From the Editorial Board of Usp. Fiz. Nauk. The papers published below are dedicated to the memory of Academician II'ya Mikhaĭlovich Frank (October 23, 1908–June 22, 1990).

Most physicists probably associate I. M. Frank with optical phenomena accompanying the passage of charged particles through matter. Indeed, Vavilov–Cherenkov radiation, transition radiation, the anomalous Doppler effect, and radiation from moving multipoles were a constant object of his scientific interests.

Joining S. I. Vavilov and P. A. Cherenkov at the beginning of the 1930's, he obtained, together with I. E. Tamm, a result which brought him worldwide fame and a Nobel prize in 1958. He worked in this field all of his life and at the end of his life he summarized the main results in a monograph (I. M. Frank, Vavilov-Cherenkov Radiation. Theoretical questions [in Russian], Nauka, M., 1988). These and other scientific accomplishments of Il'ya Mikhaĭlovich are fully described in a paper dedicated to his eightieth birthday (Usp. Fiz. Nauk, 156, 373 (1988) [Sov. Phys. Usp. 31, 960 (1988)]). Referring to I. M. Frank's post-war scientific work, the authors of this paper indicate primarily the results obtained at the Laboratory of Nuclear Physics at the Institute of Physics of the Academy of Sciences and the Neutron Laboratory at the Joint Institute of Nuclear Research in Dubna, which II'ya Mikhaĭlovich created and directed for many years. One of these remarkable results is the discovery of ultracold neutrons at the end of the 1960's. This led to the appearance of a new field of neutron physics.

Educated and primarily interested in optics and devoting several decades of his life to nuclear and neutron physics, Il'ya Mikhailovich probably saw this event as a unique opportunity for joining together two of his favorite fields of physics. At the beginning of the 1970's, he carried out several investigations on the optics of ultracold neutrons. Unfortunately, these papers, which are still of value, have remained unknown, except within a quite narrow circle of specialists.

Wishing to fill this lacuna and paying tribute to the memory of Il'ya Mikhailovich Frank, we are publishing a memorial lecture given by A. I. Frank at the VI International School on Neutron Physics in Alushta in 1990 and a lecture delivered by Il'ya Mikhailovich at the II Alushta School in 1974 (this paper was prepared for publication by A. I. Frank).

Modern optics of long-wavelength neutrons

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I. V. Kurchatov Institute of Atomic Energy, Moscow (I. M. Frank Memorial Lecture at the 6th International Neutron School, Alushta, Crimea, October 1990) (Submitted July 24, 1991) Usp. Fiz. Nauk 161, 95-108 (November 1991)

This paper is dedicated to the memory of Academician II'ya Mikhailovich Frank (1908– 1990). The papers of I. M. Frank for the period 1972–1974 concerning the optics of ultracold neutrons (UCN) are briefly reviewed and compared with later results obtained in this field. The basic stages of the development of a neutron microscope based on UCN are briefly described. The possibility of using neutron-optical methods for fundamental investigations with long-wavelength neutrons is discussed.

It is well known that F. L. Shapiro and his coworkers first started to work with ultracold neutrons (UCN) at the end of the 1960s. In these studies it was demonstrated that such neutrons can indeed propagate along curved neutron guides and that they can even be stored for a long time in closed vessels. At the same time in Munich A. Steyerl started to experiment with very slow neutrons. This was in fact the discovery of UCN.

The first period of research was summarized by F. L. Shapiro in his report at the conference in Budapest during the summer of 1972.¹

At that same conference in Budapest, Il'ya Mikhaĭlovich made some remarks² as a supplement to the report of F. L. Shapiro. He called attention to the fact that together with the approach based on the introduction of the optical potential

$$V_{\rm opt} = \frac{\hbar^2}{2m} \frac{Nb}{\pi},\tag{1}$$

which F. L. Shapiro employed, into the Schroedinger equation, the index of refraction n can be introduced immediately. In so doing the squared index of refraction was compared with the permittivity ε for light and the imaginary part of ε was determined by the cross section of all processes resulting in the vanishing of UCN, namely, radiative capture and inelastic scattering. Here the index of refraction is a complex number: n = n' + in'' and

$$n^{2} = \varepsilon' + i\varepsilon'' = 1 - \lambda^{2} \frac{N}{\pi} (b' - ib''), \qquad (2)$$

where b' and b'' are the real and imaginary parts of the scattering amplitude.

It is easy to obtain expressions for the real and imaginary parts of the index of refraction (see the formulas (4.1)– (4.5) of Ref. 6). I. M. Frank did this in his subsequent works.

For ultracold neutrons, when the velocity is less than the limiting value, $n''^2 > n'^2$ and the real part of ε is negative. If, as is usually the case, absorption is small, then the real part of ε , though it remains negative, is much greater in absolute magnitude than the imaginary part. This situation occurs in optics when light is reflected from a highly conducting metal.

Thus the reflection of UCN from the surface of a material was compared with reflection of light by metals. The Fresnel coefficients, which are the amplitudes of the reflected and transmitted waves (formulas (5.3) and (5.4) of Ref. 6), were simply introduced by analogy to the optics of metals.

Reference 2 was essentially the first work on the optics of UCN. I. M. later developed these ideas in several publications³⁻⁵ and he gave a memorable lecture at the 2nd Alushta School in 1974.⁶ It seems quite natural that I. M., as one who loved optics and contributed much to the field, would turn to the optical analogy. The optical way of thinking was very natural to him.

