The charged particle multiplicity distributions in e^+e^- annihilation processes in the LEP experiments[1)]

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A review is presented of the results obtained in an investigation of the charged particle multiplicity distributions in e^+e^- annihilation processes in experiments on the LEP accelerator at CERN. Universality in the energy dependence for the average charged particle multiplicity in e^+e^- and $p^\pm p$ collisions, the KNO scaling for the e^+e^- data, the structure in the multiplicity distribution and its relation to the number of jets in an event, the average particle multiplicities in quark and gluon jets, the pattern of "clans," and other questions are discussed.

INTRODUCTION

The advance towards higher energies after the entry into operation of each new accelerator inevitably led to the revival of interest in the regularities which characterize the multiplicity distributions of the particles formed during interactions. This was also true after the start-up of the LEP e^+e^- collider at CERN.

The experimental results for the charged particle multiplicities obtained for this accelerator in investigating the processes of e^+e^- annihilation into hadrons in the region of a Z^0 boson, and for the ALEPH[1], DELPHI[2], L3[3], and OPAL[4] facilities, are presented and discussed in the present review.

But first a few words about the situation which took shape about the time when the experiments on the LEP accelerator were performed to investigate the multiplicity distributions during the collision of high energy hadrons. Here one may pick out three interesting features.

1) At energies not exceeding the *ISR* energy ($\sqrt{s}<62$ GeV), the KNO scaling was fulfilled with a sufficient degree of accuracy[5,2)]: the probability P_n for the formation of n particles in the entire phase volume was described by a universal, energy-independent function of $z = n/\langle n\rangle$ for its representation in the form

$$\langle n\rangle P_n \sim \Psi(n/\langle n\rangle). \qquad (1)$$

The fulfillment of KNO scaling also indicates independence of the energies of the adjusted factorial and normalized moments of the multiplicity distribution:

$$\gamma_k = \mu_k/\mu_1^k = \langle n(n-1)...(n-k+1)\rangle/\langle n\rangle^k, \qquad (2)$$

$$C_k = \langle n^k\rangle/\langle n\rangle^k. \qquad (3)$$

The theoretical basis for KNO scaling[5] was based on the assumption of the validity of Feynman scaling[7], according to which the particle density in a unit interval of rapidity dn/dy must not depend on energy (i.e., the plateau in the dependence of dn/dy on y broadens with increasing energy and its height remains constant). The experimentally observed significant violation of Feynman scaling nevertheless did not lead to violation of KNO scaling. In connection with this, attempts were undertaken to justify KNO scaling independently of the fulfillment of Feynman scaling. It was shown in a number of papers (see, for example, Refs. 8–11) that KNO scaling is a property intrinsic to a broad class of branching problems.

However, in the energy region of $\sqrt{s} = 200$ GeV, 560 GeV, and 900 GeV of the CERN S$\bar{p}$pS collider the UA5 cooperation (collaboration) has detected[12] a violation of KNO scaling which showed up as an appreciable broadening of the multiplicity distribution, especially at $z > 2$.

2) The detection of the extensive applicability of the negative binomial distribution (NBD) to describe the multiplicity distributions in interactions of different types, in different rapidity intervals, and at different energies was the second interesting observation (see, for example, Refs. 13, 14, and 15 and the references given there). The negative binomial distribution has the form

$$P_n(\bar{n}, k) = \frac{k(k+1)...(k+n-1)}{n!}\frac{(\bar{n}/k)^n}{[1+(\bar{n}/k)]^{n+k}} \qquad (4)$$

with the two parameters $\bar{n}$ (the average multiplicity)[3)] and k, which are related to the mean square deviation D by the relation

$$D^2/\bar{n}^2 = 1/\bar{n} + 1/k \qquad (5)$$

[it is evident from Eq. (5) that the negative binomial distribution is broader than the Poisson distribution and goes over into the latter as $(1/k)\to 0$].

Of the several possible interpretations of the successful use of the negative binomial distribution for describing the multiplicity distributions in hadron, lepton, and semilepton reactions, the pattern of "clans" suggested by Giovannini and van Hove[14,16,17] and independently by Ekspong[18] has received the most extensive acceptance.[4)] It is based (see, for example, Ref. 14) on the introduction of the new parameters (instead of $\bar{n}$ and k)

$$\bar{N}_c = k\ln[1+(\bar{n}/k)], \qquad (6)$$

$$\bar{n}_c = \bar{n}/\bar{N}_c. \qquad (7)$$

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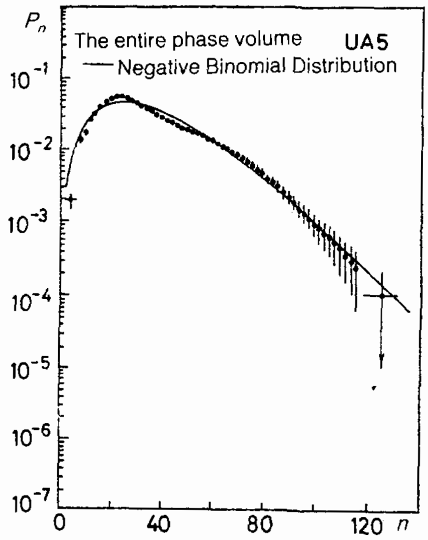

FIG. 1. The charged particle multiplicity distribution in $\bar{p}p$ collisions at $\sqrt{s} = 900$ GeV.[27] The solid curve is a fitting to a negative binomial distribution.

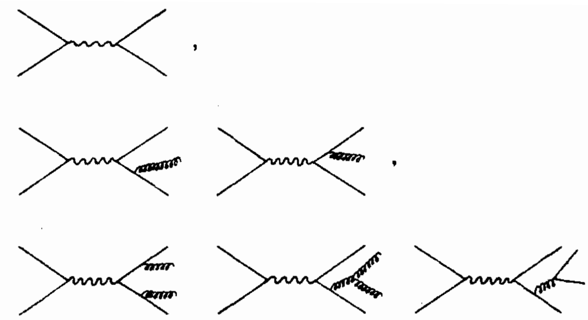

FIG. 3. The Feynman diagrams for the $e^+e^- \to$ hadrons process in the second approximation of $O(\alpha_s^2)$ in quantum chromodynamics (QCD).

A negative binomial distribution is obtained here if the number of clans (N_c) is distributed according to a Poisson distribution (with an average $\bar{N}_c$), and the average number of particles in a clan ($\bar{n}_c$) is subject to a logarithmic distribution. There would not have been a profound meaning in such a substitution of variables if it had not turned out here that, in hadron collisions, the average number of clans $\bar{N}_c$, in a fixed rapidity interval is independent of energy over the broad interval of energy from $\sqrt{s} = 22$ GeV to 900 GeV[14] (in other words, scaling for the average number of clans occurs instead of the Feynman scaling for particles). And the increase of the average multiplicity with increasing energy is due in this approach to an increase of the number of particles in a clan (apart from the natural increase of the phase volume).

But what is meant by a clan? In accordance with Refs. 14 and 16, a clan represents a group of particles of common origin in a cascade mechanism of particle formation. Clans are formed independently of each other (they have a Poisson distribution) and contain at least one particle. One would not regard this interpretation as complete. However, it turned out[22,23] that a negative binomial distribution also describes the multiplicity distributions of the partons in the Lund model of the fragmentation of a string with parton showers (LUND PS)[24] developed to describe the processes of e^+e^- annihilation. This agrees with the concept of local parton-hadron duality and allows one to suggest the interpretation of clans as one of decelerating gluon jets[14] (also see Refs. 25 and 26).

3) The detection of structure in the multiplicity distribution of charged particles at $\sqrt{s} = 900$ GeV by the UA5 collaboration was the third interesting observation[27] (Fig. 1). The nature of this structure is, in any case from the experimental point of view, as yet not understood, although it evidently also can be explained by processes of exchange not with one but several pomerons, with n-pomeron branchings in the language of the model.[28]

The observations listed also raised interest in studying the particle multiplicity distributions in processes of e^+e^- annihilation into hadrons at high energies (and, in particular, in experiments on the LEP accelerator), in which the initial state is well determined and the quarks and gluons form clear hadron jets which retain, to a good approximation, the directions and energies of the original partons, which significantly simplifies their experimental analysis.

The processes of e^+e^- annihilation into hadrons are characterized by four different stages differing in time that are shown schematically in Fig. 2:[29]

1) formation of a $q\bar{q}$ pair (and of photons) (electroweak theory);
2) emission of hard gluons (perturbative quantum chromodynamics (QCD);
3) fragmentation of quarks and gluons into hadrons (non-perturbative QCD); and
4) the decay of unstable particles (electroweak theory and QCD).

The second stage can be calculated in perturbative quantum chromodynamics by means of two approaches: the "matrix element" (exact calculations in second order

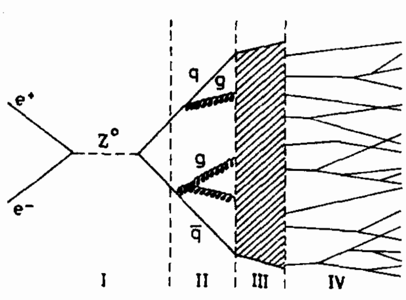

FIG. 2. The process of e^+e^- annihilation into hadrons.

