Andrei Sakharov's KANDIDAT NAUKI Dissertation

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I did not ever meet Andrei Dmitrievich Sakharov, unfortunately. The nearest I came to this was through my participation in the conference "CP Violation in Particle Physics and Astrophysics," held in the Chateau de Blois, France, during 22–26 May 1989. The dates were chosen such that the participants could meet on the evening before the Conference to celebrate the 68th birthday of Andrei Sakharov, whom the Conference was to honour. We had a fine celebration, but the guest of honour was not there, since he had been elected a member of the Soviet Parliament, whose first session was to be held on May 25. Since he had an important role to play in its organizing committee in the days before its first session, it was beyond possibility for him to be with us at Blois. Later, in June, he made a short visit to England, during which he received honorary degrees, first at the University of Sussex and then at the University of Oxford. At Oxford, after the award ceremony, he attended the annual Encaenia Luncheon at my College, but I was not present, as I had to be abroad during that week.

However, I have known the name "A. D. Sakharov" for a rather long time. My Ph.D. dissertation, submitted to Cambridge University in 1950, listed his name among its references, for a paper[1] entitled "The Interaction of the Electron and the Positron in Pair Production," which he had submitted for publication on 26 December 1947. I had noticed his remark that the arguments he was making could be applied very directly to the case of the decay of a $J = 0$ nucleus for which photon emission was forbidden. On this point he referred the reader to his dissertation, completed in 1947 at the P. N. Lebedev Physics Institute. At the end of his paper, he stated that it formed part of his dissertation, and expressed gratitude to his supervisor Professor I. E. Tamm. I did not know the scope of his dissertation, nor even its title. I did not hear the name of A. D. Sakharov again until 1957, after the observation[2] of muon catalysis in the hydrogen-deuterium bubble chamber of the Radiation Laboratory of the University of California (Berkeley) at the end of 1956, when he published with Ya. B. Zeldovich[3] a discussion of the muonic atoms and molecules involved in the processes which took place in the course of this catalysis, and of the nuclear reaction rates occurring in a $d\mu p$ molecule. In this paper with Zeldovich, he referred in a general way to work he had done in 1948 at the Lebedev Institute, mentioning an internal report he had written then but not published.

In 1982, the Marcel Dekker press published[4] "A. D. Sakharov: Collected Scientific Works," containing almost all of Sakharov's unclassified scientific reports in English translation, including discussions giving the background and significance of each group of papers by various scientific experts and some notes and discussion about them by Sakharov himself. On page 165 of this book, he gave the title of his dissertation: "Theory of nuclear transitions of the type $0\to 0$," which was the first time I had seen it, and he wrote some brief comments on the two major ideas put forward in this dissertation, one of them being the electron–positron paper published in 1948 and referred to above. I was much intrigued by this news, because my Cambridge dissertation was entitled: "Zero-zero transitions in nuclei". Some time after *glasnost* began, I took steps to seek a copy of Sakharov's dissertation. The Lebedev Institute kindly sent me a copy of it earlier this year and it is my purpose here to describe its contents, its ideas, and calculations, in the light of subsequent knowledge.

To set the scene, let me begin by giving the physics background which led to this work. Why did two students so far apart happen to land on the same topic in this way, immediately after World War II?

The decade of the 1930s was a period in which there was a great growth of our knowledge and understanding of electromagnetic and nuclear interactions with atoms and their nuclei, especially of the α-particles, the β particles and the γ-rays emitted from the latter. The radiations emitted by various nuclear species were examined and compared quantitatively with theoretical calculations of their rates and other characteristics. The long review articles by Bethe and Bacher on the Stationary States of Nuclei in 1936, by Bethe on Theoretical Nuclear Dynamics and by Livingstone and Bethe on Experimental Nuclear Dynamics in 1937, were characteristic of this period, as was Heitler's 1936 book on the Quantum Theory of Radiation. The data on nuclear decay were quickly and systematically fitted into Gamow's picture of α-decay, into the beta-decay theory of Fermi and others later, and into a pattern of electromagnetic multipoles for the γ-rays emitted by nuclei in their transitions from one nuclear state to another. For the latter, secondary processes became recognized, such as "internal conversion" when the energy of the electromagnetic field induced by a nuclear transition is given to an atomic electron, ejecting it from the atom, and "internal pair conversion" when the energy of this electromagnetic field exceeds $2m_e$ and is all given to the creation of an electron–positron pair in the vicinity of the nucleus, both of these particles generally escaping from the atom. The study of these processes developed into a large industry devoted to measuring the energy spectra for photons, electrons, positrons and α-particles emitted from nuclei. An electron spectrum would generally consist of a continuous component from beta-decay, and perhaps from internal pair conversion, with narrow lines arising from internal conversion electrons ejected from the K and L shells of the atom, the two internal conversion processes being specif-

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ic to electromagnetic transitions for the nucleus. By fitting all these measurements together as appropriate, a nuclear spectroscopy gradually became built up, in a variety of ways. This process is still going on today, although mainly for the heavier, more complicated nuclei, and for the more highly excited states of the light nuclei.

