Cosmological models of the Universe with reversal of time's arrow

A. D. Sakharov

P. N. Lebedev Physics Institute, Acad. Sci. USSR

(Submitted on 31 January 1980; resubmitted after revision on 24 April 1980)
Zh. Eksp. Teor. Fiz. **79**, 689–693 (1980) [Sov. Phys. JETP **52**, 349–351 (1980). Also S12, pp. 131–136]

Usp. Fiz. Nauk **161**, 81–85 (May 1991)

Cosmological models of the universe with reversal of time's arrow are considered. Formulations are given of the hypothesis of cosmological *CPT* symmetry suggested earlier by the writer, and of the hypothesis of an open model with many sheets, with negative spatial curvature, and with possible violation of *CPT* symmetry by an invariant combined charge. The statistical paradox of reversibility is discussed for these models. The small dimensionless parameter δ^2/a^2, which characterizes the mean spatial curvature of the universe, is explained as the result of the evolution of the universe through many successive cycles of expansion and contraction.

The equations of motion of classical mechanics and of nonrelativistic quantum mechanics admit time reversal; so also do the equations of quantum field theory (along with the *CP* transformation). The statistical equations, however, are irreversible. This contradiction has been known since the end of the nineteenth century. We shall speak of it as the "global paradox of reversibility" of statistical physics. The traditional explanation ascribes irreversibility to the initial conditions. However, the nonequivalent status of the two directions of time is still retained in the picture of the world.

Present-day cosmology opens up the possibility of eliminating this paradox. The idea of an expanding universe is now generally accepted in cosmology; according to it, a certain instant in time is characterized by the vanishing of the spatial metric tensor (this time of the "Friedmann singularity" will here be denoted for brevity by the symbol Φ). In 1966–1967 the writer suggested that one may consider in cosmology not only later times than Φ, but also earlier times, but then the statistical properties of the state of the universe at the instant Φ are such that the entropy increases not only going forward in time from this instant, but also going backward in time:

$$\begin{aligned} dS/dt > 0, \quad S(t) > S(0) \quad \text{for} \quad t > 0, \\ dS/dt < 0, \quad S(t) > S(0) \quad \text{for} \quad t < 0. \end{aligned} \tag{1}$$

Thus it is assumed that for $t > 0$ the ordinary statistical equations hold, but for $t < 0$ the time-reversed equations hold. This reversal is valid for all nonequilibrium processes, including those concerned with information, i.e., the processes of life. The author has named this sort of situation the "reversal of time's arrow." Reversal of time's arrow eliminates the reversibility paradox; in the picture of the world as a whole equivalence is restored between the two directions of time, as inherent in the equations of motion.

Despite the absence of dynamical interaction between the regions of the world with $t > 0$ and with $t < 0$, the assumption of the reversal of time's arrow has physical content; some necessary conclusions about the character of the initial conditions at the point Φ follow from it.

As a model example of reversal of time's arrow let us consider the classical kinetic theory of gases. At the time $t = 0$ we postulate a spherically symmetrical velocity distribution of the molecules at each point in space and nonuniform density and temperature distributions in space. We assume (and this is particularly important) that at $t = 0$ there is no correlation between the relative positions and relative velocities of the molecules; in this case this is the "statistical condition" by means of which one proves that the value of the entropy at the point $t = 0$ is a minimum.

In an earlier paper[1] the writer put forward the hypothesis that the universe possesses cosmological *CPT* symmetry. According to this hypothesis, all events in the universe are symmetric relative to the hypersurface that corresponds to the instant Φ of cosmological collapse. Setting $t = 0$ for this instant, we require that there by symmetry under the transformation $t \to -t$. The only exact symmetry that includes time inversion is *CPT* symmetry. It follows from *CPT* symmetry that the point Φ is singular and is neutral with respect to all invariant charges. We shall define *CPT*-conjugate fields on the auxiliary half-space

$$x_0 = |t| \geq 0, \quad -\infty < x_\alpha < +\infty,$$

and denote these fields with the indices a and b. We postulate: for spinors, $\psi^a = \gamma_5 \psi^b$; for the components of a unit tetrad, $e^a_{i(j)} = -e^b_{i(j)}$ (*PT* reflection). (The index referred to the tetrad is put in parentheses.)

