The general theory of relativity: familiar and unfamiliar

L. P. Grishchuk

P. K. Shternberg State Astronomical Institute at the Moscow State University

Usp. Fiz. Nauk **160**, 147–160 (August 1990)

The general theory of relativity agrees excellently with all available experimental data. It is used in applications, and new observations merely increase the accuracy with which this theory is confirmed. This is evidence of its practical value. But it is not by chance that the aesthetic beauty of general relativity is also emphasized, in particular its role as a paradigm for the construction of generalized theories of all physical interactions. To agree with this point of view—to recognize the beauty of general relativity in full measure—it is necessary to study this theory deeply. It is necessary to travel the entire journey from first acquaintance, through disbelief and disappointment, to recognition of its harmony and depth. At the first glance, general relativity possesses serious defects. It appears that it rejects certain fundamental principles, principles so precious and necessary which it would be impossible to give up.

It is probable that one of the most difficult features to accept is the absence in the structure of general relativity of any direct mention of ordinary flat space–time (the Minkowski four-dimensional world). Usually, one specifies the fields (for example, the classical electromagnetic field) on the background of a global Minkowski world and uses the symmetries of the background space–time for the traditional formulation of conservation laws. In ordinary physical theories, there is both a metric $\eta_{\mu\nu}$ of flat space–time as well as physical fields defined on its background. But in general relativity (in its well-known, geometrical formulation) only a curved world is discussed, and the components of the metric $g_{\mu\nu}$ of this space–time play a dual role. They appear both as quantities that determine the relations between time intervals and lengths and as potentials of the gravitational field.

It is apparently not too well known that Poincaré already attempted in 1905 a relativistic generalization of Newton's law, i.e., he attempted to reconcile the principles of the special theory of relativity (Lorentz transformations) with the law of gravitation (see, for example, in the book of Ref. 1). Einstein also began his construction of a relativistic theory of gravitation with a description of gravitational forces in a flat world. He traversed a path from accelerated (curvilinear) coordinate systems and a homogeneous gravitational field through the equivalence principle and an analysis of the procedure of physical measurements to the concept of a curved world in which we live. The beauty and economy of the geometrical formulation of general relativity is precisely that everything redundant is eliminated. Measurements of time intervals and lengths in the presence of a gravitational field always lead to the metric relations of a curved geometry; the obsolete notion of a global flat world turns out to be unnecessary. To say that in fact space–time is flat and only "effectively" observed by us as curved is as artificial as to assert that in fact the earth is flat (as in the immediate neighborhood of Moscow) and the table of distances covered by the long-range flights of Aeroflot indicates merely an effective curvature of the earth's surface due to the fact that the standards of length vary from point to point with increasing distance from Moscow.

The general theory of relativity, which uses the notion of a curved space–time continuum, is a natural, correct, and consistent method of describing gravitation and the phenomena in which gravitation is important. In insisting on the correctness of general relativity, we do not mean that this theory is some absolute truth dependent on nothing else. Like every physical theory, general relativity is incorrect in the same sense in which Newtonian mechanics was found to be incorrect from the point of view of quantum mechanics and the special theory of relativity, and the special theory of relativity was found to be incorrect from the point of view of the general theory of relativity. But here we are speaking of specific objections against general relativity and of specific theories proposed in place of it, moreover in the region of the same physical conditions that are covered by general relativity.

Geometrical general relativity, with its idea of a curved space–time, replaced the notions of a flat Minkowski world. However, there sometime arises a desire to turn backward for a few steps in the historical development. One would like to begin with the traditional notions of a field in the Minkowski world and "derive" general relativity, establishing the decisive aspects that lead to general relativity, rather than to certain other theories. (And, perhaps, one could construct an even better theory?!) One would like to compare the theory of the gravitational field with classical electrodynamics and see in detail where and why the global flat space–time "disappears" in a consistently developed theory of a relativistic gravitational field.

There are also more specific motives for such an investigation. Let us mention two of them. Far from gravitational sources, where the gravitational field disappears asymptotically, space–time is practically flat. The $g_{\mu\nu}$ differ by only small corrections $h_{\mu\nu}$ from $\eta_{\mu\nu}$:

$$g_{\mu\nu}\approx\eta_{\mu\nu}+h_{\mu\nu}. \quad (1)$$

The desire arises to formulate an exact theory of gravitation on the background of precisely such a flat space–time, i.e., one has the desire to introduce a mathematical relation that is like the relation (1), not as an approximate, but as an exact one. On the scales characteristic of electrons, protons, neutrons..., space–time is also flat to a stupendous accuracy despite the fact that on much larger scales its curvature may be appreciable. One has a desire to use the concept of a flat world by analogy with the way it is done in the theory of elementary particles and apply it to gravitation.

We shall not intrigue the reader any longer, but say immediately that all this has already been done. It is well known how naturally and beautifully Einstein's equations arise, and why the original global Minkowski space–time becomes auxiliary and fictitious, and why the concept of a flat world is unnecessary. Our list of studies[2–5] in which this

 0038-5670/90/080669-09$01.00

theme is discussed is most likely far from complete.

From the point of view of the "finished" geometrical general relativity, the metric of the background world is a certain auxiliary structure. The introduction of additional structures is nothing new: it is often used. Well known, for example, is the tetrad formulation of general relativity (see, for example, Ref. 6). It is based on the introduction of a quartet of vectors e^a_μ $(a = 1,2,3,4)$, which are defined at each point of space–time. The metric $g_{\mu\nu}$ is represented in the form $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$, where $\eta_{00} = 1$, $\eta_{11} = \eta_{22} = \eta_{33} = -1$, and the remaining η_{ab} are zero. When the e^a_μ are introduced, an additional symmetry arises–the possibility of subjecting the vectors e^a_μ to spatial and Lorentz rotations at each point of space–time. Also well known is the spinor formulation of general relativity,[7] which is based on the introduction of additional structures: spinors. Also discussed is a formulation of Einstein's equations based on the symplectic calculus of Regge (see, for example, Ref. 8), etc. Included in this list is the "field" formulation of general relativity, which uses an additional structure—a background space–time, which need not necessarily be flat.[5]

Each of these approaches is helpful for the solution of particular specific problems; makes it possible to exploit additional symmetries at one's discretion; enables one to transform certain expressions from nontensorial to tensorial nature; and serves the purposes of better understanding of the theory of the relativistic gravitational field. Of course, the introduction of additional structures does not by itself change the physical content of general relativity. The appearance of these structures in particular mathematical relations does not mean *a priori* that one is speaking about a different theory. In this light, a forceful assertion like "as a matter of principle general relativity does not contain the metric tensor $\gamma^{\mu\nu}$ of Minkowski space and it is therefore completely meaningless to speak about it in general relativity" does not sound convincing. Using the earlier example with the earth, one can say that just as the geometrical properties of the surface of the earth can be studied in projection onto a plane, so can the geometrical properties of a curved world be studied in its projection onto a flat world (for more details see, for example, Ref. 9).