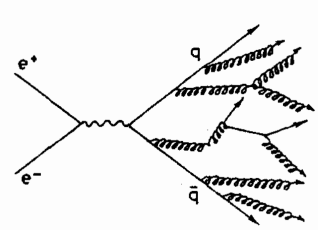

FIG. 4. A schematic representation of a parton shower in a process of e^+e^- annihilation into hadrons.

TABLE I. Average charged particle multiplicities in processes of e^+e^- annihilation into hadrons at $\sqrt{s} = 91$ GeV.

Experiment	$\langle n_{ch} \rangle$	$\langle n_{ch} \rangle / D$	Reference
DELPHI	20,71 ± 0,04 ± 0,77	3,30 ± 0,02 ± 0,20	[32]
L3	20,7 ± 0,7	—	[33]
ALEPH	20,85 ± 0,24	3,29 ± 0,01 ± 0,06	[34]
OPAL	21,40 ± 0,02 ± 0,43	3,30 ± 0,01 ± 0,11	[35]
MARK II	20,1 ± 1,0 ± 0,9	—	[36]

QCD), and the "parton showers"(using a logarithmic approximation). In event generation programs based on the use of the matrix element approach, up to four partons are formed in accordance with the basic Feynman diagrams shown in Fig. 3. In generator with parton showers in the region of a Z^0 boson, an average of about nine partons are generated with virtual gluon masses up to ≈ 1 GeV (Fig. 4). On the whole, parton shower generators describe the data (and especially the energy dependences) better than the matrix element generators.

The fragmentation of quarks and gluons in the third phase is modeled by means of the fragmentation schemes for a string or for clusters. The LUND JETSET[24,30] with different variations of a matrix element or with parton showers, in most cases using string fragmentation, and HERWIG[31] with fragmentation clusters are the most successful models which describe hadronization processes (and the decays of short-lived particles and resonances).

The availability of these models which, after suitable correction (and moreover, this correction can be performed on data at lower energies in models with parton streams) describe the experimental data well in processes of e^+e^- annihilation into hadrons, is a very important circumstance, enabling one, in contrast to hadron collisions, to understand and explain specific experimentally observed phenomena.

The detection of universality in the energy dependence of the average charged particle multiplicity in e^+e^- and $p^\pm p$ collisions over a broad energy range, the fulfillment of KNO scaling for the e^+e^- data, the detection of structure in the charged particle multiplicity distribution and its explanation by superposition of events with different numbers of jets, the equal average particle multiplicities in quark and gluon jets, and the problematic nature of intepreting the experimental data in the concept of the pattern of "clans" turned out to be the most unexpected and interesting results among those obtained during the investigation of the charged particle multiplicity distributions in processes of e^+e^- annihilation into hadrons in experiments on the LEP that are discussed in the present review.

The review consists of 11 sections. Average charged particle multiplicities and their energy dependences are discussed in Sec. 2, and a comparison is made with the $p^\pm p$ data. The behavior of the moments of the multiplicity distribution and of the forward and back correlations are analyzed in Secs. 3 and 4, KNO scaling and the parameterizations of the multiplicity distributions are discussed in Secs. 5 and 6. The multiplicity distributions in restricted rapidity intervals, for fixed numbers of jets, and for individual jets, and also their interpretation in a model of "clans" are considered in Secs. 7, 8, and 9. Average particle multiplicities in quark and gluon jets are discussed in Sec. 10. Finally, basic summaries of the research conducted are presented in the last Sec. 11.

2. AVERAGE CHARGED PARTICLE MULTIPLICITIES

The average charged particle multiplicities measured in processes of e^+e^- annihilation into hadrons in the region of the Z^0 peak in different experiments[32–36] are presented in Table I. The average value of the multiplicity averaged over the experiments is $\langle n_{ch} \rangle = 20.9 \pm 0.2$.

The dependence of $\langle n_{ch} \rangle$ on the energy $\sqrt{s}$ is shown in Fig. 5, which has been taken from Ref. 35, where references are given for all the data used. Different parameterizations were used in Refs. 32, 34, and 35 to describe this dependence:

$$\langle n_{ch} \rangle = as^b, \tag{8}$$

$$\langle n_{ch} \rangle = \beta s^\alpha - 1, \tag{9}$$

$$\langle n_{ch} \rangle = a + b \ln s + c \ln^2 s, \tag{10}$$

$$\langle n_{ch} \rangle = a + b \exp\, [c\sqrt{\ln(s/Q_0^2)}\,], \tag{11}$$

$$\langle n_{ch} \rangle = a\alpha_s^b \exp(c\sqrt{\alpha_s})(1 + O(\sqrt{\alpha_s})). \tag{12}$$

The first of them goes back to the statistical models of Fermi[37] and of Pomeranchuk[38] and to Landau's hydrodynamic model[39] (see, for example, Ref. 40 for their present development). The second has been suggested recently in Ref. 41, Eq. (10), which is also used to describe hadron reaction data, is purely empirical. Equation (11) follows from perturbation QCD in the leading logarithmic approximation[42] ($Q_0 = 1$ GeV is the cutoff parameter in the perturbation calculations connected with the start of hadronization); a, b, and c are fitting parameters. Equation (12) also follows from perturbation QCD[43,44], but with allowance for corrections to the leading logarithmic approximation. The parameter a in Eq. (12) is not calculated in quantum chromodynamics, and the running coupling constant α_s is written in the form

$$\frac{\alpha_s(s)}{4\pi} = \frac{1}{\beta_0 \ln(s/\Lambda^2)} - \frac{\beta_1 \ln\ln(s/\Lambda^2)}{\beta_0^3 \ln^2(s/\Lambda^2)} \tag{13}$$

with the parameters b [in Eq. (12)] and β_0 and β_1 [in Eq. (13)] dependent on the number of quark flavors. In an approximation when one may neglect the $O(\sqrt{\alpha_s})$ term, the two fitting parameters a and Λ remain in Eq. (12). The last one must be close to the parameter $\Lambda_{\overline{MS}}$ if the correction term $O(\sqrt{\alpha_s})$ is actually small.

The fitting of Eqs. (9), (10), and (11) to the experimental data is illustrated in Fig. 5. Equations (10), (11),

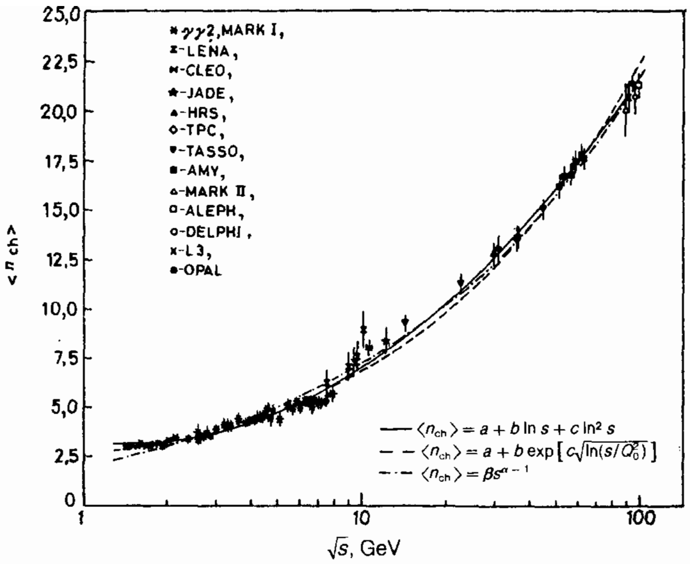

FIG. 5. The dependence on energy of the average charged particle multiplicity in processes of e^+e^- annihilation into hadrons. The curves are fittings of Eq. (9), (10), and (11).

and (12) describe the experimental data well. The fitting of Eqs. (8) and (9) is less satisfactory,[32,35] but this is possibly connected with systematic errors in the data at low energies that have not been sufficiently allowed for: an increase of the systematic errors from 5% to 10% in the $\gamma\gamma^2$ and MARK I experiments leads to a reduction of χ^2/NDF from ≈ 2 to ≈ 1.[35] Thus, the set of experimental data existing today does not allow one to prefer or to exclude even one of Eqs. (8) through (12). It is best of all to take the values obtained for the fitting parameters from Ref. 35, where the most recent data from all experiments were used. We shall restrict ourselves to quoting only the values $a = 0.065 \pm 0.010$ and $\Lambda = 136 \pm 50$ MeV that have been obtained by fitting Eq. (12) at $\sqrt{s} \geqslant 10$ GeV.[35] LUND PS (the JETSET 6.3 or 7.2 version) also describes the data well, just as do the parameterizations quoted above. HERWIG also matches the general trend of the data, but it is somewhat worse than LUND PS.

In light of the new data about charged particle multiplicities in processes of e^+e^- annihilation into hadrons at high energies, a comparison of them with the $p^\pm p$ data is of interest. Attempts to find some universal dependence of $\langle n_{ch}\rangle$ on energy for the e^+e^- and $p^\pm p$ data were also undertaken earlier[45,46] and were based on redetermining the energy going into particle formation in $p^\pm p$ collisions with allowance for the effect of "leading". However, extrapolating the results obtained into the range of the higher energies accessible on the Sp̄pS collider and the LEP turned out to be unsuccessful.