It is sometimes said that the approach employing the index of refraction is "old-fashioned" and it is more correct to employ the Schroedinger equation with an optical potential. I. M. himself emphasized that both methods are equally well-founded, and this is undoubtedly true. However, when an attempt is made to solve quite rigorously the problem of propagation of a neutron in a medium, a solution (always approximate) is obtained for some self-consistent neutron field (see, for example, Refs. 7 and 8). The quantity kb is the small parameter of the theory. The first approximation is a plane wave with a definite wave number k_1 , whence the index of refraction $n^2 = k_1^2/k^2$, where k is the wave number in vacuum, is determined in a natural manner. The optical potential, however, is simply adjusted so as to obtain the same solution. In fact, this is precisely how Fermi introduced the concept of a quasipotential, which correctly describes a wave scattered by a single nucleus. The optical potential is usually obtained, however, by averaging Fermi's quasipotential over the scatterers (see, for example, Ref. 9).

Returning to the works of I. M., we recall that at the beginning of the 1970s one of the most important problems in the physics of UCN was the storage problem. Ultracold neutrons could be stored in closed vessels for significantly shorter periods of time than predicted by the theory, and the question "Where did the UCN go?" was one of the most urgent questions. Naturally, this question also worried I. M., and he discussed it in the papers cited above.⁴⁻⁶ A number of considerations stated in this regard are, it seems to me, still important today, even though they may not be directly related with the problem of storage of UCN. I shall discuss one of them here.

In 1945 Foldy obtained a simple expression for the scattering of scalar waves:¹⁰

$$k_1^2 = k^2 + 4\pi N f_0, \tag{3}$$

where f_0 is the forward-scattering amplitude. For slow neutrons the substitution $f_0 = -b$ into the formula (3) leads,

after simple transformations, to Eq. (2). In Refs. 4 and 6, I. M. calls attention to the fact that the expression (3) is also valid for light in rarefied media, where the index of refraction is close to 1:

$$n^2 = 1 + 4\pi N \alpha, \quad |n^2 - 1| \ll 1,$$
 (4)

where α is the polarizability.

On the other hand, if n^2 is not too close to 1, Eq. (4) must be replaced by the Lorentz-Lorenz formula.

Lax investigated this question in a quite general form.^{11,12} Instead of Foldy's formula (3) he gives an expression—valid for both light and neutrons—in which the difference between the effective field acting on a scatterer and the coherent field is taken into account:

$$k'^2 = k^2 + 4\pi NCf_0.$$
(5)

The correction factor C introduced by Lax can, generally speaking, be a complex number. Then instead of Eq. (2) we have

$$n^{2} = 1 - \lambda^{2} \frac{N}{\pi} (C' - iC'') (b_{0}' - ib_{0}''), \qquad (6)$$

where $b_0 = b'_0 - ib''_0$ is the amplitude of scattering by an isolated nucleus. Thus the quantities $b' = C'b'_0$ and $b'' = C''b'_0 + C'b''_0$ must be employed as the scattering length in a medium (in the expression for b' we neglect the quantity C''b'').

It would be easy to explain the discrepancy in the storage time of UCN if b'' differed from the value b''_0 employed in the calculation by approximately $3 \cdot 10^{-4}$. For this reason, I. M. suggested that this discrepancy could be associated with the effects of the coherent field, if Lax's coefficient is indeed different from unity and different for thermal neutrons and UCN. Since $b''/b' \approx 10^{-4}$, this contradiction would be eliminated if C'' is also of the order of 10^{-4} .

Since the publication of the works of I. M., the question has been investigated in greater detail.^{7,8} It has now been firmly established that small corrections do indeed exist and they indeed depend on the wavelength of the neutron. These corrections are associated with the fact that near each nucleus the neighboring scattering nuclei are not arbitrarily distributed, since in all substances, even in liquids and amorphous bodies, there is at least short-range order. Some correlation even exists in the model when the scatterers are hard spheres. In this case it is simply said that the distance between their centers cannot be less than the diameter of the spheres. Incidentally, it is precisely in this model that an exact calculation is most easily performed. In this connection we shall present the results of Sears:⁷ C = 1 + J = 1 + J' + iJ'',

$$J' = J_0 \left(\frac{\sin ka}{ka}\right)^2, \quad J'' = \frac{J_0}{2(ka)^2} (2ka - \sin 2ka), \tag{7}$$
$$J_0 = 2\pi N ba^2,$$

where a is the radius of an atom. It is easy to see that if $ka \rightarrow 0$, then

$$C' = 1 + 2\pi N b' a^{2},$$
(8)
$$C'' = \frac{4}{2}\pi N b' k a^{3}.$$

Analogous results were obtained in Ref. 8. One can see from Eq. (8) that in the case of UCN, when n^2 is close to zero and $4\pi Nb' \approx k^2$,

$$C' \approx 1 + k^2 a^2, \quad C'' \approx k^3 a^3, \tag{9}$$

where the characteristic parameter $ka \approx 10^{-2}$. Therefore, for UCN the quantity C" can indeed be of the order of $10^{-5}-10^{-6}$.

It is significant that the local-field correction C'' depends on the wave number k. This means that the use of the value of b'' obtained from data on scattering and absorption of very cold neutrons for estimating absorption of UCN can lead to an error. A small deviation from the 1/v law should also be expected for very slow neutrons.

Thus the optical analogy has turned out to be correct on the whole. In neutron optics coherent-field effects result in a deviation from Foldy's simple formula, though this deviation is small.

