How the concept of acceleration took shape in the mechanics of Galileo

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We are all accustomed to the modern concept of acceleration. Therefore, it is difficult to imagine that there was a time when clarity was lacking in defining this important concept. Galileo vacillated between two definitions of acceleration; the change of velocity with the path traveled, or with time. First, Galileo chose the first definition and used it to analyze the law of falling bodies. Before conducting fairly accurate experiments using an inclined plane, Galileo thought that the value of acceleration corresponding to the first definition is constant during fall. However, even before making the experiments, by means of a purely logical analysis, Galileo came to the correct conclusion that such a law of motion generally cannot be achieved for falling bodies. As we shall see, the original hypothesis formulated by Galileo was incorrect, although it later led him to the correct conclusion. This is by no means the only example when a great scientist, at first reasoning incorrectly, arrives afterwards at the correct conclusion.

Let us quote suitable excerpts from Galileo's famous composition, "The Dialogues," the full name of which is: "Dialogues and Mathematical Proofs Concerning Two New Branches of Science Which Treat Mechanics and Local Motion, by Signor Galileo Galilei, Philosopher and First Mathematician of The Illustrious Grand Duke of Tuscany" (1638).[1] Galileo gave two conversationalists the names of his friends, the Florentine Salviati and the Tuscan Sagredo; the third one he named Simplicio, i.e., a simpleton. The question of interest to us about acceleration is discussed on the third day of "The Dialogues."

Sagredo says: "Uniformly accelerated motion is such, in which the velocity increases in proportion to the path traveled." In modern notation, this means $dv/ds = b$, with b a constant.

Salviati: "It is a great consolation for me that I have such a companion in error; moreover your reasoning seems so simple and plausible that, when I explained it to our Author, the latter informed me that he himself also at one time believed in this false hypothesis. But what in the long run turned out to be the most amazing is the sufficiency of only four simple words for the proofs of not only the faultiness, but also of the simple impossibility of these statements that are so plausible that, among many people to whom I have explained them, no one was found who would not at once acknowledge their correctness.

Simplicio: "It is probable I, too, would have turned out to have been among the latter. Indeed, the fact that a falling weight acquires a force during its fall and moreover, its velocity increases in proportion to its traveled path, and that its impulse of impact is twice as great upon falling from twice the height; these conclusions can be accepted without objections and doubts."

Salviati: "And at the same time, they are also incorrect and impossible, as if one would state that motion occurs instantaneously, and here is a clear proof of that for you. If velocities were proportional to distances traveled or to those having to be traveled, then such distances would be traveled in equal time intervals; thus, if the velocity with which a falling body travels a distance of four cubits were twice the velocity with which it travels the distance of the two first cubits (on the basis that the one distances is twice as large as the other), then the time intervals for traveling that distance and the other one would have had to have been the same.....

But we see, that a falling body executes its motion in time, and that it travels two cubits in a shorter time than it travels four cubits. Consequently, the statement that velocities increase in proportion to the paths traveled is false."

Sagredo then made a very curious remark: "You introduce too much clarity and too much simplicity into the explanation of obscure things; in the final analysis, the accessibility of conclusions has the consequence that knowing them seems less valuable to us."

Galileo implicitly uses an average velocity. For zero initial velocity, the velocity averaged over the path will equal half the final velocity. He assumes that the average velocity in this case due to its linear increase, simply equals half the sum of the initial and final velocities. This is correct, but one may not use the thus averaged velocity to calculate the time to travel the path. One must take the velocity averaged with respect to time to calculate the time of fall. The dependence of velocity on time will not be linear for Galileo's original hypothesis: $v = Ae^{bt}$. A linear law for the increase of velocity with time is, according to the equation shown, valid only for short values of time.

Details of Galileo's final experiments are of interest: "In order to measure time he was no longer satisfied, as in other cases, to count pulse beats, but he took a bucket with water and inserted in its bottom a small thin tube which he opened upon setting a small sphere in motion and covered with a finger after the sphere traversed distances which he had marked. The water which flowed out was collected in a cup placed below and was weighed, the amounts of water being proportional to corresponding time intervals. The paths traversed turned out to be proportional to the squares of the time intervals." This, of course, is correct, and the time-average value of velocity is valid. A. N. Krylov[2] and G. Lipson[3] did not notice the original definition of acceleration by Galileo.

[1] G. Galilei, Works [Russ. transl. Vol. I. GTTI, M., L., 1934].
[2] A. N. Krylov, "*Galileo as the founder of mechanics*," Sobranie Trudov (Collected Works), Vol. 1, Pt. 2, Izdatel'stvo Akad. Nauk SSSR, M., L., 1951.
[3] H. S. Lipson, *The Great Experiments in Physics*, Oliver and Boyd, Edinburgh, 1968. [Russ. tansl. Mir, M., 1972].

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