However, by the end of the 1930s, there were two exceptional cases known:

(a) RaC′. This name specifies a state in ^{214}Po with excitation energy 1.415 MeV.[5] It received this name because it is a notable contributor to the radiations emitted from RaC, the name used for ^{214}Bi in those days. Its α-decay to ^{210}Pb gave a particularly energetic α-particle, so it became well-known. It also emitted a well-identified internal conversion electron, in strong competition ($\sim$200:1) with this α-emission. However, no γ-ray could be found with an energy corresponding to this 1.415 MeV internal conversion electron.

(b) ^{16}O* (6.049 MeV). This state was studied in the reaction

$$p + {}^{19}F \to {}^{20}Ne^* \to \begin{cases} \alpha + {}^{16}O\,(g.\,s.)\,, & (1a) \\ \alpha + {}^{16}O^*. & (1b) \end{cases}$$

There are many ^{20}Ne* states excited in this reaction, as the incident kinetic energy T_p of the proton is varied, and many final states of ^{16}O* reached in their subsequent α-decay. Following Fowler and Lauritsen[6] in 1939, Shreib, Fowler and Lauritsen[7] systematically measured the energies and yields of the α-particles, γ-rays, internal conversion electrons, and electron–positron pairs (denoted by "π") as function of T_p. Usually, these quantities could be correlated; the internal conversion and/or electron–positron yields corresponded to particular γ-ray energies which could be matched to some particular α-particle energy.

However, there was a strong peak (corresponding to the excitation of Ne (13.649 MeV)) in the low-energy α-particle yield [i.e. process 1(b)] for $T_p = 849$ keV, where the electron–positron yield also peaked, but with no associated γ-ray. The yield of energetic α-particles also peaked there but with intensity about an order of magnitude lower. These data implied the existence of an excited state of ^{16}O* at energy 6.049 MeV, whose decay gave predominantly electron–positron pairs but no γ-ray.

Both of these states led to a nuclear transition which produced strongly a marked effect normally associated with an electromagnetic transition, but which gave no γ-ray. The most natural interpretation was that they were both $0^+ \to 0^+$ transitions.[8,9] Such a transition can generate only a time-dependent spherically symmetric electromagnetic field, with frequency $\nu = \Delta/h$ corresponding to the transition energy Δ. Its vector potential $A(r)$ can only be radially directed and such a vector potential can always be reduced to zero by a gauge transformation. There is therefore no magnetic field. Denoting this transition as $i \to f$, its electric field E_{fi} is necessarily radial, as is also the case for its transition current. Their relationship is

$$(-i\Delta/hc)\mathbf{E}_{fi} = -4\pi\mathbf{J}_{fi}, \qquad (2)$$

so that the time-dependent electric field E_{fi} is zero outside the source of the current J_{fi}, i.e. outside the nucleus. The scalar potential V_{fi} satisfies the equation

$$\nabla^2 V_{fi} = -4\pi Q_{fi} = -4\pi e(\phi_f^*(\mathbf{r})\phi_i(\mathbf{r})), \qquad (3)$$

where e is the proton charge, and Q_{fi} denotes the charge density associated with the nuclear transition. For momentum transfer k, the component of V_{fi} is given by

$$V_{fi}(\mathbf{k}) = -4\pi e \int \phi_f^*(\mathbf{r}) e^{i\mathbf{k}\mathbf{r}} \phi_i(\mathbf{r})\, \frac{d^3r}{k^2}. \qquad (4)$$

Since ϕ_f and ϕ_i are orthogonal, and spherically symmetric, the nuclear matrix-element involved is

$$M_{fi}(\mathbf{k}) = 4\pi e \sum_{\alpha} \int \phi_f^*(\mathbf{r}_\alpha) e^{i\mathbf{k}\mathbf{r}} \phi_i(\mathbf{r}_\alpha) d^3 r_\alpha \qquad (5)$$

sufficiently approximated, for nuclear transitions at low momentum transfer, by

$$M_{fi}(\mathbf{k}) = -\frac{4\pi e}{3} k^2 \langle f | \sum_{\alpha} r_\alpha^2 | i \rangle, \qquad (6)$$

where the sum α is to be taken over all the protons in the nucleus.

Sakharov's dissertation follows a number of threads connected with these two transitions, that for RaC′ where the nuclear charge Z is large and the energy Δ is (relatively small) and that for ^{16}O* where Z is small and Δ is large. His purpose was to make sure that we could understand quantitatively all aspects of these data in terms of established theory, to exclude the possibility that we had overlooked some sign of its inadequacy, and to provide some guidance as to where we might find other instances of $0 \to 0$ transitions and how we might best recognize them.

