Self-action of partly coherent laser radiation

V. A. Aleshkevich, G. D. Kozhoridze, and A. N. Matveev

M. V. Lomonosov State University, Moscow
(Submitted 19 Mar. 1990; resubmitted 12 May 1991).
Sov. Phys. Usp. **161**, 81–131 (September 1991)

This review pictures the current state of the problem of propagation of random light fields in optically transparent media having Kerr and thermal nonlinearity. Models are presented of random laser beams and pulses; the correlation properties of the fluctuating light fields and the fundamental theoretical methods of solving the stochastic nonlinear wave equations of quasioptics are described. Spatial instability, self-focusing, and defocusing, the transformation of the spatial statistics of incoherent beams, and the wind refraction of random fields are examined. Quadratic and cubic dispersion, lag of the nonlinear response, non-steady-state nature of the nonlinear polarization, and compression of random pulses are analyzed. Attention is paid to problems of the mutual influence of spatial and temporal fluctuations of the light field in a nonlinear medium.

INTRODUCTION

The creation and practical application of high-power lasers has over the past two decades stimulated theoretical and experimental studies in the field of statistical nonlinear optics, since the radiation field of such lasers fluctuates both in space and in time.

Statistical problems in optics were formulated in the first half of our century. They have facilitated the development of the theory of coherence of light fields of ordinary sources, which are generators of random waves.

We are indebted for the understanding of the fundamental statistical phenomena of linear optics mainly to Rayleigh, Zernike, Brown, Twiss, etc.[117,118,120]

The range of physical phenomena and processes accompanying the interaction of partly coherent light fields with nonlinear media is considerably more varied and extensive, while being incomparably more complex from the mathematical standpoint. This has enabled the singling out from nonlinear optics of an independent field of studies–statistical nonlinear optics.[6,121]

We should include as one of the most important approaches in this field, above all, the analysis of the propagation of partly coherent laser radiation under conditions of self-action in nonlinear media. The fundamental physical essence of this nonlinear phenomenon is that the fluctuating light field induces fluctuating optical inhomogeneities, yet correlated with the field. These inhomogeneities, along with the existing natural random inhomogeneities of the medium itself (not correlated with the field), can substantially transform the most important statistical characteristics of the laser radiation, among which primarily the spatial (angular) and frequency spectra are considerably modified, while the energy is redistributed over the spectrum. In combination with diffraction of light beams and dispersion of light pulses, such a nonlinear-diffraction-dispersion propagation of partially coherent radiation is accompanied by a set of laws, which are generally statistical, and which are more complex and varied than in the propagation of single-mode and single-frequency wave beams and pulses.

Fluctuations of the width of a beam and the duration of a pulse, distortion of the transverse profile of a beam down to the appearance of a speckle structure, transformation of the envelope of a pulse, fluctuation of the intensity of radiation, restriction of the limiting possibilities of compression of pulses in nonlinear dispersive elements, and nonlinear focusing of beams–this is far from a complete list of the fundamental physical phenomena the taking into account of which proves decisive, in a number of cases, in creating high-power laser systems, energy-transport systems, information systems, environmental probing, etc.

The complexity of the problem being studied arises, on the one hand, from the optimal choice of models of fluctuating light fields adequate to the emission of high-power lasers, and on the other hand, from the rather great laboriousness of solving stochastic nonlinear wave problems.

While the model representation of light fields is rather generally accepted, the methods and approaches applied for solving a problem under study are varied and approximate. For this reason difficulties arise in a number of cases in the quantitative comparison of the results obtained by them, although, without question, the fundamental physical laws established by using them offer a correct picture of the physical phenomena and processes.

This review presents a picture of the current state of the problem of propagation of light waves bounded in space and fluctuating in time in physically transparent media having a cubic nonlinearity of Kerr type and a thermal nonlinearity caused by the heating of the medium owing to dissipation of the energy of the laser radiation. Naturally the selection of, and the concept used in the presentation of the material in the five sections are not free from the influence of our own scientific interests.

Section 1 discusses the generally adopted models of random laser beams and pulses, and describes the correlation properties of light fields.

Section 2 formulates the mathematical posing of the problem under study, which is based on the quasioptical description of propagation of diffracted beams and dispersed wave packets. The fundamental theoretical methods are discussed that are applicable for solving the stochastic nonlinear wave equations of quasioptics. Their merits and defects

 0038-5670/91/090777-27$01.00

are pointed out, and the regions of applicability are determined.

Section 3 is devoted to problems of nonlinear diffractive propagation of randomly modulated laser beams and pulses. Spatial instability, self-focusing and defocusing, and the transformation of the spatial statistics of beams incoherent in space in media having a Kerr nonlinearity are discussed. Steady-state and non-steady-state thermal self-action and the wind refraction of random fields resulting from self-action are discussed.

The great variety of physical phenomena that determine the character of the propagation of partly coherent wave packets in media having a Kerr-type nonlinearity is discussed in Sec. 4. Here the broad set of problems of nonlinear transformation of picosecond random pulses in fiber light guides is analyzed, with account taken of the lag of the nonlinear response, the non-steady-state nature of the nonlinear polarization, quadratic and cubic dispersion, etc. Results are also given of the theoretical study of the limiting possibilities of compression of random pulses, which unquestionably is of great practical significance.

Finally, the closing Sec. 5 gives the results of studies of the past three years on the mutual influence of spatial and temporal fluctuations of a light wave in nonlinear media. Much attention is paid to elucidating the factors that convert certain forms of fluctuations into others, and vice versa.

To keep within a reasonable size of the article, the presentation of all five sections, at times concise, is conducted according to the following general scheme: physical formulation of the problem, substantiation of the applied method of study, and omitting intermediate calculations, discussion of the final result and treatment of it on the basis of a unified approach to analyzing the problem of self-action (SA) of high-power laser radiation.

Anticipating the presentation of the main material, we shall give a brief bibliographic presentation of the fundamental studies whose authors have made a substantial contribution to the study of the problem under consideration.

The first studies of the self-action of partly coherent radiation in regular media were performed analytically, mainly by the method of perturbation theory.

Bespalov and Talanov[16] first studied the instability of perturbations of the variation in media having a cubic nonlinearity. Application of the perturbation method enabled them to trace the initial stage of the transformation of the spatial structure of a plane wave modulated in the entrance cross section[16] and distributed in the medium.[6] They showed that such radiation in a nonlinear medium is unstable with respect to small amplitude-phase perturbations. The experiments of Refs. 19, 20, and 41, in which small-scale self-focusing, or decomposition of the beam into filaments was observed, were subjected to such an interpretation.

Somewhat later Akhmanov and Chirkin,[6] and also Lyakhov[18] conducted an analysis of the stability of a laser beam in a nonlinear medium. It was established that the critical power for self-focusing of partly coherent beams is larger than for single-mode beams.

At this same time Vlasov *et al.*[24] established the quadratic dependence of the critical power on the initial divergence of the beam.

The aberration-free description of the behavior of the width and transverse correlation radius of a beam having a broad space-time spectrum performed by Pasmannik[21] and by Akhmanov and Lyakhov[22] was the next step in understanding the laws of nonlinear propagation of partly coherent beams.

The moments of the light field provide a considerable amount of information on the statistics of light beams.[23] Vlasov, Petrishchev, and Talanov[24] developed a method of moments to study the self-action of spatially incoherent beams. However, the fundamental difficulty that arises here is the need to solve an infinite system of coupled equations for the moments of the field. It takes the introduction of extra restrictions to uncouple them.

Numerical methods have enabled giving a more complete description of the evolution of a multimode beam, and began to be developed effectively at the beginning of the eighties.[32-36] The use of the method of random trials (Monte Carlo) made it possible to study the transformation of the statistics of the distribution of the intensity and phase of the radiation, the dispersion of its fluctuations, and to reveal the conditions under which the distribution laws of the wave field are transformed.

At the same time, numerical methods, being highly laborious, have by no means displaced analytical studies.

Chirkin and his associates proposed a method of integrating along trajectories,[26-28] and on this basis studied the influence of weak noise[26] and random phase modulation[27] on the nonlinear propagation of ultrashort pulses. They predicted and found experimentally[28] an initial decrease in the range of spatial coherence in the thermal defocusing of a beam. An analogous result was also obtained using the method of successive approximations.[31]

In the mid-eighties emphasis was placed on elucidating the role of induced fluctuations of the refractive index in the process of nonlinear transformation of random fields.[31,49,50] It was shown that, in the nonlinear-diffraction and nonlinear-dispersion transformation of fields, fluctuations of the refractive index in the channel of propagation lead to a broadening of the beam and spreading of the pulse, independently of the sign of the nonlinearity. The study of the nonlinear mutual influence of the signal and noise components in the "signal + noise" field model has lent clarity to the understanding of the course of such incoherent nonlinear effects.[30,57]

The invention and widespread practical application of quartz fiber light guides has stimulated both experimental and theoretical studies in the field of statistical nonlinear fiber optics.

Progress in experimentation in the creation of optimal fiber-optic systems for information transfer and the obtaining of ultrashort light pulses, etc., were made possible by the widespread application of the methods of nonlinear optics, especially in the branch involving the self-action of light. At the same time, it presented a set of problems to the theory involving the choice of the optimal parameters of the optical waveguides as a function of the initial characteristics of the light pulse to be transmitted.

The existence of a space-time analogy made it possible to a certain degree to transfer a number of the results on the self-action of diffracted light beams to the propagation of dispersed wave packets. However, the analogy disappears if the light pulse propagates in a fiber in a region of anomalous dispersion of the material of the latter.[70]

Already in the early studies,[73,74] without taking account of the fluctuations of the nonlinear increment to the refractive index, the nonlinear-dispersion propagation of random pulses was studied in the second approximation of dispersion theory. These fluctuations were taken into account in Refs. 26, 27, and 75 by using the method of integration over trajectories. The complex character of the transformation of a "signal + noise"-type pulse was studied by the authors of Refs. 26 and 29.

For reducing optical losses, it is promising to use radiation with a wavelength of the order of the critical value. In this case, in which the cubic dispersion becomes dominant, the space-time analogy disappears, and we are dealing with a new class of physical phenomena that accompany the propagation of the light pulse.[30,82]

Analytical and numerical studies have shown that the peak of the pulse becomes asymmetrized and retarded, with redistribution of energy over the frequency spectrum, and complex transformation of the time coherence over the duration of the pulse, etc.

The random fluctuations of the fields of light pulses have compelled reexamination of the limiting possibilities of fiber systems for pulse compression.[77,94,112,113] For example, a slight impairment of the time coherence of the initial pulse substantially decreases its coefficient of self-compression.[112,113]

The authors of the studies cited above have examined separately the transformation of either the space or the time structure of the laser radiation.

For light fields with space-time modulation (both regular and random) under conditions of competition of nonlinear, diffraction, and dispersion effects, the nonlinear transformations of the space and time characteristics are interrelated. Therefore a number of studies[29,57,109] has appeared in recent years in which the effects of this reciprocal influence has been studied.

However, the solution of this type of space-time nonlinear wave problems is still far from completion.

1. STATISTICAL DESCRIPTION OF RANDOM LIGHT FIELDS

The need for a statistical description of the radiation of high-power lasers, which generally are multimode optical generators, arises from the existence of quantum fluctuations of the field and the polarization. The spectral properties of the radiation of lasers depend on the probability of excitation of the intrinsic oscillations and the magnitude of the interaction between them. The superposition of the latter at the output of the laser amounts to laser radiation modulated in time and space with a limited time and space coherence.

The view that has evolved up to now on the space and time statistics of laser multimode radiation, which is widely used in this and in many other studies, stems from an entire series of experimental and theoretical studies, e.g., Refs. 1–11. We note only that the problems of formation of the time statistics of multimode radiation have been reflected in Refs. 1–3 and the review, Ref. 4, its transformation in nonlinear media in Refs. 5–7; a study of the spatial statistics was conducted in Refs. 8–11.

1.1. The field of multimode lasers

The field intensity of the light wave at the output of a laser amounts to a superposition of the intrinsic oscillations of the resonator

$$E(\mathbf{r}, t) = \sum_{m,n,l} E_{0,mnl} \exp[i(\omega_{mnl} t + \varphi_{mnl})] \tag{1.1}$$

having random amplitudes E_0 and phases that are functions of the coordinate $\mathbf{r}$ and the time t. Therefore, in the general case the complex amplitude A of the light field $E = A(\mathbf{r}, t)e^{i\omega t}$ (ω is the mean frequency) is a random function. Most often one uses its general representation as a superposition of signal and noise

$$A(\mathbf{r}, t) = I_0^{1/2} F(\mathbf{r}, t)(1 + \xi(\mathbf{r}, t)), \tag{1.2}$$

Here I_0 is the characteristic value of the mean intensity, the complex function $F(\mathbf{r}, t)$ determines the mean space-time modulation of the light wave, and $\xi(\mathbf{r}, t)$ is a random complex field having a zero mean value [over the ensemble (1.2)].

1.2. Fluctuations of the light field

The transformation of the space-time statistics and the mean scales of the field in the nonlinear medium depend in a certain way on the character of the fluctuations of the field $\xi = \rho e^{i\varphi}$, which are determined both by the fluctuations of ρ (amplitude fluctuations), and by the fluctuations of φ (phase fluctuations).

Two situations can occur here:

a) As is generally known, with a large number of weakly coupled modes, the statistics of the complex field ξ approaches a normal distribution. This implies the presence of amplitude-phase fluctuations, with the probability densities of the envelope and the phase

$$\omega(\rho) = \frac{\rho}{\langle\rho\rangle} e^{-\rho_2/2\langle\rho\rangle^2}, \quad \omega(\varphi) = \frac{1}{2\pi} \tag{1.3}$$

corresponding to a Rayleigh and a uniform distribution.

b) Models are widely used of light fields with non-Gaussian statistics. In this situation one considers either amplitude or phase fluctuations with normal distribution laws. The difference in the course of the physical phenomena in the nonlinear medium here is highly significant, since only the amplitude fluctuations (in contrast to the phase fluctuations) induce random optical inhomogeneities in the propagation channel.

The fundamental laws of transformation of the space and time statistics in the nonlinear medium are characterized by the lowest moments of the amplitude of the field–the space-time correlation function (STCF) of the field and the intensity. To do this, one must define the corresponding lowest moments of the field $\xi(\mathbf{r}, t)$. If the statistics of the latter is Gaussian, then all the moments are defined in terms of the second moment of the STCF of the field. Under the assumption of statistical homogeneity, and an isotropic, and steady-state nature of the field, the second moment is

$$\langle\xi(\mathbf{r}_1, t_1)\xi^*(\mathbf{r}_2, t_2)\rangle = \sigma^2 \gamma_\xi(\Delta\mathbf{r}, \Delta t), \tag{1.4}$$

where $\Delta\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\Delta t = t_1 - t_2$, and σ^2 is the variance of the fluctuations.

1.3. Numerical simulation of light fields

The computer simulation of light fields having a given space-time statistics is based on various algorithms,[12] among which the most effective and frequently used are:

a) The method of sliding summation,[13] which enables one to construct the fields $\xi(\mathbf{r}, t)$ in the process of obtaining realizations of the light field with a given spectrum, which makes it possible to solve stochastic applied problems of nonlinear and atmospheric optics. The systematic error of the method declines rapidly with increasing overlap δ of the correlated random fields used in the formation of the random realizations of $\xi(\mathbf{r}, t)$.

b) The algorithm of linear transformation, in which the model field is given by a set of random realizations having a given covariation matrix $\langle \xi(\mathbf{r}_i, t_i)\xi^*(\mathbf{r}_j, t_j)\rangle$, which corresponds to the correlation function of the field (1.4) at a discrete set of points.[14] Figure 1 shows the histograms of the intensity distribution $I = \rho^2$ and the phase at the axis of the beam as obtained by averaging over 200 realizations. Their small deviation from exponential and uniform distributions indicates that the field being simulated $\xi = e^{i\varphi}$ has a normal distribution law.

c) The method of canonical expansions,[15] which employs a representation of the form

$$\xi(\mathbf{r}, t) = \sum_n (a_n + ib_n)\left(\frac{\lambda_n}{2}\right)^{1/2} \psi_n(\mathbf{r}, t), \tag{1.5}$$

Here a_n and b_n are statistically independent series of pseudorandom numbers having a normal distribution of probability density, and λ_n and ψ_n are the eigenvalues and the eigenfunctions of the homogeneous Fredholm equation

$$\int \langle \xi\xi^*\rangle \varphi_n(\Delta\mathbf{r}, \Delta t)\mathrm{d}\Delta\mathbf{r}\mathrm{d}\Delta t = \lambda_n \varphi_n(\Delta\mathbf{r}, \Delta t)\,, \tag{1.6}$$

whose kernel is the STCF of the random field ξ. This method is more economical than the previous ones, but can be used only for simulating fields having a normal distribution law.

1.4. Mean statistical space-time characteristics

The complex amplitude in (1.2) is modulated both in space and in time, while the characteristic scales of this modulation are the mean-square values of the width of the beam and the pulse duration

$$a_0^2 = \langle a^2\rangle = \int \mathbf{r}^2\langle |A|^2\rangle \mathrm{d}\mathbf{r}\left(\int \langle |A|^2\rangle \mathrm{d}\mathbf{r}\right)^{-1}, \tag{1.7}$$

$$\tau_0^2 = \langle \tau^2\rangle = \int t^2\langle |A|^2\rangle \mathrm{d}t\left(\int \langle |A|^2\rangle \mathrm{d}t\right)^{-1}. \tag{1.8}$$

These relationships are used if the beams and pulses have substructures. For the smooth mean envelope $F(\mathbf{r}, t)$ e.g., the reference Gaussian

$$F(\mathbf{r}, t) = \exp\left(-\frac{\mathbf{r}^2}{2a_0^2} - \frac{t^2}{2\tau_0^2}\right), \tag{1.9}$$

a_0 and τ_0 are determined at the intensity level e^{-1}.

The mean fine-scale structure of beams and pulses is characterized by the correlation radius r_{c0} and the coherence time τ_{c0}. Here the number of spatial inhomogeneities over the cross section of the beam $N_r = (a_0/r_{c0})^2$ is related to the number of excitable transverse laser modes,[10] while the number of temporal inhomogeneities over the pulse duration $N_\tau = \tau_0/\tau_{c0}$ is related to the number of excitable longitudinal modes of the resonator.

In many studies the degree of coherence γ_ξ in (1.4) is written in factored Gaussian form

$$\gamma_\xi(\Delta\mathbf{r}, \Delta t) = \exp\left(-\frac{\Delta r^2}{r_{c0}^2} - \frac{\Phi t^2}{\tau_{c0}^2}\right). \tag{1.10}$$

Here r_{c0} and τ_{c0} are determined at the e^{-1} level. Although the factored form of γ_ξ is not conserved in a nonlinear medium, the criterion for determining r_c and τ_c of statistically inhomogeneous and nonstationary fields remain as before.

2. THE PHYSICS OF SELF-ACTION OF PARTLY COHERENT LASER RADIATION IN MEDIA WITH CUBIC NONLINEARITY

The physical causes of the perturbation of the refractive index of the medium caused by the action of a high-intensity light field are varied. Among the fundamental mechanisms of nonlinearity we should distinguish anharmonicity of the electronic and vibrational responses, electrostriction, orientation of the molecules in the external field (Kerr effect), thermal heating of the medium, etc. At present the values of the nonlinear perturbations of the refractive index n_{nl} ($n = n_0 + n_{nl}$) that give rise to self-action effects are well known for various types of nonlinearity.[15]

Naturally the induced optical inhomogeneities in our case are random and fluctuating in space and in time, and the problem of self-action becomes essentially stochastic. One can conduct its analysis both analytically and numerically without departing from the framework of the widely used quasioptical description.

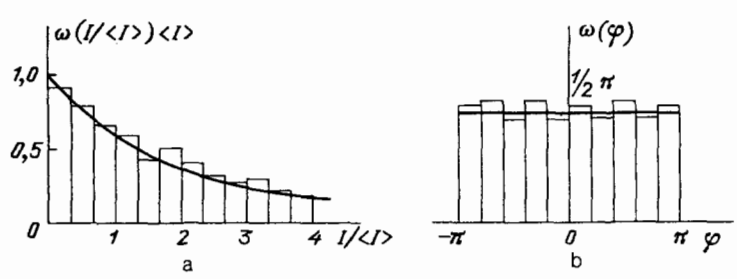

FIG. 1. Histograms of the probability distribution of the intensity (a) and the phase (b) of the random field ξ (r). The theoretical distributions are shown by the solid curves.

