The relativistic theory of gravitation

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A critical analysis of the general theory of relativity shows that adoption of its concepts leads, first, to abandonment of the conservation laws for energy, momentum, and angular momentum of matter and the gravitational field taken together, and, second, to the abandonment of the notion of the gravitational field as a classical field of the type of Faraday and Maxwell that possesses and energy–momentum density. Disagreeing with what has just been said, the authors of Ref. 3 assert that it is possible to give a field formulation of general relativity with "all the necessary attributes of such a theory – and action and equations of motion, energy–momentum tensor of the gravitational field, and conservation laws that reflect the symmetry of the background space–time." The error of this assertion can already be seen from the fact that general relativity in principle does not contain in its field equations a background Minkowski space–time, so that there cannot be any talk of a 10-parameter group of motions of space–time, as a consequence of which conservation laws for matter and the gravitational field cannot exist in general relativity, and it is also impossible to introduce the concept of an energy–momentum tensor of the gravitational field. All this is now obvious and has been considered in detail in our monograph,[1] which also gives references to original studies.

In connection with the publication of the paper of Ref. 3, it has become necessary to give a brief exposition of the basic propositions of the relativistic theory of gravitation so that the reader can more readily understand what is under discussion; in the course of the exposition I shall consider, as briefly as possible, the main errors of the authors of Ref. 3. Of course, I do not intend to analyze all their errors contained in Ref. 3, since I see no need for that.

The relativistic theory of gravitation (RTG),[1] which completed the development of the ideas advanced in Ref. 2, is based on the following physical requirements:

Proposition I. Minkowski space (pseudo-Euclidean geometry of space–time) is the fundamental space for all physical fields, including the gravitational field. This proposition is necessary and sufficient for the existence of conservation laws for energy, momentum, and angular momentum for matter and the gravitational field taken together. In other words, Minkowski space reflects dynamical properties common to all forms of matter. This ensures that they have the same physical properties. The idea of using Minkowski space to construct a theory of gravitation arose half a century ago in the studies of Rosen. It was he who introduced a Minowski space metric alongside a Riemannian metric. The introduction of two metrics immediately led to the possibility of constructing numerous scalar densities. As a result, the general form of the Lagrangian density for the free gravitational field became too complicated. During several decades that followed, Rosen constructed several theories, taking a different form of the Lagrangian as the basis in each case. Such an approach did not lead to the construction of a theory of gravitation, since Rosen did not succeed in formulating a principle that would lead to a unique Lagrangian for the free gravitational field.

Proposition II. The gravitational field is described by a symmetric second-rank tensor $\Phi^{\mu\nu}$ and is a real physical field that possesses an energy–momentum density; for generality, we shall assume that it has rest mass m and possesses the spin states 2 and 0.

The elimination from the states of $\Phi^{\mu\nu}$ of representations corresponding to the spin values 1 and 0′ is achieved by making $\Phi^{\mu\nu}$ satisfy the field condition.

$$D_\mu \Phi^{\mu\nu} = 0, \tag{1}$$

where D_μ is the covariant derivative with respect to the Minowski space metric $\gamma^{\mu\nu}$. Besides eliminating the unphysical states, this equation introduces into the theory the Minkowski space metric $\gamma^{\mu\nu}$, and this makes it possible to separate inertial forces from the effects of the gravitational field. By the choice of a diagonal metric $\gamma^{\mu\nu}$ one can completely eliminate the inertial forces. The Minkowski space metric makes it possible to introduce the concepts of a standard length and time interval in the absence of a gravitational field.

Proposition III. The geometrization principle, the essence of which is that the interaction of the gravational field with matter is, by virtue of its universality, realized by "adjoining" the gravitational field $\Phi^{\mu\nu}$ to the Minowski space metric tensor $\gamma^{\mu\nu}$ in the matter Lagrangian density in accordance with the rule

$$L_M(\tilde{\gamma}^{\mu\nu}, \Phi_A) \to L_M(\tilde{g}^{\mu\nu}, \Phi_A), \tag{2}$$
$$\tilde{g}^{\mu\nu} = (-g)^{1/2} g^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\Phi}^{\mu\nu},$$
$$\tilde{\gamma}^{\mu\nu} = (-\gamma)^{1/2}\gamma^{\mu\nu}, \quad \tilde{\Phi}^{\mu\nu} = (-\gamma)^{1/2}\Phi^{\mu\nu},$$