The first thread concerns the nuclear transition. Here, Sakharov had the case of ^{16}O in mind. He begins in an unexpected way. Realizing that the light nuclei he was concerned with (α, ^{16}O, ^{20}Ne) all had $N = Z = A/2$, he raised the possibility of a new quantum number t associated with the operation T of interchanging the neutrons with the protons. This operation necessarily satisfies the relation $T^2 = 1$, and therefore has eigenvalues ± 1. If charge symmetry holds for nuclear forces, the operation T does not change a nuclear state, but may reproduce it with the factor $t = +1$ or -1. Thus, with charge symmetry, the eigenstates of a nucleus with $N = Z$ can be classified as even or odd under this operation. He introduced the name "isotopic parity" for this eigenvalue t. He did not associate it with the charge independence of nuclear forces; indeed, I found the term "isotopic spin" used only twice in the dissertation, and then only incidentally. He does not mention isotopic spin because he did not need to do so.

This is the first illustration of his unusual mind and of his great confidence in himself and in the power of the logic of physics. This concept of "isotopic parity" was not known to physicists[2)] in Western Europe and America until Kroll and Foldy[10] pointed out in 1952 that many "tests of charge independence" would be satisfied if the nuclear forces were only charge symmetric, and that if they were charge symmetric, the states of nuclei with $N = Z$ could be classified by a discrete quantum number having eigenvalues ± 1, for which they proposed the name "charge parity," still in use today. This concept was not known in the physics literature at the time Sakharov pointed it out and gave it the name "isotopic parity." This was a most remarkable achievement for a young research student!—four years ahead in such a central field as Nuclear Physics. It is difficult to understand why this work was not published at the time.[3]

Sakharov was not satisfied with the ideal case of charge symmetry, since he realized that with the Coulomb potential in the proton–proton interaction, the states with definite isotopic parity might become mixed, to such a degree that the concept might be useless. He therefore went on to estimate the admixtures which the Coulomb interaction might generate. He took as an example the case of two levels (labelled 0 and 1) with the same spin-parity but opposite values for t, and showed that the crucial parameter was V_{10}/Δ, where V_{10} is the matrix-element of the Coulomb energy between the two states and Δ is their separation energy. He concluded that the mixing was not large in the cases of immediate interest, but that the mixing could have quite observable effects, if the two levels were close in energy.

Going further, Sakharov noted that the long-range α-particles emitted in the decay $^{20}Ne^* \to \alpha + {}^{16}O$ (g.s.) were an order of magnitude less intense than the short-range α-particles from the decay $^{20}Ne^* \to \alpha + {}^{16}O^*(6.049)$, for the narrow $^{20}Ne^*$ resonance at 13.649 MeV excitation, associated with the strong emission of electron–positron pairs without γ-rays. He asked whether it might not be possible that this $^{20}Ne^*$ state and the $^{16}O^*$ state at 6.049 MeV could both have isotopic parity $t = -1$, in contrast with $t = +1$ for ^{16}O (g.s.). Perhaps this could account also for the narrowness (≈ 20 keV) of this $^{20}Ne^*$ level. He did not insist on this interpretation, but still considered (in the concluding chapter) that it is not excluded.[4]

He also considered the systematics of the first excited 0^{+*} level as function of nuclear species. He argued that, as the mass number A increases, the contributions to its excitation energy Δ from the spin-dependent and exchange forces will also increase. His underlying speculation appears to be that these 0^{+*} excited states may *all* have $t = -1$, opposite to $t = +1$ for the ground states. We know today that these speculations are not correct. Isotopic spin I is a good quantum number and the isotopic parity is then given by $t = (-1)^I$, but the levels under discussion for ^8Be, ^{12}C and ^{16}O are far below the lowest level with $I = 1$. In any event, the levels in ^8Be and ^{12}C have spin-parity 2^+, not 0^+. However, it seems typical of Sakharov that he should develop a plausible idea very seriously, unwilling to put it aside until it could be disproved or proved by experiment. In this case, he left the matter there, assuming $t = +1$ for all these excited states in his later work, without further comment, until the Resumé, where he remarks again that the possibility that $t = -1$ may hold for some of them is still an interesting idea, worthy of further experimental test.