A new attempt in this same direction has been undertaken in Ref. 47. It is based on the assumption that, if the dependence of $\langle n_{ch}\rangle$ on $\sqrt{s}$ for e^+e^- collisions is described by some function

$$\langle n_{ch}\rangle = f(\sqrt{s}) \tag{14}$$

[as which, for example, one of the Eqs. (8) through (12), which describe the data well, may be used], then for $p^\pm p$ collisions, this dependence is transformed into

$$\langle n_{ch}\rangle = n_0 + f(\sqrt{s}/k), \tag{15}$$

where n_0 and k are unknown parameters which can be determined from a simultaneous fit of Eqs. (14) and (15) to the e^+e^- and $p^\pm p$ data [along with the parameters of the function $f(\sqrt{s})$]. The dependence of $\langle n_{ch}\rangle - n_0$ on $\sqrt{s}/k$ found in this manner [by using the function $f(\sqrt{s})$ in the form of Eq. (11)] with the values found for the parameters

$$n_0 = 2{,}57 \pm 0{,}72, \quad k = 3{,}00 \pm 0{,}32 \tag{16}$$

(for the e^+e^- data, by definition, $n_0 = 0$ and $k = 1$) is shown in Fig. 6. As is evident, for such a redetermination of Eq. (15) with the parameters (16) for the $p^\pm p$ data, the character of the dependences of the average multiplicities on energy in e^+e^- and $p^\pm p$ collisions turns out to be surprisingly congruent.

Also the interpretation of the values found for the parameters (16) is fairly interesting.[5] One may consider the parameter n_0 as the average multiplicity for the leading particles[47] [it can be greater than 2 because of the possible fragmentation of the original protons (antiproton) into resonances]. The value $k \approx 3$ finds a natural explanation[47] within an additive quark model and a hypothesis of an equal, on the average, energy distribution between the three clothed valence quarks of the original protons (antiprotons). This also means that the energy in the center of mass system for the formation of particles is effectively three times higher in e^+e^- collisions than in $p^\pm p$ collisions, so that the energy $\sqrt{s} \approx 270$ GeV in $p^\pm p$ collisions corresponds to the energy $\sqrt{s} = 91$ GeV in the LEP accelerator.

3. MOMENTS OF THE MULTIPLICITY DISTRIBUTION

The mean square deviations D of the charged particle multiplicity distribution have been measured in three experiments on the LEP.[32,34,35] The $\langle n_{ch}\rangle/D$ values found are shown in Table I, and a comparison of these results with data at lower energies (see the references in Refs. 32, 34, and 35) is presented in Fig. 7(a); data obtained for one hemisphere in the center of mass system are shown in Fig. 7(b). As is

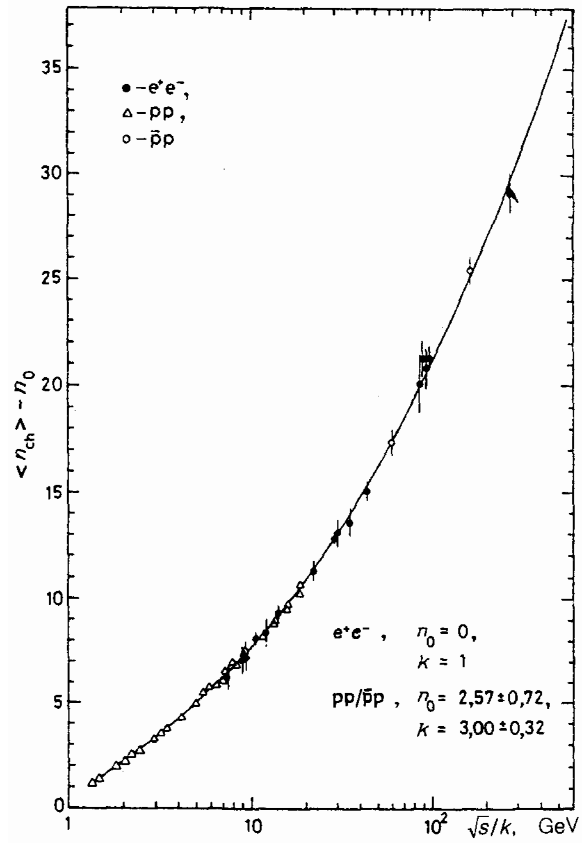

FIG. 6. The dependence of $\langle n_{ch}\rangle - n_0$ on $\sqrt{s}/k$ for the $p^{\pm}p$ data (for the values of the parameters of Eqs. (16) and the e^+e^- data (for $n_0 = 0$, $k = 1$). The solid curve is Eq. (11) that has been obtained (together with the parameters n_0 and k) by a simultaneous fitting to the e^+e^- and $p^{\pm}p$ data.

evident, for the e^+e^- data, the ratio $\langle n_{ch}\rangle/D$ is independent of energy within the error limits in the interval $\sqrt{s} = 10$ GeV to 91 GeV.

A comparison of the normalized moments C_k [Eq. (3)] measured in the DELPHI[32] and OPAL[35] experiments

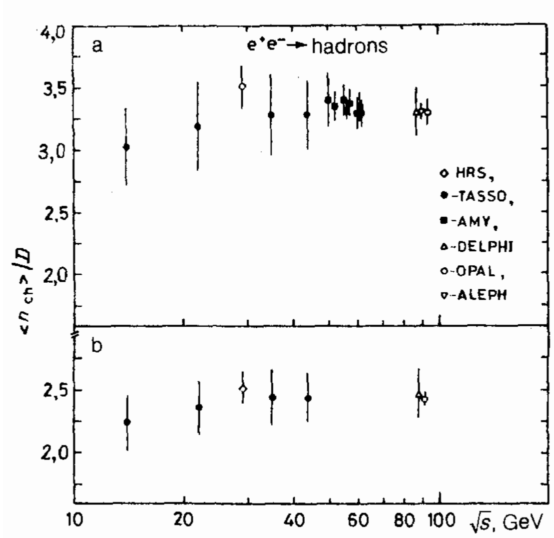

FIG. 7. The dependence on energy of the ratio $\langle n_{ch}\rangle/D$ (a) for complete events, and (b) for one hemisphere in the center of mass system.

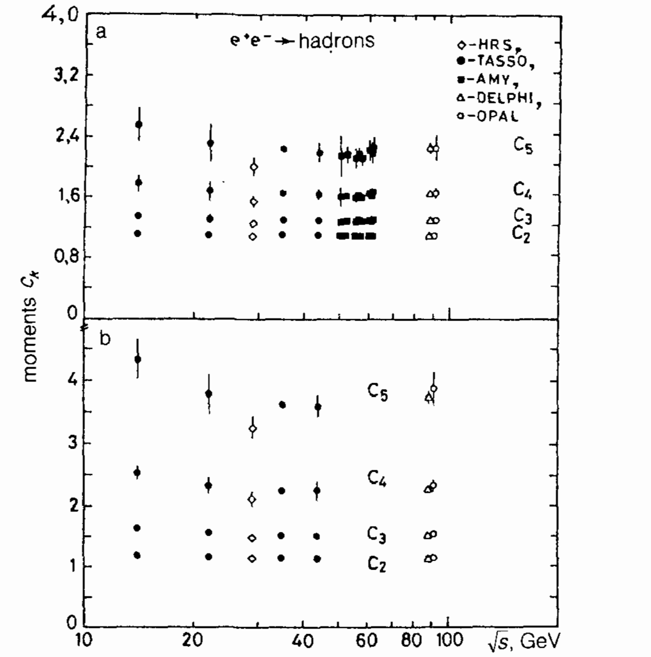

FIG. 8. The dependence on energy of the normalized moments $C_k = \langle n^k\rangle/\langle n\rangle^k$ (a) for complete events, and (b) for one hemisphere in the center of mass system.

with the results obtained at lower energies is shown in Fig. 8(a) [and in Fig. 8(b) for one hemisphere in the center of mass system]. The normalized moments C_k are independent of energy in the interval $\sqrt{s} = 10$ GeV to 91 GeV, which must indicate fulfillment of KNO scaling (see Sec. 5).

4. THE CORRELATION OF FORWARD AND BACK MULTIPLICITIES

In studying correlations between the particles emitted into different hemispheres in the center of mass system (the forward one F and the back one B), the average particle multiplicity in one hemisphere is measured as a function of the particle multiplicity in the other hemisphere. This dependence is usually parameterized as

$$\langle n_F\rangle = a + bn_B, \qquad (17)$$

where b is the correlation parameter.

Correlations of such a type between charged particles have been measured in the DELPHI experiment.[32] The dependence obtained of $\langle n_F\rangle$ on n_B (Fig. 9, curve *1*) is described well by Eq. (17) and the value of the parameter $b = 0.118 \pm 0.009$, upon comparison with the TASSO data,[48] turns out to be independent of energy within the error limits in the interval $\sqrt{s} = 14$ GeV to 91 GeV. The forward and back correlations turned out to be strongest in the central range in the rapidity interval $|y| < 1$ (Fig. 9, curve *2*), where $b = 0.289. \pm 0.012$, and for oppositely charged particles (in agreement with the earlier TASSO[48] and NA22[49] results), when $b = 0.177 \pm 0.009$ [Fig. 10(a)], whereas the correlations are low for particles of the same charge sign, and $b = 0.020 \pm 0.006$ [Fig. 10(b)]. The measured correlations are in excellent agreement with the LUND PS predictions in all cases (see Figs. 9 and 10).