The foundations of general relativity, possible ways of generalizing this theory, the comparison of general relativity with other theories of gravitation (alternatives to general relativity either in content or only in form), and the questions of physical measurements and their interpretation have already been considered in the previous papers of Refs. 10 and 11. It was noted that the field formulation of general relativity is constructed and used in specific investigations.

Here, it should be emphasized that the mere concept of a tensor gravitational field defined on the background of a Minkowski world does not lead automatically to general relativity irrespective of the specific form of the Lagrangian of the theory. For example, prior to 1984 A. A. Logunov and his collaborators developed and proposed in place of general relativity a theory of a tensor gravitational field "as a classical field of Faraday–Maxwell type" defined in a Minkowski world. This theory differs from general relativity both in its formal structure and in its physical predictions. (The main features of this theory are summarized in the paper cited as Ref. 2 in the paper of Ref. 12. This theory is a version of early constructions of Deser and Laurent.[13]) According to this theory, electromagnetic waves are deflected by a gravitating body but gravitational waves are not. So far as one can judge from subsequent publications, this theory has been abandoned even by its authors.

The field formulation of general relativity is less familiar than the usual, geometrical formulation. In the geometrical formulation of general relativity there occur only the components of the metric tensor $g_{\mu\nu}$ of a curved space–time. The constructions of the field formulation of general relativity contain both the components of the metric $\gamma_{\mu\nu}$ of a certain auxiliary (background) space–time, for example, Minkowski space, as well as the components of a tensor gravitational field $h_{\mu\nu}$. As was already noted in Refs. 10 and 11, the field formulation of general relativity has the form of an exact and rigorous field theory on a given background. It possesses all the necessary attributes of such a theory—an action and equations of motion, an energy–momentum tensor of the gravitational field, and conservation laws which reflect the symmetry of the background space–time; it possesses coordinate and gauge invariance, etc. The question of the physical meaning of the auxiliary Minkowski world also necessarily arose. It was established that attempts to give the significance of observable quantities to the metric relations of the Minkowski world leads merely to contradictions with experiment.

The subject of the present discussion is the case when the background space–time is flat. In presenting the field formulation of general relativity, we shall, for definiteness, follow Ref. 5, which, as is noted there, follows the path of previous publications on this subject by other authors. It should be emphasized that in Ref. 5 the general theory of relativity was not postulated but derived—in the sense that the construction began with the definition of background and dynamical variables; the specification of a Lagrangian of the theory, the variational derivation of the equations of motion and the energy–momentum tensor of the gravitational field, the investigation of gauge symmetries and conservation laws, etc. An arbitrary background geometry was considered, moreover in arbitrary coordinates. The requirements that lead ultimately to general relativity rather than some other theory were noted. The equivalence of the constructed "field" theory and the ordinary geometrical general relativity was demonstrated at the end of the derivation by appropriate identifications.

The paper of Ref. 5 was published[1)] before the appearance in 1984 in the papers of Logunov and his collaborators of the first references to the "relativistic theory of gravitation" (RTG). This paper contains everything needed to demonstrate the position occupied in the world of the field formulation of general relativity by the collection of propositions that are combined under the name of the RTG. Neither Ref. 5 nor the similar preceding publications of other authors were divided into sections such as Proposition 1, Proposition 2, etc., as is done in Ref. 12, but for the convenience of the reader we shall also do this here. We shall cite the paper of Ref. 5 in the translation back into Russian.

Thus:

Proposition 1. We consider a flat background space–time (Minkowski space). This case "... is privileged, since then general relativity resembles the theories of other physical fields to the greatest degree" (Ref. 5, p. 381). The Minkowski metric in Lorentz (rectilinear) coordinates has the form

$$d\sigma^2=\eta_{\mu\nu}dx^\mu dx^\nu=c^2dt^2-dx^2-dy^2-dz^2. \qquad (2)$$

In arbitrary curvilinear coordinates, which can also be introduced in a flat world, we shall have in place of the components $\eta_{\mu\nu}$ ($\eta_{00}=1, \eta_{11}=\eta_{22}=\eta_{33}=-1$, the remaining $\eta_{\mu\nu}$ are equal to zero) a set of functions $\gamma_{\mu\nu}$:

$$d\sigma^2=\gamma_{\mu\nu}dx^\mu dx^\nu, \qquad (3)$$

but the curvature tensor constructed from the metric components $\gamma_{\mu\nu}$ is of course identically equal to zero.

"It is well known that in flat space–time the use of curvilinear coordinates does not prevent the obtaining of integral conservation laws. . . . Since general relativity can always be formulated as a theory on a flat background..., the number of integral quantities (for a system with suitable conditions at infinity) is equal to 10, in accordance with the number of Killing vectors" (Ref. 5, p. 392). The ten Killing vectors reflect the presence of the 10-parameter group of motions that acts in the background Minkowski world.

Proposition 2. In the flat Minkowski world we specify the gravitational field in the form of a symmetric second-rank tensor $h^{\mu\nu}$ and other (nongravitational) fields.

There is no need to impose in advance any constraints on the components $h^{\mu\nu}$. However, in what follows it will transpire (see below) that the theory possesses a gauge symmetry, i.e., a possibility to change the $h^{\mu\nu}$ without changing the field equations and without changing the predictions for the results of any physical experiment. This symmetry is entirely similar to gauge invariance in classical electrodynamics. One can use the gauge freedom to make the components $h^{\mu\nu}$ satisfy subsidiary conditions. In practical investigations, one often uses the condition

$$h^{\mu\nu}{}_{;\nu}=0, \qquad (4)$$

where the semicolon denotes the covariant derivative with respect to the background metric $\gamma_{\mu\nu}$. This condition is used, for example, in the theory of gravitational waves[8] and in quantum gravity.[14] The condition (4) is not the only one possible or necessary and does not even exhaust the complete gauge freedom. Frequent use is made of other gauge conditions, for example, $h_{\mu\nu}u^\nu=0$ (where u^ν is some 4-vector, usually chosen in the form $u^\nu=1,0,0,0$), $h^{\mu\nu}\gamma_{\mu\nu}$, etc. (For a complete set of gauge conditions, see Refs. 15, 16).

