Interference of reactive components of an electromagnetic field

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This article discusses the formation of an interference energy flux as a result of changing the phase shift between the oscillations of the reactive components of the vectors of the electric and magnetic field intensities of radiation when spatially superposed. On the basis of a model of the interference of the reactive components of the electromagnetic field from a common physical standpoint, the passage of light through a transparent plane-parallel plate at an angle of incidence exceeding the critical angle of total internal reflection, the formation of a refracted wave in the region of total internal reflection from a semiinfinite medium, and the radiationless transport of energy between excited and unexcited atoms are described.

In describing the interference of electromagnetic waves, one usually restricts the treatment to the case of in-phase oscillations of the electric and magnetic field intensity vectors of the interfering waves. The spatial superposition of such waves conserves the in-phase character of the oscillations of the total field intensity vectors, while only their local amplitude varies. As a result the intensity of radiation is spatially redistributed, with the appearance of a characteristic alternation of maxima and minima.[1]

In the general case the interference of electromagnetic fields can alter both the local amplitude and the phase shift between the oscillations of the total electric and magnetic field intensity vectors. It is of interest to examine in a certain sense the other limiting case of interference, in which the main effect is associated with a change in the phase shift, which leads to a spatial redistribution of the radiation intensity.

Such a situation is realized in the interference of the so-called reactive components of the electric and magnetic field intensity vectors. In the case of the reactive components the phase shift between the corresponding oscillations equals $\pi/2$. Therefore the contribution of the reactive components of the electromagnetic field to the intensity of the original waves equals zero. When certain conditions are fulfilled, the spatial superposition of the reactive components of the electromagnetic field of two or more waves can make the phase shift between the oscillations of the total electric and magnetic field intensity components differ from $\pi/2$. As a result an interference flux of energy in a new direction is formed, where energy transport for the original waves can be completely absent.

In the present Methodological Notes the features of the interference of the reactive components of the electromagnetic field are treated using the example of three well known physical phenomena: 1) the passage of light through a transparent plane-parallel plate incident at an angle exceeding the critical angle of total internal reflection, 2) the formation of a refracted wave in a region of total internal reflection from a transparent semiinfinite medium, 3) radiationless energy transport between atoms.

Let a plane monochromatic wave of frequency ω polarized perpendicular to the plane of incidence be incident from a transparent medium with the refractive index n_1 onto a transparent plane-parallel plate with the refractive index $n_2 < n_1$ at an angle $\theta > \theta_{\rm cr} = \sin^{-1}(n_2/n_1)$. The plate occupies the region of space $0 \leqslant z \leqslant d$, where d is the thickness of the plate, the z axis is oriented perpendicular to the surface of the plate, the x axis is oriented parallel, and the y axis is oriented perpendicular to the plane of incidence. The incident wave propagates in the positive direction of the z axis. For the sake of definiteness we shall assume that beyond the plate, where $z > d$, a transparent medium lies having a refractive index $n_3 = n_1$. As is well known, under these conditions the incident radiation partially passes through the plate, despite the fact that the angle of incidence $\theta > \theta_{\rm cr}$.

Let us examine in greater detail the formation of the energy flux transported by the radiation inside the plate. The electric field intensity E_y inside the plate is written in the form of the sum of the corresponding components of the electric field intensity vectors E_{1y} and E_{2y} of two inhomogeneous plane waves whose amplitudes vary according to an exponential law along the z axis:

$$E_y = E_{1y} + E_{2y} = A\exp[i(k_x x - \omega t) - (k_x^2 - k_2^2)^{1/2} z] + \rho A \exp[i(k_x x - \omega t) + (k_x^2 - k_2^2)^{1/2} z], \qquad (1)$$

where t is the time, $i = \sqrt{-1}$ is the imaginary unit, k_x is the projection of the wave vector of the incident wave on the x axis, $k_2 = kn_2 < k_x$ ($\theta > \theta_{\rm cr}$), $k = \omega/c$, and c is the speed of light in vacuum. The quantity A is determined from the boundary conditions at the front surface of the plate $z = 0$, while the coefficient

$$\rho = \frac{i(k_x^2 - k_2^2)^{1/2} - k_{3z}}{i(k_x^2 - k_2^2)^{1/2} + k_{3z}} \exp[-2(k_x^2 - k_2^2)^{1/2} d] \qquad (2)$$

is determined by using the boundary conditions at the rear surface of the plate $z = d$. Here $k_{3z} = k_{1z} > 0$ is the projection of the wave vector on the z axis for the wave after being transmitted through the plate.

