Holographic methods for regulating the sensitivity of interference measurements for transparent media diagnostics

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This is a review of holographic methods which enable one to regulate the sensitivity of interference measurements. Methods for increasing measurement sensitivity by using waves reconstructed from a hologram in higher diffraction orders and from a rerecording of a hologram, and also methods for regulating sensitivity that are based on recording holograms in two wavelengths, are considered. A review is given of methods for increasing sensitivity during the multiple passage of a beam through a volume being investigated or a hologram. Electronic phase measurement methods which enable one to increase the threshold of measurement sensitivity are considered. A review of the use of holographic interferometry with regulated measurement sensitivity for transparent media diagnostics is given.

1. INTRODUCTION

Holographic methods for increasing the sensitivity of interference measurements are of great interest for transparent media diagnostics. Here one may include investigations of gas flows near models in wind tunnels and ballistic trails at low pressures, rarefied flows in shock tubes, checking accurate final measurements and small deviations from a plane, and interference spectroscopy. Studying them by the methods of ordinary two-beam interferometry does not appear to be possible. At the same time, one must reduce the measurement sensitivity in a number of cases, for example, in studying inhomogeneities with large gradients of the index of refraction.[1-6]

A review is given in the present paper of holographic methods which enable one to regulate the sensitivity of interferometric measurements. The possibility for *a posteriori* processing of wave fronts reconstructed from holograms is the basis of holographic methods for regulating sensitivity. For example, changing the deformation of a wave front reconstructed from a hologram enables one to regulate the shift of interference bands.

In measurement technology, one uses the concept of method sensitivity, the ratio of the changes of the recorded signal K that is created at the instrument's output to the change of the parameter being measured[3]

$$S = \frac{\Delta K}{\Delta \chi}.$$

In accordance with this definition, by holographic interference method sensitivity is meant the ratio of the deformation of the wave front reconstructed from a hologram ($\Delta\Phi$) to the deformation of the wave front which passed by the irradiated object ($\Delta\varphi$):

$$S_r = \frac{\Delta\Phi}{\Delta\varphi}.$$

Upon obtaining interferograms with the same tuning, for finite width bands

$$S_r = \frac{\Delta P(x, y)}{\Delta p(x, y)},$$

where $\Delta P(x,y)$ and $\Delta p(x,y)$ are the shifts of an interference band at the point of the interference field with the coordinates (x,y) that has been obtained 1) by reconstruction of wave fronts from a hologram, and 2) by interference of the wave front which passed by the object with the standard wave front, respectively. Upon tuning for an infinitely wide band, the sensitivity equals

$$S_r = \frac{M}{m}$$

where M and m are the number of interference bands in the object zone for the cases of interference for a wavefront reconstructed from a hologram and for a wavefront which directly passed by the object with the standard wave front, respectively.

Unique possibilities for increasing the sensitivity of interference analysis by compensating for the aberrations of an interferometer's optical elements appear by using holographic methods to obtain an interference pattern.

The threshold measurement sensitivity, which is determined by the minimum measurable phase difference ($\Delta\varphi_{\min}$) between the wave which passed by the irradiated object (the object wave) and the comparison (standard) wave which passes outside the object zone, is well known in classical interferometry. Upon obtaining interferograms with bands of finite width, the threshold sensitivity will be determined by the minimum measurable shift of an interference band. For the visual determination of the position of an interference band, $\Delta\varphi_{\min}$ is assumed equal to 0.2π.[1] One can improve the threshold sensitivity significantly by using electronic methods of processing.[4,5]

2. NONLINEAR RECORDING OF HOLOGRAMS

The principles of holography were first used to increase the interference band shift (i.e., to increase sensitivity) in Ref. 7. Here a hologram was recorded with one object and two reference beams set at the angles $\pm\alpha$ to the object beam. Two object waves with complex-conjugate phases propagate in the same direction during reconstruction of the wave fronts from the hologram. An interference pattern

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with doubled sensitivity is formed by the interference of these waves. However, the aberrations of the interferometer's optical system are also doubled in this method.

The method suggested in Ref. 8, which is based on the properties of a nonlinearly recorded hologram, turned out to be considerably more promising. This method is based on the increase of the wave front deformation in the higher diffraction orders.

In the usual applications of holography in order to obtain an undistorted image, one strives to achieve a linear dependence on exposure of the amplitude transmission coefficient for the hologram. However, in investigating phase objects, nonlinearity turns out to be very useful for increasing the sensitivity of interference measurements.

Let us consider the amplitude transmission (τ) of the hologram of a focused image recorded under nonlinear conditions:[6]

$$\tau(\mathbf{r}_1) \propto a_0 + a_1 \cos[(\mathbf{k}_0 - \mathbf{k}_r)\mathbf{r}_1 + \varepsilon] + a_2 \cos^2[(\mathbf{k}_0 - \mathbf{k}_r)\mathbf{r}_1 + \varepsilon] + \ldots + a_n \cos^n[(\mathbf{k}_0 - \mathbf{k}_r)\mathbf{r}_1 + \varepsilon] \ldots, \quad (2.1)$$

where a_n ($n = 0,1,2,\ldots$) are constant coefficients, ε is the phase change caused by the inhomogeneity being investigated, $\mathbf{k}_{0,r} = (2\pi/\lambda)(\cos a_{0,r}\mathbf{i} + \cos v_{0,r}\mathbf{j})$ are the components of the wave vectors for the object ($\mathbf{K}_0$) and reference ($\mathbf{K}_r$) waves on the plane of the hologram, and $\mathbf{r}_1 = x\mathbf{i} + y\mathbf{j}$ is the radius vector of the point with the coordinates (x,y) in the plane of the hologram.

If, at the reconstruction stage, such a hologram is illuminated by the wave

$$A_c = \exp(i\mathbf{K}_r\mathbf{r})$$

(where $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$), then the waves reconstructed from the hologram are described by the expression:

$$A_{rec}(\mathbf{r}) \propto \sum_{n=0}^{N} c_n \{\exp[i(n\mathbf{K}_0\mathbf{r} + n\varepsilon) + \exp[-i(n\mathbf{K}_0\mathbf{r} + n\varepsilon)]\}. \quad (2.2)$$

Expression (2.2) describes a harmonic series with wave front deformations that are multiples of ε. As has first been indicated in Ref. 8, one can use these harmonics to increase the small phase shifts ε. For example, if an nth order diffraction wave is superimposed on a plane wave of the same amplitude, then the resulting interference pattern is described by the expression (for tuning to a band of infinite width)

$$I_{rec} \propto 1 + \cos n\varepsilon. \quad (2.3)$$

It follows from Eq. (2.3) that the phase difference has been increased by n times with respect to the actual shift caused by the object under investigation. One may consider such an interferogram as obtained at a wavelength n times shorter than that which was used in recording the hologram, and the method itself as possessing a sensitivity increased by n times.

A modification of the method for increasing the phase difference was suggested in Ref. 9. In this method, a hologram whose transmission is described by Expression (2.1) is illuminated simultaneously by the two waves $\exp(i\mathbf{K}'\mathbf{r})$ and $\exp(-i\mathbf{K}'\mathbf{r})$, where $\mathbf{K}'$ is the wave vector whose component on the plane of the hologram is $\mathbf{k}' = n(\mathbf{k}_0 - \mathbf{k}_r)$. In this case, the two waves $\exp(in\varepsilon)$ and $\exp(-in\varepsilon)$ are reconstructed in the direction of the normal to the hologram; as a result of their interference, a pattern is formed with an increase of sensitivity by $2n$.

One can form two restoring waves which illuminate the hologram by means of a Mach–Zehnder type interferometer[10] or a diffraction grating.[11] The use of the nonlinear characteristics of the hologram enabled one to obtain a 14-fold sensitivity increase.[10] Another approach by recording two holograms is considered in Ref. 12.