In accordance with its origin, the considered gauge freedom is the same as the freedom to represent the earth's surface on an arbitrary plane and describe on this plane straight or curved coordinate lines. It is clear that the intrinsic properties of the earth's surface are unchanged by this, and it does not lead to any new physical effects.

Since in arbitrary curvilinear coordinates the metric of the flat background world is determined by Eq. (3), a detailed expression of the condition (4) can be represented in the equivalent form

$$(-\gamma)^{-1/2}\frac{\partial(-\gamma)^{1/2}h^\nu_\mu}{\partial x^\nu}-\frac{1}{2}h^{\alpha\beta}\frac{\partial\gamma_{\alpha\beta}}{\partial x^\mu}=0, \qquad (5)$$

where γ is the determinant of the matrix $\gamma_{\mu\nu}$. But if the coordinates are taken to be Lorentzian, i.e., $\gamma_{\mu\nu}=\eta_{\mu\nu}$ (see (2)), then the condition (4) reduces to the very simple form

$$h^{\mu\nu}{}_{,\nu}=0 \qquad (6)$$

where the comma denotes the ordinary derivative.

The Lorentz gauge condition can be expressed similarly in classical electrodynamics:

$$\frac{\partial A^\nu}{\partial x^\nu}=0. \qquad (7)$$

The same condition can be expressed in a covariant (valid in an arbitrary coordinate system) form:

$$A^\nu{}_{;\nu}=0. \qquad (7')$$

Of course, Maxwell's equations and the theory of the electromagnetic field do not cease to be what they are if they are equipped with the admissible gauge condition (7) or (7').

Equation (5) contains the symbols $\gamma_{\alpha\beta}$ and γ. Examining the equation, one might suppose that it "introduces the Minkowski space metric $\gamma_{\mu\nu}$ into the theory" and that Eq. (4) itself has some fundamental significance, in particular that it "leads to a number of radical differences from the conclusions deduced from general relativity."[17] However, in fact this equation is only one of many admissible gauge conditions that fixes (incompletely) the gauge freedom, and the symbols $\gamma_{\alpha\beta}$ and γ which occur in it reflect an altogether insignificant fact—the manner in which the coordinate lines are drawn, i.e., whether the letters x^ν are regarded as curvilinear or rectilinear coordinates in the background flat world.

Proposition 3. The specific theory of the free gravitational field $h^{\mu\nu}$ is determined by the Lagrangian L_g of this field. If in addition to the field $h^{\mu\nu}$ we consider other fields ϕ_A (they are called matter fields), then it is necessary to specify the Lagrangian L_m of these fields and also the specific form of the interaction of the fields $h^{\mu\nu}$ and ϕ_A. We discuss the question of interaction.

The constructions in Ref. 5 began with the assumption that ". . . the Lagrangian density L_m of the matter fields, including their interaction with gravitation, has the general form...," which depends on $\gamma^{\mu\nu}, h^{\mu\nu}, \phi_A$ and their derivatives. Further, it was specially shown that the form of the interaction cannot be arbitrary if one imposes the natural condition that the source for the linear part of the gravitational field is the total energy–momentum tensor, including the energy–momentum tensor of the gravitational field itself. It was shown that $\gamma^{\mu\nu}$ and $h^{\mu\nu}$ must occur in L^m in the form of a sum. It was emphasized that "this condition symbolizes the universal coupling of the gravitational field to the remaining physical fields." In other words, this condition expresses Einstein's equivalence principle and has decisive significance in the obtaining of precisely general relativity, and not some other theory. "The desire to have the total energy–momentum tensor as source for the linear part of the gravitational field leads to universal coupling of the gravitational field to the other fields (and it also leads to self-interaction) and, ultimately, to Einstein's theory" (Ref. 5, p. 379).

The sum of $\gamma^{\mu\nu}$ and $h^{\mu\nu}$ was used in Ref. 5 in the concrete form

$$(-g)^{1/2}g^{\mu\nu}=(-\gamma)^{1/2}(\gamma^{\mu\nu}+h^{\mu\nu}). \qquad (8)$$

The advantages of taking precisely this representation rather than, say $g^{\mu\nu}=\gamma^{\mu\nu}+h^{\mu\nu}$ or $g_{\mu\nu}=\gamma_{\mu\nu}+h_{\mu\nu}$, was demonstrated particularly clearly in the well-known paper of Deser.[2] The relation (8) was also encountered in earlier studies. In Ref. 2, the physical significance of this relation was also clarified (see also Ref. 18). (Modifications in the form of expression of the theory associated with transition from the representation (8) to other possible representations were analyzed in Ref. 19).

Thus, the relation (8) itself, its origin, and its physical significance were known before the relation (8) appeared in papers on the RTG under the name "geometrization principle."

Proposition 4. To complete the construction of the theory, it is necessary to specify the Lagrangian L_g of the free gravitational field and the action $S = -(1/2c\varkappa)\int d^4x L_g$. In Ref. 5 this was done by means of the scalar curvature formed from the $g_{\mu\nu}$, with allowance for the relation (8). The motives for this choice are well known—it is the simplest scalar that leads to field equations with second-order derivatives. The variation of L_g with respect to the field variables $h^{\mu\nu}$ gives the equations of the gravitational field, while variation with respect to the background metric $\gamma_{\mu\nu}$ determines the energy–momentum tensor $t_{\mu\nu}$ of the gravitational field:

$$\varkappa t_{\mu\nu} \equiv -(-\gamma)^{-1/2}\frac{\delta L^g}{\delta\gamma^{\mu\nu}}.$$

The equations of the free gravitational field given in Ref. 5 have the form

$$h_{\mu\nu}{}^{;\alpha}{}_{;\alpha} + \gamma_{\mu\nu}h^{\alpha\beta}{}_{;\alpha;\beta} - h_{\nu}{}^{\alpha}{}_{;\mu;\alpha} - h_{\mu}{}^{\alpha}{}_{;\nu;\alpha} = \frac{16\pi G}{c^4}t_{\mu\nu}. \quad (9)$$

