METHODOLOGICAL NOTES

Radiation by charges moving faster than light

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1. INTRODUCTION

Soon after Maxwell formulated his celebrated equations, it was realized that the field of a moving source (e.g., a point charge in uniform motion) differed in geometry from the field of a source at rest, and the differences would become more pronounced as the velocity of the source approached the velocity of light. An exact solution of Maxwell's equations for the field of a point charge in uniform motion was first derived by Heaviside. He was apparently also the first to understand that the case in which a charge is moving at a velocity greater than the velocity of light in a medium would be of particular interest.[1] Des Coudres and Sommerfeld[2] later took up a problem related to that studied by Heaviside: the motion of a charge not in a refractive medium but in empty space, in the case in which the charge is moving at a velocity above the velocity of light in free space. A common aspect of the results of Heaviside, on the one hand, and those of Des Coudres and Sommerfeld, on the other, was that the field propagation velocity (the phase velocity of the electromagnetic waves) played the role of a threshold: When this velocity was crossed, the field of the source changed abruptly in nature. It was shown that if the source velocity goes above the velocity of light in the medium in which the source is moving (Heaviside[1]) or in free space (Des Coudres and Sommerfeld[2]) a characteristic type of radiation should arise.

Soon after the papers by Des Coudres and Sommerfeld were published, the special theory of relativity was formulated. It turned out that material objects could not move at velocities above the velocity of light in free space. The results of Des Coudres and Sommerfeld were thus no longer taken seriously. Their work was soon forgotten, and the work of Heaviside along with it, although Heaviside had studied a physically realizable case: the motion fo a charge in a medium (rather than in free space) at a velocity above the phase velocity of waves in the medium. The work by Heaviside as well as that by Des Coudres and Sommerfeld lay forgotten until the discovery of the Čerenkov effect (or the "Vavilov-Čerenkov effect") and the derivation of a theory for this effect by Tamm and Frank.[3]

For the discussion below it is important to recall one aspect of the prohibition against motion faster than light in the special theory of relativity: The special theory does not forbid *all* motions faster than light but only motions which could lead to a disruption of cause-and-effect relationships. For example, as a material object (an elementary particle) moves, its position and velocity at a certain time determine its entire subsequent motion and in this sense are the "cause" of its positions at all subsequent times. In turn, the position and velocity of an object at any time are a "consequence" of its positions and velocities at earlier times. These and similar motions cannot occur at velocities above the velocity of light in vacuum. There are, however, objects (completely material) whose motion is not a manifestation of cause-and-effect relationships of this sort and which thus *can* move at velocities above the velocity of light in vacuum. The best-known object of this sort is a reflected spot of light (of sunlight, in particular). This and other examples of superluminal motion have been discussed in several places, in particular, in a book by Ginzburg.[4] Interestingly, one object capable of moving at velocities above the velocity of light in vacuum, is a charge. We are of course not talking about a charged material object (e.g., an elementary particle); we have in mind instead an effective charge formed in some special way. Such a charge arises, for example, when a plane electromagnetic wave is incident obliquely on the surface of a metal. If the electric vector of the incident wave lies in the plane of incidence, a nonzero surface charge will form at the metal surface. This charge will form an alternating-sign periodic structure, which will move as a whole along the metal surface at a velocity above the velocity of light. The radiation excited by this structure will evidently produce a reflected wave also. Yet another example of a source moving faster than light will be constructed below.

In this methodological note we would like to call attention to the circumstance that the electrodynamics of effective "superluminal" charges of this sort is a comparatively new and interesting field of research, possibly more interesting than the field of subliminal motions.

The most interesting feature of the radiation by sources moving at a superluminal velocity is that an observer at rest sees not a single real radiator but several spatially separated radiating objects.[5] They will be called "images" below. This multiplicity of images is seen in all the examples discussed below.

Although many studies of superluminal motions, particularly in connection with the Čerenkov effect, have been carried out, most have used a spectral expansion of the fields in Fourier integrals (or series). In the present note we use a method of retarded potentials. This approach leads to a clear space-time picture of the radiation.

2. MOTION OF A CHARGE AT A VELOCITY ABOVE THE VELOCITY OF LIGHT IN VACCUM

In this section of the paper we use a specific example to demonstrate that a lumped effective charge distribution can be formed and put into motion at a velocity above the veloc-

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ity of light in vacuum by means of subliminal motions of real charges. Our purpose in discussing this example is not to give a blueprint of some real device which would realize a distribution of this sort (although we do not rule out the possibility that this could be done) but simply to show that distributions of this type are theoretically possible.

We will first illustrate the idea of a lumped charge distribution moving at a velocity higher than the velocity of light by considering two examples. Let us assume that we have two dielectric bars, differing in length, whose faces bear negative and positive charges, respectively. If the charge per unit length is the same for the two bars, and if the bars are pressed tightly against each other, making contact with their charged faces, then the charges will cancel out over the distance L (Fig. 1,a). It will look as though there were only some effective negative charge at the left end of the distribution. We now assume that the positively charged bar is moved a distance ΔL to the left over a time Δt in such a way that its velocity $V = \Delta L / \Delta t$ is below the velocity of light in vacuum. As a result of this motion, the effective negative charge appears at the right end of the distribution (Fig. 1,b); i.e., the effective charge has moved a distance L. The velocity at which it has moved is

$$v_{\rm eff} = \frac{L}{\Delta t} = V \frac{L}{\Delta L}.$$

Since $L \gg \Delta L$, this velocity can be, in particular, higher than the velocity of light. The velocity ratio

$$\frac{v_{\rm eff}}{V} = \frac{L}{\Delta L}$$

is roughly equal to the ratio of the average length of the two bars to the difference between their lengths, and it can be made arbitrarily large. As in the case of ordinary charges, the motion of this effective charge is accompanied by a current. This current is formed by the positive charges which are moving along with the bar.

It is a straightforward matter to generalize these arguments to charge distributions which are infinitely long. Let us assume that a negatively charged bar is infinitely long (Fig. 2). We cover the top of this bar by positively charged bars of length L in such a way that there is a small gap ΔL in one place. There will obviously be an effective negative charge in this place. If the bars L to the right of the gap are now moved in turn to the left, to fill the gap ΔL, the effective negative charge will move to the right, at a velocity

$$v_{\rm eff} = \frac{L}{\Delta t} = V \frac{L}{\Delta L},$$

where $\Delta t = \Delta L / V$ is the time over which one bar L moves to the left, and V is its velocity in the process. Since the ratio $L / \Delta L$ can be made arbitrarily large, the velocity $v_{\rm eff}$ can be made higher than the velocity of light in vacuum.

This idea of a superluminal motion of an effective charge can be realized as a continuous process also. Let us assume that positive and negative charges are distributed along some straight line. The positive charge per unit length is a constant

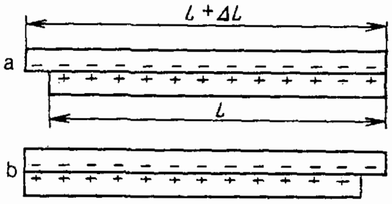

FIG. 1. Model of a charge moving a finite distance at a superluminal velocity.

$$\sigma^{(+)} = \bar{\sigma}, \tag{1}$$

where $\bar{\sigma}$ is some constant value, independent of the coordinate and the time. The negative charges are assumed to be distributed in accordance with

$$\sigma^{(-)} = -\{\bar{\sigma} + \sigma_0 \exp[-\alpha(x - vt)^2]\}, \tag{2}$$

where α and v are constants. We thus have a net negative charge, in a bell-shaped distribution around the point $x = vt$ at time t. This distribution is moving along the x axis at a velocity $v = v_{\rm eff}$, which, as we will show below, can be higher than the velocity of light in vacuum. Summing $\sigma^{(+)}$ and $\sigma^{(-)}$, we find the distribution of the total charge along the straight line:

$$\sigma(x, t) = \sigma^{(+)} + \sigma^{(-)} = \sigma_0 \exp[-\alpha(x - vt)^2]; \tag{3}$$

this is an effective lumped charge distribution which is capable of moving a velocity $v = v_{\rm eff}$ which is higher than the velocity of light in vacuum, as we will see below.