The expression for the x component of the magnetic field intensity component H_x inside the plate has the form

$$H_x = \frac{i}{k}\frac{\partial E_y}{\partial z} = H_{1x} + H_{2x}$$
$$= -i\frac{(k_x^2 - k_2^2)^{1/2}}{k} A \exp[i(k_x x - \omega t) - (k_x^2 - k_2^2)^{1/2} z]$$
$$+ i\frac{(k_x^2 - k_2^2)^{1/2}}{k} \rho A \exp[i(k_x x - \omega t) + (k_x^2 - k_2^2)^{1/2} z]. \quad (3)$$

The indices 1 and 2 of the components of the electromagnetic field intensity denote whether they belong to the inhomogeneous plane wave excited in the front or rear surface of the plate, respectively.

According to Eqs. (1) and (3) the phase shift between E_{1y} and H_{1x} equals $-\pi/2$, while that between E_{2y} and H_{2x} is $+\pi/2$. The time-averaged energy flux density (the intensity) *transported by each inhomogeneous plane wave along* the x axis,

$$I_z = \frac{c}{8\pi}\mathrm{Re}[\mathbf{EH}^*]_z = -\frac{c}{8\pi}\mathrm{Re}(E_y H_x^*), \quad (4)$$

where Re denotes taking the real component, and * in the superscript denotes the complex conjugate, proves to equal zero: $I_{1z} = I_{2_z} = 0$. Consequently, with regard to energy transport along the z axis, the components of the electromagnetic field intensity (E_{1y}, H_{1x}) and (E_{2y}, H_{2x}) are reactive.

The physical mechanism of transport of radiation energy inside the plate along the z axis involves the interference of the reactive components of the electromagnetic field, as a result of which, as we see from Eqs. (1)–(3), the phase shift between the total intensities of the electric field E_y and the magnetic field H_x come to differ from $\pi/2$. Substituting Eqs. (1)–(3) into Eq. (4), we can obtain

$$I_z = \frac{c}{8\pi}\frac{(k_x^2 - k_2^2)^{1/2}}{k}|A|^2 2\,\mathrm{Im}\,\rho$$
$$= \frac{c}{2\pi}\frac{k_{3z}}{k}\frac{k_x^2 - k_2^2}{k_x^2 - k_2^2 + k_{3z}^2}|A|^2 \exp[(-2(k_x^2 - k_2^2)^{1/2} d)], \quad (5)$$

where Im denotes taking the imaginary component. We see from Eq. (5) that, for an arbitrary plane $0 \leqslant z = \mathrm{const} \leqslant d$, the intensity of the interference energy flux I_z is a constant, while the alternation of maxima and minima characteristic of ordinary interference is absent.

In the example that we have discussed the amplitudes of the interfering inhomogeneous plane waves declined exponentially in the opposite directions of the z axis. The question arises of the possibility of interference energy transport in the case in which the amplitudes of the inhomogeneous plane waves decline exponentially in the same direction. Analysis shows that an interference energy flux along the z axis can arise also in this case if the condition is satisfied that

$$\frac{E_{1y}}{H_{1x}} \neq \frac{E_{2y}}{H_{2x}}. \quad (6)$$

The condition (6) implies that the interfering waves must differ either in frequency $(\omega_1 \neq \omega_2)$ or in the x components of the wave vector $(k_{1x} \neq k_{2x})$. Sets of inhomogeneous plane waves of this type arise in total internal reflection of radiation bounded in time or in the transverse direction, and correspond to individual Fourier components of the incident radiation.