The processes investigated in his dissertation have two aspects, nuclear and electro-dynamic. Nuclear models are needed for calculation of their rates. Without comment, although his steps are justified by my brief remarks above, he used the Coulomb potential for the calculation of the matrix-element which couples the nucleon charge with the electron density $\rho_{fi} = (\Psi_f^*(\mathbf{r})\Psi_i(\mathbf{r}))$ appropriate for the electronic transition. He emphasized that the process depends on the electron density only within the nucleus, and that the relevant nuclear matrix element is given by expression (6) above. In passing, he points out that $0 \to 0$ would be forbidden even through the electromagnetic interaction if there were a nuclear parity change. This is one of many significant "asides" in the course of his dissertation. Another is his remark that agreement of these calculations with experiment provides a test of Coulomb's law; a form $V = -e^2(1 - \exp(kr))/r$ proposed by Bopp and Podolsky, with $h/k = 14m_e$ according to Kikuchi's fit to the fine structure of hydrogen, was certainly excluded by the distribution observed for the e^+e^- opening angle θ for $^{16}O^*(6.049)$. He drew attention to the singularity of the Dirac electron wavefunctions for the static Coulomb field $-Ze^2/r$ of the nucleus, pointing out that there would be no singularity if the finite size of the nucleus had been taken into account. Quoting a mean value theorem, he replaces the leading term $\rho_{fi}(r) \sim (r)^{2s-2}$ of the product Dirac wavefunctions, where $s = [1 - (Z\alpha)^2]$, by the constant value R^{2s-2} within the nucleus, matched to the external value of $\rho_{fi}(R)$ at the nuclear surface; this is not a small effect for RaC′, where $s = 0.79$ holds rather than unity. Sakharov could have spread the charge Ze out uniformily over the nuclear volume, and calculated the Dirac wavefunctions numerically for the resulting potential $V(r)$, but he estimated that this would not change the result by more than 10%. With these matrix-elements, he then obtained rate expressions for K-electron conversion and e^+e^- pair creation, using the Golden Rule.

He next calculated the rates using Dirac plane waves for the outgoing electron and positron, which is a good approximation for ^{16}O, where Z is small, and useful for orientation. The use of relativistic wavefunctions was essential, of course, since v/c is not small at the relevant energies. The energy and angle distribution he obtained for the e^+e^- pairs is, apart from a multiplying constant,

$$p_+p_-(E_+E_- + p_+p_-\cos\theta - m_e^2)dE_+dE_-. \tag{7}$$

This result was stated by Oppenheimer in 1941 in an APS Meeting abstract[12], who noted that it was in good qualitative agreement with the data. In the limit $m_e = 0$, not considered by Sakharov, $E_\pm = p_\pm$ in (7) and the distribution vanishes at $\theta = 180°$. Today, we recognize this as a consequence of helicity conservation in the electromagnetic interaction $e\int \Psi_f \gamma_\mu \Psi_i A_\mu(r) d^3r$ for the electron–positron field. When the outgoing electron helicity is $\pm 1/2$, the outgoing positron helicity is then $\mp 1/2$, the sign change arising because the positron is antiparticle to the electron. For $\theta = 180°$, the electron and positron are moving in opposite directions along the same axis; the total angular momentum component m along the electron direction is then $m = (\pm 1/2 - (\mp 1/2)) = \pm 1$. A $0 \to 0$ transition allows only $m = 0$, so that there are no non-zero matrix-elements for $\theta = 180°$ and the decay rate to this configuration must vanish. This also holds true for the case of RaC′, where the Born approximation ($Z = 0$) is inappropriate, since the initial and final Coulomb interactions due to the nucleus also obey helicity-conservation for the electron and positron. The two facts, that the transition amplitude involves the electron and positron wavefunctions only within the nucleus, and that helicity conservation holds for $m_e = 0$, for all electromagnetic interactions, are responsible for the very simple form (7) obtained.

Sakharov pointed out that if there were a heavy pseudoscalar field (PS, say) coupling directly with the electron–positron field, and with the nucleons, e^+e^- pairs could be produced through the sequence

$$^{16}O^*(0^+) \rightarrow {}^{16}O\,(\text{g.s.}) + PS \quad \llcorner\!\!\rightarrow e^+e^-, \tag{8}$$

but that this sequence would lead to the distribution

$$p_+p_-(E_+E_- - p_+p_-\cos\theta + m_e^2)\mathrm{d}E_+\mathrm{d}E_-, \tag{9}$$

whose θ-distribution has the opposite trend, being greatest for $\theta = 180°$ and small (zero, if $m_e^2 = 0$) for $\theta = 0°$. Since $\overline{\cos\theta} = 0$, and m_e^2 is small, expression (9) gives an $E_\pm$ distribution rather similar to (7). The θ distribution for $^{16}O^*(6.049)$ clearly excludes this possibility. Sakharov noted:

(i) that the observed θ-distribution also excludes e^+e^- emission through any electromagnetic multipole, even if the γ-ray intensity were low for some unknown reason,

(ii) that the calculated ratio λ_e/λ_π of internal conversion electrons to e^+e^- pairs is about 3.5×10^{-5}, a reliable prediction since both processes involve the same nuclear matrix-element. It is of interest to mention here that the recently measured ratio[13] $4.0(5)\times 10^{-5}$ is in good agreement with his value.

This simple plane-wave calculation is quite inadequate for RaC′, where $Z = 82$. The nuclear Coulomb field affects both electron and positron strongly and it must be taken into account adequately for an e^+e^- pair of 1.414 MeV, the mean kinetic energy for e^+ and e^- being about 0.2 MeV. Whereas above we had low Z and $E \gg m_e$, we now have the opposite extreme, high Z and small (but relativistic) kinetic energies. The Coulomb field repels e^+ from the vicinity of the nucleus and attracts the electron strongly, the net result being an interesting but well-known step at $E_{+\max}$ in the positron energy spectrum.