Since the current associated with this motion of charge is expressed in terms of actual charges and their velocities, once we have determined this current we can find the velocities of the actual charges and compare them with v. The current can be found from charge conservation, i.e., from the continuity equation

$$\frac{\partial \sigma}{\partial t} + \frac{\partial J}{\partial x} = 0. \tag{4}$$

Using (3), we can verify that this equation holds for

$$J(x, t) = v\sigma_0 \exp[-\alpha(x - vt)^2]. \tag{5}$$

We can also write another expression for this current. Since the positive charges are at rest, we have

$$J(x, t) = \sigma^{(-)}(x, t) V^{(-)}(x, t), \tag{6}$$

where $V^{(-)}(x,t)$ is the velocity at which the actual negative charges move (it is because this is the velocity of actual charges that it cannot be greater than the velocity of light in vacuum). Comparing (5) and (6), and using (2), we find

$$V^{(-)}(x, t) = v \frac{\sigma_0}{\sigma_0 + \bar{\sigma} \exp[\alpha(x - vt)^2]}. \tag{7}$$

The maximum value of $V^{(-)}$ is evidently

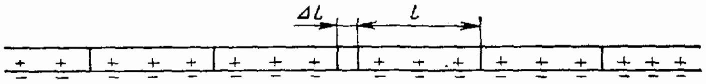

FIG. 2. Model of a charge moving along a straight line at a superluminal velocity.

$$V^{(-)}_{\max} = v\,\frac{\sigma_0}{\sigma_0+\bar{\sigma}}\,, \qquad (8)$$

so we have

$$v = v_{\rm eff} = V^{(-)}_{\max}(1+\gamma). \qquad (9)$$

Since the ratio of the constant part of the negative charge density to its net part,

$$\gamma = \frac{\bar{\sigma}}{\sigma_0}\,, \qquad (10)$$

can be arbitrarily large—much greater than unity—the velocity $v_{\rm eff}$ can thus also be made much larger than $v^{(-)}_{\max}$. In particular, the velocity $v_{\rm eff}$ can be higher than the velocity of light in vacuum. According to (3), the velocity $v_{\rm eff}$ is the velocity at which the effective charge distribution and the accompanying current distribution $J(x,t)$ move. It has thus been shown that although the actual charges are moving at velocities below the velocity of light the effective total charge is moving at a velocity $v_{\rm eff}$ greater than the velocity of light. These arguments could of course been generalized to more-complex, nonrectilinear trajectories.

Could effective charges and currents of this sort serve as sources of radiation in Maxwell's equations? They evidently could, since effective charges and currents are the sums of corresponding actual charges and currents, so the substitution of such quantities into Maxwell's equations as sources would be equivalent to the substitution of actual charges and currents into these equations.

To realize lumped effective charges and currents of this sort, we need a special mechanism to arrange a subliminal motion of the actual charges and currents (a mechanism to move the dielectric bars in the first example, and a mechanism to move the charges $\sigma^{(-)}$ at a velocity $V^{(-)}$ in the second). We might assume that, ideally, this mechanism has no effect on the electromagnetic field (e.g., it might be made of a material which is transparent to electromagnetic radiation over a broad spectral range). The radiation by these lumped effective superluminal charges and currents would then be the same as the radiation by point superluminal charges in vacuum. In this sense, the old results found by Des Coudres and Sommerfeld can be rehabilitated.

3. LIÉNARD-WIECHERT POTENTIALS FOR SUPERLUMINAL MOTION

In this section of the paper we derive Liénard-Wiechert potentials for superluminal motion, basically following the arguments of Ref. 5. We recall that the Liénard-Wiechert potentials are the vector and scalar potentials of the electromagnetic field which is produced by a point charged particle moving in accordance with some arbitrary prespecified law.

In general, the vector potential **A** and the scalar potential φ obey the equations

$$\Delta\mathbf{A} - \frac{1}{c^2}\frac{\partial^2 A}{\partial t^2} = -\frac{4\pi}{c}j, \quad \Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -4\pi\rho, \qquad (11)$$

where **j** is the current density, and ρ the charge density. System (11) for the potentials **A** and φ is valid when the potentials satisfy the well-known Lorentz gauge condition

$$\operatorname{div}\mathbf{A} + \frac{1}{c}\frac{\partial\varphi}{\partial t} = 0.$$

Solutions of these equations can be expressed in terms of retarded potentials:

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{c}\iint d\mathbf{r}'\,dt'\,\mathbf{j}(\mathbf{r}',t')\frac{\delta(t-t'-(|\mathbf{r}-\mathbf{r}'|/c))}{|\mathbf{r}-\mathbf{r}'|}\,\vartheta(t-t'), \qquad (12)$$

$$\varphi(\mathbf{r},t) = \iint d\mathbf{r}'\,dt'\,p(\mathbf{r}',t')\frac{\delta(t-t'-(|\mathbf{r}-\mathbf{r}'|/c))}{|\mathbf{r}-\mathbf{r}'|}\,\vartheta(t-t'). \qquad (13)$$

For point charged particles moving in accordance with

$$\mathbf{r} = \mathbf{r}_0(t), \qquad (14)$$

the current and charge densities are

$$\mathbf{j}(\mathbf{r},t) = q\mathbf{v}(t)\,\delta(\mathbf{r}-\mathbf{r}_0(t)), \qquad (15)$$

$$p(\mathbf{r},t) = q\delta(\mathbf{r}-\mathbf{r}_0(t)), \qquad (16)$$

where

$$\mathbf{v}(t) = \dot{\mathbf{r}}_0(t) \qquad (17)$$

is the velocity of the charged particle. It is assumed below that there is no limitation

$$|v(t)|<c\,. \qquad (18)$$

In other words, it is assumed that the charge is capable of moving at an arbitrary velocity, including velocities above the velocity of light.

Substituting expressions (15) and (16) for **j** and ρ into (12) and (13), and integrating over $\mathbf{r}'$, we find

$$\mathbf{A}(\mathbf{r},t) = \frac{q}{c}\int dt'\,\mathbf{v}(t')\frac{\delta(t-t'-(1/c)|\mathbf{r}-\mathbf{r}_0(t')|)}{|\mathbf{r}-\mathbf{r}_0(t')|}\,\vartheta(t-t'), \qquad (19)$$

$$\varphi(\mathbf{r},t) = q\int dt'\,\frac{\delta(t-t'-(1/c)|\mathbf{r}-\mathbf{r}_0(t')|)}{|\mathbf{r}-\mathbf{r}_0(t')|}\,\vartheta(t-t'). \qquad (20)$$

For the integration over t' we use the well-known formula

$$\delta(F(x)) = \sum_\alpha \frac{\delta(x-x_\alpha)}{|F'(x_\alpha)|}\,, \qquad (21)$$

where $F(x)$ is a function of the variable x, and x_α is the root of index α of the equation

$$F(x) = 0. \qquad (22)$$

An important point is that the summation on the right side of (21) is over all roots of Eq. (22). From (21) we have

$$\delta\left(t-t'+\frac{1}{c}|\mathbf{r}-\mathbf{r}_0(t')|\right) = \sum_\alpha \frac{\delta(t-t'_\alpha)}{|1-(1/c)[\mathbf{v}(t'_\alpha)(\mathbf{r}-\mathbf{r}_0(t'_\alpha))/|\mathbf{r}-\mathbf{r}_0(t'_\alpha)|]|}\,, \qquad (23)$$

where t'_α is the roots of index α of the equation

$$t-t' = \frac{1}{c}|\mathbf{r}-\mathbf{r}_0(t')|. \qquad (24)$$

Using (23), we integrate over t' in expressions (19) and (20) for the potentials:

$$\mathbf{A}(\mathbf{r},t) = \frac{q}{c}\sum_\alpha \frac{\mathbf{v}(t'_\alpha)}{|R(t'_\alpha)[1-(1/c)\,\mathbf{n}(t'_\alpha)\,\mathbf{v}(t'_\alpha)]|}\,; \qquad (25)$$

$$\varphi(\mathbf{r},t) = q\sum_\alpha \frac{1}{|R(t'_\alpha)[1-(1/c)\,\mathbf{n}(t'_\alpha)\,\mathbf{v}(t'_\alpha)]|}\,; \qquad (26)$$

here

$$\mathbf{R}(t'_\alpha) = \mathbf{r} - \mathbf{r}_0(t'_\alpha) \tag{27}$$

is a vector directed toward the observation point **r** from the point of the radiating charge at the time t'_α, and

$$\mathbf{n}(t'_\alpha) = \frac{\mathbf{R}(t'_\alpha)}{R(t'_\alpha)} \tag{28}$$

is the unit vector in the same direction. Since the unit step function (Heaviside function) $\vartheta(t - t')$ appears in the integrands in (19) and (20), the summation in (25) and (26) is over those roots of Eq. (24) for which the condition $t > t_\alpha$ holds.