Compensation for aberrations

In the methods for increasing the measurement sensitivity that are considered above, there is no compensation for phase distortions caused by the aberrations of the optical system that is used to record the holograms. Furthermore, all the errors which appeared because of imperfection of the optical elements are increased by the same factor. Therefore, there is no sense to increase sensitivity without compensating for optical aberrations.

A number of different methods for compensating for optical aberrations by using nonlinearly recorded holograms have been developed at present. A method, which essentially consists of recording on a hologram a wave distorted by the aberrations of the optical system but with the inhomogeneity being studied absent, has been suggested in Ref. 13. After photographic processing, one places the hologram in its original position, introduces the inhomogeneity being investigated, and illuminates them with a light beam. This beam diffracts on the first hologram in the direction of the complex conjugate of the original wave. The wave front deformations in the diffracted beam that are caused by the aberrations of the optical system are compensated for in this case. The diffracted object wave is recorded on the second hologram, which is subjected to nonlinear photographic processing. The hologram obtained in this manner is used to increase the sensitivity for the diffraction of the $\pm$ nth orders. A more convenient method for compensating for aberrations consists of obtaining a secondary hologram.[3] An interferogram obtained by the method of double exposure with tuning for close bands of finite width is called a secondary hologram. The double exposure enables one to obtain interferograms that are free of aberrations. A secondary hologram is recorded under nonlinear conditions.

In many experiments, it does not appear to be possible to record a double exposure hologram, for example, in investigating the development of a process with time. A series of holograms of the object is recorded in this case. One may use the method of two separated, optically conjugate holograms to compensate for the aberrations.[14,15,16] In addition to the object hologram, a reference hologram (a compensator hologram) is also exposed in this case under the same nonlinear conditions but without the object being investigated. These holograms are placed in optically conjugate planes of a telescope system with a magnification of one (see Fig. 1). The hologram compensator *1* is illuminated by a plane wave, and a number of waves are reconstructed from it. These waves are focused in the focal plane of the objective lens *2*, and the diaphragm *3* picks out the symmetric $\pm$ nth diffraction orders. The object hologram *5* is illuminated by two waves. The two waves $\exp(in\varepsilon)$ and $\exp(-in\varepsilon)$ are reconstructed from the hologram *5* along its normal; by their interference, an interference pattern is formed in which the aberrations do not distort the band pattern, and the sensitivity is increased

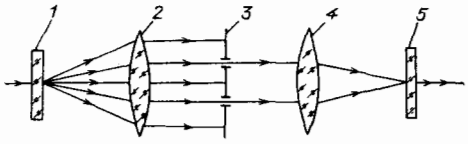

FIG. 1. An optical layout for achieving a method of two separated, optically conjugate holograms. *1*) the compensator hologram; *2*), *4*) objective lenses of the telescope system; *3*) diaphragm; *5*) the object hologram.

by $2n$. One must notice that the aberrations from the telescope optical system *2* and *4* are not compensated for. There are complications in obtaining an arbitrary tuning of the interference bands. These drawbacks can be removed by the spatial superposition of holograms *1* and *5* (see Fig. 1). In this case one records the object and reference holograms under nonlinear conditions with different band tunings. One should note that one can record holograms by the double exposure method on to a common photographic material.[17]

The distribution of the complex wave amplitude reconstructed in the nth diffraction order from the reference hologram has the form

$$A_n = \exp\,[i\,(n\mathbf{K}_0\mathbf{r} + n\varphi)],$$

where φ are the phase distortions of the wave front caused by the aberrations of the optical recording system. A wave with a complex amplitude A_n undergoes secondary diffraction by the object hologram. The distribution of the complex amplitude of a wave diffracted by the object hologram in the $-n$th diffraction order is determined by the expression

$$A_{-n,n} = \exp\,[i\,(n\mathbf{K}_r\mathbf{r} + n\varepsilon)],$$

where $\mathbf{K}_r$ is the wave vector of the reconstructed wave. The $A_{-n,n}$ wave is free of aberrations. In order that the $A_{-n,n}$ wave not overlap the other waves, the directions of the band grids of the object and reference holograms are set at some angle to each other. Two superposed holograms (or a double exposure hologram) are illuminated by two plane waves to obtain an interference pattern. If one selects the angle between these waves so that the $A_{n,0}$ and $A_{0,n}$ waves reconstructed in the nth diffraction order for the reference and object holograms propagate in the same direction, then an interference pattern with an n-fold sensitivity increase and which compensates for aberrations is formed. If one selects the angle between the two beams so that the $A_{-n,n}$ and $A_{n,-n}$ waves propagate in the same direction, then an interference pattern with a $2n$-fold sensitivity increase and which compensates for aberrations is formed.

For a double exposure of the holograms, the direction of the bands of the holographic grids may be the same. However, the frequencies of the bands from the object and reference holograms must differ significantly in this case. Reconstruction of the wave fronts and superposition in space of the object and standard waves of the nth diffraction order is accomplished by varying the angle between the two beams illuminating the hologram.[18]

In the two exposure method, one can also make the "carrier" frequency of the reference hologram n times ($n = 2,3,\ldots$) greater than the "carrier" frequency of the object hologram and illuminate the hologram with one beam.[19] The drawback of this method is that it is impossible to obtain an arbitrary tuning of the interference bands.

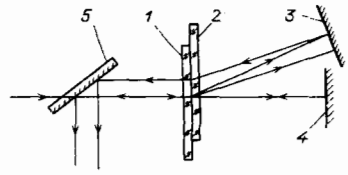

FIG. 2. An optical layout for processing superposed holograms. *1*), *2*) the reference and object holograms; *3*), *4*) mirrors; *5*) a semi-transparent plate.

Compensation for aberrations and arbitrary tuning of the interference bands can be realized by using the scheme for processing superposed holograms shown in Fig. 2.[20] The carrier frequencies for the reference *1* and object *2* holograms equal $\nu_1 = \sin\alpha_1/\lambda_0$ and $\nu_2 = \sin\alpha_2/\lambda_0$, respectively, where λ_0 is the wavelength of the source used to record the holograms. A wave is propagated from the reference hologram *1* at the angle $n\alpha_1$ in the nth diffraction order with recording system aberrations that equal $n\psi$, passes through the hologram *2*, and is returned back at the angle $n\alpha_2$ by the mirror *3*. For diffraction of the wave by the hologram *2* in the nth order, the wave will be reconstructed free of the recording system aberrations. This wave interferes with the wave which passed directly through holograms *1* and *2* and has been returned back by the mirror *4*. One tunes for finite bands by tilting mirror *4*. The interfering waves are brought out of the processing system by the semi-transparent plate *5* to increase the contrast of the interference bands.

3. RERECORDING HOLOGRAMS

The increase of sensitivity by using the higher diffraction orders for a hologram is practically limited to the $\pm$ 7-th diffraction orders. The limitation is associated with the decreasing intensity and increasing noise in the higher diffraction orders.

