Radiation from and energy loss by charged particles in moving media

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(Submitted 26 November 1991)
Usp. Fiz. Nauk **162**, 177–190 (February 1992)

1. Interest in moving media has increased in the last decade due to the expansion of research on various methods of accelerating charged particles. Tamm[1] and Veksler[2] were the first to focus attention on the possibility of accelerating a charge in streams of matter moving faster than light. They indicated that charges could also be accelerated in dense beams of relativistic electrons. With the appearance of high-current beams[3,4] these investigations have been developed into an entire new field, collective methods of particle acceleration.

Moving media are mainly understood to be media which are moving as a whole with a constant velocity **u**. Electromagnetic phenomena in these media are described as usual by Maxwell's equations[5–7]

$$\operatorname{curl} \mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t} + \frac{4\pi}{c}\mathbf{j}, \quad \operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial B}{\partial t}, \tag{1}$$

$$\operatorname{div} \mathbf{D} = 4\pi\rho, \quad \operatorname{div} \mathbf{B} = 0;$$

here ρ and **j** are the density of external charges and the density of currents and c is the speed of light in vacuum. For moving media, the main issue is the writing of the physical equations which link the electric displacement **D** and magnetic inductions **B** with fields **E** and **H**. These equations were obtained by Minkowski (see Ref. 8 or Ref. 5) for homogeneous isotropic and steady-state media

$$\mathbf{D} + \left[\frac{\mathbf{u}}{c}\mathbf{H}\right] = \varepsilon\left(\mathbf{E} + \left[\frac{\mathbf{u}}{c}\mathbf{B}\right]\right), \quad \mathbf{B} + \left[\mathbf{E}\frac{\mathbf{u}}{c}\right] = \mu\left(\mathbf{H} + \left[\mathbf{D}\frac{\mathbf{u}}{c}\right]\right), \tag{2}$$

where ε and μ are the permittivity and magnetic permeability of the medium measured in its rest system, and **u** is the constant velocity of the medium. The physical formulas in Eq. (2) are obtained from analogous equations

$$\mathbf{D}' = \varepsilon\mathbf{E}', \quad \mathbf{B}' = \mu\mathbf{H}', \tag{3}$$

written in the rest system of the medium (all quantities in it are marked with primes) if one performs a Lorentz transformation on the field and induction (see, for example, Ref. 9) into the laboratory system of coordinates. In media with dispersion, the formulas in Eq. (3) are written for a Fourier component in the expansion of all quantities in terms of two-dimensional electromagnetic waves of frequency ω' and with a wave vector $\mathbf{k}'$. Then ε and μ are functions of ω' and $\mathbf{k}'$, which are linked with ω and **k** by the following equations[10] in the laboratory system, where the medium moves with a velocity **u**

$$\omega' = \gamma(\omega - \mathbf{k}\mathbf{u}), \quad \mathbf{k}' = \mathbf{k} + \gamma\frac{\mathbf{u}}{u^2}\left[\left(1 - \frac{1}{\gamma^2}\right)(\mathbf{u}\mathbf{k}) - \omega\frac{\mathbf{u}^2}{c^2}\right], \tag{4}$$

where $\gamma = (1 - \boldsymbol{\beta}^2)^{-1/2}$, and $\mathbf{u} = c\boldsymbol{\beta}$. For cold electron plasma or a beam of relativistic electrons

$$\varepsilon = \left(1 - \frac{\omega_p'^2}{\omega'^2}\right), \quad \mu = 1, \quad \omega_p'^2 = \frac{4\pi e^2}{m'}N' = \omega_p^2 = \frac{4\pi e^2}{m}N, \tag{5}$$

because $m = \gamma m' \simeq \gamma m_0$ and $N = \gamma N'$, where e, m', and N' are the charge, mass, and electron concentration. For quantitative estimates $N \approx 1.4\cdot 10^8 j$ and $\omega_p^2 \approx 4.4\cdot 10^{18} j/\gamma$, where j is the current density in the beam in amperes per square centimeter.

Thus, using Eqs. (1) and (2) and given functions ε and μ of the arguments specified by Eq. (4) one can in principle solve any electrodynamic problems in isotropic and homogeneous moving media. However, the solution of the equations is complex because of the vector nature of the equations. For simplicity, and in media at rest, one can introduce the potentials[5,6,11] φ and **A**

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \operatorname{grad}\varphi, \quad \mathbf{B} = \operatorname{curl}\mathbf{A}, \tag{6}$$

because the velocity of the medium does not enter Maxwell's equations in Eq. (1). The equations for the potentials in four-dimensional form are[11–13]

$$\begin{gathered} \hat{\mathcal{L}} A_l = -\frac{4\pi}{c}\mu S_{lm} j_m, \\ \hat{\mathcal{L}} = \frac{\partial^2}{\partial x_k^2} - \varkappa\left(u_k \frac{\partial}{\partial x_k}\right)^2, \quad S_{lm} = \delta_{lm} + \frac{\varkappa}{1+\varkappa}u_l u_m, \end{gathered} \tag{7}$$

where the following four-dimensional vectors are introduced

$$\begin{gathered} x_1 = x, \ x_2 = y, \ x_3 = z, \ x_4 = ict; \ A_{1,2,3} = A_{x,y,z}, \ A_4 = i\varphi; \\ j_{1,2,3} = j_{x,y,z}, \ j_4 = ic\rho; \\ u_{1,2,3} = \frac{1}{c}\gamma u_{x,y,z}, \ u_4 = i\gamma, \ \gamma = (1-\beta^2)^{-1/2}, \ \mathbf{u} = c\boldsymbol{\beta}, \end{gathered} \tag{8}$$

$\varkappa = \varepsilon\mu - 1$, and repeating indices indicate summation from 1 to 4; δ_{lm} is Kronecker's symbol. As a result, finding the potentials from Eq. (7), calculating the vectors **E** and **B** with them in Eq. (6), and using the physical formulas in Eq. (2) in the form

 0038-5670/92/020143-08$03.00

$$\mathbf{D} = \varepsilon\mathbf{E} + \frac{1}{\mu}\varkappa\gamma^2[\beta^2\mathbf{E} - \boldsymbol{\beta}(\boldsymbol{\beta}\mathbf{E}) + [\boldsymbol{\beta}\mathbf{B}]],$$
$$\mathbf{H} = \frac{1}{\mu}\mathbf{B} + \frac{1}{\mu}\varkappa\gamma^2[\boldsymbol{\beta}(\boldsymbol{\beta}\mathbf{B}) - \beta^2\mathbf{B} + [\boldsymbol{\beta}\mathbf{E}]], \qquad (9)$$

one can completely define the electromagnetic fields of any sources.