We see that Eq. (24) is playing an important role here. It determines the number of terms in the expressions for the potentials, i.e., the number of spherical waves coming in toward the observer at the given time t, and also the properties of these waves. We know that Eq. (24) has a unique solution in the case in which a radiation source is moving at a velocity below the velocity of light, and the condition $t > t'$ holds.[6] For superluminal motion of the source, in contrast, this equation may have several roots.

To prove this assertion, we will build slightly on arguments presented by Landau and Lifshitz in their *Classical Theory of Fields*.[6] When relation (24), thought of as a functional dependence of **r** on t for a given position of the source, $\mathbf{r}_0$, and for a given radiation time t', is combined with the condition $t > t'$, it describes that part of a light cone with a vertex $(\mathbf{r}_0, t')$ which is in the absolute future. If we instead look at relation (24) as a function dependence of $\mathbf{r}_0$ on t' for given **r** and t, we see that it, combined with the condition $t' < t$, also describes part of a light cone, but the vertex in this case is at the observation point $(\mathbf{r}, t)$ and lies in the absolute past. Equation (24) $(t > t')$ is thus simply the condition that the source is on the past part of the light cone of the observer or the condition that the observer is on the future part of the light cone of the source. These two conditions are equivalent; we will make use of the first of them below.

In the coordinates x, ct, these parts of the light cones degenerate into two pairs of rays. One pair emerges from the radiating source, while the other goes into the position of the observer (Fig. 3). The motion of this source at a subluminal velocity is imaged in these coordinates by some world line AB. The tangent to this world line at any point on this line makes an angle no greater than 45° with the ct axis. An observer at the point x_0 has as world line a straight line running parallel to the ct axis. It can be seen from this figure that only

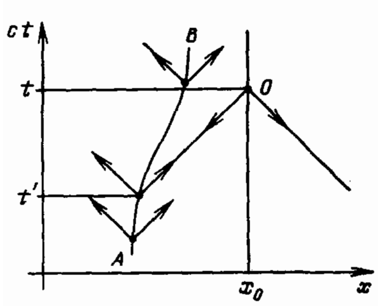

FIG. 3. World lines of an observer and a radiator which is moving at a subluminal velocity.

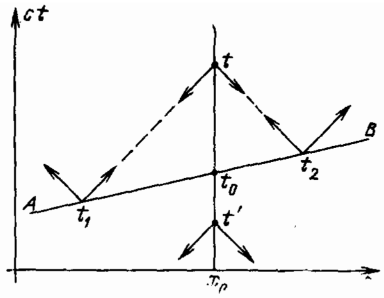

FIG. 4. World lines of an observer and of a radiator which is in rectilinear motion at a superluminal velocity.

the radiation which is emitted by the source as its world line intersects the light cone of the observer will reach the observer at the time t. This intersection will be unique, since after the very first intersection the world line of the source can only move away from the light cone of the observer.

The situation changes radically if the velocity of the charge can exceed the velocity of light. In this case Eq. (24), which determines the radiation time t', generally has more than one root. Simple geometric arguments demonstrate this point. Line AB in Fig. 4 is the world line of a source whose velocity exceeds the velocity of light (the angle between AB and the ct axis is greater than 45°). As before, an observer at the point x_0 has as world line a straight line running parallel to the time axis.

We denote by t_0 the time at which the world line of the source, AB, intersects the world line of the observer. Up to the time t_0, say, at the time t', the observer obviously receives no signals at all from the moving charge. If $t > t_0$, on the other hand, the observer receives signals from two points on the world line of the source simultaneously: from the pionts t_1 and t_2 in the diagram. At the times corresponding to these two points the source radiates light signals which arrive at the point x_0 (at the observer) at the same time t. Consequently, in the case shown in Fig. 4, Eq. (24) either has no solutions at all or has two solutions. These kinematic arguments regarding the behavior of world lines and radiated light signals, like the arguments to follow, do not depend on whether this point source is a charge, a dipole, a quadrupole, or something more general. In particular, we will see below that the diagram in Fig. 4 corresponds to the classical problem of the radiation of sources which are moving at a superluminal velocity (see, for example, the work by Tamm and Frank[3]).

Furthermore, it is not difficult to imagine cases in which Eq. (24), which determines the radiation time t', has more than two solutions. One such example is shown in Fig. 5. Line AB in this figure is the world line of an oscillator which is oscillating along the x axis. The velocity of the oscillator exceeds the velocity of light on certain parts of the trajectory. Let us assume, as in the preceding example, that an observer is at the point x_0. We see that at the time t_1 the observer receives signals radiated at three different points on the trajectory of the oscillator, while at the time t_2 the observer receives signals from five different points. Signals radiated at different times and at different points along the trajectory of the oscillator arrive at the observer at the same time. The evolution of the picture in Fig. 5 is interesting. As the observation time increases, the radiating points in a sense

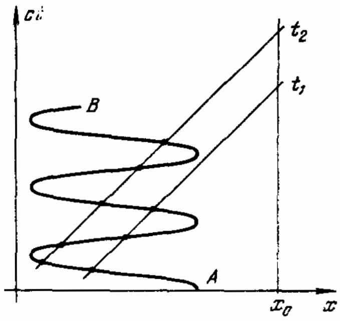

FIG. 5. World lines of an observer and of a radiator which is oscillating at a superluminal velocity.

appear and disappear in pairs.

To conclude this section of the paper, we present without derivation expressions for the Liénard-Wiechert potentials for the case in which a charge is moving through a dispersionless medium with a dielectric constant ε and a magnetic permeability μ:

$$\mathbf{A}(\mathbf{r}, t) = \frac{q\mu}{c} \sum_{\alpha} \frac{v(t'_\alpha)}{|R(t'_\alpha)\{1 - [\varepsilon\mu/c]^{1/2}\, \mathbf{n}(t'_\alpha)\, \mathbf{v}(t'_\alpha)\}|}, \qquad (29)$$

$$\varphi(\mathbf{r}, t) = \frac{q}{\varepsilon} \sum_{\alpha} \frac{1}{|R(t'_\alpha)\{1 - [\varepsilon\mu/c]^{1/2}\, \mathbf{n}(t'_\alpha)\, \mathbf{v}(t'_\alpha)\}|}. \qquad (30)$$

Here t'_α are the roots of the equation

$$t - t' = \frac{(\varepsilon\mu)^{1/2}}{c} R(t'), \qquad (31)$$

which satisfies the condition $t > t'_\alpha$, and the summation in (29) and (30) is over all such roots. The electric field **E** and the magnetic field **H** are expressed in terms of the potentials as follows:

$$\mathbf{E} = -\operatorname{grad}\varphi - \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \frac{1}{\mu}\operatorname{rot}\mathbf{A}, \quad \operatorname{div}\mathbf{A} + \frac{2\mu}{c}\frac{\partial \varphi}{\partial t} = 0. \qquad (32)$$

4. UNIFORM RECTILINEAR SUPERLUMINAL MOTION OF A CHARGE AND A DIPOLE

The problem of the radiation by a charge in uniform rectilinear motion at a superluminal velocity has been examined in many places.[1,3,7] It has become a standard problem, since many of its conclusions can be generalized, at least qualitatively, to more-complex motions of a charge. This problem is usually studied by a spectral approach. In the present paper we solve it by means of Liénard-Wiechert potentials, and in so doing we obtain a clear space-time picture of the radiation by the charge.