It is easy to see that substituting the quantity C'b for b in Eq. (1) leads to a wavelength-dependent optical potential.¹³ Apparently, it is precisely this dependence that at the present time provides the only possibility for observing experimentally the effects associated with the difference between the coherent and effective fields. Such an experiment, it seems, would be of fundamental significance.

As regards the possible characteristics of these effects in the case of reflection of UCN, the present status of the problem of storage of UCN is significantly different from the situation in the 1970s, and there are no longer any grounds for expecting significant effects here. In addition, the expressions (8) and (9) seemingly imply that in the limit $ka \rightarrow 0$ the corrections are small. Even here, however, some care must be exercised. As I. M. pointed out in Ref. 6, the case of reflection of UCN requires a special theoretical analysis. The point is that in the case of total reflection the wave decays rapidly in the matter, and in addition the decay constant is of the order of the wavelength. It is still not entirely clear how this affects the corrections, and this question is not answered in Refs. 7 and 8. Now I would like to discuss one of the most often cited works of I. M. on neutron optics-Ref. 3. A wide range of problems, including some of the problems discussed above, was studied in this work. In addition, the question of the possibility of a neutron microscope was first posed in it. In order to get a clearer picture of how the problem was viewed at that time, I quote in full the corresponding part of this paper.

"In future, when it becomes possible to do so, the simplest optical experiments will be performed. For example, one can imagine the following experiment. Ultracold neutrons pass through a small opening, strike a concave mirror, and after being reflected from the mirror collect at the focal point (Fig. 1). In the process, the neutrons will acquire an additional vertical velocity due to their free-fall in the earth's gravitational field. As a result at the mirror they will appear to have emanated from a point 0 lying somewhat above the opening A, and they will collect at the focus C below the geometric focus B. This unique velocity-dependent chromatic aberration must be taken into account in optical devices based on ultracold neutrons. It seems to me that the production of an optical image with the help of reflection and refraction of very cold neutrons is such an important



FIG. 1.

experiment that it absolutely must be performed. One can, after all, dream that in some distant future the optics of very slow neutrons will make it possible to build a neutron microscope."

I remember well the impression that this suggestion made at this time. At first glance, it was simple, almost trivial. On the other hand, the state of sources of UCN was such at that time that it was very difficult to dream seriously about a microscope and the suggestion seemed hopeless. Third, the problem of gravity-induced chromatism presented a definite challenge and one wanted to find at least theoretical approaches to its solution. Without this it was impossible to think about a microscope. This is probably why the simple experiment, proposed by I. M., with a concave mirror was never performed, and the first experiments performed by A. Steyerl and then later by our group were directed toward demonstrating the possibility of achromatization.

I had to talk to I. M. several times about this subject long before there appeared any prospect for such experiments. The experimental arrangement proposed by him could be discussed in a completely classical, i.e., corpuscular language. On the other hand, in order to understand the question the problem of forming a neutron image had to be understood from the wave standpoint. In the classical approach it was obvious that a neutron requires different amounts of time to reach the focus along different trajectories. The consequences of this in the wave analysis of the problem were not completely understood. These discussions led to the idea that gravity can be taken into account by using purely optical language, and it is possible to introduce the concept of a "gravitational index of refraction"¹⁴

$$n(r) = n(z) = \left(1 - \frac{2gz}{v_0^2}\right)^{1/2}, \quad v_0 = v \quad (z = 0). \tag{10}$$

Thus the space where gravity acts on the neutrons can be regarded as an optically nonuniform medium where one of the fundamental principles of optics—Fermat's principle—holds without any restrictions.

The validity of Fermat's principle implies an image can be formed with neutron waves in a potential field.¹⁾ The idea of an optically nonuniform medium made it possible to employ a number of well-known results in optics.¹⁶ However the correct answer to the question of the role of nonisochronicity of the classical trajectories was not obtained. At the same time it became clearer that this is an important question. In specific optical calculations the classical time of flight appeared directly in the expressions for the basic characteristics of the optical apparatus, for example, the magnification.^{15,17} Further investigations clarified the situation, and the role of the classical propagation time of the particle was understood better. I shall present below the current understanding of this question.^{18,19}

In all cases Fermat's principle can be written in the following form:

$$\delta \int_{A}^{B} k dl = 0, \quad \text{or} \quad \delta \int_{A}^{B} n dl = 0.$$
 (11)

In the standard optics it can also be written in the form

$$I = \int_{A}^{B} n dl = \int_{A}^{B} \frac{c}{v} dl = c \int_{A}^{B} dt, \quad \delta I = 0, \quad (12)$$

and expresses the fact that the propagation time of the wave between optically conjugate points A and B is minimum or stationary. In the case of a massive particle, such as a neutron, however, $k dl = (m/h)v^2 dt$ and Fermat's principle assumes the form²⁰

$$\delta \int_{A}^{B} v^2 dt = 0, \quad \text{or} \quad \delta \int_{A}^{B} n^2 dt = 0.$$
(13)

In this case Fermat's principle expresses only the fact that the phases, but not the time, are minimum or stationary (compare Eqs. (12) and (13)). This actually means that a "classical" particle requires different amounts of time to reach a surface of equal phase. We note that Fermat's principle in the form (11) follows from the quasiclassical solution of the stationary Schroedinger equation. Starting from the stationary wave equation, however, we seemingly made the question of the propagation time meaningless. Indeed, the wave pattern of the field is determined only by the amplitudes and phases of the waves arriving at the observation point. Analysis showed, however, that the question of the stationarity of this wave pattern is directly related with the question of the isochronicity of the classical trajectories.