For a system of high Z, Dirac wavefunctions must be used for both bound electron states and continuum electron and positron states. These are well-known but were calculated here *ab initio* and in a very clear way, and inserted into the formulae where previously Schrödinger wavefunctions and plane waves were used. The value Sakharov obtained for RaC′ was $\lambda_\pi/\lambda_e = 2.4\times 10^{-3}$.

The history of calculation and experiment in RaC′ is worth outlining briefly. The internal K-conversion line, without any corresponding γ-ray, was clearly demonstrated in 1937 by Alichanow and Spivak.[14] Earlier, in 1934, Alichanow and Kosodaew[15] had published a e^+ energy spectrum for RaC′, which they interpreted as having a step at an $E_{+\max}$ value corresponding to a 1.414 MeV excitation in RaC′, although its separation from the strong step observed to correspond to a known γ-ray line at 1.390 MeV was very difficult. This observation led Yukawa and Sakata[9] to calculate both λ_π and λ_e for the 1.414 MeV level in RaC′; they realized that both processes were governed by the same nuclear matrix-element, and that their calculated ratio $\lambda_\pi/\lambda_e = 4.2\times 10^{-3}$ should therefore be rather reliable. However, this result was two orders of magnitude below the value required by Alichanow and Kosodaew. In 1940, not aware of Yukawa and Sakata's calculation, Thomas[16] carried out the same calculations for RaC′, obtaining the value 6.0×10^{-3}, although there is good reason to believe that there is an error in his result. Alichanow and Latyshev[17] repeated these positron measurements in 1940 but found that their improved spectrum could be well accounted for without need for any e^+e^- emission from the 1.414 MeV level. In his 1947 review of the γ-radiations from RaC′ (which includes those from RaC′), Latyshev[18] does not refer to any emission of positrons from this 1.414 MeV level. The internal conversion electrons from this level have been investigated quite recently by Bengtson, Nielsen, and Rud[20] and the situation is now much clarified.[5] The 0^{+*} excited level of ^{214}Po is the fifth excited level, located at 1.4155 MeV; the first excited level is 2^+ at 0.6093 MeV. They also made a good measurement of the total decay rate for the 0^{+*} level, with the result $1.01(3)\times 10^{10}\ \mathrm{s}^{-1}$; the branching fractions are 26% for the $0^{+*}\rightarrow 0^+$ internal conversion, 0.12% for α-decay to ^{210}Pb(g.s), and the rest goes to the 0.6093 MeV level by an E2 γ-transition. The other levels play little role here. Bengtson *et al.* do mention the possibility of electrons from e^+e pair decay of the 1.415 level but note that theoretical estimates of their rate lie well below the levels observable in their experiment. From these figures, the rate for the $0^{+*}\rightarrow 0^+$ transition in RaC′ is now known to be $2.6(1)\times 10^9\ \mathrm{s}^{-1}$.

The estimation of absolute rates for these processes requires the use of a specific nuclear model. The 0^{+*} states to be represented correspond to breathing-mode-excitations for the nucleus. A liquid drop model is very natural but requires that the nuclear fluid be compressible, with a velocity of sound in the nuclear medium related with this compressibility but less than the velocity of light. However, Sakharov chose to replace the nuclear mass by an effective mass and calculated the kinetic energy, determining the range of oscillation by equating this with the energy of excitation. Using this method for internal conversion from RaC′, he obtained the decay rate $\lambda_e = 2.0\times 10^{13}\ \mathrm{s}^{-1}$, a rather large value. He therefore proposed a second method, which treated the nucleus ^{214}Po as an α-particle moving in a box of radius R. Its excited state RaC′ was represented as the result of exciting the α-particle from its s-wave ground state to its first excited s-level in this box. This estimation is straightforward, with result $\lambda_e = 3.2\times 10^{11}\ \mathrm{s}^{-1}$. At that time, all that was known was the branching ratio between α-emission and electron emission, so that an empirical estimate for λ_e required a reliable estimate for the α-decay rate for RaC′. Bethe[19] had already made this calculation in 1937, giving $\lambda_e = 1.3\times 10^{10}\ \mathrm{s}^{-1}$, some 25 times below Sakharov's lower estimate, a discrepancy which Sakharov was inclined to attribute to uncertainty in α-particle barrier penetration factors. However, we know today that Bethe's semi-empirical value was only about 4 times larger than the value measured by experiment.

Sakharov also estimated $\lambda_\pi = 4.6\times 10^{12}\ \mathrm{s}^{-1}$ for $^{16}O^* \rightarrow {}^{16}Oe^+e^-$, using his kinetic-energy method. Its experimental value has subsequently been determined in two different ways:

(a) by direct measurement of the lifetime by Birks, Sokolowski and Wolfson[21], using time-of-flight, giving a net decay rate of $1.03(7)\times 10^{10}\ \mathrm{s}^{-1}$. This is about half the early value of $2.0(2)\times 10^{10}\ \mathrm{s}^{-1}$ obtained by Devons, Hereward and Lindsey[22] at Cambridge in 1948, at the time of my dissertation.