2. The formulas in Eq. (7) for the potentials A_i make it possible to write their solution using the Green's function $G_0(\mathbf{r}-\mathbf{r}', t-t')$ of an instantaneous point source in the moving medium. Calculations (see Refs. 12 and 13) done as in Refs. 14–16 yield the following expressions

$$A_i(\mathbf{r}, t) = \frac{1}{(2\pi)^4}\int d\mathbf{r}' \int_{-\infty}^{t} dt' G_{im}(\mathbf{r}-\mathbf{r}', t-t') j_m(\mathbf{r}', t'), \qquad (10)$$

where the tensor Green's function G_{im} is linked with G_0 by the equation

$$G_{im} = \frac{1}{c} S_{im} G_0; \qquad (11)$$

here S_{im} is as presented in Eq. (7), and G_0 is the solution of the equation

$$\hat{\mathcal{L}}(\mathbf{r}, t) G_0(\mathbf{r}-\mathbf{r}', t-t') = 4\pi\mu(2\pi)^4 \delta(\mathbf{r}-\mathbf{r}')\delta(t-t'), \qquad (12)$$

which satisfies the radiation condition

$$G_0(\mathbf{r}-\mathbf{r}', t-t') = 0 \quad \text{for} \quad t < t'. \qquad (13)$$

Integration of Eq. (12) by expansion into Fourier integrals yields the following two equivalent expressions[8,9] for the Green's function in a medium without dispersion moving along the z axis ($\mathbf{u} = u\mathbf{e}_z$)

$$G_0(\mathbf{R}, \tau) = \frac{16\pi^4\mu}{\tilde{R}} \delta\left(\tau - \frac{\varepsilon\mu - \beta^2}{(1-\beta^2)(\varepsilon\mu)^{1/2}} \frac{\tilde{R}}{c}\right)$$
$$= \frac{16\pi^4\mu}{R_e}\left(\frac{1 + \operatorname{sgn}\tau_1}{2}\delta(\tau - \tau_1) + \frac{1 + \operatorname{sgn}\tau_2}{2}\delta(\tau - \tau_2)\right), \qquad (14)$$

where

$$\mathbf{R} = \mathbf{r} - \mathbf{r}' = \tilde{\boldsymbol{\rho}} + \tilde{z}\mathbf{e}_z, \quad \tilde{\boldsymbol{\rho}} = \boldsymbol{\rho} - \boldsymbol{\rho}',$$

$$\tilde{z} = z - z', \quad \tau = t - t',$$

$$\tilde{R} = \left[\frac{\varepsilon\mu(1-\beta^2)}{\varepsilon\mu - \beta^2}\tilde{\rho}^2 + (\tilde{z} - \eta u\tau)^2\right]^{1/2},$$

$$\tilde{\rho}^2 = \tilde{x}^2 + \tilde{y}^2, \quad \beta = \frac{u}{c},$$

$$\tilde{x} = (x - x'), \quad \tilde{y} = (y - y'), \quad \eta = \frac{\varepsilon\mu - 1}{\varepsilon\mu - \beta^2},$$

$$R_e = \left(\tilde{z}^2 + \frac{1 - \varepsilon\mu\beta^2}{1 - \beta^2}\tilde{\rho}^2\right)^{1/2},$$

$$c\tau_1 = \frac{(1-\beta^2)(\varepsilon\mu)^{1/2} R_e - (\varepsilon\mu - 1)\beta\tilde{z}}{1 - \varepsilon\mu\beta^2},$$

$$c\tau_2 = \frac{(1-\beta^2)(\varepsilon\mu)^{1/2} R_e + (\varepsilon\mu - 1)\beta\tilde{z}}{\varepsilon\mu\beta^2 - 1},$$

$$\operatorname{sgn} x = \frac{x}{|x|} = \begin{cases} +1 & \text{for} \quad x > 0, \\ -1 & \text{for} \quad x < 0. \end{cases} \qquad (15)$$

Since the Green's function in Eq. (14) is nonzero only at τ, the second term with $\delta(\tau - \tau_2)$ contributes only when the medium is moving faster than light, $u > c/(\varepsilon\mu)^{1/2}$. In a medium at rest ($u = 0$) or in vacuum ($\varepsilon = \mu = 1$) only the term with $\delta(\tau - \tau_1)$ remains in Eq. (14), and the Green's function G_0 is nonzero on a spherical surface which is expanding at $\tau > 0$ in all directions from the point $R = 0$, where the source of disturbance is located, with a speed $c/(\varepsilon\mu)^{1/2}$.

Now, if the medium in which the instantaneous point source is located begins to move as a whole with a constant velocity $\mathbf{u}$ in the direction of the z axis, due to Fresnel entrainment of light by the moving medium, the spherical surface for the Green's function in the medium at rest will be deformed and will move as a whole in the direction of motion of the medium. It is clear from the first expression for G_0 in Eq. (14) that the Green's function in a moving medium without dispersion will be nonzero on a surface representing an ellipsoid of revolution with an axis of symmetry along the velocity of the medium. The equation of this ellipsoid has the form

$$\frac{\tilde{\rho}^2}{a^2} + \frac{(\tilde{z} - \tilde{z}_0)^2}{b^2} = 1, \qquad (16)$$

where

$$a = c\tau\left(\frac{1 - \beta^2}{\varepsilon\mu - \beta^2}\right)^{1/2}, \quad b = c\tau\frac{(1-\beta^2)(\varepsilon\mu)^{1/2}}{\varepsilon\mu - \beta^2},$$

$$\tau = t - t', \quad \beta = \frac{u}{c}, \qquad (17)$$

$$\tilde{\rho}^2 = \tilde{x}^2 + \tilde{y}^2, \quad \tilde{x} = x - x',$$

$$\tilde{y} = y - y', \quad \tilde{z} = z - z', \quad \tilde{z}_0 = \eta u\tau.$$

