On some low energy experiments on hadron spectroscopy

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Possibilities to carry out low energy experiments (at $E^{\rm lab} \sim 2$–10 GeV) on the searches and studies of exotic hadrons and rare decays of mesons are considered: 1) search for baryonic states of qqqs$\bar{\rm s}$, qqs$\bar{\rm s}$, and qqqg types in direct channel; 2) search for exotic mesons in baryon exchange processes $pN \to (M)_f + (pN; p\pi N)$; 3) study of exotic mesons in OPE hadron reactions and in photoproduction processes; 4) study of the rare electromagnetic decays of mesons; 5) study of the isotopic invariance violation.

1. INTRODUCTION

The last decade may be referred to as a hadron spectroscopy renaissance. Recent advances in the experimental techniques allowed one to carry out investigations in this field at a qualitatively new level. Experiments were performed in intense hadronic beams, using the wide aperture apparatuses capable of recording and identifying both charged and neutral secondaries and able to handle processes with cross sections in the nanobarn range. New areas of research have been developed as the results of measurements with e^+e^--colliding beams (studies of meson production in $\gamma\gamma$-collisions and in gluon enriched decay channels for the J/ψ-particles), with the selection of the processes in the Coulomb field of nuclei, with the search of new mesons in $p\bar{p}$ annihilation in pure antiproton beams of LEAR. The projects of the φ factories, which will yield pure and intense sources of φ, ω, and ρ mesons produced in the resonance e^+e^- reactions at high luminosities, are being brought into reality now.

Studies of hadron spectroscopy have led to the observation of several hundreds of new particles—baryons and mesons. A brief summary of their properties fills now a weighty volume—a special issue of Phys. Lett. B.[1] It has become clear that contrary to previously held views, none of these hadrons can be regarded as an elementary particle and that the "elementary level" lies much deeper. Colored quarks and gluons have been discovered in the course of these investigations and quantum chromodynamics, which describes the interactions between these fundamental objects, was created. It has been found that quarks are indeed the structural elements of hadronic matter and are responsible for the diversity of the hadronic world. It has been established that all known hadrons fit into the $SU(3)$ systematics that reflects their very simple quark structure: mesons are q$\bar{\rm q}$ systems consisting of quark-antiquark valence pair, whereas baryons consist of three valence quarks (qqq).[1)] Of course, these valence structures are surrounded by a virtual "sea" of quark-antiquark pairs and gluons, but it is the fundamental valence particles that determine the principal quantum numbers and the hadron systematics.

As it follows from the quark model, the mesons, consisting of light u,d,s quarks, group into mesonic nonets with definite values for the quantum numbers: spin J, parity $P = -(-1)^L$, charge parity (for the neutral members of the nonet) $C = (-1)^{L+S}$, G-parity $G = C(-1)^I$. Here S is the summed spin of quarks, L is the q$\bar{\rm q}$ system orbital angular momentum ($\vec{J} = \vec{S} + \vec{L}$), I is the meson isotopic spin. The (q$\bar{\rm q}$) mesons admit only some certain combinations of the quantum numbers: $C = P = (-1)^J$, or $C = P = (-1)^{J+1}$, or $C = (-1)^J$, $P = (-1)^{J+1}$.

Besides orbital excitations the quark model also predicts radial excitations of the q$\bar{\rm q}$ system, which implies that there should exist mesonic nonets with the same value of J^{PC} but differing from each other in their quantum number n, which determines the radial part of the meson wave function. For the ground state $n = 1$ (as a rule, it is not indicated), and the radially excited mesonic families are characterized by the values of $n = 2,3$, etc. The meson mass grows with n. The first radially excited state observed experimentally was ψ' meson, which is a ($2\,^3S_1$) state of a charmed quark-antiquark pair $c\bar{c}$ (the J/ψ is the ground ($1\,^3S_1$) state (or simply $^3S_1$) of the $c\bar{c}$ system).[2)]

At present, already several radially excited states have been observed. Their existence complicates the systematics of hadrons and makes it necessary to define their quantum numbers more thoroughly. Further searches and studies of radially excited mesonic families form a very important branch of modern hadron spectroscopy.

Since the early days of the quark era, and almost up to the present time, it was not clear whether there were hadrons with more complicated valence composition, i.e., multiquark mesons (qq$\bar{\rm q}\bar{\rm q}$) or baryons (qqqq$\bar{\rm q}$) and dibaryons (qqqqqq). The advance of quantum chromodynamics led to the natural assumption that gluons could also act as fundamental valence structural elements, i.e., there should be mesons consisting only of gluons (they are referred to as glueballs[2]) or mixed hadrons consisting of valence quarks and gluons. The latter are the so-called hybrids q$\bar{\rm q}$g or qqqg.[3] All these new types of particles are usually referred to as exotic hadrons and can be subdivided into three groups.

1.1. Exotic states of the first kind

These are states with explicitly exotic quantum numbers such as the electric charge, strangeness, isotopic spin (mesons with $|Q| \geqslant 2$ or $|S| \geqslant 2$ or $I > 1$; baryons with $|Q| > 1$ or $I > 3/2$ or $S > 0$). Such particles simply cannot have the usual quark structure such as q$\bar{\rm q}$ or qqq, and must necessarily be exotic multiquark states.

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1.2. Exotic states of the second kind

These are particles with exotic combinations of quantum numbers such as spin J, parity P, and charge parity C, which hadrons with ordinary quark structure cannot have. As it was said before, for neutral $(q\bar{q})$ mesons with total quark spin S and orbital angular momentum L, the parity and charge parity were known to be given by $P=-(-1)^L$, $C=(-1)^{L+S}$, so that such mesons could only have the following combinations of quantum numbers: $C=P=(-1)^J$ or $(-1)^{J+1}$, and also $C=(-1)^J$, $P=(-1)^{J+1}$. There cannot be $(q\bar{q})$ states with $C=(-1)^{J+1}$ and $P=(-1)^J$ or $J=0$ and $C=-1$ (if $J=0$ then $S=L=0,1$ and $C=+1$). The exotic sets of meson quantum numbers are therefore as follows: $J^{PC}=0^{+-},0^{--},1^{-+},2^{+-},3^{-+}$, etc. All forms of exotic mesons such as multiquark states, hybrids and glueballs[3)] can have such values of J^{PC}.

1.3. Exotic states of the third kind

These are hadronic states with hidden exotic properties (the so-called cryptoexotic hadrons). They do not have any visible exotic features, and their complex internal structure can only be revealed indirectly by examining particular features of their characteristics, e.g., anomalously small widths, anomalous decay channels, special production modes, and so on. Exotic hadrons of all types can belong to this class.

Search for exotic states plays a particular role in the recent experiments on nanobarn hadron spectroscopy. One of the most complicated problems is the identification of cryptoexotic hadrons, since only indirect dynamic features can give us ideas about the exotic valence structure of these particles. The experimental facilities assigned for the study of exotic hadrons should have large acceptance and high sensitivity, since the cross sections for their production reactions are, as a rule, not very large.

The success of the experiments on the search for exotic hadrons is, to a great extent, determined by the proper choice of the hadronic processes for which from some qualitative arguments it may provide more distinct manifestations of the exotic states. For the reliable selection of the signal against the background the studied process should have as distinct a signature as possible.As a rule, exclusive and semiexclusive reactions of hadron production are chosen when studying their effective mass spectra. Though the cross sections for the exclusive reactions are usually small at high energies, as compared with the cross sections for inclusive reactions, nevertheless the background conditions are more favorable for the search for new resonances in exclusive channels due to the large combinatorial background in the inclusive reactions. This background arises in the effective mass spectrum for the hadron systems under study for all the combinations of secondary particles (many combinations per one event). The combinatorial background makes the selection of resonance signals difficult. In the exclusive and semiexclusive reactions with limited multiplicity the combinatorial background is either absent or strongly reduced.

In spite of all the difficulties in searching for cryptoexotic particles mentioned above, significant progress has been achieved during recent years and several serious candidates for exotic hadrons have been observed [see, e.g., the Proceedings of the Conferences of Hadronic Spectroscopy at KEK (1987),[4] BNL (1988),[5] Ajaccio (1989),[6] and St. Goar-on-Rein (1990) as well as the reviews of Refs. 7–11, where relevant results are summarized]. Most favorable conditions for the selection of exotic hadronic states as compared with usual $(q\bar{q})$ mesons and (qqq) baryons are realized in the analysis of the processes enriched with the gluon contribution. It is also true for the reactions where the production of ordinary particles is suppressed due to some dynamic singularities of the phenomena under study (e.g., by OZI suppression, by big momentum transfer, by the t-channel exchange mechanism, etc.). Reference 11 gives numerous examples of such processes.

Search for cryptoexotic states is linked closely with the experiments in which the structure of the families of ordinary mesons and baryons (radially excited states including) are clarified. Very often the problem of interpreting new resonances cannot be unambiguously solved without such a clarification since exotic particles should be "extra" states, not fitting the scheme of standard mesonic nonets.

The most exciting and impressive events of hadron spectroscopy are taking place now in the mass region of 1–2 GeV, where several states have recently been observed. These states may be particles of a new type—exotic hadrons with composite valent color structure. The experiments on studying rare decays of ordinary $q\bar{q}$ mesons are of great interest as well as the further development of hadron systematics and searches for radially excited $q\bar{q}$ mesons in the given mass region.

In this country the investigations of hadron spectroscopy are mainly being carried out at the IHEP 70 GeV machine at intermediate energies (30–40 GeV), which yielded some interesting and impressive results (see e.g., reviews of Refs. 11, 13, and 14). Unfortunately, at low energy machines (ITEP, JINR) in spite of their previous successes (observation of the $\omega\to\pi^0\gamma$ and $V\to e^+e^-$ decays) the efforts in this very important field are obviously insufficient now. However, the study of mesons and baryons with masses <2 GeV may successfully be carried out at low energies (<10 GeV), where the cross sections for exclusive processes are considerably larger than those for the corresponding reactions at Serpukhov energies. For example we can mention the experiments at BNL and *KEK* at 5–8 GeV, which are quite productive.

It goes without saying that at low energies the experimental difficulties increase as a rule, and the requirements for the apparatus become even more strict as compared with the experiments at the 30–40 GeV energies. In the last case the kinematical forward collimation of the secondary particles and comparatively high tolerances for their energy thresholds allow one to have a more compact and simple setup. It is the aperture of the magnetic spectrometer in the facility that should be enlarged for the experiments at low energies; besides the energy threshold for photons and charged particles should be lowered. Here the γ spectrometers may use scintillation counters of CsI, NaI, BGO, BaF_2 or even simpler and faster systems such as sandwiches of plastic scintillators and lead. High pressure gas Cherenkov counters, solid body wide-aperture multichannel spectrometers to measure Cherenkov light rings (these are known as RICH counters) as well as systems to measure the time-of-flight with ultimate time resolution may be used to identify charged particles. When detecting muon pairs in the electromagnetic decays of the type $\eta \to \mu^+ \mu^- \gamma$ one should use the range detectors to stop and identify muons by their $\mu \to e$ decays (primarily it is true for μ^-).

The decrease of the detection efficiency for the hadronic decays at low energies is particularly unpleasant, since the limited acceptance results in the distortion of the relevant mass spectra. It means that a very thorough calibration with well known processes is necessary. Though all this makes the experiments on hadronic spectroscopy at low energies rather difficult, a significant increase in the yields of the investigated particles in the exclusive processes with small combinatorial background justifies the construction of sophisticated facilities.

This article does not claim to be a systematic review of hadronic spectroscopy in the range of low energies and is somewhat different from the usual style. It gives an extended version of the invited talk at the Conference of the Nuclear Science Department of the Academy of Sciences of the USSR devoted to the problem of fundamental interactions of elementary particles (November, 1990) and the lecture delivered by the author at the ITEP School of Physics (March, 1991). My main goal was to attract attention of those who work at the hadron machines at low energies (and first of all, the ITEP and JINR physicists) to some very interesting unresolved problems of hadron spectroscopy, which may be studied very effectively in experiments in the beams of particles with energy of several GeV. This explains some peculiarities of the article and its tendency to discuss certain problems which reflect the author's personal partiality. The choice of the problems does not claim to be complete and certainly can be enlarged considerably.

2. BARYONIC SPECTROSCOPY IN THE DIRECT CHANNEL

We discuss here resonance reactions with the production of baryonic states in the direct channel (Fig. 1), i.e., generally speaking, very well known processes, which have already been studied for decades, since the first experiments on πp-scattering carried out by Fermi's group, which resulted in the discovery of the first hadronic resonance—$\Delta(1238)$-isobar.

