Models in statistical physics and quantum field theory

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H. Grosse. *Models in Statistical Physics and Quantum Field Theory.* Springer-Verlag, Berlin; Heidelberg; New York; London; Paris; Tokyo, 1988. 151 pp. (Trieste Notes in Physics).

The book under review is part of the "Trieste Notes in Physics" series edited by J. Parisi, W. Thirring and others. It is based on a series of lectures delivered by the author at several European universities and research centers.

The book consists of six chapters which summarize the results obtained by examining various models in statistical physics and quantum field theory. The connection between these two fields receives particular emphasis.

The first chapter contains a brief historical introduction, a description of the gas–liquid phase transition based on the Van der Waals equation, a formulation of the model approach to the statistical mechanics of phase transitions, and a review of critical exponents.

The second chapter, entitled "Spin systems," is subdivided into four subchapters: "Ising model–general results", "Heisenberg model", "The ϕ^4 model", and "Two-dimensional Ising model." The author first treats the Ising model in an external field, establishes the connection between lattice systems and field-theoretic models, and presents the Kramers–Wannier method of calculating the critical temperature. The contour constructions of Peierls are examined in detail, together with the subsequent results due to Griffiths (unfortunately, interesting work by Sinaĭ and Pirogov is omitted). Further, the author analyzes the simplest correlation inequalities in ferromagnetic models to demonstrate the existence of phase transitions. The analysis of the Heisenberg model is based on the Bogolyubov equation: the author proves that spontaneous magnetization is absent in one and two dimensions and that a phase transition exists in the $d = 3$ case. The discussion of the ϕ^4 model commences with an analysis of random walks on a lattice, which leads to Symanczik's demonstration that a system of interacting spins is equivalent to a polymer gas. Further, the derivative functional is employed in the derivation of the correlation inequality which relates the four-point correlation function to a product of two-point functions. This inequality is subsequently used to treat the limiting transition between the lattice and continuum. The two-dimensional Ising model is addressed by the transfer matrix method and the Klein–Jordan–Wigner and Bogolyubov transformations.

The third chapter, "Two-dimensional field theory", is subdivided into "Solitons" and "Field-theoretic models" subchapters. The former contains an analysis of the direct and inverse problems for the one-dimensional Schrödinger equation. Several techniques of solving selected nonlinear equations with partial derivatives by the Gel'fand–Levitan–Marchenko and Lax methods are discussed. Further, the author examines the phenomenon of broken symmetry, concentrating in particular on the model problem of an electron in a polyacetilene chain. The second subchapter begins with a discussion of the Dirac equation, the Fock space and the second quantization of fermions. The gauge transformations and charge algebras are studied in detail, as well as the Schwinger model (two-dimensional quantum electrodynamics).

The fourth chapter, "Lattice gauge models," contains two subchapters: "Definitions" and "Rigorous results." The former discusses the axiomatic limit of Gording–Wightman and the classical theory of gauge fields. The gauge-invariant Heisenberg model and fermions on a lattice are also examined. The rigorous results of the second subchapter include the Faddeev–Popov method, the Osterwalder–Schröder theorem, a thorough treatment of cluster decomposition, a discussion of the confinement problem, and finally a brief survey of numerical results and recent advances.

The fifth chapter, "String models," presents the classical mechanics of strings (definitions, equations of motion, covariant solution of these equations, and the Hamiltonian formalism). The quantization of boson strings, fermion strings, and superstrings are also examined.

The sixth and final chapter, "Renormalization groups", opens with the formulation of the similarity laws (Widom), describes the spin domain method of Kadanoff, Wilson's renormalization group approach, and returns yet again to the one-dimensional Ising model. After a preliminary discussion of the central limit theorem of probability theory, the author then proceeds to apply the renormalization group method to concrete models. He describes and rigorously analyzes the hierarchical model, the two-dimensional Ising model on a triangular lattice, the Ginzburg–Landau–Wilson model, and the various scenarios of chaotic behavior exhibited by dynamical systems, including Feigenbaum's results.

The book is written in a very interesting fashion. The author has succeeded in presenting a multitude of important results in a fairly slender volume. The presentation is clear and thorough, and all intermediate steps can be reconstructed easily. A large part of the material has never been published in monograph form and is unavailable in Russian. This book will be an excellent reference for students, graduate students, and scientists specializing in statistical physics, field theory, or solid state physics.

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