to $z$. As in charged-particle channeling theory, this replacement is admissible if a typical change in the particle trajectory occurs within a distance $\Delta z \gg d_{0}$. Because of this, and also because of the rapid reduction in the strength of the dipole-dipole interaction with increasing distance, we neglect edge effects at the ends of the channel. The final result is

$$
\begin{aligned}
W= & \left(\mu_{0} \mu_{1} g / d_{0}\right) \sum_{n_{x} n_{y}} \int_{-\infty}^{\infty} \mathrm{d} z^{\prime}\left[1-3 \cos ^{2} \theta\left(z^{\prime}\right)\right] \\
& \times\left\{1-(1 / 4) \delta_{\mathrm{s}}^{2}\left[1+\cos 2\left(k z^{\prime}+\varphi\right)\right]\right\}\left[\left(z^{\prime}\right)^{2}+r_{\perp}^{2}\right]^{-3 / 2},
\end{aligned}
$$

where $\quad r_{1}^{2} \equiv r_{1}^{2}\left(n_{x}, n_{y}\right)=\left[n_{x} a_{0}+(a / 2)+x\right]^{2}+\left[n_{y} b_{0}\right.$ $+(b / 2)+y]^{2}$ and $g= \pm 1$ for parallel and antiparallel directions of $\vec{\mu}_{0}$ and $\vec{\mu}_{l}$, respectively.

It is now convenient to transform this expression to the form


FIG. 2. Particle in a magnetic channel.

$$
\begin{align*}
& W=\left(\mu_{0} \mu_{1} g / d_{0}\right) \sum_{n_{x}, n_{y}} \int_{-1}^{1}\left(1-3 \tau^{2}\right) \\
& \times\left[1-(1 / 4) \delta_{s}^{2}(1+\cos \beta(\tau) \cdot \cos 2 \varphi\right. \\
&-\sin \beta(\tau) \cdot \sin 2 \varphi)] d \tau / r_{\perp}^{2}, \beta(\tau)=2 k r_{\perp} \tau /\left(1-\tau^{2}\right)^{1 / 2}, \tau=\cos \theta . \tag{3.3}
\end{align*}
$$

Let us examine the structure of the integrand in (3.3). It is clear that for all atomic moments $\vec{u}_{l}$ lying within the solid angle corresponding to $|\tau|<\tau_{0}$, we have $\beta(\tau) \approx 1$, and for $|\tau|>\tau_{0}$ the function $\cos \beta(\tau)$ exhibits rapid oscillations with a change of sign, where $\tau_{0}=\left(1+16 k^{2} r_{1}^{2} / \pi^{2}\right)^{-1 / 2}$.

As a result, the integral involving $\cos \beta(\tau)$ in the first interval $|\tau|<\tau_{0}$ can be evaluated in terms of elementary functions, whereas in the second interval it is approximately equal to zero. Inspection of the integral containing the factor $\sin \beta(\tau)$ shows that it vanishes because the integrand is odd. All this leads to the following final result:
$W \approx-8 \mu_{0} \mu_{1} g \delta_{s}^{2} k^{2} \cos 2 \varphi \sum_{n_{x} n_{y}}\left\{\pi^{2} d_{0}\left[1+\left(16 k^{2} r_{\perp}^{2} / \pi^{2}\right)\right]^{3 / 2}\right\}^{-1}$.
Evaluation of this sum yields the following expressions for the force components acting on the magnetic particle in the channel:

$$
\begin{align*}
& F_{\xi}=-\partial W / \partial \xi \approx 256 \mu_{0} \mu_{1} g \delta_{s}^{2} \cos 2 \varphi \cdot k^{3} \lambda_{\xi} \xi / \pi^{3} V_{0} \\
& a k, b k<2 \\
& F_{\xi} \approx 0, a k, b k>2,  \tag{3.4}\\
& F_{z^{\prime}} \approx 0, \xi=x, y, \lambda_{\xi}=a / 2, b / 2
\end{align*}
$$

For the sake of clarity, we shall now describe the motion of the particles by classical equations whose validity will be examined later. The equation of motion of a particle in the channel is

$$
m \mathrm{~d}^{2} \mathrm{r} / \mathrm{d} t^{2}=\mathbf{F}
$$

subject to initial conditions

$$
\begin{equation*}
\xi(0)=\xi_{0}, \quad z(0)=0,\left.\quad \frac{\mathrm{~d} \xi}{\mathrm{~d} t}\right|_{0}=v_{\xi},\left.\quad \frac{\mathrm{d} z}{\mathrm{~d} t}\right|_{0}=v_{z}=v_{\mathrm{sw}} \tag{3.5}
\end{equation*}
$$

The solution is
$\xi=\left(\xi_{0}^{2}+v_{\xi}^{2} / \Omega^{2}\right)^{1 / 2} \cos \left\{\left(\Omega z / v_{z}\right)-\arccos \left[\xi_{0} /\left(\xi_{0}^{2}+v_{\xi}^{2} / \Omega^{2}\right)^{1 / 2}\right]\right\}$,
where

$$
z=v_{z} t, \quad \Omega=\Omega_{s \xi} \approx 3 \delta_{s} k\left(-\mu_{0} \mu_{\mathrm{I}} g \cos 2 \varphi k \lambda_{\xi} / m V_{0}\right)^{1 / 2}
$$

It is clear from (3.6) that, for real $\Omega_{s \xi}$, the particle executes translational motion in the channel while at the same time oscillating in the transverse direction $O \xi$ about the axis. The path of the particle in the transverse plane takes the form of two amplitude-limited, independent, and mutually orthogonal oscillations with frequencies $\Omega_{s \xi}$ and initial phases determined by the system parameters and initial conditions. It follows from (3.6) that, in a medium with moving, periodically inhomogeneous magnetization and comoving magnetic particles, the particles can take part in ordered
motion that is restricted by the channel size, i.e., channeling takes place. The corresponding capture angle is $\theta_{0}=v_{\xi}^{\max } / v_{z}=\lambda_{\xi} \Omega_{s \xi} / v_{z}$.

Let us consider in greater detail the motion in the $\xi, z$ plane, assuming that the particles enter the channel along the Oz direction, i.e., $v_{\xi}=0$. The character of the motion depends on the phase of the comoving spin wave and the orientation of the particle moment $g$. Particles with $g=-1$ that travel with spin-wave phases in the ranges $-\pi / 4<\varphi<\pi / 4,3 \pi / 4<\varphi<5 \pi / 4\left(\Delta \varphi_{1}\right)$ are periodically focused at

$$
z=\pi v_{z}(2 s+1) / 2 \Omega(\varphi), \quad s=0,1,2, \ldots
$$

and the beam exhibits maximum broadening for $z=\pi v_{z} s / \Omega(\varphi)$.

It is interesting to consider the integral focusing characteristics of a time-homogeneous beam in a channel of optimum length $z_{0} \equiv z(\varphi=0, \pi / 2)$. If the total particle current evaluated over one half of the spin-wave period at exit from the channel is $J_{0}$, the current density $\rho\left(z_{0},\left|\xi_{1}\right|\right)$ and the total current $J\left(z_{0},|\xi| \leqslant\left|\xi_{1}\right|\right)$ for $|\xi| \leqslant\left|\xi_{0}\right|$ and the same period of time in the section $z_{0}$ are given by

$$
\begin{align*}
& \rho\left(z_{0},\left|\xi_{1}\right|\right)=2 J_{0} \int_{\varphi_{1}\left(\xi_{1}\right)}^{\pi / 4} \mathrm{~d} \varphi / \pi a \cos \left[\pi(\cos 2 \varphi)^{1 / 2} / 2\right],  \tag{3.7}\\
& J\left(z_{0},|\xi| \leq\left|\xi_{1}\right|\right)=2 \int_{0}^{\left|\xi_{1}\right|} \rho\left(z_{0},|\xi|\right) \mathrm{d} \xi,
\end{align*}
$$

where $\varphi(\xi)$ is the inverse of $\xi(\varphi)$. Figure 3 shows the corresponding graphs. The initial homogeneous current is focused near the channel axis and $40 \%$ of all the particles are localized in the region $|\xi| \leqslant 0.05 \lambda_{\xi}$ whereas the corresponding figure for $|\xi| \leqslant 0.1 \lambda_{\xi}$ is $53 \%$.

Similar focusing is obtained for particles with parallel orientation ( $g=1$ ) if they enter the channel within the phase interval

$$
\pi / 4<\varphi<3 \pi / 4, \quad 5 \pi / 4<\varphi<7 \pi / 4 \quad\left(\Delta \varphi_{2}\right) .
$$

Particles corresponding to phases $\varphi= \pm \pi / 4, \pm 3 \pi / 4$, travel in straight lines. The other particles $g=1$ for $\Delta \varphi_{2}$ and


FIG. 3. Focusing of particles in the channel of optimum length. Curve 1 normalized particle flux density $\rho\left(z_{0},\left|x_{1}\right|\right)$, curve 2 -normalized total flux $J$ for $|x| \leqslant\left|x_{1}\right|$.
$g=1$ for $\Delta \phi_{1}$ ) have a different motion. For them $\operatorname{Re} \Omega=0$, $\operatorname{Im} \Omega=|\Omega|$, and

$$
\xi=\xi_{0} \cosh \left(|\Omega| z / v_{z}\right)+\left(v_{z} /|\Omega|\right) \sinh \left(|\Omega| z / v_{z}\right)
$$

For the distance $z$ satisfying the condition $|\Omega| z / v_{z} \gg 1$ we have

$$
\xi=\left\{\left[\xi_{0}+\left(v_{z} /|\Omega|\right)\right] / 2\right\} \exp \left(|\Omega| z / v_{z}\right),
$$

which for $v_{z}>-|\Omega| \xi_{0}$ corresponds to particles leaving the channel without crossing its axis, and for $v_{z}<-|\Omega| \xi_{0}$ to particles that cross the channel axis and leave through the other channel 'wall'. An interesting situation arises when the transverse velocity of a particle at exit from the channel is $v_{z}=-|\Omega| \xi_{0}$. When this happens, we find that, in an arbitrary section $z$,

$$
\xi=\xi_{0} \exp \left(-|\Omega| z / v_{z}\right)
$$

i.e., we have asymptotic focusing with an abrupt rise in density on the channel axis. This regime can be produced, for example, by changing the spin-wave phase by $\pi$ near the point $z_{0}$.

We must now consider a few numerical estimates. The characteristics of the system can be determined from the dispersion relation for a longitudinal spin wave ${ }^{32}$

$$
\omega=\gamma H_{0}+\eta k^{2}
$$

where $\eta$ is the exchange parameter and $H_{0}$ is the magnetic field. Wide channels are best used to achieve maximum focusing. Since the maximum channel dimensions are $a, b<2 / k$, it is best to use spin waves with the relatively low value $k \approx 10^{4} \mathrm{~cm}^{-1}$ (these are actually magnetostatic waves). For the realistic estimates $H_{0}=10^{3}$ Oe, $V_{0}=10^{-23} \mathrm{~cm}^{3}, \delta_{s}=0.1$ (Ref. 33), and $\mu_{1} \approx(2-8) \mu_{\mathrm{B}}$, we have $\Omega_{\text {max }} \approx(1-2) \times 10^{5} \mathrm{~s}^{-1}$ and $z_{0} \approx 2-4 \mathrm{~cm}$.

Much higher channeling frequencies and, hence, shorter focal lengths correspond to neutral atoms and paramagnetic molecules. Since, for such particles, $m=2 \times 10^{-24} A$ where $A$ is the atomic weight, we find that, for $k=10^{5}$ and $\delta_{s}=0.1$, we have $\Omega\left(\varphi_{0}\right) \approx 30-130 A^{-1 / 2}$ $\mathrm{MHz}, z_{0} \approx(1-0.2) \times 10^{-2} A^{1 / 2} \mathrm{~cm}$, and for $k=10^{4} \mathrm{~cm}^{-1}$ we obtain $\Omega\left(\varphi_{0}\right) \approx 5-15 A^{-1 / 2} \mathrm{MHz}$ and $z_{0} \approx(1.2-$ $0.3) \times 10^{-1} A^{1 / 2} \mathrm{~cm}$.

For the above case of a wide channel with transverse dimensions in the range $1-0.1 \mu \mathrm{~m}$, the maximum capture angle is $\theta_{0} \approx 2-20$ seconds of arc for neutrons and $\theta_{0} \approx-10$ minutes of arc for atoms and paramagnetic molecules.

Technically, it is much simpler to perform experiments with channels of length $l<z_{0}$ but $l \gg 1 / k$, which corresponds to a ferromagnetic film.