(b) by determining the matrix-element $\langle {}^{16}O^*|r^2|{}^{16}O\rangle$ from data on the cross-section for the excitation of the state $^{16}O^*(6.049)$ by high-energy electrons incident on ^{16}O, as function of the momentum transfer, reported by Miska *et al.*[23]. The value obtained was $3.55(21)\ \mathrm{fm}^2$, which is in good agreement with the value of 3.24(30) fm^2 derived from the direct measurement of the lifetime, just mentioned under (a).

It is known that the net decay rate is dominated by λ_π; the internal conversion rate has been measured,[13] giving $\lambda_e/\lambda_\pi = 4.0(5)\times 10^{-5}$, and the rate for two-photon decay is given by $\lambda_{E1,E1}/\lambda_\pi = 2.5(1.1)\times 10^{-4}$. The $^{16}O^* \rightarrow {}^{16}O$ transition rate now known is about 400 times lower than Sakharov's kinetic-energy estimate.

My dissertation also estimated these nuclear transition rates, predicting decay rates much larger than the experimental values. The reason for these discrepancies is that the models used do not correspond with reality. These states are best described today by the shell model. Boeker[24] has obtained the value $\langle {}^{16}O^*|r^2|{}^{16}O\rangle = 2.6\ fm^2$ for ^{16}O, using two-particle two-hole (2p)(2h) and (4p)(4h) configurations, with particles in the $1d_{5/2}$ shell and holes in the $1p_{1/2}$ shell. Zuker *et al.*[25] have made an excellent over-all fit to a large part of the $^{16}O^*$ spectrum using all configurations $(1p_{1/2}, 2s_{1/2}, 1d_{5/2})^4$, and obtained 3.2 fm^2, in excellent agreement with the data. For ^{214}Po, it appears that the 0^{+*} state corresponds to nucleon pair excitations to higher shells.[20] It has been pointed out by Tape *et al.*[26] that the nuclei ^{206}Pb, ^{208}Po, ^{212}Po, and ^{214}Po all have a low-lying 0^{+*} state and that the values obtained for $\langle 0^{+*}|r^2|0^+\rangle$ are almost the same in all four cases.

The next step was to include the effect of the nuclear Coulomb field on the distribution of the e^+e^- opening angle θ. This requires the use of Dirac Coulomb wavefunctions in the transition amplitude, for both electron and positron, since their motions are necessarily relativistic. Sakharov naturally used the second-quantisation formalism for the electron–positron field. These calculations need great care and Sakharov carried them out in a masterly way. The net (E_+, E_-) energy distribution he calculated above, used time-dependent perturbation theory. The matrix-elements were calculated separately for each partial wave, but the total rate was obtained by summing their modulus squares over all initial and final spin states. However, the θ-angular distribution depends on interference terms between these amplitudes and their relative phases come into play. Its calculation could have been based on the matrix-elements already calculated (as my calculation did), but Sakharov chose to carry out the calculation by a different method, using the stationary form of perturbation theory, with a Hamiltonian which includes a term creating an electron and a positron in a spin state appropriate to their formation by a localised, radially-symmetric electric field. Since the physical states in question have relatively long lifetimes, the energy E was taken to be real. The electron and the positron each interact with the static Coulomb field of the nucleus, and move independently after they leave the source. The only requirement on them jointly is that their wavefunctions consist of outgoing waves only. This is achieved in a well-known way, by requiring that the e^+e^- wavefunction should have the form

$$\int_{m_e}^{\infty}\int_{m_e}^{\infty} dE_+ dE_- \sum_{j,m} W_{jm}(E_+, E_-)\delta^{(+)}(E - E_+ - E_-)\Psi_2, \quad (10)$$

where

$$\delta^{(+)}(\varepsilon) = P\frac{1}{\varepsilon} - i\pi\delta(\varepsilon), \quad (11)$$

and P denotes a principal value integration. Ψ_2 is the source function which specifies the initial e^+e^- spin state at the source. Although this method is described as "well-known," the fact is that it is almost always used for systems consisting of only one particle. Its use for multiparticle systems is usually limited to formal discussion of general theoretical questions, and is not often used for practical calculations. In fact, I am unable to recall any similar treatment in detail, in the literature. The difficult points concern the relative location of the wavefunctions of the electron and positron—necessary because they move outward with different velocities—and the proper calculation of the decay rate, as function of (E_+, E_-). These points are all carefully explained, so that this part of the dissertation has much educational value for the reader. The final expression obtained consists of a sum over four quadratic terms of the form $W^*_{\pm 1} W_{\pm 1} \exp(\pm i\varphi)$, where φ is a known Coulomb phase which is zero if W^* and W have the same suffix, with coefficients Σ_i for $i = 1,...4$, which are of a geometrical character, consisting of summations over products of spherical harmonics. These Σ_i can be readily calculated in a pedestrian way, but Sakharov decided to obtain them from his earlier plane-wave calculations, by a most ingenious method. Since he had before him a complete expression for the rate λ_π, for the case of the Coulomb field for a nuclear charge Ze, he realized that he could now take it to the limit $Z\rightarrow 0$ and obtain the Σ_i functions by comparing his Born approximation result with this limiting expression, a most elegant procedure which also gives a useful direct check on some elements of both calculations, since the two expressions were obtained by completely different methods. The result he obtained for the differential rate has the form