A method of successive interferogram rerecording based on the filtering of the $\pm$ first diffraction orders was suggested in Ref. 21. Here an increase of sensitivity by a factor of eight was attained, but without compensation for aberrations. A more general method for rerecording holograms by using the $\pm$ nth diffraction orders of light (see Fig. 3) and with compensation for the aberrations of the optical system for recording and rerecording holograms was suggested in Ref. 22. The original object hologram *1* is illuminated by a plane wave. Waves of the $\pm$ nth diffraction orders, which are focused by the objective lens *2* into the plane of the diaphragm *3* with two openings which pick out these two orders, are reconstructed from the hologram at the angles $\pm\alpha$. A secondary hologram is recorded in the plane *5* by interference of the complex conjugate waves

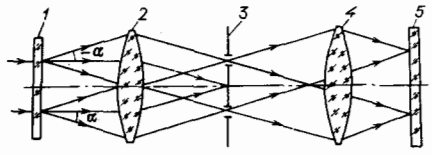

FIG. 3. An optical layout for rerecording holograms with one light beam. *1*) the original hologram; *2*), *4*) objective lenses of the telescope system; *3*) diaphragm; *5*) the secondary hologram.

$$A_{\sec} = \exp\,[in\,(\mathbf{K}_0\mathbf{r} + \varepsilon + \varphi)],$$
$$A^*_{\sec} = \exp\,[-in\,(\mathbf{K}_0\mathbf{r} + \varepsilon + \varphi)].$$

The intensity distribution on the secondary hologram has the form

$$I(\mathbf{r}_1) \propto 1 + \cos\,(2n\mathbf{k}_0\mathbf{r}_1 + 2n\varepsilon + 2n\varphi). \qquad (3.1)$$

If one installs this secondary hologram at the position *1* of the primary hologram (see Fig. 3) and again illuminates it with a plane wave, then two waves will be reconstructed from this hologram at the angles $\pm 2\alpha$:

$$A_{\sec 1} = \exp\,[i2n\,(\mathbf{K}_0\mathbf{r} + \varepsilon + \varphi)],$$
$$A^*_{\sec 1} = \exp\,[-i2n\,(\mathbf{K}_0\mathbf{r} + \varepsilon + \varphi)].$$

For these complex conjugate waves, there are the phase changes $2n\varepsilon$ caused by the object, and $2n\varphi$ from aberration. A new hologram is formed by the interference of these waves in the plane *5*. If one repeats this process N times, then the amplitude transmission coefficient of the last hologram has the form

$$\tau_0(\mathbf{r}_1) \propto 1 + \cos\,[(2n)^N(\mathbf{k}_0\mathbf{r}_1) + (2n)^N\varepsilon + (2n)^N\varphi]. \qquad (3.2)$$

In an analogous manner, if a reference hologram is rerecorded N times, its amplitude transmission coefficient equals:

$$\tau_r(\mathbf{r}_1) \propto 1 + \cos\,[(2n)^N(\mathbf{k}_0\mathbf{r}_1) + (2n)^N\varphi]. \qquad (3.3)$$

The object and reference holograms are spatially superposed and illuminated by one or two beams. The intensity distribution in the resulting interference pattern has the form:

$$I(\mathbf{r}_1) \propto 1 + \cos\,[(2n)^N(\mathbf{k}_0 - \mathbf{k}_0)\mathbf{r}_1 + (2n)^N\varepsilon]. \qquad (3.4)$$

Expression (3.4) describes an interferogram with a sensitivity increase by a factor of $(2n)^N$ and with compensation for the aberrations of the optical system for recording and rerecording holograms. After a definite number of rerecording cycles, the frequency of bands on the secondary holograms increases, and the objective lens *4* (see Fig. 3) vignettes the diffraction orders. This factor limits the number of rerecording cycles and, consequently, limits the possibilities for increasing sensitivity. The author of Ref. 22 increased the sensitivity by 24 times with compensation for aberrations. This limitation is removed if the hologram *1* (see Fig. 4) is illuminated by two plane waves which propagate at such angles with respect to each other that the $\pm$ nth diffraction orders pass through the diaphragm *3* at the angles $\pm\alpha/2$. In this case the secondary hologram *5* is recorded with the same band carrier frequency as the primary hologram. After N rerecording cycles, the wave front deformation increases by $(2n)^N$ times for a constant band carrier frequency at the hologram. The sensitivity of interference measurements was increased by a factor of 512 by using this method for processing spectroscopic holograms.[23] A similar method was used in Ref. 24 to process holograms and a 100-fold increase of phase difference was obtained.

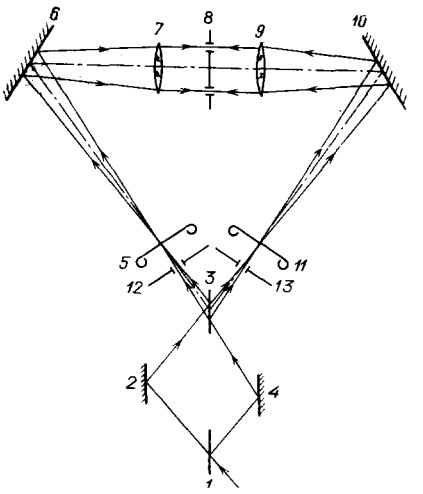

FIG. 5. An optical layout for rapid rerecording of holograms. *1*), *3*) beam splitters; *2*), *4*), *6*), and *10*) mirrors; *5*), *11*) the holograms; *12*), *13*) shutters; *7*), *9*) objective lenses of the telescope system; *8*) diaphragm.

The fundamental optical layout for fast rerecording of holograms illuminated by two beams is shown in Fig. 5.[25,26] In this layout, two plane waves alternately illuminate the holograms *5* and *11*. If the primary hologram is placed in plane *5*, then the shutter *12* is open and shutter *13* is closed. In this case, a secondary hologram is formed in plane *11* by the interference of the $\pm$ nth diffraction orders of light at the hologram *5*. Next, the primary hologram is removed (or erased), shutter *13* is opened and shutter *12* is closed, and the secondary hologram *11* is illuminated by the same two beams which were used to illuminate the primary hologram. A tertiary hologram is formed in plane *5*, and the wave front deformation recorded on the hologram is increased. Rerecordings of the object and reference holograms are made in sequence, retaining their exact mutual positions with respect to the optical rerecording system.

One can make rerecordings of holograms by illuminating them with two beams by using sources of coherent radiation, lasers. However, in this case, coherent phase noise increases rapidly in rerecording. A method of rerecording holograms by using an incoherent light source has been suggested to reduce this noise (see Fig. 6).[27] The object hologram *1* is illuminated by a single beam from an incoherent light source. A secondary hologram is recorded in plane *5*. Here the $\pm$ nth diffraction orders of light at the hologram *1* are transmitted by the openings (a) and (d) of the diaphragm *3*. The carrier frequency increases by a factor of $2n$ at the secondary hologram *5*. The secondary hologram *5* is illuminated from the opposite side by two coherent beams so

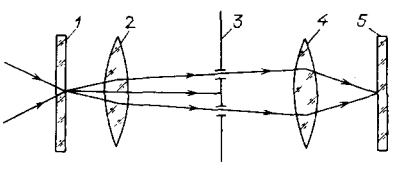

FIG. 4. An optical layout for rerecording holograms with two coherent light beams. *1*) the original hologram; *2*), *4*) objective lenses of the telescope system; *3*) diaphragm; *5*) the secondary hologram.

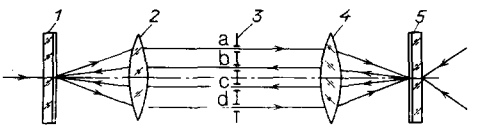

FIG. 6. An optical layout for rerecording holograms by using one incoherent beam and two coherent beams. *1*), *5*) holograms; *2*), *4*) objective lenses of the telescope system; *3*) diaphragm.

that the $+n$th diffraction order of one beam passes through the opening (b) and the $-n$th diffraction order passes through the opening (c) of the diaphragm *3*. The distance between the openings (b) and (c) is selected to be $2n$ times less than the distance between openings (a) and (d). In this case the primary hologram *1* is erased, and a tertiary hologram with a carrier frequency equal to the carrier frequency of the original hologram is recorded in its place. The hologram rerecording process is repeated until the necessary sensitivity increase is obtained.