A parallel incident beam is then transformed at exit from the channel (on the other side of the film) into a convergent beam with a small convergence angle

$$
\begin{equation*}
2 \psi(\varphi)=2 \arctan \left(\xi_{0} \Omega^{2} l \cos 2 \varphi / v_{z}^{2}\right) \tag{3.8}
\end{equation*}
$$

where $l$ is the channel length. In the course of this transformation, the transverse size of the beam is reduced so that it becomes

$$
2 \xi=2\left(\xi_{0}-z \tan \psi(\varphi)\right)=2 \xi_{0}\left(1-\Omega^{2} l \cos 2 \varphi \cdot v_{z}^{-2}\right)
$$

and the beam is focused at

$$
\begin{equation*}
z=\tilde{z}(\varphi) \equiv \xi_{0} \cot \psi(\varphi) . \tag{3.9}
\end{equation*}
$$

Let us now examine the form of the solution for $\tilde{z}_{0}=\tilde{z}\left(\varphi_{0}\right)$. Particles with $g$ and $\Delta_{1,2}$ corresponding to focusing in a long channel are focused at $\tilde{z}_{0}$ for $\xi\left(\hat{z}_{0}, \varphi\right) \leqslant \xi_{0}(1-\cos 2 \varphi)$.
The focusing characteristics $\rho\left(\tilde{z}_{0}, \xi_{1}\right)$ and $J\left(\tilde{z}_{0},|\xi| \leqslant\left|\xi_{1}\right|\right)$ calculated numerically for this case are practically indistinguishable for all values of $\xi$ from thecorresponding characteristics for a long channel, i.e., a channel of length $z=l \ll z_{0}$ focuses just as effectively as one with $z=z_{0}$. For the parameter values estimated above and film thicknesses $l=0.1 \mathrm{~cm}$, the focal length for neutrons in the case of spin waves with $k=2 \times 10^{4}$ is $z_{0} \approx 4-20 \mathrm{~cm}$.

The spatial phase locking condition for the progressive spin wave and the particles channeled in a channel of length $z$ is

$$
\omega\left|v_{z}^{-1}-\left(v_{z} \pm \Delta v_{z}\right)^{-1}\right| z \leq \pi
$$

which can be used to estimate the admissible departure of the stream of particles from the monochromatic state:

$$
\left|\Delta v_{z} / v_{z}\right| \leq \pi v_{z} / \omega z_{0}
$$

In the cases analyzed above, channeling is possible if $\left|\Delta v_{z} / v_{z}\right| \leqslant 10^{-4}-10^{-1}$. We note that when a standing spin wave is established, an effective interaction occurs with only one of the two progressive waves forming the standing wave, i.e., with the wave accompanying the particle flux. In particular, it is possible to use a spin wave with very high $k$ and small film thickness $l$, which happens when a spin-wave resonance with $\tilde{z}_{0} \approx 0.1$ is excited in a thin film with $l \approx 10^{-5}-$ $10^{-6}$ and the focus is at the point $\tilde{z}_{0} \approx 0.1 \mathrm{~cm}$.

### 3.2. Directional interaction of particles with an eiastic wave in a magnetic channel

We shall now consider the motion of particles having a magnetic moment when a longitudinal ultrasonic elastic wave of frequency $\omega$ and amplitude $\Delta z$ is excited in the direction of alignment of the magnetic moments in a saturated ferromagnet (the $O z$ direction). This wave modulates the separation between the atomic magnetic moments in accordance with the expression
$d=d_{0}\left[1+\delta_{\mathrm{p}} \cos \left(k z_{n_{z}}-\omega t+\varphi\right)\right], \mu_{1 z n_{z}}=\mu_{1}, \delta_{\mathrm{p}}=k \Delta z \ll 1$.

Proceeding by analogy with spin waves, we obtain solutions that are identical to (3.5), but differ from the spin-wave case by the expression for the frequency, i.e.,

$$
\Omega=\Omega_{p \xi} \approx 6\left(-\mu_{0} \mu_{1} g \delta_{\mathrm{p}} \cos \varphi k^{3} \lambda_{\xi} / m V_{0}\right)^{1 / 2}
$$

The motion and focusing of particles in the ultrasonic field correspond to the case of spin waves if we allow for the fact that focusing and channeling occur for $g=-1$, $-\pi / 2<\varphi<\pi / 2$ and $g=1, \pi / 2<\varphi<3 \pi / 2$. Estimates show that the excitation of an ultrasonic wave with frequency 10 GHz , velocity $v_{\mathrm{Uw}}=2 \times 10^{5} \mathrm{~cm} / \mathrm{s}$, and amplitude $\Delta z=5 \times 10^{-9} \mathrm{~cm}$ gives rise to channeling with $\Omega_{p} \approx 10^{8}$ $A^{-1 / 2} \mathrm{~Hz}$ and $z_{0} \approx 10^{-3} A^{1 / 2} \mathrm{~cm}$ for channel dimensions $a, b<10^{-5} \mathrm{~cm}$. Accordingly, for an ultrasonic wave or frequency 100 MHz and amplitude $\Delta z \approx 2 \times 10^{-8} \mathrm{~cm}$, we have
$\Omega_{\mathrm{p}} \approx 10^{5} \mathrm{~A}^{-1 / 2} \mathrm{~Hz}, z_{0}=A^{1 / 2} \mathrm{~cm} ; a, b<10^{-3} \mathrm{~cm}$. This estimated set of parameters of the ultrasonic wave is quite realistic, which means that it is actually possible to observe orientational features when neutrons synchronized with the wave pass through a modulated magnet. In the case of a beam of paramagnetic atoms, all the parameters inprove in the ratio $\left(\mu_{B} / \mu_{1}\right)^{1 / 2} \approx 30$.

### 3.3. Focusing of particles by a domain wall

The foregoing discussion shows that the elastic restoring force responsible for channeling and focusing effects is due to the alternating longitudinal component of the magnetization of the channel walls. This spatial component can appear not only in spin waves and ultrasonic waves, but also in the Néel transitional domain wall. We shall now analyze the motion of a channel consisting of magnetic moments $\vec{\mu}_{l}$ whose orientation corresponds to the rotation of the magnetization of the domain wall.

Consider a domain wall between two layers (domains) of a saturated ferromagnet, in which the magnetic moments are at angles $\phi_{1}$ and $\phi_{2}$ to the axis, respectively. We shall suppose that the wall (thickness $l_{w}$ ) lies between $z=0$ and $z=l$, and that the variation of the angle $\phi(z)$ and the longitudinal component of magnetization $\mu_{1 z n_{z}}$ is described by

$$
\begin{equation*}
\varphi(z)=\varphi_{1}+x z, \mu_{1 z n_{z}}=\mu_{1} \cos \varphi(z), \varphi(z=l)=\varphi_{2} \tag{3.11}
\end{equation*}
$$

where $x=\left(\varphi_{2}-\varphi_{1}\right) / l_{w}$ is the rotational period of the magnetization in the domain wall of thickness $l_{w}$.

If we use the method presented in Sec. 3.1 to evaluate $W$, we obtain

$$
\begin{align*}
W= & \mu_{0} \mu_{1} g \sum_{n}\left(1-3 \cos ^{2} \theta_{n}\right) \cos \left(\varphi_{1}+x z_{n_{2}}\right) / \\
& \left(z-z_{n_{z}}\right)^{2}+r_{1}^{2} j^{1 / 2}, \tag{3.12}
\end{align*}
$$

where $g= \pm 1$ for parallel and antiparallel orientations of $\vec{\mu}_{0}$ relative to the $O z$ axis, respectively.

Transforming to the coordinate frame coupled to the longitudinal motion of the particle, $z-z_{n_{z}}=z_{n_{z}}^{\prime}$, and replacing summation over $n_{z}$ in (3.12) with integration, we find that the transverse and longitudinal components of the force acting on the channeled particle are respectively given by

$$
\begin{aligned}
& F_{\xi} \approx 10^{3} \mu_{0} \mu_{1} g \cos \left(\varphi_{1}+x z\right) x^{3} \lambda_{\xi} \xi / \pi^{3} V_{0} \\
& F_{z} \approx \pi \mu_{0} \mu_{1} g \sin \left(\varphi_{1}+x z\right) x / V_{0}, a, b<2 / x
\end{aligned}
$$

The solution of the longitudinal equation of motion

$$
m \frac{\mathrm{~d}^{2} z}{\mathrm{~d} t^{2}}=F_{z}(z) \equiv F_{z 0} \sin \left(\varphi_{1}+x z\right), F_{z 0}=\pi \mu_{0} \mu_{1} g x V_{0}^{-1}
$$

is
$z+F_{z 0}\left\{\left[\sin \left(\varphi_{1}+x z\right)-\sin \varphi_{1}\right] x^{-1}-z \cos \varphi_{1}\right\}\left(m x v_{z}^{2}\right)^{-1}=v_{z} t$.
For small $f_{z 0}$, the iteration method yields the approximate solution

$$
\begin{align*}
& z \approx v_{z} t-F_{z 0}\left\{\left[\sin \left(\varphi_{1}+x v_{z} t\right)-\sin \varphi_{1}\right] x^{-1}\right. \\
& \left.\quad-v_{z} t \cos \varphi_{1}\right\} /\left(m x v_{z}^{2}\right)^{-1},  \tag{3.13}\\
& \begin{aligned}
v_{z}(t) & \approx v_{z}-\left\{F_{z 0}\left[\cos \left(\varphi_{1}+x v_{z} t\right)-\cos \varphi_{1}\right] / m x v_{z}\right\} \\
& \equiv v_{z}-\Delta v_{z}(t) .
\end{aligned}
\end{align*}
$$

For the usual domain-wall thickness $l_{\mathrm{w}} \approx 10^{-5}-10^{-6} \mathrm{~cm}$ and an initial longitudinal particle velocity $v_{z} \gtrsim 10^{5} \mathrm{~cm}$, we have $\left|\Delta v_{z}^{\text {max }}\right| \leqslant 10^{-3} \mathrm{~cm}$.

The equation of motion in the transverse direction corresponds to an oscillation with variable frequency $\Omega(t)$. To find the solution in this direction, i.e., the $O \xi$ direction, in the first approximation, we use the zero-order approximation for the longitudinal motion $z=v_{z} t$. The final result is
$\frac{\mathrm{d}^{2} \xi}{\mathrm{~d} t^{2}}=-\Omega_{0}^{2} \cos \left(\varphi_{1}+x v_{z} t\right) \xi, \Omega_{0}=6\left[\mu_{0} \mu_{1} g x^{3} \lambda_{\xi}\left(m V_{0}\right)^{-1}\right]^{1 / 2}$,
the nature of the solution depends on the ratio of $\Omega_{0}$ and $\varkappa v_{z}=\Omega_{0}$. When $\left|\Omega_{0}\right| \gg \omega_{0}$, the system then corresponds to an oscillator with a slowly varying frequency, and the general solution becomes

$$
\xi=\xi(t) \exp \left[ \pm i \int_{0}^{t} \Omega(\tau) \mathrm{d} \tau\right], \Omega(\tau)=\Omega_{0} \cos \left(\varphi_{1}+x v_{z} \tau\right)
$$

where the amplitude $\tilde{\xi}(t)$ varies slowly in comparison with the phase factor. Transforming in (3.14) to the truncated equation, we find that

$$
\begin{align*}
\xi= & {\left[\xi_{0}^{2}+\left(v_{\xi}^{2} / \Omega^{2}(t)\right)\right]^{1 / 2} \cos \left\{\int_{0}^{t} \Omega(t(z)) \mathrm{d} z / v_{z}\right.} \\
& -\arccos \left[\xi_{0} /\left[\xi_{0}^{2}+\left(v_{\xi}^{2} / \Omega^{2}(t)\right)\right]^{1 / 2}\right\} \tag{3.15}
\end{align*}
$$

which is qualitatively similar to the channeling of spin and ultrasonic waves in (3.6).

For the other limiting case of a very rapid variation in instantaneous frequency $\Omega(t)$, when $\left|\Omega_{0}\right|<\omega_{0}$, the system cannot, even approximately, be interpreted as an oscillator, and the solution can be found by the small parameter method with $\Omega_{0} / \omega_{0}$ as the small parameter. The solution is then given by

$$
\begin{align*}
\xi= & \xi_{0}+\left(v_{\xi} z / v_{z}\right)+\Omega_{0}^{2}\left[( \xi _ { 0 } / v _ { z } x ) \left\{\left[\cos \left(\varphi_{1}+x z\right)-\cos \varphi_{1}\right] x^{-1}\right.\right. \\
& \left.+z \sin \varphi_{1}\right\}+\left(v_{\xi} / v_{z}^{3} x^{2}\right)\left\{z\left[\cos \varphi_{1}+\cos \left(\varphi_{1}+x z\right)\right]\right. \\
& \left.\left.+2\left[\sin \varphi_{1}-\sin \left(\varphi_{1}+x z\right)\right] x^{-1}\right\}\right] . \tag{3.16}
\end{align*}
$$

Let us consider some numerical estimates to enable us to choose the solution. For the average value $l_{\mathrm{w}} \approx 3 \times 10^{-6} \mathrm{~cm}$, we have $\left|\Omega_{0}\right| \approx 10^{9} \mathrm{~Hz}$ and $\left|\Omega_{0}\right| \approx 3 \times 10^{10} A^{1 / 2} \mathrm{~Hz}$ for neutrons and paramagnetic atoms, where $A$ is the atomic weight of the atoms passing through the crystal. For all particles with velocities $v_{z}>10^{5} \mathrm{~cm} / \mathrm{s}$ we obtain $\omega_{0} \gg\left|\Omega_{0}\right|$, i.e., the second case occurs and we shall examine it in greater detail. We note that this situation will also occur for lower velocities in the case of neutrons.

