Theory of the magnetic thermonuclear reactor, part II[1)]

A. D. Sakharov

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The properties of a high-temperature plasma in a magnetic field were discussed in a paper by Tamm,[1] in which he demonstrated the possibility of the realization of a magnetic thermonuclear reactor (MTR). In this paper we shall consider other questions concerning the theory of MTRs.

I. THERMONUCLEAR REACTIONS. BREMSSTRAHLUNG

The following reactions may take place in an MTR:

$$\left.\begin{array}{l} D^2 + D^2 \to H^3 + p + \underline{3\ \text{MeV}} + \underline{1\ \text{MeV}}, \\ D^2 + D^2 \to He^3 + n + \underline{0{,}82}\ \text{MeV} + 2{,}46\ \text{MeV} \end{array}\right\}\ \text{primary reactions}$$

$$\left.\begin{array}{l} He^3 + D^2 \to He^4 + p + \underline{14{,}5\ \text{MeV}} + \underline{3{,}7\ \text{MeV}}, \\ H^3 + D^2 \to He^4 + n + 14\ \text{MeV} + \underline{3{,}5\ \text{MeV}} \end{array}\right\}\ \text{secondary reactions}$$

Energies supplied by the charged particles and which maintain the thermonuclear reaction in the MTR are underlined. The time τ required for a collision which results in a reaction is given by

$$\begin{aligned} \tau_D^{-1} &= N_D([\overline{\sigma v}]_1 + [\overline{\sigma v}]_2), \\ \tau_{H^3}^{-1} &= N_D[\overline{\sigma v}]_{H^3+D}, \\ \tau_{He^3}^{-1} &= N_D[\overline{\sigma v}]_{He^3+D}. \end{aligned} \tag{1.1}$$

where N_D is the number of deuterons per cubic centimeter and $[\sigma v]$ is the Maxwellian distribution of the cross section for relative velocity.

Table 1 gives, to indicate the order of magnitude of quantities, the times τ corresponding to $N_D = 0.77\times 10^{14}$ cm^{-3} (using 1950 data).

Figure 1 of the Appendix shows, as a temperature function, the amount of energy Q_1 per cubic centimeter per second supplied by charged particles at the same density $N_D = 0.77\times 10^{14}$ (only primary reactions are included).[2]

The energy Q_2, radiated by Bremsstrahlung (x-rays), and calculated by the formula

$$Q_2 = aN_D^2\sqrt{T(\text{MeV})}(1 + \alpha T) \tag{1.2}$$

is given for comparison.

Here $\alpha = 5.05$ MeV^{-1} (this term results primarily from electron-electron collisions), and $a = 1.7\times 10^{-22}$. Figure 1 of the Appendix shows that a self-sustaining reaction $Q_1 > Q_2$ is possible in a system of large dimensions.

II. CALCULATION OF THE LARGE MODEL. CRITICAL RADIUS. PHENOMENA NEAR THE WALLS[2)]

Let us consider a straight infinite cylinder of radius R. We disregard effects which depend on neutral particles. With this assumption there is no radial ion current j present (in the stationary case).

According to Ref. 1,

$$j \sim \frac{n^2}{T^{1/2}}\left(\frac{\nabla n}{n} + \frac{1}{4}\frac{\nabla T}{T}\right); \tag{2.1}$$

from which we have for $j = 0$,

$$nT^{1/4} = \text{const} = n_0 T_0^{1/4}, \tag{2.1a}$$

where n_0 and T_0 are the number of deuterons in 1 cm^3 and the temperature on the axis of cylinder, respectively. From (2.1a) it can be seen that the pressure is maximal in the center and falls off to a small value at the edge. According to Ref. 1, the magnetic field in a system with cylindrical symmetry varies in such a way that the sum of the gas pressure and magnetic field pressure

$$\frac{H^2}{8\pi} + 2nT = \text{const.} \tag{2.1b}$$

Further, we write the equation for the conservation of energy. Let π, in erg/cm^2 sec, be the heat flow, due to the thermal conductivity of the plasma. We have