To study this case, it is convenient to use the method of slowly varying amplitudes, which enables one to take account of the integral effect of interference of the entire continuum of inhomogeneous plane waves that describes the refracted radiation. Let a quasimonochromatic plane pulse of radiation polarized perpendicular to the plane of incidence (along the y axis) and bounded along the x axis, which lies in the plane of incidence, be incident from a transparent medium of refractive index n_1 onto the plane phase boundary $z = 0$ with a transparent medium having the refractive index $n_2 < n_1$ at an angle $\theta > \theta_{\mathrm{cr}} = \sin^{-1}(n_2/n_1)$.

The electric field intensity of the incident pulse is written in the form

$$E_{1y} = \mathscr{E}_1(x,z,t)\exp[i(k_{1x}x + k_{1z}z - \omega_0 t)], \quad (7)$$

where $k_{1x} = k_1 \sin\theta > k_2 = k_0 n_2$, $k_1 = k_0 n_1$, $k_0 = \omega_0/c$, $k_{1z} = k_1 \cos\theta > 0$, and ω_0 is the fundamental frequency of the pulse. The complex amplitude $\mathscr{E}_1(x,z,t)$ is a slowly varying function of its arguments

$$\left|\frac{\partial^2 \mathscr{E}_1}{\partial x^2}\right| \ll |k_{1x}|\left|\frac{\partial \mathscr{E}_1}{\partial x}\right|, \quad \left|\frac{\partial^2 \mathscr{E}_1}{\partial z^2}\right| \ll k_{1z}\left|\frac{\partial \mathscr{E}_1}{\partial z}\right|,$$
$$\left|\frac{\partial^2 \mathscr{E}_1}{\partial t^2}\right| \ll \omega_0\left|\frac{\partial \mathscr{E}_1}{\partial t}\right| \quad (8)$$

and satisfies the boundary conditions

$$\lim_{|x|\to\infty} \mathscr{E}_1 = \lim_{|z|\to\infty} \mathscr{E}_1 = 0. \quad (9)$$

Moreover, at the phase boundary of the two media $z = 0$ we have

$$\lim_{|t|\to\infty} \mathscr{E}_1 = 0. \quad (10)$$

The electric field intensity of the refracted wave is written in analogous fashion:

$$E_{2y} = \mathscr{E}_2(x,z,t)\exp[i(k_{1x}x - \omega_0 t)], \quad (11)$$

where the complex amplitude $\mathscr{E}_2(x,z,t)$ is a slowly varying function of the arguments x and t

$$\left|\frac{\partial^2 \mathscr{E}_2}{\partial x^2}\right| \ll |k_{1x}|\left|\frac{\partial \mathscr{E}_2}{\partial x}\right|, \quad \left|\frac{\partial^2 \mathscr{E}_2}{\partial t^2}\right| \ll \omega_0\left|\frac{\partial \mathscr{E}_2}{\partial t}\right|. \quad (12)$$

The conditions (12) allow one to write the equation for finding $\mathscr{E}_2(x,z,t)$ in the parabolic approximation in the variables x and t:

$$2i\frac{\omega_0 n_2^2}{c^2}\frac{\partial \mathscr{E}_2}{\partial t} + 2ik_{1x}\frac{\partial \mathscr{E}_2}{\partial x} + \frac{\partial^2 \mathscr{E}_2}{\partial z^2} + (k_2^2 - k_{1x}^2)\mathscr{E}_2 = 0. \quad (13)$$

The solution of Eq. (13) must satisfy the boundary conditions at the phase boundary of the two media $z = 0$ and the condition at infinity

$$\lim_{z\to\infty} \mathscr{E}_2 = 0. \quad (14)$$

We can derive from Eq. (13) and the boundary conditions the energy conservation law for the refracted wave in the following form:

$$I_z(0) = \frac{\partial}{\partial t}\left(\int_0^{\infty} dz \int_{-\infty}^{\infty} w dx\right), \qquad (15)$$

where the linear density of the time-averaged energy flux along the x axis

$$I_z(0) = \frac{c}{8\pi k_0} \int_{-\infty}^{\infty} \left[\frac{1}{2i}\left(\frac{\partial \mathscr{E}_2}{\partial z}\mathscr{E}_2^* - \frac{\partial \mathscr{E}_2^*}{\partial z}\mathscr{E}_2\right)\right] dx, \ z = 0, \qquad (16)$$

equals the energy transported by radiation per unit time through the band $-\infty < x < \infty$, $\Delta y = 1$ of the boundary surface of the two media oriented parallel to the x axis, and

$$w = \frac{n_2^2}{8\pi} |\mathscr{E}_2|^2 \qquad (17)$$

is the energy density of the refracted wave.