For an initially parallel beam, the convergence angle and the focal length are respectively given by

$$
\begin{equation*}
\psi \approx \tan \psi=-\frac{\mathrm{d} \xi}{\mathrm{~d} z} \approx \Omega_{0}^{2} \xi_{0}\left(\sin \varphi_{2}-\sin \varphi_{1}\right)\left(x v_{z}^{2}\right)^{-1} \tag{3.17}
\end{equation*}
$$

and

$$
\begin{equation*}
\tilde{z}_{0}=\xi_{0} \cot \psi \approx \xi_{0} / \psi . \tag{3.18}
\end{equation*}
$$

We must now elucidate the character of this focusing. When $g=1$ and $\sin \varphi_{2}>\sin \varphi_{1}$ (and, correspondingly, $g=-1$ and $\sin \varphi_{1}>\sin \varphi_{2}$ ) we have $\psi>0$ and $\tilde{z}_{0}>0$, which
corresponds to a thin converging lens with a real focal point at $\hat{z}_{0}$. It is clear that the largest beam convergence angle and, correspondingly, the shortest focal length $\tilde{z}_{0}$, occur for a $180^{\circ}$ domain wall between two domains with magnetization directions $\varphi_{1}=-\pi / 2, \varphi_{2}=\pi / 2$ for $g=1$ and $\varphi_{1}=\pi / 2$, $\varphi_{2}=3 \pi / 2$ for $g=-1$. The focal length of this optimumsystem in the case of neutrons is $\tilde{z}_{0} \approx 0.04 \mathrm{~cm}$ for $v_{z} \approx 2 \times 10^{5}$ $\mathrm{cm} / \mathrm{s}$. For light atoms $z_{0} \approx 2 \times 10^{-3} \mathrm{~cm}$ for $v_{z} \approx 10^{6} \mathrm{~cm} / \mathrm{s}$.

For a different choice of parameter values ( $g=-1$, $\sin \varphi_{2}>\sin \varphi_{1}$ and $g=1, \sin \varphi_{2}<\sin \varphi_{1}$ ), we have $\psi<0$, which, corresponds to a thin diverging lens with virtual focal point at $\tilde{z}_{0}<0$.

It is possible to produce in the laboratory a two-domain magnetic structure of this kind with one domain wall. ${ }^{34}$ The unsaturated ferromagnet, even if it is thin, usually consists of a large number of domains with linear dimensions $L_{0} \gtrsim 1$ $\mu \mathrm{m}$. Let us consider the motion of particles in this type of multi-domain structure with successive magnetization directions $\varphi_{1}, \varphi_{2}, \varphi_{1}, \varphi_{2}, \ldots$, bearing in mind the fact that, usually, $L_{0} \ll \tilde{z}_{0}$. We shall assume that the first and the second (in the direction of motion) domains are characterized by a combination of values of $g, \varphi_{1}$, and $\varphi_{2}$, such that the domain wall will focus, i.e., $\psi>0, \tilde{z}_{0}>0$. The beam aperture falls by a factor of ( $1-2 L_{0} \psi / \xi_{0}$ ) as the particles cross the second domain and before they reach the second wall. The second domain wall, for which we have to substitute $\varphi_{1} \rightarrow \varphi_{2}$, $\varphi_{2} \rightarrow \varphi_{1}$, is analogous to a diverging lens with divergence angle $-\psi<0$. Since this domain wall receives a converging beam with $\psi>0$, the emerging beam is parallel, but has a smaller aperture than the incident beam. By repeating this cycle for each successive pair of domain walls (Fig. 4), we find that an initially parallel beam traversing a channel of length $z$ remains parallel (after an even number of domains) or becomes converging (after an odd number of domains), and its transverse size is reduced by a factor $K_{0}$ given by

$$
K_{0}^{-1}=\left[1-\left(2 L_{0} \psi / \xi_{0}\right)\right]^{z / 2 L_{0}} \approx \exp \left(-z / \widetilde{z_{0}}\right)
$$

For example, for a magnet with a total thickness $z \approx 0.2$ cm and the above estimated value $\tilde{z}_{0} \approx 0.04 \mathrm{~cm}$, we have $K_{0} \approx 150$.

This type of motion corresponds qualitatively to the above asymptotic focusing of particles in the case of spin and


FIG. 4. Asymptotic focusing of a particle beam by a multidomain structure with alternating directions of magnetization.
ultrasonic waves, but occurs without any external intervention in the natural multimode structure.

In the other limiting case, in which the first and second domains are characterized by values of $g, \varphi_{1}, \varphi_{2}$ such that $\psi<0$ when the beam crosses the first domain wall, we find that the beam expands and the particles escape from the channel. In this situation, $K_{0}{ }^{-1} \approx \exp \left(z / \tilde{z}_{0}\right)$.

### 3.4. Capture of particles into a magnetic channeling regime and possible experimentation realizations

The above discussion refers to the motion of particles whose trajectory is confined to a single channel, and it is clear that this solution can be valid.

The limiting capture angle for such particles

$$
\theta_{0} \approx v_{\xi} / v_{z} \leq \Omega\left(\lambda_{\xi}^{2}-\xi_{0}^{2}\right)^{1 / 2} / v_{z}
$$

is very small and, for example, amounts to a fraction of a second of arc for an elementary internal crystal channel. Nevertheless it follows from the analysis given later that capture into channeling is also possible for particles incident on the crystal within the angular range $\Delta \theta>\theta_{0}$ relative to the channel axis.

To explain the 'macroangular' capture problem, we shall for the moment ignore microscopic variations in the trajectory of uncaptured particles within the confines of each elementary channel and consider the averge macroscopic motion in which the crystal lattice consisting of a set of planes is replaced by a continuous medium. It is clear that this replacement is valid for particles whose transverse velocity $v_{\xi}$ is greater than the velocity range within which capture into the channel is possible.

Let us consider the macroscopic motion of particles incident on the surface of a magnet at an angle $\theta=\arctan$ ( $v_{x} / v_{z}$ ). We shall choose the magnet in the form of a plate of thickness $x_{0}$ (Fig. 5). The expression for the two-dimensional distribution density of the atomic magnetic moments in the continuum approximation can be used to show that the average interaction energy between a particle and the magnet ${ }^{30}$ is given by the following formula (which can be used directly to evaluate an average over any coordinate):

$$
\begin{align*}
\langle W\rangle= & -2 \mu_{0} \mu_{1} g P V_{0}^{-1}\left\{\arctan \left[8 k\left(\frac{x_{0}}{2}+x\right) \pi^{-1}\right]\right. \\
& \left.+\arctan \left[8 k\left(\frac{x_{0}}{2}-x\right) \pi^{-1}\right]\right\} \tag{3.19}
\end{align*}
$$



FIG. 5. Capture of particles into a magnetic channel: 1 --reflection of a slow particle, 2, 3-capture into an elementary channel for particles with transverse velocity $\left|v_{x 2}\right|<\left|v_{x 3}\right|<\left|\Delta v_{x}^{\max }\right|, 4$-motion of uncaptured particle with $\left|v_{x 4}\right|>\left|\Delta v_{x}^{\max }\right|$.
where $P=\delta_{\mathrm{s}}^{2} \cos 2 \varphi$ for spin waves and $P=4 \delta_{\mathrm{p}} \cos \varphi$ for ultrasonic waves.

The solution of the averge equation of motion for a particle then takes the form

$$
\begin{equation*}
v_{x}^{2}=v_{x}^{2}(x= \pm \infty)+\Delta v_{x}^{2}(x), \tag{3.20}
\end{equation*}
$$

where

$$
\begin{aligned}
\Delta v_{x}^{2}= & 8 \mu_{0} \mu_{1} g P\left(m V_{0}\right)^{-1}\left\{\arctan \left[8 k\left(\frac{x_{0}}{2}+x\right) \pi^{-1}\right]\right. \\
& \left.+\arctan \left[8 k\left(\frac{x_{0}}{2}-x\right) \pi^{-1}\right]\right\}
\end{aligned}
$$

It is clear from (3.20) that a particle with orientation $g$ traveling in step with the spin or ultrasonic wave phase such that $g \cos 2 \varphi_{\mathrm{sw}}>0$ or $g \cos \varphi_{\mathrm{UW}}>0$ will become accelerated as it approaches the magnet, and its velocity will reach its maximum value in the $x=0$ plane. Thereafter, the particle begins to slow down and its asymptotic value $v_{x}^{2}( \pm \infty)$ reaches the initial value $v_{x}^{2}(\mp \infty)$. It is clear that, if the initial velocity exceeds the value $v_{0 x}=\Omega a / 2$ necessary for channeling, the above macroscopic acceleration effect is even less likely to lead to channeling.

A different sitution obtains for particles whose motion is such that $g \cos 2 \varphi_{\mathrm{Sw}}<0$ or $g \cos \varphi_{\mathrm{Uw}}<0$. The transverse velocity of such particles decreases as they approach the magnet. This retardation is physically due to the interaction between the magnetic moment and the magnetic field of the propagating spin or ultrasonic wave. In particular, if $\left|\Delta v_{x}^{\max }\right|$ $\geqslant\left|v_{x}( \pm \infty)\right|$, the particle can come to rest (in the transverse velocity space). When this condition is satisfied for $|x|>\left|x_{0}\right| / 2$, the particle will not enter the interior of the magnet and its motion will be reversed by reflection (curve 1 in Fig. 5). On the other hand, when $\left|\Delta v_{x}(x)\right|=\left|v_{x}( \pm \infty)\right|$ inside the magnet, i.e., for $0<|x|<\left|x_{0}\right| / 2$, the transverse velocity $\left|v_{x}\right|$ falls to $\left|v_{0 x}\right|$, for which the conditions for capture into microchanneling begin to be satisfied (curves 2 and 3 in Fig. 5). Since the above phase conditions for channeling $g \cos 2 \varphi_{\mathrm{sw}}<0, g \cos \varphi_{\mathrm{Uw}}<0$ are the same as the conditions for slowing down, such particles are captured into the channel. We note that the longitudinal velocity of the particles in the magnet remains unaltered during the transverse-velocity anomalies.

Thus, the conditions for macrocapture into a microchannel are

$$
\begin{align*}
& g \cos 2 \varphi_{\mathrm{sw}}<0, g \cos \varphi_{\mathrm{sw}}<0, \\
& \left|\Delta v_{x}^{\max }\right|^{2} \geq v_{x}^{2}( \pm \infty)-\left(\Omega^{2} a^{2} / 4\right) \tag{3.21}
\end{align*}
$$

Let us estimate the retardation effect. For the above magnet parameter values, we have $\left|\Delta v_{x}^{\max }\right| \approx 300 \delta_{\mathrm{s}} \mathrm{cm} / \mathrm{s}$ and $\left|\Delta v_{x}^{\max }\right|$ $\approx 600 \delta_{\mathrm{p}}^{1 / 2} \mathrm{~cm} / \mathrm{s}$ for spin and ultrasonic waves, respectively, in the case of neutrons, and $\left|\Delta v_{x}^{\max }\right| \approx 5 \times 10^{3} \delta_{\mathrm{s}} \mathrm{cm} / \mathrm{s}$ and $\left|\Delta v_{x}^{\max }\right| \approx 10^{4} \delta_{\mathrm{p}}^{1 / 2} \mathrm{~cm} / \mathrm{s}$ for paramagnetic atoms.

It is clear that in the case of thermal neutrons with $v_{z}$ $>10^{5} \mathrm{~cm} / \mathrm{s}$, the magnetic retardation gives rise to channeling with capture angles up to a few dozen of minutes of arc, i.e., much greater than the critical angle for capture by an individual microchannel. For atoms with uncompensated magnetic moments, a very large macrocapture angle of a magnetic channel will hardly facilitate channeling by means
of very strong retardation and scattering by lattice atoms. This type of experiment can probably be performed only with exotic atoms such as mesic atoms.

To conclude, we must consider the conditions for the validity of our classical approach to the motion of particles in microchannels.