Numerous experiments on elastic scattering, charge exchange and polarization effects in πp and $K^- p$ interactions have made it possible to obtain extensive data on the properties of these processes, to perform a detailed phase-shift analysis, and to observe several tens of baryon states of the type $N(I=1/2)$ and $\Delta(I=3/2)$, as well as $\Lambda^*(I=0)$ and $\Sigma^*(I=1)$ (with strangeness $S=-1$), which appear as s-channel resonances (see Figs. 1a and 1b). A distinctive feature of all these baryons is their strong coupling to the elastic channels ($BR(B \to N\pi)$ or $BR(Y \to K^- N)$ greater than 5%–10%), and, as a consequence, large resonance cross sections. The Breit–Wigner resonance cross section in the direct channel has the well known form

$$\sigma_{res} = \frac{2J+1}{(2s_1+1)(2s_2+1)} \cdot \frac{4\pi}{k^2} BR_{in} BR_{out}; \qquad (1)$$

Here, J is the spin of the resonance, s_1 and s_2 are the spins of the colliding particles, k is their momentum, and BR_{in} and BR_{out} are the branching ratios for the incoming and outgoing channels. Therefore, the resonance cross sections for all known baryons that appear in elastic scattering are estimated as

$$\sigma_{res} = \frac{2J+1}{2} \cdot \frac{4\pi}{k^2} [BR(B \to \pi p)]^2 > 10^{-27} + 10^{-28}\ cm^2. \qquad (2)$$

If there are states weakly coupled to the elastic channel, they cannot be seen in the analysis of the elastic processes because of the large nonresonance background. For instance, for cryptoexotic baryons with hidden strangeness, $B_\varphi = (qqqs\bar{s})$, the decay mode $B_\varphi \to \pi N$ is suppressed by the OZI rule and estimated as $BR(B_\varphi \to \pi p) \sim 10^{-3}$. For those baryons, in elastic πp scattering $\sigma_{res} \sim 10^{-31}$–$10^{-32}$ cm^2, and they cannot be seen in that process. On the other hand, there are good prospects for searches for resonance production of cryptoexotic baryons B_φ with hidden strangeness by high-precision scanning of the cross section in the energy in a number of exclusive processes:

$$\pi^- + p \to B_\varphi(I=1/2) \to \varphi + n, \quad \varphi \to K^+K^- \qquad (3)$$

$$\pi^- + p \to B_\varphi(I=3/2) \to \varphi + \Delta^0(1232), \quad \varphi \to K^+K^-,\ \Delta^0 \to p\pi^- \qquad (4)$$

$$\pi^+ + p \to B_\varphi(I=3/2) \to \varphi + \Delta^{++}(1232), \quad \varphi \to K^+K^-,\ \Delta^{++} \to p\pi^+ \qquad (5)$$

$$\pi^- + p \to B_\varphi(I=1/2) \to \Lambda + K^0, \qquad (6)$$

$$\pi^- + p \to B_\varphi(I=1/2;3/2) \to \Sigma^{*-}(1385) + K^+, \quad \Sigma^{*-} \to \Lambda\pi^- \qquad (7)$$

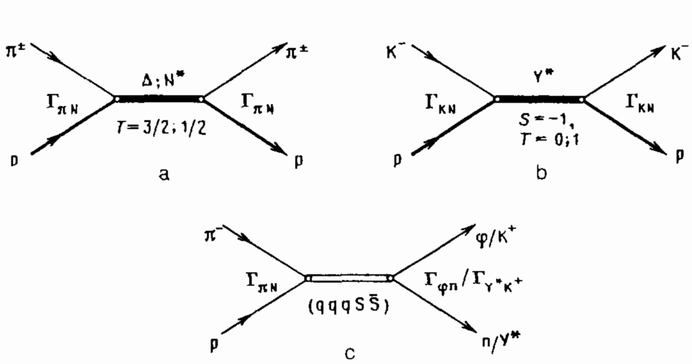

FIG. 1. Diagrams for formation of baryonic resonances in the direct channel: a—baryons in elastic $\pi^- p$ scattering; b—baryons in elastic $K^- p$ scattering; c—exotic baryons B_φ with hidden strangeness in the reactions $\pi^- p \to B_\varphi \to \varphi n$, $K^+ Y^*$.

$$\pi^+ + p \to B_\varphi(I = 3/2) \to \Sigma^{*+}(1385) + K^+, \quad \hookrightarrow \Lambda\pi^+ \tag{8}$$

$$\pi^- + p \to B_\varphi(I = 1/2) \to \Lambda(1520) + K^0 \quad \hookrightarrow pK^- \tag{9}$$

(see Fig. 1c). For that purpose, the reactions (3)–(5), forbidden by the OZI rule, are especially interesting. In fact, nonresonance processes of that kind may be more strongly suppressed by the OZI rule than processes of formation of multiquark hadrons with a complex internal color structure. Therefore the background conditions for distinguishing exotic resonances B_φ in the direct channel (3)–(5) may be much more favorable than that for the usual reactions with large cross sections. This issue was examined in Ref. 11. From the existing experimental data for (3), shown in Fig. 2 (see Ref. 15), it is clear that to search for B_φ baryons the accuracy of these measurements must be considerably improved. For baryons with hidden strangeness the expected branching ratio for this process is $BR[B_\varphi \to \varphi n] > 0.05$–$0.1$. From this, and from Eq. (1), the resonance cross section in (3) should be $\sigma_{(3)}(B_\varphi)_{\rm res} > 10^{-29}$–$10^{-30}$ cm^2.

Simple estimates based on the data in Fig. 2 show that for a scan in the effective-mass range $2.0 < M(\varphi n) < 3.5$ GeV ($1.6 \leqslant p_\pi^{\rm lab} \leqslant 6.0$ GeV/c) and with the step $\Delta m \simeq 30$ MeV, i.e., for measurement of the cross section for the reaction (3) at 50 points with statistics $> (3$–$4) \cdot 10^3$ events at each point, the required data taking time on the accelerator is > 150 h. It is assumed that the π^- beam intensity is $1.5 \cdot 10^6\, \pi^-$ per cycle, that the hydrogen target has thickness 5 g/cm^2 (at least for measurements at $p_\pi > 4$ GeV/c), that the experimental luminosity is $6 \cdot 10^3\, \mu{\rm b} - 1 \cdot h^{-1}$, and that the efficiency of detection of the decay $\varphi \to K^+K^-$ is $\varepsilon_\varphi \simeq 0.25$. The sensitivity of the searches for narrow resonances in the direct channel (3) ($\Gamma \leqslant 30$ MeV) will be about $\sigma(B_\varphi) \simeq 1$–$0.025$ μb in such an experiment for the mass range $2.0 \leqslant m(B_\varphi) \leqslant 3.5$ GeV.[4)] This is 3 to 4 orders of magnitude higher than the sensitivity in searches for baryon resonances in the direct channel in elastic πp scattering or charge exchange. Therefore, one may hope to detect exotic states B_φ for which ${\rm BR}(B_\varphi \to \pi^- p) > 10^{-3} - 10^{-4}$, which appears quite sufficient if we bear in mind the degree to which the decay $B_\varphi \to \pi^- p$ may be suppressed by the OZI rule.

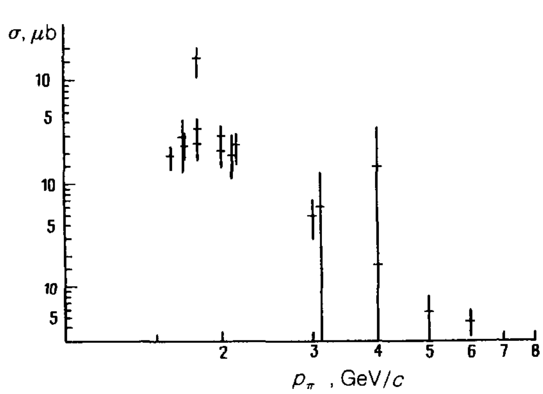

FIG. 2. Compilation of cross sections data for the reactions $\pi^- p \to \varphi n$ at low energies (Ref. 15).

The process (4) should be studied together with (3). For its identification one may use a special guard system surrounding the hydrogen target. It is also necessary to measure the missing mass for the recoil particles.

Besides the processes (3)–(9), it would also be of interest to scan the cross sections for the reactions

$$\pi^- + p \to \omega + n, \quad \hookrightarrow \pi^+\pi^-\pi^0;\ \pi^0\gamma \quad , \tag{10}$$

$$\pi^- + p \to \eta' + n \quad \hookrightarrow \pi^+\pi^-\gamma;\ \pi^+\pi^-\eta \tag{11}$$

and for the corresponding reactions involving the $\Delta(1232)$ isobar. The examination of (10) together with (3) is essential for the interpretation of the measured cross section for this last process from the point of view of the OZI rule. The data for (11) may be used to search for hybrid baryons $B_G = qqqg$ that decay as $B_G \to \eta' n$. This decay mode may have an anomalously large branching ratio. We should mention that some evidence for the possible existence of a very narrow baryon with such a decay mode was obtained in Ref. 16.

We assume that in experiments with scanning of the cross sections for exclusive reactions universal equipment will be used. It should include a wide-aperture magnetic spectrometer with proportional and drift chambers, a γ spectrometer of the hodoscope type, and a system for identification of the secondary particles. With such equipment, together with scanning of the cross sections for the reactions (3) and (4), one can also obtain data on the processes (6), (7), and (9)–(11). The main limitations here are associated with the flux of the trigger events. This places rigid requirements on the data-acquisition system, which should be able to detect $\sim 10^3$ events per cycle, and also on the multilevel system for the production of the triggering signals. Since the cross sections for the other reactions are considerably higher than for (3) and (4), a number of experimental solutions are possible here (the use of a high level of intelligence of the trigger logic, separated exposures, recoil-neutron detection, work with "diluted" trigger signals, and so on).

We should mention that beside a global energy scan, which requires a relatively long measuring time, a scan in a limited mass range is also possible in the direct channel for particular reactions—namely, those for which there are already some indications of the existence of interesting objects with anomalous features. We list several examples of such processes:

1. The reaction (7), in the vicinity of a narrow baryon $N_\varphi(1960)$, which has been observed with the spectrometer BIS-2 in the process of diffractive production $n + N \to N_\varphi^0(1960) + N$, $N_\varphi^0(1960) \to \Sigma^{*-}(1385)K^+$, with $\langle E_n \rangle \simeq 40$ GeV.[17] The scan should be made in the momentum range $1.2 \leqslant p_{\pi^-}^{\rm lab} \leqslant 1.8$ GeV/ c [$1.887 \leqslant m(\Sigma^*(1385)\,K) \leqslant 2.071$ GeV].

2. The reaction $K^- p \to (qqss\bar{s}) \to \Lambda\varphi$ + pions; ΣKK + pions; $\Lambda K\bar{K}$ + pions; $\Xi\bar{K}$ + pions, in the vicinity of a narrow resonance $R(3170)$ with $\Gamma < 20$ Me$\bar{\rm V}$, which has been seen[18] in the backward scattering processes with K beams. The scan should be made in the range $4.5 \leqslant P_K^{\rm lab} \leqslant 5.2$

GeV/c [$3.058 \leqslant m(YK\bar{K} + n\pi) \leqslant 3.270$ GeV].

3. The reaction (11) near the threshold, where there is an indication[16] of the possible existence of a narrow baryonic resonance with mass 1889.3 MeV and $\Gamma < 2$ MeV.

In the phase-shift analysis of the reactions (3)–(10) a major role may be played by angular correlations in cascade decays of the secondary particles (for example, in $\varphi \to K^+K^-$ or $\Sigma^*(1350) \to \Lambda\pi, \Lambda \to p\pi^-$).

3. SEARCH FOR EXOTIC HADRON PRODUCTION IN THE BARYON EXCHANGE PROCESSES

There were many discussions about the possibilities of more effective excitation of inner color degrees of freedom at which the exotic multiquark or hybrid states can be formed, in the processes with large momentum transfers and, in particular, in backward scattering reactions, caused by baryon exchange (see, for example, Refs. 19–21 and review of Ref. 11). The production of exotic states in such processes is expected to be characterized by the cross sections comparable with those of ordinary particles. As an example of such a backward scattering reaction we present the diagram for the hybrid meson production in the process[3a] $\pi^- p \to n_f H$, where $H = (q\bar{q}g)$—see Fig. 3.

Several experiments were carried out in the search for exotic mesonic states in the reactions with baryon exchange—for example $\pi^+ + p \to \Lambda_f + S^{++}$; $\pi^- + n \to p_f + X^{--}$; $(p_f\pi^-)(\bar{p}p_s)$. Here h_f and p_s represent the forward (f) and spectator (s) particles in the laboratory frame.