Let us write the complex amplitude of the refracted wave $\mathscr{E}_2(x,z,t)$ by using the Fourier integral

$$\mathscr{E}_2(x,z,t) = \iint_{-\infty}^{\infty} A_2(k_x,\omega) \exp[i(k_x x + k_z(k_x,\omega)z - \omega t)] dk_x d\omega. \qquad (18)$$

Here, according to Eq. (13) and the boundary condition (14), we have

$$k_z(k_x,\omega) = i[k_{1x}^2 - k_2^2 + 2k_{1x}k_x - (2\omega\omega_0 n_2^2/c^2)]^{1/2}, \ \mathrm{Im}\, k_z > 0. \qquad (19)$$

The function $A_2(k_x,\omega)$ is found from the boundary conditions at the phase boundary of the two media $z = 0$.

The conditions in (12) imply that the function $|A_2(k_x,\omega)|$ has sharp maxima at $k_x = 0$ and $\omega = 0$. Therefore in the integrand in (18) near the surface $z = 0$ we can expand the quantity k_z in a power series in k_x and ω and take account of only the first three terms of the expansion ($k_{1x} \neq k_2$, $\theta \neq \theta_{cr}$)

$$k_z = i(k_{1x}^2 - k_2^2)^{1/2}\left[1 + \frac{k_{1x}}{k_{1x}^2 - k_2^2}k_x - \frac{\omega_0 n_2^2}{c^2(k_{1x}^2 - k_2^2)}\omega\right]. \qquad (20)$$

After uncomplicated transformations using Eqs. (16), (18), and (20), one can obtain

$$I_z(0) = \frac{c}{8\pi k_0} \int_{-\infty}^{\infty} \left[\frac{k_{1x}}{(k_{1x}^2 - k_2^2)^{1/2}} \frac{\partial |\mathscr{E}_2|^2}{\partial x} + \frac{\omega_0 n_2^2}{c^2(k_{1x}^2 - k_2^2)^{1/2}} \frac{\partial |\mathscr{E}_2|^2}{\partial t}\right] dx. \qquad (21)$$

In the parabolic approximation the boundary conditions at the phase boundary $z = 0$ of the two media imply that

$$\mathscr{E}_2(x,0,t) = \frac{2k_{1z}}{k_{1z} + i(k_{1x}^2 - k_2^2)^{1/2}} \mathscr{E}_1(x,0,t). \qquad (22)$$

Therefore the final expression for the linear energy flux density through the phase boundary of the two media acquires the form

$$I_z(0) = \frac{ck_{1z}^2}{2\pi k_0(k_1^2 - k_2^2)} \times \int_{-\infty}^{\infty} \left[\frac{k_{1x}}{(k_{1x}^2 - k_2^2)^{1/2}} \frac{\partial |\mathscr{E}_1|^2}{\partial x} + \frac{\omega_0 n_2^2}{c^2(k_{1x}^2 - k_2^2)^{1/2}} \frac{\partial |\mathscr{E}_1|^2}{\partial t}\right] dx. \qquad (23)$$

According to Eq. (23) an energy flux through the phase boundary of the two media in the region of total internal reflection arises whenever the quantity $|\mathscr{E}_1|$ varies either in time or along the phase boundary of the two media. Each Fourier component taken separately in the expansion (18) gives a zero contribution to the energy flux through the phase boundary of the two media. However, the interference of the continuum of these Fourier components causes the phase shift between the total electric field intensity E_{2y} and the total magnetic field intensity

$$H_{2x} = \frac{i}{k_0}\frac{\partial E_{2y}}{\partial z} = \frac{i}{k_0}\frac{\partial \mathscr{E}_2}{\partial z}\exp[i(k_{1x}x - \omega_0 t)] \qquad (24)$$

to differ from $\pi/2$. Consequently an interference energy flux arises through the phase boundary of the two media, which makes possible the formation of a refracted wave.