Quantum mechanical analysis of the motion of particles in a magnetic potential well such as (3.3)-(3.4) leads to quantization in a parabolic potential $V=m \Omega^{2} \xi^{2} / 2$, which results in an energy spectrum of the form $E_{n}$ $=\hbar \Omega(n+1 / 2), n=0,1,2, \ldots$. It then flows from the correspondence principle that the classical and quantum-mechanical analyses become identical for $n \gg 1$. It is only then that all the channeling directions form a quasicontinuous distribution, which corresponds to classical capture into the channel for all angles smaller than the critical value. By equating the height of the potential barrier $V^{\text {max }}$ $=m \Omega^{2} \lambda_{\xi}^{2} / 2$ to $E_{n}^{\max }$, we readily obtain the following estimate for the criterion that indicates the validity of the classical solution:

$$
n^{\max } \approx m \Omega \lambda_{\xi}^{2} / 2 \hbar \gg 1
$$

It is clear that this condition is compatible with the requirement of greater channel dimensions that arises if we require maximum focusing, for which the only restriction is $a k, b k<2$. Let us estimate the minimum channel dimensions for which classical channeling can take place. Estimates carried out for neutrons show that for domain walls and magnetic helicons, and for spin-wave resonance, the minimum channel size for the highest possible $k$ is 50 and $5 \AA$, respectively. The latter figure suggests that it may be possible to achieve 'true' channeling in an internal elementary magnetic crystal channel with dimensions up to $10 \AA$. Total 'trapping' of particles in narrower channels is not possible, and all the properties of magnetic channeling then correspond to the quasichanneling that was considered above in connection with photon channeling in crystals.

## 4. PROPERTIES AND THEORY OF MAGNETIC CHANNELING OF NEUTRONS IN NONMAGNETIC CRYSTALS

### 4.1. Coherent Schwinger mechanism for the channeling of neutral particles in crystals

The magnetic channeling in crystals, considered above for neutrons and other particles with a magnetic moment, requires, at the very least, an ordered magnetic structure. We now present an analysis of a fundamentally different magnetic channeling mechanism that is equally effective in magnetic and nonmagnetic crystals. Although, to be specific, we shall confine our attention to neutrons, the discussion will be valid for any particles that possess a magnetic moment. ${ }^{35}$

Leaving on one side purely nuclear processes, the energy of interaction between a neutron and a nonmagnetic lattice is determined by the electrodynamics of moving media. A magnetic field $\mathbf{H}=\mathbf{E} \times \mathbf{v}$ is produced in the coordinate frame in which the particle moving inside the crystal, in the atomic electric field, is at rest.

Consider a planar channel and the electric field $\mathbf{E}(\mathbf{r})$ pointing away from the crystal plane and in the direction of the middle of the channel. The average magnitude of this field, evaluated over the area $S_{0}$ of a unit cell, falls to zero in
the middle of the channel. Consequently, the mean value of the field $\mathbf{H}(\mathbf{r})$ is then a maximum (in amplitude) near the plane, but falls to zero and changes sign at the center of the channel (in the middle of the interplanar channel), and its numerical magnitude then rises to the same maximum value when we approach the next channel plane. As we cross this plane, the change in the sign of the electric field vector produces a discontinuous change in the sign of the magnetic field $\mathbf{H}$, and the subsequent spatial variation of this field repeats its behavior in the preceding channel.

Thermal motion of the lattice naturally broadens these discontinuous changes in the field across the channel plane. The final result is that a magnetic moment moving through the field experiences a sequence of magnetic wells and barriers.

The maximum field $H^{\text {max }}$ on a plane can be found from the formula

$$
\oint H d l=4 \pi i / c
$$

since, in the frame in which the neutron is at rest, the motion of the atomic nuclei and electrons that lie on a crystal plane is equivalent to a current $i$. The maximum current corresponds to the case where the contour of integration runs close to the static ('frozen') crystal plane. As the size of the contour is increased (so that it departs from the immediate neighborhood of the plane and encloses parts of adjacent channels), the contour encompasses not only the nuclei but also some of the atomic electrons. Since their signs are different, this is equivalent to a reduction in the current threading the contour. Since for a plane of width $L$, on which nuclear charges $Z e$ lie at the sites of unit cells of area $S_{0}$, the maximum effective current is $i^{\text {max }}=Z e v L / S_{0}$, we have $H^{\text {max }}$ $=2 \pi Z \mathrm{Zev} / S_{0} c$. This field reaches $H^{\text {max }} \approx 10^{5}$ Oe for resonance neutrons and $H^{\text {max }} \approx 10^{8} \mathrm{Oe}$ for fast neutrons, which should lead to a change in the trajectory in the crystal.

In traditional terminology, the neutron-lattice interaction can be characterized as a coherent Schwinger interaction and constitutes a generalization of the well-known neu-tron-atom Schwinger scattering.

### 4.2. Structure of the potential well for the magnetic interaction of a neutron in a crystal channel

It follows from the preceding discussion that the efficacy of the neutron-lattice interaction depends on the form of the lattice field $\mathbf{E}$ that in turn determines the magnetic field H.

For the sake of simplicity, we shall confine our attention to an atomic field in the form of the screened Coulomb potential

$$
V_{e}=Z e \exp (-r / R) / r, \quad R=\hbar^{2} / m_{\mathrm{e}} e^{2} Z^{1 / 3}
$$

The use of more accurate potentials would complicate the solution very considerably without significantly changing the overall picture of channeling. Since $E_{r}=-\nabla V_{e}$, we can use the above form of $V_{e}$ to find the field $\mathbf{H}$ due to an atom and acting on the neutron:

$$
\begin{equation*}
H_{r}=Z e\left[\left(1 / r^{2}\right)+(1 / r R)\right]\left[\mathrm{e}_{\mathrm{r}} \mathrm{v}\right] \exp (-r / R) / c \tag{4.1}
\end{equation*}
$$

The expression for the magnetic potential energy of a neutron $V_{\mu}=\mu \hat{o} H_{r}$, where $\hat{\sigma}$ represents the Pauli spin matrices, will now be examined in the laboratory frame with the $O y$
axis lying along the channel axis and $O x$ perpendicular to the crystal planes (the $O z$ axis lies in the crystal plane).

The common assumption in the case of channeling is that the potential $V_{\mu}$ that contains the alternating-sign components $\mu \sigma_{x} H_{x}, \mu \sigma_{y} H_{y}$ due to the alternating-sign electricfield components $E_{z}, E_{y}$, and the constant-sign variable component $\mu \sigma_{z} H_{z}$ associated with $E_{x}$, can be replaced with the average potential $\left\langle V_{\mu}\right\rangle$. In the classical treatment, this averaging is due to the insensitivity of the trajectory of massive particles to rapid and numerically small variations in the periodic field $H_{r}$. Since the neutron-atom interaction is weak, the trajectory is wholly determined by the average integral parameters of the field. A quantum-mechanical treatment will be presented below.

Since the field components $H_{x}, H_{y}$ have alternating signs, and vary symmetrically within each longitudinal crystal lattice period, we have $\left\langle H_{x}\right\rangle=\left\langle H_{y}\right\rangle=0$. Using the standard procedure of averaging over a crystal plane by integrating with the weight $1 / S_{0}$ over the transverse coordinate $\rho=\left(r^{2}-x^{2}\right)^{1 / 2}$, we obtain the following magnetic potential of a static ('frozen') crystal plane:

$$
\left\langle V_{\mu}\right\rangle=\mu \sigma_{z}\left(\mathrm{\xi}_{x}\right) \int_{0}^{\infty} H_{z}\left(\left(x^{2}+\rho^{2}\right)^{1 / 2}\right) \cdot 2 \pi \rho \mathrm{~d} \rho / S_{0}\left(x^{2}+\rho^{2}\right)^{1 / 2}
$$

where $\vec{\xi}$ is the unit vector along the normal to the $y z$ plane. Integration with $H_{r}$ given by (4.1) yields

$$
\begin{equation*}
\left(V_{\mu}\right\rangle=\mu \sigma_{z}\left(\overrightarrow{\xi e_{x}}\right) \cdot 2 \pi \cdot e v Z \exp (-x / R) / S_{0} c \tag{4.2}
\end{equation*}
$$

which ignores the thermal motion of the lattice. We now evaluate an average of this expression, using the normal distribution with variance $u^{2}$ for the fluctuations in the positions of the atoms, and allow for the structure of the potential well produced by two adjacent planes forming the channel, which finally yields

$$
\begin{equation*}
\left\langle\left\langle V_{\mu}\right\rangle\right\rangle=\sigma_{z}\left[V_{0}(x)-V_{0}(x \pm d)\right] v / c \tag{4.3}
\end{equation*}
$$

where


FIG. 6. Potential energy of a neutron in a planar channel of width $q=5 R$, reduced to a static ('frozen') lattice: 1 -static lattice with $u=0$, 2( $u=R / 6$, 3- $u=R / 4,4-u=R / 3,5-u=R / \sqrt{2}, 6-u=R, 7-$ $u=R / \sqrt{2}$.)

$$
\begin{aligned}
V_{0}(x)= & \left.\left(\pi \mu Z e / S_{0}\right) \exp \left(u^{2} / 2 R^{2}\right)\right\rfloor \exp (-x / R) \\
& \times[1-\Phi((u / R \sqrt{2})-(x / u \sqrt{2}))] \\
& -\exp (x / R)[1-\Phi((u / R \sqrt{2})+(x / u \sqrt{2}))]\}
\end{aligned}
$$

in which $\Phi(\alpha)$ is the probability integral and $d$ is the crystal plane separation. Henceforth we shall denote the double average $\left\langle\left\langle V_{\mu}\right\rangle\right\rangle$ by $V(x)$.

Figure 6 shows a plot of the potential given by (4.3). The well structure has an interesting dependence on the crystal parameters. When $u \ll R, d$, the position of the minimum of the potential well is given by

$$
\begin{equation*}
x_{0} \approx \sqrt{2} u\{\ln [\sqrt{2} R / \sqrt{\pi} u[1+\exp (-d / R)]\}\}^{1 / 2} \tag{4.4}
\end{equation*}
$$

In the other limiting case, $R<u<d$, we have

$$
\begin{equation*}
x_{0} \approx u\left[1+\left(R^{2} / 2 d^{2}\right)+(u / 2 R) \exp \left(-d^{2} / 2 u^{2}\right)\right] \tag{4.5}
\end{equation*}
$$

It follows from (4.4) and (4.5), and also from the numerical solution shown in Fig. 7, that an increase in the root mean square amplitude $u$ of thermal oscillations is accompanied by a shift of the center of the well away from the plane and toward the middle of the channel, while the channel depth decreases. The same shift with a simultaneous increase in the well depth and barrier height occurs when the channel width $d$ is increased.

### 4.3. Wave functions and neutron channeling

The motion of a neutron in the above potential magnetic field is decribed by the following two-compound Pauli equation that also takes into account the neutron-nucleus interaction potential:

$$
\left(-\frac{\hbar^{2}}{2 m} \Delta+V_{n}+V(x)\right)\left|\begin{array}{l}
\Psi_{1}  \tag{4.6}\\
\Psi_{2}
\end{array}\right|=E\left|\begin{array}{l}
\Psi_{1} \\
\Psi_{2}
\end{array}\right|, V(x) \equiv\langle\langle\overrightarrow{\hat{\sigma}} \mathrm{H}\rangle\rangle
$$

The wave functions $\Psi_{1,2}$ represent the two possible orientations of the neutron magnetic moment along the quantization axis (which is parallel to $\mathbf{H}$ ). The nuclear nucleon-scat-


FIG. 7. Relative change $\eta=V / V_{\mu}^{\max }$ in the shape of the normalized potential well due to neutrons as a function of the ratio ( $u / R: I-u=R / 6$, $2-u=R / 3,3-u=R, 4-u>=R, 5-$ spatial distribution of localization density for neutrons near the middle of the crystal plane $x=0$.
tering potential $V_{n}$ is characterized (especially at moderate energies) by a slightly anisotropic function, namely, the scattering amplitude $f_{n}(\theta)$. In particular, for $s$-scattering, $f_{n}(\theta)=R_{0}$ where $R_{0}$ is the nuclear radius. The second part of the potential, which was discussed in detail above, is due to the Schwinger interaction ${ }^{3,36,37}$ and finally reduces to coherent scattering by a set of atoms. The Schwinger scattering amplitude is highly anisotropic and, for small angles, is determined by $f_{\mu}(\theta)=i Z \mu e / \hbar c \theta$. We know (see, for example, Ref. 38) that the channeling of particles, and the use of the average crystal-plane potential to describe it, is possible only for small-angle coherent forward scattering by a segment of a plane, which is much greater (in the longitudinal direction) than interatomic scattering, followed by the interference of scattering amplitudes. The magnetic part of the potential $V(x)$ in (4.3) is the dominant part in this process. This is so because for the angles of incidence typical for this process, $\theta<10^{-3}$, we have $\left|f_{\mu} / f_{n}\right| \gg 1$. The zero-order approximation to the solution of (4.6) can be found by using the single potential $V(x)$ and without taking the nuclear interaction into account. The spacial distribution of particles over the cross section of the channel $|\psi(x)|^{2}$, found in this way can be used to calculate the change in the yield of a nuclear reaction involving the participation of channeled particles. This remark also applies to the other neutron channeling (quasichanneling) mechanisms discussed above.