The problem of the stationariness of the interference pattern was analyzed more carefully in connection with the problems of neutron interferometry. Chue and Stodolsky²¹ showed that near the so-called ideal configuration, when the classical trajectories bring the particles to the same observation point simultaneously, the first derivative of the differences of the phases with respect to the experimental parameters is equal to zero.

In Ref. 22 the validity of this conclusion was confirmed by a direct calculation for an interferometer, designed for operating with UCN in the earth's gravitational field. The parameter of the problem was the neutron velocity. It was shown that for isochronous trajectories the first derivative of the phase difference with respect to the velocity does indeed vanish. This condition, however, is satisfied when the phase difference itself is significant, i.e., under conditions when interference of high order is observed. For this reason, the observation of an interference pattern still requires significant monochromatization because of effects which are of second order in the velocity. On the other hand, in order to observe an interference pattern under conditions when the trajectories are not isochronous it is necessary to have a very high degree of monochromatization, even in zeroth order. This conclusion follows in a more general form from the

results of Ref. 23, where it is proved that when the conditions of the ideal configuration are not satisfied the effect necessarily follows in first order of variation of the parameters.

In order to solve the problem of achromatization of the interference pattern, it was proposed²² that a geometry be employed in which isophaseness, i.e., zero-order interference, and isochronicity are achieved simultaneously. Such a geometry can be realized in a double-loop interferometer scheme.

It is obvious that analysis of the operation of a neutron interferometer in the earth's graviational field has a direct bearing on the problem of formation of an image in an optical device, where not two waves (rays), as in an interferometer, but rather an entire family of rays interfere. Thus the question of the propagation time of a neutron in an optical device turned out to be very directly related with the problem of the gravity-induced chromatic aberrations, about which Il'ya Mikhaĭlovich wrote in 1972.

Since the publication of I. M.'s paper³ in the journal Priroda, a great deal has been accomplished in the practical optics of UCN. Since quite complete reviews are available,^{18,19,24} I shall list only briefly the basic stages which were passed on the path toward a neutron microscope.

The basic characteristics of gravity-induced aberrations are now quite well understood. Chromatic aberrations which result in displacement of the image plane (position chromatism) are distinguished from chromatic aberrations which change the optical magnification (magnification chromatism). In addition, gravity-induced geometric aberrations, associated with different curvature of the rays which make a different angle with the vertical, occur even for perfectly monochromatic waves. In a real situation all these types of aberrations can be present simultaneously.

The honor of obtaining the first image of a simple source with the help of UCN, i.e., almost in the manner proposed by I. M., belongs to A. Stereyl and his coworkers.²⁵ An important difference, however, was that they were the first to find a method for compensating the chromatic positional aberrations. To do so, instead of a simple mirror, a more complicated optical element—a zonal mirror—was employed; in this element a concave mirror was combined with a zonal interference system (1980).

In 1984 our group tested an optical device with four mirrors, which also give positional monochromatization.²⁶ The optical resolution in both of these experiments, strictly speaking, was not determined, but they marked the beginning of experimental work in technical optics of UCN.

In 1984 there appeared a report in which it was stated that work on a two-mirror high-magnification microscope, developed by A. Steyerl's group,²⁰ had begun. The first results of the tests were published in 1985.²⁷ This device had double achromatization (with respect to magnification and position), and the use of a parabolic instead of a spherical mirror significantly reduced the usual aberrations. A resolution of approximately $100 \,\mu$ m was obtained with a magnification of \times 78. In all these studies the image was analyzed by mechanically scanning the image plane or the object plane.

In 1986 our group reported on our work with a lowmagnification (\times 1.4) device, which also exhibited double achromatization. Here an image detector was employed for the first time and images of a number of simple objects were obtained, including a cross-shaped diaphragm and a twodimensional object specially prepared by the method of photolithography on silicon with the image of a neutron reflected from the mirror—the emblem of the Alushta Neutron School. In spite of the low magnification the resolution of the device was equal to 70–100 μ m.²⁰

At the same time A. Steyerl and his coworkers were the first to attempt to use for purposes of microscopy neutrons which are faster than UCN.²⁹ It is obvious that in this case total-reflection mirrors cannot be used. For this reason, the normal-incidence mirror optics had a multilayer reflecting interference coating. Although a quite modest resolution (230 μ m) was obtained, the experiment marks the transition to a very promising, from the practical viewpoint, energy range.²⁾

In 1988 A. Steyerl's group reported that they achieved in their microscope a resolution of the order of $10 \,\mu$ m with a magnification of $\times 79$. In addition, a mode with an even higher magnification, right up to $\times 280$, was tested.³² I want to mention the fact that the paper containing these results was included in a collection which the coworkers and friends of Il'ya Mikhaĭlovich dedicated to his 80th birthday. This collection also contains a review of neutron microscopy.²⁴

Now I shall discuss in somewhat greater detail the latest work performed by our group in this field. Thus far, in all experiments on the practical optics of very slow neutrons, schemes with a vertical optical axis were employed, and this tendency, undoubtedly, originated with I. M.'s ideas which we discussed above. However it recently became clear that the problem of gravity-induced chromatism is somewhat easier to solve in schemes with horizontal ray paths. In this arrangement of the device there is only one form of chromatic distortions—displacement of the image as a whole as the velocity of the neutron changes.³³ One method for compensating such chromatism is proposed in Ref. 34.