An equivalent form of these equations was also specified:

$$(h_{\mu\nu}\gamma^{\alpha\beta} + h^{\alpha\beta}\gamma_{\mu\nu} - h_{\mu}{}^{\beta}\delta_{\nu}{}^{\alpha} - h_{\nu}{}^{\beta}\delta_{\mu}{}^{\alpha})_{;\alpha;\beta} = \frac{16\pi G}{c^4}t_{\mu\nu}.$$

In the presence of matter fields, $t_{\mu\nu}$ in these equations is replaced by the total energy—momentum tensor $T^{\text{tot}}_{\mu\nu}$. An obvious consequence of the field equations are the differential conservation laws

$$t^{\mu\nu}{}_{;\nu} = 0$$

or

$$T^{\mu\nu}{}_{\text{tot};\nu} = 0$$

with the integral conservation laws that follow from them and reflect the fact that the flat world admits a 10-parameter group of motions (the Poincaré group).

We have in fact constructed a field formulation of general relativity (namely, the general theory of relativity), although the equivalence of this theory and the metric formulation of general relativity based on the metric tensor $g_{\mu\nu}$ may not yet be obvious. The decisive elements in the construction of the theory were the choice of L_g and the choice of the type of interaction of gravitation with other fields (Einstein's equivalence principle). We continue our study of the theory.

In Ref. 5 gauge transformations of the theory were introduced and investigated, and their origin was demonstrated. It is clear that the only symmetry of the theory is the group of diffeomorphisms, i.e., the possibility of mapping space–time onto itself in an arbitrary manner, or, expressed even more simply, the possibility of covering space–time with arbitrary coordinate meshes. This symmetry can be cast in the form of gauge transformations, in which case the coordinate system is regarded as fixed, while the tensor and other fields defined on space–time are transformed. In Ref. 5, two types of gauge condition were introduced: "ordinary transformations," under which both the background and dynamical variables are transformed, and "true transformations" (intrinsic), under which only the dynamical variables are transformed. In Ref. 5 the explicit form of these transformations is given, in, moreover, not only infinitesimally small but also finite form. We give an infinitesimally small true gauge transformation for the $\tilde{h}^{\mu\nu}$:

$$\tilde{h}^{\mu\nu} \to \tilde{h}^{\mu\nu\prime} = \tilde{h}^{\mu\nu} + [\xi^{\alpha}(\tilde{h}^{\mu\nu} + \tilde{\gamma}^{\mu\nu})]_{,\alpha} - \xi^{\mu}{}_{,\alpha}(\tilde{h}^{\alpha\nu} + \tilde{\gamma}^{\nu\alpha}) - \xi^{\nu}{}_{,\alpha}(\tilde{h}^{\alpha\mu} + \tilde{\gamma}^{\alpha\mu}), \quad (10)$$

where $\tilde{h}^{\mu\nu} \equiv (-\gamma)^{1/2}h^{\mu\nu}, \tilde{\gamma}^{\mu\nu} \equiv (-\gamma)^{1/2}\gamma^{\mu\nu}$, and ξ^{α} is an arbitrary vector. The dynamical matter fields are similarly transformed.

A remarkable property of the gauge invariance of the theory is that the substitution (10) does not change the field equations (9), i.e., if $h^{\mu\nu}$ gives a solution of these equations, then so does $h^{\mu\nu\prime}$. The Lagrangian L_g changes by a divergence (a total derivative) and a term that contains the background equations of motion (in the given case, the background Ricci tensor).

Using the arbitrary vector ξ^{α}, one can achieve fulfillment of the gauge conditions. As we noted above, a convenient (but not necessary) choice of the gauge conditions is Eq. (4). With allowance for the conditions (4), Eqs. (9) simplify to

$$h_{\mu\nu}{}^{;\alpha}{}_{;\alpha} = \frac{16\pi G}{c^4}t_{\mu\nu}. \quad (11)$$

The specification of the gauge transformations and their significance completes the construction of the theory. As we see, this theory possesses all the necessary attributes of a field theory (see the beginning of the paper).

It should also be said that the tensor $t_{\mu\nu}$ is a generally covariant generalization (due to the introduction of an additional structure—the tensor $\gamma_{\mu\nu}$) of the energy–momentum pseudotensor that is used in the geometrical formulation of general relativity. The tensor $t_{\mu\nu}$ does not reduce to an expression that depends solely on $g_{\mu\nu}$ and the derivatives of $g_{\mu\nu}$. The tensor $t_{\mu\nu}$ itself is gauge noninvariant, but precisely to the same degree to which the left-hand side of Eqs. (9) is gauge noninvariant.

The equivalence of the constructed field theory and the geometrical formulation of general relativity is established by identifying the $g^{\mu\nu}$ in the relation (8) with the components of the metric tensor of the curved (physical) space–time. Substitution of the relations (8) in Eqs. (9) reduces them to Einstein's equations for the free field:

$$R_{\mu\nu} = 0.$$

In the presence of matter fields, Einstein's equations with energy–momentum tensor $T_{\mu\nu}$ can be obtained similarly:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}. \quad (12)$$

The Lagrangian L_g is transformed into the Hilbert Lagrangian. The Lagrangian L^m of the matter fields contains $h^{\mu\nu}$

and $\gamma^{\mu\nu}$ only in the form of the sum (8), and variation of L^{m} with respect to $g^{\mu\nu}$ determines the energy–momentum tensor $T_{\mu\nu}$.

The gauge symmetry of the field formulation of the theory is transformed into the coordinate symmetry of the geometrical formulation. Indeed, combining $\gamma^{\mu\nu}$ with $h^{\mu\nu}$ in accordance with the rule (8) we obtain $g^{\mu\nu}$, and combining the same $\gamma^{\mu\nu}$ with the gauge-transformed $h^{\mu\nu'}$, we obtain $g^{\mu\nu'}$, where

$$g^{\mu\nu'} = g^{\mu\nu} + \xi^{\alpha} g^{\mu\nu}{}_{,\alpha} - \xi^{\mu}{}_{,\alpha} g^{\alpha\nu} - \xi^{\nu}{}_{,\alpha} g^{\mu\alpha}. \tag{13}$$

But the $g^{\mu\nu}$ and $g^{\mu\nu'}$ obtained in this manner are related by the ordinary tensor law

$$g^{\mu\nu'}(x^{\alpha'}) = \frac{\partial x^{\mu'}}{\partial x^{\alpha}} \frac{\partial x^{\nu'}}{\partial x^{\beta}} g^{\alpha\beta}(x^{\alpha}) \tag{14}$$

for the case of coordinate transformations $x^{\alpha'} = x^{\alpha} - \xi^{\alpha}$ with the same vector ξ^{α}. The arguments of the functions $g^{\alpha\beta}$ and $g^{\mu\nu'}$ denote the point at which they are taken. With allowance for this, we return from formula (14) to formula (13).