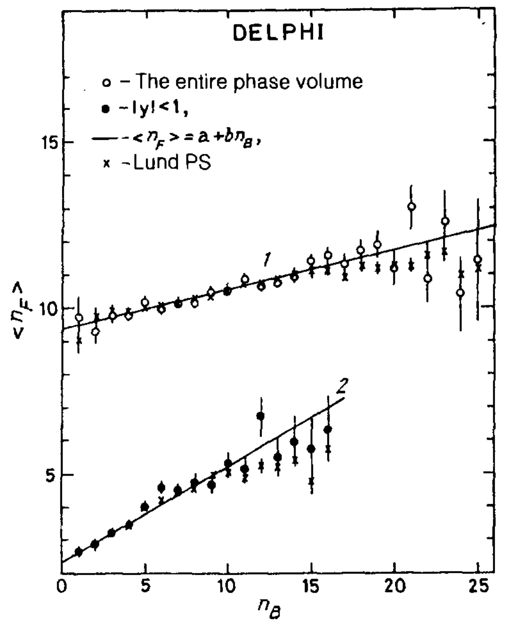

FIG. 9. Correlations between the charged particles $\langle n_F \rangle$ as a function of n_B. *1*) for all events, and *2*) in the middle rapidity range $|y| < 1$. The crosses are the LUND PS predictions. The straight lines are the result of fitting to Eq. (17).

5. THE KNO SCALING

As was already noted in Sec. 3, the independence from energy of the normalized moments C_k must indicate the validity of the KNO scaling. Actually, it was shown in the DELPHI experiment[32] that a charged particle multiplicity distribution represented in the form of a dependence $\Psi(z) = \langle n \rangle P_n$ on $z = n/\langle n \rangle$ is independent of energy from $\sqrt{s} = 14$ GeV to $\sqrt{s} = 91$ GeV (both for complete events and also for particles emitted into one hemisphere in the center of mass system). ALEPH[34] and OPAL[35] confirmed this result. Figure 11, in which the DELPHI and OPAL results are compared with the HRS[50], TASSO[48], and AMY[51] data at lower energies, illustrates the fulfillment of KNO scaling for the e^+e^- data.

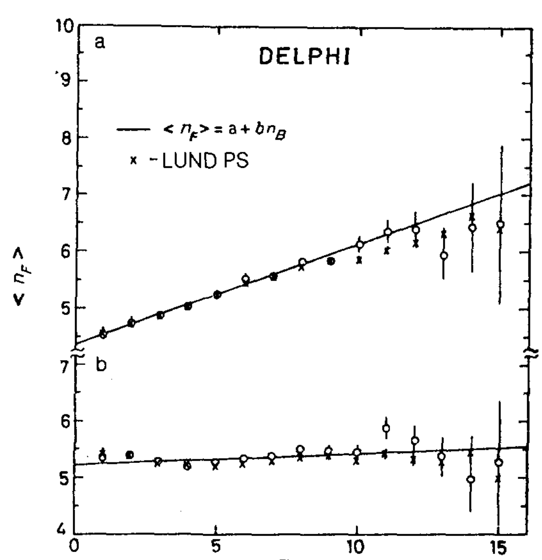

FIG. 10. Correlations between the charged particles $\langle n_F \rangle$ as a function of n_B (a) for oppositely charged particles, and (b) for particles of the same charge. The crosses are the LUND PS predictions. The straight lines are the result of fitting to Eq. (17).

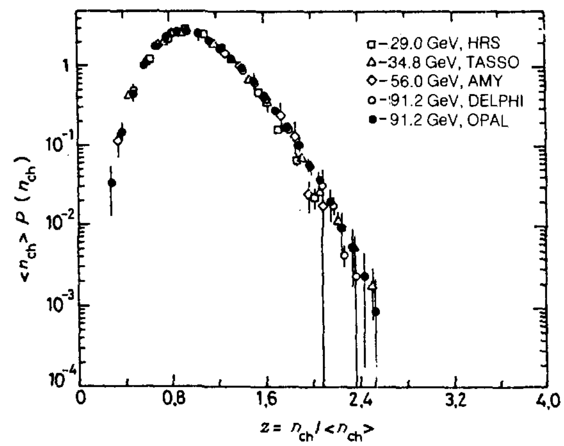

FIG. 11. Charged particle multiplicity distributions $\langle n \rangle P_n$ as a function of $n/\langle n \rangle$ for e^+e^- data at different energies.

LUND PS describes the multiplicity distributions in e^+e^- collisions over the entire energy interval investigated, and consequently, it also describes KNO scaling.[32-35] An analysis carried out within this model shows that hard processes lead to broadening of a multiplicity distribution in terms of the KNO variable z, whereas on the other hand, soft processes lead to a narrowing of it. The compensation of these two phenomena yields approximate KNO scaling so that, for example, the ratio $\langle n \rangle / D$ is practically constant for $\sqrt{s} = 15$ GeV to 1,000/GeV.[52] The trend towards broadening the distribution in z shows up only at energies much higher than $\sqrt{s} = 1$ TeV. This is also confirmed by the direct generation of events by means of the LUND PS at extremely high energies.[53] We also already referred to the fact that a broad class of branching processes also leads to KNO scaling.[8-11] All this agrees with the experimental observation of KNO scaling in e^+e^- collisions in the energy interval in the center of mass system from 20 GeV to 91 GeV. With that, the violation of KNO scaling in $\bar{p}p$ collisions at the energies of the CERN S$\bar{p}$pS collider that has been determined in the UA5 experiment appears puzzling.

6. THE FORM OF A MULTIPLICITY DISTRIBUTION

An investigation of the form of a charged particle multiplicity distribution over the entire phase volume that has been carried out in experiments of Refs. 32, 34, 35 showed that it agrees well with that predicted in LUND PS (the JETSET 6.3 and 7.2 versions). Just as at lower energies,[54] the experimental data are also described well[34,35] by a normal logarithmic distribution:

$$P_n(\mu, \sigma, c) = \int_{n/\langle \tilde{n} \rangle}^{(n+1)/\langle \tilde{n} \rangle} \frac{N}{\sqrt{2\pi}\sigma} \frac{1}{\tilde{z} + c} \exp\left\{ -\frac{[\ln(\tilde{z} + c) - \mu]^2}{2\sigma^2} \right\} d\tilde{z}, \tag{18}$$

where $\tilde{z} = n/\langle \tilde{n} \rangle$ and, because of normalization, of the pa-

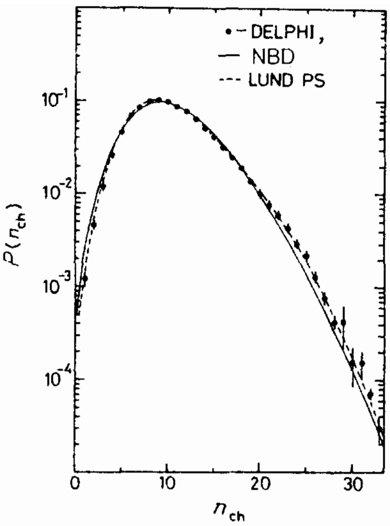

FIG. 12. The charged particle multiplicity distribution in one hemisphere in the center of mass system measured in the DELPHI experiment;[32] the solid curve is a fitting to a negative binomial distribution (NBD), and the dashed curve is the LUND PS prediction.

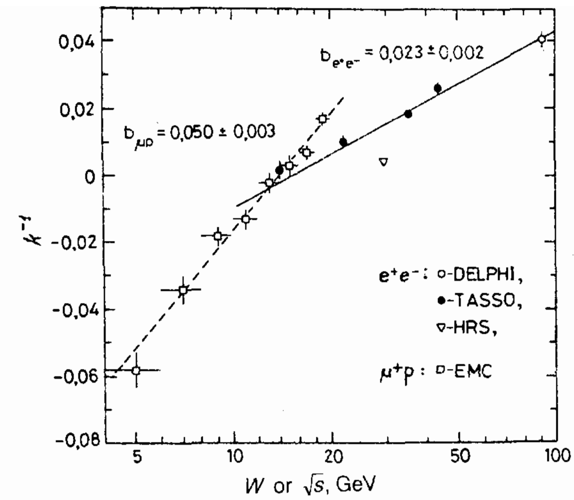

FIG. 14. The dependence on energy of the parameter $1/k$ obtained by fitting negative binomial distributions to the e^+e^- and μ^+p data. The straight lines are approximations using the dependence $1/k = a + b\ln(\sqrt{s}/Q_0)$.

rameters σ, μ, and c, only two are independent. The use of this distribution, as well as the so-called modified negative binomial distribution[55] in the experiment of Ref. 32, is based on the hypothesis that the particle formation process may be considered as a scale-invariant branching process.

In light of the successful use of the negative binomial distribution (NBD) to describe the data in hadron reactions, also including those in the UA5 experiment at $\sqrt{s} = 200$ GeV and 500 GeV (see the Introduction), the less satisfactory use of the negative binomial distribution for describing e^+e^- data turned out to be unexpected. While with allowance for the possibly not fully understood systematics, one may still speak of fairly satisfactory agreement of the negative binomial distribution with the data for the entire phase volume,[32,34,35] there is already no such agreement for charged particle multiplicity distributions in one hemisphere in the center of mass system ($\chi^2/NDF = 66/23$ for DELPHI[32] and $\chi^2/NDF = 101/33$ for OPAL[35]). Figures 12 and 13 graphically illustrate this situation.