The setup that stabilizes the position of the image here is a turning system, consisting of two mirrors positioned at right angles to one another. If a turning system were not present, gravity would have the effect that for every pair of rays making with the axis an angle of the same magnitude and different sign the upper ray would correspond to the smaller wave number. Phase equality in the image plane is ensured in this case by the large length of the top ray; this is connected with the bending of the rays in the gravitational field. In this case both rays have the same phase, but they are not isochronous, and it is to this that the chromatism of the pattern corresponds.

The turning system interchanges the top and bottom rays, and in the process the phases at the point corresponding to the standard optical image are equalized. The optimal position of this system corresponds to isochronous trajectories. As one can see, the situation is entirely similar to a twoloop layout of a neutron interferometer. This situation is illustrated in Fig. 2.

A microscope based on this idea has been developed,³⁵ and it has recently been tested. The objective in this microscope is a Schwarzschild objective from a standard optical microscope. The microscope has an optical magnification $M = \times 47$. The microscope is equipped with a new coordinate-sensitive UCN detector with electronic data acquisition,³⁶ which makes it possible to perform continuous exposure over a period of several days.

The new detector was used to record images of two slits



FIG. 2. The optical layout of a horizontal microscope with a turning system (b) as compared with a two-loop interferometer (a).

of width 40 and 13.5 μ m.³⁷ Analysis of the form of the image gave an estimate of the resolution of the microscope (and of the measuring system), which was equal to 17 μ m. An image of a periodic test object, consisting of 33- μ m-wide transparent and reflecting stripes, was also recorded. The contrast of the image is satisfactory.

This then is the current state of affairs in this field. It appears that we shall soon witness the first experimental work on the use of a neutron microscope for the investigation of matter.

Of course, the questions discussed above by no means exhaust the modern optics of long-wavelength neutrons. The relation between neutron optics and the quantum mechanics of a slow particle is obvious (see, for example, Ref. 38). Optical methods also provide new possibilities for investigating the fundamental interactions of a neutron. This, in particular, has been clearly demonstrated in experiments concerned with the investigation of the electric neutrality of the neutron.^{39,40}

I shall discuss these questions in greater detail. The question of the linearity of the Schroedinger equation for a free particle is undoubtedly of fundamental significance. In all models of nonlinear quantum mechanics the degree of nonlinearity is characterized by some fundamental quantity b having the dimension of energy. In the popular model of logarithmic nonlinearity⁴¹ the Schroedinger equation contains the term

$$F = -b\ln|\psi|^2. \tag{14}$$

The spreading of the wave packet in space is bounded by the quantity

$$L = \hbar / (2mb)^{1/2}.$$
 (15)

The last relation is apparently quite universal and does not depend strongly on the model, since it relates the linear and energy constants of any quantum-mechanical problem. The best experimental limit $b \leq 3.3 \cdot 10^{-15}$ eV has now been obtained in an optical experiment on the observation of Fresnel diffraction of neutrons with wavelength 20 Å by the edge of a screen.⁴²

Note that in an optical device an image is formed by means of constructive interference. Thus it is also closely related with the principle of superposition, i.e., with the question of the linearity of the theory.^{19,3)} This fact is direct-

ly employed in optics for determining the limiting resolution of an optical device. The standard diffraction relation, for example, for the resolution of a microscope, is

$$\delta \approx \frac{0.6\lambda}{A},\tag{16}$$

where δ is the limiting resolution determined by the size of the diffraction spot, A is the numerical aperture $A = \sin \theta$, and θ is the aperture angle. The meaning of this limit is associated with the fact that the maximum size of a coherent wavefront at the first lens (mirror) is

$$L_{\rm coh} = 2f \tan \theta. \tag{17}$$

In a linear theory, which the standard optics is, $L_{\rm coh}$ is simply identical with the diameter of the corresponding optical element (here *f* is the focal length). If the optical device is imperfect and has aberrations, then the corresponding size of the region of coherence, called the Fresnel zone and determined separately in each specific case, plays the role of the diameter of the lens.

It is obvious that the inverse problem can be formulated. Once the resolution of the device has been determined experimentally, the coherence length of the wave in the first optical element can be determined. Although the problem does not arise in standard optics, for a neutron-optic device this question can be important. Using the data for Steyerl's microscope,³² having a resolution of $10 \,\mu$ m, and substituting the corresponding quantities into the expressions (3) and (24), we find that the coherence length¹⁹ is equal to $L_{\rm coh} \approx 0.2$ cm.

This means that there are no fundamental limitations on the coherence length of a neutron wave at the level indicated. If an attempt is made to relate this quantity with an energy constant, then from the expression (22) we obtain the estimate $b \le 10^{-17}$ eV.

The question of image formation in the nonlinear theory thus requires a more complete analysis. Analogous calculations for the analysis of a diffraction experiment⁴² were recently performed in Ref. 43.

Significant new possibilities for studying the wave properties of the neutron would open up with the construction of a neutron interferometer for long-wavelength neutrons. The first such suggestion was discussed back in 1979.⁴⁴ Some questions regarding the theory of an interferometer with UCN in the earth's gravitational field were studied in Ref. 22. Today interferometers based on diffraction gratings probably offer the most realistic prospects.^{45–47,38} Such an interferometer could be used for a quite extensive program of research. In particular, with its help it would be possible, as suggested by Sears,¹³ to perform an experiment of the Fizeau type,⁴⁸ in which the dependence of the optical potential on the wave number would be manifested. Progress in the study of the linearity of quantum mechanics and much else could be made.