where Φ_A denotes the matter fields. By matter we understand all forms of matter except for the gravitational field. In accordance with the geometrization principle, the motion of matter under the influence of the gravitational field $\Phi^{\mu\nu}$ in the Minkowski space with metric $\gamma^{\mu\nu}$ is identical to its free motion in the effective Riemannian space with metric $g^{\mu\nu}$. the metric tensor $\gamma^{\mu\nu}$ of Minkowski space and the tensor $\Phi^{\mu\nu}$ of the gravitational field in this space are primary concepts, while the Riemannian space and its metric are secondary concepts that owe their origin to the gravitational field and its universal effect on matter fields. The effective Riemannian space has in the literal sense of the word a field origin due to the presence of the gravitational field $\Phi^{\mu\nu}$. In this proposition, Einstein's idea of a Riemannian geometry of space–time finds a partial reflection. Since the metric properties in the presence of a gravitational field are determined

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by the tensor $g^{\mu\nu}$, and without a field by the tensor $\gamma^{\mu\nu}$, the present theory is capable of establishing how the sizes of bodies and the rates of clocks change under the influence of the gravitational field. The general theory of relativity cannot give an answer to such questions, since in principle it does not contain the Minkowski space metric $\gamma^{\mu\nu}$, and it is therefore quite meaningless to speak of it in general relativity. Since the effective Riemannian space is produced by the gravitational field $\Phi^{\mu\nu}$ acting in Minkowski space, it can always be specified (and this is very important) in one coordinate system. This means that we shall be dealing with Riemannian spaces that can be represented in a single chart. From our point of view, closed Riemannian spaces are completely ruled out, since they do not have a field origin. In accordance with the geometrization principle, the matter Lagrangian density depends on the gravitational field only through the density of the metric tensor g^{mn}, and, since the action for any Lagrangian density is a scalar, the variation of the action δJ_M corresponding to an arbitrary infinitesimally small change of the coordinates will be equal to zero:

$$\delta J_M = \delta \int d^4x L_M(\tilde{g}^{mn}, \Phi_A) = 0, \quad (3)$$

where Φ_A are the matter fields.

From this condition, bearing in mind that under infinitesimally small coordinate transformations

$$x'^i = x^i + \xi^i(x), \quad (4)$$

where $\xi^i(x)$ is an infinitesimally small displacement four-vector, the variations $\delta_L \tilde{g}^{mn}$ and $\delta_L \Phi_A$ transform in accordance with the rules

$$\delta_L \tilde{g}^{mn} = \tilde{g}^{kn} D_k \xi^m + \tilde{g}^{km} D_k \xi^n - D_k(\xi^k \tilde{g}^{mn}),$$
$$\delta_L \Phi_A = -\xi^k D_k \Phi_A + F^{B;n}_{A;k} \Phi_B D_n \xi^k, \quad (5)$$

we can obtain the identity[1]

$$g_{mn} \nabla_k T^{kn} = -D_k \left(\frac{\delta L_M}{\delta \Phi_A} F^{B;k}_{A;m} \Phi_B \right) - \frac{\delta L_M}{\delta \Phi_A} D_m \Phi_A, \quad (6)$$

where $T^{kn} = -2\delta L_M / \delta g_{kn}$ is the matter energy–momentum tensor. If the equations of motion for matter are satisfied,

$$\frac{\delta L_M}{\delta \Phi_A} = 0, \quad (7)$$

then on the basis of the identity (6) we have the equations

$$\nabla_k T^{kn} = 0, \quad (8)$$

where ∇_k is the covariant derivative with respect to the Riemannian metric g_{ik}.

If there are four equations for matter, and only in this case, we can use instead of equations (7) for matter the equivalent equations (8). In what follows, when constructing the equations of the gravitational field, we must have in mind Eqs. (8).