As we have repeatedly emphasized, gauge freedom and the possibility of choosing some particular gauge condition are foreseen in the field formulation of general relativity. With or without gauge conditions, the theory continues to be general relativity. But we shall assume that for some reason condition (4) is chosen. Substituting in it the relation (8), we can obtain the relation

$$[(-g)^{1/2} g^{\mu\nu}]_{;\nu} = 0. \tag{15}$$

If, in addition, we require $\gamma_{\mu\nu} = \eta_{\mu\nu}$, then this relation can be expressed in the form

$$\frac{\partial (-g)^{1/2} g^{\mu\nu}}{\partial x^{\nu}} = 0. \tag{16}$$

In terms of the components of the metric $g^{\mu\nu}$, the relation (15) (or (16)) appears as some additional condition on the $g^{\mu\nu}$ that are determined from Einstein's equations (12). We shall discuss this question below, but now we draw certain conclusions.

The rather laborious rewriting of the formulas and definitions from Ref. 5 into the present paper was needed in order to make a transparent comparison of the theory that the author of Ref. 12 calls the RTG with the field formulation of general relativity.

We introduce new notation: $\Phi^{\mu\nu}$ in place of $h^{\mu\nu}$ and the symbol D_{μ} for covariant differentiation instead of the semicolon.

The construction of the field formulation of general relativity began with the concept of the tensor gravitational field $h^{\mu\nu}$ defined in a Minkowski space with metric $\gamma^{\mu\nu}$. The authors of the RTG proceed from a similar conception. It is shown in Ref. 5 that under certain natural requirements the Lagrangian L^{m} must contain $\gamma^{\mu\nu}$ and $h^{\mu\nu}$ in the form of a sum, and, following earlier studies, the specific relation (8) is used. In studies on the RTG, the same relation is introduced under the designation "geometrization principle" (Eq. (2) of Ref. 12). On the way to the field formulation of general relativity the specific Lagrangian L_g was used in Ref. 5. Precisely the same Lagrangian is given in Ref. 12, Eq. (20).[2)] After omission of a total derivative, this Lagrangian [Eq. (22), Ref. 12] is identical to Rosen's well-known Lagrangian.[20] In Ref. 5, by analogy with earlier studies, the energy–momentum tensor of the gravitational field is introduced and the existence of conservation laws is demonstrated. Precisely the same $t_{\mu\nu}$ occurs in Ref. 12.[3)] In Ref. 5, gauge transformations of the theory are introduced, and the gauge invariance of the theory and the possibility of imposing gauge conditions are proved.[4)] Finally, field equations are derived in Ref. 5, and these can, in a specific form (that takes into account a gauge condition), be represented in the form (11), (4). Exactly the same equations occur in Ref. 12 [Eqs. (41) and (42)], where they are called the RTG equations.

After this comparison it is clear that, taking as his point of departure Propositions 1–4, the author of Ref. 12, after other authors, followed a path that leads to the gravitational theory known as the general theory of relativity of A. Einstein (in the field formulation). There are no grounds for changing the authorship of the general theory of relativity or giving it a different name.

The paper of Ref. 12 begins with the strong assertion that general relativity "leads, first, to abandonment of the conservation laws for energy, momentum, and angular momentum of the matter and the gravitational field taken together." This claim is not addressed to the theory that reduces to Eqs. (41)–(42) of Ref. 12 (Eqs. (11) and (4) in the present paper). The question of the "dependence of the inertial mass on the method of arithmetization of three-dimensional space" (see Ref. 17) is not raised. Moreover, it is said[17] that this theory "agrees completely with the fundamental physical principles."

Such a state of affairs can only be welcomed. Since Eqs. (41)–(42) of Ref. 12 are also the concrete expression of the equations of general relativity (in the field formulation), the assertion that "general relativity does not possess. . .conservation laws"[17] is automatically eliminated by the author of Ref. 12 himself. From the point of view of the field of formulation of general relativity (and, incidentally, the geometrical formulation too), the quoted claims are also refuted without necessary choice of the gauge condition (4) (see Ref. 5), but there is also no prohibition on its use.

The reader may have become weary of repeated consideration of the somewhat unusual field formulation of general relativity. He is justified in asking what all this means in terms of the ordinary geometrical general relativity, in terms of the (physical) metric $g^{\mu\nu}$. Let us consider the equations. As regards Eqs. (12) [Eqs. (39) in Ref. 12)], everything is clear—they are Einstein's equations. But suppose that for some reason the gauge conditions (4) have been chosen. In terms of $g^{\mu\nu}$, they take the form (15). The authors of the RTG call these conditions, together with Einstein's equations, four further equations from the "field system of equations" of the RTG [Eq. (40) in Ref. 12]. It is these equations, according to the opinion of the authors of the RTG,[17] that "make it impossible to eliminate the Minkowski space metric $\gamma^{\mu\nu}$ from the theory." It is in connection with these equations that it is said[17] that "the Minkowski space metric. . . occurs organically in the theory. This is the fundamental difference between the RTG and general relativity." It is because of these equations that the RTG "leads to the physical consequences that differ qualitatively from general relativity."[17]

To what shattering of the foundations of general relativity do Eqs. (40) of Ref. 12 lead? I would like to reassure the reader: none at all.