The low energy of UCN presents a number of unique possibilities for investigating a quite wide class of quantum gravity-induced effects. Among the theoretically predicted phenomena I should mention the quantization of the energy of UCN in a gravitational field^{49,50} and the observation of wave interference of the structure near the gravitational caustic of a point source of UCN.⁵¹ It should also be noted that the gravitational phase shift observed for thermal neutrons can be measured with the help of a UCN interferometer.^{52,53} It is possible that by transferring to very slow neutrons can be used to establish, with high reliability, for the neutron the equivalence of the inertial and gravitational masses.

In connection with the latter problem it is pertinent to discuss another possibility, existing in the optics of very slow neutrons. This is the possibility of investigating the interaction of a neutron with matter and weak fields by observing the precession of a neutron in a magnetic field.⁵⁴

The Larmor precession of the neutron spin can be interpreted in terms of the interference of the two spin components of the neutron wave function. The angle of precession is identified with the phase difference between the components:

$$\vartheta = k(n_{+} - n_{-})L = k \left[\left(1 + \frac{\mu B}{E} \right)^{1/2} - \left(1 - \frac{\mu B}{E} \right)^{1/2} \right] L,$$
(18)

where n_+ and n_- are the indices of refraction of each of the spin components in the magnetic field.

If in the region of propagation of the wave there exists, in addition to the magnetic field, a potential V of some nature, then the character of the refraction of the two components of the wave will be determined by the combined effect of both fields and the angle of precession will be expressed as follows:

$$\vartheta = k \left[\left(1 + \frac{\mu B}{E} - V \right)^{1/2} - \left(1 - \frac{\mu B}{E} - V \right)^{1/2} \right] L. \quad (19)$$

Thus if the precessing neutron traverses a path d in a region where the potential V operates, then the action of this potential results in the appearance of an additional angle of precession. For $\mu B / E \ll 1$ and $V / E \ll 1$ this additional angle is determined by the relation

$$\varphi = k \frac{\mu B}{E} d \frac{V}{2E}.$$
 (20)

Recalling that the index of refraction of a neutron in a potential is determined in the same approximation by the expression $n \approx 1 - (V/2E)$, we write Eq. (27) in the form

$$\varphi = \omega_{\rm L} \frac{d}{v} \left(1 - n \right), \tag{21}$$

where v is the velocity of the neutron and $\omega_L = 2\mu B/h$ is the Larmor precession frequency. The expression (21) was derived in Ref. 55 for the case of the refraction of long-wave-length neutrons in ordinary matter. In this case the additional precession can be termed optical rotation of spin.

We note the angle of the additional spin rotation is proportional to the quantity $1/v^3$ and is small for thermal neutrons, but in the case of long-wavelength neutrons it can be significant even for small thicknesses d.

In order to prepare a state with a precessing spin, the spin must be turned by an angle $\pi/2$ even before the neutron enters the range of the potential V. It is for this reason that we speak about not one neutron wave, but rather two neutron waves corresponding to the two spin components. Thus the procedure of rotation by the angle $\pi/2$ is similar to the coherent separation of the wave into two spatial components in the standard interferometer. After the region of the potential

tial, which by virtue of the dispersion acts on the two components differently, has been traversed and after a reverse $\pi/2$ rotation, playing a role similar to that of a combiner in standard optics, the polarization of the beam is analyzed. The relation between the intensity and the angle of rotation is described by the same cosine law as in the standard interferometer. For this reason, in Ref. 54 the additional spin precession in a potential of any nature was regarded as the basis of the experimental method of neutron spin interferometer (NSI). The effect in which we are interested-the additional spin rotation-can be separated against the background of a large angle of Larmor precession with the help of a difference method, similar to the neutron-spin-echo technique.⁵⁶

The quantity d(n-1) appearing in the expression (21) is the difference between the geometric and optical thicknesses of the refracting specimen (or the range of the potential). For this reason, neutron spin interferometry can serve as a foundation for the development of an observation method analogous to the phase-contrast method in standard optics. It could also be applicable in neutron microscopy.19,24,57

The method of NSI makes it possible, for example, to perform an experiment on the observation of the gravitational phase shift and the phase shift in a noninertial coordinate system.58

The magnitude of the gravitational phase shift in the same approximations as the formula (28) is determined by the expression

$$\varphi_{g} = \omega_{L} \frac{m_{g}}{m_{in}} \frac{gL^{2}}{2v^{3}},$$
(22)

while the phase shift in a device moving with acceleration a is given by the expression

$$\varphi_{\rm in} = \omega_{\rm L} \frac{aL^2}{2v^3},\tag{23}$$

where m_g and m_{in} are the gravitational and inertial masses of the neutron. It has not been excluded that the NSI methods can also be used to perform a Fizeau experiment.

There is one other feature in the physics of long-wavelength neutrons. This feature is associated with the smallness of the characteristic quantum time for UCN

$$\tau = \hbar/E \approx 5 \cdot 10^{-9} \,\mathrm{c.} \tag{24}$$

This suggests that it could be possible to observe nonstationary quantum processes. This possibility was discussed theoretically in Refs. 59-62. One effect of this type is studied in Ref. 63.

In conclusion I wish to emphasize once again that many of the ideas put forth by I. M. Frank in his first works on the optics of ultra-cold-neutrons have turned out to be very fruitful and are still important today.