Proposition IV. The Lagrangian density of the free gravitational field must be constructed as a quadratic function of the first-order covariant (with respect to the Minkowski space metric $\gamma^{\mu\nu}$) derivatives $D_\lambda g^{\mu\nu}$. As gauge group for the field $\Phi^{\mu\nu}$, we take the local noncommutative group of supercoordinate transformations of the form

$$\delta_\varepsilon \tilde{\Phi}^{\mu\nu} = \delta_\varepsilon \tilde{g}^{\mu\nu} = \tilde{g}^{\mu\lambda} D_\lambda \varepsilon^\nu + \tilde{g}^{\nu\lambda} D_\lambda \varepsilon^\mu - D_\lambda(\varepsilon^\lambda \tilde{g}^{\mu\nu}), \quad (9)$$

where $\varepsilon^\nu(x)$ is an infinitesimal four-vector. It is easy to show that the operators δ_ε form a Lie algebra, and that their commutator is equal to

$$(\delta_{\varepsilon_1}\delta_{\varepsilon_2} - \delta_{\varepsilon_2}\delta_{\varepsilon_1}) \tilde{g}^{\mu\nu}(x) = \delta_{\varepsilon_3} \tilde{g}^{\mu\nu}(x), \quad (10)$$

where

$$\varepsilon_3^\mu = \varepsilon_2^\nu D_\nu \varepsilon_1^\mu - \varepsilon_1^\nu D_\nu \varepsilon_2^\mu.$$

We now introduce a gauge principle, the essence of which is that under the transformations (9) the Lagrangian of the gravitational field L_g changes only by a divergence:

$$L_g \to L_g + D_\nu B^\nu(x). \quad (11)$$

Note that although the expression (9) for $\Delta_\varepsilon \tilde{g}^{\mu\nu}$ is formally, i.e., in its form, identical to the expression for the infinitesimal increment of $\tilde{g}^{\mu\nu}$ under a coordinate transformation

$$x^\mu \to x^\mu + \xi^\mu(x), \quad (12)$$

for the field $\Phi^{\mu\nu}$ it differs essentially from the infinitesimal increment

$$\delta_\xi \tilde{\Phi}^{\mu\nu} = \tilde{\Phi}^{\mu\lambda} D_\lambda \xi^\nu(x) + \tilde{\Phi}^{\nu\lambda} D_\lambda \xi^\mu(x) - D_\lambda(\xi^\lambda \tilde{\Phi}^{\mu\nu}) \quad (13)$$

which arises under the transformations (12). Thus, the gauge transformations that we have introduced have a fundamentally different content from coordinate transformations. On the basis of Propositions I–IV the relativistic theory of gravitation is constructed uniquely if the field equations have order not higher than the second.

We now turn to the construction of the Lagrangian of the free gravitational field. It is easy to show that under the transformations (9) the unique simplest scalar densities $(-g)^{1/2}$ and $\tilde{R} = (-g)^{1/2} R$, where R is the scalar curvature of the effective Riemannian space, change in accordance with the law

$$(-g)^{1/2} \to (-g)^{1/2} - D_\nu[\varepsilon^\nu (-g)^{1/2}], \quad (14)$$
$$\tilde{R} \to \tilde{R} - D_\nu(\varepsilon^\nu \tilde{R})$$

and, therefore, satisfy the gauge principle.

The scalar density $\tilde{R}$ can be represented in the form

$$\tilde{R} = -\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu} G^\sigma_{\lambda\sigma} - G^\lambda_{\mu\sigma} G^\sigma_{\nu\lambda}) - D_\nu(\tilde{g}^{\mu\nu} G^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma} G^\nu_{\mu\sigma}), \quad (15)$$

where we have the third-rank tensor

$$G^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}) \quad (16)$$

or

$$\tilde{R} = -\tilde{g}^{\mu\nu}(\Gamma^\lambda_{\mu\nu} \Gamma^\sigma_{\lambda\sigma} - \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\nu\lambda}) - D_\nu(\tilde{g}^{\mu\nu} \Gamma^\sigma_{\mu\sigma} - \tilde{g}^{\mu\sigma} \Gamma^\nu_{\mu\sigma}), \quad (17)$$

and Christoffel symbols

$$\Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}). \quad (18)$$

Note that in (15) each group of terms separately behaves as a scalar density under coordinate transformations. At the same time, it should be noted that, whereas in the complete expression $\tilde{R}$ the dependence of the metric on $\gamma^{\mu\nu}$ is identically eliminated, in the separately taken first and second group of terms in (15) it cannot be eliminated. Since by virtue of (1) the gauge principle is also satisfied by a scalar density of the form

$$\gamma_{\mu\nu}\tilde{g}^{\mu\nu}, \tag{19}$$

the total Lagrangian density of the free gravitational field with spin states 2 and 0 satisfying the gauge principle will be

$$L_g=\lambda_1(\tilde{R}+D_\nu Q^\nu)+\lambda_2(-g)^{1/2}+\lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu}+\lambda_4(-\gamma)^{1/2}; \tag{20}$$

where the divergence term with vector density Q^ν is added in order to eliminate from L_g terms with derivatives of higher than first order. This last requirement is achieved by the choice

$$Q^\nu=\tilde{g}^{\mu\nu}G^\sigma_{\mu\sigma}-\tilde{g}^{\mu\sigma}G^\nu_{\mu\sigma}. \tag{21}$$