When the diagonal form of the matrix $\hat{\sigma}_{z}$ is taken into account, equation (4.6) splits into two independent Schrödinger equations for neutrons of different polarization, i.e.,

$$
\begin{equation*}
\Delta \Psi_{1,2}+\frac{2 m}{\hbar^{2}}(E \pm V(x)) \Psi_{1,2}=0 \tag{4.7}
\end{equation*}
$$

When the neutron polarization is changed, the form of the magnetic potential transforms as a mirror reflection in the $V=0$ line in Fig. 6.

In view of the complicated form of the potential $V(x)$, the solution of the problem can be sought by quasiclassical or numerical methods. The results obtained by both methods are reproduced below. We begin with the quasiclassical method.

The standard multidimensional quasiclassical method yields the following result:

$$
\begin{equation*}
\Psi_{1,2}=C_{1,2}\left(p_{0 x} / p_{x}\right)^{1 / 2} \exp \left\{i\left[p_{y} y^{\prime} \pm \int_{x_{0}}^{x} p_{x} \mathrm{~d} x\right] / \hbar\right\}, \tag{4.8}
\end{equation*}
$$

where $p_{x}=\left[p_{0 x}^{2} \pm 2 m V(x)\right]^{1 / 2}$ is the transverse momentum of a particle on a quasiclassical trajectory between the turning points $x_{1}, x_{2}$, determined from the condition

$$
p_{0 x}^{2}=2 m\left|V\left(x_{1,2}\right)\right|
$$

The criterion for the validity of the quasiclassical approximation in the multidimensional case is ${ }^{39}$

$$
\begin{gathered}
I(\Delta p / p)-\left[3(\nabla p)^{2} / 2 p^{2}\right]-[(n \nabla p) \nabla n / p] \\
+n \Delta n+\left[(\nabla n)^{2} / 2\right] 4 \ll 2 p^{2} / \hbar^{2} .
\end{gathered}
$$

This relation establishes the connection between the possible values of the momentum $p$ and the spatial variation of the unit vector $\mathbf{n}=\mathbf{p} / p$. For two-dimensional motion we have $n=\left(e_{x} p_{x}+e_{y} p_{y}\right)$, and if we use the explicit form of $p_{x}$, the above validity condition becomes

$$
\begin{equation*}
\left|2 \frac{\partial^{2} V}{\partial x^{2}}+m\left(\frac{\partial V}{\partial x}\right)^{2} p_{x}^{-2}\right| \ll \frac{4 p^{4}}{m \hbar^{2}} \tag{4.9}
\end{equation*}
$$

It is clear that (4.8) is not valid in the immediate neighborhood $\delta x$ of the turning points $x_{1,2}$. It is shown in Ref. 35 that

$$
\delta x \approx \hbar^{2} v_{0}^{\max } / 8 m^{3} v^{3} c r_{0}
$$

where $r_{0} \approx u$ for the well wall closer to the plane, and $r_{0} \approx R$ for the wall closer to the channel center. Direct estimates show that, for thermal and faster neutrons, $\delta x \approx 10^{-14} \mathrm{~cm}$ for parameter values in the ranges $Z=5-50, S_{0}=10^{-16}$ -$10^{-15} \mathrm{~cm}^{2}, u=10^{-10}-2 \times 10^{-9} 10^{-9} \mathrm{~cm}, R=(1-3) \times 10^{-9}$ cm . Near the bottom and the top of the well, where $\partial V /$ $\partial x \approx 0$, the quantity $\delta x$ is smaller still.

There is one further limitation that follows from the second term in (4.9), which is greater than the first everywhere except for values of $\delta x$ near $x_{1,2}$. Direct analysis of (4.3) shows that

$$
\partial^{2} V / \partial x^{2} \approx V_{0}^{\max } / u R
$$

near the bottom and the top of the barrier, and

$$
\partial^{2} V / \partial x^{2} \approx V_{0}^{\max } / r_{0}^{2}
$$

elsewhere. If we use these estimates, we find that (4.8) is valid throughout the domain of $x$ if the longitudinal velocity of the neutron is

$$
v>\left(\hbar^{2} V_{0}^{\max } / 2 m^{3} u R c\right)^{1 / 3}
$$

for energies of transverse motion near the bottom of the well, and

$$
v>\left(\hbar^{2} V_{0}^{\max } / 2 m^{3} r_{0}^{2} c\right)^{1 / 3}
$$

for all other energies. For the same parameter ranges as above, this leads to the condition $v>10^{-4} \mathrm{~cm} / \mathrm{s}$, which is satisfied for thermal and faster neutrons.

Since the potential is periodic, the solution given by (4.8) must satisfy the Bloch theorem $\Psi(x)$ $=f \Psi(x \pm d),|f|=1$. Moreover, since $\Phi_{1,2}$ must be continuous at all points (for example, at $x_{2}$ ), we find that the dispersion relation that determines the admissible values of the neutron energy in the channel is given by

$$
\begin{equation*}
\left|\cos \varphi_{1} \cdot \cosh \varphi_{2}-G(x) \sin \varphi_{1} \cdot \sinh \varphi_{2}\right| \leq 1, x \rightarrow x_{2} \tag{4.10}
\end{equation*}
$$

where

$$
\begin{aligned}
G(x)= & \left\{\left[p_{x}\left(x<x_{2}\right) / p_{x}\left(x>x_{2}\right)\right]+\left[p_{x}\left(x>x_{2}\right) /\right.\right. \\
& \left.\left.p_{x}\left(x<x_{2}\right)\right]\right\} / 2
\end{aligned}
$$

and

$$
\varphi_{1}=\int_{x_{1}}^{x_{2}} p_{x} \mathrm{~d} x / \hbar, \varphi_{2}=\int_{x_{2}}^{x_{1}+d}\left|p_{x}\right| \mathrm{d} x / \hbar
$$

are real functions. Passing to the limit as $x \rightarrow x_{2}$, we find that

$$
G\left(x_{2}\right)=0,\left|\cos \varphi_{1} \cdot \cosh \varphi_{2}\right| \leqslant 1 .
$$

Since the maximum value is

$$
\varphi_{1,2}^{\max } \leq\left|p_{x}^{\max }\right| \equiv\left(2 m V_{0}^{\max }\right)^{1 / 2} \mathrm{~d} / \hbar \ll 1
$$

for all possible parameters (which corresponds to highly transparent barriers between channels and the absence of
levels of bound motion in the channel), we find that (4.10) is satisfied if

$$
\int_{x_{1}}^{x_{2}} p_{x} \mathrm{~d} x \geq \int_{x_{2}}^{x_{1}+d}\left|p_{x}\right| \mathrm{d} x
$$

Since the potential curve is symmetric, we find that the last condition is satisfied only when $\left|x_{2}-x_{1}\right| \geqslant d / 2$. It is clear that this is possible only for the energy of transverse motion that lies above the middle of the potential, i.e., for $V>0$ in Fig. 6. Motion with $V<0$ corresponds to zero transmission and is forbidden. In the case of motion above the barrier, for which $\varphi_{2}=i\left|\varphi_{2}\right|, G\left(x_{2}\right)=1$, condition (4.10) assumes the form of the identity relation $\left|\cos \left(\varphi_{1}+\left|\varphi_{2}\right|\right)\right| \leqslant 1$ which corresponds to the absence of forbidden bands and states in the quasiclassical approximation.

For $\varphi{ }_{1,2}^{\max } \ll 1$, which corresponds to narrow channels and moderate-energy neutrons, the state of a neutron in the lattice is delocalized and corresponds to the geometric optics of a plane-layered medium with a variable refractive index. For light paramagnetic atoms, the well depth rises rapidly and may reach a few electron volts for fast particles. In such cases, $\varphi_{1,2}^{\max } \geqslant 1$ and the number of discrete levels in pure channeled (localized) motion is determined by the number of nodes of the dispersion relation given by (4.10). The same result is obtained for very high energy neutrons in wide channels. Estimates show that, for paramagnetic atoms, the first discrete levels appear in the channel for longitudinal velocity $v \approx 10^{6} \mathrm{~cm} / \mathrm{s}$ and the number of such levels reaches a few dozen for $v \approx 10^{9} \mathrm{~cm} / \mathrm{s}$. The precise threshold energies and the number of levels are simpler to determine from the numerical calculations presented below.

The characteristic angle for near near-barrier motion (i.e., for transverse energy equal to the barrier height) in the case of neutrons and the limiting channeling angle (Lindhard angle) for paramagnetic atoms is given by

$$
\begin{equation*}
\theta_{0}=\left[4 V_{0}^{\max } /\left(p^{2} / 2 m\right)\right]^{1 / 2}=4\left(\pi \mu Z e \eta_{0} / S_{0} m c v\right)^{1 / 2} \tag{4.11}
\end{equation*}
$$

where $\eta_{0} \approx 0.3-0.7$ is the relative depth of the potential well relative to the 'frozen' lattice for all possible values of $R$ and $u$.

The reduction in the angle given by (4.11) with increasing longitudinal velocity is due to the more rapid rise in longitudinal energy as compared with the linear increase in the well depth and barrier height. It is useful to note that the reduction in the angle $\theta_{0}$, described by $\theta_{0} \sim 1 / v^{1 / 2}$, with in. creasing velocity in the case of magnetic channeling, which is slower than the Coulomb channeling of heavy particles, for which $\theta_{0} \sim 1 / v$, is found to bring the values of this parameter for the two types of channeling closer together; it also means that magnetic effects have to be taken into account in the channeling of paramagnetic ions. For the above parameter values we have $\theta_{0} \approx 3^{\prime}-5^{\prime}$ for $v \approx 4 \times 10^{6} \mathrm{~cm} / \mathrm{s}, E_{0} \approx 10$ eV and $\theta_{0} \approx 15^{\prime \prime}-20^{\prime \prime}$ for $v \approx 10^{9} \mathrm{~cm} / \mathrm{s}, E_{0} \approx 1 \mathrm{MeV}$ in the case of neutrons, whereas $\theta_{0} \approx 0.5-1^{\circ}$ and $\theta_{0} \approx 3^{\prime}-5^{\prime}$ for the same velocities in the case of light paramagnetic atoms. The spectral problem is solved by numerical methods in Ref. 40 for the channeling of neutrons and other particles having a magnetic moment in a nonmagnetic lattice. The energy spectrum was determined by a pseudospectral method for the following set of equations that are equivalent to the original Schrödinger equation:
$\left[(k d+2 n \pi)^{2}-\lambda\right] \Psi_{n}+\sum_{s=-N}^{N} \tilde{V}_{n-s} \Psi_{s}=0, s, n=0, \pm 1, \ldots, \pm N ;$
where $\Psi_{n}$ and $\widetilde{V}_{n-s}$ are the coefficients in the Fourier expansion of the wave function and the dimensionless potential $\widetilde{V}(x)=V_{0}(x) 2 m / \hbar^{2} d^{2}$, in which $k$ is the quasimomentum of the particles and $\lambda=2 m E_{1} / \hbar^{2} d^{2}$ gives the energy levels in the channel in dimensionless units. The solution was sought using $N=64$ eigenwaves and a variable-step grid. Figure 8 shows the structure of the energy spectrum in units of the potential $\widetilde{V}^{\text {max }}(x) \equiv \widetilde{V}^{\text {max }}$.

Analysis of the band structure shows that the dimensionless amplitude $\widetilde{V}^{\text {max }}$ is the dominant parameter that, in the final analysis, determines all the features of directed motion. The transverse motion is completely delocalized for small $\widetilde{V}^{\text {max }}<1$ (which corresponds to low longitudinal velocities or small particle magnetic moments). The first bound state (i.e., purely channeled motion) occurs for $\widetilde{V}^{\text {max }} \approx 10$. As the potential increases to $\widetilde{V}^{\text {max }} \approx 100$, the number of quasidiscrete levels (or, more precisely, bands of allowed motion), increases to 3 , and the correspondence principle ensures that the problem has an equivalent classical solution for $\widetilde{V}^{\max } \gtrsim 10^{3}$.