Take an arbitrary solution $g_{\mu\nu}(x)$ of Einstein's equations. Substitute it in Eqs. (15). From this equation find the $\gamma_{\mu\nu}$. Show that suitable functions are

$$\gamma_{\mu\nu}(x) = \eta_{\alpha\beta}\frac{\partial f^{\alpha}}{\partial x^{\mu}}\frac{\partial f^{\beta}}{\partial x^{\nu}}, \tag{17}$$

where the four functions $f^{\alpha}(x)$ are solutions of the four equations

$$\frac{\partial(-g)^{1/2}g^{\mu\nu}\partial f^{\alpha}/\partial x^{\mu}}{\partial x^{\nu}} = 0. \tag{18}$$

Your solution $g^{\mu\nu}(x)$ automatically satisfies Eqs. (15) for $\gamma_{\mu\nu}$ chosen in the form (17)–(18), Equation (17) shows how the coordinate lines must be drawn in the flat world that has been constructed as an "extension." For the intrinsic properties of space–time with the physical metric $g^{\mu\nu}(x)$ this has no significance.

But if for some reason it is not immaterial to the reader how the coordinate lines are drawn in the flat world "extension" and he has agreed on the additional requirement $\gamma_{\mu\nu} = \eta_{\mu\nu}$, i.e., on Eq. (16), then it is necessary to proceed as follows. Take your solution $g^{\mu\nu}(x^{\alpha})$. First transform it in accordance with the tensor law to the coordinates $x^{\alpha'} = f^{\alpha}(x^{\beta})$, where f^{α} are found from Eqs. (18). In the expression $g^{\mu'\nu'}(x^{\alpha'})$ obtained for your solution remove the primes and substitute in Eqs. (16). They will be satisfied. Return to work on your solution.

In Eqs. (16) one can readily recognize the so-called harmonic coordinate conditions on the functions $g^{\mu\nu}$. V. A. Fock greatly loved harmonic coordinates. Emphasizing their advantages for the solution of particular problems, but without asserting that their choice overthrows general relativity, Fock obtained by means of them important concrete results.[5)]

Working with equations that are completely equivalent to those of general relativity, the authors of the RTG nevertheless arrive at a number of nonstandard conclusions. In papers of the author of Ref. 12, it is said that the RTG leads to a prediction of "exceptional strength" (the term is taken from the summary of Ref. 21) and "strikes a blow against the dogmatism that has so widely permeated general relativity" (from the summary of Ref. 22).

We list several very strong assertions of the authors of the RTG.

1. It is asserted[17] that an argument of Einstein, and also of the mathematician Klein "contains a simple but fundamental error," as a result of which the quantity that Einstein identified as the energy and momentum of an isolated system "is seen in a more careful examination to be identically equal to zero."

In Ref. 11 it is shown that this assertion can be explained solely by mathematical misunderstanding on the part of its authors.

In Ref. 12 this assertion is not renewed.

2. It is asserted that "nonuniqueness of the predictions for gravitational effects is an organic feature of general relativity"[17] and that "its inability to give uniquely determined predictions about gravitational phenomena necessarily" leads "to the abandonment of general relativity as a physical theory" (from the summary of Ref. 23).

In Ref. 11 it is shown that the assertion of "nonuniqueness of the predictions of general relativity" is based on a misunderstanding on the part of the authors of the assertion. It is shown in what the misunderstanding consists. There is no nonuniqueness in the observational predictions of general relativity.

In Ref. 12, in conjunction with the characteristic reproach that "the authors of the paper... did not understand the matter in hand," the assertion about "nonuniqueness of the predictions of general relativity" (i.e., that there are too many answers) is replaced by the assertion of the "impossibility of general relativity giving an answer to the question" (i.e., inability to give a single answer). As can be seen from the text of Ref. 12, the author himself doubts the significance of the question he poses and the possibility of testing the answer—for this it would be necessary to "switch on" and "switch off" the gravitational field of the sun. But if one speaks of the proposed calculation in its own right, which according to the belief of the author of Ref. 12 gives a "quite definite answer to this question, since the Minkowski space metric tensor occurs in the system of equations (39) and (40)," then, of course, the same calculation can also be made in general relativity [see Eqs. (12) and (15) of the present paper]. There is no calculation that could be made in the RTG but could not be made in general relativity; for the RTG equations are just a special form of expression of the equations of general relativity (in the field formulation). Thus, as can be seen from Ref. 12, the thesis of the "nonuniqueness of the predictions of general relativity" is overturned by the author of Ref. 12 himself, and there is therefore no need for the thesis of the "need to abandon general relativity as a physical theory."

3. It is asserted[17] that by virtue of the RTG equations a "homogeneous and isotropic Friedmann universe can only be... flat." This is the assertion that is characterized as a "prediction of exceptional strength."[21]

As we have seen above, the complete set of RTG equations in terms of the metric $g^{\mu\nu}$ consists of Einstein's equations plus the condition (15), which does not restrict $g^{\mu\nu}$ at all. In the most burdensome case the complete set of RTG equations in terms of the metric $g^{\mu\nu}$ reduces to Einstein's equations plus the harmonic coordinate condition (16). Therefore, to overthrow the thesis of a "flat universe" it is sufficient to find the metric of the spatially open (and not spatially flat) homogeneous and isotropic Friedmann universe in harmonic coordinates. Such a metric satisfies the complete set of RTG equations. It is discussed explicitly in §94 of Fock's book[24] and was given in Ref. 11.

In Ref. 12 it is said that "this assertion is incorrect, since it is easy to show, in particular, that Fock's solution does not satisfy the causality principle...." It can be seen from this remark that the attempt to save the third important assertion of the authors of the RTG is associated with a certain new causality principle, which is also formulated in Ref. 12.

We now turn to the discussion of this causality principle.

We recall that in Ref. 11 the artificial, formal nature of the Minkowski metric was explained. It was emphasized that the causality cone is determined by the metric of the curved world, and not the flat world. It was said that the causality cone and world lines of real bodies may be situated both inside and outside the light cone of the formally determined Minkowski metric.

In Ref. 12, after the ritual "this assertion of the authors indicates that they did not understand the essence of the RTG," it is said that "a situation such as this . . .cannot occur in the RTG, since any physical field (including the gravitational field) is incapable of carrying the world lines of test particles outside the causality cone of the Minkowski space." But a page later this is formulated as a "causality principle that makes it possible to select solutions of the system of equations (39) and (40) that have physical meaning," and it is recognized that "the causality principle is not satisfied automatically."