The upshot is that we obtain a scalar density with respect to all coordinate transformations:

$$L_g=-\lambda_1\tilde{g}^{\mu\nu}(G^\alpha_{\mu\nu}G^\beta_{\alpha\beta}-G^\alpha_{\mu\beta}G^\beta_{\nu\alpha})+\lambda_2(-g)^{1/2}+\lambda_3\gamma_{\mu\nu}\tilde{g}^{\mu\nu}+\lambda_4(-\gamma)^{1/2}; \tag{22}$$

we shall determine the unknown constants λ_1, λ_2, λ_3, λ_4 below.

A direct general method for constructing this Lagrangian is given in the monograph of Ref. 1.

In accordance with the principle of least action, we obtain from this

$$\frac{\delta L_g}{\delta\tilde{g}^{\mu\nu}}=\lambda_1R_{\mu\nu}+\frac{1}{2}\lambda_2g_{\mu\nu}+\lambda_3\gamma_{\mu\nu}=0, \tag{23}$$

with Ricci tensor

$$R_{\mu\nu}=D_\lambda G^\lambda_{\mu\nu}-D_\mu G^\lambda_{\nu\lambda}+G^\sigma_{\mu\nu}G^\lambda_{\sigma\lambda}-G^\sigma_{\mu\lambda}G^\lambda_{\nu\sigma}. \tag{24}$$

Since in the absence of a gravitational field Eq. (23) must become an identity, we have

$$\lambda_2=-2\lambda_3. \tag{25}$$

Separating the energy–momentum tensor of the gravitational field in the Minkowski space,

$$t_g^{\mu\nu}=-2\frac{\delta L_g}{\delta\gamma_{\mu\nu}}=2(-\gamma)^{1/2}\left(\gamma^{\mu\alpha}\gamma^{\nu\beta}-\frac{1}{2}\gamma^{\mu\nu}\gamma^{\alpha\beta}\right)\frac{\delta L_g}{\delta\tilde{g}^{\alpha\beta}}+\lambda_1J^{\mu\nu}-2\lambda_3g^{\mu\nu}-\lambda_4\tilde{\gamma}^{\mu\nu}, \tag{26}$$

where

$$J^{\mu\nu}=D_\alpha D_\beta(\gamma^{\alpha\mu}\tilde{g}^{\beta\nu}+\gamma^{\alpha\nu}\tilde{g}^{\beta\mu}-\gamma^{\alpha\beta}\tilde{g}^{\mu\nu}-\gamma^{\mu\nu}\tilde{g}^{\alpha\beta}), \tag{27}$$

and taking into account (23), we arrive at a different equivalent form of the dynamical equations of the free gravitational field:

$$\lambda_1J^{\mu\nu}-2\lambda_3\tilde{g}^{\mu\nu}-\lambda_4\tilde{\gamma}^{\mu\nu}=t_g^{\mu\nu}. \tag{28}$$

If this equation is to be satisfied identically in the absence of a gravitational field, we must set

$$\lambda_4=-2\lambda_3. \tag{29}$$

Since for the free gravitational field we always have

$$D_\mu t_g^{\mu\nu}=0, \tag{30}$$

we obtain from Eq. (28)

$$D_\mu\tilde{g}^{\mu\nu}=0. \tag{31}$$

Thus, Eqs. (1), which determine the spin states of the field, follow directly from Eqs. (28). With allowance for Eqs. (31), the field equations (28) can be written in the form

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu}-\frac{\lambda_4}{\lambda_1}\tilde{\Phi}^{\mu\nu}=-\frac{1}{\lambda_1}t_g^{\mu\nu}. \tag{32}$$