If we take account of absorption in the reflecting medium and consider the quantity n_2 to be complex, then the phase shift between $E_{2y}(k_x,\omega)$ and $H_{2x}(k_x,\omega)$ of the separately taken Fourier components in the expansion (18) comes to differ from $\pi/2$, and each such Fourier component transports energy along the z axis. Consequently the following additional term appears in the expression for the linear energy flux density $I_z(0)$:

$$\Delta I_z(0) = \frac{ck_{1z}^2 \,\mathrm{Re}(k_2^2 - k_{1x}^2)^{1/2}}{\pi k_0(k_1^2 - k_2^2)} \int_{-\infty}^{\infty} |\mathscr{E}_1|^2 dx, \qquad (25)$$

where $k_2 = k_0(n_2' + in_2'')$, $n_2'' > 0$, $n_2' \gg n_2''$, $\mathrm{Re}(k_2^2 - k_{1x}^2)^{1/2} > 0$, and $\mathrm{Im}(k_2^2 - k_{1x}^2)^{1/2} > 0$.

Equations (23) and (25) correspond to different mechanisms of transport of radiation energy through the phase boundary of the two media in the region of total internal reflection. Equation (23), which describes the energy flux associated with formation of a refracted wave, corresponds to a collective mechanism of energy transport where the entire continuum of Fourier components of the refracted wave acts as a unit. Equation (25), which describes the energy flux associated with excitation of atoms of the medium, corresponds to an individual mechanism of energy transport, where each Fourier component of the refracted wave can transfer its energy to atoms of the medium.

According to Eq. (23) the radiation energy is transported into the reflecting medium for that fraction of the incident beam where the condition is fulfilled that $k_{1x}\partial|\mathscr{E}_1|^2/\partial x > 0$, and is transported from the reflecting medium into the original medium for the fraction of the incident beam where the condition is fulfilled that $k_{1x}\partial|\mathscr{E}_1|^2/\partial x < 0$. In view of the boundary condition (9), the total energy flux through the entire phase boundary of the two media $-\infty < x < \infty$ is zero. Thus, while from one side of the incident beam a certain amount of energy "flows into" the reflecting medium, from the other side of the incident beam exactly the same amount of energy "flows out" of the reflecting medium, which leads to a lateral shift of the reflected beam.[2]

We see from Eq. (23) that radiation energy is transported into the reflecting medium for the fraction of an incident pulse where the condition is fulfilled that $\partial |\mathscr{E}_1|^2/\partial t > 0$, and is transported from the reflecting medium into the original medium for the fraction of the pulse where the condition is fulfilled that $\partial |\mathscr{E}_1|^2/\partial t < 0$. In the case of incidence of a pulse whose envelope has a single maximum, this leads to a time shift (retardation) of the reflected pulse. According to the condition (10) the total energy flux through the phase boundary of the two media for the entire time of incidence of the pulse equals zero. An analogous result is obtained for the total internal reflection of an acoustic pulse.[3]

The last example of interference of the reactive components of an electromagnetic field pertains to the radiationless resonance transport of energy from an excited to an unexcited atom.[4] In describing this phenomenon one usually pays major attention to the dynamics of the atomic states in taking account of the dipole-dipole interaction of the atoms as a perturbation. At the same time, the electrodynamics of the radiationless transport of energy in the space between the atoms is practically not taken into account. We shall show below that radiationless energy transport is described by an interference flux that arises in the spatial superposition of the reactive components of the electromagnetic field of the dipoles in the near zone.