### 4.4. Relative efficacy of the neutron-nucleus (optical) and coherent Schwinger (electromagnetic) interaction of a neutron and a crystal plane

We noted above that the neutron-nucleus interaction is described by the Fermi quasipotential $V(r)=2 \pi \hbar^{2} b \delta(r$ $\left.-r_{n}\right) / m$, where $b$ is the neutron-nucleus scattering length and $m$ is the neutron mass. When this potential is averaged over the area $S_{0}$ of the unit cell in the channeling plane and over the thermal oscillations of the lattice, we obtain the average neutron-nucleus potential of a crystal plane in the form

$$
\begin{equation*}
V_{s}=\left(\sqrt{2 \pi} \hbar^{2} b / m u S_{0}\right) \exp \left(-x^{2} / 2 u^{2}\right) \tag{4.13}
\end{equation*}
$$

In contrast to the channeling of charged particles, for which the type of incident particle uniquely determines the type of the potential (well or barrier) and the character of the interaction (attraction, as for electrons, or repulsion, as for protons and positrons), both attraction (for $b<0$ ) and repulsion ( for $b>0$ ) is possible in this case for the same particles (neutrons). The potential $V_{s}$ is usually small.

For example, for typical parameter values $u \approx 0.1-0.2$ $\AA, S_{0} \approx 10^{-15} \mathrm{~cm}^{2},|b| \approx 10^{-12} \mathrm{~cm}$, we have $V_{\mathrm{s}}^{\max } \approx 10^{-6} \mathrm{eV}$. It is clear that this potential cannot ensure the localized mo-


FIG. 8. Band structure of the spectrum of channeled neutral particles.
tion of the neutron, which will be delocalized but ordered (quasichanneling).

The moving particle experiences the coherent Schwinger interaction in addition to the neutron-nucleus interaction. It was shown above [see (4.3)] that $V(x) \sim v$. At low neutron energies $E<E_{0}$, we have $V_{\mathrm{s}}^{\text {max }}>V^{\text {max }}$.

The characteristic energy $E_{0}$ is determined by the condition $V_{\mathrm{s}}^{\text {max }}>V^{\text {max }}$ and is given by

$$
\begin{equation*}
E_{0}=\left[\left(\hbar^{2} b c / u \mu Z e\right)^{2} / \pi m\right] \exp \left(u^{2} / R^{2}\right) . \tag{4.14}
\end{equation*}
$$

At low neutron energies (for example, for ultracold neutrons ), $V_{\mathrm{s}}^{\text {max }}>V^{\text {max }}$. In the opposite case of fast neutrons with $E>E_{0}$, we have $V_{\mathrm{s}}^{\max }<V^{\text {max }}$. In particular, when $E=10 \mathrm{MeV}$, we have $V^{\text {max }} \approx 10^{3} V_{\mathrm{s}}^{\text {max }}$.

For $Z \approx 30, u \approx 0.1-0.2 \AA R \approx 0.2-0.3 \AA$, we find that $E_{0} \approx 100 \mathrm{eV}$.

The resultant potential for the neutron-crystal interaction is determined by both mechanisms. The final total potential is then asymmetric (especially for $E \approx E_{0}$ ).

There are several points that can be made in connection with the energy structure of the scattering length $b$. It is well known (see, for example, Refs. 1 and 2) that

$$
b=b_{0}+\left\{\Gamma_{n}(E) /\left[2 k\left(E-E_{n}+i \Gamma\right)\right]\right\}
$$

where $b_{0}=R_{0}, R_{0}$ is the radius of the scattering nucleus. $\Gamma_{n}(E)$ and $\Gamma$ are the neutron and total widths of the resonance with energy $E_{n}$ (Refs. 1 and 2 ), and $k=(2 m E)^{1 / 2} / \hbar$ is the wave number of the moving neutron. Since $\Gamma_{n}(E) \sim E^{1 / 2}$, the second term in the expression for $b$ is described by the Breit-Wigner resonance

$$
\left(E-E_{\eta}+i \Gamma\right)^{-1}
$$

where $E$ is the neutron energy.
For most nuclei, neutron resonances lie in the range $E_{n}$ $=10-100 \mathrm{keV}$. For thermal and slow neutrons with $E \ll E_{n}$, the second (dispersive) term in $b$ is insignificant, and $b \approx b_{0}$. It is precisely this result that was used in the above calculations. The influence of the dispersive term becomes apparent only in the neighborhood of the resonance with $E \approx E_{n} \gg E_{0}$, but the attendant very strong absorption makes channeling very difficult to observe. Moreover, when $E \gg E_{0}$, the main effect is the coherent Schwinger interaction, which makes the contribution of $b$ relatively unimportant.

## 5. MECHANISM AND THEORY OF INDUCED-DISPERSIVE CHANNELING OF NEUTRAL PARTICLES WITH INTERNAL ELECTROMAGNETIC RESONANCES

### 5.1. Particles with a resonant structure in a longitudinally periodic potential

In addition to the above neutral-particle channeling $m$ schanisms, which rely on nuclear forces or the magnetic moment of the particles, there is one further mechanism of ordered focusing and confinement to a channel that is due to the internal energy spectrum of the particles. The ordering force is due to the steady interaction between the alternating magnetic moment induced by the periodic interaction and the periodic (alternating in the rest frame of the moving particle) field of the lattice potential that induces this magnetic moment.

Specifically, the mechanism is as follows. The excitation of the atom is possible if the transition frequency $\omega_{\mathrm{k} 1}$
between levels $E_{k}$ and $E_{l}$ of the moving atom or other particle is close to one of the frequencies of the external disturbance (these are the frequencies of 'collision' with periodicially distributed lattice atoms) $\omega_{n}=2 n \pi v / d$, where $n=1,2,3, \ldots, v$ is the velocity of an atom and $d$ is the longitudinal lattice period. This effect was first discussed in Refs. 41-43 and, in greater detail, in Ref. 44.

However, in addition to the trivial excitation of internal degrees of freedom of the atom there is a further previously unnoticed effect. The point is that the periodic field of the lattice induces in the atom an electric moment $\mathbf{p}=\alpha \mathbf{E}$ (in a magnetic lattice, there is also a magnetic moment), which interacts with the lattice field $\mathbf{E}$ that induces the moment. Since the moment varies in step with the field, the interaction continues throughout the entire motion. The interaction depends in a complicated way on the ratio $\omega_{k l} / \omega_{n}$, the transverse structure of the field $E\left(r_{\perp}\right)$ of the lattice within the cross section of the channel, and the relaxation parameters of the moving atom. The possibility of this induceddispersive interaction (IDI) was first noted and estimated in Ref. 45, and in greater detail in Ref. 46.

We now turn to the quantitative consideration of the IDI potential, assuming that we are dealing with a two-level atom. Since the particle size can be of the same order as the lattice constants and the size of the lattice atoms, we have, in general, a nondipole interaction.

We shall perform the analysis in the frame coupled to the longitudinal motion of the particle. Consider first the case of planar channeling. Suppose that the $y, z$ plane is a crystal plane. The motion of a particle in the field due to the lattice is described by the Hamiltonian $\widehat{H}=\widehat{H}_{0}+\widehat{V}$ in which $\widehat{H}_{0}$ characterizes the internal state of the particle and $\widehat{V}$ describes the interaction with the lattice. The latter operator can be expanded in terms of the two-dimensional recipro-cal-lattice vectors $g=\left\{2 \pi n_{y} / a_{y}, 2 \pi n_{z} / a_{z}\right\}$ averaged over the thermal oscillations of lattice atoms at the points $r_{n}$ :

$$
\begin{align*}
& \hat{V}\left(R ; r_{1}, \ldots, r_{Z} ; t\right) \equiv \sum_{\mathrm{g}} V_{\mathrm{g}} \exp \left(i \omega_{\mathrm{g}} t\right) \\
& =\left(2 \pi u^{2}\right)^{-3 / 2} \int_{-\infty}^{\infty} \exp \left(-r^{2} / 2 u^{2}\right) \sum_{n}\left[\mathcal{E e \Phi}\left(!\mathrm{r}_{l}-\mathbf{r}-r_{n} \mid\right)\right. \\
& -\sum_{l=1}^{Z} e \Phi\left(\left|\mathrm{r}_{l}-\mathrm{r}-\mathrm{r}_{n}\right|\right) \mid \mathrm{d} r=\left(\pi Z_{0} e^{2} / S\right) \\
& \times \sum_{g} \gamma_{g}^{-1} \exp \left[i g v t+\left(u^{2} / 2 a^{2}\right)\right] \\
& \times \sum_{j=0}^{1}\left\{Z \exp \left[x \gamma_{g}(-1)^{j}\right] \operatorname{erfc}\left(\left(u_{g} / \sqrt{2}\right)+\left[u(-1)^{j} / u \sqrt{2}\right]\right)\right. \\
& -\sum_{l=1}^{z} \exp \left[i \mathrm{~g}_{l}{\overrightarrow{p_{l}}}^{z}+x_{l} \gamma_{\mathrm{g}}(-1)^{j}\right] \operatorname{erfc}\left(\left(u \gamma_{\mathrm{g}} / \sqrt{2}\right)\right. \\
& \left.+\left[x_{l}(-1)^{j / u \vee} 2\right]\right) ; \tag{5.1}
\end{align*}
$$

where $a_{y}, a_{z}$ are the atomic lattice constants in the plane of channeling, $n_{y}, n_{z}$ are nonnegative integers, $\mathbf{R}=\{x, 0,0\}$
and $r_{l}=\left\{x_{l}, \rho_{l}\right\}$ are, respectively, the coordinates of the nucleus with charge $Z e$ and the $l$ th electron in the moving atom, $S$ is the area per atom in the plane of channeling, $u$ is the amplitude of thermal oscillations, and $\gamma_{\mathrm{g}}$ $=\left[g^{2}+\left(1 / a^{2}\right)\right]^{1 / 2}$.

The expression given by (5.1) was evaluated for the screened Coulomb potential $\varphi(r)=\left(Z_{0} e / r\right) \exp (-r / a)$ in which $a=0.885 a_{0} \prime\left(Z^{2 / 3}+Z_{0}^{2 / 3}\right)^{1 / 3}$ is the screening radius and $a_{0}=\hbar^{2} / m e^{2}$. The contribution of the magnetic Schwinger-type interaction can be neglected in $H$ since the vector potential is smaller than the scalar potential by the factor $c / v \gg 1$.

It is clear that the main contribution to the IDI is provided by the resonant components of the expansion given by (5.1) for which the frequency $\omega_{\mathrm{g}} \equiv \omega$ is close to the transition frequency between levels 1 and $2, \omega_{12} \equiv \omega_{0}$, and also the zeroth (steady) component that corresponds to $g=0$.

Since the result of the resonant coherent interaction is that the atom is found in a mixed state, the atom-lattice interaction potential must be found from the equation for the density matrix
$i \hbar \frac{\partial \hat{\rho}}{\partial t}=\left\{\hat{H}_{0}+\hat{V}_{0}+\hat{V}_{\mathrm{g}} \exp \left(i \omega_{\mathrm{g}} t\right), \hat{\rho}\right\}-i T \hat{\rho}, V_{0} \equiv V_{\mathbf{g}=0}$.
Relaxation processes in this system are described by the phenomenologically introduced matrix

$$
T \hat{\rho}=\left(\begin{array}{ll}
\left(\rho_{11}-\rho_{11}^{(0)}\right) / T_{1} & \rho_{12} / T_{2}  \tag{5.3}\\
\rho_{21} / T_{2} & \left(\rho_{22}-\rho_{22}^{(0)}\right) / T_{1}
\end{array}\right)
$$

where $T_{1}$ and $T_{2}$ are the longitudinal and transverse relaxation time constants, the relaxation process being described for the two-level system by the rate $\omega_{21}$ of the transitions between states 1 and 2 under the noncoherent influence of the lattice, the rate $\Lambda$ of $2 \rightarrow 1$ spontaneous transitions, and the rates $\kappa_{1}$ and $\kappa_{2}$ of phase relaxation of both levels:

$$
1 / T_{1}=2 w_{21}+A \equiv \gamma, \quad 1 / T_{2}=\left(1 / T_{1}\right)+x_{1}+x_{2} \equiv \Gamma
$$

In the absence of the external disturbance ( $\gamma_{g}=0$ ), the equilibrium level populations are given by

$$
\rho_{11}^{(0)}=\left(w_{21}+A\right) /\left(2 w_{21}+A\right), \rho_{22}^{(0)}=w_{21} /\left(2 w_{21}+A\right) .
$$

The evaluation of $w_{1}, \varkappa_{1}, \chi_{2}, A$ will be considered below. The component form of the operator equation (5.2) is

$$
\begin{align*}
\frac{\partial \rho_{11}}{\partial t}= & {\left[\left(V_{12} \rho_{21}-\rho_{12} V_{21}\right) / i \hbar\right]-\gamma\left(\rho_{11}-\rho_{11}^{(0)}\right) } \\
\frac{\partial \rho_{21}}{\partial t}= & {\left[V_{21}\left(\rho_{11}-\rho_{22}\right) / i \hbar\right] } \\
& +\left[\rho_{21}\left(V_{22}-V_{11}\right) / i \hbar\right]-\dot{\omega}_{21} \omega_{0}-\Gamma \rho_{21} \tag{5.4}
\end{align*}
$$

where

$$
\begin{aligned}
V_{21} & =V_{12}^{*}=\left(V_{g}\right)_{21} \exp (-i \omega t) \\
& \equiv \hbar \Omega(x) \exp (-i \omega t), V_{11,22}=\left(V_{0}\right)_{11,22} .
\end{aligned}
$$

Substituting $\quad \Delta \omega=\omega-\omega_{0}, \quad \alpha=\left(V_{22}-V_{11}\right) / \hbar \quad$ and $\rho(t)=\rho_{11}-\rho_{22}, \rho_{0}(t)=\rho_{21} \exp (i \omega t)$, we find from (5.4) that
$\frac{\partial \rho}{\partial t}=4 \Omega \operatorname{Im} \rho_{0}-\gamma \rho+A, \frac{\partial \rho_{0}}{\partial t}=(i \beta-\Gamma) \rho_{0}-i \Omega \rho, \beta=\Delta \omega-\alpha$.