Therefore, although hitherto it was asserted that Eqs. (39) and (40) form the "complete system of equations" of the RTG and in, say, the previous paper of Ref. 17 the causality principle was not even mentioned, it is now explained that solutions of the RTG must in addition be selected on the basis of an additional criterion, they must be tested, so to speak, to see if they are physically meaningful—as this is understood in the RTG. Here, it should be noted that the mutual disposition of the light cones of the $g_{\mu\nu}$ and the $\eta_{\mu\nu}$ metric can be of interest only in the case when an attempt is made to interpret the metric relations of the flat world as observable. But in any case such an attempt is doomed to failure. Moreover, the mutual disposition of the light cones is gauge noninvariant—it can be changed without in any way changing the physical properties of the solution. Moreover, it can be changed even if one insists rigorously on the fulfillment of Eqs. (15), i.e., in one and the same gauge (4), due to the remaining gauge freedom.

Nevertheless, it is firmly stated in Ref. 12 that "only those solutions of Eqs. (39) and (40) have physical meaning that satisfy the causality condition (48)-(49)." The condition itself is formulated as follows: ". . .for any four-vector u^i that is isotropic in Minkowski space,"

$$\gamma_{\mu\nu}u^\mu u^\nu=0 \tag{48}$$

the following causality condition must be satisfied:

$$g_{\mu\nu}u^\mu u^\nu\leqslant 0. \qquad (49)''$$

(Since in the remaining text of Ref. 12 it is the Greek indices that take the values 0, 1, 2, 3, we have made an appropriate change of indices here too.) We immediately verify that the only remaining (according to the assertions of the RTG) Friedmann solution is flat. The expression for the interval of the flat solution derived from the requirements of the RTG is given, for example, in the book of Ref. 25 (see also Ref. 17):

$$ds^2=c^2v^3(t)dt^2-v(t)(dx^2+dy^2+dz^2). \tag{19}$$

It is said there that ct, x, y, z "are the coordinates of a pseudo-Euclidean space and are chosen in accordance with the values (1, − 1, − 1, − 1) of the Minkowski metric."[6)] In other words, it is assumed that the solution (19) is considered on the background of a Minkowski world with metric (2). In the book of Ref. 25 it is emphasized that the RTG, by virtue of Eqs. (15), "uniquely leads to a prediction that the universe. . . is flat. Since this conclusion is a consequence of only equations" (15), "this general conclusion does not depend on the value of the graviton rest mass."

To verify the conditions (48) and (49), we take a vector with the components $u^0=1,\ u^1=1,\ u^2=0,\ u^3=0$. This vector satisfies Eq. (48). We now substitute this vector in (49). We obtain

$$g_{\mu\nu}u^\mu u^\nu=v(v^2-1). \tag{20}$$

We recall that the function v must be strictly positive and for flat Friedmann solutions varies monotonically from 0 to ∞. At the present epoch $v\gg 1$, about which the following is said in Ref. 25: ". . . it is natural to assume that if $\tau=\tau_0$ then $R(\tau_0)\gg 1$," where τ_0 denotes the present time, and $R^2(\tau)\equiv v(\tau)$. It can be seen from the relation (20) that "the causality condition (49)" is not satisfied for the flat universe (19); moreover, as follows from Ref. 25, we probably live in an epoch in which it is strongly violated. Thus, the assertion of the RTG, which is characterized there as a "prediction of exceptional strength," does not stand up to the test for being "physically meaningful," as it is defined in the same RTG.

There is one further remark in Ref. 12 that cannot be passed by. Logunov believes it is possible to reproach Ya. B. Zel'dovich as follows: "Academician Ya. B. Zel'dovich did not critically consider the work of his coauthor, since otherwise he could have easily seen that the Minkowski space metric in the equations of motion of the gravitational field. . .simply cancels." For my part, I merely remark that this question was specially discussed in our first joint paper on the subject.[10] Of course, the Minkowski metric "cancels" in the equations of motion of the gravitational field, and they reduce to Einstein's equations. In precisely the same sense it "cancels" in the RTG, irrespective of whether or not the authors of the RTG recognized the equations of the general theory of relativity in their own equations.

To summarize, it must be recognized that the accusations against general relativity that have been thought up and frequently repeated by the author of Ref. 12, such as the "absence of conservation laws," the "nonuniqueness of predictions," the "impossibility of description in the spirit of Faraday–Maxwell fields," etc., are ultimately refuted by the authors himself.

I should like to end this paper with words of gratitude to and respect for Ya. B. Zel'dovich.

[1)] The publication of the paper Ref. 5 has a curious history. In a letter of the editorial board of ZhÉTF [the Russian original of Soviet Physics-JETP] of August 16, 1983 it was said that the bureau of the editorial board of ZhÉTF had considered the paper and agreed that ". . . in the nature of its content, this paper does not correspond to the currently established profile of ZhÉTF; we attempt to publish papers. . .of a less formal nature. At the same time, the exceptional overloading of the portfolio that we currently experience . . .forces us to approach the selection of papers accepted for publication with a special rigor, also as regards their subject matter. Under these conditions, we. . .have been forced to agree that its publication in ZhÉTF would not be expedient. The paper should probably be sent to some other journal of a more mathematical direction, for example, TMF (Teoreticheskaya i Matematicheskaya Fizika: Theoretical and Mathematical Physics). . . ." In a letter from the editorial board of TMF of November 10, 1983 it was said that the board had rejected the paper "on the basis of the report of the referee." In the report itself it is stated that: "the profile of this paper does not suit the subjects that have been established for TMF, and it should be sent to ZhÉTF. As regards its contents, it should be said that the paper presents the well known. . . bimetric formulation of Einstein's theory, in which the background for the gravitational interaction is taken to be the metric γ_{ik}, and the Hilbert–Einstein equations are expressed in the form. . . [;] it is well known that such an approach is not capable of solving the energy–momentum problem in Einstein's theory. . . [;] depending on the method of arithmetization of three-dimensional space one can obtain for the inertial mass, for example, any value in the given formulation of the theory. Therefore the arguments of the authors are incorrect. . . ." I should like to use this opportunity to thank S. P. Novikov and Ya. G. Sinai for supporting the publication of this paper in the journal Commun. Math. Phys.

2) Here and below, we set the parameter $m^2 = 0$. It is well known that the attempt to introduce a graviton mass leads to serious difficulties. But we intend to leave this question entirely on the side, so that the discussion of the RTG is not transformed into a continuous transition from one modification to another.