2.1. The quasioptical description

The quasioptical description widely used in optics is based on the assumption that the field of a quasiplane and quasimonochromatic wave (in other words, the field of a light beam and a wave packet)

$$\mathbf{E} = \mathbf{e}\frac{1}{2}A(\mathbf{r}, t)e^{i(\omega_0 t - k_0 z)} + \text{c.c.}, \tag{2.1}$$

is propagating along the z axis and is characterized by the complex amplitude A, which not only varies slowly on the scale of the wavelength and the period of the optical oscillations, but also varies along z more slowly than in the perpendicular xy plane. The propagation of intense electromagnetic waves in nonmagnetic media within the framework of classical electrodynamics is described by the wave equation

$$\text{curl curl } \mathbf{E} + \frac{1}{c^2}\frac{\partial^2 \mathbf{D}}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial^2 p^{\text{nl}}}{\partial t^2} \tag{2.2}$$

together with the material equations for the vectors of the linear and nonlinear polarization.

The linear polarization of an isotropic medium is related to the field intensity $\mathbf{E}$ by the functional

$$\mathbf{D} = \int_0^\infty dt'\varepsilon_0(t')\mathbf{E}(t - t'), \tag{2.3}$$

which reflects the existence of dispersion in the medium due to the time-dependence of the linear permittivity $\varepsilon_0(t')$, while the nonlinear polarization is usually represented in the form of an expansion in powers of $\mathbf{E}$. For media having cubic nonlinearity this is written in the form

$$p^{\text{nl}} = \int\int\int_0^\infty dt'dt''dt'''\chi^{(3)}(t', t'', t''') \times \mathbf{E}(t - t' - t'')\mathbf{E}(t - t' - t'' - t'''). \tag{2.4}$$

The slowness of variation of the amplitude in space and time as compared with the fast variations of the eikonal [the argument of the exponential in (2.1)] of the plane wave allows one to transform (2.2) into a nonlinear parabolic equation as follows: the first term in (2.2) is transformed into the form

$$\text{curl curl } \mathbf{E} = \text{grad div } \mathbf{E} - \Delta\mathbf{E}$$

$$\approx -\frac{\mathbf{e}}{2}\left(-2ik_0\frac{\partial A}{\partial z} + \frac{\partial^2 A}{\partial \mathbf{r}^2}\right)e^{i(\omega_0 t - k_0 z)} + \text{c.c.} \tag{2.5}$$

Here we have div $\mathbf{E} = 0$, and $\partial^2 A/\partial \mathbf{r}^2$ is the Laplace operator in the xy plane. The slowness of the variation of the amplitude in time allows one to transform (2.3) by expanding $A(t - t')$ in a series in t' to yield the expression

$$\mathbf{D} = \frac{\mathbf{e}}{2}\left[\varepsilon_0(\omega_0)A + \sum_{m=1}^{\infty}\frac{(-1)^m}{m!}\left(\frac{\partial^m \varepsilon_0(\omega)}{\partial \omega^m}\right)_{\omega_0}\frac{\partial^m A}{\partial t^m}\right]e^{i(\omega_0 t - k_0 z)} + \text{c.c.} \tag{2.6}$$

The terms with $m = 1, 2, 3, \ldots$ correspond successively to the first, second, third, etc. approximations of dispersion theory.

Finally, the wave of the nonlinear polarization (2.4) at a frequency far from the resonance frequencies, at which the dispersion of the nonlinearity is small, is

$$\mathbf{p}^{\text{nl}} = \frac{3}{4}\mathbf{e}\chi^{(3)}|A|^2\frac{A}{2}\exp\left[i(\omega_0 t - k_0 z)\right] + \text{c.c.} \tag{2.7}$$

Substitution of (2.5)–(2.7) into (2.2) makes it possible to write the equation for the complex amplitude

$$\left(\frac{\partial}{\partial z} + \frac{1}{u}\frac{\partial}{\partial t} + \frac{i}{2k_0}\frac{\partial^2}{\partial \mathbf{r}^2} - \frac{ik''_\omega}{2}\frac{\partial^2}{\partial t^2}\right)A - \frac{1}{2k_0}\sum_{m=3}^{\infty}\frac{(-1)^{m+1}}{m!}\left(\frac{\partial^m k^2}{\partial \omega^m}\right)_{\omega_0}\frac{\partial^m A}{\partial t^m} = \frac{4\pi}{c^2}\frac{\omega_0^2}{2ik_0}\cdot\frac{3}{4}\chi^{(3)}|A|^2 A, \quad u = \left(\frac{\partial k_0}{\partial \omega}\right)^{-1}, \quad k''_\omega = \left(\frac{\partial^2 k_0}{\partial \omega^2}\right)_{\omega_0}. \tag{2.8}$$

Let us transform to the running system of coordinates $\mathbf{r} = \mathbf{r}$, $z = z$, and $\tau = t - (z/u)$, adopt the representation $k_0 = n_0\omega_0/c$, and write the nonlinear increment to the refractive index $n' = n_0 + n_{\text{nl}}$ in the form

$$n_{\text{nl}} = n_2|A|^2 = \frac{3\pi\chi^{(3)}}{2n_0}|A|^2.$$

This enables us to derive the fundamental equation of quasioptics that describes simultaneously the nonlinear self-modulation and refraction, diffraction, and dispersion:

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k_0}\frac{\partial^2}{\partial \mathbf{r}^2} - \frac{ik''_\omega}{2}\frac{\partial^2}{\partial \tau^2}\right)A - \frac{1}{2k_0}\sum_{m=3}^{\infty}\frac{(-1)^{m+1}}{m!}\left(\frac{\partial^m k^2}{\partial \omega^m}\right)_{\omega_0}\frac{\partial^m A}{\partial \tau^m} = -i\frac{n_{\text{nl}}}{n_0}k_0 A. \tag{2.9}$$

We note that, in the propagation of ultrashort pulses of light in fiber light guides, one must take account of the dependence of $\chi^{(3)}$ on the time in (2.4); in the last equation terms appear that are associated with the lag of the nonlinear response. This situation is studied in the analysis of the propagation of ultrashort pulses in fiber light guides.

2.2. Theoretical methods of study

Since Eq. (2.9) has no exact solution, while the problem is aggravated also by the need in a broad set of phenomena that it should describe to employ a statistical approach, it is urgent to design approximate analytical and numerical methods to solve it. However, in a number of cases the fluctuations of the field of the light wave make it possible to describe the behavior of several of its mean characteristics more simply than the analogous characteristics of a regular wave.

We present below a brief description of the various theoretical approaches. For convenience of presentation we shall write Eq. (2.9) in the operator form

$$\frac{\partial A}{\partial z} + \hat{\mathscr{L}}_{\rm lin} A = \hat{\mathscr{L}}_{\rm nl} A, \tag{2.10}$$

Here $\hat{\mathscr{L}}_{\rm lin} = \hat{\mathscr{L}}_{\rm dif} + \hat{\mathscr{L}}_{\rm dis}$ is the linear operator, which represents a superposition of the diffraction and dispersion operators, and $\hat{\mathscr{L}}_{\rm nl}$ is the nonlinear operator, whose form is determined by the type of nonlinearity and the regime of self-action.

2.2.1. Perturbation method

Random radiation amounts to a superposition of a plane, quasistationary regular light wave and a weak noise that is homogeneous in space and stationary in time: $A = A_0 + \xi$. The equation for the deterministic wave, which does not undergo diffraction and dispersion, has the form

$$\frac{\partial A_0}{\partial z} = \hat{\mathscr{L}}_{\rm nl} A_0. \tag{2.11}$$

The action of the nonlinear operator $\hat{\mathscr{L}}_{\rm nl}$ leads to a phase self-modulation of the wave without change in its intensity. For weak noise, the variance of the fluctuation $\sigma^2 = \langle \xi^2 \rangle$ is weak in comparison with the intensity of the deterministic wave, $\sigma^2 \ll |A_0|^2$. Hence the optical inhomogeneities are induced practically solely by the regular wave $n_{nl} = n_{nl}(|A_0|^2)$. Consequently the transformation of the noise component in the nonlinear medium is described by the equation

$$\frac{\partial \xi}{\partial z} + \hat{\mathscr{L}}_{\rm lin} \xi = \hat{\mathscr{L}}_{\rm nl} \xi, \tag{2.12}$$

where the nonlinear operator

$$\hat{\mathscr{L}}_{\rm nl} = -\frac{ik_0}{n_0} n_{\rm nl} (|A_0|^2 + A_0 \xi)$$

is determined by the intensity of the regular wave.

The solution of Eq. (2.12) is sought in the form of the integral

$$\xi(\mathbf{r}, \tau, z) = \int d\mathbf{k} \int d\omega \xi(\omega, \mathbf{k}) \exp(-i\mathbf{k}_\perp \mathbf{r} - ik_{\|} z), \tag{2.13}$$

where $\xi(\omega, \mathbf{k})$ is the Fourier amplitude of the noise. Substituting (2.13) into Eq. (2.12) and integrating it leads to the solution for the Fourier amplitude for the noise field.

We note that the perturbation method is applicable only for weak noise and short tracks of propagation. In a nonlinear medium the variance of the fluctuation of the noise component can increase as it propagates, and then the induced fluctuations of the refractive index will be substantial. However, the perturbation method imposes no restrictions involving the assumption of conservation of the statistics of the light field in the nonlinear medium.

2.2.2. The method of moments

This method is intended for analyzing the correlation functions of second and higher orders. The equation for

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau_1, \tau_2, z) = \langle A(\mathbf{r}_1, \tau_1, z) A^*(\mathbf{r}_2, \tau_2, z) \rangle \tag{2.14}$$

is derived by transforming from the equation (2.10) for the complex amplitude to the equation for the second-order STCF of the field. To do this, the equations of quasioptics for $A_1 = A(\mathbf{r}_1, \tau_1, z)$ and $A_2^* = A^*(\mathbf{r}_2, \tau_2, z)$ are multiplied respectively by A_1 and A_2:

$$\begin{aligned} &\frac{\partial A_1}{\partial z} A_2^* + \hat{\mathscr{L}}_{\rm lin1} A_1 A_2^* = {}_{\rm nl1} A_1 A_2^*, \\ &\frac{\partial A_2^*}{\partial z} A_1 + \hat{\mathcal{L}}^*_{\rm lin2} A_2^* A_1 = \hat{\mathcal{L}}^*_{\rm nl2} A_2^* A_1. \end{aligned} \tag{2.15}$$

Here $\hat{\mathscr{L}}_{\rm lin1,2}$ and $\hat{\mathscr{L}}_{\rm nl1,2}$ are the linear and nonlinear operators of the arguments $(\mathbf{r}_1, \tau_1)$ and $(\mathbf{r}_2, \tau_2)$. After summing the two equations of (2.15) and averaging over the ensemble, we obtain the equation

$$\left(\frac{\partial}{\partial z} + \hat{\mathscr{L}}_{\rm lin1} + \hat{\mathscr{L}}^*_{\rm lin2}\right) \langle A_1 A_2^* \rangle = \langle (\hat{\mathcal{L}}_{\rm nl1} + \hat{\mathcal{L}}^* \quad) A_1 A_2^* \rangle. \tag{2.16}$$

We easily note that the right-hand side of Eq. (2.16) contains the fourth-order moments of the field. One can easily show that the equation for the correlation function of the intensity will, in turn, "engage" the sixth-order moments, etc. Therefore the fundamental difficulty of the method of moments is the disengagement of the correlator on the right-hand side of (2.16) and the subsequent equations. In particular, when the space-time scales of the correlation of the random functions $A_1 A_2^*$ and $\hat{\mathscr{L}}_{\rm nl1} + \hat{\mathscr{L}}_{\rm nl2}$ differ strongly, then we have

$$\langle (\hat{\mathcal{L}}_1 + \hat{\mathcal{L}}^* \quad) A_1 A_2^* \rangle = \langle \hat{\mathcal{L}} \quad + \hat{\mathcal{L}}^* \quad \rangle \langle A_1 A_2^* \rangle \tag{2.17}$$

and Eq. (2.16) is closed. Along with this condition, closure of the equation is attainable upon applying a strong restriction that assumes the invariance of the statistics of the field existing at the entrance into a medium having a normal distribution law.

2.2.3. The aberration-free description

This method employs a self-similar representation of the STCF of Gaussian-type in a model of radiation of the "noise burst" type $A = I_0^{1/2} F \xi$:

$$|\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau_1, \tau_2, z)| = \frac{I_0}{f_r^2 f_\tau} \exp\left(-\frac{\mathbf{r}_1^2 + \mathbf{r}_2^2}{2 f_r^2 a_0^2} - \frac{\tau_1^2 + \tau_2^2}{2 f_\tau^2 \tau_0^2} - \frac{\Delta \mathbf{r}^2}{g_r^2 r^2} - \frac{\Delta \tau^2}{g_\tau^2 \tau^2}\right). \tag{2.18}$$

Here the mean-square width and duration of the noise are given as $a(z) = a_0 f_r(z)$ and $\tau_u(z) = \tau_0 f_\tau(z)$ and correspond to the form of F in (1.9), while the correlation radius and the coherence time are $r_c(z) = r_{c0} g_r(z)$ and $\tau_c(z) = \tau_{c0} g_\tau(z)$.

Upon substituting (2.18) into (2.16), we obtain an equation for the four mean-statistical parameters $f_{r,\tau}$ and $g_{r,\tau}$ that are functions of the time and the coordinates of propagation.

2.2.4. Integration over trajectories

The solution of the nonlinear equation of quasioptics (2.9) is represented in the form

$$A(\mathbf{r}, \tau, z) = \int d\mathbf{r}' \int d\tau' A_0(\mathbf{r}', \tau') G(\mathbf{r}, \mathbf{r}', \tau, \tau', z), \tag{2.19}$$

where the Green's function in the nonlinear medium with allowance for diffraction and quadratic dispersion is given by the integral over trajectories[26–28]

$$G(\mathbf{r}, \mathbf{r}', \tau, \tau', z) = \int D^2(\mathbf{r}(z')) \int D(\tau(z')) \exp\left\{ \int_0^z \left[-\frac{i}{2k_0}\left(\frac{d\mathbf{r}}{dz'}\right)^2 + i\frac{k''_\omega}{2}\left(\frac{d\tau}{dz'}\right)^2 - i\frac{k_0}{n_0} n_{nl}(|A(\mathbf{r}(z'), \tau(z'))|^2) \right] \right\}. \tag{2.20}$$

Here $D^2(\mathbf{r}(z'))$ and $D(\tau(z'))$ are differentials that denote integration over a set of trajectories that pass through the space point $\mathbf{r}(z')$ and the time point $\tau(z')$. Here we have $\mathbf{r}(z'=0) = \mathbf{r}'$, and $\tau(z'=0) = \tau'$.

The fundamental difficulty of the method is the choice of the trajectories of integration. However, if we take account of the fact that the main contribution comes from the trajectories for which

$$|A(\mathbf{r}(z'), \tau(z'))|^2 \approx |A_0(\mathbf{r}', \tau')|^2, \tag{2.21}$$

then, by using the iteration method, we can obtain a solution for the correlation function of the field.

The method of integrating over trajectories under these restrictions is essentially a fixed-channel method, since the nonlinear perturbation of the refractive index arises from the input distribution of the field intensity.

2.2.5. Nonlinear phase channel

In the case of strong nonlinearity, when the characteristic length L_{ph} of nonlinear phase modulation is much smaller than the diffraction length L_{dif} amd the dispersion length L_{dis}, one can solve the nonlinear equation of quasioptics (2.9) in two stages. In the first stage one treats only the nonlinear self-modulation

$$\frac{\partial A_1}{\partial z} = \hat{\mathscr{L}}_{nl} A_1, \tag{2.22}$$

and in the second stage solves the linear equation

$$\frac{\partial A_2}{\partial z} + \hat{\mathscr{L}}_{lin} A_2 = 0, \tag{2.23}$$

which describes the diffraction and dispersion of the phase-modulated field A_1 distributed in the medium.

The nonlinear phase-channel method is applicable for light fields having an arbitrary law of space-time modulation. It is suitable for analyzing the behavior of a light wave that amounts to a superposition of signal and noise.

2.2.6. The method of successive approximations

For a weak nonlinearity, when $L_{ph} > L_{dif}, L_{dis}$, one can treat as the first approximation the solution of the linear equation

$$\frac{\partial A_1}{\partial z} + \hat{\mathscr{L}}_{lin} A_1 = 0. \tag{2.24}$$

In the second approximation one solves the equation

$$\frac{\partial A_2}{\partial z} + \hat{\mathscr{L}}_{lin} A_2 + \hat{\mathscr{L}}_{nl} A_2, \tag{2.25}$$

in which the operator $\hat{\mathscr{L}}_{nl}$ is determined by the intensity of the diffracted and dispersed waves A_1. In each of the subsequent stages of approximation one can correct the induced optical inhomogeneities for the nonlinear operator $\hat{\mathscr{L}}_{nl}$ by using the wave intensity of the previous approximation. However, the procedure of finding the solution in the third and subsequent approximations is difficult owing to the complexity of determining the Green's function of the corresponding equation.

2.2.7. Numerical simulation

In a numerical experiment a direct integration of the nonlinear equation of quasioptics is performed by the method of separation with respect to the physical factors.[33,35,36] Here one applies a stepwise linearization of the nonlinear equation (2.9) while using the fast Fourier transform algorithm in the dispersion step. The numerical simulation of the initial conditions of the input equation is based on the orthogonality of the expansion of the mutual-coherence function of the field, and is described in Sec. 1. The statistical characteristics at the distance z are determined by the Monte-Carlo method by averaging over the ensemble of realizations of the obtained solutions together with the pseudorandom ones with respect to conservation of the three first integrals of the equation: energy, momentum, and the Hamiltonian.[34]

Numerical calculation of the mutual coherence function Γ in (2.14) is performed by using the method of finite differences. The function is a complex quantity, and the equation for the mutual coherence function breaks down into a system of two equations, which are solved numerically together with the material equation of the medium. To construct the finite-differences scheme, the region of the solution is divided by a three-dimensional grid, and equidistant surfaces in cylindrical coordinates are constructed. At the points of intersection of the planes the nodes of the grid—one calculates the sought function.

3. NONLINEAR-DIFFRACTION PROPAGATION OF RANDOMLY MODULATED LIGHT BEAMS AND PACKETS

The initial equation (2.9) describes the nonlinear propagation of light waves in the presence of diffraction and dispersion. The scales of development of these phenomena along the line of propagation (the z axis) depend on the transverse dimensions of the beam a_0 and the correlation radius r_{c0} and also the pulse duration τ_0 and its time coherence τ_{c0}, and are characterized by the diffraction and dispersion lengths:

$$L_{dif}^{inc} \sim k_0 a_0 r_{c0}, \quad L_{dis}^{inc} \sim \tau_0 \tau_{c0} |k''_\omega|^{-1}.$$

The theme of this section will be the analysis of the nonlinear propagation of diffracted waves in the absence of dispersion of the group velocity, which is equivalent to the condition $z \ll L_{dis}^{inc}$. Indeed, this inequality holds both for quasicontinuous radiation and for pulses of picosecond duration in the atmosphere, where $k''_\omega \sim 10^{-28}$ cm^2 m^{-1}, $L_{dis}^{inc} \sim 10$ km, which substantially exceeds the extent of real propagation tracks. In liquids and solids, where k''_ω is larger by 3–5 orders of magnitude, the dispersion effects are insignificant for pulses up to nanosecond duration.

In optical transparent, weakly absorbing media with an attenuation coefficient α, the nonlinear-diffraction propagation is described by the equation

$$\frac{\partial A}{\partial z} + \frac{i}{2k_0}\frac{\partial^2 A}{\partial \mathbf{r}^2} + \frac{\alpha A}{2} = -i\frac{k_0}{n_0} n_{nl} A, \tag{3.1}$$

In a linear medium ($n_{nl} = 0$) this converts into the parabolic equation introduced in Ref. 37, which describes the transverse diffusion of the amplitude of the wave beams.

The nonlinear increment to the refractive index n_{nl} is determined by the mechanisms of nonlinearity, the principal ones of which–Kerr and thermal–will be discussed in succession in this article. Thus, the first part of this section presents the laws of propagation of partly coherent waves in a medium having a Kerr-type local nonlinearity, while the problems of thermal self-action (TSA) are discussed in the second part.