We thus assume that a charge is moving along the z axis. Its equations of motion are

$$x_0 = 0, \quad y_0 = 0, \quad z = vt. \qquad (33)$$

If the observation point has the coordinates x,y,z, then Eq. (24), which determines the number of terms in expressions (25) and (26) for the Liénard-Wiechert potentials, takes the form

$$[x^2 + y^2 + (vt' - z)^2]^{1/2} - c(t - t') = 0. \qquad (34)$$

This equation obviously has two roots:

$$t'_{1,2} = \frac{1}{c(\beta^2 - 1)}\{\pm[(vt - z)^2 - (\beta^2 - 1)(x^2 + y^2)]^{1/2} - ct + \beta z\}, \qquad (35)$$

where $\beta = v/c$. The denominators in (25) and (26) are

$$c[(vt - z)^2 - (\beta^2 - 1)(x^2 + y^2)]^{1/2}; \qquad (36)$$

in the case at hand, they are identical for the two roots of Eq. (24). Consequently, under the condition

$$(vt - z)^2 \geqslant (\beta^2 - 1)(x^2 + y^2) \qquad (37)$$

the potentials in (25) and (26) are

$$\mathbf{A}(\mathbf{r}, t) = \frac{2e\mathbf{v}}{c[(vt - z)^2 - (\beta^2 - 1)(x^2 + y^2)]^{1/2}}, \quad \varphi(\mathbf{r}, t) = \frac{2e}{[(vt - z)^2 - (\beta^2 - 1)(x^2 + y^2)]^{1/2}}. \qquad (38)$$

If condition (37) does not hold, Eq. (24) has no roots at all, and the potentials are zero. The factor of 2 in the expressions for **A** and φ appears specifically because Eq. (24) has two roots, which make different contributions. In the subluminal case there is only a single root, and the factor of 2 does not appear. The equation

$$(vt - z)^2 - (\beta^2 - 1)(x^2 + y^2) = 0 \qquad (39)$$

describes the Mach cone: the surface which separates the part of the space in which there is field from the part in which there is not. The vertex angle of this cone is specified by

$$\operatorname{tg}\psi = \left|\frac{dx}{dz}\right|_{y=0} = \left|\frac{vt - z}{(\beta^2 - 1)x}\right| = \frac{1}{(\beta^2 - 1)^{1/2}}, \quad \sin\psi = \frac{1}{\beta} = \frac{c}{v} \qquad (40)$$

In other words, this is the cone characteristic of Čerenkov radiation.

The position of the source of the radiation on the z axis at the time of the radiation is specified by the coordinate

$$z_{1,2} = vt'_{1,2}. \qquad (41)$$

It follows from this result and relations (35) that the observer receives signals from two sources, although there is actually only a single source: a charge moving at a superluminal velocity. The number of roots of Eq. (24) which satisfy the condition $t > t'_\alpha$ is thus equal to the number of radiation sources which the observer sees in the case of a single real radiation source. We will call these sources which are "seen" by the observer "images." It can be seen from (41) and (35) that when the radiation first reaches the observer, i.e., on the Mach cone, the two images are at the same point

$$z_{1,2} = \frac{\beta^2 z - vt}{\beta^2 - 1}, \qquad (42)$$

since the expression in the radical in (35) is zero at this time. The images subsequently move off in different directions, since t'_1 decreases with increasing t, while t'_2 increases (Fig. 6).

Let us take a brief look at the question of whether it is possible to observe the images separately. This question is particularly pertinent to our problem of the radiation by a

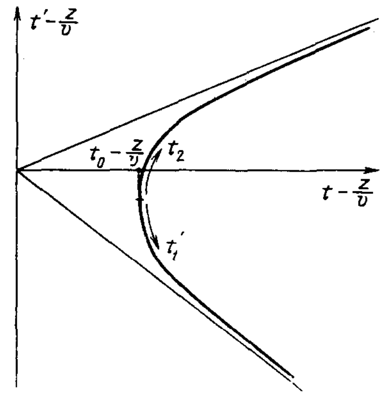

FIG. 6. The times t_1' and t_2' as functions of t. The time t_2' increases, while t_1' decreases.

superluminal charge, since the two images make absolutely identical contributions to the field throughout the space–it is for this reason that there is a 2 in (38). The question should be answered in the affirmative in the sense that it is possible to separately observe the images. To demonstrate this point, we assume that a charge moves uniformly at a superluminal velocity only up to the point

$$z_0 = \frac{\beta^2 z - vt}{\beta^2 - 1} \tag{43}$$

and then comes to a halt or begins some other type of motion. The observer will then detect only the field radiated by the image moving in the negative z direction and also the field from the rest of the trajectory, which now differs from the field of the image which is moving in the negative z direction. This simple argument resolves the question of whether the images can be observed separately. In other cases, in particular, in cases discussed below, there are further possibilities for a separate observation of images. For example, separate observation would be made possible by a difference in the frequencies at which the images radiate.

To complete the picture, we write expressions for the Liénard-Wiechert potentials in the case in which the charge is in uniform rectilinear motion at a superluminal velocity in a dispersionless medium with a permittivity ε and a magnetic permeability μ;

$$\mathbf{A}(\mathbf{r}, t) = \frac{q\mu}{c} \frac{2\mathbf{v}}{[(z-vt)^2 - (\varepsilon\mu\beta^2 - 1)(x^2 + y^2)]^{1/2}}, \tag{44}$$

$$\varphi(\mathbf{r}, t) = \frac{q}{\varepsilon} \frac{2}{[(z-vt)^2 - (\varepsilon\mu\beta^2 - 1)(x^2 + y^2)]^{1/2}}. \tag{45}$$

These expressions obviously describe the Čerenkov effect in a dispersionless medium. The fields in this case are expressed in terms of the potentials as in (32), and the Mach angle ψ (the vertex angle of the Čerenkov cone) is determined by the usual relation:

$$\sin\psi = \cos\theta_0 = \frac{c}{nv}. \tag{46}$$

Here θ_0 is the angle between the wave vector of the radiation and the velocity of the charge.

The problem of the radiation by a dipole in uniform rectilinear motion at a superluminal velocity was solved in 1942 by Frank.[8] He treated the case in which an electric dipole whose magnitude is a sinusoidal function of the time moves through a medium with a refractive index $n > 1$ at a velocity v greater than the phase velocity of light, c/n. In this case, Frank showed the following: If the observer receives waves radiated by the dipole which are propagating in a direction which differs from that in which the dipole is moving by an angle $\theta < \theta_0$, where θ_0 is the Čerenkov radiation angle ($\cos\theta_0 = c/nv$, where v is the velocity of the dipole), then the frequency ω of the wave which is received is given by

$$\omega = \frac{\Omega}{(nv/c)\cos\theta - 1} \qquad \left(\theta < \arccos\frac{c}{nv} = \theta_0\right). \tag{47}$$

In this expression, Ω is the dipole oscillation frequency. Under these conditions the denominator in expression (47) for the wave frequency is obviously positive.

If waves propagating at a direction making an angle $\theta > \theta_0$ with the direction in which the dipole is moving are received, their frequency is given by

$$\omega = \frac{\Omega}{1 - (nv/c)\cos\theta} \quad (\theta > \theta_0). \tag{48}$$

Expression (48) is characteristic of the ordinary Doppler effect, which occurs if the velocity of the sources does not exceed that of the signal. Expression (47), on the other hand, holds in the case of a superluminal velocity of the source and at observation angles $\theta < \theta_0$. The behavior described by (47) was called an "anomalous Doppler effect" by Frank.