The center of the ellipsoid ($\tilde{\rho} = 0, \tilde{z} = \tilde{z}_0$) moves in the direction of the motion of the medium with a speed

$$u_0 = \frac{d\tilde{z}_0}{dt} = \frac{d\tilde{z}_0}{d\tau} = \eta u, \quad \eta = \frac{\varepsilon\mu - 1}{\varepsilon\mu - \beta^2}. \qquad (18)$$

A medium moving at this speed entrains any electromagnetic disturbance in the direction of its movement. The coefficient of entrainment η for $\beta \ll 1$ coincides with the Fresnel coefficient of entrainment,[9] and at $\beta \approx 1$, $\eta \approx 1$, that is, in a medium without dispersion moving at a relativistic speed, any disturbance is completely entrained.

The phenomenon of entrainment appears differently depending on the speed of the medium. When the medium is moving slower than light, $u < c/(\varepsilon\mu)^{1/2}$, the instantaneous source of the disturbance is always within the expanding ellipsoid (Eq. (16)), because in this case $\tilde{z}_0 < b$, that is, the velocity of drift of the center of the ellipsoid in the direction of motion of the medium is less than the velocity of expansion of the ellipsoid in the opposite direction, in the $-z$ direction. Then a disturbance from the coordinate origin ($R = 0$), where the instantaneous point source is located, always reaches any point of observation P. It is only due to entrainment that the velocity of propagation of the disturbance in the direction of motion of the medium is greater than the velocity of this disturbance against the direction of motion of the medium: the moving medium carries all disturbances "downstream." At $u = c/(\varepsilon\mu)^{1/2}$ (medium moving at the speed of light) the expanding ellipsoids at any time τ are tangent to the plane $\tilde{z} = 0$ at the coordinate origin ($\tilde{\rho} = 0, \tilde{z} = 0$), where the instantaneous point source is located. In this case the entrainment is such that $\tilde{z}_0 = b$, that is, the drift velocity of the center of the ellipsoid "downstream" in the medium is exactly equal to the velocity of propagation of the disturbance against the motion of the medium. Then all disturbances from the coordinate origin may reach only those observation points P that are in the half-plane $\tilde{z} > 0$ ("downstream" in the medium). In the region $\tilde{z} < 0$ the signal from the instantaneous point source located at the coordinate origin ($\tilde{\rho} = 0, \tilde{z} = 0$) is always identically equal to zero. Finally, when the medium is moving faster than light ($u > c/(\varepsilon\mu)^{1/2}$) the entrainment by the moving medium is so strong that the instantaneous point source at the coordinate origin is always outside ($\tilde{z}_0 > b$) all expanding ellipsoids, on the surface of which the disturbance is nonzero. Then outside the conical surface

$$\tilde{z} = \tilde{\rho}\left(\frac{\varepsilon\mu\beta^2 - 1}{1 - \beta^2}\right)^{1/2} = (\tilde{x}^2 + \tilde{y}^2)^{1/2}\left(\frac{\varepsilon\mu\beta^2 - 1}{1 - \beta^2}\right)^{1/2},$$

$$\tilde{z} > 0, \quad \varepsilon\mu\beta^2 > 1, \tag{19}$$

the field from the instantaneous point source is identically equal to zero at any time τ. If the point of observation P is within the cone (Eq. (19)) then the disturbance passes through it twice: first by the leading edge of the expanding ellipsoid, and afterwards by its trailing edge. According to Eq. (15), the time interval between these two signals is equal to

$$\Delta\tau = \tau_2 - \tau_1 = 2(1 - \beta^2)(\varepsilon\mu)^{1/2} R_e / c(\varepsilon\mu\beta^2 - 1). \tag{20}$$

On the cone itself (Eq. (19)) the signal passes through the point of observation only once. If one knows the Green's function, one can calculate the fields of any sources placed in the moving medium (see Refs. 12 and 13).

3. The fields of different types of sources in a moving medium (see Refs. 12 and 13).

3.1. A point and extended charge at rest

Let there be a point charge q at the coordinate origin. Then the charge density and the current density in Eqs. (7), (8), and (10) have the form

$$\rho(\mathbf{r}', t') = q\delta(\mathbf{r}') = q\delta(x')\delta(y')\delta(z'), \quad \mathbf{v}(\mathbf{r}', t') = 0, \quad \mathbf{j}(\mathbf{r}', t') = 0. \tag{21}$$

Substituting these expressions into Eqs. (14) and (15), and then into Eqs. (10) and (11), we obtain

$$\varphi(\mathbf{r}) = q\frac{1 - \varepsilon\mu\beta^2}{\varepsilon(1 - \beta^2)}\frac{f(\mathbf{r}, \beta)}{R_0}, \quad \mathbf{A}(\mathbf{r}) = -\frac{\varepsilon\mu - 1}{1 - \varepsilon\mu\beta^2}\frac{\mathbf{u}}{c}\varphi(\mathbf{r}), \tag{22}$$

where

$$R_0 = \left(z^2 + \frac{1 - \varepsilon\mu\beta^2}{1 - \beta^2}\rho^2\right)^{1/2}, \quad \rho^2 = x^2 + y^2, \quad \mathbf{r} = \boldsymbol{\rho} + z\mathbf{e}_z,$$

$$f(\mathbf{r}, \beta) = \begin{cases} 1 & \text{for } \varepsilon\mu\beta^2 < 1 \text{ and any } z \text{ and } \rho, \\ 2 & \text{for } \varepsilon\mu\beta^2 > 1 \text{ and } z > b_0(x^2 + y^2)^{1/2}, \\ 0 & \text{for } \varepsilon\mu\beta^2 > 1 \text{ and } z < b_0(x^2 + y^2)^{1/2}, \end{cases} \tag{23}$$