Kerr nonlinearity

In media with Kerr nonlinearity the perturbation of the refractive index is described by the relaxation equation

$$\left(\tau_{rel}\frac{\partial}{\partial\tau}+1\right)n_{nl} = n_2|A|^2. \tag{3.2}$$

For pulses of picosecond or greater duration, one can consider the self-action regime to be lag-free, and we have $n_{nl} = n_2|A|^2$. It is necessary to take account of the lag in the nonlinear response for quasicontinuous radiation with a coherence time $\tau_c < \tau_{rel}$.

3.1. Spatial instability

The nonlinear refraction of radiation having a regular transverse field distribution in combination with diffraction leads to nonlinear aberrational distortions of light beams,[38] while amplitude-phase fluctuations are the cause of onset of instability–separation of the beam into filaments. The first observations of filamentous structure of light in liquids owing to self-focusing[17,39–41] are described by the authors of Ref. 41.

A theory of the formation of filaments is given in Ref. 16. It was shown by the perturbation method described in Sec. 2 that, in a nonlinear dielectric, the amplitude-phase perturbations of a plane electromagnetic wave lead to its breakdown into individual beams having different self-focusing lengths, depending on the scale of the initial perturbation.

Representation of the dimensionless amplitude of the perturbed plane wave in the form

$$E(\mathbf{r}', 0) = E_0 + \xi_0 e^{-i\vec{\varkappa}_1\mathbf{r}'}$$

$$(E = (n_2/n_0)^{1/2}A, \quad \vec{\varkappa}_1 = \mathbf{k}_1/k_0, \quad \mathbf{r}' = k_0\mathbf{r}; \quad E_0 = \text{const}) \tag{3.3}$$

and solution of Eq. (2.12) with the initial condition (3.3) shows that the perturbation in the medium varies as

$$\xi(\mathbf{r}', z') = \xi_0 e^{-i\vec{\varkappa}_\perp\mathbf{r}'-i\varkappa_{\|}z'}, \tag{3.4}$$

where we have

$$\varkappa_{\|}^2 = \frac{\varkappa_\perp^2}{4}(\varkappa_\perp^2 - \varkappa_{rp}^2), \quad \varkappa_{rp}^2 = 2E_0^2, \quad z' = k_0 z. \tag{3.5}$$

Perturbations having transverse wave numbers $0 < \varkappa_\perp < \varkappa_{gr}$ are unstable and grow according to an exponential law with the increment $(2E_0^2 - \varkappa_\perp^2)^{1/2}\varkappa_\perp/2$, which has its maximum value $E_0^2/2$ when $\varkappa_\perp = \varkappa_{fil} = E_0$. This corresponds to formation of a fine-scale structure–filaments–having the transverse dimension

$$r_{fil} = \frac{\tau}{\chi_{fil}k_0} = \frac{\lambda_0}{2}(n_0 n_2 A_0^2)^{-1/2}. \tag{3.6}$$

An estimate of the filament dimensions for a beam of diameter 2 mm and power 1 MW yields $r_{fil} \sim 180\ \mu$m, which exceeds by severalfold the experimentally observed dimensions,[42] since the theory that was developed took no account of the self-focusing of the main wave.

The power of an individual filament $P_{fil} = \pi r_{fil}^2 A_0^2/4 \sim P_{cr}$ coincides apart from a coefficient with the critical power for self-focusing of a coherent light beam.[40] An increase in the mean power of a partly coherent light beam leads to an increase in the number of filaments, but not in their intensity.

3.2. Self-focusing of beams of incoherent light

Taking account of the finiteness of the cross section of beams complicates the pattern of the self-action of light. An analysis of the self-focusing of such beams was performed first in Ref. 21, which studied the nonlinear transformation of the spatial correlation function. The closed equation for the second moment $\Gamma_{12} = \langle A(\mathbf{r}_1, z)A^*(\mathbf{r}_2, z)\rangle$ upon transforming to the variables $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\vec{\rho} = \mathbf{r}_1 - \mathbf{r}_2$ obtained from (2.16) is

$$\left(\frac{\partial}{\partial z} + \frac{i}{k_0}\nabla_R\nabla_\rho\right)\Gamma_{12} = \frac{ik_0 n_2}{n_0}\Gamma_{12}(\Gamma_{11} - \Gamma_{22}). \tag{3.7}$$

The procedure for uncoupling the correlator on the right-hand side of (2.16) is based on the assumption that the Gaussian statistics of the field is conserved in the nonlinear medium. This is justified for a weak additional correlation between the random fields introduced by the nonlinear interaction when the power of the input radiation on the scale of the transverse correlation of the beam (in essence the power of an individual inhomogeneity) is

$$P_{fil} \ll P_{cr}(\tau_{rel}/\tau_{c0})^{1/2} \tag{3.8}$$

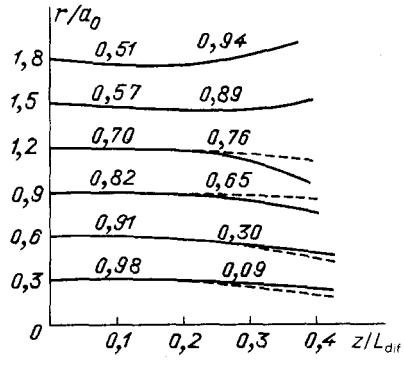

FIG. 2. Modification of the tubes of equal coherence (solid lines) and equal power (dashed lines) along the track of propagation of the beam for $P_0/P_{cr} = 2$ and $N_r = 1$. Column of numbers at left–values of the modulus of the degree of coherence; at right–values of the power (the total power of the beam is taken to be unity).

($P_{cr} = c\lambda_0^2/16\pi^2 n_2$ is the critical power for self-focusing of a coherent beam, and $\tau_{c0} > \tau_{rel}$).

The solution of Eq. (3.7) with the initial condition

$$\Gamma_{12}(0) = \Gamma_0 \exp\left(-\frac{\vec{\rho}^2}{r_{c0}^2} - \frac{\mathbf{r}^2}{a_0^2}\right) \tag{3.9}$$

is sought in the self-similar form

$$\Gamma_{12}(z) = I_0 \exp\left(-\frac{\vec{\rho}^2}{r_c^2(z)} - \frac{\mathbf{R}^2}{a^2(z)} + i\varphi(z)\mathbf{r}\vec{\rho} + \psi(z)\right). \tag{3.10}$$

In the paraxial approximation the width $a(z)$ of the beam and the correlation radius $r_c(z)$ vary according to

$$a(z) = a_0\left[1 + \frac{z\sqrt{2}}{L_{\text{dif}}^{\text{inc}}}\left(1 - \frac{P_{\text{fil}}}{P_{cr}}\right)\right]^{1/2}, \quad r_c(z) = \frac{r_{c0}}{a_0}a(z), \tag{3.11}$$

where we have

$$L_{\text{dif}}^{\text{inc}} = \frac{1}{\sqrt{2}}k_0 a_0 r_{c0}.$$

Equation (3.11) directly implies that the number of spatial inhomogeneities in the linear medium remains invariant:

$$N_r(z) = \left(\frac{a(z)}{r_c(z)}\right)^2 = \left(\frac{a_0}{r_{c0}}\right)^2 = N_r(0). \tag{3.12}$$

Self-focusing of a beam of incoherent light is possible when $P_{\text{fil}} > P_{cr}$, while the input power is

$$P_0 = N_r P_{\text{fil}} > P_{cr} N_r. \tag{3.13}$$

This means that the critical power for self-focusing of an incoherent beam increases proportionally to the number of inhomogeneities in the original cross section of the beam.

The paraxial (aberration-free) description gives a physically correct quantitative picture of the self-focusing of an incoherent beam. The numerical solution of Eq. (3.7) performed in Ref. 43 enabled calculating more accurately the critical power for self-focusing, which increases with increasing N_r:

$$P_{cr}^{\text{inc}} = P_{cr}(1 + 0{,}6N_r) \tag{3.14}$$

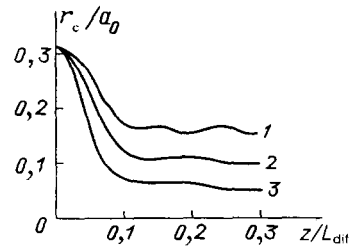

FIG. 4. Variation of the correlation radius near the axis of the beam along the track of propagation ($N_r = 3, \delta = 0$) for $P_0/P_{cr} = 10$ (*1*), 20 (*2*), and 40 (*3*).

and analyzing the aberrational character of the self-focusing (Fig. 2). As we see from the diagram, the tubular surfaces of equal coherence and equal power with an initial radius $r \leqslant 1.2a_0$ are narrowed by self-focusing, while they expand in the region $r > 1.2a_0$, where diffraction predominates. The lines of the tubes of the two types coincide only for the peripheral region of the beam ($r > 1.5a_0$), where the nonlinear effects are not significant. Characteristically, the paraxial region of the beam ($r \leqslant 0.8a_0$) self-focuses more rapidly than the dimension of the transverse inhomogeneities decreases. Therefore, at the focus of the nonlinear lens the spatial structure of the focused beam is better than the original–this conclusion has also been confirmed experimentally.

We should note that the procedure of deriving the closed equation (3.7) in Ref. 43 is justified for any statistics of the field by the smallness of the fluctuations of the nonlinear increment of the refractive index as compared with its mean value: $\sigma_{n_{\text{nl}}}^2 \ll \langle n_{\text{nl}}\rangle^2 = n_2^2\langle I\rangle^2$. This permits one to remove the quantity n_{nl} that enters into the operator $\hat{\mathscr{L}}_{\text{nl}}$ outside the angle brackets when averaging the right-hand side of (2.16).

Thus the correctness of this approach requires the fulfillment of the conditions

$$\tau_c \ll \tau_{\text{rel}}, \quad \frac{\tau_{\text{rel}}\langle I\rangle^2}{\pi G_I(0)} \gg 1, \tag{3.15}$$

where $G_I(\omega)$ is the spectral density of the intensity. From the physical standpoint the condition (3.15) means that, for broadband radiation (with a small coherence time τ_c compared with τ_{rel} and a small spectral density of fluctuation at low frequencies), the lag in the nonlinear response restrains

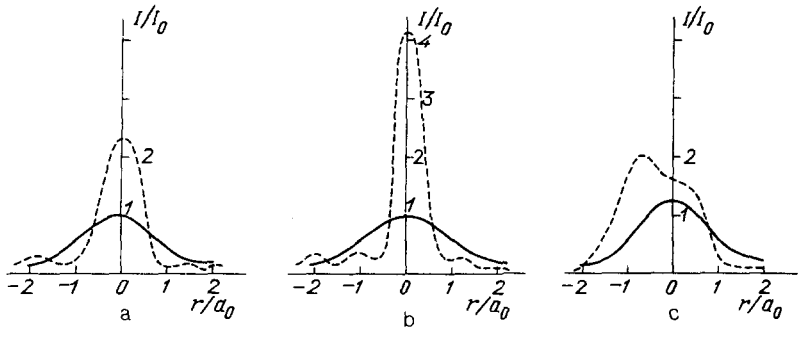

FIG. 3. Mean (solid lines) and instantaneous (dashed lines) profiles of the intensity of a beam with $P_0/P_{cr} = 20$ and $N_r = 3$ at the entrance to the medium (a) and in the cross section $z/L_{\text{pl}} = 0.15$ for $\delta = 0$ (b) and 100 (c).

the growth of the fluctuation $\sigma^2_{n_{nl}}$, and the nonlinear lens is formed by the mean intensity of the field.

If the fluctuations of the initial field are Gaussian, then, when (3.15) is satisfied, they remain so also in the nonlinear medium. This conclusion is confirmed also by calculations based on the method of random trials.

3.3. Transformation of the spatial statistics (Monte-Carlo method)

The total amount of the information on self-focusing of light beams having a limited space and time coherence can be obtained only by the method of random trials (the Monte-Carlo method).[35]

The fact is highly important that a broad-band beam ($\delta = \tau_{rel}/\tau_{c0} \gg 1$) focuses as a whole, whereas for a narrow-band beam ($\delta \ll 1$) an additional spatial modulation appears, which leads to separation of the latter into filaments (Fig. 3). Since a filament is a nondeterministic inhomogeneity of the radiation, with its maximum of intensity appreciably exceeding the mean value, while the intensity at its boundaries is close to zero, its transformation is characterized in full measure also by the corresponding transformation of the initial exponential law of intensity distribution, especially in the region of small intensities and those considerably exceeding the mean. The phase distribution always remains invariant in a nonlinear medium.

The decomposition into filaments of a narrow-band multimode beam, while initially progressing rapidly, continues until each filament contains the power P_{cr}. The characteristic transverse dimension of a filament is determined essentially by the radius of spatial coherence, the variation of which (near the axis of the beam) is shown in Fig. 4. The rapid breakdown into filaments then ceases independently of the initial spatial structure of the beam.

Fine-scale self-focusing is accompanied by rapid growth of the relative fluctuations of intensity with subsequent saturation. With increasing beam power, the aforementioned fluctuations decline owing to the increase in the mean intensity upon self-focusing.

For broad-band radiation ($\delta \gg 1$), the nonlinear refraction will be determined by the mean intensity profile, and spatial modulation completely disappears (Fig. 5). We see that an increase in δ entails a decrease in the mean intensity at the axis of the beam. At the same time, the variation of the spatial coherence radius and of the relative variance of the intensity fluctuations in going to broad-band beams are smooth in character. This indicates the absence of fine-scale self-focusing. The statistics of the radiation in the medium approaches Gaussian as δ increases.

A comparison of the character of the decline in the effective width of the beam $a \propto \langle I \rangle^{-1/2}$ and the coherence radius (curves *3* in Fig. 5) shows that the ratio $N_r = (a/r_c)^2$ declines owing to aberration noises, while the structure of the beam improves.

3.4. Nonlinear refraction of beams by induced random optical inhomogeneities

The correlation of the field and the optical inhomogeneities that it induced leads to a number of new, interesting laws of propagation of partly coherent waves. Their manifestation is most easily observed by using the nonlinear phase-channel method to analyze the self-action of a partly coherent beam of the form

$$A(\mathbf{r}, z = 0) = I_0^{1/2}\xi(\mathbf{r})\exp\left(-\frac{\mathbf{r}^2}{2a_0^2}\right), \tag{3.16}$$

$$\langle \xi(\mathbf{r}_1)\xi^*(\mathbf{r}_2)\rangle = \sigma^2\exp\left(-\frac{\Delta r^2}{r_{c0}^2}\right).$$

The modulus of the correlation function of the field in the paraxial approximation[29]

$$|\Gamma_{12}(z)| = \sigma^2 I_0\left(\frac{a_0}{a(z)}\right)^2\exp\left(-\frac{\mathbf{r}_1^2+\mathbf{r}_2^2}{2a^2(z)} - \frac{\Delta \mathbf{r}^2}{r_c^2(z)}\right) \tag{3.17}$$

together with the expressions for the beam width and the correlation radius

$$a(z) = a_0\left[\left(1 \mp \frac{1}{2}\frac{z^2}{L_{nl}^2}\right)^2 + \frac{z^2}{L_{dif}^2} + 2N_r\frac{z^2}{L_{dif}^2} + N_r\frac{z^4}{L_{nl}^4}\right]^{1/2}, \tag{3.18}$$

$$r_c(z) = r_{c0}\left[\left(1 \mp \frac{1}{2}\frac{z^2}{L_{nl}^2}\right)^2\left(1 + \frac{1}{2}\frac{z^2 L_{dif}^2}{L_{nl}^4}\right)^{-1} + \frac{z^2}{L_{dif}^2} + 2N_r\frac{z^2}{L_{dif}^2}\right] \tag{3.19}$$

enables one to draw a number of important conclusions (here $L_{dif} = k_0 a_0^2$ $L_{nL} = a_0(n_0/n_2\sigma^2 I_0)^{1/2}$ are the characteristic diffraction length and nonlinear self-action length of a coherent beam, while the sign (−) corresponds to $n_2 > 0$, and (+) to $n_2 < 0$).

In a linear medium ($L_{nl} \to \infty$) the beam width $a(z)$ and its correlation radius $r_c(z)$ increase in the same way owing to coherent and incoherent diffraction at the corresponding distances L_{dif} and $L_{dif}^{inc} = l_{dif}/(2N_r)^{1/2}$, so that $N_r(z) = \text{const}$.

In a nonlinear medium two nonlinear effects arise in addition that are caused by the existence of a regular and a

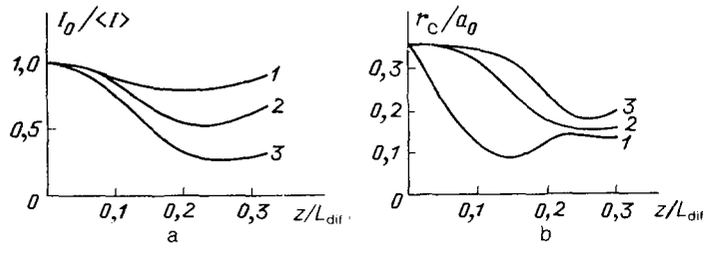

FIG. 5. Variation of the intensity (a) and correlation radius (b) on the axis of the beam for $N_r = 3$, $P_0/P_{cr} = 20$ and various widths of the frequency spectrum $\delta = 0$ (*1*), 10 (*2*), and 100 (*3*).

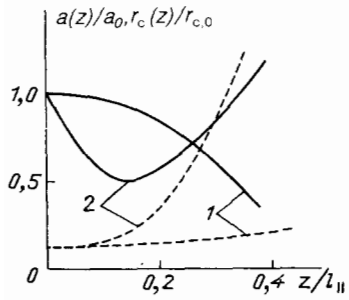

FIG. 6. Variation of the radius of the beam (dashed lines) and the correlation radius for $P_0/P'_{cr} = 5$ (*1*) and 50 (*2*) for $\alpha z = 1.6$ (for the beam radius the scale along the axis of ordinates has been decreased tenfold).

random profile of the induced lens, with the latter correlated with the fluctuations of the field. The coherent nonlinear refraction that develops over the length L_{nl} gives rise to the same increase ($n_2 < 0$) or decrease ($n_2 > 0$) in a and r_c, while as before, as in Ref. 21, we have $N_r(z) = \text{const}$. The existence of fluctuations of the nonlinear lens, independently of the sign of the nonlinearity, favors broadening of the beam [the last term in (3.18)] and decrease in the correlation radius [denominator of the first term in (3.19)].

The decrease in the correlation radius, which can occur in the initial stage in a defocusing medium, is a somewhat unexpected result. However, it has been predicted and confirmed experimentally in the thermal defocusing of a partly coherent beam by the authors of Ref. 28.

Thermal nonlinearity. The dissipation of the energy of laser radiation as it propagates in the atmosphere, in liquids, and in solids leads to the appearance of thermal self-action (TSA) of light–a number of nonlinear phenomena involving self-modulation in space and in time of a wave with induced optical inhomogeneities, whose origin is a consequence of the inhomogeneous heating of the medium.[44] The temperature variations, which entail the appearance of a nonlinear increment to the refractive index, are determined by the multitude of regimes of heating of the medium. In the general case they can be calculated on the basis of joint solution of the equation of heat conduction.[45] The first of them is written most often in the form

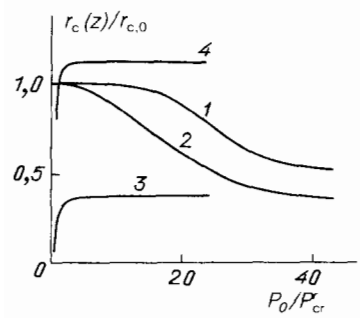

FIG. 7. Dependence of the correlation radius on the power of the beam for $\delta = 0.1$ (*1*), 0.2 (*2*), 4 (*3*), and 10 (*4*) (for curves *3* and *4* the scale along the axis of ordinates has been decreased tenfold).

$$\frac{\partial}{\partial\tau}(\Delta T) + \mathbf{v}\frac{\partial}{\partial\mathbf{r}}(\Delta T) - \chi\frac{\partial^2}{\partial\mathbf{r}^2}(\Delta T) = \frac{\alpha}{\rho c_p}|A|^2. \tag{3.20}$$

This reflects the fact that the isobaric change in the energy per unit volume having the heat capacity ρc_p is determined by the dissipation of energy per unit time $\alpha|A|^2$ (the intensity is $I = |A|^2$) and by the heat transport, the fundamental mechanisms of which are heat conduction (χ is the heat-conductivity coefficient) and convection (v is the rate of flux across the beam).