The upper limits for the cross sections of these processes have been found to lie within the interval from several *nb* up to ≈ 150 nb.

Data on backward production of a number of known mesonic states, on backward elastic $\pi^\pm$ p-scattering and charge exchange reactions ($\pi^- p \to n\pi^0$) have also been obtained.

Extensive research on the backward production of mesons generally, and exotic mesons in particular, in pion-nucleon interactions is impeded by experimental difficulties. The point is that the mesons are emitted in these processes in backward directions in the center of mass system (cf. Fig. 3), so that, in the laboratory frame, the decay products have a soft momentum spectrum and a wide angular distribution. The detection efficiency for these mesons in experiments performed with magnetic spectrometers is usually low. There are also difficulties with the identification of the decay products of mesonic resonances.

All these difficulties can be obviated by investigating the production of mesonic resonances in baryon exchange processes (with N- and Δ-poles) in a proton beam, using reactions of the form

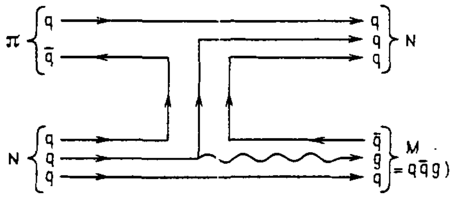

FIG. 3. Diagram for the hybrid meson production in the baryon exchange π^- p reaction.

$$p + N \to M^{++} + [N\pi^- n] \quad (\Delta^-\text{-pole}), \tag{12}$$

$$p + N \to M^+ + [Nn], \; M^+ + [Np\pi^-] \quad (\text{n-pole}, \Delta^0\text{-pole}), \tag{13}$$

$$p + N \to M^0 + [Np], \; M^0 + [Np\pi^0] \quad (\text{p-pole}, \Delta^+\text{-pole}), \tag{14}$$

$$p + N \to M^- + [Np\pi^+] \quad (\Delta^{++}\text{-pole}) \tag{15}$$

(see Fig. 4 and Ref. 21). For example, Fig. 5 is the diagram for the production of hybrid meson $H = q\bar{q}g$ in reactions (13) or (14).

The mesons produced in these reactions are emitted in the forward direction, and can be relatively readily recorded in a large-aperture magnetic spectrometer. The charged particles that emerge from these decays can be identified by a set of gas-filled Cherenkov counters. When reactions such as (12)–(15) are recorded, the experiments have to be carried out at proton energies that are not too high because the cross sections of exclusive processes with baryon exchange fall rapidly with increasing primary energy. The ITEP proton accelerator ($E_p \simeq 10$ GeV) seems quite optimal for this type of experiments.

The cross sections of the processes (13) can be normalized using the data on the two reactions

$$pp \to d\pi^+, \tag{16}$$

$$pp \to d\rho^+ \tag{17}$$

which also involve baryon exchange. The corresponding cross sections were measured in the experiments with CERN precise magnetic spectrometer at $P_p = 21.1$ GeV/c. They are equal to $\sigma(pp \to d\pi^+) = 15.1 \pm 1.5$ nb and $\sigma(pp \to d\rho^+) = 15.9 \pm 2.4$ nb, respectively.[22] The cross sections for the processes $pp \to dA_2(A_1)$, which were also observed in this experiment (see Fig. 6), have similar values. For the reaction (16) there are also some data at $P_p \leqslant 5$ GeV/c, e.g., $\sigma(pp \to d\pi^+)|_{5\ \mathrm{GeV}/c} = (3.4 \pm 0.2)\ \mu$b. It is possible to estimate roughly the value of the cross section $\sigma(pp \to d\pi^+) \simeq 100$–$200$ nb at $P_p \simeq 10$ GeV/c and the similar values of the cross sections for the reactions $pp \to dA_2^+$, dA_1^+, $d\rho^+$.

If one takes in consideration that p and n fusion into deuteron this leads to a decrease of the cross section by more than an order of magnitude, then the expected cross sections for the ordinary meson production in baryon exchange reactions (12)–(15) at the incident proton momentum $P_p \simeq 10$ GeV/c can be at the level of several μb. Thus the search for exotic mesons in baryon exchange reactions (12)–(15) seems to be very promising.

It should be noted that the investigation of the reaction of type (15) with the Δ^{++} isobar exchange opens up new possibilities in searching for charged members of exotic isovector mesonic triplets. We shall demonstrate these possibil-

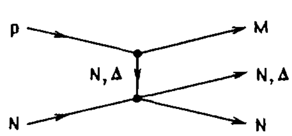

FIG. 4. Diagram for exclusive reactions (12)–(15) with baryon exchange.

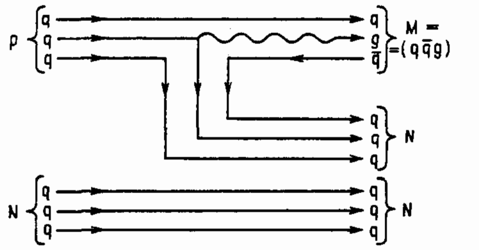

FIG. 5. Diagram for the hybrid meson production in the baryon exchange pN reaction.

ities on the example of C(1480) meson, which is a serious candidate for exotic hadrons. A neutral state, belonging to this exotic multiplet, $C(1480)^0$ has been observed in the "Lepton-F" experiment[23] in the charge-exchange reaction

$$\pi^- p \to C(1480)^0 n; \quad C(1480)^0 \to \varphi\pi^0 \tag{18}$$

The C(1480) meson is characterized by the following basic parameters: $M_C = (1480 \pm 40)$ MeV, $\Gamma_C = (130 \pm 60)$ MeV, $J^{PC} = 1^{--}$; $I^G = 1^+$. At $P_\pi = 32.5$ GeV/c the cross section for the reaction (18) is equal to (40 ± 15) nb. The C(1480) meson exotic nature seems to manifest itself in its strong coupling with the $\varphi\pi$. The properties of this hadron could be explained most naturally in the model which presents C(1480) either as a multiquark meson $|C(1480)\rangle = |(u\bar{u} - d\bar{d})s\bar{s}/\sqrt{2}\rangle$ or as a hybrid state $|C(1480)\rangle = |(u\bar{u} - d\bar{d})g/\sqrt{2}\rangle$.[22-25] The search for the charged partners of C(1480) meson is of great interest in connection with the problem of the exotic meson existence. The results obtained by the SIGMA group[26] point to a possibility of C(1480) to manifest itself in the inclusive reaction $\pi^- + p \to \varphi\pi^- X$; $\varphi \to \mu^+\mu^-$. The selection of φ mesons by their rare decays $\varphi \to \mu^+\mu^-$ has been chosen because of the difficulties in identifying hadronic channels of the φ decays in the inclusive processes due to the combinatorial background.

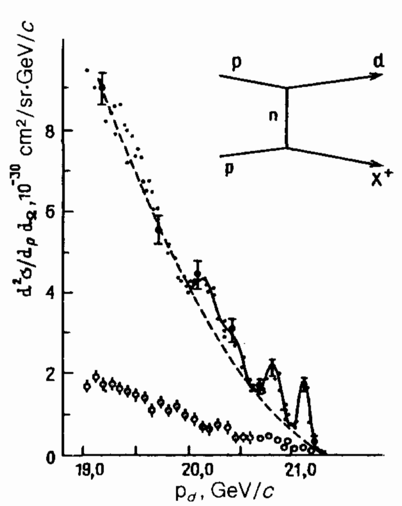

FIG. 6. The momentum spectrum of deuterons in the baryon exchange reaction $p + p \to d + X$ at $p_p = 21.5$ GeV/c and the deuteron angle of emission $\alpha = 40$ mrad (in the laboratory frame). O–background from the hydrogen target walls.[22]

It is obvious that more precise data on $C(1480)^-$ are needed, which in principle may be obtained in the exclusive process

$$\pi^- p \to C(1480)^- p. \quad C(1480)^- \to \varphi\pi^- \tag{19}$$

Similarly to the ρ^- meson production in $\pi^- p \to \rho^- p$, it seems that in the reaction (19) OPE dominance will take place, as the diffractional mechanism is forbidden here because of the G-parity conservation.

The cross sections of the reactions (18) and (19) should be of the same order, decreasing with energy as E_π^{-2}. Therefore for (19) the background situation will be considerably worse than for (18), since the OZI allowed background process $\pi^- p \to (K^+K^-\pi^-)p$ is of diffraction nature and does not decrease with the energy E_π (contrary to the charge-exchange process $\pi^- p \to (K^+K^-\pi^0)n$ which is the main source of the background for the process (18)). The expected background is particularly large when detecting the decay $\varphi \to K^+K^-$ (for $E_\pi \sim 30$ GeV it exceeds the background for the process (18) by almost two orders of magnitude). For the decay $\varphi \to K_S^0 K_L^0$ the background decreases by severalfold because of the fixed charge parity of $K_S^0 K_L^0$ pair ($C = -1$), nevertheless it still remains quite significant.

When using reaction (15) with Δ^{++}-exchange, the search for $C(1480)^-$ mesons will be carried out in the reactions with double charge exchange

$$p + N \to C(1480)_f^- + (Np\pi^+). \tag{20}$$

The background situation in this reaction may be much better than in (19). It is also of great interest to study the production processes of the $\eta\pi, \eta'\pi, \eta\eta, \eta'\eta, \eta\omega$ states[27-29] and of some other systems convenient for the selection of exotic mesons, in proton reactions (13)–(15) with baryon exchange. One should also remember about the search for double charged mesons with open exotics in reaction (12), e.g., in the $\rho^+\rho^+$ system for which very interesting predictions have been obtained (in the study of $\gamma\gamma \to \rho^+\rho^-$; $\rho^0\rho^0$, see Ref. 30 and the review of Ref. 31). The production of the exotic meson $\bar{U}^{++} \equiv (\bar{s}\bar{d}uu) \to \bar{\Lambda}p\pi^+$ (Ref. 32) in the reactions with Y-exchange

$$p + N \to \bar{U}_f^{++} + [\Sigma^- \text{ (or } \Sigma^*(1385))N] \quad \bar{U}_f^{++} \to \bar{\Lambda}p\pi^+ \tag{21}$$

is some modification of the process (12). These reactions would require a somewhat higher proton energy, since for them $E_{p\ \text{thresh}} = 13.6(14.7)$ GeV.

Hence, the investigations of the proton reactions with baryon exchange at the machines with moderate energies (ITEP, JINR, KEK, BNL) open up new possibilities in searching for exotic mesonic states.

4. PRODUCTION OF EXOTIC STATES IN OPE REACTIONS

As is known, a number of recently observed mesons which are candidates for exotic states, are produced in exclusive quasi two-body charge-exchange reactions $\pi^- p \to M^0 n$ with OPE dominance (C(1480)-meson,[23] G(1590) meson,[33] $g_T(2010)$, $g_{T'}(2300)$, $g_{T''}(2340)$ Ref. 34, etc.). The cross sections for these reactions decrease very rapidly with an increase of primary energy (as $\sim E_\pi^{-2}$). Therefore, the experiments in the pion beams with momenta of several GeV/c seem to be very promising for a detailed study of the properties of these states and a search for new exotic mesons. To realize such a program successfully one should provide, as it was already mentioned in the Introduction, sufficiently high detection and identification efficiencies for the decay products of the states under study.

As an example we shall consider the possibilities of investigating C(1480) meson at low energies. During 250 hours of data taking, using the accelerator at the Institute for Theoretical and Experimental Physics, about $2.5\cdot 10^4$ events of the reaction (18) could be detected in the experiments with 5 GeV/c π^- beam at the intensity $\sim 10^6$ π/cycle and detection efficiency for (18) $\varepsilon \simeq 0.10$ (see Table I). Such statistics allow one to carry out partial-wave analysis of this process and to perform a sensitive search for the decays $C(1480)\to\omega\pi^0, K^*, \bar{K}, K^+K^-\pi^0, \rho\eta$, etc., which are essential for the interpretation of the C(1480) meson nature.