An experimental investigation of the factors limiting the possibilities for increasing sensitivity in rerecording holograms with one beam of light from an incoherent and two beams of light from a coherent source has been conducted in Ref. 28. A KGM 12–100 quartz halogen incandescent lamp was used as the incoherent light source, and a LG-38 helium-neon laser was used as a coherent source. The requirements for accuracy in superposing the reference and object holograms increase with increasing sensitivity. For an aberration magnitude from 50λ to 100λ, the holograms must be superposed with from 3 μm to 5 μm accuracy. Failure of the requirements for accurate superposition of the holograms, and also settling of the photographic emulsion during drying the holograms lead to residual aberrations which distort the interference pattern. These factors also limit the possibilities for increasing measurement sensitivity. For an increase of sensitivity by a factor of 96, the residual wave aberrations did not exceed 0.5λ, and for an increase of sensitivity by a factor of 192, the residual wave aberrations were already 2λ.

4. THE TWO-WAVELENGTH METHOD

Let us consider a method which enables one to change the sensitivity of interference measurements. The method is based on recording holograms in two wavelengths (see Fig. 7a).[29-32] One can use the two-wavelength exposure method to obtain interferograms with either reduced or increased sensitivity. Either two lines from one laser or two different lasers are used to form the object and reference beams (containing light of two wavelengths λ_1 and λ_2). The amplitude transmission of such a hologram has the form[6]

$$\tau(\mathbf{r}_1) \propto 2 + \cos[(\mathbf{k}_{01} - \mathbf{k}_{r1})\mathbf{r}_1 + \varepsilon_1] + \cos[(\mathbf{k}_{02} - \mathbf{k}_{r2})\mathbf{r}_1 + \varepsilon_2], \tag{4.1}$$

where

$$\mathbf{k}_{01,2} = 2\pi/\lambda_{1,2}(\cos v_0 \cdot \mathbf{i} + \cos u_0 \cdot \mathbf{j})$$

are the components on the plane of the hologram of the wave vectors of the object and reference waves for the wavelengths λ_1 and λ_2:

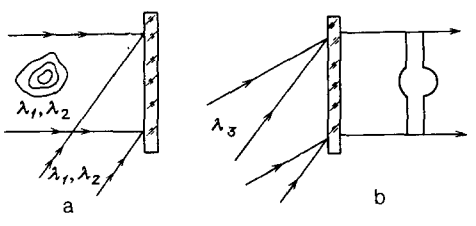

FIG. 7. An optical layout for recording holograms by using a) radiation with two wavelengths, and b) reconstruction of wave fronts.

$$\varepsilon_{1,2} = 2\pi\Delta R/\lambda_{1,2}, \tag{4.2}$$

$$\Delta R = \int_0^l (n(x, y, z) - n_0)\,dz$$

is the optical path difference caused by the inhomogeneity being investigated. We assume that the medium does not possess dispersion.

At the stage of reconstruction (Fig. 7b), the hologram is simultaneously illuminated by two beams with wavelength λ_3. The amplitudes of the restoring waves equal $A_{c1} = \exp(i\mathbf{K}_{1c}\mathbf{r})$ and $A_{2c} = \exp(i\mathbf{K}_{2c}\mathbf{r})$. If $\mathbf{K}_{1c}$ and $\mathbf{K}_{2c}$ are selected so that their components on the plane of the hologram equal $\mathbf{K}_{1c} = -(\mathbf{k}_{01} - \mathbf{k}_{r1})$ and $\mathbf{k}_{rc} = -(\mathbf{k}_{02} - \mathbf{k}_{r2})$, then the two waves $\exp(i\varepsilon_1)$ and $\exp(i\varepsilon_2)$ are reconstructed along the normal to the hologram. During the interference of these waves, the intensity distribution is

$$I_{rec} \propto 1 + \cos(\varepsilon_2 - \varepsilon_1). \tag{4.3}$$

Using Relation (4.2), let us determine

$$\varepsilon_2 - \varepsilon_1 = \frac{2\pi}{\lambda_{eff}}\Delta R, \tag{4.4}$$

where $\lambda_{eff} = \lambda_1\lambda_2/(\lambda_1 - \lambda_2)$. Thus, the observed interferogram is identical to one obtained for the interaction of the object and reference waves at a wavelength equal to λ_{eff}. Let us introduce the concept of the coefficient of sensitivity change[3]

$$M_r = \frac{\lambda_1}{\lambda_{eff}} = \frac{\lambda_1 - \lambda_2}{\lambda_2}. \tag{4.5}$$

If the amplitudes of the restoring waves equal $A_{c1} = \exp(i\mathbf{K}_{1c}\mathbf{r})$ and $A_{c2} = \exp(i\mathbf{K}_{3c}\mathbf{r})$, and moreover, the component of $\mathbf{K}_{3c}$ equals $\mathbf{k}_{3c} = \mathbf{k}_{02} - \mathbf{k}_{r2}$, then the two waves $\exp(i\varepsilon_1)$ and $\exp(-i\varepsilon_2)$ are reconstructed along the normal to the hologram, and during their interference the intensity distribution equals

$$I_{rec} \propto 1 + \cos(\varepsilon_1 + \varepsilon_2).' \tag{4.6}$$

In this case, the coefficient of sensitivity change is $M_H = (\lambda_1 + \lambda_2)/\lambda_2 > 1$. It is shown in Ref. 33 that one can obtain an even larger range of sensitivity change by using nonlinear hologram recording.

Let us illuminate a hologram that has been recorded in two wavelengths (see Fig. 4) with two beams so that one obtains in plane *5* a secondary hologram with the amplitude transmission

$$\tau \propto 1 + \cos(\mathbf{k}_r\mathbf{r}_1 + \varepsilon_1 - \varepsilon_2), \tag{4.7}$$

where $\mathbf{k}_r$ determines the carrier frequency for the bands of the secondary hologram. If one uses a method of successive rerecording in the $\pm n$th diffraction orders for this secondary hologram, then one can obtain a coefficient of sensitivity change equal to[6]

$$M_H = (2n)^N(\lambda_1 - \lambda_2)\lambda_2^{-1}.$$

Compensation for aberrations

One of the methods of compensating for the aberrations of the optical system consists of recording compensator holograms in two wavelengths.[6,34] A compensator hologram is recorded without the object being investigated and without rearranging the optical layout.

The amplitude transmission of the reference compensator hologram equals

$$\tau_1 \propto 2 + \cos[(\mathbf{k}_{01} - \mathbf{k}_{r1})\mathbf{r}_1 + \varphi_1] + \cos[(\mathbf{k}_{02} - \mathbf{k}_{r2})\mathbf{r}_1 + \varphi_2], \quad (4.8)$$

where $\psi_1 = 2\pi\Delta R_a/\lambda_1$, $\psi_2 = 2\pi\omega_a/\lambda_2$, and ΔR_a is the path difference caused by imperfection of the optical elements. The amplitude transmission of the object hologram has the form

$$\tau_2 \propto 2 + \cos[(\mathbf{k}_{01} - \mathbf{k}_{r1})\mathbf{r}_1 + \varepsilon_1 + \varphi_1] + \cos[(\mathbf{k}_{02} - \mathbf{k}_{r2})\mathbf{r}_1 + \varepsilon_2 + \varphi_2]. \quad (4.9)$$

Let us illuminate a compensator hologram with a plane wave. Two waves are reconstructed in the first diffraction order: $\exp\{i[(\mathbf{K}_{01} - \mathbf{K}_{r1})\mathbf{r} + \varphi_1]\}$ and $\exp\{i[(\mathbf{K}_{02} - \mathbf{K}_{r2})\mathbf{r} + \varphi_2]\}$. If one picks out these two waves by means of spatial filtering and records a secondary hologram in a plane that is optically conjugate with the original hologram, then the transmission of such a hologram has the form

$$\tau_3 \propto 1 + \cos(\mathbf{k}_\Sigma \mathbf{r}_1 + \varphi_2 - \varphi_1) \quad (4.10)$$

where

$$\mathbf{k}_\Sigma = (\mathbf{k}_{01} - \mathbf{k}_{r1}) - (\mathbf{k}_{02} - \mathbf{k}_{12}).$$