$$b_0 = \left(\frac{\varepsilon\mu\beta^2 - 1}{1 - \beta^2}\right)^{1/2}$$

As before, the z axis is directed along the velocity of the medium $\mathbf{u}$, and the two-dimensional vector $\boldsymbol{\rho}$ lies in the x,y plane perpendicular to $\mathbf{u}$. The potentials φ and $\mathbf{A}$ of the charge at rest do not depend on time. When the medium is moving slower than light ($\varepsilon\mu\beta^2 < 1$), the equipotential surfaces on which the potentials are constant are a set of ellipsoids of revolution with the axis along the velocity $\mathbf{u}$ of the medium

$$\frac{z^2}{l_z^2} + \frac{\rho^2}{l_\rho^2} = 1, \quad \frac{l_z}{l_\rho} = \left(\frac{1 - \varepsilon\mu\beta^2}{1 - \beta^2}\right)^{1/2} = \left(1 - \frac{\varepsilon\mu - 1}{1 - \beta^2}\beta^2\right)^{1/2}. \tag{24}$$

The ratio of semiaxes l_z/l_ρ is such that these ellipsoids are "flattened" in the direction of motion of the medium ($l_z < l_\rho$). When the medium is moving faster than light ($\varepsilon\mu\beta^2 > 1$) the equipotential surfaces are hyperboloids of revolution with the axis along the velocity $\mathbf{u}$

$$\frac{z^2}{m_z^2} - \frac{\rho^2}{m_\rho^2} = 1, \quad \frac{m_z}{m_\rho} = \left(\frac{\varepsilon\mu\beta^2 - 1}{1 - \beta^2}\right)^{1/2}. \tag{25}$$

The envelope of this set of hyperboloids is a conical surface given by Eq. (19). The potentials are nonzero only within this surface at $z > 0$. Outside of it, the potentials and fields are identically equal to zero.

Knowing the potentials in Eq. (22) one can use Eqs. (6) and (9) to determine the fields and inductions (see the equations in Refs. 12 and 13). It turns out that the vector of the electric field $\mathbf{E}$ lies in the (ρ, z) plane, and is perpendicular to the surfaces of Eqs. (24) or (25). When the medium is moving slower than light ($\varepsilon\mu\beta^2 < 1$) the vector $\mathbf{E}$ is pointed away from the coordinate origin, that is, it forms an acute angle with the radius vector $\mathbf{r}$ drawn from the coordinate origin. When the medium is moving faster than light, the vector $\mathbf{E}$ is pointed toward the coordinate origin, that is, it forms an obtuse angle with vector $\mathbf{r}$. Due to the equation div $\mathbf{D} = 4\pi\rho$, the vector $\mathbf{D}$ is always pointed along radius $\mathbf{r}$. Only for $\varepsilon\mu\beta^2 < 1$ it is parallel to $\mathbf{r}$, and for $\varepsilon\mu\beta^2 > 1$ it is antiparallel to $\mathbf{r}$. The magnetic induction $\mathbf{B}$ is proportional to the velocity of the medium and to the electric field, and the vector $\mathbf{B}$ *is perpendicular to the vectors* $\mathbf{u}$ *and* $\mathbf{E}$. *The force lines* of magnetic induction, to which the vector $\mathbf{B}$ is tangent, are circles whose centers lie on the z axis, and the planes are perpendicular to this axis. The magnetic field of a charge at rest in the moving medium is always exactly equal to zero.

When the medium is moving faster than light the potentials and fields of the point charge (see Eq. (22)) have a discontinuity on a surface specified by Eq. (19). In order to track the transition through this special surface more accurately, let us examine the potentials and the fields of an infinitely thin charged segment of length l with a total charge q situated along the velocity of the medium. The details of the calculations and the formulas for the potentials and fields are given in Ref. 13. Here we describe their features qualitatively. First, as in the case of a point charge at rest, the magnetic field is identically equal to zero. Second, the entire space is divided into three regions by conical surfaces (for $\varepsilon\mu\beta^2 > 1$)

$$z^2 = b_0^2(x^2 + y^2), \quad (z - l)^2 = b_0^2(x^2 + y^2), \quad b_0^2 = \frac{\varepsilon\mu\beta^2 - 1}{1 - \beta^2}, \tag{26}$$

whose apexes are at the beginning $(z = 0, \rho = 0)$ and end $(z = l, \rho = 0)$ of the segment. Then the potentials become continuous functions, of the coordinates and the fields and inductions, as before, have a discontinuity on the surfaces specified by Eq. (26). The fields are identically equal to zero in the first region $z < b_0\rho = b_0(x^2 + y^2)^{1/2}$. In the remaining regions, $b_0\rho \leqslant z \leqslant b_0\rho + l$ (second region) and $z \geqslant b_0\rho + l$ (third region), they are nonzero. In the second region, the vector $\mathbf{E}$ is perpendicular to the radius vector $\mathbf{r}$ and is pointed toward the axis of motion of the medium (the z axis). In the third region the vector $\mathbf{E}$ is pointed toward the coordinate origin. As $l \to 0$, the two conical surfaces (Eq. (26)) contract into one $z = b_0\rho$ and the field on it goes to infinity. However, Gauss's theorem is always satisfied for the charged segment, because the infinite contribution from the fields in the second region always cancelled by an analogous infinite contribution from fields in the third region. Heaviside[17] solved a similar inverse problem for the fields of a charged segment moving with a constant speed greater than light in a medium at rest. Using analogous methods one can calculate the fields of point electric and magnetic dipoles and fields far from an arbitrary set of charges (see Refs. 12 and 13).