In Galilean coordinates, this equation has a particularly simple form:

$$\Box\tilde{\Phi}^{\mu\nu}-\frac{\lambda_4}{\lambda_1}\tilde{\Phi}^{\mu\nu}=-\frac{1}{\lambda_1}t_g^{\mu\nu}; \tag{33}$$

it is natural to ascribe to the numerical factor $-\lambda_4/\lambda_1=m^2$ the significance of the square of a graviton rest mass, and the value of $-1/\lambda_1$ must, in accordance with the correspondence principle, be taken equal to 16π. Thus, all the unknown constants that occur in the Lagrangian density (22) will be determined on the basis of (25) and (29):

$$\lambda_1=-\frac{1}{16\pi},\quad \lambda_2=\lambda_4=-2\lambda_3=\frac{m^2}{16\pi}. \tag{34}$$

Equations (33) are nonlinear, since the gravitational field itself is also a source.

In the general case, the Lagrangian of the free gravitational field constructed on the basis of the gauge principle will have the form

$$L_g=\frac{1}{16\pi}\tilde{g}^{\mu\nu}(G^\lambda_{\mu\nu}G^\sigma_{\lambda\sigma}-G^\lambda_{\mu\sigma}G^\sigma_{\nu\lambda})-\frac{m^2}{16\pi}\times\left[\frac{1}{2}\gamma_{\mu\nu}\tilde{g}^{\mu\nu}-(-g)^{1/2}-(-\gamma)^{1/2}\right]. \tag{35}$$

The dynamical equations for the free gravitational field that correspond to it can be written in the form

$$J^{\mu\nu}-m^2\tilde{\Phi}^{\mu\nu}=-16\pi t_g^{\mu\nu} \tag{36}$$

or

$$R^{\mu\nu}-\frac{m^2}{2}(g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta})=0. \tag{37}$$

Equations (1) are a consequence of these equations.

We emphasize especially that Eqs. (36) or (37) are not invariant with respect to the gauge transformations (9). This means that the presence in the Lagrangian of a mass term makes it possible to determine uniquely the metric tensor of the effective Riemannian space, and also the energy–momentum tensor of the gravitational field. From the point of view of the logic of the theory, it is very probable that the graviton rest mass is nonzero.

The total Lagrangian density for matter and the gravitational field will be

$$L=L_g+L_M(\tilde{g}^{\mu\nu},\Phi_A), \tag{38}$$

where Φ_A are the matter fields, and L_g is determined by the expression (35).

On the basis of (38), the complete system of equations for the gravitational field will have the form

$$R^{\mu\nu}-\frac{m^2}{2}(g^{\mu\nu}-g^{\mu\alpha}g^{\nu\beta}\gamma_{\alpha\beta})=\frac{8\pi}{(-g)^{1/2}}\left(T^{\mu\nu}-\frac{1}{2}g^{\mu\nu}T\right), \tag{39}$$

$$D_\mu\tilde{g}^{\mu\nu}=0, \tag{40}$$

or, in a somewhat different form,

$$\gamma^{\alpha\beta}D_\alpha D_\beta\tilde{\Phi}^{\mu\nu}+m^2\tilde{\Phi}^{\mu\nu}=16\pi t^{\mu\nu}, \tag{41}$$

$$D_\mu \Phi^{\mu\nu} = 0. \quad (42)$$

In (39) and (41), the energy–momentum tensor density of matter and the density of the total energy–momentum tensor of matter and the gravitational field in the Minkowski space are defined as follows:

$$T^{\mu\nu} = -2\frac{\delta L_M}{\delta g_{\mu\nu}}, \quad t^{\mu\nu} = -2\frac{\delta L}{\delta \gamma_{\mu\nu}}.$$

Using the obvious equations

$$\nabla_\lambda \gamma_{\mu\nu} = -G^\sigma_{\lambda\mu}\gamma_{\sigma\nu} - G^\sigma_{\lambda\nu}\gamma_{\mu\sigma},$$
$$D_\mu \tilde{g}^{\mu\nu} = (-g)^{1/2}\,(D_\mu g^{\mu\nu} + G^\lambda_{\mu\lambda} g^{\mu\nu}), \quad (43)$$

we can find from Eqs. (39) the relation

$$16\pi \nabla_\mu T^{\mu\nu} = m^2 \gamma_{\lambda\sigma} g^{\lambda\nu} D_\mu \tilde{g}^{\mu\sigma}, \quad (44)$$

from which it is directly seen that Eqs. (40) are necessary, since they ensure fulfillment of the matter equations of motion (8). In other words, the matter equations are contained in the gravitational equations (39) and (40) if matter is described by four field variables.