If one rerecords a secondary object hologram in a similar way, then its transmission is

$$\tau_4 \propto 1 + \cos(\mathbf{k}_\Sigma \mathbf{r}_1 + \varepsilon_2 - \varepsilon_1 + \varphi_2 - \varphi_1). \quad (4.11)$$

The object and compensating holograms are either superposed or placed into optically conjugate planes in the stage of obtaining the resulting interference pattern. In the method of superposed holograms, the object wave, $\exp[i(\mathbf{K}_\Sigma \mathbf{r} + \varepsilon_2 - \varepsilon_1 + \varphi_2 - \varphi_1)]$ interferes with the wave reconstructed from the compensator hologram, $\exp[i(\mathbf{K}_\Sigma \mathbf{r} + \varphi_2 - \varphi_1)]$ with the intensity distribution

$$I_{rec} \propto 1 + \cos(\varepsilon_2 - \varepsilon_1). \quad (4.12)$$

Thus, the phase distortions $(\varphi_2 - \varphi_1)$ are compensated for.

Another method of compensating for aberrations consists of an arrangement of the primary object and compensating holograms that have been recorded simultaneously at the wavelengths λ_1 and λ_2 in the optically conjugate planes of lenses *2* and *4* of the telescope system in Fig. 1, and moreover, the holograms have been recorded under nonlinear conditions and hologram *1* has been illuminated by two coherent beams. By means of a filtering diaphragm, one can, for example, pick out the waves $\exp\{i[n(\mathbf{K}_{01} - \mathbf{K}_{r1})\mathbf{r} + n\varphi_1]\}$ and $\exp\{\pm i[m'(\mathbf{K}_{01} - \mathbf{K}_{r1})\mathbf{r} + m'\varphi_2]\}$, where n and m' are the diffraction order numbers. These waves illuminate the object hologram *5*, from which two waves, $\exp(in\varepsilon_1)$ and $\exp(\pm im'\varepsilon_2)$, are reconstructed along the normal, and by their interference an interferogram is formed with the intensity distribution

$$I_{rec} \propto 1 + \cos(n\varepsilon_1 \mp m'\varepsilon_2). \quad (4.13)$$

Expression (4.13) shows that one can regulate the measurement sensitivity within wide limits by picking out suitable diffraction orders and with compensation for optical system aberrations and arbitrary tuning of the interference bands.

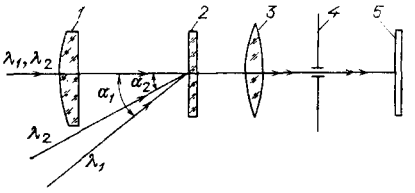

FIG. 8. An optical layout for an interferometer to check optical components at two wavelengths in real time. *1*) the component being checked; *2*) hologram; *3*) objective lens; *4*) diaphragm; *5*) screen.

Residual aberrations caused by elements *2* and *4* of the optical system are a drawback of such a method.

The method of double exposure (with the object and without it) on an ordinary photographic recorder enables one to get rid of this drawback.[6] The angle between the object and reference beams is changed significantly before the second exposure. The hologram is simultaneously illuminated by two plane waves at the reconstruction stage (see Fig. 7b). By turning one of the light beams illuminating the hologram, one can superpose the following waves in the nth diffraction order ($n = 1,2,\ldots$):

1) $\exp[in(\varepsilon_1 + \varphi_1)]$ and $\exp(in\varphi_1)$;
2) $\exp[in(\varepsilon_2 - \varphi_2)]$ and $\exp(in\varphi_2)$;
3) $\exp[-in(\varepsilon_{1,2} + \varphi_{1,2})]$ and $\exp[in(\varepsilon_{1,2} + \varphi_{1,2})]$;

and

4) $\exp[in(\varepsilon_2 + \varphi_2)]$ and $\exp[in(\varepsilon_1 + \varphi_1)]$.

Interference patterns with changeable sensitivity are obtained for interference of the indicated pairs of waves. Variants 1) and 2) correspond to ordinary interferograms recorded at wavelengths λ_1 and λ_2. Variant 3) has an increase of sensitivity by $2n$ times. The interference patterns of Variant 4) have coefficients of sensitivity change $M_{H3} = n(\lambda_2 - \lambda_1)/\lambda_2$, but with the residual aberrations $n(\varphi_2 - \varphi_1)$. If one uses waves which underwent double refraction[17], $A_{-n\lambda_1, n\lambda_1} = \exp(in\varepsilon_1)$ and $A_{-m\lambda_2, m\lambda_2} = \exp(in\varepsilon_2)$, then one can obtain an interference pattern with coefficients of sensitivity change $M_{H3} = n(\lambda_2 - \lambda_1)/\lambda_2$, but with compensation for aberrations.

Holographic interferometry in two wavelengths can be used to check optical components in real time[35] (see Fig. 8). Ordinary interferometry may be unsuitable because of the large phase difference introduced by the component being checked. In this case the component being checked *1* is illuminated by radiation with wavelength λ_1, and the hologram is recorded in the plane *2*. The band carrier frequency at the hologram is $\nu = \sin\alpha_1/\lambda_1$; α_1 is the angle between the object and reference waves (wavelength λ_1). This same component is illuminated at the wavelength λ_2 in the second stage, and the angle between the object and reference beams changes to the value $\alpha_2 = \arcsin(\lambda_2 \sin\alpha_1/\lambda_1)$. The two waves

$$\exp\frac{i2\pi\Delta R}{\lambda_1}, \quad \exp\frac{i2\pi\Delta R}{\lambda_2}$$

are reconstructed along the normal to the hologram in this case. An interference pattern is formed in the plane *5* which is identical to the interferogram that is obtained by illuminating the component at the wavelength

$$\lambda_{eff} = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2}. \quad (4.14)$$

It is convenient to use photochromes and thermal plastics as the photographic material.[36]

5. MULTIPLE PATH INTERFEROMETRY

Classical interferometry with multiple passages of the beam through the object being studied enables one to increase the sensitivity of the interference measurements and is very useful in checking optical components.[5] The main drawback of classical interferometry is the lack of compensation for the aberrations of the optical system of a multiple passage interferometer.

A multiple path holographic interferometer enables one to increase measurement sensitivity and to compensate for the aberrations of the optical elements.[37,38] According to the method explained in Ref. 37, the inhomogeneity being investigated is placed between two semi-transparent mirrors set parallel to each other and separated by a distance L. The object light beam, which is successively reflected from the semi-transparent mirrors, illuminates the inhomogeneity 1, 3, 5,... times, respectively. The path difference between the object and reference beams equals 0, $2L$, $4L$,..., respectively. If $2L > l_{co}$, where l_{co} is the coherence length of the laser radiation, then only the first of them forms a hologram by interference with the reference beam. If the initial path difference between the reference and object beams is selected to equal $2L$, then a hologram is recorded by the interference of the reference beam with the object beam which illuminated the inhomogeneity three times. If an ordinary double exposure hologram is recorded by means of such a system, then a threefold sensitivity increase takes place; similarly, one can obtain sensitivity increases of 5, 7,... times.