$$\operatorname{div} \pi = \left(\frac{n}{N_D}\right)^2 Q(T), \tag{2.1c}$$

where $Q = Q_1 - Q_2 = 0.77\times 10^{14}$ erg/cm^3 sec. Then, for T in ergs,

$$\pi = \frac{3{,}6\cdot 10^{-8}n^2}{H^2\sqrt{T}}\left(\nabla T + \frac{7}{2\left(\sqrt{\frac{M}{2m}} + \frac{1}{\sqrt{2}} + \frac{41}{8}\right)}\frac{T}{n}\nabla n\right). \tag{2.2}$$

From (2.1a) we have $(T/n)\nabla n = -\frac{1}{4}\nabla T$, i.e.,

$$\pi = -\frac{3{,}5\cdot 10^{-8}n^2}{H^2\sqrt{T}}\nabla T. \tag{2.1d}$$

By examining Eqs. (2.1a) and (2.1d) together, it is possible to find the distribution of all quantities as a function of the tube radius as well as the critical value of the radius at which the release of nuclear energy is equal to the heat lost to the walls. Dimensionless variables were introduced for discussion purposes. The magnetic field appears in the combination HR; from this it follows that the critical radius R_c is inversely proportional to H_0.[3)] Neglecting secondary reac-

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TABLE I.

T, keV	Lifetime of D with respect to both reactions (primary)	Lifetime of H^3	Lifetime of He^3	T, keV	Lifetime of D with respect to both reactions (primary)	Lifetime of H^3	Lifetime of He^3
10	7620	134	27400	100	202	13,8	89,9
20	1770	34	8850	200	112	18,0	51,2
50	421	15,3	322	300	85,5	23,0	46,4

tions and boundary effects, it was found that for $T_0 = 107$ keV,

$$R_\kappa H_0 \approx 10^7 \text{ G cm}. \tag{2.3}$$

For $H_0 = 25{,}000$ G, we have $R_c = 400$ cm.

We note that the numerical value of the coefficient of thermal diffusion, equal to $\frac{1}{4}$ and appearing in front of $\nabla T/T$ in formula (2.1), has fundamental importance for the realization of MTR. This can be seen easily from the following considerations. The order of magnitude of the heat emission of any body is proportional to (in the stationary case, when the emission of energy is not particularly inhomogeneous)

$$I = L\int_{T_1}^{T_0} \varkappa(T)\mathrm{d}T,$$

where L is the linear dimension, $\varkappa$ the thermal conductivity, T_0 the temperature at the center, and T_1 the temperature at the boundary. In our case $\varkappa \sim n^2/T^{1/2}$. Let $n \sim T^{-\alpha}$, where α, in general, is not necessarily equal to $\frac{1}{4}$ [generalization of formula (2.1a)].

We have $\varkappa \sim 1/T^{1/2+2\alpha}$. For $\alpha = \frac{1}{4}$, $I \sim \ln(T_0/T_1) \approx 15$; when $\alpha > \frac{1}{4}$, I can be rather large. For example, for $\alpha = \frac{1}{2}$, $I \sim (T_0/T_1)^{1/2} \approx 10^3$, while for $\alpha < \frac{1}{4}$, the integral converges for $T_1 \to 0$. It has been proved that the presence of additives in deuterium plasma having $Z > 1$ (helium, air, etc.) decreases α, i.e., it acts in a favorable direction (calculations by Fradkin).

In Figs. 2 and 3 (in the Appendix) the distributions of temperature T and particle density n for the case of $H_0 = 25{,}000$ G are shown; T_1 is taken as 1000° (see below). The magnetic field in the center falls off to a small value and therefore n_0 can be computed from the formula

$$n_0 = \frac{H_0^2}{16\pi T}. \tag{2.4}$$

For $H_0 = 25{,}000$ G and $T = 100$ keV $= 1.6 \times 10^{-7}$ ergs, we have $n_0 = 0.77 \times 10^{14}$ cm^{-3}. Figure 4 (in the Appendix) shows the energy release per unit of volume with n_0 given by formula (2.4) and $H_0 = 25{,}000$ G. Obviously, it is most reasonable to operate at the lowest temperature at which the thermonuclear reaction is self-sustaining.