On the other hand, as often happens, in reality this field of science has turned out to be much richer than could have been imagined in that distant past and Il'ya Mikhailovich was sincerely pleased with this development.

that such an objective be used in a neutron microscope for UCN.

- ³⁾ See also the lecture by A. Steyerl in Proceedings of the 6th School on Neutron Physics (in press).
- ¹F. L. Shapiro, Preprint R3-7135, Joint Institute of Nuclear Research, Dubna, 1973; Neutron Investigations [in Russian], Nauka, M., 1976, p. 229.
- ² I. M. Frank, "Remark of Professor I. M. Frank" in Proceedings of International Conference on Nuclear Study with Neutrons, Budapest, 1972, p. 285
- ³I. M. Frank, Priroda, No. 9, 24 (1972).
- ⁴I. M. Frank, "Wave properties of neutrons," Preprint R3-7809, Joint Institute of Nuclear Research, Dubna, 1974.
- ⁵I. M. Frank, "Absorption and reflection of ultracold neutrons," Pre-
- print R3-7810, Joint Institute of Nuclear Research, Dubna, 1974. ⁶I. M. Frank, "Neutron optics and ultracold neutrons" in *Collection of* Lectures at the 2nd International School on Neutron Physics, Alushta, Joint Institute of Nuclear Research, Dubna, 1974, p. 19; Usp. Fiz. Nauk, this issue, p. 109. [Sov. Phys. Usp., this issue, following this paper]. ⁷V. F. Sears, Phys. Rep. **82**, 1 (1982).
- ⁸ M. Warner and J. E. Gubernatis, Phys. Rev. B 32, 6347 (1985).
- ⁹V. I. Lushchikov, "Ultracold neutrons" in Neutrons: On the Fiftieth Anniversary of Their Discovery [in Russian], Nauka, M., 1983, p. 132.
- ¹⁰ L. I. Foldy, Phys. Rev. 67, 107 (1945)
- ¹¹ M. Lax, Rev. Mod. Phys. 23, 287 (1951).
- ¹² M. Lax, Phys. Rev. 85, 621 (1952).
- ¹³ V. F. Sears, Phys. Rev. A 32, 2624 (1985).
- ¹⁴ I. M. Frank and A. I. Frank, Pis'ma Zh. Eksp. Teor. Fiz. 28, 559 (1978) [JETP Lett. 28, 516 (1978)].
- ¹⁵ A. Steyerl and G. Shutz, Appl. Phys. 17, 45 (1978).
- ¹⁶ Yu. A. Krivtsov and Yu. I. Orlov, Geometrical Optics of Inhomogeneous Media [in Russian], Nauka, M., 1980.
- ¹⁷ A. I. Frank, "Hamilton's analogy and the optics of ultracold neutrons," Preprint IAE-3203, Institute of Atomic Energy, Moscow, 1979.
- ¹⁸ A. I. Frank, Usp. Fiz. Nauk 151, 229 (1987) [Sov. Phys. Usp. 30(2), 110 (1987)].
- ¹⁹ A. I. Frank, Nucl. Instrum. Methods A 284, 161 (1989).
- ²⁰ A. Steyerl, J. Phys. (Paris) 45, 255 (1984).
- ²¹C. Chue and L. Stodolsky, Phys. Rev. D 22, 1337 (1980).
- ²² A. I. Frank, Zh. Tekh. Fiz. 53, 935 (1983) [Sov. Phys. Tech. Phys. 28(5), 600 (1983)].
- ²³ Y. Z. Pei, J. S. Zhong, and S. Yuo, Phys. Rev. D 32, 368 (1985).
- ²⁴ A. I. Frank, "Neutron microscopy based on ultracold neutrons", At. Energ. 66, 93 (1989) [Sov. At. Energy (USA) 66(2), 106 (1989)]; Some Problems in Modern Nuclear Physics. Dedicated to the Eightieth Birthday of I. M. Frank [in Russian], Nauka, M., 1989, p. 213.
- ²⁵G. Shutz, A. Steyerl, and W. Mampe, Phys. Rev. Lett. 44, 1400 (1980).
- ²⁶S. S. Arzumanov, S. V. Masalovich, A. N. Strepetov, and A. I. Frank, Pis'ma Zh. Eksp. Teor. Fiz. 39, 486 (1984) [JETP Lett. 39, 590 (1984)].
- ²⁷ P. Hermann, K.-A. Steinhauser, R. Cahier, and A. Steyerl, Phys. Rev. Lett. 54, 1969 (1985).
- ²⁸S. S. Arzumanov, S. V. Masalovich, A. N. Strepetov, and A. I. Frank, Pis'ma Zh. Eksp. Teor. Fiz. 44, 213 (1986) [JETP Lett. 44, 271 (1986)
- ²⁹ A. Steyerl and S. S. Malik, Physical B + C 137, 270 (1986).
- ³⁰C. R. Burch, Proc. Phys. Soc. London 59, 41, 47 (1947).
- ³¹ M. Born and E. Wolf, *Principles of Optics*, Pergamon Press, New York, 1965 [Russ. transl., Nauka, M., 1970].
- ³² A. Steyerl, W. Drexel, T. Ebisawa, E. Gustmiedl, K.-A. Steinhauser, R. Gahler, W. Mampe, and P. Ageron, Rev. Appl. Phys. 23, 171 (1988); Some Problems in Modern Nuclear Physics. Dedicated to the Eightieth Birthday of I. M. Frank [in Russian], Nauka, M., 1989, p. 4.
- ³³S. V. Masalovich, VANT. Ser. Obshchaya i yadernaya fizika, No. 3(36), 69 (1986).
- ³⁴ I. A. Karaseva, S. V. Masalovich, M. M. Rusinov, A. N. Strepetov, and A. I. Frank, "Neutron physics" in Proceedings of the International Conference on Neutron Physics, Kiev, 1987, TsNII atominform, Moscow, 1988, Vol. 1, p. 216.
- ³⁵S. S. Arzumanov, I. A. Karaseva, Yu. V. Kudryashov, S. V. Masalovich, M. M. Rusinov, A. N. Strepetov, and A. I. Frank, Horizontal neutron microscope," Preprint IAE-4968/14, Institute of Atomic Energy, Moscow, 1989.
- ³⁶ T. A. Arkhipova, A. M. Mechetin, M. T. Pakhomov et al., "Coordinatesensitive ultracold neutron detector for a neutron microscope," Preprint IAE-5009/14, Institute of Atomic Energy, Moscow, 1990.
- ³⁷S. S. Arzumanov, S. S. Masalovich, A. A. Sabel'nikov, A. N. Strepetov, and A. I. Frank, Pis'ma Zh. Eksp. Teor. Fiz. 52, 981 (1990) [JETP Lett. 52, 369 (1990)].
- ³⁸ A. Steyerl, W. Drexell, S. S. Malik, and E. Gudsmiedl, Physica B 151, 36 (1988).