Precise experiments on the C(1480) meson production in charge-exchange reaction (18), which proceeds through the OPE mechanism, would allow one (after thorough study of the OPE contribution) to determine the product of branching ratios $K_C = \mathrm{BR}[C(1480)\to\pi\pi]\ \mathrm{BR}[C(1480)\to\pi\varphi]$. The analysis of the data[23] on reaction (18) at 32 GeV/c would yield a rough estimate $K_C = \mathrm{BR}[C(1480)\to\pi\pi]\mathrm{BR}[C(1480)\to\pi\varphi]\simeq(1\text{–}3)\ 10^{-3}$ (Grudtsin, Khadzhimiryan, private communication, see Ref. 25). More precise data which may be obtained in new experiments at low energies and, maybe, also in studying reaction (19) seem to be of great importance. A detailed study of the probabilities of different channels of the C(1480) decay (including the K_C measurement) and the comparison of these results with similar data for the ρ_1'(1450) meson would allow one to investigate the nature of these vector states and at last to determine whether these are two close in mass but different mesons with different valent content or we are dealing with the manifestations of the same state (which is hardly probable). For the discussions of this C(1480)/ρ_1'(1450) problem see Refs. 24,25,35–37.

Since the C(1480) quantum numbers $J^{PC}=1^{--}$ coincide with those of a photon, it would be interesting to look for the production of this particle in electromagnetic processes. In the Ω-spectrometer experiment at CERN ~25 events of the reaction $\gamma p\to(\varphi\pi^0)p$ have been detected within the energy range $20<E_\gamma<70$ GeV and some excess of these events in the region of $M\sim 1.5$ GeV has been observed.[38] From the data of Ref. 38 one may obtain the photoproduction cross section $\sigma[\gamma p\to C(1480)p]$ BR $[C(1480)\to\varphi\pi^0] = (3\pm 1.5)$ nb. More precisely, the upper limit for this cross section is ⩽6 nb, 95% C.L.

The vector meson photoproduction may proceed through the diffraction mechanism (the cross section for this process is practically energy independent) and through OPE. The cross section for the latter process has the form

$$\sigma(\gamma p\to C(1480)p)|_{\mathrm{OPE}}\mathrm{BR}(C(1480)\to\varphi\pi^0) = [0.13\,\mu\mathrm{b/s^2}\,(\mathrm{GeV}^4)]\,(\mathrm{BR}(C\to\varphi\pi^0))^2 \qquad (22)$$

(N. N. Achasov, private communication, see Ref. 25).[5)]

Hence, if the sensitivity of these photoproduction experiments at $E_\gamma\simeq 5$–8 GeV will be ~3 *nb*, one may hope to single out the OPE photoproduction of the C(1480) meson if $\mathrm{BR}[C(1480)\to\varphi\pi^0]>0.10$, and to estimate this branching. Note, that the value of $\mathrm{BR}[C(1480)\to\varphi\pi^0]$ is very important for the interpretation of the C(1480) nature. If this decay probability is large enough (⩾5%), then the exotic nature of C(1480) meson would be quite evident (see Refs. 21 and 37).

5. INVESTIGATION OF RADIATIVE DECAYS OF LIGHT MESONS

Studies of the electromagnetic decays of hadrons play an important role in elementary particle physics. These processes which are determined by the interaction of real or virtual photons with the electric charges of quark fields, permit one to obtain unique information on the nature of various quark configurations in hadrons, on mixing mechanisms in mesonic nonets, on the electromagnetic structure of strongly interacting particles and on a number of their phenomenological characteristics such as magnetic moments, form factors, polarizability and so forth. Recent years are remarkable for a considerable success in this field.

In the investigation of the radiative decays $A\to B+\gamma$

TABLE I. The possibilities of the experimental study of the rare decays of light mesons in the experiments on pion beams with momenta $3\leqslant P_\pi\leqslant 5$ GeV/c.

p_{π^-}, (GeV/c)	$\sigma(\pi^-p\to\omega n)$	N_ω	$\sigma(\pi^-p\to\eta n)$	N_η	$\sigma(\pi^-p\to\eta' n)$	$N_{\eta'}$	$\sigma(\pi^-p\to f_1 n)$	N_{f_1}
3	500	$3\cdot 10^9$	150	$0,9\cdot 10^9$	50	$3\cdot 10^8$	100	$6\cdot 10^8$
4	220	$1,3\cdot 10^9$	100	$0,6\cdot 10^9$	30	$2\cdot 10^8$	50	$3\cdot 10^8$
5	150	$0,9\cdot 10^9$	70	$0,4\cdot 10^9$	25	$1,5\cdot 10^8$		
p_{π^-}, (GeV/c)	$\sigma(\pi^-p\to\varphi n)$	N_φ	$\sigma(\pi^-p\to C^0 n)$	N_C	$\sigma(\pi^-p\to a_2^0 n)$	N_{a_2}	$\sigma(\pi^-p\to f_2 n)$	N_{f_2}
3	5	$3\cdot 10^7$			750	$5\cdot 10^9$	400 – 2000	$(2-10)\cdot 10^9$
4	2	$1,2\cdot 10^7$			350	$2,5\cdot 10^9$	1000	$6\cdot 10^9$
5	1	$0,6\cdot 10^7$	~0,7	$4\cdot 10^6$	300	$2\cdot 10^9$	600	$4\cdot 10^9$

1. σ–reaction cross section (μb).
2. N_ω; N_η; $N_{\eta'}$; N_φ, etc., the number of mesons produced at the setup target during 1000 hours of data taken at the ITEF accelerator.
3. The target: 12 g/cm² LiH or 5 g/cm² liquid hydrogen ($n_p\sim 3\cdot 10^{24}$ p/cm²; the effective number of protons is 2.5 per LiH molecule).
4. The intensity of the pion beam is $n_\pi\sim 1.5\cdot 10^6$ π^-/burst; $2\cdot 10^9$ π^-/hour.
5. The luminosity of the exposure is $6\cdot 10^3\,\mu b^{-1}$ hour^{-1}.
6. The notation of mesons: $f_1 = D/f_1(1285)$; $f_2 = f_2(1270)$; $a_2^0 = a_2(1320)^0$; $C^0 = C(1480)^0$.

both direct and indirect methods of measurements are used. In the direct experiments the particles under study (A) are produced in hadron collisions or in electromagnetic interactions and all their decay products (including photons) are detected immediately in the experimental setups. Among the indirect methods is the Coulomb production method (the "Primakoff effect") based on studying coherent production of hadrons in the Coulomb field of nuclei $B + (Z,A) \rightarrow A + (Z,A)$, i.e., in the process of interaction between the beam particles with virtual ("almost real") photons. The cross section for the Coulomb process is proportional to the radiative width $\Gamma(A \rightarrow B + \gamma)$.

For a straightforward detection of radiative meson decays of the type $A \rightarrow B + \gamma$ (or of more complicated processes) it is necessary to suppress the background from $\pi^0 \rightarrow \gamma\gamma$ with one lost photon. Such a situation occurs in selecting the radiative decay $\eta \rightarrow \pi^+ \pi^- \gamma$ against the background from $\eta \rightarrow \pi^+ \pi^- \pi^0, \pi^0 \rightarrow \gamma(\gamma)$ or in searching for the decay $\omega \rightarrow \pi^+ \pi^- \gamma$ with the background from the main decay channel $\omega \rightarrow \pi^+ \pi^- \pi^0, \pi^0 \rightarrow \gamma(\gamma)$. Here and in what follows (γ) stands for the lost photon. The reliable selection of rare radiative decays requires complete detection of all the decay secondaries (both charged particle and photons), measurement of their momenta and reconstruction of the effective masses with the highest possible resolution. The experimental facility should include a counter guard system with a low photon energy detection threshold, which would cover as large an acceptance as possible. This is quite necessary in order to maximally suppress the background from lost photons. The hodoscope γ-spectrometer which is used for the detection of the radiative decay photons in the working acceptance of the setup (the selection of the events with single photons from $A \rightarrow B + \gamma$, and so on) also serves as a part of the complete guard system. It is worth mentioning that the background from the events with π^0 with lost photons does not produce a narrow peak at the mass position of the particle under study. As an example, we give here the effective mass spectrum of the $\pi^+ \pi^- \gamma$ system obtained in Ref. 39 at the "Lepton-F" setup. This spectrum which is dominated by $\omega \rightarrow \pi^+ \pi^+ \pi^0, \pi^0 \rightarrow \gamma(\gamma)$ decay was used in search for the process $\pi^- p \rightarrow \omega n$ with radiative decay $\omega \rightarrow \pi^+ \pi^- \gamma$. The upper limit for the branching ratio of this decay $BR(\omega \rightarrow \pi^+ \pi^- \gamma) < 4 \cdot 10^{-3}$ (95% C.l.) was obtained in this experiment. This makes it clear that the searches for radiative decays of hadrons may be carried out almost up to the probability level of $< 1\%$ of the main background processes.

Another effective method of suppressing the background from processes with lost photons, which is necessary to select hadron radiative decays, is based on the kinematical constraints of the events and demands an independent determination of the decaying particle emission angles. Missing photons may lead to an uncompensated component of summed transverse momentum of detected decay products of the particle under study with respect to its direction. The corresponding background may be considerably reduced by a cutoff along the uncompensated p_T component. The experiment on the search for the $\omega \rightarrow \pi^+ \pi^- \gamma$ decays at the ASTERIX setup in the annihilation of stopping antiprotons[40] $\bar{p}p \rightarrow \omega\pi^+ \pi^-; \omega \rightarrow \pi^+ \pi^- \gamma$ is an example of successful application of this kinematical method of background suppression. Here the ω meson emission direction is determined very nicely from the kinematics of the annihilation reaction. The upper limit $BR(\omega \rightarrow \pi^+ \pi^- \gamma) < 3 \cdot 10^{-3}$ (95% C.l.) was obtained in these measurements, in which the kinematic cutoffs allowed a suppression of the background by a factor of more than 20.

This kinematic method cannot be applied to search for the radiative decays $M \rightarrow \text{hadrons} + \gamma$ in the charge-exchange reaction $\pi^- + p \rightarrow M + n$ if no recoil neutron is detected and there is no independent method of determining the true direction of M meson emission. This difficulty may, in principle, be overcome, if one detects the neutron n or studies the reaction $\pi^- p \rightarrow M\Delta^0; \Delta^0 \rightarrow p\pi^-$. However in this case the experimental apparatus becomes more complicated and its acceptance is significantly reduced. Therefore for the charge-exchange reactions a maximal increase of the guard system efficiency is more promising.

Recently data on numerous radiative decays were obtained in experiments with hadron beams, in the study of the ω, ρ and φ decays in resonance reactions $e^+ e^- \rightarrow \rho, \omega, \varphi \rightarrow$ (radiative decays) in $e^+ e^-$ colliding beams, in experiments on meson production in the Coulomb field of nuclei and in $\gamma\gamma$ collisions $\gamma\gamma \rightarrow M$ (in the reaction $e^+ e^- \rightarrow e^+ e^- \gamma\gamma \rightarrow e^+ e^- M$). The results of these investigations were examined in the reviews of Refs. 14,41–43.

Bearing in mind the prospect of the study in the very near future of radiative and electromagnetic decays of light mesons in experiments with intense low energy hadron beams and at φ factories with high luminosity we shall deal with some problems existing in this field in more detail. As a starting point for our discussions of proposed low-energy experiments we shall mainly use the results of studying a number of electromagnetic processes, obtained with the "Lepton-G" and "Lepton-F" setups at the Serpukhov machine using meson beams with momenta of 25–32 GeV/c. These results are presented in Table II and in Figs. 7, 8, 10–17 (see Refs. 39, 51 and the reviews of Refs. 14,25, where one can also find the original references). These and some other experiments have demonstrated a possibility of direct study of rare radiative decays of mesons, produced in hadron beams, at values of their branching ratios of $5 \cdot 10^{-3}$–10^{-5} (depending on the background conditions).

5.1. Vector and pseudoscalar meson decays

The radiative decays of vector and pseudoscalar mesons

$$V \rightarrow P + \gamma, \qquad (23)$$

and

$$P \rightarrow V + \gamma \qquad (24)$$

are the simplest electromagnetic processes, initiated by dipole magnetic transitions. They are essentially determined by the quark magnetic moments.

The radiative widths for (23) and (24) have the form

$$\Gamma(V \rightarrow P\gamma) = (4/3)\mu_{VP}^2 K_\gamma^3, \qquad (25)$$

and

$$\Gamma(P \rightarrow V\gamma) = 4\mu_{VP}^2 K_\gamma^3; \qquad (26)$$

Here K_γ is photon energy ($K_\gamma = (M_V^2/2M_P^2)/2M_V$ for (23) or $K_\gamma = (M_P^2 - M_V^2/2M_P$ for (24)), μ_{VP} is the

TABLE II. The results of studying rare electromagnetic decays of mesons in the experiments using the "Lepton G" and "Lepton F" setups.