At the present time calculations do not take account of secondary reactions and of the increase of Bremsstrahlung if He^3 and He^4 appear in the system. True, the first of the factors mentioned (lowering R_c) is more important. Evidently one can additionally decrease R_c by forced burning of He^3. One can obtain only very preliminary qualitative estimates of the method of solution in the vicinity of the wall, where an important role is played by the neutral atoms and molecules which come from the wall. (A similar problem was solved by Tamm for the case of a small model, with the occurrence of a temperature jump.)

We disregard both recombination in the plasma volume (we will consider only a recombination at the walls) and collision of neutral atoms with one another.

With the above assumptions one can find a qualitative diagram of the solution (Fig. 1) and also check the validity of the starting assumptions. The ion temperature jumps to T' in the vicinity of the wall. Slow neutral particles leaving the wall undergo charge transfer at a very small distance from the wall (of the order of 1 mm). Fast neutral particles emerge with ranges of the order of 1 cm. The difference in the ranges of fast and slow neutral particles is due to the fact that the mean free path is equal to the product of the time of free flight and the particle velocity. The time of free flight is determined by the relative velocities of neutral and charged particles and changes with the velocity of the neutral particle by only a factor of $\sqrt{2}$ at most.

Let us discuss a limiting case, when the probability of ionization is equal to the product of some α and the probability of charge exchange; $\alpha \ll 1$. Let the total current of fast neutral particles be j_n. We have $\pi_0 \sim \frac{3}{2} T' j_n + \pi_1$, where π_0 is the heat flow in the region $x > x_1$, and π_1 is the heat flow in the region $x < x_1$, which, according to the analysis below, is many times smaller and can be disregarded.

According to the theory of albedo, the probability of ionization of fast neutral particles is $1 - 1/(1 + \sqrt{\alpha}) \simeq \sqrt{\alpha}$[4]. Therefore the ion current is $j_i = \sqrt{\alpha} j_n$. The probability of ionization of slow particles is αj_n, i.e., smaller than the probability of ionization for a fast particle; it can be neglected. At $x < x_1$, n is small. Therefore $\pi_1 \sim \nabla n^2$, and the temperature

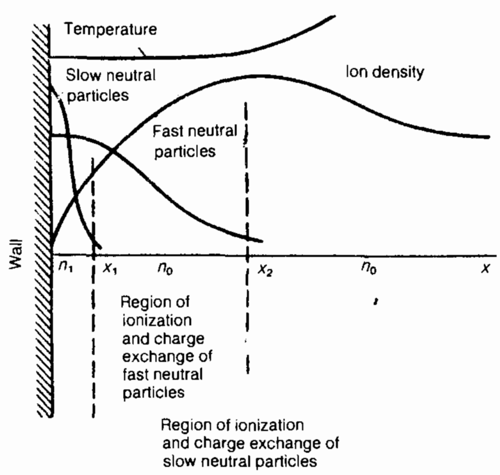

FIG. 1. Region of ionization and charge exchange of slow neutral particles.

can be considered as constant:

$$\pi_1 = \frac{7}{2} T' j_i = \frac{7}{2} \frac{4\cdot 10^{-10}\sqrt{T'}\nabla(n^2)}{H^2}, \qquad (2.5)$$

$$\pi_1 = \frac{7}{3}\sqrt{\alpha}\pi_0,$$

$$\nabla(n^2) = \frac{n_1^2}{x},$$

where n_1' is the number of ions at point x_1.

$$x_1 = \frac{1}{\sigma n_1}\frac{v_0}{v} \qquad (2.6)$$

where σ is the charge-transfer cross section, v_0 is the velocity of slow neutral particles, and v_1 is ion velocity.