$$D\eta_+^2\eta_-^2(p_+p_-)^{2s-1}R^{4s-4}(E_+E_- + p_+p_-\cos\theta - m_e^2)dE_+dE_-\sin\theta d\theta, \quad (12)$$

where

$$\eta_p = |\Gamma(s + iE_p)/\Gamma(2s + 1)|\exp(\pi E_p/2),$$

with $s = [1 - (Z\alpha)^2]$ as before, R is the nuclear radius and the constant coefficient D need not be specified here. This expression agrees with Sakharov's earlier result for the (E_+, E_-) spectrum in the total rate, after integration over θ, and with the angular distribution obtained by Oppenheimer in the limit $Z\rightarrow 0$. The angular and energy distributions in this expression agree with those reported in my dissertation, apart from an additional factor independent of E_+, E_-, and θ, which reduces to 1 in these two limiting cases (it is most probably an error, on my part). Clearly the Coulomb correction to the angular distribution is a rather minor change. However, the details of Sakharov's calculation illustrate very well his directness of calculation and the breadth of his knowledge and ability. The methods used are unusual and quite economical; he managed to avoid the evaluation of a long complicated expression by building the final calculation on his earlier calculations. Although it is not important for this particular calculation, his method was ingenious and provided internal checks which help to avoid errors.

There is one further force in the final three-body state $(^{214}Po + e^+ + e^-)$ still to be considered, the e^+e^- Coulomb interaction. If the electrons and positrons have low relative velocity v, this potential C_{+-} can have a large effect, since it can continue to act after e^+ and e^- have left the nuclear field. Its effect is not perturbative, because it leads to the existence of Coulombic bound states (positronium). For ^{16}O, the limit $Z \to 0$ is a good approximation for λ_π. In this case, the e^+e^- system is created by a longitudinal (virtual) photon, which puts it into a $^3S_1$ configuration in its rest-frame. Sakharov's argument corresponds to the hypothesis that the differential rate calculated with the inclusion of C_{+-} is given by

$$R(e^+e^-, C_{+-})$$
$$= R(e^+e^-, C_{+-} = 0)|\Psi(r=0, C_{+-})/\Psi(r=0, C_{+-}=0)|^2, \quad (13)$$

where Ψ is here the c.m. wavefunction for the e^+e^- pair. Since this effect is important only for low relative velocities, Ψ may be calculated using the Schrödinger equation; the correction factor in (12) can then be given explicitly,

$$|\Psi(r=0, C_{+-})/\Psi(r=0, C_{+-}=0)|^2 = \frac{2\pi e^2/\hbar v}{1-\exp(-2\pi e^2/\hbar v)}. \quad (14)$$

This result is quite convincing, especially for $^{16}O^*$, where the nuclear Coulomb field produces little effect on the outgoing electron and positron. As $v \to 0$, the magnitude of this correction becomes rather large. Sakharov noted that for $p_+ = p_-$ and angle θ of about 1°, v/c is about 1/4. Integrating over angles $0 \leqslant \theta \leqslant 1°$, he found that the e^+e^- Coulomb field C_{+-} increases the rate of e^+e^- production by about 50%. Although this effect is large, it would be quite difficult to check empirically, in view of the small opening angles θ involved. We may note that the enhancement factor (13) cannot be expanded in powers of $\alpha = e^2/\hbar c$ for arbitrarily small values of v since the expression has poles at $e^2/\hbar v = \pm i$. The argument given for this enhancement factor in his dissertation is simple but quite convincing for the case for which it is used. In RaC′ decay, where the Coulomb field of the nucleus is strong and of long range, his argument for (13) would appear valid only for much smaller angles θ; the e^+e^- Coulomb potential can only have a dominating role after the pair has gone far out from the nuclear field. Sakharov's published paper on this topic[1] is a considerable elaboration of this chapter of his dissertation, for his endeavour in that paper was to establish some general principles for judging the validity of the use of this correction. It is, of course, difficult to cover all possible cases, and some individual cases may be easier to consider.

Sakharov also remarked that it would be possible for the e^+e^- system to emerge from $^{16}O^*(6.049)$ as a positronium atom, either in its ground state or in some excited state, with a total energy of about 5 MeV, but he did not make any quantitative estimate of these rates, relative to λ_π. It is of interest to note that internal pair conversion occurs at a well-established rate in π^0 decay, the ratio $\lambda_{\gamma+-}/\lambda_{\gamma\gamma}$ being 1.20(3)%. The corresponding positronium emission

$$\pi^0 \to \gamma + (e^+e^-)_{\mathrm{bd}} \quad (15)$$

has been detected in experiments at Serpukhov[26] and measured to have a branching fraction $1.84(29) \times 10^{-9}$.