It can be directly seen from Eqs. (39) and (40) of the relativistic theory of gravitation that the Minkowski space metric occurs both in the system of equations (39) and the system (40). The choice of the physically equivalent coordinate systems is completely determined by the specification of the Minkowski space metric tensor $\gamma^{\mu\nu}$. All the field variables that occur in Eqs. (39) and (40) are functions of the space–time variables of the Minkowski world.

The authors of Ref. 3 assert "...that the mathematical content of RTG reduces entirely to the mathematical content of general relativity in the field formulation." This assertion is also incorrect, since the basic equations of the RTG, (39) and (40) (and they are 14 in number), are generally covariant and contain the Minkowski space metric tensor, all the field variables being, moreover, functions of the Minkowski space coordinates. In general relativity, there is a system of ten generally covariant equations, and it cannot be augmented, if one remains in the framework of general relativity, by additional generally covariant equations. In the RTG only those Riemannian spaces that can be specified in a single chart are possible. General relativity admits Riemannian spaces that have complicated topology and can be covered only by an atlas of charts. Since the RTG is based on Minkowski space, it has 10-parameter group of motions that leaves all the gravitational equations, (39) and (40), form invariant. In general relativity, such a group is in principle impossible. Thus, the mathematical and physical contents of the RTG and general relativity are quite different, although, of course, in constructing the RTG we have used Einstein's idea of a Riemannian geometry of space–time.

Further, the authors of Ref. 3 write of the artificial, formal nature of the Minkowski metric, since, as they assert: "The causality cone and the world lines of real bodies may be situated both inside and outside the light cone formally determined by the Minkowski metric."

This assertion of the authors indicates that they did not understand the essence of the RTG, in which the gravitational field is a classical physical field, and the theory itself is constructed completely in the framework of the special theory of relativity, and therefore the Minkowski space metric occurs in the system of gravitational equations (39) and (40) and in the formulation of the causality principle. A situation such as the one described by the authors cannot occur in the RTG, since any physical field (including the gravitational field) is incapable, in accordance with the special theory of relativity, of carrying the world lines of test particles outside the causality cone of the Minkowski space, otherwise such a "gravitational field" would not be physical. In principle, it is not possible to give any field formulation of general relativity, since it is based solely on Riemannian geometry. The assertion of the authors of Ref. 4, that they have constructed an "exact theory (the Einstein theory) of the gravitational field," is incorrect, since the background Minkowski metric that they use is not contained in the equations of general relativity for the gravitational field, and it is therefore meaningless to speak of such a background in general relativity. The incorrect assertions of the authors of Ref. 4 passed in essence in their entirety into the content of Ref. 3 as well.

In my opinion, Academician Ya. B. Zel'dovich did not critically consider the work of his coauthor, since otherwise he could have easily seen that the Minkowski space metric in the equations of motion (2.18) of the gravitational field in Ref. 4 simply cancels. It is also absent in Eqs. (2.20) of that paper. It is here that we find the origin of the basic error of the authors of Ref. 3.

Let us now consider what is the basic difference between the RTG equations (40) and the well-known harmonic coordinate conditions in general relativity. In general relativity, the harmonic conditions are expressed in the form

$$\frac{\partial \tilde{g}^{\mu\nu}}{\partial x^\mu} = 0,$$

where x^μ are arbitrary coordinates in Riemannian space. These conditions are not generally covariant. If in Eqs. (A), for example, we use spherical coordinates, then we must recognize that the obtained results will not have any physical meaning. Fock understood this, and therefore in perturbation theory for island problems he took the coordinates in (A) to be Cartesian coordinates, although in Riemannian geometry such coordinates do not exist. In this way, he could, in principle, have arrived at the introduction of a tensor gravitational field embedded in Minkowski space and at the construction of a theory of gravitation in the framework of the special theory of relativity. However, he did not go in the direction, since he did not believe in the success of such a direction of research. General relativity cannot answer the following question: Why is it that (A) is used, the coordinates x^μ must necessarily be taken to be Cartesian coordinates? In the RTG there is no such problem, since it is obvious from Eqs. (40) that they have the form of equations (A) provided the coordinates x^μ are Galilean (Cartesian) coordinates of the Minkowski space. It is for just such a choice of coordinates that inertial forces are completely eliminated, i.e., everything that holds in any other physical theory is realized. Equations (40) are generally covariant and universal and determine the polarization properties of the gravitational field. The harmonic condition in general relativity cannot be generally covariant. Therefore, the assertion of the authors of Ref. 3: "The complete set of RTG equations in terms of the metric $g_{\mu\nu}$ of the curved space–time can be reduced to Einstein's equations plus the harmonic coordinate condition" is simply false.