We map the field a onto the region $t \geqslant 0$ and the field b onto the region $t \leqslant 0$ [with the corresponding change of signs of $e^b_{0(j)}$]. From the condition of continuity at the hypersurface we have $e_{\alpha(j)}(0) = 0$ (the point Φ is singular) and $\psi(0) = \gamma_5 \psi(0)$, so that the current vanishes,

$$j(0) = \bar{\psi}\gamma_j\psi = \bar{\psi}\gamma_5\gamma_j\gamma_5\psi = -\bar{\psi}\gamma_j\psi = 0$$

(neutrality condition at the point Φ).

The neutrality of the universe requires that the observed baryon asymmetry arose in the course of nonequilibrium processes of expansion of the universe. For this it is necessary to assume breaking of baryon charge conservation, but it is possible to have conservation of a combined charge of the type $3B \pm L$ (see Refs. 1 and 2), where B is the baryon charge and L is the lepton charge. We note, however, that in the currently most popular schemes for unifying the

 0038-5670/91/050401-03$01.00

strong, weak, and electromagnetic interactions [for example, the $SU(5)$ scheme] there is no such conservation (the conservation of $B - L$ is also approximate in most schemes).

CPT symmetry is not the only possible realization of the reversal of time's arrow. It suffices to assume that at the instant Φ the statistical conditions that there be no correlations is satisfied. The most natural assumption is the one according to which violation of *CPT* symmetry in reversal of time's arrow is due to the presence of a finite invariant combined charge (of course, provided such a charge exists and does not possess a gauge field). The numerical size of the combined charge here has no direct connection with the residual baryon asymmetry, which arises dynamically in the course of the expansion of the universe.

The reversal of time's arrow (with or without *CPT* symmetry) is possible either in the ordinary open model of the universe, or in models with infinitely repeating cycles of expansion and contraction (in pulsating models, or, in the present writer's terminology, in many-sheeted models; see Ref. 2). Owing to their inherent singularities, these latter models seem to us more interesting, and we shall consider them in more detail.

First of all we emphasize that in these models cycles close to the instant Φ must be decidedly different from the "later" cycles, for which all the main statistical characteristics asymptotically approach their limiting values for $|n| \to \infty$ (n is the number of the cycle, $-\infty < n < +\infty$). These limiting "self-reproducing" values correspond to the many-sheeted model without reversal of time's arrow (cf. Ref. 2). In the many-sheeted model without reversal of time's arrow, according to Ref. 2, the spatial curvature and all of the invariant charges must be equal to zero (in the sense of average values). In the model with reversal of time's arrow these quantities must become zero only asymptotically. In this sense the many-sheeted sort of model is more general.

Accordingly, let us examine a model with a finite spatial curvature $-a^{-2}$ and, possibly, a finite combined charge. We shall suppose that the curvature is negative (a is the hyperbolic radius), which evidently corresponds to the observations. We shall also assume that the Einstein cosmological constant is different from zero, with its sign corresponding to a vacuum energy density $\varepsilon < 0$. We make no assumption about the absolute value $|\varepsilon|$, but it is very probable that $|\varepsilon|$ is small in comparison with the mean density of matter at the present time. The negative sign corresponds to breaking of the symmetry of the vacuum state with $\varepsilon = 0$.

The dynamics of the universe is determined by the Einstein equation

$$8\pi G T_0^0 = R_0^0 - \frac{1}{2} R,$$

which we write in the form (with c, the speed of light, set equal to 1)

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}(\rho + \varepsilon) + \frac{1}{a^2}, \tag{2}$$

where H is the Hubble parameter, ρ is the density of "ordinary" matter, and ρ and $1/a^2$ go to zero as $a \to \infty$. Since $\varepsilon = \text{const} < 0$, at some value of a the quantity H goes to zero and expansion is replaced by contraction. Accordingly, the universe experiences an infinite number of cycles of expansion and contraction.