At present, matching solutions in the regions $x < x_1$; $x_1 < x < x_2$, and $x > x_2$ have not been considered. We limit ourselves to a preliminary evaluation of the thermal current, for which it is possible to have a temperature jump of 10 eV (applicable to conditions of a large model; with a small model a temperature jump is certain to occur). We take $n_1 = 1.4\times 10^{15}$ cm^{-3}, $\alpha = 1$ (i.e., we are near the limit of applicability of the above-mentioned theory), $T' = 1.6\times 10^{-11}$ ergs, $\sigma = 3\times 10^{-15}$ cm^{-2}, $H = 50{,}000$ G, $v_0/v_1 = 0.05$ (wall at room temperature). We obtain $x_1 = 0.01$ cm (i.e., order of magnitude of the Larmor circle for ions, and in this case we are near the limit of applicability of the theory); $\pi = 5\times 10^8 \sim 50$ W/cm^2, which has the correct order of magnitude.

III. POWER OF MAGNETIZATION. OPTIMAL CONSTRUCTION. PERFORMANCE OF ACTIVE MATTER

The basic parameters of MTR are shown in Fig. 2. We will find the optimal relation of ∂ to d, securing a minimum mass of copper and power of magnetization in the self-sustaining region. The ratio D/∂ is obviously determined by engineering considerations and is of the order 3–5.

The product $d(\partial - d)$ is proportional to $H_0 R_0$ and should be considered as given. We are looking for a minimum of $D(\partial^2 - d^2) \sim \partial(\partial^2 - d^2)$, which can be found to occur at $\partial \approx 2.2d$. We take

$$\partial = 2d, \quad D = 6d; \qquad (3.1)$$

and the power of magnetization $P \sim H_0^2 d$. Power emitted from nuclear reaction and yield from active matter is

$$W \sim n_0^2 d^3 \sim H_0^4 d^3 \sim P^2 d.$$

For characterization of numerical coefficients in these formulas we will consider the following example (in the following, we will always have this particular example in mind whenever we mention numerical parameters):

$$\begin{cases} H_0 = 50000 \text{ G} \\ d = 4 \text{ m}, \\ \partial = 8 \text{ m}, \\ D = 24 \text{ m}, \\ n_0 = 3{,}0\cdot 10^{14} \text{ cm}^{-3}, \\ T_0 = 100 \text{ keV}, \end{cases}$$

Volume $= 0{,}96\cdot 10^9$ cm^3,
Area $= 0{,}96\cdot 10^7$ cm^2,
Release of thermonuclear power
$17{,}6\cdot 10^6$ erg/cm^3 sec.

The weight of the copper in the winding (taking a filling factor $k = 0.5$) is 13,000 tons. The current density in the winding is 400 amp/cm^2 (average 200 amp/cm^2).

The power of magnetization is about 400,000 kW (and slightly larger when the nonuniform wrapping of the coil and other structural considerations are taken into account). Release of thermonuclear energy (assuming that, on the average, reaction takes place in half the volume of the tube with the above-mentioned speed of reaction) is

$$W = 8{,}8\cdot 10^{15} \text{ erg/sec} = 880000 \text{ kW}.$$

With this, the number of burned D nuclei per second is

$$\frac{8{,}8\cdot 10^{15}\text{ erg/sec}}{1{,}6\cdot 10^{-6}\text{ erg MeV}}\frac{4}{3{,}3\text{ MeV} + 4\text{ MeV}} = 3\cdot 10^{22}\,\frac{\text{D nuclei}}{\text{sec}}$$

(which amounts to 150 g/24 hours). One can expect to obtain about 100 g/24 hours of tritium or 80 times more than U^{233}.[5] Increasing the power P and the weight of the copper by a factor of 2.5 increases this output 8.5 times (without change in current density). Increasing current density n times, we can reduce linear dimensions by a factor of $n^{1/2}$ without changing the product $H_0 R_0$. In this case the weight of copper will be reduced by a factor of $n^{3/2}$, and the power of magnetization and the yield of active substances will increase by a factor of $n^{1/2}$.