If one records a single exposure hologram under nonlinear conditions during multiple illuminations of the object, then one can obtain a sensitivity increase of $2nm'$ times by using the $\pm$ nth diffraction orders, where m' is the number of passages of the beam through the object.[39] A further sensitivity increase is possible if one rerecords the object hologram many times in the $\pm$ nth diffraction orders. One can obtain a sensitivity increase of $m'(2n)^N$ times in this case.[6]

Another method for picking out a wave which has repeatedly passed through a zone being investigated lies in the fact that waves illuminate an object at small angles to each other. This leads to a lateral shift of the image of the light source in the focal plane of the receiving objective lens. This enables one to select, by means of spatial filtering, a wave which has passed through the object a specified number of times and, after superposing the reference wave, to record an interference pattern.[38,40] Repeated illuminations and spatial filtering of the object wave can be achieved by means of an optical system of two partially reflecting mirrors arranged at small angles to each other. This method enables one to obtain interferograms with significantly higher band contrast than by using a coherent reference wave; however, errors connected with the lateral shift of the beams can appear.[3]

It was suggested in Ref. 41 to illuminate repeatedly a hologram on which a wave front is "frozen" instead of the object being investigated. The hologram was placed between the mirrors so that a diffracted beam, after reflection from the mirrors, was returned back to the hologram. One of the layouts which enables one to increase the measurement sensitivity with compensation for the inhomogeneities of the film support of the hologram, is depicted in Fig. 9. The hologram *1* is placed between the semi-transparent mirror *2* and the mirrors *3* and *4*. Furthermore, mirrors *3* and *4* are arranged in the $\pm$ first diffraction orders. Such a layout enables one to form two waves (during multiple illuminations of the hologram) with complex conjugate phases and having the same distortions caused by the film support of the hologram. The beams which passed through mirror *2* are removed from the system by the semi-transparent plate *5*, are filtered by means of the objective lens *6* and the diaphragm *7*, and an interference pattern with increased sensitivity is observed in the plane *8*.

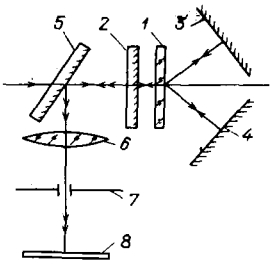

FIG. 9. An optical layout for a multiple pass holographic interferometer. *1*) hologram; *2*) semi-transparent mirrors; *3*), *4*) mirrors; *5*) semi-transparent plate; *6*) objective lens; *7*) diaphragm; *8*) screen.

6. AN ELECTRONIC PHASE MEASUREMENT METHOD

Increasing the threshold sensitivity and automating the measurement process are the present problems in processing holographic interferograms. The ordinary two-step process of obtaining interferograms by reconstructing wave fronts from a hologram assumes further processing of interferograms, just the same as in classical interferometry. Such a standard procedure does not allow one to use the advantages of holography in full measure. The electronic phase measurement method, which possesses high accuracy for measuring small deformations, was first used in holographic interferometry of diffusely reflecting objects.[42,43] In this method, which has been named the heterodyne method, two different reference beams are used to record a hologram. Two beams are also used to reconstruct the hologram; however, the frequency of one of the restoring waves is shifted by a small amount. In this case, the intensity in the reconstructed picture varies harmonically with time. For example, one can achieve the necessary frequency shift by means of a rotating diffraction grating,[44] with a half-wave plate,[45] or with an electrooptical modulator.[4]

A phase difference measurement with an accuracy of 0.004π was reported in Refs. 42 and 43. An argon ion laser was used in these papers, and a frequency shift amounting to about 100 Hz was introduced by means of a rotating grating. This system was used to determine deflections, deformations, and bending moments.

A heterodyne instrument for processing holograms, which enables one to study low density flows, since the threshold sensitivity of heterodyne processing is two to three orders of magnitude higher in comparison with ordinary methods of holographic interferometry, was suggested in Ref. 46. The hologram is recorded by means of two reference beams in a double exposure. In the instrument, a beam from a single frequency laser is split by a beam splitter and is shifted in frequency by means of acousto-optical modulators.

Two restoring beams illuminate the hologram at a definite angle by means of mirrors. The object A_0 and standard A_{st} waves are reconstructed from the hologram; their amplitudes have the form

$$A_0 \propto \exp[i(\mathbf{K}_0\mathbf{r} + \varepsilon + \varphi)] \exp(i\omega_0 t),$$
$$A_{st} \propto \exp[i(\mathbf{K}_{st}\mathbf{r} + \varphi)] \exp(i\omega_{st} t), \quad (6.1)$$

where ε and φ are the phase distributions caused by the object and the optical system aberrations, respectively, ω_0 and ω_{st} are the circular frequencies of the object and standard waves, and $\mathbf{K}_0$ and $\mathbf{K}_{st}$ are their wave vectors.

A pattern is formed by superposition of these waves; its intensity distribution has the form

$$I \propto 2\ \{1 + \cos[(\mathbf{k}_0 - \mathbf{k}_{st})\mathbf{r} + \varepsilon + (\omega_0 - \omega_{st})t]\}, \quad (6.2)$$

where $\mathbf{k}_0$ and $\mathbf{k}_{st}$ are the components of the vectors $\mathbf{K}_0$ and $\mathbf{K}_{st}$ on the plane of the hologram. Expression (6.2) describes the running interference pattern during tuning for bands of finite width. If one places a photo-detector in an undisturbed field, then it will produce an alternating current signal whose frequency equals $(\omega_0 - \omega_{st})$. If one places a second photo-detector at some point (x,y) in a field disturbed by the object, then it will also produce an alternating current signal of frequency $(\omega_0 - \omega_{st})$, but which is phase shifted by the amount $\varepsilon(x,y)$ in comparison with the signal of the first photo-detector. One can measure the phase distribution $\varepsilon(x,y)$ for the object by scanning the second photo-detector over the zone disturbed by the object. A dissector camera which is connected to an electronic computer fulfills the role of a scanning photo-detector. The electronic computer controls the process of scanning the image in the camera and stores the values of $\varepsilon(x,y)$ in its memory.

In addition to high threshold sensitivity for measurements of the phase $\varepsilon(x,y)$, one must count convenience and rapid automated processing among the merits of the instrument. Among the drawbacks are the lack of a visual check on the nature of the data flow. Besides, as follows from Relation (6.2), the phase difference of the signal that is measured by the reference and object photo-detectors is recorded with accuracy to $2q\pi$ ($q = \pm 1, \pm 2, \ldots$). This means that if there are sudden density changes (large λ) in the object, one must correct the values of $\varepsilon(x,y)$ that are obtained.

The use of two beams at the stages of recording and reconstructing holograms requires using complicated interferometers and processing facilities. Besides, the aberrations of the facility for reconstructing holograms are not compensated for. This limits the range of usefulness of the method.

A holographic heterodyne processing method not requiring the use of two beams and acousto-optical modulators and which enables one to compensate for the aberrations of the hologram processing facility has been suggested in Ref. 47. The essence of the suggested method is explained (see Fig. 10). The reference *2* and object *1* holograms are accurately superposed with each other and are illuminated by a plane wave. Filtering of the first diffraction order of light is done in the focal plane of the objective lens *4*. The superposition of the holograms is checked in the optically conjugate plane *7* by tuning to an infinitely wide interference band. One moves one of the holograms, for example the reference one, back and forth with some velocity v. Conversion of the light signal into an electrical one is accomplished in

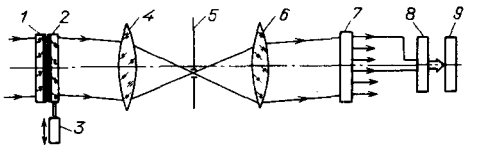

FIG. 10. An optical layout for heterodyne processing of two superposed holograms. *1*), *2*) the object and reference holograms; *3*) motor; *4*), *6*) objective lenses; *5*) diaphragm; *7*) photo-detector matrix; *8*) electronic system; *9*) electronic computer.

the plane *7* by means of a photo-detector, and the phase difference of the electrical signals is measured by means of the electronic system 8. If they are recorded on one holographic interferometer, the reference and object holograms have the same aberrations if they are accurately superposed, and their transmission coefficients are determined by the expressions

$$\tau_1 \propto 1 + \cos(2\pi\nu x + \varphi(x, y)), \quad (6.3)$$
$$\tau_2 \propto 1 + \cos(2\pi\nu x + \varphi(x, y) + \varepsilon(x, y)), \quad (6.4)$$

where x and y are coordinates in the plane of a hologram, one of whose axes is perpendicular to the lines of the hologram, and the other axis is parallel to them.