A. Decay	Number of events	Branching ratio (BR) BR (exp)	BR (VDM)	Parametrization of the transition formfactor F (q^2)	Formfactor slope b, GeV^{-2}	b (VDM), GeV^{-2}
$\eta \to \mu^+\mu^-\gamma$	600	$(3{,}1 \pm 0{,}4)\cdot 10^{-4}$	$(3{,}08 - 3{,}13)\cdot 10^{-4}$	$(1 - q^2/\Lambda_\eta^2)^{-1}$, $\Lambda_\eta = 0{,}72 \pm 0{,}09$ GeV	$1{,}9 \pm 0{,}4$	1,8
$\eta' \to \mu^+\mu^-\gamma$	33	$(8{,}9 \pm 2{,}4)\cdot 10^{-5}$	$(7{,}0 - 8{,}7)\cdot 10^{-5}$	In qualitative agreement with ρ-pole approximation	$1{,}7 \pm 0{,}4$	1,5
$\omega \to \pi^0\mu^+\mu^-$	60	$(9{,}6 \pm 2{,}3)\cdot 10^{-5}$	$8\cdot 10^{-5}$	$[1 - (q^2/\Lambda_\omega^2)]^{-1}$, $\Lambda_\omega = 0{,}65 \pm 0{,}03$ GeV	$2{,}4 \pm 02{,}$	1,7
$\eta \to \mu^+\mu^-$	27	$(6{,}5 \pm 2{,}1)\cdot 10^{-6}$	$(4 - 5)\cdot 10^{-6}$			
$\eta \to \pi^0\mu^+\mu^-$	—	$\le 5\cdot 10^{-6}$	90%-c. l.			
$\eta' \to \pi^0\mu^+\mu^-$	—	$\le 6\cdot 10^{-5}$				
$\eta' \to \eta\mu^+\mu^-$	—	$\le 1{,}5\cdot 10^{-5}$				
$\eta \to \pi^0\mu^+\mu^-\gamma$	—	$\le 3\cdot 10^{-6}$				
B. Decay	Number of events	BR (exp.)		Γ(exp) KeV		
$D/f_1(1285) \to \varphi\gamma$	19	$(0{,}9 \pm 0{,}2 \pm 0{,}4)\cdot 10^{-3}$		$23 \pm 5 \pm 10$		
$D/f_1(1285) \to \rho\gamma$	—	$\le 5\cdot 10^{-2}$		≤ 1250		95%-c. l.
$\omega \to \pi^+\pi^-\gamma$	—	$\le 4\cdot 10^{-3}$		≤ 34		
$f_2'(1525) \to \varphi\gamma$	—	$\le 4\cdot 10^{-3}$		≤ 304		

[1] The table presents not only the experimental data, but the predictions for BR and transition formfactors using the VDM.
[2] Formfactor slope $b = dF/dq^2|_{q^2=0} = 1/\Lambda^2$.
[3] The values for the formfactor slopes obtained in the experiments on studying formfactors in the reaction $e^+e^- \to e^+e^-\eta(\eta')$ with tagged virtual photons (i.e., $\gamma\gamma^* \to M$) in the spacelike region are found to be $b_\eta = (1.4 \pm 0.3)$ GeV^{-2} and $b_{\eta'} = (1.6 \pm 0.2)$ GeV^{-2} (Ref. 53). Within the error bars these data agree with the results obtained using the "Lepton G" spectrometer and the VDM predictions.
[4] The radiative decay characteristics of $f_1(1285)$ meson obtained by MARK III are $\Gamma(f_1(1285) \to \rho\gamma) = 1700 \pm 700$ KeV; BR$(f_1(1285) \to \rho\gamma) = (6.8 \pm 2.8)\cdot 10^{-2}$ (Ref. 52).
[5] The upper limit obtained by ASTERIX is BR$(\omega \to \pi^+\pi^-\gamma) < 3\cdot 10^{-3}$ (95% c.l.).

effective magnetic moment of the VP transition, determined by the quark magnetic moments, by coefficients for the superpositions of different $\bar{q}q$ pairs in the wave functions of V and P mesons and by overlapping integrals I_{VP}. Table III presents the main results of studying these decays (see Refs. 1,14,41–44 and references cited there) and the comparison of the experimental data with some theoretical models.[45] As is seen from Table III these theoretical estimates differ quite significantly among themselves. However, it is expected that, in the near future, the lattice QCD calculations will allow one to improve considerably the accuracy of the theoretical predictions for decays (23) and (24) and make it

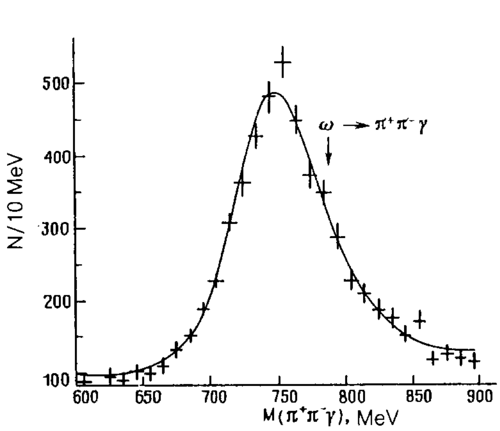

FIG. 7. Effective mass spectrum of $\pi^+\pi^-\gamma$ events in the region of the spurious ω peak, i.e., the events mainly due to the process $\pi^- p \to \omega n, \omega \to \pi^+\pi^-\pi^0, \pi^0 \to \gamma(\gamma)$ with one lost photon ("Lepton-F" data, see Ref. 39). The arrow shows the expected position of the peak for the radiative decay $\omega \to \pi^+\pi^-\gamma$.

almost $\sim 5\%$.[47] This is the reason why the precision study of decays (23) and (24) seems to be very important now.

The prospects of studying radiative decays of some neutral mesons in the experiments where the charge exchange reaction $\pi^- p \to Mn$ at $P_\pi \simeq 3$–5 GeV/c is the source of such mesons may be estimated from the data of Table I, which gives the effective meson fluxes, produced in the setup target during 1000 hours of measurements at the ITEP accelerator. These fluxes make up $N_\omega = (1\text{–}3)10^9\omega$; $N_\eta = (0.5\text{–}0.9)\cdot 10^9\eta$; $N_{\eta'} = (1.5\text{–}3)\cdot 10^8\eta'$, $N_\varphi = (0.6\text{–}3)\cdot 10^7\ \varphi$, $N_{f_1(1285)} = (3\text{–}6)\cdot 10^8\ f_1(1285)$, $N_{a_2} = (2\text{–}5)\cdot 10^9 a_2$.

In order to compare the opportunities provided by the experiments on the search for rare mesonic decays in hadron beams and those available at the φ factories with the luminosity $(10^{32}\text{–}10^{33})$ cm^{-2}s^{-1}, let us estimate the resonance cross sections for the vector meson e^+e^- production in the direct channel. The resonance cross sections have the form

$$\sigma_V = \sigma(e^+e^- \to V) \doteq \frac{12\pi}{M_V^2}\Gamma_{ee}/\Gamma_{tot}.$$

From here and the data of Ref. 1 it follows that $\sigma_\varphi = 4.5\cdot 10^{-30}$ cm^2, $\sigma_\omega = 1.7\cdot 10^{-30}$ cm^2, $\sigma_\rho = 0.6\cdot 10^{-30}$ cm^2. During 1000 hours of data taking at the φ factory the effective fluxes of vector mesons will amount to $N_\varphi = 1.6\cdot(10^9\text{–}10^{10})\varphi$, $N_\omega = 6\cdot(10^8\text{–}10^9)\omega$, $N_\rho = 2\cdot(10^8\text{–}10^9)\rho^0$. The $\varphi \to \eta\gamma$ decay may be used as a tagged η meson source at the φ factory, with $N_\eta = 1.3\cdot 10^{-2}N_\varphi = 2\cdot(10^7\text{–}10^8)\eta$. *Hence the experiments* at the φ factories seem to be much more advantageous for the study of the φ meson rare decays. The sensitivity of

TABLE III. Branching ratios and partial widths for the radiative decays of vector and pseudoscalar mesons.

Decay	$BR_{exp}\cdot(rad)$ [1]	Γ exp·(rad) (KeV) [1]	Theoretical predictions [2]			Perspective of new experiments [3]		
			[45a]	[45 b]	[45c]	ε	N events in hadron beams	N events on φ-factory
$\omega\to\pi^0\gamma$	$(8,5\pm0,5)\cdot10^{-2}$	717 ± 42	825	649	$\underline{800}$			
$\omega\to\eta\gamma$	$(4,7^{+2,2}_{-1,8})\cdot10^{-4}$	$4,0^{+1,8}_{-1,6}$	8,3	5,2	5,8(5,0)	0,06	$(3-10)\cdot10^4$	$(2-20)\cdot10^4$
$\rho^\pm\to\pi^\pm\gamma$	$(4,5\pm0,5)\cdot10^{-4}$	67 ± 7	58	68	84(73)			
$\rho^0\to\pi^0\gamma$	$(7,9\pm2,0)\cdot10^{-4}$	118 ± 30						
$\rho^0\to\eta\gamma$	$(3,8\pm0,7)\cdot10^{-4}$	57 ± 10	54	40	52			
$\varphi\to\pi^0\gamma$	$(1,31\pm0,13)\cdot10^{-3}$	$5,8\pm0,6$	6,0	0	6,1	0,2	$(1,6-8)\cdot10^3$	$(4-40)\cdot10^5$
$\varphi\to\eta\gamma$	$(1,28\pm0,06)\cdot10^{-2}$	56 ± 3	71	72	71(67)	0,05	$(4-20)\cdot10^3$	$(1-10)\cdot10^6$
$\varphi\to\eta'\gamma$(4)	$<4,1\cdot10^{-4}$	$<1,8$	0,86		0,31(0,29)	0,05	20 — 100	$(0,6-6)\cdot10^4$
$\eta'\to\pi^+\pi^-\gamma$(5) ($\rho\gamma$)	$(30,0\pm1,5)\cdot10^{-2}$	62 ± 7	78	87	103			
$\eta'\to\omega\gamma$	$(3,00\pm0,31)\cdot10^{-2}$	$6,2\pm0,9$	7,2	8	10,9(9,6)	0,10	$(5\div10)\cdot10^5$	
$K^{*\pm}\to K^\pm\gamma$	$(0,101\pm0,009)\cdot10^{-2}$	$50\pm4,5$	33	78	80(83)			
$K^{*0}\to K^0\gamma$	$(0,230\pm0,020)\cdot10^{-2}$	115 ± 10	130	122	135(137)			

1) Averaged experimental data for branching ratios and widths for the radiative decays of the type $V\to P\gamma$(23) and $P\to V\gamma$(24); $BR_{exp.}$ (rad.) and Γ_{exp} (rad) are obtained from Ref. 1 (PDG).

2) Theoretical estimates (Refs. 45a–45c) are based on different versions of the quark model. They differ in the values for a number of model parameters (quark magnetic moments, mixing angles in nonets, $SU(3)$-breaking, etc.). Besides, different assumptions on the transition from the analyses at the quark level to real hadron decays are used in these models. The underlined value for Γ in Ref. 45c is used for absolute normalization.

3) The table gives estimates of the expected statistics for some radiative decays (23) and (24), which may be obtained in the experiments with low energy hadronic beams (see Table I)) and at φ factories of DAΦNE type (with the expected luminosity of $L=10^{32}$–10^{33} cm^{-2} s^{-1}) per 10^3 hours of data taking; ε—detection efficiency for the investigated decay (estimate, which takes into account BR for secondary decays, e.g., $\varphi\to K^+K^-$; $\eta\to\gamma\gamma$, etc.).

4) Limits for BR($\varphi\to\eta'\gamma$) and $\Gamma(\varphi\to\eta'\gamma)$ are given with 90% c.l. For the estimation of the possibilities to investigate this process the expected value for BR($\varphi\to\eta'\gamma)\simeq1.7\cdot10^{-5}$ ($\Gamma(\varphi\to\eta'\gamma)\simeq0.3$ KeV) was used.

5) For the decay $\eta'\to\pi^+\pi^-\gamma$ the contribution from $\eta'\to\rho\gamma$ is dominating; however the experimental data admits a noticeable nonresonance contribution to this process ($\lesssim25\%$,—see Ref. 46).

the e^+e^- colliding beams experiments with ω mesons turns out to be approximately the same, but with η mesons it is by an order of magnitude worse than that with the hadron beams. When planning the experiments on rare decays (particularly on rare φ meson decays) one should bear in mind that the experiments at the φ factory DAΦNE (Frascati) may start in 1995 (at the luminosity $L=10^{32}$ cm^{-2} s^{-1}).