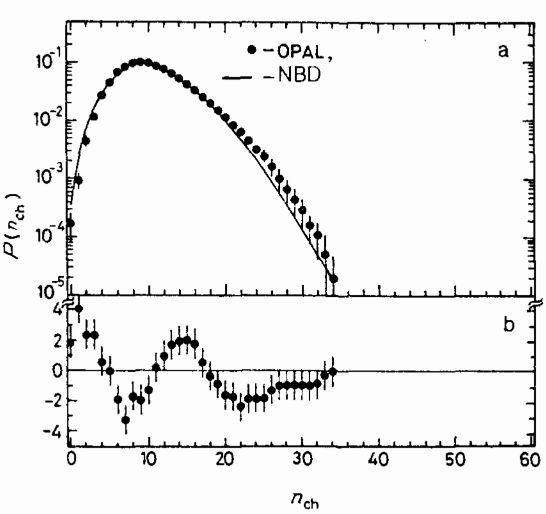

FIG. 13. The same as in Fig. 12, but from the results of the OPAL experiment.[35] The deviations of the experimental points from a negative binomial distribution (NBD) are shown in the lower part (b) of the figure.

The value of the parameter $\bar{n}$ in the negative binomial distribution [Eq. (4)], upon its fitting to the experimental data,[32,34,35] turned out to be close to the measured value of the average charged particle multiplicity over the entire phase volume (see Table I). Figure 14 illustrates the character of the energy dependence of the parameter $1/k$ for the e^+e^- data[48,50,32] in comparison with μ^+p data for the EMC collaboration.[56] The slope parameter in the parameterization $k^{-1} = a + b\ln(\sqrt{s}/Q_0)$ for the e^+e^- data at $Q_0 = 1$ GeV ($b = 0.023 \pm 0.002$) turned out to be half that for the μ^+p data ($b = 0.050 \pm 0.003$) and for the $p^{\pm}p$ data in the interval $\sqrt{s} \approx 10$ GeV to 91 GeV ($b = 0.058 \pm 0.001$).[12]

7. MULTIPLICITY DISTRIBUTIONS IN RESTRICTED RAPIDITY INTERVALS

In light of the not very successful description of a charged particle multiplicity distribution by the negative binomial distribution, especially for particles emitted into one hemisphere in the center of mass system, a detailed analysis was undertaken in the DELPHI experiment[57] (also see Ref. 58) of the multiplicity distributions in a restricted rapidity interval, and they were compared with the negative bimodal distribution. The detection of structure in the multiplicity distributions which showed up most strongly in the middle rapidity intervals and was very reminiscent of the structure detected in the $\bar{p}p$ collisions at $\sqrt{s} = 900$ GeV in the UA5 experiment (see Fig. 1) was an unexpected result of this analysis.[6)] As is evident from Fig. 15, the multiplicity distributions (represented in KNO form) have maxima at small values of $z = n/\langle n\rangle$ at $|y| < 1.0$, $|y| < 1.5$, and $|y| < 2.0$, and some "excess" at large z values. It is not surprising that the negative binomial distribution does not describe these distributions ($\chi^2/NDF \approx 10$). This structure shows up even more clearly for the multiplicity distributions of the particles

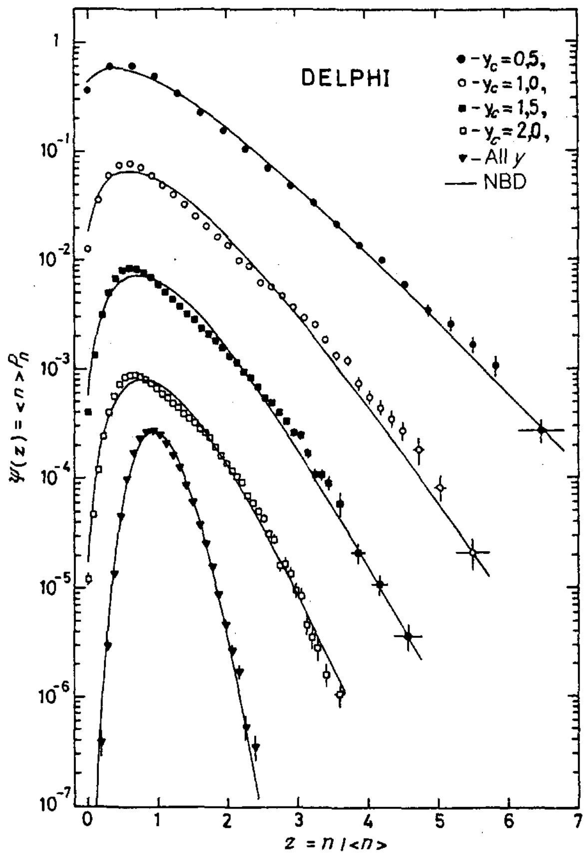

FIG. 15. The charged particle multiplicity distributions in the form of the dependence of $\langle n \rangle P_n$ on $n/\langle n \rangle$ in different rapidity intervals.[57] The solid curves are fittings of negative binomial distributions (NBD).

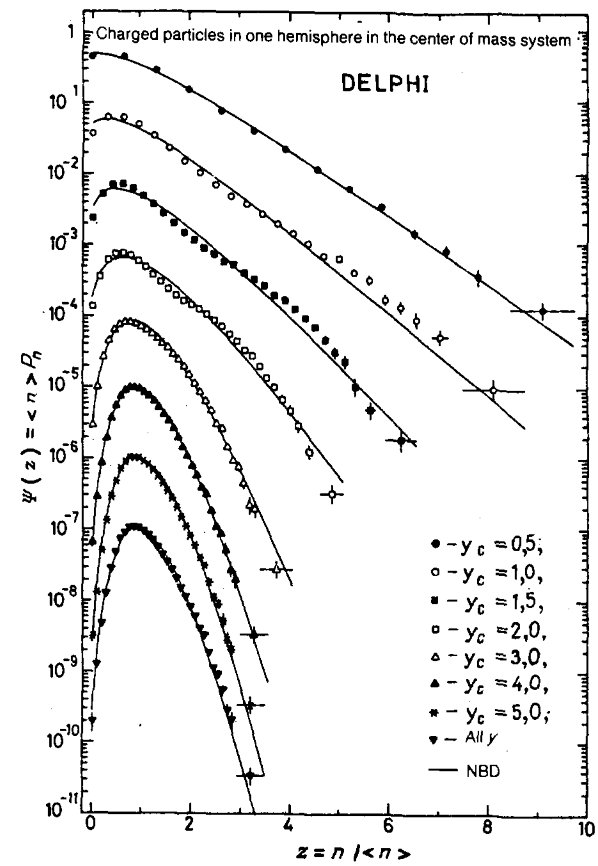

FIG. 16. The same as in Fig. 15, but for charged particles in one hemisphere in the center of mass system in different rapidity intervals.[57]

emitted into one hemisphere in the center of mass system that are shown in Fig. 16. The structure observed in the experiment agrees with the LUND PS prediction.[57]

Although the negative binomial distribution does not describe the structure in the multiplicity distributions, it qualitatively reproduces the forms of these distributions in different rapidity intervals. Therefore, it is of interest to investigate the dependence of the parameters $\bar{N}_c$ and $\bar{n}_c$ [Eqs. (6) and (7)] in the concept of clans on the rapidity interval $|y| < y_c$ and their dependence on energy. The dependences of the average number of clans $\bar{N}_c$ and of the average number of particles $\bar{n}_c$ in a clan on y_c obtained in the TASSO[48] and DELPHI[57] experiments are shown in Figs. 17 and 18. As is evident, the parameter $\bar{N}_c$ in a fixed rapidity interval is practically independent of energy. This means that scaling, which is not observed for the particle density over rapidity,[59,60] occurs for the density of clans. On the other hand, the parameter $\bar{n}_c$ increases appreciably with increasing energy. Thus, the increase of the particle multiplicity with energy in the pattern of clans is connected (apart from the simple increase of the phase volume) not with increasing density of clans, but with the increasing number of particles in a clan. This situation is in accord with the conclusions drawn by Van Hove and Giovannini[23] in their analysis of e^+e^- collisions by means of events generated according to the LUND PS model and with the behavior of the parameters $\bar{N}_c$ and $\bar{n}_c$ in hadron reactions (see, for example, Ref. 61). Let us note, however, that the average number of clans for a fixed y_c

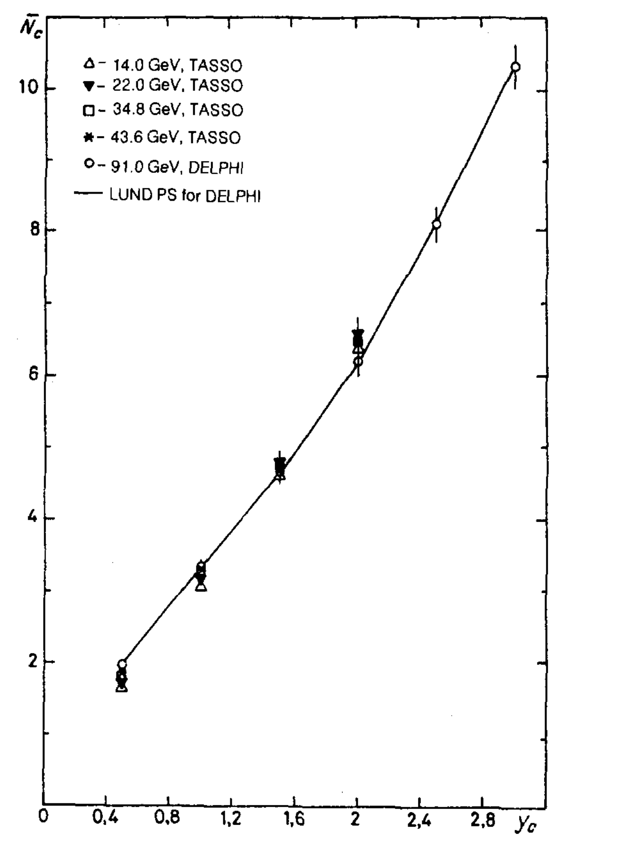

FIG. 17. The dependences of the average number of clans $\bar{N}_c$ on the value of the rapidity interval y_c in the TASSO and DELPHI experiments. The solid curve joins the points that are predicted at $\sqrt{s} = 91$ GeV by the LUND PS model.