The RTG is constructed like the theories of other physical fields in the framework of the special theory of relativity. In accordance with that theory, any motion of any point test body always takes place within the causal light cone of Minkowski space. Therefore, noninertial frames of reference realized by test bodies must also be situated within the causality cone of the pseudo-Euclidean space–time. This itself determines the class of possible noninertial frames of reference. Local equivalence of inertial and gravitation acting on a material point will hold if the light cone of the effective Riemannian space does not go outside the causality light cone of Minkowski space.

It is in precisely this case that the gravitational field acting on a test body can be locally eliminated by going over to an allowed noninertial frame of reference attached to the body. But if the light cone of the effective Riemannian space were to pass outside the causality light cone of Minkowski space, this would mean that for such a "gravitational field" there would not exist an admissible noninertial frame of reference in which this "field" could be locally eliminated as regards its effect on a material point. In other words, local "equivalence" of inertia and gravitation is possible only when the gravitational field, as a physical field acting on particles, does not carry their world lines outside the causality cone of the pseudo-Euclidean space–time.

This condition should be regarded as a causality principle that makes it possible to select solutions of the system of equations (39) and (40) that have physical meaning and correspond to gravitational fields. The causality principle is not satisfied automatically. This is due to the fact that because the gravitational field is "adjoined" to the Minkowski space metric γ^{ik} it occurs in the coefficients of the second derivatives in the field equations, i.e., it changes the original space–time geometry. Only the gravitational field has this property. The interaction of all other known physical fields usually never affects the second derivatives of the field equations, and therefore does not change the original pseudo-Euclidean geometry of space–time. We shall now give an analytic formulation of the causality principle in the RTG.

Since in the RTG the motion of matter under the influence of the gravitational field in the pseudo-Euclidean space–time is equivalent to free motion of matter in the corresponding effective Riemannian space–time, for causally connected events (allowed world lines of particles and light) in the effective Riemannian space, we must, on the one hand, have fulfillment of the following condition in Galilean coordinates of the Minkowski space:

$$ds^2=g_{ik}dx^i dx^k\geqslant 0, \tag{45}$$

and, on the other hand, for such events the condition of nonnegativity of the Minkowski space interval must hold. In the same Galilean coordinates, this can be expressed in the form

$$d\tau^2=(dx^0)^2-(dx^1)^2-(dx^2)^2-(dx^3)^2\geqslant 0. \tag{46}$$

We represent the velocity $v^\alpha = dx^\alpha/d^t$ in the form $v^\alpha = ve^\alpha$, where e^α is an arbitrary unit vector in Euclidean space in Cartesian coordinates.

From the expressions (45) and (46) we find the causality condition[1]

$$g_{00}+2g_{0\alpha}e^\alpha+g_{\alpha\beta}e^\alpha e^\beta\leqslant 0. \tag{47}$$

Its covariant generalization is trivial: For any four-vector u^i that is isotropic in Minkowski space,

$$\gamma_{ik}u^i u^k=0, \tag{48}$$

the following causality condition must be satisfied:

$$g_{ik}u^i u^k\leqslant 0. \tag{49}$$

The condition (49) means that the light cone of the effective Riemannian space does not pass outside the causality light cone of the pseudo-Euclidean space–time.

From Eq. (48) we readily find

$$u^i=(1, ve^\alpha),\quad v=\frac{\gamma_{00}^{1/2}}{1-(\gamma_{0\beta}e^\beta/\gamma_{00}^{1/2})}, \tag{50}$$

where v is the coordinate velocity, and e^α is an arbitrary three-dimensional unit vector

$$\chi_{\alpha\beta}e^\alpha e^\beta=1,\quad \chi_{\alpha\beta}=-\gamma_{\alpha\beta}+\frac{\gamma_{0\alpha}\gamma_{0\beta}}{\gamma_{00}}; \tag{51}$$

here $\chi_{\alpha\beta}$ is a metric tensor that enables us to determine the square of the spatial distance:

$$dl^2=\chi_{\alpha\beta}dx^\alpha dx^\beta. \tag{52}$$

Thus, only those solutions of Eqs. (39) and (40) have physical meaning that satisfy the causality condition (48)–(49).

