Consistent classical supergravity theories

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M. Muller. *Consistent Classical Supergravity Theories.* Springer-Verlag, Berlin; Heidelberg; New York; London; Paris; Tokyo; Hong Kong, 1989 pp. 125. (Lecture Notes in Physics. V. 336)

The book by M. Muller under review contains a systematic presentation of all the known theories of four-dimensional supergravity off the mass shell.

According to the theorem of Haag, Lopushanskiĭ and Sonius the algebra of the symmetries of the massive quantum field theory (the Poincaré algebra) can be expanded nontrivially only to the Poincaré superalgebra with N supersymmetries. This superalgebra contains the fermion symmetries, the generators of which are the N Majorana spinors Q^i_α (α is the spinor index).

Similarly, the conformal algebra (the algebra of the symmetries of a massless field theory) can be expanded up to the superconformal algebra with N supersymmetries which contains the Poincaré superalgebra as a subalgebra.

The theories which are symmetric with respect to local super-transformations are called supergravity theories. All such theories in accordance with their symmetry algebra can be divided into two classes: Poincaré supergravity and conformal supergravity. While the Poincaré supergravity is a supersymmetric expansion of Einstein's gravitational theory, the conformal supergravity is an expansion of the conformal gravity of Weil which contains terms with higher derivatives of the fields and a dimensionless coupling constant. Therefore the Poincaré supergravity is rather more suitable for a description of the actually observed world. However the theory of conformal supergravity has a larger symmetry group and is therefore simpler for theoretical investigations. The second reason leading us to investigate the conformal supergravity is the possibility of constructing a Poincaré supergravity by means of violating the superconformal algebra.

If the classical field theory is specified not by a Lagrangian, but by a system of equations of motion of physical fields, then it is said that it is formulated on the mass shell; in this case the transformations of the symmetry of the theory transform the solutions of the equations of motion into themselves. But if for a classical theory there exists such a set of auxiliary (nonphysical) fields that there exists a Lagrangian which is invariant under the transformations of the symmetry of the theory, then it is said that the theory is formulated off the mass shell. In this case symmetric equations of motion follow automatically from the Lagrangian for all fields (both the physical and auxiliary fields); a portion of these equations does not exhibit a dynamic character, but they enable one to express the auxiliary fields in terms of the physical fields. Thus, from the theory off the mass shell there arises a theory on the mass shell.

The existence of an invariant Lagrangian in the theory off the mass shell opens the way to the investigation of the quantum structure of the theory.

As a rule, formulation of a theory off the mass shell is associated with the establishment of the geometrical nature of the symmetry. In supergravity theories the geometrical source of the symmetries are the general coordinate transformations in superspace.

At the present time the following supergravity theories are known off the mass shell: conformal supergravities for $N = 1,2,3,4$ and the Poincaré supergravities for $N = 1$ and for $N = 2$. In the latter case both for $N = 1$, and for $N = 2$ several nonequivalent theories have been constructed. While two ($N = 1$) Poincaré supergravities off the mass shell differ only in their sets of auxiliary fields (and therefore coincide in the course of transition to the mass shell), the situation for $N = 2$ turns out to be more complicated. The theories with $N = 2$ can be divided into two groups. The theories belonging to the first group contain a minimal set of physical fields (graviton, two gravitino and a graviphoton) and quite a large set of auxiliary fields. The number of auxiliary fields in theories of the second type is significantly smaller, but these theories contain additional physical fields and in the transition to the mass shell have the appearance of theories of interaction of physical fields of a minimal set with matter fields.

The book under review consists of three parts. In Part I the geometry of the standard superspace (the Wess-Zumino superspace) which is described by four boson coordinates and $4N$ fermion Q^i_α is investigated. Supergravity is regarded as a theory of supercoupling in this space. After imposing couplings on some components of twisting it becomes possible to construct a theory of conformal supergravity for $N = 1,2,3,4$.

In Part II the author constructs a theory of Poincaré supergravity off the mass shell with the aid of violation of the superconformal group in conformal supergravity. For this multiplets of the matter fields are introduced into the theory of the conformal supergravity. Then a coupling is imposed on the matter fields which violates the superconformal group. It turns out to be possible to carry out this procedure for theories with $N = 1$ and $N = 2$ with different ways of violating the superconformal group for these values of N (with the aid of introduction of different matter multiplets) being available, and this leads to nonequivalent theories. Part II also gives a review of Poincaré supergravity theories on the mass shell for $N > 2$.

Part III is entirely of a review nature. It presents the theory of the interaction of matter fields with Poincaré supergravity fields for $N = 1$ and $N = 2$.

The book is aimed at specialists in the field of supergravity theory. It can also be used as a reference on the geometry of the extended superspace and on the theories of expanded supergravity.

The book is not written as a textbook: although it contains detailed calculations and a considerable part of the material is internally complete, it clearly lacks explanations.

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