3.2. Fields of a charged particle in uniform motion

Let there be a point particle with charge q moving in a medium moving with a constant velocity $\mathbf{v}$. Then the charge density ρ and current density $\mathbf{j}$ in Eqs. (7), (8), and (10) acquire the form

$$\rho(\mathbf{r}', t') = q\delta(\mathbf{r}' - \mathbf{v}t'), \quad \mathbf{j}(\mathbf{r}', t') = q\mathbf{v}\delta(\mathbf{r}' - \mathbf{v}t'). \tag{27}$$

Substituting these expressions and the Green's function from Eq. (14) into Eqs. (11) and (10) we obtain

$$\varphi(\mathbf{r}, t) = \left(\frac{\mu}{\varepsilon}\right)^{1/2} q\left[1 - \frac{\varkappa\gamma^2}{1+\varkappa}\left(1 - \frac{\mathbf{uv}}{c^2}\right)\right]\frac{f(\mathbf{r}, t, \boldsymbol{\beta})}{\widetilde{R}_1},$$

$$\mathbf{A}(\mathbf{r}, t) = \frac{\dfrac{\mathbf{v}}{c} - \dfrac{\varkappa\gamma^2}{1+\varkappa}\dfrac{\mathbf{u}}{c}\left(1 - \dfrac{\mathbf{uv}}{c^2}\right)}{1 - \dfrac{\varkappa\gamma^2}{1+\varkappa}\left(1 - \dfrac{\mathbf{uv}}{c^2}\right)}\varphi(\mathbf{r}, t), \tag{28}$$

where

$$\widetilde{R}_1 = \left[(\boldsymbol{\beta}_1\mathbf{r}_1')^2 + \frac{1-\beta^2}{\varepsilon\mu - \beta^2}(\mathbf{r}_1')^2\left(1 - \frac{\varepsilon\mu - \beta^2}{1-\beta^2}\boldsymbol{\beta}_1^2\right)\right]^{1/2}, \quad \mathbf{u} = c\boldsymbol{\beta},$$

$$\mathbf{r} = x\mathbf{e}_x + y\mathbf{e}_y + z\mathbf{e}_z, \quad \mathbf{r}' = \mathbf{r} - \mathbf{v}t = x'\mathbf{e}_x + y'\mathbf{e}_y + z'\mathbf{e}_z,$$

$$x' = x - v_x t, \quad y' = y - v_y t, \quad z' = z - v_z t, \quad \beta_{x,y,z} = \frac{v_{x,y,z}}{c}, \tag{29}$$

$$\boldsymbol{\beta}_1 = \beta_x\mathbf{e}_x + \beta_y\mathbf{e}_y + \beta_{\text{rel}}\left[\frac{\varepsilon\mu - \beta^2}{\varepsilon\mu(1-\beta^2)}\right]^{1/2}\mathbf{e}_z, \quad \beta_{\text{rel}} = \beta_z - \eta\beta,$$

$$\mathbf{r}_1' = x'\mathbf{e}_x + y'\mathbf{e}_y + z'\left[\frac{\varepsilon\mu - \beta^2}{\varepsilon\mu(1-\beta^2)}\right]^{1/2}$$

$$\varkappa = \varepsilon\mu - 1,$$

$$\mathbf{e}_z, \quad \gamma = (1-\beta^2)^{-1/2}, \quad \eta = \frac{\varepsilon\mu - 1}{\varepsilon\mu - \beta^2},$$

and the function

$$f(\mathbf{r}, t, \boldsymbol{\beta}) = \begin{cases} 0 & \text{for} \quad t - t_{1,2}' < 0, \\ 2 & \text{for} \quad t - t_{1,2}' > 0, \\ 1, & \text{if} \quad t - t_{1,2}' \text{ have opposite signs} \end{cases}$$

here

$$c(t - t_{1,2}') = \frac{(\varepsilon\mu - \beta^2)\gamma^2}{1 - (\varepsilon\mu - \beta^2)\gamma^2\boldsymbol{\beta}_1^2}\left[(\mathbf{r}_1'\boldsymbol{\beta}_1) \mp \widetilde{R}_1\right], \tag{30}$$

where the upper sign before $\widetilde{R}_1$ refers to t_1', and the lower sign refers to t_2'. The formulas in Eq. (28) give potentials at the point of observation $\mathbf{r}$ and at time t. The delay of the signal is taken into account in these formulas by using the function $f(\mathbf{r}, t, \boldsymbol{\beta})$: according to Eq. (29), the inequalities $t - t_{1,2}' > 0$ define space-time regions in which the field is nonzero. The reverse inequalities $t - t'_{1,2} < 0$ define regions where the fields are identically equal to zero at any time t. Equations (28)–(30) for a charge at rest ($\mathbf{v} = 0$) in a moving medium reduce to the formulas of the previous section 3.1 with all the features which have already been studied. If, on the other hand, the charge moves in a medium at rest ($\mathbf{u} = 0$), then we obtain a picture (see Ref. 16, Vol. 2, Chapter 5) which is analogous to the picture of the fields of a charge at rest in a moving medium (inverse problem), so we can switch from one problem, a charge moving in a medium at rest, to another problem, a charge at rest in a moving medium, using the ordinary Lorentz transformation. Indeed, for a medium at rest ($\mathbf{u} = c\boldsymbol{\beta} = 0$) for $v < c/(\varepsilon\mu)^{1/2}$ the equipotential surfaces are ellipsoids of revolution with their axis directed along the velocity of the charge. The center of these ellipsoids is at the location of the charge at observation time t. As $v \to c/(\varepsilon\mu)^{1/2}$ these ellipsoids "flatten" in the direction of motion of the charge in full accordance with the Lorentz length contraction. In the transition of the charge speed v through light speed in the medium, $c/(\varepsilon\mu)^{1/2}$, the field rearranges itself so that for $v > c/(\varepsilon\mu)^{1/2}$ it is identically equal to zero in front of the moving charge, where $\mathbf{r}'\mathbf{v} > 0$. Behind this charge, which is moving faster than light, the field is nonzero only within the conical surface with an apex at the location of the charge at time t and with an aperture of half-angle φ_0, such that $\sin\varphi_0 = c/v(\varepsilon\mu)^{1/2} = 1/\beta_v(\varepsilon\mu)^{1/2}$. The normal to this surface forms an angle $\theta_0 = (\pi/2) - \varphi_0$ with the velocity of the particle $\mathbf{v}$. For this angle

$$\cos\theta_0 = \frac{c}{v(\varepsilon\mu)^{1/2}}. \tag{31}$$

Cherenkov radiation is generated at this angle when the charged particle is moving uniformly faster than light.[18] The equipotential surfaces within this cone (potentials φ and $\mathbf{A}$ are constant on them) at time t are a set of hyperboloids of revolution with their axis along the velocity of the charge $\mathbf{v}$ (z' axis)

$$z'^2 - \left(\varepsilon\mu\frac{v^2}{c^2} - 1\right)(x'^2 + y'^2) = \text{const}, \quad z' = z - v_z t < 0, \tag{32}$$
$$x' = x - v_x t, \quad y' = y - v_y t.$$