Let us assume that near an excited atom, for which the transition to the ground state is allowed in the electric dipole approximation, an unexcited atom of exactly the same type is found. In the classical theory the interaction of these atoms can be described by using the model of electric dipoles undergoing harmonic oscillations at the frequency ω of the atomic transition. In a spherical system of coordinates the components of the electric and magnetic field intensity vectors of the dipole undergoing harmonic oscillations at the frequency ω along the z axis are written as follows:

$$E_r = 2\cos\theta\cdot\left(\frac{1}{k^2r^2}-\frac{i}{kr}\right)k^2p,\ E_\theta = \sin\theta\cdot\left(\frac{1}{k^2r^2}-\frac{i}{kr}-1\right)k^2p,$$

$$H_\varphi = -i\sin\theta\cdot\left(\frac{1}{kr}-i\right)k^2p,\ E_\varphi = H_r = H_\theta = 0, \tag{26}$$

where $p=(p_0/r)\exp[i(kr-\omega t)]$, p_0 is the complex amplitude of the oscillations of the dipole moment, $\mathbf{r}$ is the radius vector drawn from the center of the dipole to the point of observation, θ is the angle between the z axis and $\mathbf{r}$, and φ is the angle between the x axis and the projection of $\mathbf{r}$ on the xy plane.

The total energy flux emitted by the dipole through an arbitrary closed surface surrounding the dipole,

$$I=\frac{\omega^4}{3c^3}|p_0|^2 \tag{27}$$

is determined by only two components of the electromagnetic field of the dipole

$$E_\theta = -\sin\theta\cdot k^2p,\ H_\varphi = -k^2p, \tag{28}$$

which describe the radiation in the far wave zone. Precisely the energy flux of (27) and the components of the electromagnetic field of (28) determine the radiative energy transport between the atoms.

The reactive components of the electromagnetic field of the dipole

$$E_r = 2\cos\theta\cdot\frac{p}{r^2},\ E_\theta=\sin\theta\cdot\frac{p}{r^2},\ H_\varphi = -i\sin\theta\cdot\frac{kp}{r}, \tag{29}$$

which describe the radiation in the near zone, yield no contribution to the energy flux of (27). However, in the presence of a second dipole that undergoes harmonic oscillations at the same frequency ω, the interference of the reactive components of the electromagnetic field of the dipoles can lead to formation of an interference energy flux between the dipoles, which describes the radiationless transport of energy from one dipole to the other.

Let us study the emission of two dipoles that undergo harmonic oscillations at the frequency ω along the z axis, and which lie symmetrically on the x axis with respect to the coordinate origin at the distance l from one another,

$$p_1 = p_{10}\exp(-i\omega t),\ p_2 = p_{20}\exp(-i\omega t), \tag{30}$$

Here p_{10} and p_{20} are the complex amplitudes of the oscillations of the first and second dipole, respectively, at the coordinates $(-l/2,0,0)$ and $(l/2,0,0)$.

The total energy flux emitted by the dipoles through the plane $x=0$,

$$I_x = I_{1x}+I_{2x}+I_{\text{int},x} \tag{31}$$

contains three terms. The first two terms

$$I_{1x}=\frac{\omega^4}{6c^3}|p_{10}|^2,\ I_{2x}=-\frac{\omega^4}{6c^3}|p_{20}|^2 \tag{32}$$

describe the energy fluxes emitted independently by each dipole. The last term on the right-hand side of Eq. (31), $I_{\text{int},x}$, describes the interference energy flux, which is determined by the electromagnetic field of both the first and second dipole.

If the distance l between the dipoles is much smaller than the wavelength of the radiation $\lambda = 2\pi c/\omega$, then we must take account in the interference flux of only the reactive components of the electromagnetic field in the near zone:[29]

$$E_z(0,y,z,t) = (3\cos^2\theta-1)\frac{p_{10}+p_{20}}{R^3}\exp(-i\omega t),$$

$$H_y(0,y,z,t) = -i\sin\theta\cdot\cos\varphi\cdot k\frac{p_{10}-p_{20}}{R^2}\exp(-i\omega t), \tag{33}$$

where $R=[(l^2/4)+y^2+z^2]^{1/2}$.