In Ref. 8 and in some subsequent studies,[9,10] attention was focused for the most part on spectral components of the field of a moving dipole, i.e., equations relating the frequency, wave vector, and amplitude of the radiated waves. Below we solve the same problem expressed in terms of the retarded potentials. The approach makes it possible to see some space-time features of the superluminal Doppler effect.[11,14]

We first consider the radiation by a dipole in uniform rectilinear motion at a superluminal velocity in vacuum: $v > c$. The moving dipole is described by the polarization vector

$$\mathbf{P}(\mathbf{r}, t) = \mathbf{p}_0 e^{i\Omega t}\delta(\mathbf{r} - \mathbf{v}t). \tag{49}$$

As before, the solution for the fields could be expressed in terms of Liénard-Wiechert potentials, but in the case at hand it is more convenient to describe the fields by a Hertz vector $\mathbf{\Pi}(\mathbf{r},t)$, which satisfies the equation

$$\left(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{\Pi}(\mathbf{r}, t) = 4\pi\mathbf{P}(\mathbf{r}, t). \tag{50}$$

The electric field $\mathbf{E}(\mathbf{r},t)$ and the magnetic field $\mathbf{H}(\mathbf{r},t)$ are expressed in terms of this Hertz vector in the following way:

$$\mathbf{E} = \operatorname{grad}\operatorname{div}\mathbf{\Pi} - \frac{1}{c^2}\frac{\partial^2\mathbf{\Pi}}{\partial t^2}, \quad \mathbf{H} = \frac{1}{c}\operatorname{rot}\mathbf{\Pi}. \tag{51}$$

A solution of Eq. (50) is expressed in terms of the retarded function

$$\mathbf{\Pi}(\mathbf{r}, t) = \iint d\mathbf{r}'\, dt'\, \mathbf{P}(\mathbf{r}', t') \frac{\delta(t - t' - (1/c)|\mathbf{r} - \mathbf{r}'|)}{|\mathbf{r} - \mathbf{r}'|}\,\vartheta(t - t'). \tag{52}$$

Substituting expression (49) for polarization vector $\mathbf{P}(\mathbf{r},t)$ into this solution, and using the rules for integrating δ-functions, we find an equation which describes the space-time features of the radiation by a dipole in superluminal motion:

$$\mathbf{\Pi}(\mathbf{r}, t) = \frac{\mathbf{p}_0}{[(z - vt)^2 - (\beta^2 - 1)(x^2 + y^2)]^{1/2}}\left(e^{i\Omega t_1} + e^{i\Omega t_2}\right). \tag{53}$$

Here it has been assumed that the dipole is moving in the positive z direction. As in the case of a charge, Eq. (34) plays an important role: It has two roots, so there are two terms in expression (53) for the Hertz vector. The times t_1 and t_2 are expressed in terms of t in accordance with (34), which can be rewritten in the following way for given coordinates of the observer (x,y,z):

$$[(\beta+1)(vt'-z)-(vt-z)][(\beta-1)(vt'-z)+(vt-z)]=\beta^2(x^2+y^2).$$

We thus see that the behavior of t_1 and t_2 as a function of t is described by a branch of a hyperbola (Fig. 6). It can be seen from this figure that the Hertz vector becomes nonzero at the time t_0 given by

$$t_0=\frac{1}{v}\{z+[(\beta^2-1)(x^2+y^2)]^{1/2}\}.$$

It is also clear that t_1 is a decreasing function of the time t, while t_2 is an increasing function. We will show below that the first term in (53), which is proportional to $e^{i\Omega t_1}$, describes the anomalous Doppler effect, while the second, proportional to $e^{i\Omega t_2}$, describes the normal Doppler effect. We formally introduce the frequencies

$$\omega_{1,2}=\Omega\frac{dt_{1,2}}{dt}=\frac{\Omega}{\beta^2-1}\left\{\pm\frac{\beta(vt-z)}{[(vt-z)^2-(\beta^2-1)(x^2+y^2)]^{1/2}}-1\right\}; \quad (54)$$

this expression has the physical meaning of a frequency only in the case in which the frequency depends weakly on the time, as the wave amplitude in (53) does. Each of these requirements is met in the limit $t\to\infty$. In this case the relationship between t and $t_{1,2}$ becomes particularly simple. Taking the limit $t\to\infty$ in (35), we find

$$t_{1,2}=\frac{t}{1\mp\beta}\quad(t\to\infty), \quad (55)$$

i.e.,

$$t_1=-\frac{t}{\beta-1},\quad t_2=\frac{t}{\beta+1}. \quad (56)$$

This result means that the phase of the term $e^{i\Omega t_1}$ in the square brackets in (53) decreases, while the phase of the term $e^{i\Omega t_2}$ increases, with the time. If the signal is radiated by the source at the time t_1, then the source is at the point $z_1=vt_1$ at the time of the emission. It is not difficult to see that as a dipole moves along the positive z direction the point z_1 moves along the negative z direction. An observer following the source of radiation, which is at the point $z_1=vt_1$, therefore sees this source move opposite to the direction in which the dipole is moving. The observer thus sees both images, as in the case of a charge. These images are seen at angles

$$\cos\theta_{1,2}=\frac{(vt-z)\pm\beta[(vt-z)^2-(\beta^2-1)(x^2+y^2)]^{1/2}}{\beta(vt-z)\pm[(vt-z)^2-(\beta^2-1)(x^2+y^2)]^{1/2}}. \quad (57)$$

When the radiated signal first reaches the observer [the expression in square brackets in (57) vanishes in this case], the two sources are seen at the same angle, namely, the Čerenkov angle (Fig. 7,a). The angle at which an observer fixed at point ϑ (Fig. 7,b) sees the first image then begins to decrease,

$$\theta_1\leqslant\theta_0$$

and it ultimately vanishes. The angle at which the observer sees the second image increases,

$$\theta_2\geqslant\theta_0,$$

and tends toward π (Fig. 7,b). The first image is at point 1 and moves to the left; the second is at point 2 and moves to the right. The actual position of the charge at the time of observation is the vertex of the cone. After a long time ($t\to\infty$), the signals from the different images thus arrive at the observer from opposite sides.

Since both the frequency ω and the angle θ depend on the time, the time t can be eliminated from these functional dependences, and we can construct a functional dependence of, say, the frequency ω on the angle θ. Curiously, this function dependence,

$$\omega_{1,2}(t)=\frac{\Omega}{1-\beta\cos\theta_{1,2}(t)},$$

is exactly the same as (48), although the physical meaning of ω and θ here is completely different from that in (48).

The first term in square brackets in (53) describes a wave with a frequency

$$\omega=\frac{\Omega}{\beta-1}; \quad (58)$$

in other words, this term describes the radiation associated with the anomalous Doppler effect. We also note that the minus sign in the argument of the exponential function in this first term [see (56)] stems from the time reversal, i.e., from the circumstance, mentioned above, that t_1 is a decreasing function of t. Figuratively speaking, in the case of the anomalous Doppler effect, the later the waves are radiated by the dipole, the sooner they reach the observer. The term with $e^{i\Omega t_2}$ in (53) describes radiation associated with the normal Doppler effect.