Now let the medium at rest begin to move with a constant velocity $\mathbf{u}$ in the direction of the z axis. Then the field of the charged particle begins to be entrained by the moving medium. This entrainment occurs in the following manner (for the details of the calculations see Ref. 13). The centers of the ellipsoids of revolution (in the case of motion slower than light) and of the hyperboloids of revolution (in the case of motion faster than light) are found as before at the location of the charge at time t. The axis of symmetry is directed along the vector

$$\mathbf{v}_1 = \mathbf{v} - \mathbf{u}\frac{(\varepsilon\mu - 1)\{[\varepsilon\mu(1-\beta^2)]^{1/2} + [1 - (\mathbf{u}\mathbf{v}/c^2)](\varepsilon\mu - \beta^2)^{1/2}\}}{[\varepsilon\mu(\varepsilon\mu - \beta^2)(1-\beta^2)]\{[\varepsilon\mu(1-\beta^2)]^{1/2} + (\varepsilon\mu - \beta^2)^{1/2}\}}, \tag{33}$$

which is rotated by an angle α_0 in a counterclockwise direction relative to the vector $\mathbf{v}$

$$\sin\alpha_0 = \frac{\beta_\rho|\beta_z - \beta_{1z}|}{(\beta_\rho^2 + \beta_z^2)^{1/2}(\beta_\rho^2 + \beta_{1z}^2)^{1/2}},$$
$$\beta_{1z} = \left[\frac{\varepsilon\mu - \beta^2}{\varepsilon\mu(1-\beta^2)}\right]^{1/2}(\beta_z - \eta\beta), \quad \beta_\rho^2 = \beta_x^2 + \beta_y^2. \tag{34}$$

Simultaneous with the rotation of the axis of symmetry, the movement of the medium "flattens" the equipotential surfaces in the direction of the velocity $\mathbf{u}$ of the medium (z axis). The rotation and deformation of the ellipsoids or hyperboloids is such that, depending on the parameters of the medium $\varepsilon\mu$ and $\mathbf{u}$, and the velocity of the charge $\mathbf{v}$, the ellipsoids of the case of motion slower than light may be transformed into the hyperboloids of the case of motion faster than light, and vice versa.

If the parameter

$$\Gamma(\boldsymbol{\beta},\boldsymbol{\beta}_v) = \left(\frac{\varepsilon\mu - \beta^2}{1-\beta^2}\beta_1^2 - 1\right)\left(\boldsymbol{\beta}_v = \frac{\mathbf{v}}{c}, \boldsymbol{\beta} = \frac{\mathbf{u}}{c}, \boldsymbol{\beta}_1 = \frac{\mathbf{v}_1}{c}\right) \tag{35}$$

is positive, then the case of motion faster than light is realized with hyperboloids of revolution, and if Γ is negative, then the case of ellipsoids of revolution is realized (for more details see Ref. 13). One can show[13] that at $v_\rho = (v_x^2 + v_y^2)^{1/2} > c/(\varepsilon\mu)^{1/2}$, Γ is positive for any velocity of the motion of the medium, and the radiation field is nonzero within the conical surface with an apex at the location of the charge at time t and with an aperture half-angle φ_1

$$\tan\varphi_1 = \frac{1}{\Gamma(\boldsymbol{\beta},\boldsymbol{\beta}_v)}. \tag{36}$$

In a medium at rest ($\mathbf{u} = 0$) this cone is located behind the moving particle, as in the case of Cherenkov radiation.[18] Now if the medium begins to move along the z axis, then according to Eqs. (33) and (34) the radiation cone begins to rotate by an angle α_0 counterclockwise from vector $\mathbf{v}$, and simultaneously the aperture angle φ_1 changes. As the velocity of the medium increases, the rotation angle α_0 increases so that when the medium is moving at relativistic speeds, the bottom of the cone is pointed in the direction of the velocity of the medium, along the z axis: the rapidly moving medium sharply "blows" the radiation cone away. When the rotation angle α_0 becomes greater than the angle φ_1, the radiation field is completely on one side of the trajectory of the uniform motion of the charge: the field of the charge "shines to the side" of the trajectory. Now if $v_\rho < c/(\varepsilon\mu)^{1/2}$, but $v = (v_\rho^2 + v_z^2)^{1/2} > c/(\varepsilon\mu)^{1/2}$, then as the velocity of the medium increases, the hyperboloids (in the case of motion faster than light) will change into the ellipsoids of the case of motion slower than light, and then back into hyperboloids.[13] The simplest way to track this is in the special example of a charge and medium moving along the same path, when both velocities $\mathbf{v}$ and $\mathbf{u}$ are parallel along the z axis ($v_\rho = (v_x^2 + v_y^2)^{1/2} = 0$). In this case, when the charge is moving faster than light ($v > c/(\varepsilon\mu)^{1/2}$) in a medium at rest its field is nonzero behind the charge within the conical surface with an apex at the location of the charge at time t. In front of the charge the field is identically equal to zero. Now if the medium begins to move in the same direction, due to entrainment of the field by the medium the cone begins to "open up": it is as if the moving medium "blows" into this cone. Since the radiation condition in this case has the form[12,13]

$$|v_{\rm rel}| > \frac{c}{(\varepsilon\mu)^{1/2}}, \quad v_{\rm rel} = \frac{v-u}{1-(uv/c^2)}, \tag{37}$$

at some velocity of the medium $u < v$ it is disrupted, and the field of the charge again acquires the characteristic form of a Coulomb field "flattened" along the z axis and being carried along with this charge. There is no radiation. A further increase in the speed of the medium u will lead to a change in the sign of $v_{\rm rel}$ so that the medium begins to overtake the charge ($u > v$). Finally, the condition in Eq. (37) will again be satisfied, but the cone of radiation (inside the cone the field is nonzero) will now be turned forward along the movement of the charge and medium. Outside this cone and behind the charge the field is identically equal to zero.