By illuminating the superposed holograms by a single reference beam and the interference of the waves reconstructed in the first diffraction order, the aberrations of the optical systems that are used both to record the holograms and also in the reconstruction process are compensated for. In the process of reconstructing the superposed holograms, one of which, for example, the reference one, has a velocity v at the moment of superposition, in plane *7* that is optically conjugate with the plane of the superposed holograms, the modulation of the radiation intensity in regions of the operational field that are undisturbed and disturbed by the inhomogeneity is determined by the expressions

$$I_{\text{und}} \propto 1 + \cos(2\pi\nu v t), \quad (6.5)$$
$$I_{\text{dis}} \propto 1 + \cos(2\pi\nu v t + \varepsilon). \quad (6.6)$$

Thus, the variation of intensity with time is accomplished with the frequency (νv). After converting the light signal into an electrical one, the phase difference ε between the two electrical signals corresponding to the disturbed and undisturbed regions is measured, and the aberrations φ are compensated for.

A similar processing method can be used to arrange the object and reference holograms in optically conjugate planes of the telescope system. One of the holograms is moved back and forth. This method combines the advantages of the method of increasing sensitivity due to the use of the nonlinear properties of holograms with an increase of threshold sensitivity determined by heterodyne processing.

A holographic interferometer with heterodyne processing, which enables one to investigate transparent media in real time, has been suggested in Ref. 48. The reference hologram is recorded in the output plane of the interferometer. After photographic processing, the hologram is illuminated by two beams, the reference and the object beam with the object being investigated inserted, and it is moved back and forth. A phase difference is measured in the output plane between electrical signals, one of which is produced by a fixed photo-detector placed in the zone that is undisturbed by the object, and the other signal is produced by a photo-

detector which scans the zone of the object. At high threshold sensitivity ($\varepsilon_{min} < 0.04\pi$), this method enables one to automate the process of measuring phase difference and to compensate for the aberrations of the interferometer in real time.

An interferometer with heterodyne processing, which uses the two-wavelength method of holography to reduce the measurement sensitivity in checking aspherical surfaces in real time has been suggested in Ref. 49. The reduction of sensitivity enables one to make the value of the measured phase difference $\varepsilon < 2\pi$ and to use heterodyne processing of interference patterns.

It is expedient to use heterodyne processing of optically conjugate holograms in studying fast flowing processes (see Fig. 1). In this case, the holograms *1* and *5* are recorded by means of the interferometer simultaneously at the wavelengths λ_1 and λ_2 (with the object and without it). One of the holograms is moved back and forth. This method enables one to combine the advantages of the two-wavelength method for regulating measurement sensitivity with heterodyne hologram processing.

7. APPLICATIONS

Holographic methods for increasing sensitivity enabled one to investigate low density gas flow. In particular, by using a method of nonlinear hologram recording, the authors of Ref. 50 succeeded in determining the position of the shock wave from a flying sphere on a ballistic track at a pressure of 5 mm Hg.

By developing these methods in Refs. 51 and 52, they determined not only the position of the shock wave, but also the density distribution through the entire shock layer. The holograms were recorded on a Mach-Zehnder interferometer. A method of separated, optically conjugate holograms was used to obtain interferograms. The interferograms were obtained by the interaction of waves of the $\pm$ first and $\pm$ second diffraction orders. The sensitivity increase was used to conduct research on the gas flow in a shock tube.[3] The holographic method of repeated illuminations was used especially effectively in research on low density gas flows.[3,53,54] It was used in research on the processes in a reflecting nozzle of a shock tube and on an aeroballistic track. In the holographic interferometer that is used in research on a ballistic track, the reference beam was formed from the object beam by means of a lateral shift interferometer. An increase of interferogram sensitivity by a factor of 22 was obtained by an eleven-fold illumination of the field under investigation and using the $\pm$ first diffraction orders; this enabled one to make a quantitative calculation of the density at the inflow pressure $p = 2$ mm Hg.

The holographic method for increasing the sensitivity of interference measurements based on repeated rerecording of object and reference holograms was also used to study the flow of a low density gas stream around a model on a ballistic track.[22] An interference pattern suitable for quantitative processing was obtained to increase the sensitivity by a factor of eight. A check on the plane-parallel nature of the ends of a thin rod was conducted in Ref. 55 by using the method of rerecording object and reference holograms. The diameter of the rod was several millimeters, and it did not appear to be possible to investigate the plane-parallel nature of its ends by the ordinary methods of interferometry or with goniometers. An interferogram obtained on a Mach–Zehnder type interferometer is shown in Fig. 11a. A reconstructed interferogram obtained by the superposition of rerecorded object and reference holograms, which increased sensitivity by a factor of 16 and enabled one to determine the nonplanarity of the ends of a rod being checked, is shown in Fig. 11b. The results of investigating the quality of manufacture of a transparent phase diffraction grating, which increased the sensitivity six-fold, are shown in this same paper. Rerecording of holograms was used to study the spatial waviness in a vitreous substrate.[56] Discrete illumination of the object at different angles was used in this paper. All the original waves which illuminated the waviness at different angles were reconstructed from the hologram. Rerecording of the holograms was done for a selected discrete direction of illumination according to a layout in the $\pm$ first diffraction orders.

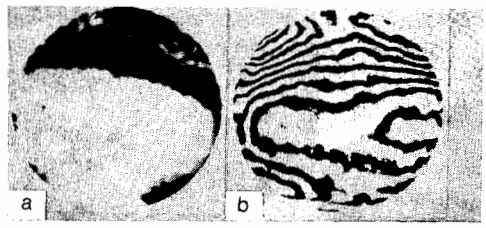

FIG. 11. Interferograms which characterize the plane-parallel nature of the ends of a thin rod with different measurement sensitivities and compensation for aberrations. a) Without an increase of sensitivity; b) with an increase of sensitivity by a factor of eight.

The origin and development of the acoustic waves which appeared during focusing of ruby laser radiation into a cell with a solution of cryptocyanine dye in ethyl alcohol have been investigated for the first time in Ref. 57 by the methods of holographic interferometry with increased sensitivity. The layout of the experimental facility included a pulsed ruby laser whose second harmonic pumped the dye laser. The radiation of this laser was used for probing. Part of the radiation of the first harmonic of the ruby laser was focused into the region near the wall of the cell with the ethyl alcohol solution of cryptocyanine placed in one of the arms of a Mach–Zehnder interferometer. The radiation from the dye laser was directed into a multi-mirror optical delay line. The holograms were recorded at different time delays between the activating and probing radiation pulses. The interferograms shown in Figs. 12a, 12b, and 12c, which have been

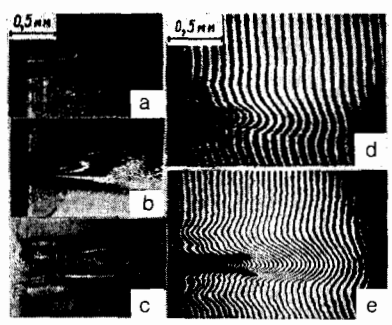

FIG. 12. Interferograms which show zones of change of the index of refraction in a dye solution. a, b, and c are tuned to a band of infinite width: a) is for $\tau_{del} = 0$; b) is for $\tau_{del} = 100$ nsec; c) is for $\tau_{del} = 200$ nsec. d and e are interferograms obtained from the same hologram ($\tau_{del} = 200$ nsec), d) without a sensitivity increase, and e) with a sensitivity increase by a factor of four.

obtained from holograms, show zones of change of the index of refraction in the dye solution with action of the ruby laser radiation on it for different delay times. There is no time delay for one interferogram (see Fig. 12a). The interferograms in Figs. 12b and 12c characterize the change of the index of refraction for delays of 100 nsec and 200 nsec, respectively. The interferograms of Figs. 12d and 12e have been obtained by rerecording a hologram ($\tau_{-\mathrm{del}} = 200$ nsec) and correspond to the sensitivity coefficients $K = 1$ and $K = 4$, respectively. The interferogram with $K = 4$ was used for quantitative processing and showed a change of the index of refraction of $\Delta n = -0.005$ in a heated region. As is evident from the interferograms in Figs. 12a through 12e, a cylindrical wave arises at the boundary separating the heated and the cold liquid regions; its propagation velocity is 1,500 m/sec.