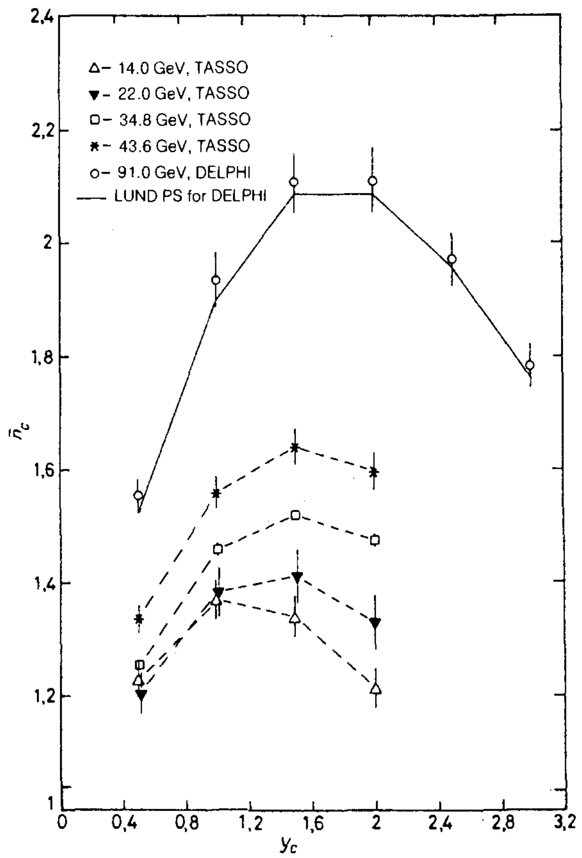

FIG. 18. The dependences of the average number of particles in a clan $\bar{n}_c$ on the value of the rapidity interval y_c in the TASSO and DELPHI experiments. The solid curve joins the points that are predicted at $\sqrt{s} = 91$ GeV by the LUND PS model. The dashed curves join the TASSO points with the same energies.

turns out to be somewhat higher in the processes of e^+e^- annihilation into hadrons than in hadron reactions.

8. THE MULTIPLICITY DISTRIBUTION FOR A FIXED NUMBER OF JETS

The detection of structure in the charged particle multiplicity distribution that is especially noticeable at fixed middle rapidity intervals and, as a consequence, the poor description of the data by the negative binomial distribution, as well as the fairly puzzling behavior of the parameters which describe the clans (the large $\bar{N}_c$ and, correspondingly, the small $\bar{n}_c$ values in comparison with hadron reactions) posed the problem of investigating the multiplicity distributions in processes of e^+e^- annihilation into hadrons for a fixed number of jets. Such an analysis has been made in the DELPHI experiment.[62]

The detection of the jets was done according to an algorithm developed by the JADE collaboration.[63] For each pair of charged particles in an event, their scaled invariant mass was calculated by

$$Y_{ij} = 2E_iE_j(1 - \cos\theta_{ij})/E_{\rm vis}^2, \tag{19}$$

where E_i and E_j are the energies and θ_{ij} is the angle between the momenta of the particles, and $E_{\rm vis}$ is the total energy of the charged particles in this event. After this, the pair of particles with minimum Y_{ij} was replaced by a pseudoparticle with the four-dimensional momentum $(p_i + p_j)$. This procedure was repeated until the values of Y_{ij} for all the pairs of pseudoparticles or particles turned out to be larger than a specified value $Y_{\min}$. The pseudoparticles or particles which remained were considered as belonging to the jet. Of course, the relative fraction of events with a specific number of jets depends strongly on the value of $Y_{\min}$ (see, for example, Refs. 64, 65, and 66), and moreover, the fraction of the events with the number of jets incorrectly identified by means of the above algorithm increases greatly with increasing $Y_{\min}$, at any rate for two-jet and three-jet events. Selected results from DELPHI for minimum values of $Y_{\min}$ are given below (see Ref. 62 about the dependence of the results on $Y_{\min}$).

Comparison of the charged particle multiplicity distributions in two-, three-, and four-jet events showed[62] that, with increasing numbers of jets, the distributions are shifted into the range of larger multiplicities (see the $\langle n_{\rm ch}\rangle$ values in Table IIa) and become broader, and the LUND PS model reproduces the experimental data [Fig. 19(a)]. Here it turned out that the multiplicity distributions for a fixed number of jets are described excellently by a negative binomial distribution [Fig. 19(b)]. And what is more, for such events a negative binomial distribution also satisfactorily describes the multiplicity distributions in restricted ra-

TABLE II. Average values of the number of charged particles $\langle n_{\rm ch}\rangle$, the number of clans $\bar{N}_c$, and the number of particles in a clan n_c [$\bar{N}_c$, and $\bar{n}_c$ are calculated directly from the experimental values $\langle n_{\rm ch}\rangle = \bar{n}$ and D (Ref. 62) by Eqs. (5) and (7)] for two-jet, three-jet, and four-jet events for $Y_{\min} = 0.04$: (a) for complete events, and (b) for individual jets.

	Parameter	Complete events with number of jets j		
		$j=2$	$j=3$	$j=4$
a	$\langle n_{\rm ch}\rangle$	19,3 ± 0,8	24,8 ± 1,0	31,4 ± 1,3
	$\bar{N}_c$	15,8 ± 0,7	19,4 ± 0,8	27,7 ± 1,1
	$\bar{n}_c$	1,22 ± 0,05	1,27 ± 0,05	1,13 ± 0,05
	Parameter	Individual jets in events with number of jets j		
		$j=2$	$j=3$	$j=4$
b	$\langle n_{\rm ch}\rangle$	9,6 ± 0,4	8,2 ± 0,3	7,8 ± 0,3
	$\bar{N}_c$	7,7 ± 0,3	6,4 ± 0,3	6,3 ± 0,3
	$\bar{n}_c$	1,25 ± 0,05	1,29 ± 0,05	1,24 ± 0,05

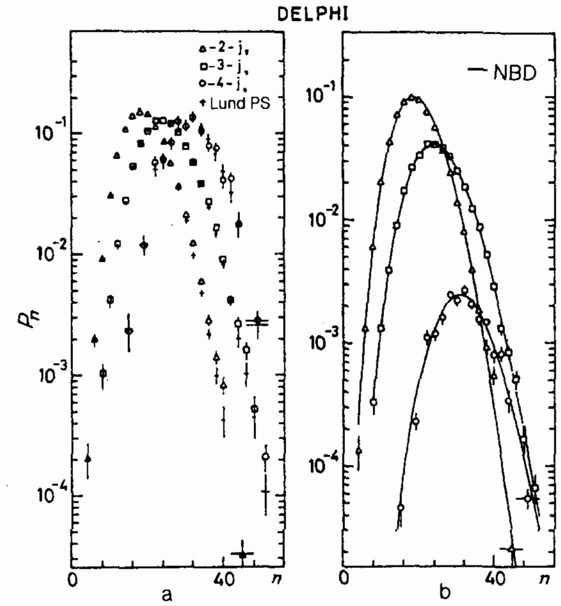

FIG. 19. The charged particle multiplicity distributions in two-jet, three-jet, and four-jet events (for $Y_{min} = 0.04$). (a) Each of the distributions is normalized to one, and the LUND PS predictions are shown by small crosses. (b) Each distribution is multiplied by the relative fraction $F(j)$ of two-jet, three-jet, and four-jet events, and the solid curves are fittings of negative binomial distributions (NBD).

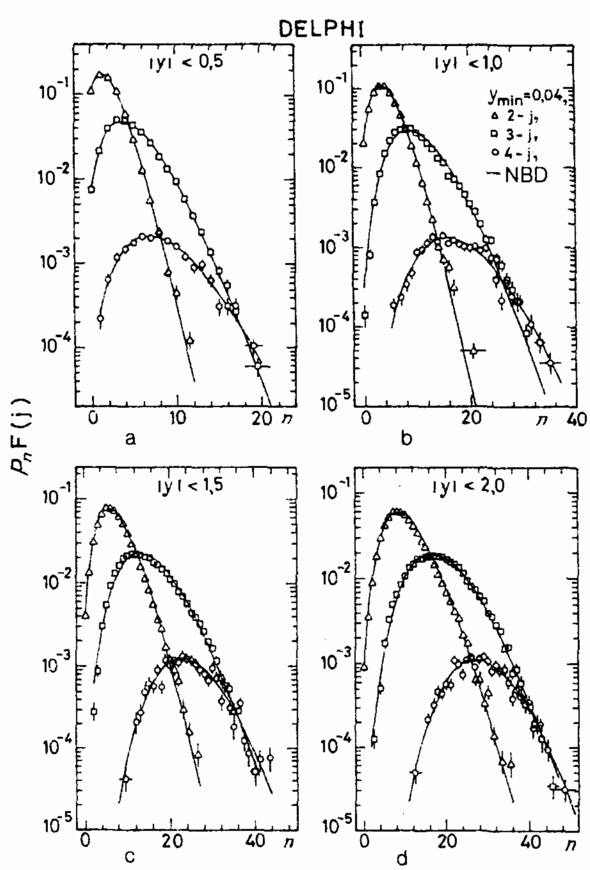

FIG. 20. The charged particle multiplicity distributions P_n in two-jet, three-jet, and four-jet events (for $Y_{min} = 0.04$) multiplied by the relative fractions $F(j)$ of two-jet, three-jet, and four-jet events in the different rapidity intervals: (a) $|y| < 0.5$, (b) $|y| < 1.0$, (c) $|y| < 1.5$, and (d) $|y| < 2.0$. The solid curves are fittings of negative binomial distributions (NBD).

pidity intervals, including those rapidity intervals where structure which is not described by the negative binomial distribution was observed in the multiplicity distributions for complete events (see Section 7). Figure 20, from which the origin of this structure also becomes obvious, illustrates this: a maximum at small multiplicities in a multiplicity distribution is basically connected with two-jet events, and the shoulder in the $n \approx 20$ region is caused by the contribution of multiple jet, basically three-jet events.