¹⁾ A. Steyerl and G. Shutz¹⁵ also employed Fermat's principle for analysis of the "zone mirror" which they proposed for UCN.

²⁾ In this work a two-mirror optical objective, which the authors call a Schwarzschild objective, was employed. Such an objective was first employed for a standard microscope by Burch,³⁰ and it is described by Born and Wolf in their book of Ref. 31. In 1974, I. M. spoke at a seminar at the Institute of Atomic Energy. In this talk he proposed, in particular,

- ³⁹ Yu. V. Borisov, N. V. Borovikova, A. V. Vasil'ev et al., Zh. Tekh. Fiz. 58, 951 (1988) [Sov. Phys. Tech. Phys. 33(5), 574 (1988)].
- ⁴⁰ J. Bauman, R. Gahler, J. Kalus, and W. Mampe, Phys. Rev. D 37, 3107 (1988).
- ⁴¹I. Bialinicki-Birula and J. Mycielsky, Ann. Phys. (N.Y.) 100, 62 (1976); Phys. Scr. 20, 539 (1979).
- ⁴² R. Gahler, A. G. Klein, and Z. Zeilinger, Phys. Rev. A 23, 1611 (1981).
- ⁴³J. Kamersberger and A. Zeilinger, Physica B 151, 193 (1988).
- ⁴⁴ A. Steyerl, S. S. Malik, K.-A. Steinhauser, and L. Berger, Z. Phys. B 36, 109 (1979).
- ⁴⁵ A. I. Ioffe, Nucl. Instrum. Methods A **268**, 169 (1988).
- ⁴⁶ K. Eder, M. Gruber, and A. Zeilinger, Nucl. Instrum. Methods A 284, 171 (1989).
- ⁴⁷ G. Drabkin, A. Ioffe, S. Kirsanov et al., Nucl. Instrum. Methods A 284, 176 (1989).
- ⁴⁸ U. Bonse and A. Rumph, Phys. Rev. Lett. 56, 2441 (1986).
- ⁴⁹ V. I. Luschicov, Phys. Today **30**, 42 (1977).
- ⁵⁰ V. I. Lushchikov and A. I. Frank, Pis'ma Zh. Eksp. Teor. Fiz. 28, 607 (1978) [JETP Lett. 28, 559 (1978)].
- ⁵¹ M. V. Berry, J. Phys. A 15, L385 (1982).
- ⁵² R. Colella, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. 34, 1472 (1975).

- ⁵³S. A. Werner, H. Kaiser, M. Arif, and R. Clothier, Nucl. Instrum. Methods A 268, 22 (1988).
- ⁵⁴ V. G. Baryshevskiĭ, S. V. Cherepitsa, and A. I. Frank, Phys. Lett. A 153, 299 (1991); Preprint IAE-4887/14, Institute of Atomic Energy, TsNIIatominform, Moscow, 1989.
- ⁵⁵ A. I. Frank, VANT. Ser. Obshchaya i yadernaya fizika, No. 3(36), 69 (1986).
- ⁵⁶ F. Mezei, Z. Phys. 255, 146 (1972).
- ⁵⁷ A. I. Frank in Collection of Lectures at the 5th School on Neutron Physics, Alushta, 1986, Joint Institute of Nuclear Research, Dubna, 1987, p. 192.
- ⁵⁸ U. Bonse and T. Wroblewsky, Phys. Rev. D 30, 1214 (1984).
- ⁵⁹ M. Moshinsky, Phys. Rev. D 88, 625 (1952)
- ⁶⁰ A. S. Gerasimov and M. V. Kazarnovskii, Zh. Eksp. Teor. Fiz. 71, 1700 (1976) [Sov. Phys. JETP 44(5), 892 (1976)].
- ⁶¹ R. Gahler and R. Golub, Z. Phys. B 56, 5 (1984).
- ⁶² J. Felber, R. Gahler, and R. Golub, Physica B 151, 135 (1988).
- ⁶³ V. G. Nosov and A. I. Frank, J. Moscow Phys. Soc. 1, 1 (1990).

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