In the general case the temperature fluctuations arise both from random variations of the beam intensity and of the velocity of motion of the medium.

On the mathematical level the TSA of randomly modulated waves is described by Eqs. (3.1) and (3.20) ($n_{nl} = (\partial n/\partial T)\Delta T = n_T\Delta T$). Here the velocity of motion of the medium is either modeled by a homogeneous flux or undergoes turbulent pulsations (coarse- and fine-scale).

Since, as we mentioned in the Introduction, the treatment of the behavior of multimode radiation in a randomly inhomogeneous medium lies outside the scope of our article, we have restricted the treatment to the case $\mathbf{v} = \text{const}$. However, even under this assumption the study of TSA remains an extremely complex problem. Therefore it is usually carried out for a concrete regime of propagation of light. The latter is defined by the relationship between the duration τ_0 of the radiation, the coherence time τ_c, and the characteristic times of heat transport $\tau_v = a_0/v$ and of heat diffusion $\tau_\chi = a_0^2/\chi$.

3.5. Thermal defocusing of random beams

For light beams of quasicontinuous radiation with a coherence time $\tau_c > \tau_\chi, \tau_v$, a regime of stationary self-action is realized. Its features have been studied both theoretically and experimentally.[28] Using the method of integrating over trajectories has enabled calculation of the spatial correlation function (SCF) of the field for coherent ($z \ll l_\parallel = (1/2)k_0 r_{c0}^2$) and incoherent ($z \gg l_\parallel$) regimes of TSA in a motionless medium for $z < L_{dif}^{inc} = k_0 a_0 r_{c0}/\sqrt{2}$.

For $z \ll l_\parallel$ in the paraxial approximation in the variables $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\vec{\rho} = \mathbf{r}_1 - \mathbf{r}_2$, we have

$$|\Gamma_{12}(\mathbf{R}, \vec{\rho}, z)| = I_0\left(\frac{a_0}{a(z)}\right)^2 \exp\left(-\frac{\mathbf{R}^2}{a^2(z)} - \frac{\vec{\rho}^2}{r_c^2(z)} - \alpha z\right), \tag{3.21}$$

where

$$a(z) = a_0\left\{\left[1 + \frac{1}{4}h(\alpha)\frac{z^2}{L_{nl}^2}\right]^2 + 2\left(\frac{z}{L_{dif}}\right)^2\left[1 + h^2\left(\frac{\alpha}{2}\right)\frac{z^2}{l_{ph}^2}\right]\right\}^{1/2}, \tag{3.22}$$

$$r_c(z) = R_{c0}\left\{\left[1 + \frac{1}{4}h(\alpha)\frac{z^2}{L_{nl}^2}\right]\left[1 + h^2\left(\frac{\alpha}{2}\right)\frac{z^2}{l_{ph}^2}\right]^{-1} + 2\frac{z^2}{L_{dif}^2}\right\}^{1/2}. \tag{3.23}$$

Here

$$L_{nl} = \left(\frac{n_0 \rho c_p \chi}{|n_T| \alpha I_0}\right)^{1/2}, \quad l_{ph} = \frac{L_{nl}^2}{l_{\parallel} \cdot \sqrt{2}} \tag{3.24}$$

are the characteristic lengths associated with the mean profile and the temperature fluctuations ΔT, and $h(\alpha) = (1 - e^{-\alpha z})/\alpha z$.

According to (3.22) and (3.23), the radius of a random beam in a self-focusing medium increases more rapidly than that of a regular beam, owing to the smaller diffraction length L_{dif} for the random beam and the existence of the additional random phase modulation caused by the finiteness of the length l_{ph}.

The critical power P_{cr}^{a} for self-focusing of the beam is determined from the condition $2L_{nl} = L_{dif}$, and equals

$$P_{cr,inc}^{a} = \left(\frac{a_0}{r_{c0}}\right)^2 P_{cr}, \quad P_{cr} = \frac{\rho c_p n_0 \chi}{\alpha k_0^2 |n_T| a_0^2} \tag{3.25}$$

(P_{cr} is the critical power for a regular beam). The critical power increases as the initial structure of the beam deteriorates. The behavior of the correlation radius r_c differs from the behavior of the radius a (Fig. 6): r_c can be either larger or smaller than the initial value r_{c0}. This is determined by the relationship of the lengths L_{nl} and l_{ph}.

The critical power for decrease of the radius is found from the condition $L_{nl} = l_{ph}$:

$$P_{cr}^{r} = \left(\frac{a_0}{r_{c0}}\right)^4 P_{cr}. \tag{3.26}$$

For a beam power $P_0 > P_{cr}^{r}$ its radius increases owing to defocusing, whereas the correlation radius decreases. Increase in the power P_0 leads to a substantial decrease in its correlation radius in the coherent regime of self-action (Fig. 7).

In the incoherent regime of TSA the role of temperature fluctuations is changed. It has been shown that in a medium the correlation radius $r_c > r_{c0}$ if $z > 8L_{nl}^2 l_{ph}^{-2} l_{\parallel}$. Here we must have $L_{nl} \geqslant l_{ph}$, which is equivalent to $P_0 > P_{cr}^{r}$. In this case the radius of the beam and the correlation radius increase with increasing z (see curves *3* and *4* in Fig. 7).

The self-defocusing of the radiation of a multimode laser based on yttrium aluminum garnet in water mixed with acetone and in castor oil has been studied in an apparatus whose block diagram is shown in Fig. 8. The measurements of the radii of the beam a and of coherence r_c at the exit from

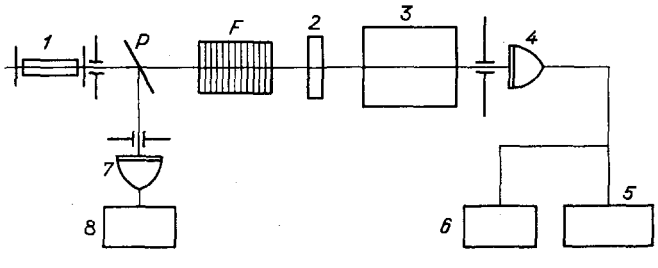

FIG. 8. Diagram of the apparatus: *1*–YAG laser, *P*–semitransparent plate, *F*–cassette with filters, *2*–cuvette containing liquid, *3*–polarization interferometer, *4*, *7*–photoreceivers, *5*, *8*–recording apparatus, *6*–chart recorder.

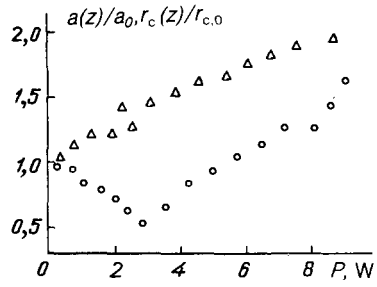

FIG. 9. Dependence of the beam radius and the correlation radius on the power at the exit from a cuvette 4-cm long.

the cuvette for various powers of the radiation showed that the radius a of the beam increases with increasing P_0, whereas r_c can initially decline (Fig. 9). The ratio of the correlation radius to the radius of the beam for $P_0 < 2$ W declines, for $2\text{ W} < P_0 < 6$ W is constant, while when $6\text{ W} < P_0 < 10$ W, it increases rapidly.

The results presented above describe the behavior of the paraxial part of the beam and are valid for weak TSA. The distribution of the field and the coherence over the cross section of the defocused beam and its spatial coherence are actually determined by the nonlinear spherical and noise aberrations.[36,46] A numerical experiment shows that, when $P_0 > P_{cr}^{r}$, the beam decomposes into filaments at distances $z \sim L_{nl}$ (Fig. 10a). The appearance of a fine-scale structure coincides with the appearance of an aberration ring in the profile of the mean intensity (Fig. 10b), as occurs in the internal defocusing of a Gaussian beam.[47]

The separation of the beam at $z > L_{nl}$ is accompanied by transformation of the distribution laws of the field, which obeys Gaussian statistics: The phase distribution remains uniform, while excursions occur in the intensity distribution when $I < \langle I \rangle$, $I > \langle I \rangle$.

The relative dispersion of the intensity fluctuation declines at first. However, after formation of a speckle structure it begins to increase sharply (Fig. 11).

The correlations of the field fluctuations and the optical inhomogeneities that they induce in the defocusing medium are manifested distinctively in the TSA of the beam, which represents a superposition of the "signal + noise" type.[29,48] Here the coherence can be impaired, not only of the noise, but also initially of the coherent signal, although on the average both beams are defocused. Characteristically, the expansion of such a beam prevails over the expansion of a noise beam having the same mean power, which in turn is defocused more strongly than the coherent beam (Fig. 12). This somewhat unexpected result arises from the nonlinear interaction of the signal and noise components via the fluctuating induced optical channel. For the same reason, the correlation radius of a "signal + noise"-type beam decreases in the initial stage more rapidly than that of a noise beam (Fig. 13).

Along with the transformation of the transverse scales (a and r_c), TSA leads to transformation of the time scales of the field fluctuations of the quasicontinuous radiation.[49,50] For a beam with $r_c \to \infty$ and an arbitrary envelope over the cross section $F(\mathbf{r})$, an initial decline in the coherence time occurs, in the absence of dispersion of the medium,

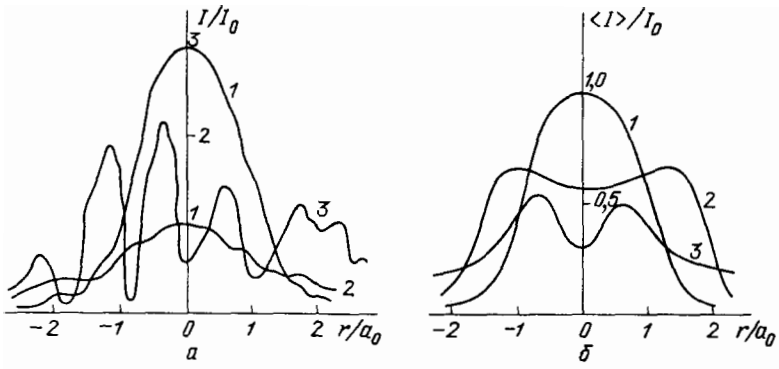

FIG. 10. Instantaneous (a) and average (b) profiles of the intensity with $P_0/P_{cr} = 120$ for $z/L_{dif} = 0$ (*1*), 0.1 (*2*), and 0.3 (*3*).

$$\tau_c(r,z) = \tau_{c0}(1 + \theta_{nl}(r,z))^{-1/2}, \tag{3.26'}$$

caused by the fluctuations (in time) of the optical inhomogeneities (Fig. 14). The decrease in τ_c calculated by the nonlinear phase-channel method for $z < (L_{nl}L_{dif})^{1/2}$ occurs in different ways over the cross section of the beam, since

$$\theta_{nl}(r,z) = h(\alpha)\frac{zL_{dif}}{\pi a_0^2 L_{nl}^2}\int d\mathbf{r}' F^2(\mathbf{r}')\lg\left(\frac{\mathbf{r}-\mathbf{r}'}{a_0}\right)^2. \tag{3.27}$$

As a result of self-action, a beam originally coherent in space with a time-fluctuating intensity loses this property—τ_c over the cross section described by (3.26) ultimately gives rise to spatial fluctuations of the field in the cross section of the beam.

Such nonlinear interrelation will be discussed in detail in the next section of this paper.

3.6. Wind refraction

Motion of the medium arising because of wind currents, autoconvection, and displacement of the laser beam upon scanning can lead to self-deflection and distortion of the profile of a coherent beam.[51–53] The self-deflection of coherent beams has been well studied, and one can make a correction for it.[13,54–56] The propagation of partly coherent beams in moving media occurs in more complex fashion.

A quasicontinuous beam induces in a transversely moving medium a temperature field described by the equation

$$v\frac{\partial(\Delta T)}{\partial x} = \frac{\alpha}{\rho c_p}|A|^2, \tag{3.28}$$

which is written under the assumption that $\tau_v < \tau_\chi$ (a situation typical of the atmosphere).

The nonlinear self-action length in this case

$$L_{nl} = \left(\frac{n_0\rho c_p a_0 v}{|n_T|\alpha I_0}\right)^{1/2} \tag{3.29}$$

is obtained from (3.24) by the substitution $\chi \to a_0 v$ (which is equivalent to the substitution $\tau_\chi = a_0^2/\chi \to \tau_v = a_0/v$).

The joint solution of Eqs. (3.1) and (3.28) for a beam having a random distribution of the field over the cross section of the form

$$A(\mathbf{r}, z = 0) = \sqrt{I_0}\xi(\mathbf{r})e^{-r^2/2a_0^2},$$

and a medium moving with constant velocity has been performed analytically[31,57] and numerically.[14]

In the typical situation of propagation of a high-power beam with $P_0 = 3$ kW, width $a_0 = 5$ cm and wavelength $\lambda = 1\ \mu$m in the atmosphere, which is moving with velocity $v = 4$ m/s, an estimate by the formula (3.29) gives $L_{nl} \sim 1$ km, whereas $L_{dif} \sim 3$ km. The condition $L_{nl} < L_{dif}$ enabled applying the nonlinear phase-channel method for $z < (L_{nl}L_{dif})^{1/2}$. Formulas were obtained by this method for the mean values of the beam width and the correlation radii along the x and y axes; it was shown that the induced temperature fluctuations lead to an additional (as compared with a coherent beam) defocusing of the beam and a decrease in the correlation radius. On the whole, owing to the motion of the medium, the defocusing and decrease in the transverse correlation scale in the yz plane occur more rapidly than in the plane of the wind xz.

A most interesting result is the independence of the displacement of the energy axis from the initial correlation radius

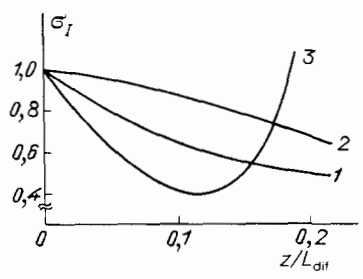

FIG. 11. Variation of σ_I on the axis of the beam for $L_{dif}/L_{nl} = 3$ (*1*), 6 (*2*), and 9 (*3*) ($N_r = 9$).

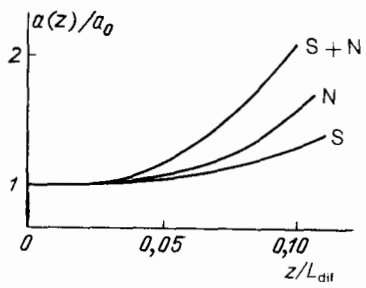

FIG. 12. Broadening of beams of the signal (S) and noise (N) of "signal + noise" type for $\alpha z \ll 1$ when $L_{nl}/L_{dif} = 0.1$, $N_r = 5$.

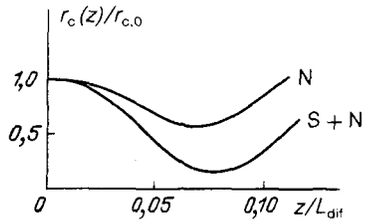

FIG. 13. Variation of the correlation radius of the noise beam and the "signal + noise" type beam for the same parameters as in Fig. 12.

$$x_0(z) = -\frac{h(\alpha)}{2}a_0\left(\frac{z}{L_{nl}}\right)^2. \tag{3.30}$$

This conclusion coincides with the results of the numerical experiment performed in Ref. 14 on the basis of the Monte-Carlo method, and enables one to identify the track of propagation of a partly coherent beam with the track of a coherent beam, which is simpler to calculate.

Fluctuations of the wind velocity caused by atmospheric turbulence substantially alter the pattern of thermal distortion.[13,58-64] Since their discussion lies outside the scope of this review, we shall restrict the treatment to the most general remarks.

The numerical simulation[59,63] studied the change in the spatial statistics of a laser beam on a track having a variable velocity of side wind. It was shown that an increase in σ_v leads to weakening of the nonlinear wind refraction, while here the sickle-shaped energy profile[44] characteristic of regular wind refraction breaks down, and local maxima arise in the envelope (speckle structure)[13,14] lying along the wind direction (Fig. 15).

For beams with an initial spatial modulation, these effects are manifested on shorter tracks. The presence of fluctuations leads to breakdown of the statistical homogeneity of the field in the cross section of the beam. The correlation radius in a direction perpendicular to the wind decreases in comparison with the initial value on the windward side and increases on the leeward side; the coherence of the radiation along the wind direction improves.[14]

3.7. Nonstationary thermal self-action

A distinguishing feature of the self-action of light pulses in the absence of relaxation of the thermal nonlinearity

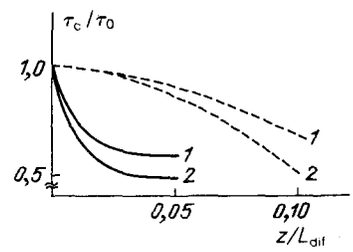

FIG. 14. Decrease in τ_c of a laser beam in its paraxial region for $\alpha z \ll 1$ (dashed) and $\alpha z \gg 1$ (solid lines) for $L_{nl}/L_{dif} = 0.2$ (*1*) and 0.1 (*2*).

($\tau_0 < \tau_\chi, \tau_v$) is the variation of the nonlinear perturbations of the refractive index, not only along the track of propagation, but also over the duration of the pulse:

$$n_{nl} = \frac{\alpha n_T}{\rho c_p}\int_{-\infty}^{\tau}|A|^2 d\tau'. \tag{3.31}$$

The nonlinear refraction accumulating over time leads to a distinctive dynamics of variation of the space and time statistics of laser radiation.

One of the first studies[65] examined theoretically and experimentally the spatial spectra of the intensity fluctuations of pulsed coherent and partly coherent laser radiation that had passed through an absorbing turbulent medium, and gave a very physical interpretation of the influence of self-action on the transformation of the spatial spectra and the variation of the dispersion of the fluctuations.

It was shown that, when $z \ll L_{nl}$ (L_{nl} is the self-action length) in a regular medium, the amplitude and phase initial perturbations of the incident wave will be suppressed by the thermal nonlinearity, while at distances $z \gg L_{nl}$ the initial perturbations are amplified. This arises from the fact that the spatial modulation of the intensity at the entrance to the nonlinear medium leads to time modulation in the interior of the medium.

Since the nonlinear length of nonstationary TSA is a function of the time

$$L_{nl}^{nst}(\tau) = L_{nl}\left(\tau_\chi \Big/ \int_{-\infty}^{\tau} F^2(\tau')d\tau'\right)^{1/2} \tag{3.32}$$

[L_{nl} is given by Eq. (3.24), and $F(\tau)$ is the envelope of the pulse], the characteristic period of modulation τ_m and its frequency $\Omega \sim \tau_m^{-1}$ can be estimated from the condition $L_{nl}(\tau_m) = z$, whence we have $\Omega \sim x^2/L_{nl}^2\tau_\chi$. In the presence of space-time modulation of the intensity $|A|^2 = I_0(\mathbf{r})\cos\Omega\tau$, the sign of the perturbations of the temperature of the medium

$$\Delta T \sim \int_{-\infty}^{\tau}|A|^2 d\tau' = I_0(\mathbf{r})\sin\Omega\tau/\Omega$$

can be opposite to the sign of the perturbations of the intensity. Therefore self-focusing and amplification of the perturbations can occur.

The nonmonotonic variation of the dispersion of the fluctuations of intensity of the radiation in a turbulent medium having a spectrum of fluctuations of the permittivity of the form

$$\Phi_\varepsilon(\varkappa) = c\varkappa^{-11/3}\exp\left(-\frac{\varkappa^2}{\varkappa_m^2}\right)$$

is reflected in Fig. 16; here we have

$$\beta = (1/2)\ln(I/I_0), \quad \xi_0 = (\varkappa_m z/L_{nl})(\chi\tau)^{1/2}.$$

The general tendency is for the thermal nonlinearity to suppress the high-frequency part of the spectrum $\Phi_\beta(\varkappa)$ and to diminish the variance $\langle\beta^2\rangle$. However, further on as the parameter ξ_0 increases, the fluctuations increase. This confirms the calculations in Ref. 32, which were performed by the method of random trials.