The reversal of time in the anomalous Doppler effect was first noted in the field of acoustics.[12] Since it is vastly simpler to realize supersonic motions than superluminal mo-

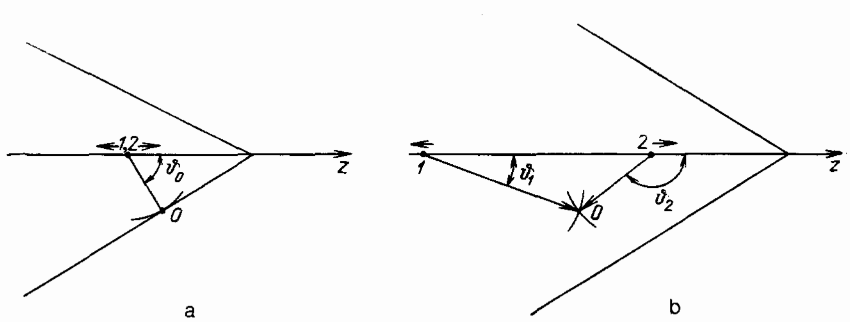

FIG. 7. The Mach cone at various observation times. a—At the time at which it passes the observer; b—after it has passed the observer. The observer sees two images.

tions, the question of the anomalous Doppler effect and time reversal is of greater interest in acoustics than in optics. This question was studied in Ref. 13, where a doubling of the images was also noted in this case.

Time reversal can be sensed even better in the following example, which is a slight alteration of the preceding example. We assume that a dipole moment is varying not in accordance with (49) but in accordance with the slightly more complex law

$$\mathbf{P}(\mathbf{r}, t) = \mathbf{p}_0 f(t)\, e^{i\Omega t} \delta(\mathbf{r} - \mathbf{v}t). \tag{59}$$

In other words, the dipole moment has a certain envelope $f(t)$ in addition to its sinusoidal time dependence. A solution of Eq. (50) in this case is

$$\Pi(\mathbf{r}, t) = \frac{p_0}{[(z - vt)^2 - (\beta^2 - 1)(x^2 + y^2)]^{1/2}} \left(f(t_1)\, e^{i\Omega t_1} + f(t_2)\, e^{i\Omega t_2}\right). \tag{60}$$

It can be seen from this solution that if, for example, the function $f(t)$ increases then the normal Doppler signal (the second term in this expression) also increases, since t_2 is an increasing function of t. The anomalous Doppler signal [the first term in (60)], on the other hand, decreases, since t_1 is a decreasing function of t. We also assume that the source is periodically radiating some asymmetric signal (so that we can distinguish between its beginning and its end; Fig. 8,a). The anomalous Doppler signal will then be time-reversed (Fig. 8,b), while the characteristics of the normal Doppler signal will occur in the same sequence as in $f(t)$ (Fig. 8,c).

The normal and anomalous Doppler effects found by Frank as spectral features of the field radiated in the course of superluminal motion can thus now be assigned to two different sources which are "seen" by the observer: two images.

We now write without derivation equations which describe the radiation by a dipole in uniform, rectilinear, superluminal motion in a refractive medium for the simplest case, in which the permittivity ε and magnetic permeability μ of the medium do not depend on the frequency. We assume as before that the dipole varies in accordance with (59). The Hertz vector $\Pi(\mathbf{r},t)$ then satisfies the equation

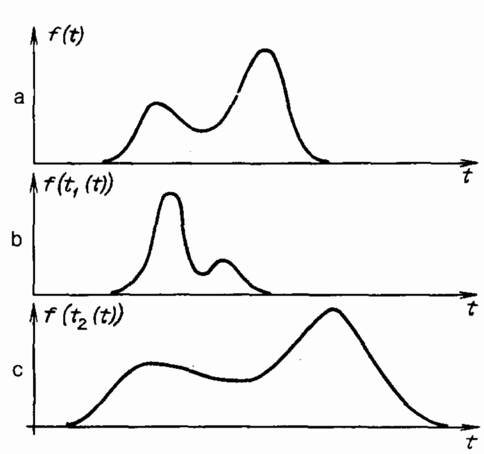

FIG. 8. a—Signal radiated by the source; b, c—signals received by the observer from the different images.

$$\left(\Delta - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)\Pi(\mathbf{r}, t) = 4\pi \mathbf{P}(\mathbf{r}, t) = 4\pi \mathbf{p}_0 f(t)\, \delta(\mathbf{r} - \mathbf{v}t), \tag{61}$$

where $n = (\varepsilon\mu)^{1/2}$ is the refractive index. The fields–electric and magnetic–are expressed in terms of Hertz vector $\Pi(\mathbf{r},t)$) in the following way:

$$\mathbf{E} = \frac{1}{\varepsilon}\,\mathrm{grad\,div}\,\Pi - \frac{\mu}{c^2}\frac{\partial^2 \Pi}{\partial t^2}, \quad \mathbf{H} = \frac{1}{c}\,\mathrm{rot}\,\frac{\partial \Pi}{\partial t}. \tag{62}$$

As in the absence of a medium, the solutioin of Eq. (61) describes the radiation of two images,

$$\Pi(\mathbf{r}, t) = \frac{p_0}{[(vt - z)^2 - (n^2\beta^2 - 1)(x^2 + y^2)]^{1/2}} \left(f(t_1)\, e^{i\Omega t_1} + f(t_2)\, e^{i\Omega t_2}\right), \tag{63}$$

and the times t_1 and t_2 are related to t by

$$t_{1,2} = -\left\{t - \frac{n^2 vz}{c^2} \pm \frac{n}{c}[(vt - z)^2 - (n^2\beta^2 - 1)(x^2 + y^2)]^{1/2}\right\}(n^2\beta^2 - 1)^{-1}. \tag{64}$$

As before, t_1 is a decreasing function of the time, and t_2 an increasing function. The observer sees images at angles θ_1 and θ_2 which satisfy

$$\theta_1 < \theta_0, \quad \theta_2 > \theta_0, \tag{65}$$

where $\theta_0 = \arccos(1/n\beta)$ is the Čerenkov angle. The frequencies of the radiation received by the observer from the different images are ($t \to \infty$)

$$\omega_1 = \frac{\Omega}{n\beta - 1}, \quad \omega_2 = \frac{\Omega}{n\beta + 1}. \tag{66}$$

We see that the frequencies in (66) and conditions (65) agree with (47) and (48) which were derived by Frank on the basis of spectral arguments.

5. UNIFORM SUPERLUMINAL MOTION OF A CHARGE ALONG A CIRCLE

The radiation by a charge in uniform motion along a circle at a superluminal velocity is a problem of considerable interest. We recall that the corresponding problem with a subluminal velocity, i.e., the synchrotron radiation problem, has been the subject of a formidable number of studies.[3] Let us examine the basic features of this problem.

A charge is moving counterclockwise along a circle in accordance with

$$x_0 = R\cos\omega t, \quad y_0 = R\sin\omega t, \tag{66'}$$

where

$$v = \omega R > c. \tag{67}$$

We also assume that the observation point 0 lies in the orbital plane of the charge and has coordinates $(0, -D, 0)$, so the overall picture is as shown in Fig. 9. The number of terms in the expressions for the Liénard-Wiechert potentials in (25) and (26) is determined by the number of roots (with respect to t') of Eq. (24). For the case in which a charge is moving along a circle, this equation becomes

$$g(t') = (R^2 + D^2 + 2RD\sin\omega t')^{1/2} - c(t - t') = 0. \tag{68}$$

This equation describes the dependence of the roots t'_α on

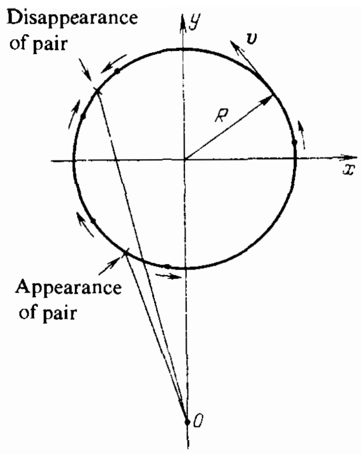

FIG. 9. The images seen by an observer at point 0 as the radiator moves along a circular orbit at a superluminal velocity.