In conclusion of this section we note that if the charged particle moves in a medium moving with a constant velocity v for a finite time interval $2T$, when the medium is moving faster than light ($\varepsilon\mu\beta^2 > 1$) a field of radiation of frequency ω is generated in the form of a wave of radiation at an angle θ to the z axis at distances

$$l_{1,2} = vT_{1,2} = \frac{\pi v}{\omega \alpha_{1,2}(\theta)},$$
$$\alpha_{1,2}(\theta) = 1 - \frac{v}{c}\frac{\varkappa\beta\gamma^2(1-\varkappa\beta^2\gamma^2\sin^2\theta)^{1/2} \mp (1+\varkappa)^{1/2}\cos\theta}{(\varkappa\beta^2\gamma^2-1)(1-\varkappa\beta^2\gamma^2\sin^2\theta)^{1/2}}, \tag{38}$$

where $\varkappa = \varepsilon\mu - 1$, $\gamma = (1-\beta^2)^{-1/2}$, $\varkappa\beta^2\gamma^2 = \gamma^2(\varepsilon\mu\beta^2 - 1)$. These quantities define the length of the path of formation (see Ref. 19) of the field of radiation with frequency ω in a moving medium. For $\varepsilon\mu\beta^2 < 1$ the path of formation is determined only by l_1.

3.3. The Lienard–Wiechert potentials in a moving medium

If a point particle with a charge q travels in a moving medium according to an arbitrary law $\mathbf{r} = \mathbf{r}_0(t)$ with a velocity $\mathbf{v} = \mathbf{v}_0(t) = d\mathbf{r}_0(t)/dt$ then the charge and current densities are given by

$$\rho(\mathbf{r}, t) = q\delta(\mathbf{r} - \mathbf{r}_0(t)), \quad \mathbf{j}(\mathbf{r}, t) = q\mathbf{v}_0(t)\delta(\mathbf{r} - \mathbf{r}_0(t)). \tag{39}$$

Substituting these expressions and the Green's function into formulas (11) and (10) we obtain (for details of calculations see Ref. 13)

$$\varphi(\mathbf{r}, t) = \frac{q\alpha}{\varepsilon}\sum_s \frac{1-\xi(t_s')}{l_0(\mathbf{r}, t, t_s')}, \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu q}{c}\alpha\sum_s \frac{\mathbf{v}_0'(t_s')}{l_0(\mathbf{r}, t, t_s')}, \tag{40}$$

where

$$\alpha = \left[\frac{\varepsilon\mu - \beta^2}{\varepsilon\mu(1-\beta^2)}\right]^{1/2},$$

$$\xi(t_s') = \varepsilon\mu\eta\alpha^2\left[\beta^2 - \frac{(\mathbf{u}\mathbf{v}_0(t_s'))}{c^2}\right], \quad \eta = \frac{\varepsilon\mu - 1}{\varepsilon\mu - \beta^2},$$

$$l_0 = R_0(\mathbf{r}, t, t_s') - \frac{\alpha(\varepsilon\mu)^{1/2}}{c}(\mathbf{R}_0(\mathbf{r}, t, t_s')\mathbf{v}_1(t_s')), \quad \mathbf{u} = c\boldsymbol{\beta} = u\mathbf{e}_z,$$
$$\mathbf{r} = \boldsymbol{\rho} + z\mathbf{e}_z, \quad \mathbf{r}_0(t_s') = \boldsymbol{\rho}_0(t_s') + z_0(t_s')\mathbf{e}_z,$$
$$R_0(\mathbf{r}, t, t_s') = |\mathbf{R}| \tag{41}$$
$$= [(\boldsymbol{\rho} - \boldsymbol{\rho}_0(t_s'))^2 + \alpha^2(z - z_0(t_s') - \eta u(t - t_s'))^2]^{1/2},$$

$$\mathbf{v}_0'(t_s') = \left\{\mathbf{v}_0(t_s') - \eta\alpha^2\mathbf{u}\left[1 - \frac{(\mathbf{u}\mathbf{v}_0(t_s'))}{c^2}\right]\right\},$$
$$\mathbf{v}_1(t_s') = \left\{\mathbf{v}_0(t_s') - \eta\alpha\mathbf{u}\left[1 - \frac{\alpha}{1+\alpha}\frac{(\mathbf{u}\mathbf{v}_0(t_s'))}{c^2}\right]\right\}.$$

The quantities $(\mathbf{r},t)$ determine the space-time coordinates of the observation point, while $(\mathbf{r}_0(t_s'),t_s')$ are similar coordinates of the point where the charged particle is situated at time t_s'. The instants t_s' over which summation is carried out in Eq. (40) are determined as solutions of equation

$$\frac{c}{(\varepsilon\mu)^{1/2}}(t - t_s') = \alpha R_0(\mathbf{r}, t, t_s'), \tag{42}$$

which satisfy the causality condition $t_s' < t$. In a vacuum Eqs. (40)–(42) reduce to the well-known ones of Ref. 6. In this case disturbances are propagated with the speed of light from each point of the trajectory as spherical waves, and Eq. (42) has only a single solution, since the spherical wave passes through the observation point only once. In a moving medium in virtue of the drift of the ellipsoids of perturbations for the Green's function (14) in the direction of motion of the medium such situations are possible, particularly when the medium moves at a speed exceeding that of light in it, and the field is identically equal to zero at any time t in the entire region of space "upstream" from the medium.