The experiment studied the spatial structure of pulsed ($\tau_0 \sim 8$ ms) multimode radiation of a GOR-300 laser after

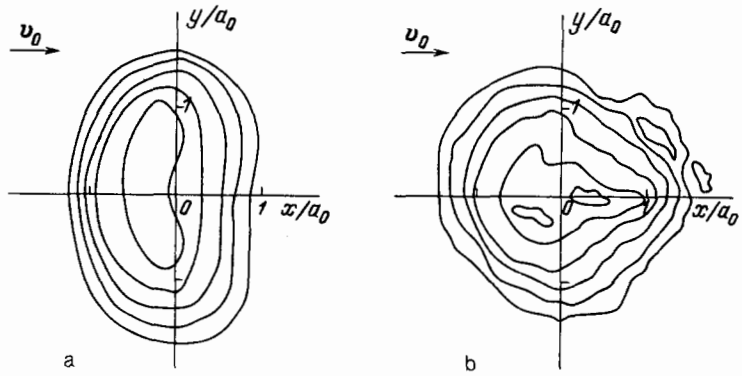

FIG. 15. Lines of equal intensity in the cross section of the beam at a distance $z = L_{\rm dif}$ for $\tau_x/\tau_v = 10$, $L_{\rm dif}/L_{\rm nl} = 7$ when $\sigma_v/v_0 = 0.1$ (a) and 0.8 (b).

passing through a cuvette containing tinted ethyl alcohol with convective turbulence created in it. The two-dimensional spectra of the fluctuations of intensity—calculated and measured—agreed well in the region of wave numbers $0.4 \leqslant \varkappa/\varkappa_{\rm m} \leqslant 1.3$.

The variation of the spatial coherence along the track of propagation and over the duration of the pulse in nonsteady-state TSA is distinguished by its variety.

Studies of the correlation radius, the variance of the fluctuations, and the correlation functions of the amplitude and the phase of small perturbations of the field of the plane wave

$$A(\mathbf{r}, z, \tau) = A_0 \exp(iq(\tau)z)(1 + \xi(\mathbf{r}, z, \tau))$$

in pulsed radiation with $\tau_0, \tau_{c0} \gtrsim r_{c0}^2/4\chi$ were conducted in Ref. 66.

The linearized system of equations with respect to the first and second correlation functions $\Gamma_1 = \langle \xi(\mathbf{r}_1, z, \tau) \xi^*(\mathbf{r}_2, z, \tau)\rangle$ and $\Gamma_2 = \langle \xi(\mathbf{r}_1, z, \tau)\xi(\mathbf{r}_2, z, \tau)\rangle$ and the correlators associated with them of the field ξ and the temperature T, $\varphi_{lm} = \langle T_l \xi_m \rangle$, of the field and the fluctuations of the permittivity $\psi_{lm} = \langle \varepsilon_l \xi_m \rangle$ was solved numerically.

Use of the perturbation method to analyze the correlation functions enabled a substantially shortened expense of machine time, which is usually large in the method of random trials. We should include among its defects the lack of account taken of diffraction of the light beam as a whole and of pumping of energy from the main part of the wave into the fluctuating part.

It was possible by this method to establish the fundamental laws of dynamics of the fluctuations and the spatial coherence of the wave for its various statistical properties.

Figures 17 and 18 show the variation of the variance $\sigma_\xi^2 = \sigma_\rho^2 + \sigma_\varphi^2$, normalized to its initial value along the coordinate of propagation $z/k_0 r_0^2$; $r_0 = (2\chi\rho c_p/k_0^2 \alpha I_0 |\varepsilon_T|)^{1/4}$ characterizes the plane-wave region that contains a power equal to the critical value, while $t_0 = r_0^2/\chi$ is the time for establishment of the temperature on the scale of r_0.

For initial amplitude modulation (AM), effective transformation of the amplitude fluctuations into phase fluctuations occurs owing to both diffraction effects and nonlinearity. The swift increase in σ_φ^2 and the decline in σ_ρ^2 (see Fig. 17) lead to a monotonic increase in the variance of the field fluctuations σ_ξ^2 (see Fig. 18). In the course of time these variations of z become ever stronger.

For initial phase modulation (PM), the transformation of the phase fluctuations into amplitude fluctuations is restrained for $z < k_0 r_0^2$ by the competition of nonlinearity and diffraction effects. This explains the initial decline in σ_ξ^2, which for $z > 1.25 k_0 r_0^2$ is then replaced by an increase in σ_ξ^2, just as in the case of initial AM. The dynamics of the variation of the radius of spatial coherence of a partly coherent wave with phase modulation is shown in Fig. 19.

With increasing distance z a characteristic minimum appears in the dynamics of $r_c(\tau)$, whose time for attainment declines with increasing z.

The retardation of the PM → AM transformation owing to nonlinearity is equivalent to the increase in r_c for small times, although subsequently the phase change induced by the random lenses increases, and r_c decreases. With increas-

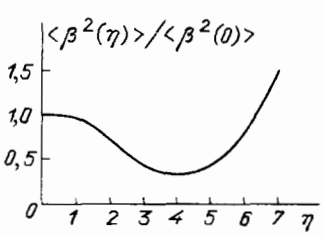

FIG. 16. Variation of the variance of the intensity fluctuation.

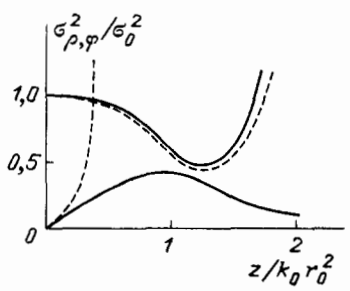

FIG. 17. Variance of the fluctuations of the amplitude σ_ρ^2 (solid curves) and the phase σ_φ^2 (dashed) for $\tau/t_0 = 2$.

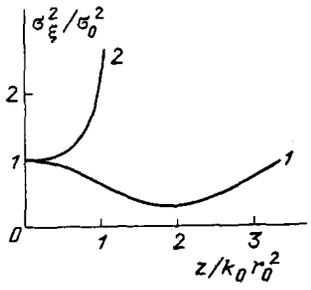

FIG. 18. Variance of the fluctuations of the field σ_ξ^2 for phase (*1*) and amplitude (*2*) fluctuations for $\tau/t_0 = 2$.

ing length z of propagation, the nonlinear effects accumulate, and the initial increase in r_c is practically absent–it immediately declines. However, toward the end of the pulse, r_c begins to increase owing to the onset of correlation between the phase and amplitude fluctuations. An analogous result was obtained by the method of random trials.[69]

In analyzing strong fluctuations the method of random trials was used in Ref. 67. The random two-dimensional fields were simulated:

$$A(\mathbf{r}, \tau, z) = A_0 e^{-r^2/2a_0^2} F(\tau) e^{i\varphi(\mathbf{r})},$$

where $F(\tau) = 1$ when $0 < \tau < \tau_0$ is the envelope of a square pulse, while the correlation function of the phase φ was assumed to be Gaussian:

$$\Gamma_\varphi(\mathbf{r}) = \sigma_\varphi^2 \exp\left(-\frac{\mathbf{r}^2}{r_\varphi^2}\right).$$

The parameters of the radiation and the medium in the numerical experiment were chosen to be close to the conditions of a laboratory experiment[68] with a variance $\sigma_\varphi^2 = 4$ and a correlation radius $r_\varphi = 0.25$ mm of the phase fluctuations on the screen.

In the experiment, whose diagram is shown in Fig. 20, the variation was studied of the spatial coherence of the pulsed radiation of the ruby laser *1* upon passing through the phase screen *5* and the cuvette *6* containing an absorbing liquid.

The experimental dependences of r_c on the emitted energy are shown in Fig. 21 along with the theoretical dependences obtained for the experimental conditions by the random-trial and perturbation methods.

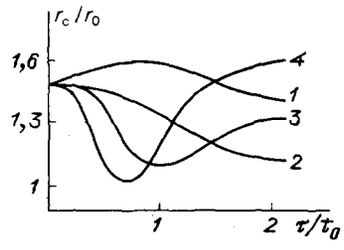

FIG. 19. Variation of the correlation radius over the duration of the pulse for $z/k_0 r_0^2 = 1$ (*1*), 2 (*2*), 3 (*3*), and 4 (*4*).

The increase and decline of r_c in the initial instants of time for $z_1 = 5$ cm are replaced by a monotonic decline for $z_{1g} = 10$ cm.

The results of the laboratory and the numerical experiments agree well with the conclusions of perturbation theory on the influence of the character of the initial fluctuations (amplitude and phase) on the dynamics of the correlation radius over the duration of the pulse.

4. NONLINEAR-DISPERSION SELF-MODULATION OF RANDOM LIGHT PULSES

The theme of this section is the description of the fundamental laws of propagation of brief light pulses in nonlinear dispersing media, among which a special place is occupied by quartz glasses–the basic material of fiber light guides.

In the region of maximum transparency of quartz glass ($\lambda \sim 1$–1.7 μm) the dispersional spreading length ($L_{\rm dis} = \tau_0^2/|k''_\omega|$) for pulses of subpicosecond duration is ($\tau_0 \sim 10^{-13}$ c) $L_{\rm dis} \sim 10$ cm.[70,71] At the same time, the diffraction effects in optically homogeneous media for beams of radius $a_0 \sim 1$ cm develop at lengths $L_{\rm dif} \sim 10^4$ cm. In optical fibers with a transverse dimension of the order of several micrometers, diffraction is compensated by the considerable transverse decline in the refractive index between the core and the cladding.

Great lengths and low optical losses diminish the threshold of the nonlinear effects in optical fibers, despite the fact that quartz glass is a material having a weak Kerr-type cubic nonlinearity. The situation has offered a wide possibility for studying a multitude of nonlinear phenomena that accompany the waveguide propagation of light pulses. The features of the broadening, compression, and generation of ultrashort pulses have been studied in detail in the papers cited above.[70,71] However, unavoidably the existing fluctuations of the field of laser radiation can appreciably affect the character of the transformation of pulses in a fiber light guide, and in the practical realization of systems for compression, frequency conversion, information transfer, etc., a substantial correction of the potentialities of such systems is needed.

As was shown in Ref. 72, the applicability of the quasioptical description is justified down to the femtosecond range of pulse durations.

Below we shall present a description of the features of diffraction-free propagation of partly coherent light pulses in optical quartz single-mode fibers.

4.1. Light pulses in media having quadratic dispersion

In the second approximation of dispersion theory the nonlinear propagation of laser pulses is described by the equation

$$\left(\frac{\partial}{\partial z} - \frac{ik''_\omega}{2}\frac{\partial^2}{\partial \tau^2}\right)A = -\frac{ik_0}{n_0} n_{\rm nl} A, \tag{4.1}$$

In form this coincides with (3.1), which reflects the existence of a space-time analogy between diffractive propagation of light beams and dispersive propagation of light pulses. The substitutions $k''_\omega \to k_0^{-1}$ and $\tau \to \mathbf{r}$ enable one to obtain much a priori information on the features of propagation of wave packets on the basis of the results presented in the previous section.

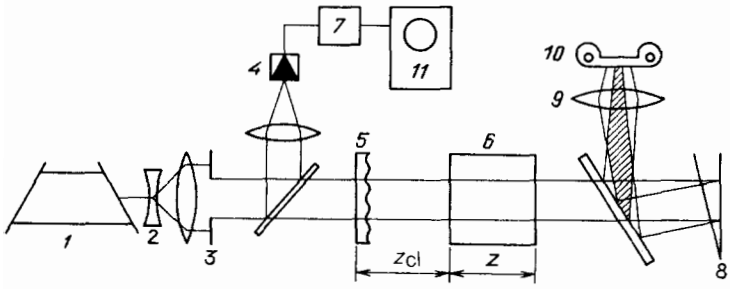

FIG. 20. *1*—beam, *2*—collimator, *3*—diaphragm, *4*—photodiode, *5*—phase screen, *6*—cuvette, *7*—amplifier, *8*—interferometer, *9*—objective, *10*—motion-picture camera, *11*—oscillograph.

However, there is also a difference between the behavior of wave beams and packets. First, in real situations one is dealing with three-dimensional beams (a wave packet is a two-dimensional field in the coordinates z and τ). Second, in contrast to k_0^{-1}, the dispersion parameter k''_ω can be either positive or negative. In the region of normal dispersion $\lambda < \lambda_{cr}$ (λ_{cr} is the critical wavelength at which the condition $k''_\omega = 0$ is realized) in media with $n_2 > 0$, the pulses can spread (become defocused in time). In the region of anomalous dispersion $\lambda > \lambda_{cr}$ self-compression (focusing in time) of pulses can occur. In the latter situation, with a certain choice of parameters of the light pulse, a soliton regime of propagation can occur. Here the soliton is stable toward small perturbations (in contrast to the self-channeling of light beams).

The propagation of a noise burst[27,75] and the influence of weak noise on the propagation of a signal[26] have been studied by the method of integration over trajectories.

For a pulse

$$A(\tau, z = 0) = \xi\sqrt{I_0}e^{-\tau^2/2\tau_0^2} \tag{4.2}$$

having an initial random phase modulation $\xi = \rho e^{i\varphi}$ ($\rho = \text{const}$)

$$\langle\varphi(\tau_1)\varphi(\tau_2)\rangle = \sigma^2 e^{-\Delta\tau^2/\tau_{c0}^2} \tag{4.3}$$

in the paraxial approximation $|\tau| < \tau_0$ an expression was derived[27] for the STDF

$$|\Gamma(\tau_1, \tau_2, z)| = V^{-1}(z)\exp\left[-\frac{\tau_1^2 + \tau_2^2}{2\tau_0^2 V^2(z)} - \sigma^2\frac{(\tau_1 - \tau_2)^2}{\tau_{c0}^2 V^2(z)}\right], \tag{4.4}$$

$$V^2(z) = 1 + \sin^2\left[(2R)^{1/2}z\right]\left[\frac{1}{2R} - \frac{2\sigma^2}{R}\left(\frac{\tau_0}{\tau_{c0}}\right)^2 - 1\right], \tag{4.5}$$

This describes both the evolution of the mean duration of a Gaussian pulse and its coherence time.

The relationship between the nonlinear phase modulation

$$L_{ph} = \frac{n_0}{k_0 n_2 I_0} \tag{4.6}$$

and the dispersion length $L_{dis} = \tau_0^2/|k''_\omega|$ is determined by the nonlinearity parameter $R = L_{dis}/L_{ph}$.

The most essential result is that, when

$$R = R_{cr} = \frac{1}{2} + 2\sigma^2\left(\frac{\tau_0}{\tau_{c0}}\right)^2 \tag{4.7}$$

a mean statistical pulse can propagate in a steady-state situation-the pulse duration and the correlation time remain constant. This is a consequence of the fact that, in individual pulses (realizations), phase modulation causes both expansion and narrowing of the pulse, which under certain conditions compensate one another on the average.

When $R \neq R_{cr}$ the pulse duration and the correlation time vary in the same way. Hence their ratio remains constant.

The presence of random amplitude-phase modulation in a "noise burst" pulse leads to the appearance of time fluctuations of the induced nonlinear optical lens that facilitates spreading of the pulse. The variation of the duration of such a noise pulse and its coherence time are shown in Fig. 22.[75] The monotonic increase in τ_{inc} and τ_c occur both in the coherent ($z < L_{coh} = \tau_{c0}^2/|k''_\omega|$), and in the incoherent ($z > L_{coh}$) regimes of propagation in a medium with normal dispersion.

The transformation of pulses of the "signal + noise" type occurs in a rather complicated way, which arises from the nonidentical behavior of the regular and noise components of the light field in the nonlinear medium. In the fixed-channel approximation, the authors of Ref. 26 studied the influence of a weak noise perturbation ($\sigma^2 \ll 1$) on the nonlinear regime of propagation of a regular pulse. Without re-

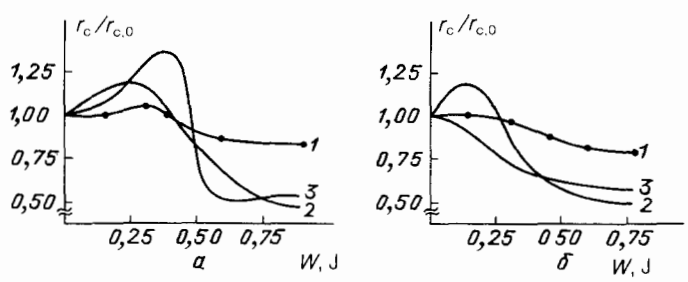

FIG. 21. Variation of the radius of spatial coherence at the exit from the cuvette for a distance between the phase screen and the cuvette of 5 cm (a) and 10 cm (b) as obtained experimentally (*1*), by the method of random trials (*2*), and by the perturbation method (*3*).

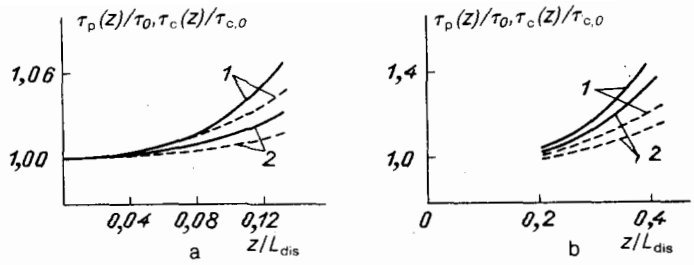

FIG. 22. Variation of the normalized pulse duration (solid curves) and of its coherence time τ_c (dashed) in coherent (a) and in incoherent (b) regimes of propagation for $R = 2$ (*1*) and 4 (*2*) ($N_{la} = 10$).

striction on the dispersion of the noise component, Ref. 29 used the nonlinear-phase-channel method to study in detail the transformation of light fields of the "signal + noise" type for lag-free and lagging regimes of self-action. For lag-free nonlinearity, the mean-square duration of the pulse

$$\tau_p(z) = \tau_0\left\{\left[1 \pm \frac{1}{2}(1+\sigma^2)\frac{z^2}{L_{ph}L_{dis}}\right]^2 + (1 + 2\sigma^2 N_\tau^2)\frac{z^2}{L_{dis}^2} + 4\sigma^2(1+\sigma^2)N_\tau^2\frac{z^4}{L_{ph}^2 L_{dis}^2}\right\}^{1/2} = \tau_0 f^{1/2}(z) \tag{4.8}$$

varies in a complicated way. However, Eq. (4.8) implies that four fundamental effects influence the variation of the pulse duration: they are the coherent and incoherent linear-dispersion spreading at lengths L_{dis} and $L_{dis}/\sqrt{2}\sigma N_\tau$ ($N_\tau = \tau_0/\tau_{c0}$ is the number of longitudinal inhomogeneities over the duration of the pulse) independently of the form of the dispersion; coherent nonlinear-dispersion spreading or compression at the length $L_L^c = [2L_{ph}L_{dis}/(1+\sigma^2)]^{1/2}$ depending on the sign of the nonlinearity and the form of dispersion; incoherent nonlinear-dispersion spreading at the length $L_L^{inc} = L_L^{coh}/2\sqrt{2}\sigma N_\tau$ independently of the sign of the nonlinearity and the form of dispersion.

The transformation of the variance of the intensity fluctuations σ_I^2 occurs so that, at the characteristic distances $z \sim L_{lg}$ the intensity fluctuations occur preferentially at the fronts of the time envelope, while in the central part of the pulse the envelope of the intensity $I(\tau)$ has a practically regular structure.[76,77] This situation enables one to stabilize the parameters of compressed pulses by spatial filtration of their spectral components in a grating compressor (????). This important result is illustrated in Fig. 23, taken from Ref. 77, where, along with the increase in the coherence time of the central part of the pulse, the variance of the fluctuation σ_I^2 decreases.