For the initial conditions in the neighborhood of the point Φ, the following four types of assumption are the most natural (σ is the density of entropy, and n_c is the density of the combined charge; $n_c a^3 = 0$ means that there is no combined charge or that it is equal to zero):

1) $\sigma a^3 \sim 1, \quad n_c a^3 \sim 1;$

2) $\sigma a^3 \sim 1, \quad n_c a^3 = 0;$

3) $\sigma a^3 \to 0, \quad n_c a^3 \sim 1;$

4) $\sigma a^3 \to 0, \quad n_c a^3 = 0.$

Types 2 and 4 correspond to cosmological *CPT* symmetry. In the case of types 1 and 3, the *CPT* symmetry is broken by the presence of combined charge, which can lead to important differences in the details of the world picture in the positive and negative cycles. Types 1 and 2 correspond to hot models of the universe, types 3 and 4, to cold models. A cold model is the natural realization of the reversal of time's arrow, but on the whole there are neither theoretical nor experimental data for the choice of a definite type.

The entropy σa^3 in a comoving volume a^3 increases in each cycle. Let us suppose that as n increases by 1 the entropy increases by a factor ν; to calculate this number, which is possible in principle, one would have to take into account the main nonequilibrium processes. At present (in "our" cycle n_1) the entropy $\sigma a_1^3 \sim n_\gamma / H^3$, where n_γ is the density of photons of the residual radiation. It is assumed that the density ρ is less than the critical density. For types 1 and 2 we have an estimate of the ordinal number n_1 of our cycle (as an example we have taken $\nu = 1.1$):

$$|n_1| \sim \frac{\ln |n_\gamma H^{-3}|}{\ln \nu} \sim \frac{\ln 10^{87}}{\ln 1{,}1} \sim 2 \cdot 10^3.$$

In the cold types of model additional cycles are necessary to produce the initial entropy; in type 4 the initial particles arise as the result of a large number of almost empty cycles, owing to the small curvature, proportional to $|\varepsilon|$.

Writing δ^{-3} for the density of the residual-radiation photons, $\delta \sim 0.1$ cm, we have a very small dimensionless number $\delta^2/a^2 \sim 10^{-58}$, which characterizes the curvature of the universe (provided, of course, that the curvature is not identically equal to zero, which still cannot be regarded as excluded). An important advantage of the many-sheeted model with reversal of time's arrow is the possibility of explaining in a natural way the appearance of this dimensionless number in the course of successive cycles of expansion and contraction.

The asymptotic situation with completely similar successive cycles is described by Eq. (2) with the term $1/a^2$ neglected. The solution of Eq. (2) is of the form

$$a = a_n \left(\sin \frac{3}{2} a_0^{-1} t \right)^{2/3}, \quad a_0 = \left\{ \frac{8\pi G}{3} |\varepsilon| \right\}^{-1/2}.$$

The maximal hyperbolic radius a_n of the nth cycle is determined from the condition $\rho(a_n) = |\varepsilon|$, and is proportional to $\nu^{|n|/3} \to \infty$ as $|n| \to \infty$. The duration of each cycle is $T_A = 2\pi a_0 / 3$. The densities of baryons, of leptons, and of entropy at corresponding times in successive cycles does not depend on $|n|$. The cycles closer to Φ are described by Eq. (2) with ρ neglected (except in relatively small intervals of time at the beginning and end of each cycle). Neglecting ρ, we have $a = a_0 \sin(t/a_0)$, and the duration of each cycle is $T_I = \pi a_0$. The transition from the initial to the asymptotic situation is defined by the condition $\rho(a_0) = |\varepsilon|$, and will occur at cycle number $n_2 > n_1$ (on the assumption that at present $\rho < \rho_c$). The baryon asymmetry n_B/n_γ, however, already has its asymptotic value, since it is determined by the initial stage of the expansion of the universe.

The stability of this pattern of successive collapses has not been investigated. In this paper we have discussed the reversibility paradox, the hypothesis of cosmological *CPT* symmetry, and the various types of many-sheeted models.

I express my gratitude to all who have taken part in discussing preliminary versions of this paper, and to my wife, E. G. Bonner, for her help.

[1] A. D. Sakharov, ZhETF Pis'ma 5:32 (1967); JETP Lett. 5:24 (1967), trans. S7.
[2] A. D. Sakharov, ZhETF 76:1172 (1979); Sov. Phys. JETP 49:594 (1979), trans. S11.

Translated by W. H. Furry