The last three columns in Table III contain the information about possible statistics for some radiative decays of type (23) and (24), which can be obtained in experiments at the low energy hadron machines and in the e^+e^- colliding beams. These statistics will be quite sufficient to improve greatly the measurement accuracy of radiative widths of corresponding decays, which will be restricted by the systematic errors. Search for the $\varphi\to\eta'\gamma$ decay which has not yet been observed, will be of particular interest.

5.2. Some other radiative decays

The study of the radiative decays $\varphi\to\gamma[\delta/a_0(980)]$ and $\varphi\to\gamma[S^*/f_0(975)]$ may play an important role in clarifying the nature of the known scalar states $\delta/a_0(980)$ and $S^*/a_0(975)$. The probabilities of these processes depend strongly on the fact, whether the scalar mesons are ordinary ($q\bar{q}$) mesons, 4 quark states of the type $(u\bar{u}-d\bar{d})s\bar{s}/\sqrt{2}$ and $(u\bar{u}+d\bar{d})s\bar{s}/\sqrt{2}$ or $K\bar{K}$ molecules.[48-50] Certain estimates for the opportunities available here are listed in Table IV.

Let us now deal with the radiative decays of the axial-vector mesons and, in particular, with the decays

$$D/f_1(1285)\to\varphi\gamma, \quad (27)$$

and

$$D/f_1(1285)\to\rho\gamma. \quad (28)$$

TABLE IV. Possible experiments on the search for radiative decays of scalar and axial-vector mesons.

Decay	Quark structure of the a_0 and f_0 mesons	BR	The expected statistics in future experiments		
			ε	Hadron beams with momenta 3–5 GeV/c	φ–factory $L=10^{32}$–10^{33} cm^{-2}s^{-1}
$\varphi\to\delta/a_0(980)+\gamma$	$(u\bar{u}-d\bar{d})s\bar{s}/\sqrt{2}$	$\sim2\cdot10^{-4}$ (theor.)	0,05	$\sim60-300$	$1,5\cdot10^4-1,5\cdot10^5$
$\to\eta\pi^0$	$(u\bar{u}-d\bar{d})/\sqrt{2}$	$\sim2\cdot10^{-5}$ (theor.)		$\sim6-30$	$1,5\cdot10^3-1,5\cdot10^4$
$\varphi\to S^*/f_0(975)+\gamma$	$(u\bar{u}-d\bar{d})s\bar{s}/\sqrt{2}$	$\sim2\cdot10^{-4}$	0,15	$\sim2\cdot10^2-10^3$	$5\cdot10^4-5\cdot10^5$
$\to\pi^+\pi^-$	$(u\bar{u}-d\bar{d})$ $s\bar{s}$	$5\cdot10^{-5}$		$\sim50-250$	10^4-10^5
$D/f_1(1285)\to\varphi\gamma$		$(0,9\pm0,4)\cdot10^{-3}$	0,05	$\sim2\cdot10^4$	
$D/f_1(1285)\to\rho\gamma$		$<5\cdot10^{-2}$ [51] $(6,8\pm2,8)\cdot10^{-2}$ [52] $>10^{-2}$ (theor.)	0,1	$\sim4\cdot10^5$	
$D/f_1(1285)\to\omega\gamma$		$>10^{-3}$ (theor.)	0,05	$\sim2\cdot10^4$	

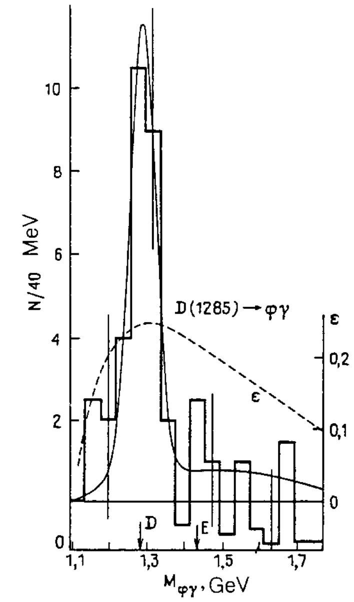

FIG. 8. Effective mass spectrum of the $\varphi\gamma$ system in the reaction $\pi^- p\to(\varphi\gamma)n$ after all the necessary cutoffs ("Lepton-F" data, see Refs. 39, 51). The arrows show the tabulated mass values for the $D/f_1(1285)$ meson and $E(1420)$ meson. The dashed curve and the right-hand scale show the acceptance of the apparatus.

In the experiments at the "Lepton-F" facility when studying the reactions $\pi^- p\to\varphi\gamma n$; $\varphi\to K^+K^-$ and $\pi^- p\to\rho\gamma n$; $\rho\to\pi^+\pi^-$ the following process was singled out

$$\pi^- p\to D/f_1(1285)n, \quad \llcorner\!\to \varphi\gamma\to K^+K^-\gamma \tag{29}$$

i.e., the decay (27) was observed (see Fig. 8) and its branching ratio was measured; at the same time the upper limit for the branching of the decay (28) was also established[39,51,52] (see all these data in Table IV). The decay (28) was detected during the investigation of the $\rho\gamma$ system mass spectrum in $J/\psi\to\gamma(\gamma\rho)$ at the MARK III setup.[52] The branching obtained for the radiative decay (28) in Ref. 52 is significantly higher than the predictions of the majority of the theoretical models and hence it should be studied further. The experiments using the low-energy hadron beams, studying the reaction (29) and analogous processes for the $\rho\gamma$ and $\omega\gamma$ systems would allow one to increase considerably the statistics for the radiative decays $D/f_1(1285)$, to study the structure of the axial meson nonet, and to determine the mixing angle in this nonet, which seems to differ greatly from the ideal one (Table IV). It will be very interesting to search for radiative decays $E(1420)\to\varphi\gamma$ and $f_1'(1530)\to\varphi\gamma$. These problems are discussed in more detail in Refs. 25,51.

5.3. Electromagnetic lepton decays and light meson structure[14]

For the conversion decays of mesons

$$A\to B+\gamma_V\to Bl^+l^- \tag{30}$$

(the examples are $\pi^0\to e^+e^-\gamma$, $\eta\to\mu^+\mu^-\gamma$, $\omega\to\pi^0\mu^+\mu^-$, etc.) the production probability for a lepton pair with the effective mass $m_{l^+l^-}$ is proportional to the probability of emitting a virtual photon with time-like 4-momentum, $q^2=m^2_{l^+l^-}$. The dynamical electromagnetic structure arising in the vertex of the $A\to B$ transition determines the probability of emitting such photon. This electromagnetic structure caused by the cloud of virtual states in the transition region is characterized by a specific formfactor, which is called the transition formfactor. Figure 9 presents the diagrams of the processes where these formfactors are determined.

The data on the electromagnetic structure of the $A\to B$ transition vertex can be obtained by studying the probability of the decay $A\to B+(l^+l^-)$ as a function of the squared effective mass of the lepton pair, $q^2=m^2_{l^+l^-}$, i.e., from the analysis of the mass spectrum $d\Gamma/dq^2$ in (30).[6)]

If particles A and B were structureless objects, it would be possible to calculate the mass spectrum of lepton pairs $[d\Gamma/dq^2]_{QED}$ with high accuracy, by the methods of quantum electrodynamics. The complicated internal structure of particles modifies this spectrum, where the contribution is now made by the transition formfactor:

$$(d\Gamma/dq^2)_{exp}=(d\Gamma/dq^2)_{QED}|f_{AB}(q^2)|^2. \tag{31}$$

By comparing the measured spectrum of lepton pairs in decays (30) with QED calculations for pointlike particles, it is possible to determine experimentally the square of the transition formfactor $|f_{AB}(q^2)|^2$ in the time-like region of the momentum transfer. As a rule, a normalized form factor is used

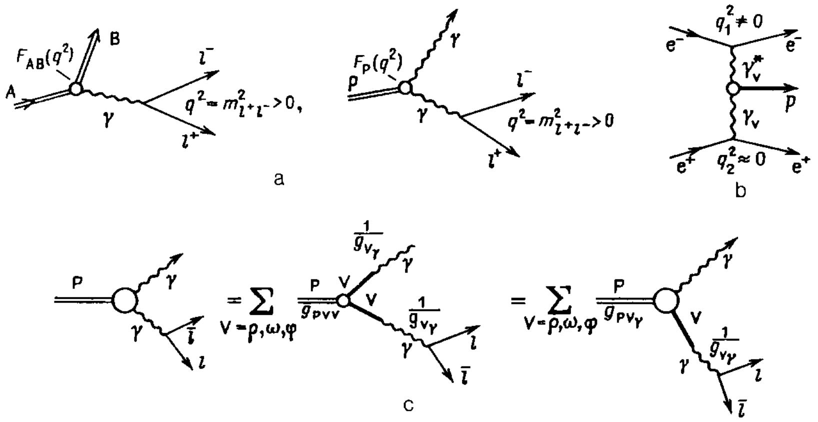

FIG. 9. Meson transition formfactors. a) Diagrams for the decays $A\to Bl^+l^-$ and $P\to l^+l^-\gamma$. b) Diagram for the pseudoscalar meson production process in $\gamma\gamma^*$ interactions with tagged virtual photon (i.e. in the reaction $e^+e^-\to e^+e^-\gamma\gamma^*\to e^+e^-P$). c) Transition formfactor in the VDM (vector dominance model) approximation.

$$F_{AB}(q^2) = f_{AB}(q^2)/f_{AB}(0) \qquad (32)$$

(where $F_{AB}(0) = 1$), determined from the normalization of the leptonic pair spectrum in (30) to the width of the corresponding radiative decay

$$\frac{d\Gamma(A \to Bl^+l^-)}{dq^2 \cdot \Gamma(A \to B + \gamma)} = \{QED\} |F_{AB}(q^2)|^2. \qquad (33)$$

The transition formfactors in the decays $\eta \to \mu^+\mu^-\gamma$, $\eta' \to \mu^+\mu^-\gamma$ and $\omega \to \pi^0\mu^+\mu^-$ were studied in the experiments on the "Lepton-G" setup (see the review of Ref. 14 and the references given there). The charge-exchange reactions $\pi^- p \to \eta(\eta',\omega)n$ at 25 and 32 GeV/c were the sources of η, ω and η' mesons. The total number of η mesons produced in the charge-exchange reactions was $2 \cdot 10^7$, and of η' and ω mesons $\sim 10^7$. The corresponding results are presented in Table V and in Figs. 10–17. These were the experiments on "Lepton-G" facility, which showed for the first time that the slope of the transition formfactors at $q^2 \to 0$ were in good agreement with the VDM (vector dominance model) predictions (see Figs. 14–17). These results were confirmed by the data obtained with colliding e^+e^- beams for the processes with tagged virtual photons in the space-like region of momentum transfer (see diagram of Fig. 9).[53]

The goal of a wide program at the SATURNE machine (Saclay) is the investigation of the rare lepton decays of η mesons at the so-called "η meson factory." The source for the tagged η mesons in these experiments was the reaction

$$p + d \to {}^3He + \eta \qquad (34)$$

on the liquid deuterium target, near the threshold, where the cross section for this process turned out to be sufficiently large, and the background was quite small.[54] To single out the recoil ^{3}He nuclei and to identify the η mesons in the rection $p + d \to {}^3He + X^0$ with the precision missing mass technique one uses a focusing magnetic spectrometer and time-of-flight measurements. If a proton beam with the intensity of 10^{12} p/s is used the η meson counting rate should be $10^5\ s^{-1}$ or $\sim 10^{10}\ \eta$/day.

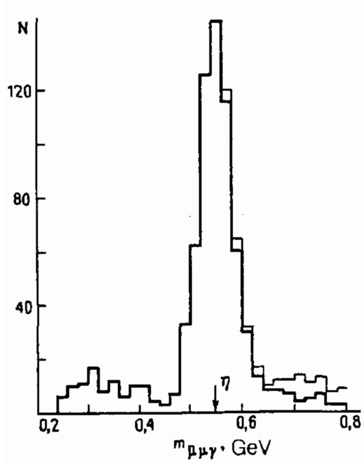

FIG. 10. The $(\mu^+\mu^-\gamma)$ mass spectrum in the reaction $\pi^- p \to (\mu^+\mu^-\gamma)n$ for the events with photon energy $E_\gamma > 1.5$ GeV. The peak corresponds to decay $\eta \to \mu^+\mu^-\gamma$. The arrow points to the table value for η mass. The thin-line histogram is for all events. The thick-line histogram is for the events with $m^2_{\mu^+\mu^-} < 0.24$ GeV^2 (used to define the transition formfactor). N is the number of events per 20 MeV mass interval.