Holographic two-wavelength interferometry with reduced sensitivity was used to check the quality of manufacture of the surfaces of optical components.[35,58-60] In the earlier stages of manufacture, a surface is not known with sufficient accuracy to make a check for corrections; the resulting interferogram contains too many bands, and it is impossible to analyze them. Technical difficulties arise in using a long wavelength source, which enables one to reduce the sensitivity. Two-wavelength holography provides the possibility of using visible light to obtain an interferogram that is identical to that which arises from a long wavelength source. As has been shown above, an interferogram obtained from a two-wavelength hologram will be identical to one which is recorded by using a source with the wavelength

$$\lambda_{\mathrm{eff}} = \frac{\lambda_1\lambda_2}{|\lambda_1 - \lambda_2|}.$$

By using a dye laser with changeable radiation wavelength, one can achieve a continuous series of λ_{eff}.[61] Two-wavelength holographic interferometry was used to check a polished glass mirror and a polished hyperbolic mirror coated with lacquer with $\lambda_{\mathrm{eff}} = 9.47\ \mu$m.[5]

The use of two-wavelength holographic interferometry to check aspherical mirrors was first suggested in Ref. 58. The shorter the spectral interval $(\lambda_1 - \lambda_2)$ is between the wavelengths λ_1 and λ_2 that are used to record the holograms, the larger is the decrease in the coefficient of sensitivity.

A dye laser with a dispersing resonator which includes a Fabry-Perot standard and a diffraction grating was used in Refs. 62 and 63. Rearranging the spectral interval between the wavelengths that are generated is made difficult and the radiation coherence length is short for a laser with such a resonator.

Two-frequency dye lasers with smoothly changeable wavelengths λ_1 and λ_2, which were used for two-wavelength holographic interferometry, have been developed in Refs. 64 and 65. Dispersing resonators containing diffraction gratings set at a grazing angle of incidence that enable one to obtain a half-width of smoothly changeable lasing lines from 0.002 nm to 0.01 nm were used in these lasers.

Crossing a sudden density change (identification of bands) is a serious problem in using a monochromatic light source to obtain holograms and interferograms of gas flows. It appears to be possible to solve this problem experimentally by means of a two-wavelength method for reducing the sensitivities of interferograms without resorting to numerical calculations. For this, one can select the wavelengths λ_1 and λ_2 so that, for a sensitivity decrease, the value of the sudden density change becomes smaller than λ. In this case, the sudden density change (in fractions of an interference band) is measured directly from an interferogram, and the true value of the sudden change is calculated by taking the sensitivity change coefficient into account.[6]

It was suggested in Ref. 23 to increase the sensitivity of interference measurements of the dispersion of the index of refraction in atomic media by rerecording spectroscopic holograms in the $\pm$ first diffraction orders. The spectroscopic holograms were recorded in the output plane of the spectrograph by using the radiation from a dye laser with an attached Michelson resonator. Two coherent beams of radiation from a helium-neon laser were used to rerecord a spectroscopic hologram of atomic sodium vapors. The original spectroscopic hologram was rerecorded eight times, which enabled one to obtain a record increase of sensitivity, by a factor of 512.

The holographic method of processing spectroscopic interferograms was used to increase the sensitivity of Rozhdestvenskii's hook method.[66] Spectroscopic interferograms near the second resonance doublet of rubidium were obtained for this purpose on a classical hook method setup. The frequency of the interference bands near the absorption lines being investigated was ~ 8 lines/mm. The spectroscopic interferograms obtained were rerecorded in the $\pm$ first diffraction orders for the purpose of increasing sensitivity. By increasing the sensitivity of the method by a factor of eight, interference "hooks" were found; upon processing them, a ratio of the oscillator strengths in the doublet has been found which agrees with the most reliable data. This paper showed the promise of the holographic method for studying the oscillator strengths of weak transitions.

The method of increasing sensitivity by rerecording spectroscopic holograms was used to study the nonlinear behavior of the dispersion of the index of refraction for neon atoms placed inside the Michelson resonator of a dye laser.[67] An unusual, asymmetric variation of the dispersion of the index of refraction for neon atoms near an absorption line ($\lambda = 594.5$ nm) caused by resonance interaction of the

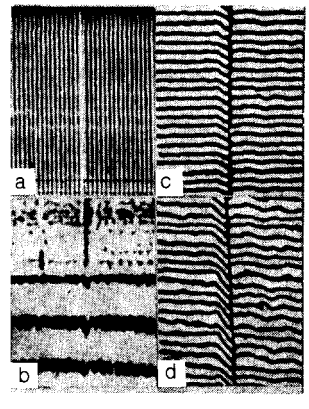

FIG. 13. Photographs obtained by *a posteriori* processing of a spectroscopic hologram (a is a photograph of the original spectroscopic hologram), and interferograms obtained by processing spectroscopic holograms (b is without a sensitivity increase; *c* and *d* are with sensitivity increases by factors of 8 and 16).

atoms with a strong light field was found by increasing sensitivity by a factor of 16. A photograph of the original spectroscopic hologram is shown in Fig. 13a, and photographs of the interferograms obtained by processing the spectroscopic holograms are shown in Figs. 13b, 13c, and 13d. Fig. 13b is with no sensitivity increase, and Figs. 13c and 13d are with sensitivity increases by factors of 8 and 16.

The spectra of the indices of refraction for barium atoms in a bichromatic pumping field and for sodium atoms in a resonance light field were investigated in Refs. 68 and 69 by processing spectroscopic holograms with increased interference measurement sensitivity.

The dispersion of the index of refraction at the intensifying transition of neon atoms ($\lambda = 632.8$ nm) was investigated with increased sensitivity in Ref. 70. An "inversion" of the dispersion curve with respect to the absorption transition was observed for the intensifying transition in the experiment. It is well known that an attempt to observe the phenomenon of negative dispersion was first made by Kopfermann and Ladenburg in 1928 and was unsuccessful because of the inadequate sensitivity of the classical method of interference spectroscopy.

Two methods were used simultaneously to increase the sensitivity of interference and holographic low density plasma diagnostics: 1) going into the infrared region, which is a selective method to increase the sensitivity for measuring the electron concentration, and 2) using higher order waves reconstructed by nonlinearly recorded holograms to obtain interferograms.[71] A method for increasing sensitivity based on rerecording the object and reference holograms recorded in the infrared region in the $\pm$ nth diffraction orders, which compensates for the aberrations of the optical systems for recording and rerecording holograms was used in Ref. 72 to investigate the plasma in a laser beam. This method is identical to the one suggested in Ref. 22, with the difference that the authors of Ref. 72 recorded the object and reference holograms on ordinary photographic material.

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