After certain transformations using the formulas of (33), we can obtain

$$\begin{aligned} I_{\text{int},x} &= \iint_{-\infty}^{\infty}\left[-\frac{c}{8\pi}\mathrm{Re}(E_zH_y^*)\right]\mathrm{d}y\mathrm{d}z \\ &= \frac{ckl}{16\pi}\mathrm{Re}[i(p_{10}p_{20}^*-p_{10}^*p_{20})]\iint_{-\infty}^{\infty}\left[\frac{(3z^2/R^2)-1}{R^6}\right]\mathrm{d}y\mathrm{d}z \\ &= \frac{\omega}{2l^3}\mathrm{Im}(p_{10}p_{20}^*). \end{aligned} \tag{34}$$

As we see from Eqs. (33) and (34), the phase shift between $E_z(0,y,z,t)$ and $H_y(0,y,z,t)$ comes to differ from $\pi/2$, while the interference energy flux $I_{\text{int},x}$ differs from zero only when the oscillations of the dipoles are phase-shifted.

Moreover, we must note that $I_{\mathrm{int},x} \neq 0$ only in the region between the dipoles $-l/2 < x < l/2$.

To describe the radiationless energy transport between the atoms, we must take account of the fact that the unexcited atom acquires a dipole moment that undergoes harmonic oscillations at the radiation frequency ω when acted on by the electric field of the emission of the excited atom. Assuming that only the component $E_z = -E_\theta(\theta = \pi/2) = -p_1/l^3$ of the reactive radiation field of (29) acts on the unexcited atom, we obtain

$$p_2 = -\frac{\alpha p_{10}}{l^3}\exp[i(kl - \omega t)] \approx -\frac{\alpha p_{10}}{l^3}\exp(-i\omega t), \qquad (35)$$

where $kl = 2\pi l/\lambda \ll 1$, l is the distance between the atoms, $\alpha = \alpha' + i\alpha''$ is the polarizability of the unexcited atom at the frequency ω, $\alpha'' > 0$, and p_{10} is the complex amplitude of the oscillations of the dipole moment of the excited atom.

By using Eqs. (34) and (35) we can easily find that the interference energy flux is

$$I_{\mathrm{int},x} = \alpha''\frac{\omega}{2l^6}|p_{10}|^2 \qquad (36)$$

and is directed from the excited to the unexcited atom, since $\alpha'' > 0$. Comparison of the quantities I_{1x} and $I_{\mathrm{int},x}$ shows that radiationless energy transport becomes the main process if

$$l \lesssim l_0 = 3\left(\alpha''\frac{c^3}{\omega^3}\right)^{1/6} = \frac{3}{\sqrt{2\pi}}(\alpha''\lambda^3)^{1/6}. \qquad (37)$$

Assuming that $\omega = 3\times10^{15}$ radians/s, $\alpha'' = 10^{-24}$ cm^3 for the optical frequency range, we obtain the estimate $l_0 \approx 30$ Å $\ll \lambda = 2.1\times10^{-5}$ cm.

Owing to the appearance of an interference energy flux between the electric dipole undergoing harmonic oscillations and the atoms of the medium, the emission power of the dipole in an absorbing medium is larger than in the case of a transparent medium.[4] The additional radiation energy is absorbed by the medium at the distance l_0 from the dipole, while its magnitude proves to be proportional to the absorption coefficient of the medium at the emission frequency of the dipole.

The energy flux density transported by the electromagnetic wave is determined by the amplitude of the wave and by the phase shift between the oscillations of the electric and magnetic field intensity vectors. Control of the energy flux density of an electromagnetic wave by varying its amplitude by using absorption, amplification, and also the interference phenomenon, is well known and widely used in practice. In the present Methodological Notes we have studied the control of the energy flux of an electromagnetic field by altering the phase shift between the oscillations of the reactive components of the electric and magnetic field intensities when spatially superposed. The interference of the reactive components of the electromagnetic field possesses a number of features, including: 1) formation of an interference energy flux in a new direction where energy transport of the original waves may be absent, 2) absence in the intensity distribution of the interference energy flux of an alternation of maxima and minima. The introduction of the concept of interference of the reactive components of an electromagnetic field enables one to treat from a common physical standpoint such phenomena, different at first glance, as total internal reflection of light and radiationless energy transport between atoms.

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