But what may one say about the clan parameters $\overline{N}_c$ and $\bar{n}_c$ for events with a fixed number of jets? It is evident from Table IIa that the values of the parameter $\overline{N}_c$ are close to the values of the average multiplicity $\langle n_{ch} \rangle$, and the parameter $\bar{n}_c$ is not much different from one (and is appreciably less than the values of $\bar{n}_c$ for the complete sample of events with different numbers of jets.[7)] In the OPAL experiment[35] it has also been shown that, for events with different numbers of jets but with small sphericity S (corresponding to a large contribution from two-jet events), the multiplicity distributions start to approach a Poisson distribution.

The closeness of the average number of particles in a clan $\bar{n}_c$ to one actually indicates the unsuitability of interpreting clans as groups of particles of common origin in a cascade mechanism of particle formation (if one emphasizes the word particle). Their interpretation at the parton level as decelerating gluon jets also seems very doubtful to me because the values of the parameter $\overline{N}_c$ are too large and are not much different from $\langle n_{ch} \rangle$.

9. MULTIPLICITY DISTRIBUTIONS IN INDIVIDUAL JETS

The multiplicity distributions of individual jets in events with a fixed number of jets have also been analyzed in the DELPHI experiment.[62] They have been obtained inclusively, i.e., the multiplicity of each of the jets was understood to be the multiplicity of an individual jet in an event with j jets (two, three, or four inputs for each two-, three-, or four-jet event). The multiplicity distributions for individual jets turned out to be narrower and shifted into the range of smaller multiplicities (smaller $\langle n_{ch} \rangle$; see Table IIb) as the number of jets was increased, and was parameterized fairly well by a negative binomial distribution. From the values of the parameters $\overline{N}_c$ and $\bar{n}_c$ that have been obtained (see Table IIb), one may draw the same conclusions as in the previous section.

10. THE MULTIPLICITIES OF QUARK AND GLUON JETS

The investigation of multiple jet and, in particular, of three-jet events in which one of the jets is due to the hard radiation of a gluon can, in principle, enable one to answer the question: are the multiplicities of particles in jets triggered by quarks and gluons different? Such a difference is predicted for isolated quark and gluon jets in first order quantum chromodynamics[67,68] (also see the review in Ref. 69):

$$\langle n \rangle_g / \langle n \rangle_q = 9/4, \tag{20}$$

and indicates the ratio of the color charges of the gluon and quark. With allowance for the higher orders in QCD[70] and for the number of flavors $N_f = 5$, the ratio (20) is transformed to

$$\langle n \rangle_g / \langle n \rangle_q = (9/4)(1 - 0{,}27\sqrt{\alpha_s} - 0{,}07\alpha_s). \tag{21}$$

For the measured value of the coupling constant $\alpha_s(M_z) = 0.112 \pm 0.0007$ (see Ref. 71, where references are given to all the experiments on the LEP), the expression in the right-hand parentheses reduces the ratio $\langle n\rangle_g/\langle n\rangle_q$ by 10% at $\sqrt{s} = 91$ GeV.

From the experimental point of view, the main problem in determining the ratio $\langle n\rangle_g/\langle n\rangle_q$ consists of the reliable separation of the quark and gluon jets in each event taken separately. The first attempt to measure this ratio in processes of e^+e^- annihilation into hadrons was undertaken by the HRS collaboration[72] and was based on an analysis of the 276 symmetric three-jet events (with angles of 120° between the jets. Their result

$$\langle n\rangle_g/\langle n\rangle_q = 1{,}29^{+0{,}21}_{-0{,}41} \pm 0{,}20 \tag{22}$$

seemed to indicate the same charged particle multiplicity in the quark and gluon jets rather than the validity of Eq. (21).

Two approaches were used in the DELPHI experiment[62] to measure the $\langle n\rangle_g/\langle n\rangle_q$ ratio. The first, inclusive one is based on comparing the average charged particle multiplicities and the mean square deviations in individual jets $(\langle n\rangle_i, D_i)$ in events with a fixed number of jets with the analogous values ($\langle n\rangle_W$ and D_W) for the entire event as a whole.

From the obvious relations

$$\langle n\rangle_W = \sum \langle n\rangle_i, \quad D_W^2 = \sum D_i^2 \tag{23}$$

and assuming that jets are formed independently and that their multiplicities are uncorrelated, it follows that

$$R = \left(\frac{D_i}{\langle n\rangle_i}\right)\left(\frac{D_W}{\langle n\rangle_W}\right)^{-1} = \sqrt{j}, \tag{24}$$

where j is the number of jets. As is evident from Table III, the relation (24) is fulfilled not only for two-jet[8)], but also for three-jet and four-jet events.

With these same assumptions, by using the definition $D^2 = \langle n^2\rangle - \langle n\rangle^2$, one can show[62] that, for three-jet events

$$\langle n\rangle_g/\langle n\rangle_q = (1+2R)/(1-R), \tag{25}$$

where

$$R^2 = \frac{1}{2}\left[\left(\frac{D_i}{\langle n\rangle_i}\right)^2 - 3\left(\frac{D_W}{\langle n\rangle_W}\right)^2\right], \tag{26}$$

and for four-jet events

$$\langle n\rangle_g/\langle n\rangle_q = (1+2R)/(1-2R), \tag{27}$$

where

$$R^2 = \frac{1}{4}\left[\left(\frac{D_i}{\langle n\rangle_i}\right)^2 - 4\left(\frac{D_W}{\langle n\rangle_W}\right)^2\right]. \tag{28}$$

As is evident from Table III, the values of $\langle n\rangle_g/\langle n\rangle_q$ are compatible with one within a 10% error for the three-jet events and within a 15% error for the four-jet events. Of course this result, which is interesting by itself alone, still cannot be directly compared with the predictions of QCD [Eq. (21)], since the energies of the quark and gluon jets may be different. A suitable upper limit for the correction coefficient was estimated,[62] ordering the jets according to energy and using the dependence of the average charged particle multiplicity on energy. It amounted to 1.4 ± 0.1.[9)]

In the second approach, the ratio $\langle n\rangle_g/\langle n\rangle_q$ in the DELPHI experiment[62] was determined in the same way as in the HRS experiment,[72] from an analysis of 451 symmetric three-jet events (that have been picked out by means of the algorithm in Ref. 63) with angles of $120° \pm 20°$ between the jets. The averaged charged particle multiplicity in a jet (assuming that the jets are identical and uncorrelated) amounted to

$$\langle n_{ch}\rangle_g = \langle n_{ch}\rangle_q = 7{,}54 \pm 0{,}15. \tag{29}$$

However, because of the impossibility of identifying quark and gluon jets, their actual multiplicities may be greatly different. To check for such a possibility, the jets were ordered according to multiplicity (the jet with the maximum multiplicity was considered as third, and the one with minimum multiplicity as first), and the ratio

$$Q = 2\langle n\rangle_3/(\langle n\rangle_1 + \langle n\rangle_2) = 1{,}68 \pm 0{,}06, \tag{30}$$

was calculated which, naturally, is greater than one because of ordering the jets according to multiplicity. Next, assuming a Poisson multiplicity distribution in a jet, the relation (30) was calculated for two assumptions: a) Eq. (29) is fulfilled, and b) $\langle n\rangle_g = 11$, $\langle n\rangle_q = 5.5$ (i.e., $\langle n\rangle_g/\langle n\rangle_q = 2$), starting from the average experimental value of multiplicity $\langle n_{ch}\rangle = 22$ for a complete event. It turned out that, in the first case, $Q = 1.57$, in good agreement with Eq. (30), but $Q = 2.10$ in the second case, i.e., it is significantly different from Eq. (30). If the values of $\langle n\rangle_g$ and $\langle n\rangle_q$ are not fixed as has been done above, but they are determined by fitting, assuming a Poisson multiplicity distribution, then

$$\langle n_{ch}\rangle_g/\langle n_{ch}\rangle_q = 1{,}21 \pm 0{,}21, \tag{31}$$

in agreement with the earlier result in Eq. (22) and at variance with the value that is predicted by Eq. (21).