4.2. The role of higher-order dispersion

The broadening of pulses in fiber light guides lowers their transmission power and restricts the information band of transmission. In multimode fibers this broadening is caused by the difference between the group velocities of propagation in the individual modes. In a single-mode light guide the fundamental mechanism of broadening is dispersion. Weakening of the quadratic dispersion in quartz glasses can be attained by a suitable choice of the wavelength of the radiation in the range $\lambda \sim 1.3$–$1.6\ \mu$m. This region of wavelengths is the most promising for realizing long-range fiber-optic communications.[78]

At a wavelength close to critical ($\lambda \sim \lambda_{cr}$), $k''_\omega = 0$, one must take account of higher-order dispersion. Under such conditions the fundamental equation of quasioptics, which describes the propagation of picosecond pulses in a single-mode quartz light guide with account taken of dispersion up to the third order, inclusive, has the form

$$\frac{\partial A}{\partial z} - \frac{ik''_\omega}{2}\frac{\partial^2 A}{\partial \tau^2} + \frac{k'''_\omega}{6}\frac{\partial^3 A}{\partial \tau^3} = -\frac{ik_0}{n_0}n_2|A|^2 A. \tag{4.9}$$

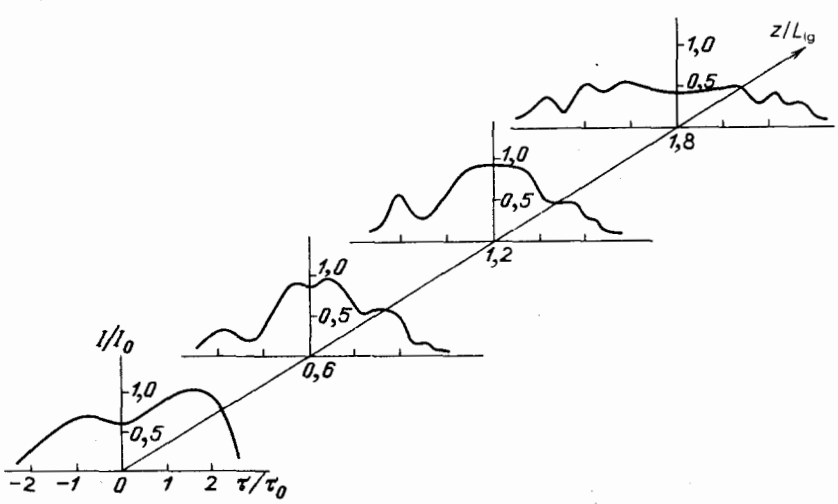

FIG. 23. Distribution $I(\tau)$ in the region $k''_\omega > 0$ for $R = 300$, $N_\tau^{-1} = 0.64$, $\sigma = 0.2$.

The contribution of the effects of cubic dispersion is determined by the characteristic length

$$L'_{\rm dis} = \frac{\tau_0^3}{k'''_\omega}. \tag{4.10}$$

In quartz glass for $\lambda \sim \lambda_{\rm cr}$ and pulse duration 1 ps, we have $L'_{\rm dis} \sim 13$ km, while for pulses with $\tau_0 \sim 0.1$ ps we already have $L'_{\rm dis} \sim 13$ m. If $|\lambda - \lambda_{\rm cr}| > 5$ nm, the second-order dispersion becomes dominant, and $L_{\rm dis} \ll L'_{\rm dis}$.[79]

The nonlinear propagation of regular phase-modulated pulses with account taken of the effects of quadratic and cubic dispersion was studied in Refs. 80 and 81. For a fixed wavelength λ the relationship between the effects of quadratic and cubic dispersion depends on the duration of the pulse: for long pulses $\tau_0 > k'''_\omega/|k''_\omega|$ quadratic dispersion dominates ($L_{\rm dis} \ll L'_{\rm dis}$), while for short pulses $\tau_0 < k'''_\omega/|k''_\omega|$ the cubic dispersion becomes decisive.

On the other hand, the contribution of self-action can be characterized by the relationship between the length of nonlinear phase self-modulation $L_{\rm ph}$ and the dispersion lengths $L_{\rm dis}$ and $L'_{\rm dis}$. Here, from the conditions $L_{\rm ph} = L_{\rm dis}$ and $L_{\rm ph} = L'_{\rm dis}$, one can respectively introduce the critical powers

$$P_{\rm cr,2} = \frac{\tau_0}{k_0 n_2 L_{\rm dis}}, \quad P_{\rm cr,3} = \frac{\tau_0}{k_0 n_2 L'_{\rm dis}}. \tag{4.11}$$

The nonlinear propagation of long and short pulses of the "noise burst" type in the region of normal dispersion has been studied analytically and numerically (by the Monte-Carlo method) in Refs. 30 and 82.

For short pulses with power $P_0 < P_{\rm cr,2}, P_{\rm cr,3}$ (regime of weak self-action), when the effect of cubic dispersion dominates ($L'_{\rm dis} < L_{\rm dis}, L_{\rm ph}$), the analytic expression for the modulus of the temporal correlation function of the field obtained by the nonlinear phase-channel method ($z < (L_{\rm dis} L'_{\rm dis} L_{\rm ph})^{1/3}$) for a distance $z \lesssim L'_{\rm dis}$ acquires the form

$$|\Gamma_\tau(\tau_1, \tau_2, z)| = \left(\frac{2L'_{\rm dis}}{z}\right)^{2/3} \frac{I_0}{\sqrt{2}N_\tau} \exp\left[-\frac{z^2}{2L_{\rm ph}^2}\frac{(\tau_1-\tau_2)^2}{\tau_c^2}\right] \times {\rm Ai}[y(\tau_1, z)]\,{\rm Ai}[y(\tau_2, z)], \tag{4.12}$$

where we have

$$y(\tau, z) = -\left(\frac{2L'_{\rm dis}}{z}\right)^{1/3}\left[\frac{\tau}{\tau_0} - \frac{zL'_{\rm dis}}{2L'^2_{\rm dis}}\right]. \tag{4.13}$$

These expressions imply that, along with the strong asymmetric spread of the pulse (Fig. 24), its peak intensity is retarded by the time interval

$$\tau_{\rm ret} = \tau_0\left(\frac{z}{2L'_{\rm dis}}\right)^{1/3}\left[1 - \left(\frac{2L'_{\rm dis}}{z}\right)^{1/3}\frac{zL'_{\rm dis}}{2L'^2_{\rm dis}}\right], \tag{4.14}$$

its energy center is displaced, and the trailing front becomes steeper. This conclusion is confirmed also by a numerical experiment, the data of which agree well with the analytic results. The maximum intensity of the pulse

$$I_0(z) \approx 0{,}3\left(\frac{2L'_{\rm dis}}{z}\right)^{2/3}\frac{I_0}{\sqrt{2}N_\tau} \tag{4.15}$$

upon propagation decreases mainly because of third-order dispersion and random time modulation, which is characterized by the parameter $N_\tau = \tau_0/\tau_{c0}$.

Analysis of (4.12) and a numerical experiment show that nonlinear self-action favors impairment, while cubic dispersion favors improvement of the time coherence of the pulse. Fragmentation occurs in the function Γ on the trailing front, where the temporal coherence function is periodically modulated with the period of the Airy function Ai, which is determined by the characteristic length $L'_{\rm dis}$ of the cubic dispersion. In a regime of weak self-action, the coherence time increases within the limits of the initial duration of the pulse (Fig. 25).

At distances $z > L'_{\rm dis}$ the weakening of the intensity peak along z is still accelerated, while the time of retardation $\tau_{\rm ret}$ gradually decreases and disappears. This results from the manifestation of quadratic dispersion, which favors restoration of the symmetry of the pulse.

In a regime of strong self-action numerical experiment shows that a substantial spreading of short pulses occurs at distances $L_{\rm ph} < z < L'_{\rm dis} < L_{\rm dis}$, and the time envelope of the profile of the mean intensity acquires a fragmented form. However, division into subpulses does not occur.[83] As the ratio $L_{\rm dis}/L'_{\rm dis}$ increases, the spreading becomes more asymmetric, and the energy center of the pulse is further retarded.

The rate of spread of the individual inhomogeneities lags behind the rate of spread of the pulse as a whole. Therefore, even on tracks $z \sim 0.1 L_{\rm dis}$, the number of time inhomogeneities N_τ increases. Here the fluctuations in the energy-bearing part of the pulse decrease, and they are expelled to the periphery (Fig. 26).

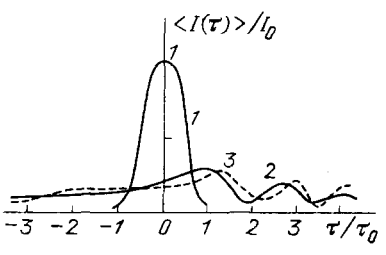

FIG. 24. Mean envelope of the intensity for $z/L_{\rm dis} = 0$ (*1*) and 0.2, calculated analytically (*2*) and numerically (*3*).

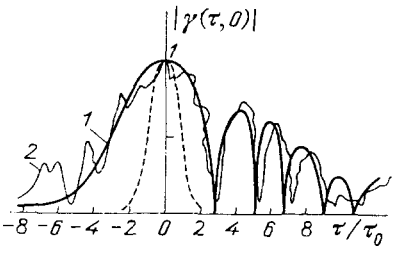

FIG. 25. Modulus of the exponent of the temporal correlation function of the field for $z/L_{\rm dis} = 0$ (dotted line) and 0.12 calculated analytically (*1*) and numerically (*2*).

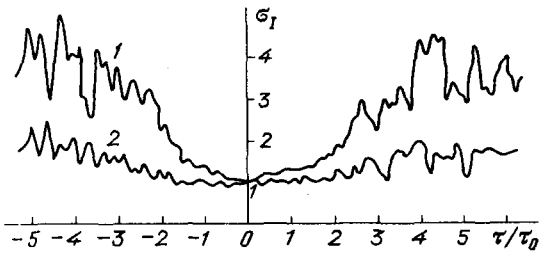

FIG. 26. Time distribution of the relative fluctuations of intensity at distances $z/L_{dis} = 0.1$ (*1*) and 0.2 (*2*) when $L_{dis} = L'_{dis} = 50L_{ph}$, $N_\tau = 2$.

The existence of strong nonlinearity radically alters the character of the transformation of the frequency spectrum. When third-order dispersion plays the dominant role, the fluctuations of the input field cause the spectral density of the pulse to be enriched with frequencies in the anti-Stokes region (Fig. 27). Such an upward waveguide frequency displacement can be effected smoothly by a suitable choice of both the power and duration of the pulse and of the material of the light guide and its length.

To sum up the results so far, we can conclude that, for randomly modulated pulses, removal of quadratic dispersion does not slow the tempo of spreading of the pulse as a whole, while asymmetric deformation of the envelope and group retardation of the pulse occur. However, the region $\lambda \sim \lambda_{cr}$ is promising for transformation of the carrier frequency of the pulse into the anti-Stokes region and retardation of the tempo of impairment of time coherence, which occurs with strong nonlinearity in a medium with normal dispersion.

4.3. Non-steady-state nature of nonlinear polarization

For intense pulses ($I \sim 10^{10}$–10^{12} W/cm^2), when the rate of variation of the mean envelope of the intensity of the laser field and the Kerr nonlinearity are rather high, the fundamental equation of quasioptics is supplemented on the right-hand side by another nonlinear term[70]

$$\frac{\partial A}{\partial z} - \frac{ik''_\omega}{2}\frac{\partial^2 A}{\partial \tau^2} = -\frac{ik_0}{n_0}n_2|A|^2A - \frac{n_2}{c}\frac{\partial}{\partial \tau}(|A|^2A). \tag{4.16}$$

The existence of this term on the right-hand side of (4.16) involves the large gradient of the field intensity over the duration of the pulse. The length

$$L_{nst} = \frac{\tau_0 c}{n_2 I_0} \tag{4.17}$$

characterizes the effect of non-steady-state behavior of the nonlinear polarization for deterministic pulses. For an intensity $I \sim 10^{10}$ W/cm^2 and a wavelength of the radiation $\lambda \sim 1$ μm, the nonlinear phase-modulation length is of the order of $L_{ph} \sim 0.1$ m, while for picosecond pulses $L_{nst} \sim 100$ m. Thus, although L_{nst} is three orders of magnitude greater than L_{ph}, yet, it is an order of magnitude smaller than L_{dis}, and the non-steady-state nature the nonlinear polarization must be taken into account. The characteristic lengths L_{ph} and L_{nst} become comparable only for a duration $\tau_0 \sim 10$ fs. In single-mode fibers, even with high-power pulses ($I \sim 10^{12}$ W/cm^2), the nonlinear perturbation of the refractive index $n_{nl} \sim 3 \times 10^{-4}$ is far smaller than the decline in the refractive index Δn between the core and the cladding. In this regard we can consider the transverse structure of the laser field along the track of propagation to be constant.

While the effect of nonlinear self-modulation is associated with the intensity of the laser field, the action of the non-steady state nature of the nonlinear polarization depends mainly on the decline in intensity on the scale of the pulse duration. For randomly modulated pulses the non-steady state nature of the nonlinear polarization can already be substantial at lengths $\sim \tau_c c/n_2\sigma_I$, which in a number of cases are smaller than (4.17).

The formation of a shock wave under the action of the non-steady state nature of the nonlinear polarization for deterministic pulses has been studied in detail, both in the lag-free[84,85] and the dispersive[86] regimes of self-action. Reference 87 studied analytically and numerically the effects of the non-steady state nature of the nonlinear polarization for random pulses, when $N_\tau \gg 1$ in single-mode light guides in the region of normal dispersion.

The non-steady-state nature of the nonlinear polarization, which is enhanced with increasing N_τ, leads to group retardation of the peak of the pulse and steepening of its trailing front (Fig. 28). The retardation of the peak of the pulse increases with increasing power in it and with decreasing initial duration τ_0.

On the initial track $z < L_{ph}$, the time of retardation of

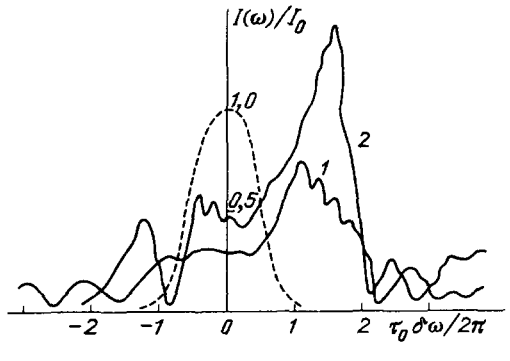

FIG. 27. Frequency spectrum of the pulse at distances $z/L_{dis} = 0$ (dotted) and 0.26 for $N_\tau = 4$, $L_{dis}/L'_{dis} = 1$ (*1*) and 5 (*2*).

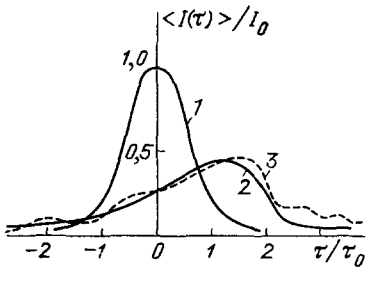

FIG. 28. Mean profile of the intensity of a noise pulse for $z/L_{dis} = 0$ (*1*) and 0.14 (*2*) calculated analytically (*2*) and numerically (*3*) for $L_{dis}/L_{ph} = 50$, $L_{dis}/L_{nst} = 2$, $N_\tau = 4$.

the peak of the pulse is found be proportional to the cube of the traversed path

$$\tau_{\rm ret}(z) = 7N_\tau^2\tau_0\frac{z^3}{L_{\rm dis}^2 L_{\rm nst}} \tag{4.18}$$

Subsequently, when $z \sim L_{\rm nst}$, the increase in $\tau_{\rm ret}$ gradually weakens:

$$\tau_{\rm ret}(z) = \frac{1}{\sqrt{2}}N_\tau^2\tau_0\frac{z^{1/2}L_{\rm dis}^{1/2}}{L_{\rm nst}} \tag{4.19}$$

At a distance $z \sim L_{\rm dis}$ it begins to decrease. This is caused by the gradual spreading of the pulse, which in turn weakens the decline in intensity in the time τ, and hence also the effect of non-steady-state self-action. The pulse becomes gradually symmetrized as the role of the quadratic dispersion increases. The result of numerical experiment agrees qualitatively with the analytic formulas (4.18) and (4.19) (Fig. 29).

For deterministic pulses ($N_\tau = 0$) the non-steady-state self-action weakens the spreading of the pulse in the region of normal dispersion of the group velocity and enhances it in the region of anomalous dispersion.[86] Random modulation of the pulse always facilitates its additional spreading, independently of the type of dispersion.

4.4. Compression of random pulses

The joint action of the effects of nonlinearity and anomalous dispersion of the group velocity can lead to a temporal compression of pulses and to formation of optical solitons. Reference 88 theoretically predicted the possibility of a soliton regime of propagation in light guides, while such a regime was realized in Ref. 89 in a light guide based on quartz glass.

The nonlinear transformation of deterministic pulses of various forms has been studied in a number of papers, including Refs. 90 and 91. The actual quantitative laws of the dispersion regime of compression of regular pulses were established in Refs. 92 and 93 by mathematical modeling methods. However, in real compression systems random factors can play a significant role. In going over to picosecond pulses, along with the irregularity of the light guide itself, the influence increases of temporal amplitude-phase fluctuations of the light field on the process of self-compression of pulses. Some regularities of these processes were analyzed numerically in Refs. 77 and 94 by the Monte-Carlo method.

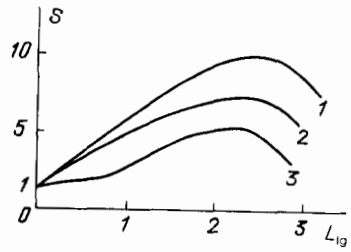

FIG. 30. Dependence of the degree of compression of regular pulses in a lag regime of self-action on the length of the light guide for $\tau_{\rm rel}/\tau_0 = 0$ (*1*); 0.1 (*2*); and 1 (*3*).

The mean-square duration of a light pulse of "signal + noise" type in a quartz light guide under the joint action of the effects of nonlinear phase self-modulation and anomalous dispersion is determined analytically by Eq. (4.8), where a minus sign is set in front of the second term in $f(z)$. In situations characteristic of fiber-optic compression, the nonlinearity parameter is $R = L_{\rm dis}/L_{\rm ph} \sim 10^2$,[95] while the pulse duration on the initial track $z < L_{\rm ph}$ practically always decreases. At the distance

$$L_{\rm opt} = L_{\rm lg}\left(\frac{2}{1+16\sigma^2N_\tau^2}\right)^{1/2}\left(1-\frac{1+2\sigma^2N_\tau^2}{R}\right)^{1/2} \tag{4.20}$$

the pulse duration reaches its minimum value

$$\tau_{\rm p,min} = \tau_0\left[1-\frac{1}{1+16\sigma^2N_\tau^2}\left(1-\frac{1+2\sigma^2N_\tau^2}{R}\right)^2\right]^{1/2}. \tag{4.21}$$

Consequently a fiber light guide of length $L_{\rm opt}$ makes possible the maximum compression of a light pulse at the exit. Here the degree of compression is $S_{\max} = \tau_0/\tau_{\rm p,min}$.

For deterministic pulses the maximum degree of compression can be increased by increase in the parameter $R = \tau_0^2 k_0 n_2 I_0/n_0|k''_\omega|$. Consequently high-power pulses are compressed more effectively than are low-power pulses. However, at intensity values $I \sim 10^{10}$ W/cm^2 and higher, the process of transformation of the pulse duration is influenced decisively by the effect of non-steady-state self-action, which favors weakening of the process of self-compression. Also the compression process occurs more effectively for long than for short pulses. Moreover, for short enough pulses $\tau_0 < 10^{-13}$ s the manifestation of lag in the nonlinear re-

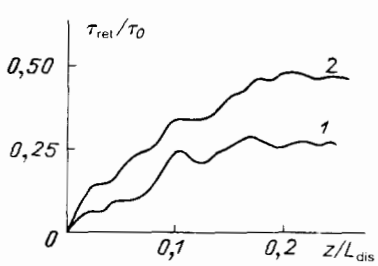

FIG. 29. Retardation of the peak of the pulse calculated numerically for $R = 50$, $N_\tau = 5$, and $L_{\rm dis}/L_{\rm nst} = 1$ (*1*) and 2 (*2*).

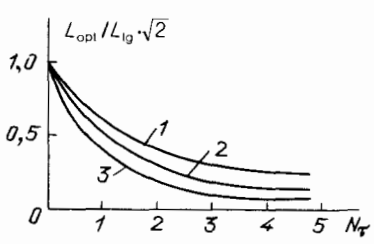

FIG. 31. Dependence of the optimal length of the light guide for random pulses on N_τ for $\sigma^2 = 0.1$ (*1*); 0.2 (*2*); and 0.3 (*3*).

sponse reduces the efficiency of compression[96] of regular pulses (Fig. 30). On the other hand the parameter R can be increased also by an increase in the ratio $k_0/|k''_\omega|$, which is attained by decreasing the wavelength λ of the radiation. However, near λ_{cr} effects of third-order dispersion, which reduce the degree of compression, begin to be manifested. Thus the most efficient self-compression of pulses is obtained in the region $\lambda \sim \lambda_{cr} + 5$ nm, where at the minimal wavelength of the radiation the anomalous quadratic dispersion still dominates over the cubic dispersion.