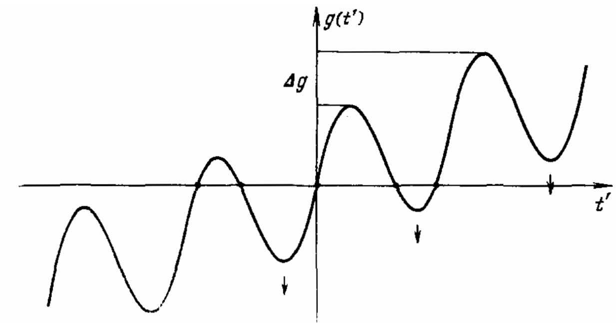

FIG. 10. Graphical solution of Eq. (24).

the instantaneous time t; it is transcendental and can be solved numerically. More important, however, is the qualitative behavior of the solutions of this equation, which can be seen from the plot of this solution in Fig. 10. This figure is a plot of the left side of Eq. (68). The intersections of this curve with the abscissa are obviously the roots of Eq. (68). Equation (68) could have a large number of roots: The higher the frequency ω or the linear velocity v of the charge, the more frequent the oscillations of the function $g(t')$ and thus the greater the number of roots for given values of R and D. The number of roots is always odd. At small values of ω, i.e., when the oscillatory curve is stretched out greaterly in the horizontal direction, there is obviously only a single root. As ω increases, the additional roots always appear in pairs, so the total number of roots always remains odd. During the superluminal motion of a charge along a circle, an observer at point 0 thus sees an odd number of images.

We have a few words to say about taking the limit from motion along a circle to motion along a straight line. This is not a totally trivial point, since for motion along a circle there are always an odd number of images, while for motion along a straight line there are only two, i.e., an even number, as was shown in the preceding section of this paper. When we take the limit of rectilinear motion, we let R and D go to infinity in such a way that the difference $D - R = d$ (the distance from the observer to the trajectory of the charge) remains finite. The retardation times $t - t'$ also remain finite. According to (67), the frequency ω vanishes at $v = \text{const}$. It then follows that the square root in Eq. (68) can be finite only near the value of t' corresponding to $\sin \omega t' = -1$. In this case, the square root is approximately $d = D - R$. Expanding $\sin \omega t'$ in a series around this value of t', and retaining the first two terms, we see that Eq. (68) becomes Eq. (34) for rectilinear motion, with the two roots of that equation. The other roots go off to infinity because the oscillations of the functions $g(t')$ become progressively more infrequent in the limit $\omega \to 0$. In the case of rectilinear motion, as in the case of circular motion, the number of images can thus be assumed to be odd, but only two of the images are at a finite distance from the observer.

Let us take a more detailed look at Eq. (68). The left side of Eq. (68) is a linear function of t, so the corresponding plot simply undergoes a uniform downward shift as t increases. Over one period of the motion of the charge along the circle, the plot of $g(t')$ moves a distance Δg downward (Fig. 10); this distance is equal to the vertical distance between two neighboring maxima. It follows that during one orbit of the circle by the charge one pair of images disappears, and another pair appears. The times at which the images appear and disappear correspond to cases in which the curve of $g(t')$ is tangent to the abscissa. At this points, the derivative $g'(t')$ vanishes; i.e., we have

$$g'(t') = c\,|1 - \beta(t_i')\cos\theta(t_i')| = 0. \tag{69}$$

This is the condition that the projection of the velocity of the charge onto the direction from the charge to the observer is equal to the velocity of light. For a given position of the observer on the trajectory of the charge (on the circle), there are thus two special points. At one of these points, two images merge and disapper, while at the other two images appear and move away from each other.

The quantity $\omega t'_\alpha$ is the azimuthal position of image α, so after a pair appears one of the images of this pair moves counterclockwise (Fig. 10; t'_α increases), while the other moves clockwise (t'_α decreases). Consequently, before a pair disappears, its constituent images are moving opposite to each other. The total number of images moving counterclockwise is an odd number, and the number of images moving clockwise is smaller by one.

The quantity in (69) appears in the denominator of the Liénard-Wiechert potentials in (25) and (26). Consequently, these potentials become infinite at the time at which a pair of images appears or disappears. This result is understandable on physical grounds. Since the projection of the velocity of the charge onto the direction to the observer is equal to the velocity of light at the points at which images appear and disappear, the charge and the wave which is radiates toward the observer move together "for a long time," so the wave amplitude has time to grow to infinity.

Unfortunately, it is not possible to write explicit expressions for the Liénard-Wiechert potentials in this case, because Eq. (68) cannot be solved explicitly. We thus turn to another characteristic of the fields: the surface on which the fields are discontinuous, and on one side of which the fields become infinite. In the case of uniform, rectilinear, superluminal motion of a charge along a circle, this surface is not a

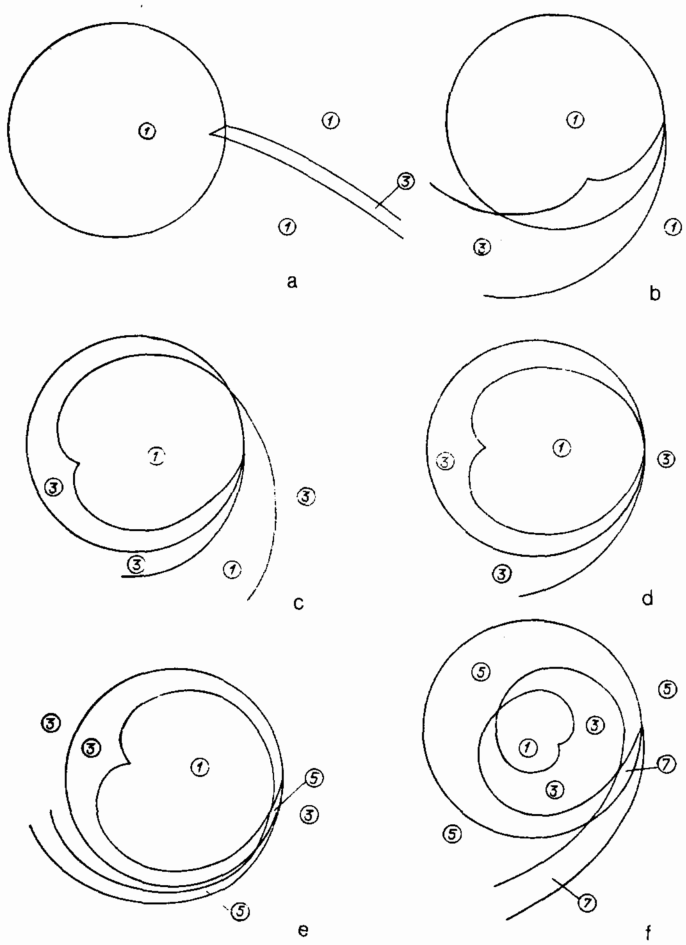

FIG. 11. Mach curve for superluminal motion of a radiator along a circular trajectory. The velocity increases in the order a–f.

cone, so we will call it simply a "Mach surface." The line along which this surface intersects the orbital plane is then a "Mach curve." The shape of the Mach surface or curve does not depend on the nature of the radiator, i.e., on whether it is a charge, a dipole, etc.