IV. DRIFT IN A NONUNIFORM MAGNETIC FIELD. SUSPENDED CURRENT. INDUCTIVE STABILIZATION

The magnetic field in a MTR (with neglected screening by plasma currents) coincides with the field of the direct current. Nonuniformity of the magnetic field leads to rather dangerous drift effects (Fig. 3). For the particle having mass M at point A the field is directed along the z-axis and the gradient of the field along the x axis, $\partial H_z/\partial x = -H_z/x$.

Suspended Current

Let us consider the motion of charged particles in the magnetic field induced by the coil of the MTR ($\sim 50{,}000$ G) and by the current on the axis (200 A) due to a ring conductor passing along the axis of the tube (Fig. 4). In such a field magnetic lines of force have spiral shape. On the basis of the numerical illustrations above, the guiding center of the Larmor circle of a particle moving along the magnetic line of

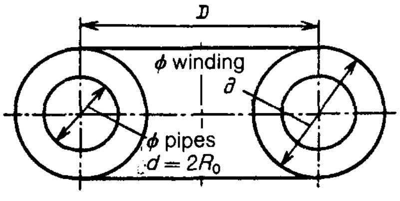

FIG. 2.

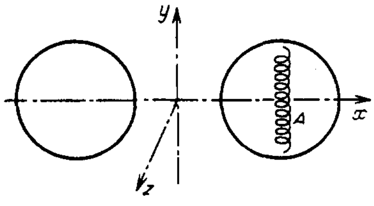

FIG. 3.

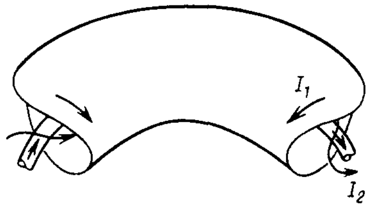

FIG. 4.

force encircles the y axis 40 times more often than the axial current (z axis). The divergence of the velocity vector field for the motion of the guiding center of Larmor circles, neglecting drift, should vanish (this follows from Liouville's theorem). On this motion drift is superimposed. The divergence of this vector field should also vanish; therefore the projection of the resulting trajectories on the cross section of the toroid (xy plane) represents closed trajectories displaced by some distance Δ from their position in the absence of drift. An estimate of the drift shows that this quantity is always sufficiently small. For example, for protons with 40 MeV energy ($v_0 = 5.2\times 10^9$ cm/sec) the spiral velocity is 2×10^7 cm/sec, and the velocity of drift is 1.5×10^6 cm/sec. From this, $\Delta \sim 20$ cm. We note that in this case we are avoiding the difficulty connected with the presence of volume charges. The question of how to produce an axial current arises. At the present time it is not clear if it is possible to put cables through the hot region which would support the axial ring and would carry current and cooling water. It is possible to form such a configuration of protective fields, for example, by means of strong current in cables, which would protect the cables from being hit by hot gas. Let us consider another possibility, i.e., suspension of the axial ring with the help of a magnetic field (an additional horizontal field with $H' \sim 100$ G will not change the qualitative picture of the magnetic field in the toroid).

The ring material should withstand high temperatures, since the only way of cooling it is by thermal radiation, corresponding, even at $T = 1400\,^\circ$C, to about 40 W/cm^2[$\pi = 5\times 10^{-5} T^4$(K)], i.e., a very small quantity. One of the possible ways of making a ring that operates at such high temperatures is to use metal tubes with a high melting point and containing melted light metal (Li, Be, Al, etc.).

Direct Current—200,000 amp. The total power necessary to sustain a direct current is 2000 to 10,000 kW. Great difficulties would be encountered in the transfer of this energy (in the form of radio frequency) to the ring and in the rectification of the alternating current.