For pulses of the "signal + noise" type, the optimal length for compression is smaller than for regular pulses. Here L_{opt} decreases substantially with increasing variance of the time fluctuations (Fig. 31). The dependence of L_{opt} on the number of temporal inhomogeneities is sharply marked when $N_\tau < 3$, while with further increase in N_τ the optimal length practically does not vary. The decrease in L_{opt} with increasing σ^2 and N_τ is explained by the enhancement of the incoherent dispersion and nonlinear effects, which even on the short track $z \sim L_{opt}$ succeed in compensating the nonlinear coherent compression of the pulse owing to phase self-modulation. The degree of maximum compression of a random pulse, on the one hand, depends weakly on the variance of the fluctuations σ^2, and on the other hand, it declines considerably with increasing N_τ when $N_\tau < 3$, while further for $N_\tau \sim 3$–5 it practically does not vary (Fig. 32). Thus, the sharply marked dependence on N_τ of the optimal length of the light guide and the maximal degree of compression is manifested when $N_\tau < 1$, i.e., when the coherence time of the noise component τ_c is comparable with the duration of the mean envelope. Here even small changes in τ_c lead to rather strong jumps in the parameters L_{opt} and S_{max}.

An appreciable increase in S_{max} with increasing R is possible only in the region $R \sim 10$–50 (Fig. 33). For large values $R \sim 10^2$, even when $N_\tau > 0.7$, an increase in the intensity of the pulse hardly facilitates an increase in S_{max}. Already a twofold compression of a random pulse is impossible, even with small values of the dispersion of the fluctuation of the noise component ($\sigma^2 \sim 0.1$) of intense pulses ($R > 10^2$). In this situation S_{max} for N_τ no longer depends on R and equals $\sim [1 + (1/16\sigma^2 N_\tau^2)]^{1/2}$.

For random pulses having a broad frequency spectrum, when $N_\tau \gg 1$, it is practically impossible to achieve temporal self-compression. This arises from the fact that a process of slight compression occurs on a short track $z \ll L_{lg}$, while further on, owing to development of the incoherent nonlinear and dispersion effects, the pulse begins to spread strongly. Consequently, for effective compression of random pulses, it is more expedient to increase the coherence than to increase the input intensity.

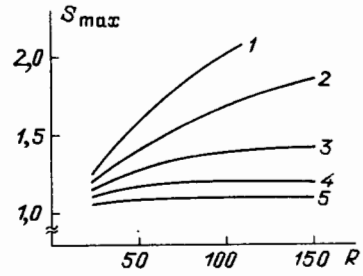

FIG. 33. Variation of S_{max} as a function of R for $\sigma^2 = 0.1$ and $N_\tau = 0.3$ (*1*); 0.4 (*2*); 0.5 (*3*); 1 (*4*); and 2 (*5*).

Reference 97 presented the results of a numerical experiment to study the statistical steady state of a wave packet under the joint action of a positive cubic nonlinearity and anomalous dispersion. A breakdown was shown of the steady state of the random process $\xi(\tau)$ in the case of a "noise burst" with a large nonlinearity parameter $R \sim 10^2$ under self-compression conditions. The possibility was demonstrated of increasing the coherence time τ_c at the boundaries of the pulse, whereas a decrease in τ_c is observed in the middle of the pulse (Fig. 34).

The evolution of the temporal correlation function of a laser field in a fiber light guide in a region of anomalous dispersion was studied[98] by the method of moments under the assumption of conservation of the statistics and form of the mean envelope. It was also shown that an increase in the number of temporal inhomogeneities N_τ reduces the efficiency of compression of a noise pulse (Fig. 35). The maximum for S_{max} is realized when $N_\tau = 0$.

On the whole the mutual compensation of the effects of coherent nonlinear self-compression and of anomalous dispersion makes it possible to stabilize light pulses in fiber light guides; under conditions of small losses it increases the transmission power and rate of transport up to 10 terabits per second.[99] Initial phase fluctuations, analogously to amplitude fluctuations, also weaken the efficiency of compression. However, the output parameters (τ_p and S_{max}) are less

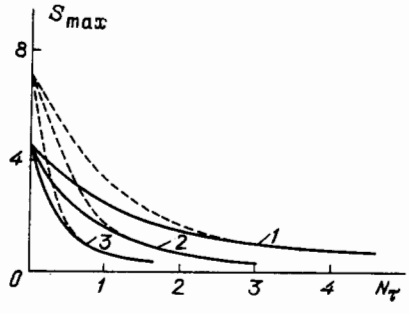

FIG. 32. Variation of S_{max} as a function of N_τ for $R = 50$ (solid lines) and $R = 100$ (dotted) for $\sigma^2 = 0.1$ (*1*); 0.2 (*2*); and 0.3 (*3*).

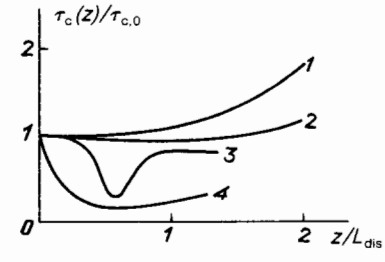

FIG. 34. Variation of τ_c of a noise pulse $N_\tau = 6.5$ in its central part ($\tau = 0$) for $R = 0.8$ (*1*), 3 (*2*), 6 (*3*), and 15 (*4*).

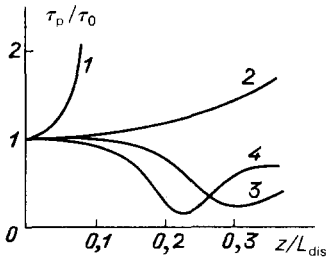

FIG. 35. Transformation of the pulse duration with propagation for $R = 5$ for $N_r = 10$ (*1*), 2 (*2*), 1 (*3*), and 0 (*4*).

sensitive to such fluctuations of the input radiation, and the variance of S_{max} in the region $z \sim L_{opt}$ is substantially weakened.[77,92]

In experiments on compression of regular pulses considerable advances have been made in recent years. For example, in Ref. 100 a 27-fold compression was achieved in 7-ps pulses for wavelength 1.55 μm, and in Refs. 101 and 102, more than 100-fold compression of 30-ps pulses. Today pulses have already been obtained experimentally that cover only several periods of optical oscillations.[103] However, the laws of self-compression of random pulses have not yet been experimentally confirmed.

5. THE MUTUAL INFLUENCE OF TEMPORAL AND SPATIAL COHERENCES OF LASER RADIATION IN A NONLINEAR MEDIUM

In many real cases the space-time boundedness of light fields, when the characteristic lengths of diffraction L_{dif} and of dispersion L_{dis} are comparable, introduces new phenomena into the process of their propagation and transformation by optical systems. The high-frequency components of a pulse are diffracted more slowly than the low-frequency components. Therefore, in the final value of the relative width of the frequency spectrum $\Delta\omega/\omega_0$, the nonidentical diffraction of the different frequency components deforms the time envelope of the pulse.[104] The effect of inequality of the diffraction lengths of the different spectral components is enhanced for laser pulses of duration of several periods of optical oscillations.[105,106] We should also expect a reverse influence of dispersion on the transformation of the width of the beam. For example, in the final value of the relative width of the wave spectrum $\Delta k/k_0$, even in a diffraction-free regime of propagation, the beam will be deformed in the cross section in the dispersion region $z \sim L_{dis}$. For randomly modulated fields the effects of the mutual influence of diffraction and dispersion are manifested on relatively short tracks, and also a transformation is observed of spatial into temporal fluctuations, and vice versa.[107]

In a nonlinear medium, when $L_{nl} < L_{dif}, L_{dis}$, even on a track $z \lesssim L_{nl}$ the transverse deformation of the beam causes a change in the induced optical inhomogeneities of the medium. Hence it leads to transformation of the pulse duration. In turn the transformation of the time envelope of the pulse facilitates the nonlinear refraction of the beam. The simultaneous space-time self-modulation of partly coherent laser radiation by the random inhomogeneities $\tilde{n}_{nl}$ in the propagation channel gives rise to a reciprocal influence of the time and space fluctuations of the laser field.[29,57,108,109]

5.1. Space-time self-modulation of regular fields

The generalized equation of diffraction and dispersion in a linear medium that describes the propagation of short light pulses with bounded transverse dimensions has the form[104]

$$\left(\frac{\partial}{\partial z} + \frac{i}{2k_0}\frac{\partial^2}{\partial \mathbf{r}^2} - \frac{ik''_\omega}{2}\frac{\partial^2}{\partial \tau^2} - \frac{i}{u_{gr}k_0}\frac{\partial^2}{\partial z \partial \tau}\right) A(\mathbf{r}, \tau, z) = 0, \tag{5.1}$$

where the last term describes the reciprocal influence of the space-time scales of the laser field.

The solution of Eq. (5.1) with account taken of all its terms involves great mathematical difficulties. References 105 and 110 have analyzed the temporal distortion of the laser field having an initial space-time envelope $F(\mathbf{r}, t)$ of Gaussian type in a dispersion-free regime of propagation. The numerical calculation[105] that was performed of the temporal envelope of the pulse at the distance $z \sim L_{dif}$ showed an appreciable increase in the duration of a femtosecond pulse at the periphery of the beam ($r \sim a_0$). This tendency persists also in the far zone $z \gg L_{dif}$, where the duration of a pulse with coordinate z varies as follows:

$$\tau_p(z) = \tau_0\left[1 + \left(\frac{a_0 r}{v_0 \tau_0 z}\right)^2\right]^{1/2}; \tag{5.2}$$

Here v_0 is the phase velocity of propagation.

Deformation of the temporal envelope can also arise upon passage of laser fields bounded in space and time through various optical elements. For example, in the passage of ultrashort pulses through a zone plate, the linear transverse refraction facilitates a change in the form and duration of the light pulse.[111]

For pulses of duration $\tau_p \gtrsim 1$ ps, the last term in (5.1) is vanishingly small and the transformation of the space-time scales of the laser field occurs independently. However, the reciprocal influence of the space and time parameters becomes substantial again for high-power laser radiation in a nonlinear medium, when $L_{nl} < L_{dif}, L_{dis}$. For example, the focusing of the beam enhances the nonlinear perturbation of the refractive index n_{nl} of the medium along the track of propagation, and correspondingly the nonlinear-dispersion distortion of the time envelope of the pulse. The total duration of a pulse having an initial Gaussian space-time envelope obtained by the method of successive approximations with account taken of the increase in the optical perturbations with propagation owing to focusing is given by the following expression[115] in the near zone $z < L_{nl}$:

$$\tau_p(z) = \tau_0\left\{1 \pm \frac{1}{2}\frac{z^2}{k_0 L_{dis} L_{nl}^2}\left[\left(1 - \frac{z^2}{2k_0 L_{dif} L_{nl}^2}\right)^2\right]^{-1}\right\}, \tag{5.3}$$

where

$$L_{nl} = \frac{1}{k_0}\left(\frac{n_0}{n_2 I_0}\right)^{1/2} \tag{5.4}$$

is the nonlinear length in a lag-free regime of self-action, which does not contain the space-time parameters of the light field.

In (5.3) the plus sign before the second term holds in a

medium with normal dispersion, and the minus sign for anomalous dispersion.

Of course, the nonlinear-dispersion compression or spread of the pulse, in turn, will correspondingly enhance or weaken the nonlinear refraction of the beam.

5.2. The reciprocal influence of time and space fluctuations

Within the framework of the non-steady-state diffraction equation (5.1), even in a linear medium the problem of the reciprocal influence of the time and space fluctuations of diffracted and dispersed fields is highly complex. In a dispersion-free regime ($k''_\omega = 0$) of linear propagation, this problem has been studied thoroughly in Ref. 107. At the entrance to the medium the STCF of the field is represented in the factored form

$$\Gamma(\mathbf{r}_1, \mathbf{r}_2, \tau_1, \tau_2) = \Gamma(\mathbf{r}_1, \mathbf{r}_2)\Gamma(\tau_1, \tau_2). \tag{5.5}$$

The space-time distribution of the mean intensity $\langle I(\mathbf{r}, t)\rangle$ that one obtains shows an additional spread of the pulse due to the initial spatial fluctuations. The coherence time is transformed in varying ways in the cross section of the beam. For example, at the axis of the beam ($r = 0$) the time coherence does not vary ($\tau_c = \text{const}$), while in its peripheral part ($r \sim a_0$) for $r_{c0} \ll a_0$ and $\tau_{c0} \gg \tau_0$, the coherence time declines upon propagation. The situation differs when $\tau_{c0} \ll \tau_0$: in this situation an initial incomplete spatial correlation does not lead to decrease in τ_c. We should expect also a reverse influence of incomplete time coherence on the decrease in the correlation radius r_c in a diffraction-free regime with $\tau_{c0} \ll \tau_0$ and $r_{c0} \gg a_0$.

Within the framework of the linear equation of quasioptics without taking account of the last term in (5.1), the transformation of the time and space coherence of the laser field occurs independently. The effects of the reciprocal influence of the time and space fluctuations begin to be manifested for high-power diffracted and dispersed light fields in nonlinear media. Both the initial time and space coherence influence the variation of the space-time parameters of the laser field with account taken of diffraction and dispersion. Therefore, to study the reciprocal influence of time and space fluctuations, it is expedient to study at the entrance to the medium a random light field with either a limited space or limited time coherence.

References 29, 57, 108, and 109 analyzed in detail various aspects of the reciprocal influence of the time and space coherence of diffracted and dispersed fields for various regimes of self-action. Let us turn to some important results of these studies.

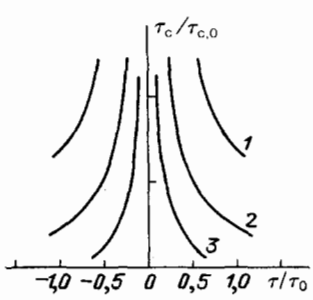

FIG. 36. Impairment of time coherence of the radiation at a wavelength $\lambda_0 \sim 1$ μm of an initially spatially incoherent structure for $L_{nl} = 1$ cm at distances $z = 10$ m (*1*), 50 m (*2*), and 100 m (*3*).

We note first of all that, owing to the reciprocal influence of the scales of the time and space coherence, the factored form of the STCF of the field of (5.5) in a nonlinear medium is not conserved. Of especial interest are the effects of space-time self-modulation of diffracted and dispersed fields in a regime of strong self-action $L_{nl} < L_{dif}$, L_{dis}.

The expression for the mean intensity obtained by the method of successive approximations shows a weakening of the nonlinear-dispersive transformation of the pulse due to the initial spatial fluctuations:

$$\tau_p(z) = \tau_0\left\{1 \pm \frac{1}{2}\frac{z^2}{k_0 L_{dis} L_{nl}^2}\left[\left(1 + N_r \frac{z^4}{k_0^2 L_{dif}^2 L_{nl}^4}\right)\right]^{-1}\right\}. \tag{5.6}$$

Here we have $N_r \gg 1$, while $N_\tau = 0$ and L_{nl} is given by Eq. (5.4) at the entrance to the medium. We should note that the transverse incoherent structure of the field always leads to weakening of the nonlinear-dispersion effects, independently of the sign of the nonlinearity.

Analysis of the STCF of the field showed an impairment of the initial time coherence of the radiation.[29] Here the coherence time varies in different ways over the duration of the pulse:

$$\tau_c = \frac{k_0 L_{nl}^2 \tau_0^2}{z|\tau|}. \tag{5.7}$$

In the central part of the pulse ($\tau = 0$) complete time coherence does not break down ($\tau_c = \infty$), while it gradually declines toward the periphery of the pulse with increasing τ (Fig. 36).

On the other hand, an initial incomplete time coherence ($N_\tau \gg 1$) also facilitates the breakdown of complete spatial correlation ($N_r = 0$) of the field. The transformation of the correlation radius occurs in different ways over the cross section of the beam:

$$r_c = k_0 L_{nl}^2 a_0^2 (z|\mathbf{r}|\cos\theta)^{-1}, \tag{5.8}$$

where θ is the angle between the vectors $\mathbf{R} = (\mathbf{r}_1 + \mathbf{r}_2)/2$ and $\vec{\rho} = \mathbf{r}_1 - \mathbf{r}_2$.

The reciprocal transformation of some types of fluctuations into others has been traced also in media having thermal nonlinearity. Initial spatial fluctuations in the stationary thermal self-action in the case $\alpha z \ll 1$ lead to a monotonic decline of τ_c along the track $z < L_{nl}$.[57] For $\alpha z \gg 1$, when the nonlinear effects are manifested in the thin layer $z < L_3 = \alpha^{-1}$, the coherence time declines in the initial stage of self-action, while further on, with weakening of the nonlinear effects, τ_c hardly varies (Fig. 37).

The physical pattern of the nonlinear reciprocal influence of space and time fluctuations of a laser field is very complex. However, we can distinguish certain common regularities of this reciprocal influence.

In the case of weak noise modulating a strong signal: $I_s \gg I_n$—the nonlinear perturbation of the refractive index of the medium in the propagation channel is determined by the intensity of the dominant signal and is practically regular. Therefore, in the propagation of a laser pulse in such a channel, an incoherent nonlinear effect is practically absent and

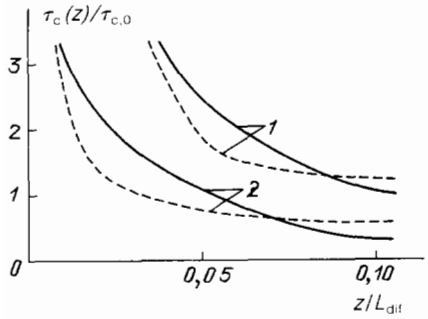

FIG. 37. Transformation of τ_c for $\tau = 0.5\tau_0$ for $\alpha z \ll 1$ (solid lines) and $\alpha z \gg 1$ (dotted) of damping for $L_{dif}/L_{nl} = 5$ (*1*) and 10 (*2*).

the reciprocal coupling of space and time fluctuations is vanishingly small.

In media having a lag-free Kerr nonlinearity and in a steady-state regime of thermal self-action, the reciprocal influences become appreciable only in those cases in which the fluctuational component of the refractive index $\tilde{n}_{nl}$, which is comparable with the regular component $\langle n_{nl} \rangle$, has a distribution inhomogeneous over the cross section of the beam and a non-steady-state one over the duration of the pulse. In a moving medium even steady-state fluctuations in time impair the spatial coherence in a plane parallel to the velocity of the moving medium. This decrease in r_c involves the inhomogeneous distribution of $\tilde{n}_{nl}$ in the cross section of the beam owing to the wind fluxes.

The effects of reciprocal influence are substantially weakened in those cases in which the manifested lag in the nonlinear response and the non-steady-state nature of the thermal self-action favor a smoothing of the fluctuations of the induced optical channel. However, such a lag and non-steady-state nature due to the character of the induced fluctuations that accumulates over the duration of the pulse can impair the time coherence of the noise signal, which has at the entrance into the nonlinear medium a modulation that is in a steady state over its duration.

CONCLUSION

In this review we have tried to reveal the most general regularities of the self-action of randomly modulated light beams and pulses, primarily in regular nonlinear media.

In the motley palette of stochastic nonlinear wave phenomena, the principal theme seemed to us to be to single out those phenomena most important in solving practical problems of transport of energy and information by laser radiation, of atmospheric optics, and of constructing high-power laser systems, etc.

We call attention to the primacy of theoretical over experimental studies, although in an actual experiment fluctuations of the field intensity of the light wave are always present, and taking correct account of them is a laborious problem. Lack of such an account impedes the comparison of the experimental and theoretical results, not only in problems of propagation of high-power light waves, but also practically in all other nonlinear-optical wave phenomena.

We note in passing that the solution of the problem of the nonlinear interaction of partly coherent beams and pulses is yet far from completion (harmonic generation, parametric interaction, Raman scattering, including the transformation of the frequency spectrum of ultrashort light pulses in optical fibers due to stimulated Raman scattering, etc.)

Returning to the problem of self-action of partly coherent light, we point out that taking account of the stratification of nonlinear media and of the natural fluctuations in their optical properties makes the pattern of self-action even more varied. We have tried not to touch upon this aspect of the problem at all, although in individual sections the results are presented of the cited studies in which the fluctuations of the field and of the medium are inseparable.

The influence of initial fluctuations of the medium, including those induced during self-action, is apparently a separate topic for another review article, which must be based on the studies of the past ten years carried out in the leading laboratories of our country and abroad.