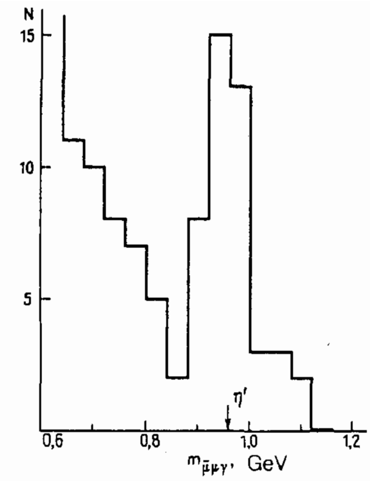

FIG. 11. Mass spectrum of the $(\mu^+\mu^-\gamma)$ system in the reaction $\pi^- p \to (\mu^+\mu^-\gamma)n$ in the range higher than the η mass. The arrow points to the table value of the η' mass. N is the number of events per 40 MeV mass interval.

At present the experiments at the "η factory" investigate the decay $\eta \to \mu^+\mu^-$, whose identification requires two range muon telescopes and measurement of the η missing mass in reaction (34). The aperture for the detection of the $\eta \to \mu^+\mu^-$ decays is $\varepsilon \simeq 3.8\%$ and the counting rate for this decay will be ~ 130 events/day. Later the range spectrometers for muon detection will be replaced by two magnetic spectrometers. This will give a possibility to identify the decays $\eta \to \mu^+\mu^-\gamma, \eta \to \pi^0\mu^+\mu^-$ using the double missing mass technique and selecting the events with $m(X^0) = 0$ (photons) and $m(X^0) \simeq m_{\pi^0}$ for the process $\eta \to \mu^+\mu^- X^0$. Precise measurements of the η formfactor in the $\eta \to \mu^+\mu^-\gamma$ decay are planned to be carried out with the statistics by two orders of magnitude higher than that in the "Lepton-G" experiments. The sensitivity in searching for

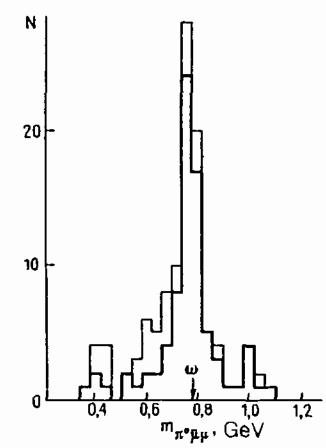

FIG. 12. The $(\pi^0\mu^+\mu^-)$ mass spectrum in the reaction $\pi^- p \to (\pi^0\mu^+\mu^-)n$ for the events with $m^2_{\mu^+\mu^-} < 0.4$ GeV^2 (this region was used for the study of the transition formfactor). The peak corresponds to decay $\omega \to \pi^0\mu^+\mu^-$. The arrow points to the table value of the ω mass. N is the number of events per 40 MeV bin. The outer and inner histograms correspond to the threshold γ energies $(E_\gamma)_{thres.}$ equal to 1 GeV and 1.4 GeV, respectively. The background reduces with the increase of $(E_\gamma)_{thres.}$.

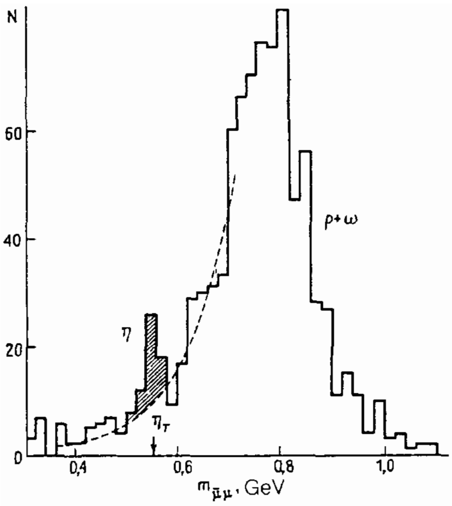

FIG. 13. The $\mu^+\mu^-$ pair mass spectrum in the reaction $\pi^- p \to \mu^+\mu^- n$. The arrow points to the tabulated value of the η mass. The dashed curve fits the background. N is the number of events per 20 MeV bin. The shaded peak corresponds to the $\eta \to \mu^+\mu^-$ decay.

the $\eta \to \pi^0\mu^+\mu^-$ will be $\simeq 5\cdot 10^{-8}$. Reference 54 gives a more detailed program for further studies of rare η meson decays at the SATURNE η factory.

In Table V we examine prospects for studying the conversion decays of the $P \to l^+l^-\gamma, V \to Pl^+l^-$ type for $\eta,\eta',\omega,\varphi$ mesons and some other rare electromagnetic decays in the low energy hadron beam experiments (see data of Table I) as well as in the experiments at the φ factories[55] and at the η factory at Saclay. Although the muon conversion decays have certain advantages for the study of the transition formfactors (see discussions of this problem in Ref. 14), in the low energy experiments the conversion decays with electrons (e.g., $\omega \to \pi^0 e^+e^-, \eta \to e^+e^-\gamma$, etc.) may also be of certain interest. In spite of the necessity to take into account the radiative corrections, electron conversion processes may be more advantageous as compared with muon conversion decays because of the difficulties in identifying soft muons.

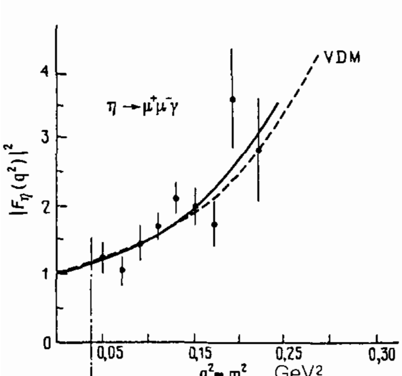

FIG. 14. Data on the η meson electromagnetic transition formfactor $F_\eta(q^2)$ in $\eta \to \mu^+\mu^-\gamma$ decay. Points are experimental values for $|F_\eta(q^2)|^2$. The solid curve is the result of fitting the experimental data with the pole dependence $K(1-q^2/\Lambda_\eta^2)^{-2}$, where $\Lambda_\eta = (0.72 \pm 0.09)$ GeV. The dashed curve is the VDM prediction. The K factor takes into account the systematic error in the absolute normalization of $|F_\eta|^2$.

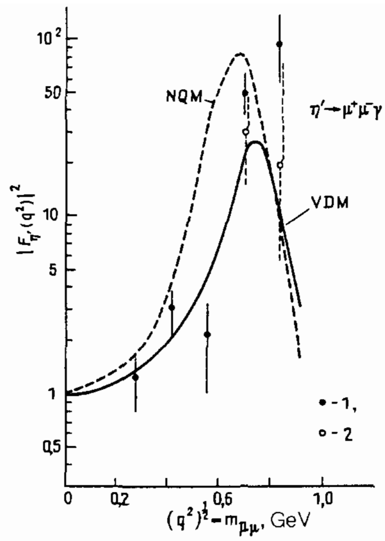

FIG. 15. Data on the electromagnetic transition formfactor of the η' meson (in the decay $\eta' \to \mu^+\mu^-\gamma$): ●—are experimental values for the formfactor squared $|F_{\eta'}(q^2)|^2$; O are the same but with a maximal correction for the 20% background under the peak of $\eta' \to \mu^+\mu^-\gamma$ in Fig. 11 (under the assumption that the whole background in the $\mu^+\mu^-$ system lies in the range of ρ meson mass). The solid curve was calculated using the VDM. The dashed curve is the prediction of the nonlocal quark model (NQM) (Ref. 60).

As the data from Table V show, all the experiments considered may provide extensive statistics for the events of the electromagnetic conversion decays, necessary for a thorough study of the transition formfactors of neutral mesons. These experiments will advance considerably the investigation of the electromagnetic structure of light mesons, which 10 years ago was performed at "Lepton-G" setup at IHEP.

6. SEARCH FOR ISOTOPIC INVARIANCE VIOLATIONS

In experiments using the ITEP machine one can also investigate a number of phenomena, connected with the vio-

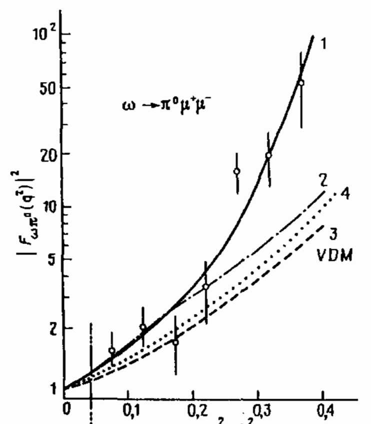

FIG. 16. Data on the electromagnetic transition formfactor for the $\omega\pi^0$ vertex. The points are the experimental values for $|F_{\omega\pi^0}(q^2)|^2$. Curve *1* is the result of fitting the experimental data by the pole formula $|F_{\omega\pi^0}(q^2)|^2 = K(1-q^2/\Lambda_\omega^2)^{-2}$; $\Lambda_\omega = 0.65 \pm 0.03$ GeV. The K factor takes into account the systematic error in the absolute normalization of the experimental quantities. Curve *2* is the prediction of the model with a modified ρ-propagator.[61] Curve *3* has been calculated using the VDM. Curve *4* is the prediction of the nonlocal quark model (Ref. 60).

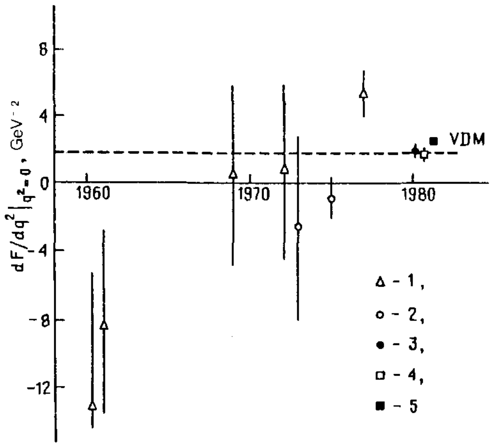

FIG. 17. The world results on measuring the electromagnetic transition formfactor slopes of some neutral mesons $[dF/dq^2]|_{q^2\simeq 0}[\mathrm{GeV}^{-2}]$: △ is for $\pi^0\to e^+e^-\gamma$ decays; ○ is for $\eta\to e^+e^-\gamma$ decays; ● is for $\eta\to\mu^+\mu^-\gamma$ decay; □ is for $\eta'\to\mu^+\mu^-\gamma$ decay; ■ is for $\omega\to\pi^0\mu^+\mu^-$ decay; dashed line is VDM prediction for $\eta\to\mu^+\mu^-\gamma$ formfactor slope $[dF_\eta/dq^2]|_{q^2\simeq 0;\mathrm{VDM}} = 1.8\ \mathrm{GeV}^{-2}$ (VDM predictions for other decays are close to this value—see Table II). For Refs. on the data see the review of Ref. 14.

lation of isotopic invariance. They are, first of all, caused by strong interactions and manifest themselves in the difference of the current masses of u and d quarks (m_u/m_d lies within the limits of 0.3–0.6). Some contribution to the violation of isotopic invariance is also made by electromagnetic processes. Although at the quark level the extent of isotopic invariance violation is determined by the ratio $(m_d - m_u)/m_d \sim O(1)$ in hadron physics this violation is greatly reduced and becomes of the order of $(m_d - m_u)/m \sim 10^{-2}$. Here m is the constituent mass of a u or d quark. However in certain cases one may expect stronger violations of isotopic invariance, e.g., in the mixing of neutral isovector and isoscalar states of a meson nonet whose masses are very close.[56]

Indeed, if isotopic invariance does take place, then in the meson nonet there exist states $M^0_{I=1} = (u\bar{u} - d\bar{d})/\sqrt{2}$ and $M^0_{I=0} = (u\bar{u} - d\bar{d})/\sqrt{2}$.[7)] Because of equal contribution from $u\bar{u}$ and $d\bar{d}$ pairs to the wave functions of $M^0_{I=1}$ and $M^0_{I=0}$ mesons the matrix elements for the decays $M\to K^+K^-$, $M\to K^0\bar{K}^0 (M = M^0_{I=1,0})$ turn out to be equal. For the widths $\Gamma(M\to K^+K^-)$ and $\Gamma(M\to K^0\bar{K}^0)$ one should take into account a slight difference in the phase spaces for these two decays. The isotopic invariance violation leads to the mixing of $M^0_{I=0}$ and $M^0_{I=1}$ states. This mixing destroys the equality of $u\bar{u}$ and $d\bar{d}$ components in the wave functions of neutral mesons. Two new physical states M^0_H and M^0_L arise with their quark compositions $M^0_H, M^0_L = (a_{H,L}u\bar{u} + b_{H,L}d\bar{d})$. Here H and L denote the heavier and lighter states in the neutral "isoscalar–isovector doublet." The fraction of the lighter $u\bar{u}$ component becomes larger than the fraction of the $d\bar{d}$ component for the M^0_L meson and smaller for the M^0_H meson, i.e., $a_L > 1, |b_L| < 1$ (for M^0_L) and $a_H < 1, |b_H| > 1$ (for M^0_H). Therefore for the $K\bar{K}$ decays of these mesons we have the following relation

$$\Gamma(M_L \to K^+K^-) > \Gamma(M_L \to K^0\bar{K}^0),$$
$$\Gamma(M_H \to K^+K^-) < \Gamma(M_H \to K^0\bar{K}^0). \tag{35}$$

In particular it has been shown in Ref. 56 that for the tensor meson nonet a considerable violation of isotopic invariance takes place in kaon decays of the $f_2^0(1270)$ and a_2^0 (1320) mesons

TABLE V. Conversion decays of light mesons $A\to Bl^+l^-$.