These equations for the three real functions $\operatorname{Im} \rho_{0}, \operatorname{Re} \rho_{0}, \rho$ determine the evolution of the two-level system in the crystal field that acts both as a thermostat and a perturbation. The final form of the potential representing the interaction between an atom and a crystal plane is

$$
\begin{align*}
V(x, t)= & \operatorname{Sp} \hat{V} \hat{\rho})=\sum_{i, j=1}^{2} V_{i j} \rho_{j i}=\left(V_{11}(x)+V_{22}(x)\right) / 2 \\
& +\alpha \hbar \rho(x, t)+2 \hbar \Omega \operatorname{Re} \rho_{0}(x, t) \tag{5.5}
\end{align*}
$$

From now on, the potential $V(x, t)$ and the associated channeling process must be analyzed for a particular particle (atom). We also note that the choice of the necessary kinematic parameters of the particle (velocity and direction) that ensure a given bounce frequency $v_{g}$ can be substantially simplified by using the concept, developed in the articles of Ref. 47 on dynamic chaos, a crystal plane as a set of crystal axes.

### 5.2Properties of induced-dispersive channeling of mesic atoms

Since the usual type of directed motion of atoms in crystals gives rise to considerable difficulties, and is relatively ineffective(because of very considerable retardation, scattering, excitation, and ionization), we shall analyze the conditions for IDI in the case of mesic atoms that are relatively small and very stable. To be specific, we shall examine the motion of these particles in the field due to the (100) plane of crystalline LiH . Motion with velocity $v$ under the influence of the $g$-component of the perturbation is accompanied by transitions between the muon energy levels, given by
$\Psi_{1} \equiv \Psi_{100}=\left(b_{0}^{3 / 2} \sqrt{\pi}\right)^{-1} \exp \left[\left(-i E_{1} t / \hbar\right)-\left(r_{0} / b_{0}\right)\right]$,
$\Psi_{2} \equiv \Psi_{210}=\left(4 b_{0}^{5 / 2} \sqrt{2 \pi}\right)^{-1} r_{0} \cos \theta_{0}$

$$
\times \exp \left[\left(-i E_{2} t / \hbar\right)-\left(r_{0} / 2 b_{0}\right)\right]
$$

in which $b_{0}=a_{0} m_{e} / m_{\mu}, m_{\mu}$ is the muon mass and $\theta$ is the angle to the $x$-axis.

The first two terms in the potential given by (5.5) depend on $V_{11}, V_{22}$ and describe the purely electrostatic interaction between the spatially distributed electric charge of the neutral mesic atom (localized nucleus and meson spacecharge 'smeared out' with density $e\left|\Psi_{1,2}\left(\mathrm{r}_{0}\right)\right|^{2}$ ) and the average and transversely inhomogeneous electric field due to the crystal plane. It is clear that, as the effective charge of the moving atom tends to zero ( $b_{0} \rightarrow 0$ ), we have $V_{11}, V_{22} \rightarrow 0$, independently of whether the atom has a static or only an induced moment. The case of the mesic atom corresponds to the very small value $b_{0} \approx 2.5 \times 10^{-11} \mathrm{~cm}$. Simple estimates based on the obvious relations $V_{11.22}^{\max }<e b_{0} E \approx V_{0} b_{0} / a_{0}$ lead to the result $V_{11.22}^{\max }<10^{-3} \mathrm{eV}$, which is identical with the very unwieldy result of direct calculations reported in Ref. 46. The amplitude of the third term in the potential given by (5.5), which is due to the pure induced interaction and is determined by $V_{12}$, is calculated in Ref. 46 and is shown to be up to 0.1 eV . These numerical results show that we can neglect the elements $V_{11}, V_{22}$ for the mesic atom.

Before we can proceed, we have to know the relaxation parameters.

The radiationless relaxation rate $w_{21}$ is determined by the interaction between the mesic atom and the nonresonant
(random) components of the static lattice field transformed to the moving frame and acting as a thermostat. To calculate $w_{12}$, we must first determine the transition probability for the muon in the mesic atom under the influence of a single atom on the crystal plane travelling past the mesic atom, and then evaluate the average over all the atoms in the plane. If the position of the perturbing lattice atom at time $t$ is defined by the position vector $R_{0}=\{0, y, v t\}$, the energy of interaction between the mesic atom and the lattice atom is

$$
\begin{aligned}
& W(t)=-\left[Z_{0} e^{2} /\left|\mathrm{R}_{0}-\mathbf{R}-\mathrm{r}_{0}\right|\right] \exp \left(-\left\{\mathrm{R}_{0}-\mathbf{R}-\mathrm{r}_{0} \mid / a\right)\right. \\
& \quad \approx-\left(Z_{0} e^{2} /\left|R_{0}-R\right|\right)\left\{1+\left[\left(y y_{0}+x x_{0}+v t z_{0}\right) /\left|R_{0}-R\right|\right]\right\} \\
& \quad \times \exp \left\{-\left|\mathbf{R}_{0}-\mathbf{R}\right|-\left[\left(x x_{0}+y y_{0}+v t z_{0}\right) /\left\{\mathbf{R}_{0}-\mathbf{R} \mid\right] a^{-1}\right\},\right.
\end{aligned}
$$

where

$$
\left|\mathbf{R}_{0}-\mathbf{R}\right|=\left[x^{2}+y^{2}+(v t)^{2}\right]^{1 / 2}
$$

In accordance with first-order time-dependent perturbation theory, the total transition probability is given by

$$
P_{21}=\left|\int_{-\infty}^{\infty} W_{21}\left(\left(x^{2}+y^{2}+v^{2} t^{2}\right)^{1 / 2}\right) \exp \left(i \omega_{0} t\right) \mathrm{d} t\right|^{2} / \hbar^{2}
$$

If we now evaluate the average over $y$, and take into account the time $a_{z} / v$ spent by a particle in traversing the unit cell, we find that the incoherent transition probability per unit time is

$$
\begin{equation*}
w_{21}=(2 v / S) \int_{0}^{\infty} P_{21}(x, y) \mathrm{d} y \tag{5.6}
\end{equation*}
$$

The transverse relaxation time $T_{2}$ is expressed in terms of the probability of dephasing of the ground and excitated states $\Psi_{1,2}$ by the moving mesic atom. The method used to analyze this dephasing can be related to the variable-frequency oscillator model.

A particle flying past a lattice atom gives rise to a timedependent perturbation. In accordance with the approach first proposed by Lenz and Weisskopf, ${ }^{48}$ this perturbation shifts the energy levels of the incident particle which subsequently (after the flypast) return to their original positions. This shift is accompanied by the following increment on the wave function (in addition to the usual $E_{1,2} t / \hbar$ )

$$
\eta_{1,2}=\int_{-\infty}^{\infty} W_{11,22}\left(\left(\rho^{2}+v^{2} t^{2}\right)^{1 / 2}\right) \mathrm{d} t / \hbar
$$

Successive random interactions of this kind result in random phase fluctuations that lead to a reduction, and then total destruction, of the coherence of the states. In accordance with general theory (see, for example, Ref. 48), the dephasing process can be described quantitatively by introducing an effective collision cross section $\sigma(\eta)$ whose form we shall now determine.

The dephasing process is characterized by the correlation function

$$
B(\tau) \equiv\langle\exp (-i \eta(t)) \exp (i \eta(t+\tau))\rangle=\langle\exp (\dot{i}(\tau))\rangle
$$

The equation for $B(\tau)$ can be found by the Anderson method described in Ref. 48.

This is done by constructing the difference
$B(\tau+\Delta \tau)-B(\tau)=\Delta B(\tau)$ whose explicit form has the structure

$$
\Delta B(\tau)=\langle\exp (i \eta(\tau)) \exp (i \eta(\Delta \tau))\rangle-\langle\exp (i \eta(\tau))\rangle
$$

Since the dephasing process is obviously ergodic and Markovian, we finally have

$$
\Delta B(\tau)=-B[\langle 1-\exp (-\dot{i}(\Delta \tau))\rangle]
$$

The moment $\langle 1-\exp (-i \eta(\Delta \tau))\rangle$ can be calculated by averaging the phase term ( $1-\exp (-i \eta)$ ) over the possible coordinates of all the thermally oscillating atoms in the crystal plane that fly past the mesic atom (in its rest frame) in time $\Delta \tau$ :

$$
\begin{aligned}
& \langle 1-\exp (-i \eta)\rangle=\Delta \tau \cdot \theta(x) \\
& \begin{aligned}
\theta(x)= & v(n\rangle a_{x} \int_{0}^{\pi} \mathrm{d} \varphi \int_{0}^{\infty} \rho d \rho[1-\exp (-i \eta(\rho))] \\
& \quad \times \exp \left\{-\{x-(\rho / \cos \varphi)]^{2} / 2 u^{2}\right\} / \sqrt{2 \pi} u
\end{aligned}
\end{aligned}
$$

where $x$ is the distance from the plane, $\langle n\rangle=1 / S a_{x}$ is the average concentration of the atoms of the crystal, and $a_{x}$ is the separation between planes. The final result is the difference equation $\Delta B=-\Delta \tau B \theta(x)$ which, for $\Delta \tau \rightarrow 0$, becomes a difference equation with the solution $B=\exp \left[-\left(\theta^{\prime}+i \theta^{\prime \prime}\right]\right.$.

The quantity $\theta^{\prime} \equiv\langle n\rangle v \sigma$, expressed in terms of the collision cross section

$$
\begin{aligned}
\sigma=\left(a_{x} / \sqrt{2} \pi u\right) & \int_{0}^{\pi} \mathrm{d} \varphi
\end{aligned} \int_{0}^{\infty} \rho \mathrm{d} \rho(1-\cos \eta(\rho)),
$$

specifies the required coherent-state dephasing time for the mesic atom and determines the parameters of the correlation matrix

$$
\begin{equation*}
x_{1,2}=\theta^{\prime}\left(\eta_{1,2}\right)=v \sigma_{1,2}\left(\eta_{1,2}\right) / S a_{x} . \tag{5.7}
\end{equation*}
$$

The rigorous but much more complicated expressions for $w_{21}(5.6), \kappa_{1,2}$ (5.7), and the spontaneous transition probability $A=(2 e / 3)^{8} / b_{0} \hbar^{4} c^{3} \approx 1.3 \times 10^{11} \mathrm{~s}^{-1}$ can be found in Ref. 46. The resulting parameter set enables us to analyze the structure of the potential well (5.5) and the motion of the mesic atom.

The resonance condition $\omega_{n} \equiv 2 \pi n v / a_{z} \approx \omega_{21} \equiv \omega_{0}$ for $\omega_{21} \approx 5 \times 10^{18} \mathrm{~s}^{-1}$ and the specific longitudinal period $a_{z}$ is satisfied for $v \approx 6 \times 10^{9} \mathrm{~cm} / \mathrm{s}$ which corresponds to mesicatom energy of about 25 MeV .