[1] Yu. I. Zaĭtsev and D. P. Stepanov, Pis'ma Zh. Eksp. Teor. Fiz. **6**, 733 (1967). [JETP Lett. **6**, 209 (1967)].
[2] H. Gerbhart *et al.*, Phys. Lett. A **40**, 191 (1972).
[3] J. A. Armstrong and A. W. Smith, Prog. Opt. **6**, 213 (1970).
[4] M. A. Duguay, J. W. Hansen, and S. L. Shapiro, IEEE J. Quantum Electron. **6**, 725 (1970).
[5] R. V. Ambartsumyan, S. P. Bazhulin, H. G. Basov, and V. S. Letokhov, Zh. Eksp. Teor. Fiz. **58**, 441 (1968) [Sov. Phys. JETP **31**, 234 (1970)].
[6] S. A. Akhmanov and A. S. Chirkin, *Statistical Phenomena in Nonlinear Optics* (in Russian), Izd-vo Mosk. Un-ta, M., 1971.
[7] S. A. Akhmanov, Yu. E. D'yakov, and A. S. Chirkin, *Introduction to Statistical Radiophysics and Optics* (in Russian), Nauka, M., 1981, p. 640.
[8] M. Young and P. L. Drewes, Opt. Commun. **2**, 253 (1970).
[9] L. Csillag, M. Janossy, and K. Kantor, Phys. Lett. **20**, 626 (1966).
[10] A. G. Arutyunyan, S. A. Akhmanov, Yu. D. Golyaev, V. G. Tunkin, and A. S. Chirkin, Zh. Eksp. Teor. Fiz. **64**, 1511 (1973) [Sov. Phys. JETP **37**, 764 (1973)].
[11] S. M. Arakelyan, S. A. Akhmanov, V. G. Tunkin, and A. S. Chirkin, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 571 (1974) [Sov. Phys. JETP **19**, 299 (1974)].
[12] V. V. Bykov, *Numerical Simulation in Statistical Radiophysics* (in Russian), Sov. Radio, M., 1971, p. 326.
[13] V. P. Kandidov, Dissertation for the doctorate in physical-mathematical sciences, M. V. Lomonosov State University, Moscow, 1987.
[14] S. I. Terzieva, Dissertation for the candidateship in physical-mathematical sciences, M. V. Lomonosov State University, Moscow, 1989.
[15] A. Starikov, J. Opt. Soc. Am. B **72**, 1538 (1982).
[16] V. I. Bespalov and V. I. Talanov, Pis'ma Zh. Eksp. Teor. Fiz. **3**, 471 (1966) [JETP Lett. **3**, 307 (1966)].
[17] V. I. Talanov, *ibid.*, **2**, 218 (1965) [JETP Lett. **2**, 138 (1965)].
[18] G. A. Lyakhov, Opt. Spektrosk. **33**, 969 (1972) [Opt. Spectrosc. (USSR) **33**, 530 (1972)].
[19] N. Blombergen and P. Lallemandt, Phys. Rev. Lett. **16**, 81 (1966).
[20] Y. T. Shaman and Y. R. Shen, *ibid.*, **15**, 1008 (1965).
[21] G. A. Pasmanik, Zh. Eksp. Teor. Fiz. **66**, 490 (1974) [Sov. Phys. JETP **39**, 234 (1974)].
[22] S. A. Akhmanov and G. A. Lyakhov, Abstracts of the 7th All-Union Conference on Quantum and Nonlinear Optics, M., 1974, p. 173.
[23] S. M. Rytov, Yu. L. Kravtsov, and V. I. Tatarskiĭ, *Introduction to Statistical Radiophysics* (in Russian), Nauka, M., 1978, p. 463.
[24] S. N. Vlasov, V. A. Petrishchev, and V. I. Talanov, Izv. Vyssh. Uchebn. Zaved. Radiofiz. **14**, 1353 (1971). [Radiophys. Quantum Electron. **14**, (1971)].
[25] V. A. Petrishchev, *ibid.*, p. 1416. [*ibid.*, **14**, (1971)].
[26] A. M. Fattakhov and A. S. Chirkin, Kvantovaya Elektron. (Moscow) **10**, 1989 (1983) [Sov. J. Quantum Electron. **13**, 1326 (1983)].
[27] A. M. Fattakhov and A. S. Chirkin, *ibid.*, **11**, 2349 (1984) [Sov. J. Quantum Electron. **14**, 1556 (1984)].
[28] A. S. Chirkin and F. M. Yusubov, *ibid.*, **10**, 1833 (1983) [Sov. J. Quantum Electron. **13**, 1210 (1983)].
[29] V. A. Aleshkevich, G. D. Kozhoridze, and A. N. Matveev, *ibid.*, **15**, 829 (1988) [Sov. J. Quantum Electron. **18**, 529 (1988)].
[30] V. A. Aleshkevich, V. A. Vysloukh, G. D. Kozhoridze, A. N. Matveev, and S. I. Terzieva, *ibid.*, p. 325 [Sov. J. Quantum Electron. **18**, 207 (1988)].
[31] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and S. I. Ter-

zieva, *ibid.*, **12**, 192 (1985) [Sov. J. Quantum Electron. **15**, 121 (1985)].
[32] V. P. Kandidov and V. I. Ledenev, *ibid.* **8**, 873 (1981) [Sov. J. Quantum Electron. **11**, 521 (1981)].
[33] V. P. Kandidov, Izv. Akad. Nauk SSSR Ser. Fiz. **47**, 1583 (1983) [Bull. Acad. Sci. USSR Phys. Ser. **47**(8), 120 (1983)].
[34] J. Satsuma and N. Yajima, Prog. Theor. Phys. Suppl. **55**, 284 (1974).
[35] V. A. Aleshkevich, S. S. Lebedev, and A. N. Matveev, Zh. Eksp. Teor. Fiz. **83**, 1249 (1982) [Sov. Phys. JETP **56**, 715 (1982)].
[36] V, A. Aleshkevich, S. S. Lebedev, and A. N. Matveev, Kvantovaya Electron. (Moscow) **9**, 2066 (1982) [Sov. J. Quantum Electron. **12**, 1340 (1982)].
[37] M. A. Leontovich, Izv. Akad. Nauk SSSR Ser. Fiz. **8**, 16 (1944).
[38] S. A. Akhmanov, A. P. Sukhorukov, and A. S. Chirkin, Usp. Fiz. Nauk **93**, 19 (1967) [Sov. Phys. Usp. **10**, 609 (1968)].
[39] G. A. Askar'yan, Zh. Eksp. Teor. Fiz. **42**, 1567 (1962) [Sov. Phys. JETP **15**, 1088 (1962)].
[40] R. Y. Chiao, E. Garmire, and C. H. Townes, Phys. Rev. Lett. **13**, 479 (1964).
[41] N. F. Pilipetskiĭ and A. R. Rustamov, Pis'ma Zh. Eksp. Teor. Fiz. **2**, 88 (1965) [JETP Lett. **2**, 55 (1965)].
[42] P. Lallemandt and N. Blombergen, Phys. Rev. Lett. **15**, 1013 (1965).
[43] V. A. Aleshkevich, S. S. Lebedev, and A. N. Matveev, Kvantovaya Elektron. (Moscow) **8**, 1090 (1981) [Sov. J. Quantum Electron. **11**, 647 (1981)].
[44] D. C. Smith, Proc. IEEE **65**, 1679 (1977) [Russ. transl. TIIÉR **65**, 59 (1977)].
[45] L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon Press, Oxford, 2nd ed. 1987, Hydrodynamics [Russ. original Nauka, M., 1986 and 1988, p. 733].
[46] V. A. Aleshkevich, S. S. Lebedev, and A. N. Matveev, Izv. Vyssh. Uchebn. Zaved. Radiofizika **35**, 1368 (1982). [Radiophys. Quantum Electron. **35**, (1982)].
[47] V. A. Aleshkevich, A. V. Migulin, A. P. Sukhorukov, and É. N. Shumilov, Zh. Eksp. Teor. Fiz. **62**, 551 (1972) [Sov. Phys. JETP **35**, 292 (1972)].
[48] V. A. Aleshkevich, V. P. Kalinin, G. D. Kozhoridze, and A. N. Matveev, Optika Atmosfery **1**, 36 (1988).
[49] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and S. I. Terzieva, Abstracts of the 12th All-Union Conference on Quantum and Nonlinear Optics, M. 1985, Part 1, p. 129.
[50] V. A. Aleshkevich, G. D. Kozhoridze, and A. N. Matveev, Kvantovaya Elektron. (Moscow) **12**, 1695 (1985) [Sov. J. Quantum Electron. **15**, 1114 (1985)].
[51] S. A. Akhmanov, D. P. Krindach, A. V. Migulin, A. P. Sukhorukov, and R. V. Khohlov, Quantum Electron. QE-4, **10**, 568 (1968).
[52] D. C. Smith and H. Gerbhart, Phys. Lett. **16**, 725 (1970).
[53] V. A. Aleshkevich and A. P. Sukhorukov, Pis'ma Zh. Eksp. Teor. Fiz. **12**, 112 (1970) [JETP Lett. **12**, 77 (1970)].
[54] S. A. Akhmanov, M. A. Vorontsov, V. P. Kandidov *et al.*, Izv. Vyssh. Uchebn. Zaved. Radiofizika **23**, 1 (1980). [Radiophys. Quantum Electron. **23** (1980)].
[55] I. M. Belouslova, N. V. Vysotina *et al.*, Izv. Akad. Nauk SSSR, Ser. Fiz. **48**, 2299 (1984) [Bull. Acad. Sci. USSR Phys. Ser. **48**(12), 9 (1984)].
[56] P. A. Konyaev, V. P. Lukin, and B. V. Fortes, Optika Atmosfery **1**, 71 (1988).
[57] V. A. Aleshkevich, G. D. Kozhoridze, and A. N. Matveev, Izv. Vyssh. Uchebn. Zaved. Radiofizika **32**, 816 (1989). [Radiophys. Quantum Electron. **32**, 816 (1989)].
[58] P. A. Konyaev and V. P. Lukin, Appl. Opt. **25**, 415 (1985).
[59] V. P. Kandidov and S. A. Shlenov, Kvantovaya Elektron. (Moscow) **12**, 1490 (1985) [Sov. J. Quantum Electron. **15**, 982 (1985)].
[60] P. A. Konyaev and V. P. Lukin, *ibid.*, **15**, 341 (1988) [Sov. J. Quantum Electron. **18**, 217 (1988)].
[61] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and M. V. Shamonin, Optika Atmosfery **2**, 933 (1989).
[62] V. V. Vorob'ev, Izv. Vyssh. Uchebn. Zaved. Fiz. No. **11**, 61 (1977) [Sov. Phys. J. **20**, 1444 (1977)].
[63] K. D. Egorov and S. S. Chesnokov, Kvantovaya Electron. (Moscow) **14**, 1269 (1987) [Sov. J. Quantum Electron. **17**, 808 (1987)].
[64] J. L. Lumley and H. A. Panofsky, *Structure of Atmospheric Turbulence*, Interscience, N.Y., 1964 (Russ. transl., Mir, M., 1966).
[65] B. S. Agranovskiĭ, V. V. Vorob'ev *et al.*, Kvantovaya Elektron. (Moscow) 7, 545 (1980) [Sov. J. Quantum Electron. **10**, 308 (1980)].
[66] S. M. Babichenko and V. P. Kandidov, *ibid.*, **11**, 1372 (1984) [Sov. J. Quantum Electron. **14**, 926 (1984)].
[67] S. M. Babichenko, V. P. Kandidov, V. A. Myakinin, and S. A. Shlenov, *ibid.*, **13**, 2183 (1986) [Sov. J. Quantum Electron. **16**, 1443 (1986).
[68] V. A. Myakinin and N. S. Tikhonova, *ibid.*, **12**, 1074 (1985) [Sov. J. Quantum Electron. **15**, 706 (1985)].
[69] V. A. Aleshkevich, S. S. Lebedev, and A. N. Matveev, *ibid.*, **11**, 1459 (1984) [Sov. J. Quantum Electron. **14**, 983 (1984)].
[70] S. A. Akhmanov, V. A. Vysloukh, and A. S. Chirkin, *Optics of Femtosecond Laser Pulses* (in Russian), Nauka, M., 1988, p. 312.
[71] S. A. Akhmanov, V. A. Vysloukh, and A. S. Chirkin, Usp. Fiz. Nauk **149**, 449 (1986) [Sov. Phys. Usp. **29**, 642 (1986)].
[72] I. Thomazeau *et al.*, Opt. Lett. **10**, 223 (1985).
[73] V. A. Babenko, B. Ya. Zel'dovich, V. I. Malyshev, and A. A. Sychev, Kvantovaya Elektron. (Kiev) **2**, 19 (1973) **3**, 97 (1973)].
[74] S. A. Akhmanov, *Nonlinear Optics* (in Russian), Mir. M., 1979, p. 365.
[75] F. M. Fattakhov and A. S. Chirkin, Izv. Akad. Nauk SSSR Ser. Fiz. **49**, 553 (1985). [Bull. Acad. Sci. USSR Phys. Ser. **49** (3), 129 (1985)].
[76] V. P. Kandidov and S. A. Shlenov, Vestn. Mosk. Univ. Ser. Fiz. Astron. **25**, 51 (1984).
[77] V. A. Vysloukh, D. N. Dovchenko *et al.*, Izv. Akad. Nauk SSSR Ser. Fiz. **50**, 1220 (1986). [Bull. Acad. Sci. USSR Phys. Ser. **50** (6), 170 (1986)].
[78] E. M. Dianov, Kvantovaya Elektron. (Moscow) **7**, 453 (1980) [Sov. J. Quantum Electron. **10**, 259 (1980)].
[79] D. Marcuse, Appl. Opt. **19**, 1653 (1980).
[80] D. Marcuse, *ibid.*, **20**, 2969 (1981).
[81] D. Marcuse and C. Lin, IEEE J. Quantum Electron, **QE-17**, 869 (1981).
[82] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and S. I. Terzieva, Abstracts of the 5th International Symposium "SPS-87", Vilnius, 1987, p. 105.
[83] V. A. Vysloukh, Kvantovaya Elektron. (Moscow) **10**, 1668 (1983) (*sic*).
[84] R. J. Joenk and R. Landauer, Phys. Rev. Lett. A **24**, 228 (1967).
[85] T. Yajima, Jpn. J. Appl. Phys. **21**, 1044 (1982).
[86] D. Anderson and M. Lisak, Opt. Lett. **7**, 394 (1982).
[87] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and S. I. Terzieva, Abstracts (In Russian) of the 13th International Conference on Quantum and Nonlinear Optics, Minsk, 1988, Part 1, p. 158.
[88] A. Hasegawa and F. Tappert, Appl. Phys. Lett. **23**, 142 (1973).
[89] L. F. Mollenhauer, R. H. Stolen, and J. P. Gordon, Phys. Rev. Lett. **45**, 1095 (1980).
[90] D. Anderson, Opt. Commun. **48**, 107 (1983).
[91] I. N. Sisakyan and A. B. Shvartsburg, Kvantovaya Elektron. (Moscow) **11**, 1703 (1984) [Sov. J. Quantum Electron. **14**, 1146 (1984)].
[92] S. A. Akhmanov, V. A. Vysloukh, L. Kh. Muradyan *et al.*, Preprint (in Russian) of the Physics Faculty of the M. V. Lomonosov State University No. 18, Moscow, 1984.
[93] W. J. Tomlinson, R. H. Stolen, and C. V. Shank, J. Opt. Soc. Am. B **1**, 139 (1984).
[94] V. A. Vysloukh and L. Kh. Muradyan, Kvantovaya Elektron. (Moscow) **14**, 1437 (1987) [Sov. J. Quantum Electron. **17**, 915 (1987)].
[95] W. H. Knox *et al.*, Appl. Phys. Lett. **46**, 1120 (1985).
[96] V. A. Vysloukh and T. A. Matveeva, Kvantovaya Elektron. (Moscow) **14**, 792 (1987) [Sov. J. Quantum Electron. **17**, 498 (1987)].
[97] V. P. Kandidov and S. A. Shlenov, Izv. Akad. Nauk SSSR Ser. Fiz. **50**, 1191 (1986) [Bull. Acad. Sci. USSR Phys. Ser. **50** (6), 142 (1986)].
[98] D. D. Klovskiĭ, I. N. Sisakyan, A. B. Shvartsburg, and S. M. Shirokov, Radiotekh. Elektron. No. 4, 740 (1987). [Radio Eng. Elec. (USSR) (1987)].
[99] A. Khasegava and Yu. Kodama, TIIÉR **69**, 57 (1981).
[100] L. F. Mollenauer, R. H. Stolen, J. P. Gordon, and W. J. Tomlinson, Opt. Lett. **8**, 289 (1983).
[101] E. M. Dianov, A. Ya. Karasik *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **40**, 148 (1984) [JETP Lett. **40**, 903 (1984)].
[102] E. M. Dianov, A. Ya. Karasik, *et al.*, *ibid.*, **41**, 242 (1985) [JETP Lett. **41**, 294 (1985)].
[103] I. Kherman and B. Vil'gel'mi, *Lasers for Ultrashort Light Pulses* (in Russian, translation from the German), ed. P. G. Kryukov, Mir, M., 1986.
[104] S. A. Akhmanov, A. P. Sukhorukov, and A. S. Chirkin, Zh. Eksp. Teor. Fiz. **55**, 1430 (1968) [Sov. Phys. JETP **28**, 748 (1969)].
[105] A. M. Bel'skiĭ and A. G. Khapalyuk, Zh. Prikl. Spektrosk. **17**, 150 (1972) [J. Appl. Spectrosc. (USSR) **17**, 947 (1972)].
[106] O. E. Martinez, Opt. Commun. **59**, 229 (1986).
[107] I. P. Christov, Optica Acta **33**, 63 (1986).
[108] V. A. Aleshkevich, G. D. Kozhoridze, and A. N. Matveev, Abstracts (In Russian) of the 5th All-Union Conference "Optics of Lasers", L., 1987, p. 131.
[109] V. A. Aleshkevich, G. D. Kozhoridze, and A. N. Matveev, Abstracts (In Russian) of the 9th All-Union Conference on Propagation and Diffraction of Waves, Tbilisi, 1985, Part 2, p. 149.
[110] I. P. Christov, Opt. Commun. **53**, 364 (1985).
[111] V. N. Lugovoĭ, Pis'ma Zh. Eksp. Teor. Fiz. **19**, 176 (1974) [JETP Lett. **19**, 110 (1974)].
[112] V. A. Aleshkevich and G. D. Kozhoridze, Kvantovaya Elektron. (Moscow) **17**, 819 (1990) [Sov. J. Quantum Electron. **20**, 737 (1990)].
[113] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and M. V. Sha-

monin, Abstracts (in Russian) of the conference "Optoelectronics-89", Baku, Institute of Physics, Academy of Sciences of the Azerbaidzhan SSR, Baku, 1989, p. 77.
[114] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and M. V. Shamonin, *Lasers and the Atmosphere* (in Russian), ed. O. A. Volkovnitskiĭ, Obninsk, 1990, Part 2, p. 132.
[115] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, and M. V. Shamonin, Abstracts (in Russian) of the All-Union Seminar, "Mathematical Simulation and Application of Diffraction Phenomena", M. V. Lomonosov State University, Moscow, 1990, p. 97.
[116] V. A. Aleshkevich, G. D. Kozhoridze, A. N. Matveev, S. I. Terzieva, and M. V. Shamonin, Kvantovaya Elektron. (Moscow) **17**, 1619 (1990) [Sov. J. Quantum Electron. **20**, 1512 (1990)].
[117] R. H. Brown and R. Q. Twiss, Nature **177**, 27 (1956).
[118] J. Rayleigh, *Wave Theory of Light* (in Russian), Gostekhizdat, M., 1940.
[119] M. Born and E. Wolf, Principles of Optics, 4th edn., Pergamon Press, Oxford, 1970 [Russ. transl., Nauka, M., 1973].
[120] F. Zernike and J. E. Midwinter, *Applied Nonlinear Optics: Basics and Applications*, Wiley Interscience, N.Y., 1973 [Russ. transl., Mir, M., 1976].
[121] S. A. Akhmanov and R. V. Khokhlov, Nonlinear Optics, Gordon and Breach, N.Y., 1972 [Russ. original, VINITI, M., 1964].

Translated by M. V. King