Of course, the question of the existence of a rest mass of the graviton remains open. However, it should be emphasized that the presence of a graviton mass, however small, leads to qualitatively new physical conclusions. For example, it can be shown that on the collapse of a spherically symmetric body of arbitrary mass the process of contraction is halted in the region near the Schwarzschild sphere and replaced by subsequent expansion. Thus, the existence of "black holes" is completely ruled out. This conclusion still holds when the graviton mass tends to zero. A homogenous and isotropic universe is infinite and "flat," and its evolution proceeds cyclicly from a maximal finite density to a minimal density, and then back to a maximal density, etc. The theory predicts the existence in the universe of a large "hidden" mass of matter.

The authors of Ref. 3 assert that in the RTG one obtains Fock's solution "for the metric of a spatially open (and not spatially flat) homogeneous and isotropic Friedmann universe in harmonic coordinates." This assertion is incorrect, since it is easy to show, in particular, that Fock's solution does not satisfy the causality principle (48)–(49).

Now a few words about nonuniqueness, or, more precisely, about the impossibility of general relativity giving an answer to the question of the delay of a radio signal in the field of the sun. We denote by t the time of propagation of a radio signal from the earth to Mercury and back, and by t_0 the time of propagation of a radio signal from the earth to Mercury and back in the case when there is no effect of the gravitational field of the sun. Of course, we are not capable of switching off any interaction, but in a theory such a possibility always exists, and, using it, we can answer this question: How does a particular field (in the given case, the gravitational field) influence the change of some particular quantity? We introduce $\Delta t = t - t_0$, which determines the delay time of a radio signal due to the effect of the gravitational field of the sun. Is it possible to calculate this quantity in the general theory of relativity? No, it is not, since the equations of general relativity do not contain the metric tensor of Minkowski space, which is what would enable one to calculate the distance and, therefore, the time of propagation of a radio signal in the case of absence of the gravitational field of the sun. In contrast to general relativity, the RTG gives a quite definite answer to this question, since the Minkowski space metric tensor occurs in the system of equations (39) and (40). Of course, one can always say that general relativity is not obliged to answer such a question. But from the general theoretical point of view such an answer would be strange, since in a theory we always have the possibility of switching off a particular interaction.

It is not possible to liquidate such ambiguity in general relativity. Everything that the authors of Ref. 3 write on this question is completely unrelated to our conclusion, since they did not understand the matter in hand.

With regard to the assertion of the authors that at a conference "excellent agreement between theory (namely, the general theory of relativity!) and observations was noted," it should be emphasized in this connection that the experimental data are usually compared with the results of post-Newtonain calculations obtained in accordance with a perturbation theory that is constructed on the basis of Minkowski space in Galilean coordinates, and in the process assumptions are made about the decrease of the Riemannian metric that are sufficient for the introduction of a classical tensor gravitational field. The perturbation theory is constructed intuitively in the same way that we solve the problem in the framework of the RTG. It is this that leads to the correct result.

The post-Newtonian perturbation theory is not an unambiguous consequence of general relativity, but it is an exact consequence of the RTG. However, it should be noted that an analogous post-Newtonian perturbation theory can also be constructed in other alternative theories of gravitation (see, for example, Ref. 2), and therefore the fact of identity of its conclusions with observations does not yet prove the correctness of any particular theory of gravitation. A genuine verification of a theory is possible only by study of phenomena in strong gravitational fields.

Since the introduction into the theory of a graviton rest mass eliminates the degeneracy with respect to the gauge group, the limit obtained with it tends to zero in the final results leads in some cases to conclusions quite different from those that would be obtained if the graviton mass were set equal to zero in the basic equations (39)–(40). This circumstance indicates that the introduction into the theory of a mass term (which is subsequently made to tend to zero in the final results) is not a purely technical device, since we arrive at quite different physical conclusions. Such an approach leads to the construction of a theory with a broken gauge group.[5]

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