Decay	BR (exp.)	BR (theor.)	The expected statistics in future experiments[1)]			
			ε	Hadron beams with momenta 3–5 GeV/c	φ–factory	η-factory
$\eta\to e^+e^-\gamma$	$(5,0\pm 1,2)\cdot 10^{-3}$		0,1	$(2,5-5)\cdot 10^5$	10^4-10^5	
$\eta\to\mu^+\mu^-\gamma$	$(3,1\pm 0,4)\cdot 10^{-4}$	$3,1\cdot 10^{-4}$	0,1	$(1,5-3)\cdot 10^4$	$6\cdot 10^2-6\cdot 10^3$	
$\eta\to\mu^+\mu^-$	$(6,5\pm 2,1)\cdot 10^{-6}$	$>4,0\cdot 10^{-6}$ (2)	0,25	$<(1-2)\cdot 10^3$	30 — 300	$\sim 10^4$
$\eta\to\pi^0e^+e^-$	$<4\cdot 10^{-5}$		0,05	$<(1-2)\cdot 10^3$	$<50-500$	
$\eta\to\pi^0\mu^+\mu^-$	$<5\cdot 10^{-6}$		0,05	$<(1-2)\cdot 10^2$	$<5-50$	$<10^2$
$\eta'\to e^+e^-\gamma$	—		0,1	—	—	—
$\eta'\to\mu^+\mu^-\gamma$	$(1,1\pm 0,3)\cdot 10^{-4}$	$(7,0-8,7)\cdot 10^{-5}$	0,1	$(1,5-3)\cdot 10^3$	—	—
$\eta'\to\pi^0e^+e^-$	$<1,3\cdot 10^2$	—	0,05	—	—	—
$\eta'\to\pi^0\mu^+\mu^-$	$<6\cdot 10^{-5}$	—	0,05	$<(5-10)\cdot 10^2$	—	—
$\eta'\to\eta e^+e^-$	$<1,2\cdot 10^{-2}$	—	0,02	—	—	—
$\eta'\to\eta\mu^+\mu^-$	$<1,5\cdot 10^{-5}$	—	0,02	$<50-100$	—	—
$\omega\to\pi^0e^+e^-$	$(5,9\pm 1,9)\cdot 10^{-4}$	$7,6\cdot 10^{-4}$	0,05	$4\cdot 10^4-10^5$	$2\cdot 10^4-10^5$	
$\omega\to\pi^0\mu^+\mu^-$	$(9,6\pm 2,3)\cdot 10^{-5}$	$6,9\cdot 10^{-5}$	0,05	$4\cdot 10^3-10^4$	$2\cdot 10^3-10^4$	
$\omega\to\eta e^+e^-$	—	$(3,4\pm 1,4)\cdot 10^{-6}$	0,02	100 — 200	40 — 400	
$\omega\to\eta\mu^+\mu^-$	—	$(8,4\pm 3,6)\cdot 10^{-10}$	0,02	—		
$\varphi\to\pi^0e^+e^-$	$1,2\cdot 10^{-4}$	$1,3\cdot 10^{-5}$	0,05	4 — 20	10^3-10^4	
$\varphi\to\pi^0\mu^+\mu^-$	—	$1,9\cdot 10^{-6}$	0,05	0,6 — 3	$1,5\cdot 10^2-1,5\cdot 10^3$	
$\varphi\to\eta e^+e^-$	$(1,3^{+0,8}_{-0,6})\cdot 10^{-4}$	$1,1\cdot 10^{-4}$	0,02	1,5 — 75	$3,5\cdot 10^3-3,5\cdot 10^4$	
$\varphi\to\eta\mu^+\mu^-$	—	$5,7\cdot 10^{-6}$	0,02	1 — 5	$2\cdot 10^2-2\cdot 10^3$	
$a_2\to\omega\mu^+\mu^-$	—	$2\cdot 10^{-4}-10^5$	0,05	10^3-10^4		

[1)] The expected statistics for several conversion decays of light mesons is evaluated here for future experiments with hadronic beams with low momenta (3–5 GeV/c), for φ factory of DAΦNE type with the luminosity $L = 10^{32}$–10^{33} cm^{-2} sec^{-1} and for SATURNE η-factory (in the reaction with tagged η: $p + d\to\eta + {}^3\mathrm{He}$). The duration of the measurements is $\sim 10^3$ hours; ε—the setup efficiency for the decays under study (taken into account branching ratios of the decays of secondary particles).
[2)] This is the unitarity limit for the branching ratio of the decay $\eta\to\mu^+\mu^-$.

$$\Gamma(a_2^0(1320) \to K^+K^-) = 0{,}86\Gamma(a_2^0(1320) \to K^0\overline{K^0}), \quad (36)$$

$$\Gamma(f_2^0(1270) \to K^+K^-) = 1{,}30\Gamma(f_2^0(1270) \to K^0\overline{K^0}). \quad (37)$$

If the isotopic spin were conserved, then the numerical multipliers in the right-hand side of (36), (37) would have been 1.05 and 1.06, which is determined by the kinematic factors ($\Gamma \sim p^5$, where p is the momentum of decaying particles). Hence, significant violations of the isotopic invariance in a_2^0 and f_2^0 decays is predicted (~20–25%).[56] This does not contradict experimental data, according to which $\Gamma(f_2^0 \to K^+K^-)_{exp} = (1.68 \pm 0.35)\cdot\Gamma(f_2^0 \to K^0\overline{K}^0)_{exp}$ (see Ref. 56, where the data of Ref. 57 are used). However one should bear in mind, that the measurements for $f_2^0 \to K^+K^-$ and $f_2^0 \to K^0\overline{K}^0$ were performed in different experiments and may contain additional systematic errors. Therefore it is of extreme importance to carry out these measurements with the same setup, paying particular attention to systematic errors.

Since the decays $M^0 \to K^+K^-$ and $M^0 \to K_S^0K_S^0, K_S^0 \to \pi^+\pi^-$ are of different topology then to reduce the systematic errors it would be expedient to study simultaneously these decays of the $f_2^0(1270)$ and the $a_2^0(1320)$ mesons, for which the effects of isotopic invariance violation have different signs. Then isospin nonconservation will manifest itself in the difference of the ratio $R = [BR(a_2^0 \to K^0\overline{K}^0)/BR(a_2^0 \to K^+K^-]/[BR(f_2^0 \to K^0\overline{K}^0)/BR(f_2^0 \to K^+K^-]^{-1}$ from unity (the value expected from (36) and (37) for R is 1.5).[8)]

Very similar masses of f_2^0 and a_2^0 mesons and small values of their decay branchings along the K meson channels make these measurements rather complicated. To separate the contributions from f_2^0 and a_2^0 one can additionally use different mechanisms of meson production in the reactions

$$\pi^- p \to f_2^0(1270)n, \quad (38)$$

and

$$\pi^- p \to a_2^0(1320)n. \quad (39)$$

The process (38) is dominated by one pion exchange that results in a very narrow t-distribution of events in this reaction. For the reaction (39) the OPE mechanism is forbidden and the t-distribution is much broader. Therefore the selection of the events with $|t'| < 0.05$ (GeV/c)^2 strongly suppresses the contribution of the reaction (39) compared with (38). On the other hand the cutoff at $|t'| > 0.25$–0.40 (GeV/c)^2 suppresses the contribution of the OPE process (38) and enriches the statistics with the events from (39). The very extensive statistics of the events with f_0^2 and a_0^2 mesons, that can be obtained in the study of (38) and (39) at low energy (see Table I) makes such measurements quite possible. At 5 GeV/c one can detect $\sim 10^6$ decays of $f_2^0 \to K\overline{K}$ or $a_2^0 \to K\overline{K}$ during ~100 hours of data taking.[9)]

Relation (37) may also be checked in the processes when only the production of particles with $I = 0$ takes place (e.g., in the reaction $dd \to {}^4He = f_2^0$). However such experiments seem to be even more complicated.

It is very interesting also to study the suppressed decays $\eta' \to 3\pi$, which are forbidden by the isotopic invariance. The decay $\eta' \to 3\pi^0$ was observed earlier in experiments using the GAMS-2000 setup[58] with the branching ratio BR $(\eta' \to 3\pi^0) = (1.6 \pm 0.4)\cdot 10^{-3}$. The decay $\eta' \to \pi^+\pi^-\pi^0$ was not seen up to now because of a strong background of $\rho\pi$ events in the region of η' mass. Only an upper limit for its branching was established: $BR(\eta' \to \pi^+\pi^-\pi^0) < 5\cdot 10^{-2}$.[1]

When selecting the reaction $\pi^- p \to \eta'(938)n$ near the threshold in experiments with the precision time-of-flight neutron spectrometer, that allows one to detect η' mesons using the missing mass technique, one can achieve a very high mass resolution ($\sigma \sim 0.05$ MeV, see Ref. 59). Such resolution will greatly reduce the nonresonance background from $\rho\pi$ and 3π and will also allow one to measure $BR(\eta' \to \pi^+\pi^-\pi^0)$ and $BR(\eta' \to 3\pi^0)$ and to find the ratio of these decay probabilities which is of great interest for the study of the mechanisms of isotopic invariance violation and for the determination of the current mass ratio for u and d quarks.

7. CONCLUSION

In the next decade precision experiments in hadron spectroscopy will be advancing on a wide scale in many laboratories. The ITEP and JINR physicists may make an important contribution in this field if they create new good facilities for this research program and do it as soon as possible.

1) Here and in what follows we use the following notation: q-light u,d,s quarks (or u,d quarks if s quarks are specially indicated), g-gluons.

2) We use ordinary spectroscopic notation for $q\bar{q}$ system: $(n^{2s+1}L_J)$. The states with $L = 0$ are referred to as S, with $L = 1$ as P, with $L = 2$ as D and so on.

3) In the case of gg glueballs, the possible values of J^{PC} from the above sets of exotic quantum numbers are $J^{PC} = (2n+1)^{-+}$, where $n = 0;1$, etc. The whole exotic set of quantum numbers is possible for ggg glueballs.

4) Here we have given the limits for the resonance cross sections at the 3σ level from the nonresonance background, determined from the data of Fig. 2.

5) This approximation is true for $BR(C \to \omega\pi^0) < BR(C \to \varphi\pi^0)$, as is expected for the four-quark exotic meson. The experimental data give a less strict limitation: $BR(C \to \omega\pi^0) < 2BR(C \to \varphi\pi^0)$.[23] Therefore for a complete analysis it is important to measure simultaneously the processes $\gamma + p \to C(1480) + p$; $C(1480) \to \varphi\pi^0$; $\omega\pi^2$.

6) For neutral A and B mesons one-photon transitions (30) are allowed if the charge parities A and B have opposite signs

7) For simplicity we assume that an ideal mixing takes place. However, this assumption is not essential: the admixture of $s\bar{s}$ quarks in $M_{I=0}^0$ practically does not change the results obtained below.

8) It is also possible to calibrate the setup for these topologies with the decays. $f_2'(1525) \to K^+K^-, K^0\overline{K}^0$, using the reaction $K^- p \to f_2'(1525)\Lambda$. The $f_2'(1525) \simeq (s\bar{s})$ meson is decaying with practically identical probabilities along the channels K^+K^- and $K^0\overline{K}^0$.

9) The analysis of the experimental data[57] shows that the contributions from f_2^0 and a_2^0 mesons in the experiments in different regions of t may be separated at least when studying the decays of $f_2^0(1270)$ mesons (measurements in the region of small t). For the $a_2^0(1320)$ meson the situation is not so obvious because of a possible contribution from the rescattering effects for (38) in the region of large t. It goes without saying that when selecting the contribution form f_2^0 (or a_2^0) the data phase analysis should be used (see, e.g., Ref. 57).

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