3) The author of Ref. 12 calls the entity defined by Eq. (26) of his paper the energy-momentum tensor. Then the basic equation, Eq. (41), of paper [12] is mathematically inconsistent; for on the right there is a tensor but on the left a tensor density. In reality, the expression (26) of [12] determines, not a tensor, but a tensor density, i.e., a quantity that differs by a factor $(-\gamma)^{1/2}$. After correction of this inaccuracy, complete agreement with the equations of Ref. 5 is achieved.

4) Without drawing distinctions between the "usual" and "true" gauge transformations given in Ref. 5, the author of Ref. 12 discusses the "gauge principle" on the basis of the relation (9) of [12], which is mathematically inconsistent—the two equations in (9) cannot be satisfied simultaneously.

5) As always, theorems about the choice of coordinate or gauge conditions are directly valid "in the small"; their validity "in the large" requires a separate investigation. This question is examined in the literature, but we shall not dwell on it here.

6) We retain here the notation and style of the papers of the authors of the RTG. The function $v(t)$ in (19) is related by a simple transformation to the cosmological scale factor $a(t)$ that occurs in the more common expressions for the metric of the flat Friedmann universe (in nonharmonic coordinates): $ds^2 = c^2dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$ or $ds^2 = a^2(\eta)(d\eta^2 - dx^2 - dy^2 - dz^2)$.

[1] *A. Einstein and the Theory of Gravitation* [Collection of Russian translations], Mir, M., 1979.

[2] S. Deser, Gen. Relat. Gravit. **1**, 9 (1970).

[3] N. Rosen, Phys. Rev. **57**, 147 (1940); Ann. Phys. (N.Y.) **22**, 1 (1963); A. Papapetrou, Proc. R. Irish Acad. Ser. A: **52**, 11 (1948); R. H. Kraichnan, Phys. Rev. **98**, 1118 (1955); S. N. Gupta, Rev. Mod. Phys. **29**, 334 (1957); W. Thirring, Ann. Phys. (N.Y.) **16**, 96 (1961); R. P. Feynman, Acta Phys. Pol. **24**, 697 (1963); L. Halpern, Bull. Acad. R. Belg. Cl. Sci. **49**, 226 (1963); D. E. Burlankov, Zh. Eksp. Teor. Fiz. **44**, 1941 (1963) [Sov. Phys. JETP **17**, 1306 (1963)]; V. I. Ogievetsky and I. V. Polubarinov, Ann. Phys. (N.Y.) **35**, 167 (1965); S. Weinberg, Phys. Rev. B **138**, 988 (1965).

[4] S. Weinberg, *Gravitation and Cosmology*, Wiley, N.Y., 1972 [Russ. transl., Mir, M., 1975]; Ya. B. Zel'dovich and I. D. Novikov, *The Theory of Gravitation and Evolution of Stars* [in Russian], Nauka, M., 1971, p. 87.

[5] L. P. Grishchuk *et al.*, Commun. Math. Phys. **94**, 379 (1984).

[6] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th English ed., Pergamon Press, Oxford, 1975 [Russ. original, 6th ed., Nauka, M., 1973].

[7] R. Penrose, "Structure of spacetime," in *Battelle Rencontres: 1967 Lectures in Mathematics and Physics,* edited by C. M. DeWitt and J.A. Wheeler, Benjamin, N.Y., 1968 [Russ. transl., Mir, M., 1972].

[8] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation*, Freeman, San Francisco, 1973 [Russ. transl., Mir, M., 1975].

[9] D. E. Burlankov, Yad. Fiz. **50**, 278 (1989) [Sov. J. Nucl. Phys. **50**, 174 (1989)].

[10] Ya. V. Zel'dovich and L. P. Grishchuk, Usp. Fiz. Nauk **4**, 695 (1986) [Sov. Phys. Usp. **9**, 780 (1986)].

[11] Ya. B. Zel'dovich and L. P. Grishchuk, Usp. Fiz. Nauk **155**, 517 (1988) [Sov. Phys. Usp. **31**, 666 (1988].

[12] A. A. Logunov, Usp. Fiz. Nauk **160**(8), 135 (1990) [Sov. Phys. Usp. **33**, (1990)] (in this issue).

[13] S. Deser and B. B. Laurent, Ann. Phys. (N.Y.) **50**, 76 (1968).

[14] N. P. Konopleva and V. N. Popov, *Gauge Fields* [in Russian], Atomizdat, M., 1980.

[15] L. P. Grishchuk and A. D. Popova, Zh. Eksp. Teor. Fiz. **80**, 3 (1981) [Sov. Phys. JETP **53**, 1 (1981)]; J. Phys. A **15**, 3525 (1982).

[16] L. P. Grishchuk, Usp. Fiz. Nauk **121**, 629 (1977) [Sov. Phys. Usp. **20**, 319 (1977)].

[17] A. A. Logunov, Yu. M. Loskutov, and M. A. Mestvirishvili, Usp. Fiz. Nauk **155**, 369 (1988) [Sov. Phys. Usp. **31**, 581 (1988)].

[18] K. S. Thorne, D. L. Lee, and A. P. Lightman, Phys. Rev. D **7**, 3563 (1973).

[19] A. N. Petrov and A. D. Popova, Vestn. Mosk. Univ. Ser. 3, Fiz. Astronomiya **28**, 13 (1987); Int. J. Mod. Phys. Ser. A: **3**, 2651 (1988).

[20] N. Rosen, *Cosmology and Gravitation*, edited by P. Bergmann and V. DeSabbata, Plenum Press, N.Y., 1980, p. 383, (NATO Ser. B. **58**).

[21] A. A. Logunov and M. A. Mestvirishvili, *The Relativistic Theory of Gravitation* [in Russian], published by Moscow State University, M., 1984.

[22] A. A. Logunov, *The Relativistic Theory of Gravitation and New Notions About Space-Time* [in Russian], published by Moscow State University, M., 1986.

[23] A. A. Logunov and Yu. M. Loskutov, *Nonuniqueness of the Predictions of the General Theory of Relativity and the Relativistic Theory of Gravitation* [in Russian], published by Moscow State University, M., 1986.

[24] V. A. Fock, *The Theory of Space, Time and Gravitation*, Oxford, 1964 [Russ. original, Gostekhizdat, M., 1955].

[25] A. A. Logunov and M. A. Mestvirishvili, *Fundamentals of the Relativistic Theory of Gravitation* [in Russian], published by Moscow State University, M., 1985.

Translated by Julian B. Barbour