A second means of antidrift stabilization, which is technically much more feasible and which is therefore necessary to examine carefully, is the formation of an axial current directly in the plasma by the method of induction. It is not clear if, in using this method, the high-temperature plasma is not destroyed at the moment when the induction current vanishes.

V. PROBLEM OF PLASMA INSTABILITY

It is necessary to determine whether in plasma with a magnetic field disturbances exist which, according to the equations of plasma dynamics, grow in time (exponentially, or according to a power law). It is necessary to consider a series of cases. Most theoretical and experimental studies have dealt with the current flow in a plasma parallel to the external magnetic field, where turbulent instabilities of the plasma were found. One might also suspect the presence of instability in a nonuniform plasma in the presence of a drift current. At the present time this problem has merely been postulated.

APPENDIX

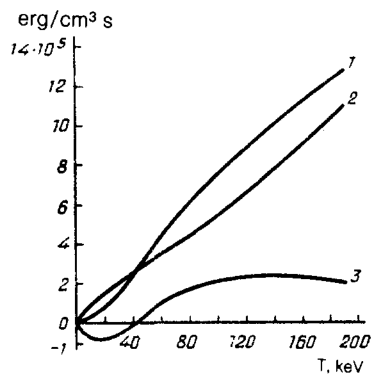

FIG. A1. 1: Energy supplied by charged particles; 2: energy of Bremstrahlung; 3: their difference.

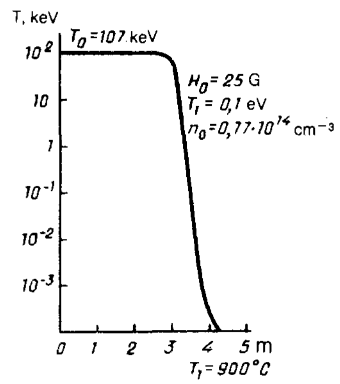

FIG. A2. Temperature distribution.

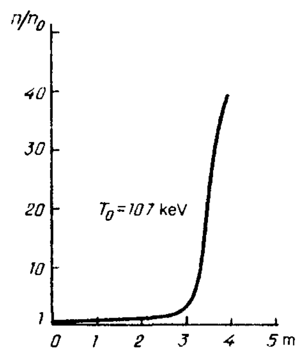

FIG. A3. Density distribution.

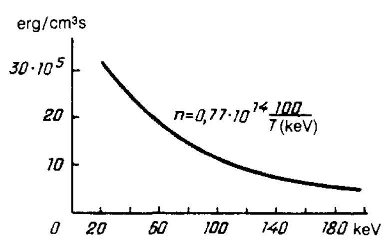

FIG. A4. Total released power.

[1)] Work done in 1951; Parts I and III were written by I. E. Tamm.

[2)] An account of a large model (disregarding boundary effect and secondary reactions) was first given by Tamm in October 1950. Results which include the effects of processes with neutral particles were obtained by him[1] for a small-density system (temperature jump).

[3)] This result can be easily understood without any calculations. Heat release per centimeter of cylinder $\sim R^2 Q n_0^2$. Heat loss is proportional to $\sim n_0^2 T^{1/2} H_0^{-2}$. In order to find R_c we equate these expressions. Then $R_c^2 \sim TQ^{-1} H_0^{-2}$.

[4)] Ionization is similar to absorption, and charge transfer is similar to scattering. The albedo of half-space is $2/(1+\sqrt{\alpha})-1$.

[5)] We note, however, that the energy value of U^{233} which can burn in simple reactors significantly exceeds the release of heat in a thermonuclear reactor.

[1] I. E. Tamm, in *Proceedings of the 1957 Geneva Conference on Peaceful Applications of Atomic Energy,* Vol. 1, *Plasma Physics and the Problem of Controlled Thermonuclear Reactions* (M. A. Leontovich, ed.), Pergamon Press, New York, 1961.