Let us consider a Mach curve which has been formed by the time $t = 0$. This curve is evidently the envelope of a family of circular fronts radiated by the charge before this time. The circular front radiated by a charge at a point with an azimuthal angle φ at the time $t = 0$ is described by the equation

$$(x-x_0)^2+(y-y_0)^2=(T\varphi)^2, \qquad (70)$$

where

$$T=\frac{Rc}{v}=\frac{R}{\beta}, \qquad (71)$$

$$x_0=R\cos\varphi, \quad y_0=R\sin\varphi. \qquad (72)$$

The phase φ is related to the radiation time by

$$\varphi=\frac{vt}{R}. \qquad (73)$$

By condition, both φ and t take on only negative values. We treat the circular wavefronts as a family of curves which depend on the parameter φ. The envelope of this family—the Mach curve—is then determined by the system of equations

$$(x-R\cos\varphi)^2+(y-R\sin\varphi)^2-(T\varphi)^2=0, \qquad (74)$$

$$R\sin\varphi\,(x-R\cos\varphi)-R\cos\varphi\,(y-R\sin\varphi)-T^2\varphi=0, \qquad (75)$$

the second of which is found by differentiating the first with respect to the parameter φ (as is easily verified). Solving this system of equations for x and y, we find

$$x=R[\gamma^2\varphi\sin\varphi\pm\gamma(1-\gamma^2)^{1/2}\varphi\cos\varphi+\cos\varphi],$$
$$y=R[\pm\gamma(1-\gamma^2)^{1/2}\varphi\sin\varphi-\gamma^2\varphi\cos\varphi+\sin\varphi], \qquad (76)$$

which are the equations of the Mach curve in parametric form ($\gamma=\beta^{-1}=c/v$). Calculating the curvature K of the Mach curve by the standard rules, we find

$$K=\frac{1}{R[\gamma\varphi\pm(1-\gamma^2)^{1/2}]}, \qquad (77)$$

where the plus sign refers to the internal branch of the Mach curve, and the minus sign to the external branch (Fig. 11). We see that in the case

$$\varphi=\tilde{\varphi}=-\frac{(1-\gamma^2)^{1/2}}{\gamma} \qquad (78)$$

the curvature of the internal branch becomes infinite. At the value of φ determined by (78), there is a cusp on the internal branch of the Mach curve.[15]

Figure 11 shows Mach curves plotted in accordance

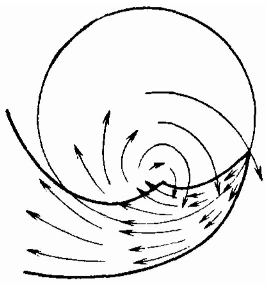

FIG. 12. Electric field in the case of superluminal motion of a radiator along a circular trajectory.

with (76) for various velocities of the circular motion of the radiation source. The velocity increases in the order a–f in Fig. 11. The Mach curve divides the entire space into regions in which the observer sees different numbers of images of the source. The entire curve is of course rotating around the center of the trajectory, at the same frequency as that of the charge. The observer thus sees different numbers of images at different times, although in all cases there is a region in which the observer always sees one image: when the observer is closer than the cusp on the Mach curve to the center of the trajectory.

The Mach curve has certain symmetries. Inside the trajectory, the Mach curve has mirror symmetry with respect to the straight line which connects the cusp to the center of the trajectory. The two external branches of the Mach curve are similar to each other; they coincide if one of them is rotated through an angle $2\tilde{\varphi}$.

In Fig. 11,a the velocity of the source is just slightly greater than the velocity of light, the angle $\tilde{\varphi}$ is small, and the observer sees three sources for only a brief interval, while it is in region 3. In Figs. 11,b and c, the Mach curve is shown for larger values of $\tilde{\varphi}$. In Fig. 11,d, the angle φ is equal to π, and the external branches of the Mach curve coincide. In this case an observer not on the trajectory always sees three images. At larger values of $\tilde{\varphi}$ (Fig. 11,e) a region appears in which the observer sees five images. At larger values, $\tilde{\varphi} > 2\pi$, a region appears in which the observer sees seven images; and so forth.

Substituting (78) into (76), we easily find that the distance from the center of the trajectories to the cusp on the Mach curve is $T = \gamma R$. As the source velocity and correspondingly the angle $\tilde{\varphi}$ increase, this distance decreases. The cusp on the Mach curve reaches the center of the trajectory only if the velocity is infinite.

As we have already mentioned, it is not possible to derive analytic expressions for the Liénard-Wiechert potentials or thus the fields in this case. However, knowing the Mach curve, we can sketch a qualitative picture of the field, drawing from the analogy with rectilinear motion. This picture of the field is shown in Fig. 12 for the case in which a negative charge is moving along a circular trajectory. The point of greater interest is the field near the cusp on the Mach curve, but efforts to calculate this field have not yet succeeded.

As it receives signals from images moving in the same direction as the radiator, the observer detects the normal Doppler effect and the normal time sequence of events. The signals from images which are moving in the direction opposite that of the radiator, in contrast, are characterized by the anomalous Doppler effect and a time reversal.

6. CONCLUSION

We hope that we have been able to demonstrate just how interesting the field of superluminal motions of charges and studies of the radiation accompanying this motion are. There is already an extensive literature on superluminal motions. For the most part, however, it deals with the radiation accompanying uniform motion, i.e., phenomena associated with the Čerenkov effect, the anomalous Doppler effect, etc. In this note we have attempted to call attention to the circumstance that a source in superluminal motion may be thought of as a set of several images, whose resultant field gives the field of the source, for both uniform and accelerated motion of the source. From this standpoint, the anomalous Doppler effect is also seen in a new light. The decay of one source into several images is seen in all the examples which have been considered.

The multiple-image effect could undoubtedly be seen experimentally; in fact, it might even be put to work in practical electronics. The topic of greatest interest for further study from that standpoint would be Čerenkov radiation in a magnetic field (i.e., on a circular trajectory of a charge) in a medium. It would not be necessary to observe the entire circular trajectory in order to observe a doubling of a radiation source or the disappearance of a pair of images; it would be sufficient to observe only the part of the trajectory which has the points at which the source doubles or the pair of images disappears.

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[1]B. M. Bolotovskiĭ, *Oliver Heaviside* [in Russian], Nauka, M., 1985.
[2]T. Des Coudres, *Archives Neerlandaises des sciences exastes et naturelles, Serie II*, Martinus Nijhoff, La Haye, 1900, T.V.P. 652 (Lorentz Übelband); A. Sommerfeld, Gött, Nachr. 99, 363 (1904); 201 (1905); G. Schott, *Electromagnetic Radiation*, Cambridge, 1912.
[3]I. E. Tamm and I. M. Frank, Dokl. Akad. Nauk SSSR **14**, 107 (1937); Tr. FIAN SSSR **2**(4), 63 (1944); I. S. Tamm, J. Phys. USSR **1**, 439 (1939).
[4]V. L. Ginzburg, *Theoretical Physics and Astrophysics*, Pergamon, Oxford, 1979 [Russ. original, Nauka, M., 1987].
[5]B. M. Bolotovskiĭ, *Theoretical Physics and Astrophysics*, (celebrating the 70th birthday of V. L. Ginzburg) [in Russian], Nauka, M., 1989, p. 24.
[6]L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th ed., Pergamon Press, Oxford, 1975 [Russian original, Nauka, M., 1973 and 1988].
[7]J. V. Jelley, *Čerenkov Radiation and Its Applications*, Pergamon, New York, 1958 [Russ. transl. IL, M., 1960]; V. P. Zrelov, *Čerenkov Radiation in High Energy Physics*, Israel Program for Scientific Translations, 1970 [Russ. original, Vols. 1 and 2, Atomizdat, M., 1968].
[8]I. M. Frank, Izv. Akad. Nauk SSSR Ser. Fiz. **6**, 3 (1942); Vavilov-Čerenkov Radiation [in Russian], Nauka, M., 1988.
[9]V. L. Ginzburg and V. Ya. Éĭdman, Zh. Eksp. Teor. Fiz. **26**, 1823 (1954).
[10]V. L. Ginzburg and I. M. Frank, Dokl. Akad. Nauk SSSR **56**, 583 (1947).
[11]B. M. Bolotovskiĭ and V. P. Bykov, Izv. Vyssh. Uchebn. Zaved., Radiofiz. **32**, 386 (1989) [Radiophys. Quantum Electron. **32** (1989)].
[12]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Addison-Wesley, Reading, Mass., 1959 [Russ. original, Gostekhizdat, M., 1953, p. 319].
[13]A. A. Kolomenskiĭ, Akust. Zh. **25**, 547 (1979) [Sov. Phys. Acoust. **25**, 312 (1979)].
[14]K. A. Barsukov and V. A. Gvozdev, Zh. Eksp. Teor. Fiz. **68**, 447 (1975) [Sov. Phys. JETP **41**, 217 (1975)].
[15]H. Ardavan, Proc. R. Soc. London, **424**, 113 (1988).

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