One first succeeded in actually clearly identifying the quark and gluon jets in three-jet events (identified by means

TABLE III. Values of R [Eq. (24)] and of $\langle n\rangle_g/\langle n\rangle_q$ [Eqs. (25) and (27)] in two-jet, three-jet, and four-jet events for three values of the parameter Y_{min} (Ref. 62).

Number of jets	Parameter	$Y_{min} = 0{,}01$	$Y_{min} = 0{,}02$	$Y_{min} = 0{,}04$
2	R	1,40 ± 0,08	1,43 ± 0,08	1,44 ± 0,08
3	R	1,74 ± 0,10	1,72 ± 0,10	1,76 ± 0,10
	$\langle n\rangle_g/\langle n\rangle_q$	1,11 ± 0,11	1,00 ± 0,10	1,17 ± 0,11
4	R	1,91 ± 0,11	2,00 ± 0,11	2,17 ± 0,13
	$\langle n\rangle_g/\langle n\rangle_q$	1,00 ± 0,15	1,00 ± 0,15	1,41 ± 0,20

of the algorithm in Ref. 63) in the OPAL experiment[73] by tagging one or even both quark jets from the semilepton decays of the charmed and charming quarks. Since such tagging of quark jets could lead to a biased sample, a method for obtaining an unbiased sample of untagged events with the usual mixture of quarks or different flavors has been developed and used in the experiment. In this sample those three-jet events were picked out in which the difference between the energies of the most energetic jet and each of the remaining jets was more than 8 GeV, the angle in the plane of the event between the most energetic jet and each of the other two jets was 150° ± 10°, and the energies of each of these two last jets were from 20 GeV to 30 GeV. The most energetic jet in an event was considered to be a quark jet (which is valid in 94% of the cases). The procedure for separating the remaining quark and gluon jets is described in detail in Ref. 73 and is based on a comparison of the tagged and untagged samples.

The average charged particle multiplicities in quark and gluon jets for events with a resulting usual mixture of quarks of different flavors amounted to[73]

$$\langle n_{ch}\rangle_g = 7{,}4 \pm 0{,}2, \quad \langle n_{ch}\rangle_q = 7{,}2 \pm 0{,}2, \tag{32}$$

in complete agreement with the results of Eq. (29) from DELPHI,[62] and their ratio is

$$\langle n_{ch}\rangle_g/\langle n_{ch}\rangle_q = 1{,}03 \pm 0{,}03^{+0{,}15}_{-0}. \tag{33}$$

Analogous results have been obtained for the average multiplicities of all (and not only for charged) particles[73]:

$$\langle n\rangle_g = 12{,}9 \pm 0{,}3, \quad \langle n\rangle_q = 12{,}5 \pm 0{,}3, \tag{34}$$

$$\langle n\rangle_g/\langle n\rangle_q = 1{,}02 \pm 0{,}04^{+0{,}06}_{-0}. \tag{35}$$

It has also been shown in the experiment that these results are consistent with practically identical energies for the quark and gluon jets. The softer energy spectrum of the particles in a gluon jet relative to a quark jet is observed only in the core of a jet, whereas the averaged energies of the particles in both jets turned out to be close to each other.

Thus, the combined results from the three HRS,[72] DELPHI,[62] and OPAL[73] e^+e^- experiments leave no doubt that the average particle multiplicities in quark and gluon jets are identical.

11. CONCLUSION

Thus, even the first investigations of the regularities which characterize the multiplicity distributions of the particles formed in processes of e^+e^- annihilation into hadrons in the region of the Z^0 boson, in experiments on the LEP, as well as comparison of them with other e^+e^- data at lower energies and with analogous regularities in hadron collisions, enabled one to obtain a number of interesting results. One may formulate basic summaries of this research in the following manner.

The dependence of the average charged particle multiplicity $\langle n_{ch}\rangle$ in e^+e^- collisions on energy is in agreement with the dependences of Eqs. (11) and (12) that are predicted by perturbation QCD. Here one succeeds in describing the dependences of $\langle n_{ch}\rangle$ on $\sqrt{s}$ for the e^+e^- and $p^{\pm}p$ data over the broad energy ranges 7.45 GeV$\leqslant\sqrt{s}\leqslant$91 GeV and 4.5 GeV $\leqslant\sqrt{s}\leqslant$900 GeV, respectively, by one universal function by parameterizing the e^+e^- data as $\langle n_{ch}\rangle = f(\sqrt{s})$ and the $p^{\pm}p$ data as $\langle n_{ch}\rangle = n_0 + f(\sqrt{s}/k)$. And the values of the parameters n_0 and k obtained from a simultaneous fit to the e^+e^- and $p^{\pm}p$ data find a natural physical interpretation. This allows one to hypothesize similarity of the basic regularities in the development and subsequent hadronization of quark-parton showers in processes of hadron formation in e^+e^- collisions and in hadron reactions. And this allows one to hope for further progress in attempts to modify LUND PS or HERWIG type models, which describe e^+e^- data well, for the tasks of describing the processes of particle formation in hadron reactions.

From the behavior of the normalized moments C_k and a comparison of charged particle multiplicity distributions in the form of dependences of $\langle n\rangle P_n$ on $n/\langle n\rangle$ at different energies, the important conclusion is drawn that KNO scaling is valid for the e^+e^- data in the $\sqrt{s} = 20$ GeV to 91 GeV energy range. Generation of events according to the LUND PS model indicates the possible fulfillment of KNO scaling at the even significantly higher energies $\sqrt{s}\geqslant 1$ TeV. In this regard, the observation of violation of KNO scaling in $\bar{p}p$ collisions at $\sqrt{s} = 200$ GeV, 540 GeV, and 900 GeV in the UA5 experiment on the CERN S$\bar{p}$pS collider requires further interpretation and experimental confirmation.

The charged particle multiplicity distributions in e^+e^- collisions at $\sqrt{s} = 91$ GeV become broader and are shifted towards significantly larger multiplicities for events with a large number of jets. The structure detected in a multiplicity distribution with a maximum at small n_{ch} values and with a shoulder at the larger $n_{ch} \approx 20$, which is especially noticeable at "middle" rapidity values, is explained by the superposition of two-jet and multiple jet (mainly three-jet) events. This is indirectly an experimental confirmation of the suggested explanation of a similar structure in the multiplicity distribution in $\bar{p}p$ interactions at $\sqrt{s} = 900$ GeV by the contribution of multipomeron showers.

The average particle multiplicities in quark and gluon jets are not different, whereas, because of the different color charges of the quark and gluon, QCD predicts $\langle n\rangle_g/\langle n\rangle_q \approx 2$ at the LEP energy. At the same time, it remains unclear whether this experimental result contradicts the QCD predictions obtained for the multiplicities of partons formed by virtual gg and $q\bar{q}$ pairs. Here one must also keep in mind the conceptual uncertainty in defining the concept of a quark or gluon jet if one considers the process of e^+e^- annihilation into hadrons from the point of view of the development of a parton shower.

The charged particle multiplicity distributions in processes of e^+e^- annihilation into hadrons is parameterized well by a negative binomial distributions for events with a fixed number of jets; this also includes selected rapidity intervals, but with parameters which depend strongly on the number of jets. In combining events with different numbers of jets, a negative binomial distribution at best describes the data only qualitatively and, of course, it does not reproduce the structure in a multiplicity distribution.

The concept of "clans", connected with the use of the negative binomial distribution for describing data, has been analyzed in detail. Just as in hadron collisions, approximate independence from energy of the average number of clans $\overline{N}_c$ and an increase with energy of the average number of particles in a clan $\bar{n}_c$ in a specified frequency interval have been established. However, an analysis of events with a fixed number of jets showed that the parameter $\bar{N}_c$ does not differ greatly from $\langle n_{ch}\rangle$ and, consequently, the parameter $\bar{n}_c$ is fairly close to one. Also no significant dependence of the parameter $\bar{n}_c$ on the number of jets has been detected. This places in doubt the physical interpretation of clans as a group of particles of common origin in a cascade mechanism of particle formation, also including the parton level, as decelerating gluon jets.

The entire set of experimental data presented in this review is described well by the LUND PS model.

In conclusion, I am pleased to thank my colleagues on the DELPHI experiment, and especially V. A. Uvarov, together with whom the results which comprise an appreciable part of this review were obtained. I also thank T. S. Kulikova for help in drawing up this paper.

1) From lecture notes at the First Moscow State University International School on Experimental High Energy Physics. Sochi, October 14–22, 1991.

2) Or its modification for non-asymptotic energies; the so-called KNO-G scaling introduced by Golokhvastov.[6]

3) That is denoted thus as opposed to the experimentally measured average multiplicity $\langle n\rangle$.

4) See Refs. 19, 20, and 21 for the relation of the negative binomial distribution to the theory of branching processes.

5) Which are independent within the error limits from the specific form of the function $f(\sqrt{s})$.[47]

6) Although, in the UA5 experiment, the structure in the multiplicity distribution becomes more noticeable at large rapidity intervals.

7) Let us recall that, at $n = 1$, a negative binomial distribution is converted into a Poisson distribution.

8) It also has been shown in the OPAL experiment[35] that the value of R for events with a small value of sphericity S (i.e., with a dominant contribution of two-jet events) also amounts to $\sqrt{2}$.

9) One must emphasize that this is specifically an upper limit. The ordering of jets according to energy leads to the situation that this coefficient must be greater than one, even if $\langle n\rangle_g/\langle n\rangle_q$.

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