4. The energy losses of charged particles in a moving medium.

Once we know the fields of charged particles in a moving medium we can calculate the energy losses per unit length or per unit time. If the energy losses ΔW_q of a charge q over a length L or during a time $\tau = L/v$ are much less than the total energy W_q of this charge, we can assume that the charge moves almost uniformly in this interval, that is, its velocity $\mathbf{v}$ is constant. In this case the energy losses of a point charged particle per unit length are defined by the braking force acting on the charge at its location, and it is directed along the velocity of the particle $\mathbf{v}$ (see, for example, Ref. 13 or Ref. 20). In a medium at rest with frequency dispersion, a charged particle moving uniformly along the z axis ($\mathbf{v} = v\mathbf{e}_z$) loses energy in Vavilov–Cherenkov (V.–Ch. radiation and in the excitation of (longitudinal-subscript l) plasma oscillations in the medium[18,20]

$$\frac{dW_q}{dz} = F_z \Big|_{\substack{z=vt,\\ \rho\to 0}} = |qE_z(\mathbf{r},t)|_{\substack{z=vt,\\ \rho\to 0}} = \left(\frac{dW_q}{dz}\right)_{\text{V.-Ch.}} + \left(\frac{dW_q}{dz}\right)_l, \qquad (43)$$

where

$$\left(\frac{dW_q}{dz}\right)_{\text{V.-Ch.}} = -\frac{q^2}{c^2} \int_{\varepsilon(\omega)\mu(\omega)\beta^2>1,\ \omega>0} \mu(\omega)\left(1-\frac{1}{\varepsilon(\omega)\mu(\omega)\beta^2}\right)\omega d\omega, \qquad (44)$$

$$\left(\frac{dW_q}{dz}\right)_l = -\frac{q^2}{v^2}\sum_s \frac{2\omega_s}{\left|\frac{\partial\varepsilon(\omega)}{\partial\omega}\right|_{\omega=\omega_s}} K_0\left(\frac{\omega_s}{v}\rho_{\min}\right), \quad \beta=\frac{v}{c},$$

ω_ε is the sth positive root of the equation: $\varepsilon(\omega_s)=0$; here $K_0(x)$ is the modified Bessel function; $\mathbf{r}=\boldsymbol{\rho}+z\mathbf{e}_z$, and $\rho_{\min}\approx r_D$ is the Debye screening radius, which in the plasma model of the medium has the form

$$r_D = \left(\frac{kT}{m_0}\right)^{1/2}\frac{1}{\omega_p}, \quad \omega_p^2 = 4\pi e^2 N/m_0,$$

where m_0, N, and T are the mass, concentration, and temperature of electrons, and k is Boltzmann's constant.

Now if the medium begins to move as a whole with a velocity $\mathbf{u}$ in the direction of the motion of the particle (along the z axis), calculations of the braking force (see Ref. 13) lead to the formulas in Eq. (44) with a small but significant addition: each of the expressions in Eq. (44) is multiplied by a sign function of the difference in the speeds of the charge and the medium: $\mathrm{sgn}(v-u)=+1$ for $v>u$ and $\mathrm{sgn}(v-u)=-1$ for $v<u$. Moreover, the speed of the particle v in the formulas in Eq. (44) is replaced with the relative speed of motion of the charge and medium (see v_{rel} in Eq. (37)), and all functions $\varepsilon(\omega)$ and $\mu(\omega)$ depend on the frequency $\omega'=\omega\gamma(v-u)/v$ measured in the rest system of the medium. The sign function points out the phenomenon of the reversal of the sign of the energy loss of a charge in a moving medium. Indeed, in a medium at rest, when $u=0$ and $v-u>0$ the right sides of the formulas in Eq. (44) are always negative. This means that the charge loses energy in radiation of waves or in excitation of plasma oscillations of electrons in a medium at rest. In a moving medium, when $u>v$, that is, when the medium is overtaking the charge, the expressions for loss change sign, because $\mathrm{sgn}(v-u)=-1$. As a result, the moving medium begins to accelerate the charge in it with simultaneous emission of Vavilov–Cherenkov radiation waves and the excitation of plasma oscillations of electrons in the medium. Physically this is explained by the instability of states with negative photon energies $\hbar\omega'=\hbar\omega\gamma(v-u)/v$ in a medium moving with a speed $u>v$ (see Ref. 21 about this). Actually the change in the sign of the braking force is linked with the following. Let the medium be at rest and let the charge move in the $+z$ direction. Then the braking force is pointed in the $-z$ direction. Now let us switch to the rest system of the charge. In this system the medium is moving with the velocity of the charge in the $-z$ direction and the charge is at rest. However, the force $F_z=qE_z$ in Eq. (43) in this system, as before, is pointed in the $-z$ direction because the component E_z remains unchanged in such Lorentz transformations. As a result, in the rest system of the charge, the moving medium accelerates the charge in the direction of its motion.

As an example of the application of the formulas obtained here we calculate the energy acquired per unit length of the path by a charge q in a dense beam of relativistic ($u\approx c$) electrons with energy $W_{\text{el}}=m_0c^2\gamma$ and with a concentration N. The electromagnetic properties of this beam are described by the formulas in Eq. (5). Since $\varepsilon<1$, $(dW_q/dz)_{\text{V.-Ch.}}=0$. Thus, losses are determined only by the excitation of plasma oscillations in the beam of relativistic electrons by charge q and acquire the form[13]

$$\frac{dW_q}{dz} = q^2\cdot\frac{4\pi r_0}{\gamma} N \ln\frac{\gamma}{\Delta\gamma}, \qquad (45)$$

where $\Delta\gamma/\gamma$ is the relative scatter of energy in the beam of electrons, $r_0=e^2/m_0c^2\approx 2.8\cdot 10^{-13}$ cm, $N\approx 1.4\cdot 10^8 j$ in cm^{-3}, j is the current density of the beam in amperes per square centimeter. The increase in the energy of a particle with charge q is independent of its mass (even heavy ions can be accelerated) and is proportional to the square of the charge of the accelerated particles. In today's high-current electron beams with $\gamma\approx 4$, $\Delta\gamma\approx 0.3\gamma$ and with current densities $j\approx 30$ kA/cm^2 ($N\approx 6\cdot 10^{12}$ cm^{-3}) the increase in energy for bunches of accelerated particles with concentrations of the order of 10^{10} cm^{-3} is 5 keV/cm per accelerated particle. Alpha particles have been obtained in this way with energies[4] of tens of MeV. Of course to calculate the real acceleration mode one must solve the problem self-consistently, that is, one must consider the inverse effect of accelerated particles on the accelerating electron beam.

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Translated by C. Gallant