If we use the dechanneling length $\Delta z_{\mathrm{d}} \approx 1-30 \mu \mathrm{~m}$ that is determined below and satisfies the relation $\gamma<v / \Delta z_{\mathrm{d}} \ll \Gamma$, we find from (5.5) that the potential assumes the following quasistationary form over the entire stable part of the trajectory $v / \Gamma<z<\Delta z_{d}$ :

$$
\begin{align*}
V(x) \approx & \frac{\hbar \Delta \omega}{2}\left\{\frac{4 A \Omega^{2}}{4 \Omega^{2} \Gamma+\gamma\left(\Gamma^{2}+\Delta \omega^{2}\right)}+\frac{\left(\lambda_{1}+\gamma\right)\left(2 \gamma-\lambda_{1}-C_{2}\right)}{\left(\lambda_{1}+\Gamma\right)\left(3 \lambda_{1}+C_{2}\right)}\right. \\
& \left.\times\left[1-\frac{A\left(\Gamma^{2}+\Delta \omega^{2}\right)}{C_{0}}\right]\right\}, \tag{5.8}
\end{align*}
$$

where $\lambda_{1}, \lambda_{2}, \lambda_{3}$ are the roots of the characteristic equation of (5.4):

$$
\lambda^{3}+C_{2} \lambda^{2}+C_{1} \lambda+C_{0}=0,
$$

in which

$$
\begin{aligned}
& C_{0}=\Gamma^{2} \gamma+4 \Omega^{2} \Gamma+\beta^{2} \gamma, C_{1}=2 \Gamma \gamma+\Gamma^{2}+4 \Omega^{2}+\beta^{2} \\
& C_{0}=2 \Gamma+\gamma
\end{aligned}
$$

Figure 9 shows the form of the induced-dispersive potential calculated from (5.8) for the interaction of a mesic atom and two crystal planes for the two symmetric values of detuning $\Delta \omega= \pm \omega_{21} / 5$. In the first case ( $\Delta \omega>0$ ), the potential well that ensures stable directed motion lies next to the crystal plane. Because of the considerable analogy with the structure of the potential well for negatively charged particles, we shall refer to it as the $\mathrm{ID}^{-}$potential. Similarly, ID $^{+}$will represent the potential corresponding to $\Delta \omega<0$. We must now determine the quantitative characteristics of the channeling of mesic atoms.

If we approximate any of the $\mathrm{ID}^{-}$potential wells by the function

$$
V^{(-)}=\alpha^{(-)} \xi^{2}+\beta^{(-)} \xi^{3}, \quad \xi=x-x_{0}
$$

where $\alpha^{(-)} \approx 1.47 \mathrm{eV} / \mathrm{A}^{2}, \beta^{(-)} \approx 1.08 \mathrm{eV} / \mathrm{A}^{3}$ and $x_{0}$ is the position of the minimum, we find that this kind of well contains the single bound state

$$
\begin{aligned}
\Psi_{0}^{(-)}= & \exp \left[-\xi^{2} / 2\left(l^{(-)}\right)^{2}\right]\left[1-\beta^{(-)}\left(l^{(-)}\right)^{3}\left(M / 2 \alpha^{(-)}\right)^{1 / 2}\right. \\
& \times\left\{\left(\xi / l^{(-)}\right)+\left[\left(\xi^{3} / 3\left(l^{(-)}\right)^{3}\right]\right) h^{-1}\right] /\left(e^{(-)} \sqrt{\pi}\right)^{1 / 2}
\end{aligned}
$$

with energy $E_{0}^{(-)}=\hbar \sqrt{\alpha^{(-)} / 2 M} \approx-0.04 \mathrm{eV}$ where $M$ is the mass of the mesic atom. The probability that this level will be populated by a particle incident at angle $\theta=0$ is $P_{0}^{(-)} \approx 0.14$. Since there are two such wells in the space between the two planes, the total probability is twice this.

The $\mathrm{ID}^{+}$potential contains in each period two wells of equal depth, but different width. The wider well can be approximated by the parabola

$$
V^{(1+)}=\alpha^{(+)} \xi^{2}, \quad \alpha^{(+)} \approx 1.54 \mathrm{eV} / \mathrm{A}^{2}
$$

and the narrower one by


FIG. 9. Potential energy and allowed energy levels in induced-dispersive channeling of neutral atoms of muonium for $\Delta \omega>0\left(V^{(+)}, E_{0}^{(+)}\right)(a)$ and $\Delta \omega<0\left(V^{(-)}, E_{0}^{(-)}\right)(b) . \rho_{e}(x)$ is the normalized average electron density of the lattice atoms on (100) planes.
$V^{(2+)}=-V_{0}^{(2+)} / \cosh ^{2}\left(\xi / \beta^{(+)}\right), V_{0}^{(2+)} \approx 0.09 \mathrm{eV}, \beta^{(+)} \approx 0,1 \AA$, and, in this case, we find that each well contains only one quantum state
$E_{0}^{(1+)} \approx 0.05 \mathrm{eV}, \Psi_{0}^{(1+)}=\exp \left[-\xi^{2} / 2\left(l^{(+)}\right)^{2}\right] /\left(l^{(+)} \sqrt{\pi}\right)^{1 / 2}$,
$E_{D^{2+}}^{(2)} \approx 0.07 \mathrm{eV}, \quad \Psi \partial^{(2+)}=\left(2 / \beta^{(+)} B(s, s)\right)^{1 / 2} /\left[2 \cosh \left(\xi / \beta^{(+)}\right]^{s} ;\right.$
where $l^{(+)}=\left[\hbar\left(2 \alpha^{(+)} M\right)^{1 / 2}\right]^{1 / 2}$

$$
s=\left[\left(2 M V_{0}^{(2+)} \beta^{(+)^{2}} / \hbar^{2}\right)+(1 / 4)\right]^{1 / 2}-(1 / 2) \approx 0,36
$$

The probabilities that these levels will be populated are ${ }^{46}$ 0.33 and 0.18 .

The dechanneling length for these states, calculated from quantum theory, ${ }^{49}$ is $z_{d}^{(1+)} \approx 30 \mu \mathrm{~m}, \quad z_{d}^{(2+)} \approx z_{d}^{(-)}$ $\approx 1.5 \mu \mathrm{~m}$.

It is readily shown that the retardation of the particle during its directed motion does not lead to a significant change in the chosen detuning for either $\mathrm{ID}^{+}$or $\mathrm{ID}^{-}$potentials. The 'detuning length' $z_{p}$ can be estimated, starting with the condition for equal frequency shift of the resonance by the retardation

$$
\delta \omega\left(z_{\mathrm{p}}\right) \approx \omega_{0} \delta v\left(z_{\mathrm{p}}\right) / c \approx z_{\mathrm{p}} g \cos \varphi_{0}(-\mathrm{d} \varepsilon / \mathrm{d} z)_{\mathrm{e}} / M c
$$

and the average width of the resonant transition

$$
\langle\Gamma\rangle=\int \Gamma(x)\left|\Psi_{0}^{-}(x)\right|^{2} \mathrm{~d} x \approx 10^{16} \mathrm{~s}^{-1}
$$

It is clear that, in specimens of length $z<z_{p}$, the shift is 'masked' by the line width and does not affect the resonance characteristics. The condition $\delta \omega\left(z_{p}\right)=\langle\Gamma\rangle$ then yields $z_{p} \approx 70-700 \mu \mathrm{~m}$, so that $z_{d}<z_{p}$ and the shift of the resonance is of little significance throughout the entire path of directed motion $z \leqslant z_{d}$.

We note that longitudinal velocity (and energy) necessary for resonance can be substantially reduced by using the higher harmonics of the bounce frequency $\omega_{0}=2 \pi n v / a_{z}$ or by exploiting the transitions between the sublevels that are produced by splitting of the $1 s$ state in the external magnetic field and are associated with the different orientations of the muon spin.

## 6. EXPERIMENTS ON SMALL-ANGLE DIRECTION MOTION OF NEUTRAL PARTICLES AND PHOTONS

There have been several experimental studies ${ }^{50-52}$ of the channeling of fast neutrons and hard $\gamma$ rays $^{50}$ as well as soft x rays. ${ }^{51}$ The most convincing experimental results that, to a large extent, confirm the possibility of channeling of fast ( 2.5 MeV ) neutrons and, to a lesser extent, hard photons with energies of about 3 MeV , were reported in Ref. 50 . The measurements were performed in the horizontal channel of the VVR-M reactor of the Institute for Nuclear Research of the Ukrainian Academy of Sciences.

The incident neutron beam and its small divergence were defined by normalized-steel collimators, 2 m long, with a channel of $3 \times 3 \mathrm{~mm}^{2}$. A suitably oriented germanium single crystal (mosaicity less than $40^{\prime \prime}$ ) in the form of a cylinder 56 mm in diameter and 20 mm tall, was mounted on a goniometer head between the collimators. Figure 10 shows the measured angular distribution of the fast-neutron transmission factor close to the [ $1 \overline{1} 0$ ] direction. Because there are no theoretical calculations of small-angle motion of neutral


FIG. 10. Fast-neutron counting rate as a function of the angle of rotation of the Ga single crystal relative to the beam. The angle $\theta$ is measured from the [110] direction (marked by the arrow).
particles near the axis, these experimental results are difficult to interpret from the point of view of channeling (the theoretical basis for this experiment was the assumption of coherent single scattering by one or a few chains ${ }^{36.37}$ which is valid for short channels), although the main features of the angular distribution can be explained in terms of the theory, developed in Section 4, of planar channeling due to the coherent Schwinger interaction.

The same apparatus was used to investigate small-angle near-axis motion of hard $\gamma$ rays accompanying the emission of fast neutrons from the reactor pile. The angular distribution obtained for $\gamma$ rays near the [110] axis is shown in Fig. 11. In contrast to the neutron channel, there is a well-defined peak with relative amplitude of $2 \%$ near the axis. Unfortunately, this result cannot be unambiguously interpreted as a direct demonstration of channeling because of the lack of theoretical results on the axial channeling (more precisely, quasichanneling) of photons. Direct application to this problem of the theory of planar channeling developed in Section 2 yields a possible peak with a relative amplitude of $0.5 \%$. Nevertheless, it is clear that this discrepancy by a factor of 4 is not too large and can be explained by the theory of axial quasichanneling. We note that our own interpretation ${ }^{50}$ of the peak as the result of diffraction during strictly axial motion is somewhat unconvincing. First, diffraction should occur well outside the half-width of the transmission peak. Second, diffraction can be accompanied not only by a


FIG. 11. $\gamma$-ray counting rate for $\hbar \omega \leqslant 3 \mathrm{MeV}$ under the same conditions as in Fig. 10.
reduction in the intensity of the direct beam due to the transfer of energy to the Bragg peak, but it may also be enhanced by the two-wave anomalous transmission effect (Bormann effect). It is likely that an unequivocal interpretation will have to await the advent of the necessary theory.

It would be desirable to formulate experiments with geometry allowing adequate interpretation within the framework of existing theory.

## 7. CONCLUSION

Our review shows that although the interaction between neutral particles and photons on the one hand and the regularly distributed atoms on crystal planes and axes on the other is clearly weak, there are several sufficiently effective mechanisms capable of ensuring strongly coupled directed motion such as channeling, or the more loosely coupled quasichanneling, with obvious experimental consequences.

The typical situation is that of only one (occasionally two or three) energy levels in the interplanar (axial) channel or a narrow channel of width of the order of 1 nm or less. In the transverse direction, the particle (photon) can become localized in a region significantly larger than the channel diameter, which leads to small quantitative differences between the final beam-transport characteristics during channeling and during undirected motion.

All this imposes much greater (as compared with charged particles) demands on the incident-beam collimation and-most importantly-on the statistics of the associated experimental data. There are also cases in which a small change in the transverse structure of the wave function (with or without channelling) can lead to large final differences (for example, in the channeling of resonance and slow neutrons in a multi-atom lattice containing an alternation of nuclei with strong and weak absorption). Moreover the very nature of channeled motion enables us to formulate experiments with new, nontraditional geometries. An example is provided by the possibility that channeled photons could be identified by the presence of a diffraction peak in a direction differing from the usual Bragg.

We note that we have not covered in this review the exceedingly weak ordering effects that cannot give rise to the observed phenomena (for example, effects due to strong and weak interactions, the higher-order multipole moments of the neutron that tend to push it toward regions of high lat-tice-field gradients, the susceptibility anomalies in the scattering of $\gamma$ rays with energies close to the electron-positron
pair production threshold, and so on).
Further studies of channeling and new applications of the phenomenon would benefit from experiments on planar channeling of neutrons by the coherent Schwinger mechanism at high energies and the 'optical' Fermi interaction at low energies. The most promising systems for induced-dispersive channeling include zeolite crystals in which microvoids periodically interrupt the microchannels and can act as a trajectory-shaping periodic disturbance. It is clear that the channeling of hydrogen atoms is possible in such structures. In all cases, it is desirable to use more highly collimated beams with divergence at the very least smaller than the Lindhard angle.

It seems to us that by satisfying the entire set of optimizing conditions it will be possible to develop practical applications of the channeling of neutral particles and photons in structure analysis and in focusing, transport, and control of neutral beams, including beam rotation by slightly curved crystals.
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