To summarize, we can certainly say that Sakharov's dissertation is quite unusual. It demonstrates his keen awareness of the importance of symmetry principles and selection rules; indeed, he proposed a new selection rule, that of "isotopic parity" (= charge parity), as a consequence of charge symmetry for the nuclear forces, at least four years before this was noticed elsewhere. He had a modern approach in pointing out how data could be used to rule out conceivable extrapolations from established theory; e.g. he used the data to exclude in several ways the possibility that there may be deviations from Coulomb's law at short distances on the nuclear scale, and excluded the possibility that $^{16}O^*(6.049)$ might have $J^P = 0^-$. He had an unusual degree of familiarity with the methods of quantum mechanics for the case of three-body states ($e^+ + e^-$ + atomic nucleus), showing a complete mastery of the details of practical calculations for a two-particle emission process. He was able to conceive of approximations which led him to estimates which are completely convincing, such as his treatment of e^+e^- Coulomb interactions in pair conversion. Even as a research student, he could see "far-out" possibilities ahead, such as the direct emission of positronium, even in 1947.

In conclusion, I would like to give thanks to Academician E. L. Feinberg and Mr. A. Lazarian for translating some parts of Sakharov's dissertation verbally to me, and to Academician L. V. Keldysh and Dr. B. L. Altshuler for giving me the opportunity of contributing this paper to the Sakharov Memorial Volume.

APPENDIX A

Chapter Headings in Sakharov's Dissertation

APPENDIX B

Perhaps I may be forgiven for adding a few words about my own dissertation of 1950. It was begun in response to the experimental work of Samuel Devons at the Cavendish Laboratory on the lifetime of $^{16}O^*(6.049)$ and the opening-angle distribution for the e^+e^- pair emitted in the dominant decay process for that state.

The results stated briefly by Oppenheimer and Schwinger concerning $^{16}O^*(6.049)$ and by Yukawa and Sakata, and by Thomas concerning RaC′, were re-derived and these calculations were extended to the Z-dependence of the opening-angle distribution.[29]

This was the time when the new Feynman methods were coming in and there was a general use of renormalisation theory. It was therefore natural for me to calculate the e^2 corrections to the above calculations, including real photon emission, but they turned out to be small and multiplicative.[29] I recognized that the largest e^2 correction arises from the e^+e^- Coulomb interaction, and that it is an approximation to $\Psi_C(0)/\Psi_{free}(0)$. However, my e^2 result was large only if e^2/hv is large, but it was not valid then, because this factor had a finite radius of convergence for a series expansion in e^2/hv.

Nuclear models were developed for breathing-mode oscillations, and their dynamics were related to the bulk energy of a compressible nuclear fluid, rather than with internal kinetic energy; α-particle models were discussed but little was known about α-α forces then.

A long Appendix used the Feynman techniques described in the dissertation to evaluate higher-order corrections for the Born approximation amplitudes for the elastic scattering of an electron by a static potential.[30]

[1)] R. H. Dalitz is a professor at Oxford University, England. He is a well-known specialist on nuclear physics, elementary particle physics, and processes with strange particles. He is the author of the widely used Dalitz diagrams. (Note by the editor of Usp. Fiz. Nauk).

[2)] Early in 1952, Trainor[11] recognised that the $N = Z$ nuclear states could be characterized by a multiplicative quantum number having values ± 1, but this observation was based on shell-model wavefunctions which were calculated for charge-independent nuclear forces and had definite values of isospin I. The eigenvalues he found actually obeyed the rule $(-1)^I$ for isospin I, but Trainor did not make this connection. He did not realise that this quantum number would remain valid if charge-independence failed and only charge symmetry survived.

[3)] It should be noted that in our country Sakharov's dissertation and the concept introduced in it of parity with respect to isotopic spin were widely known. In particular, a separate section was devoted to this subject in the monograph by L. V. Groshev and I. S. Shapiro "*Spectroscopy of atomic nuclei*" (in Russian) published in 1952. (Note by V. M. Kolybasov, translator of present article for Usp. Fiz. Nauk).

[4)] Today it appears probable that Sakharov was correct on the last point. The $^{20}Ne^*$ level at 13.642(3) MeV, with width 17(1) keV, and now believed to be 0^+ is now assigned $I = 1$, and hence $t = -1$. It corresponds to a known level in ^{20}F at excitation energy 3.526 MeV. Within ± 0.4 MeV of this level, four 0^+ states with $I = 0$ are known, so that it is plausible that it should have appreciable $I = 0$ admixtures; indeed it must have, since it decays to $(\alpha + {}^{16}O\ (g.s.))$, if these assignments are correct. This level decays dominantly (99.6%) to $(p + {}^{19}F)$, so that its partial widths for α-decay are less than 0.1 keV.

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