METHODOLOGICAL NOTES

Emission of radiation by particles in media with inhomogeneities and coherent bremsstrahlung

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A theoretical investigation is reported of coherent bremsstrahlung generated by a relativistic particle together with noncoherent Bethe–Heitler radiation in a medium characterized by macroscopic density inhomogeneities. These inhomogeneities may be due to the structure of the medium or due to fluctuations excited in an initially macroscopically homogeneous medium. The conditions for the observation of the coherent bremsstrahlung in various systems (plasma channel, pile of plates) are considered. The relationship between the coherent bremsstrahlung and some already known effects (inverse Compton effect involving Langmuir oscillations and transition radiation in media with inhomogeneities) is considered.

1. COHERENT MECHANISMS OF THE EMISSION OF RADIATION BY RELATIVISTIC PARTICLES IN A MEDIUM

It is well known[1,2] that a relativistic electron moving in a single crystal generates coherent bremsstrahlung and that the coherence in this case is related to the ordered distribution of atoms. In an ideal unbounded single crystal we have what is known as long-range order, i.e., the positions of atoms are fully correlated irrespective of the distance.

In a plasma and in noncrystalline condensed media there are also situations when the distribution of particles is ordered over macroscopic distances. Such an order may be established if a medium exhibits some random inhomogeneities of the density (for example, turbulence in a plasma), if acoustic waves propagate in a liquid or a solid, or if structures are formed deliberately (this is true of a pile of plates made of different materials). However, in such cases the correlation is different (statistical) from that in a single crystal and the ordered distribution of particles is local (within the limits of a correlation length).

Macroscopic inhomogeneities in a medium are found to affect considerably the generation of bremsstrahlung by relativistic particles and give rise to coherent bremsstrahlung in addition to the familiar Bethe–Heitler radiation. This can happen in spite of the fact that the wavelength of the emitted radiation is short compared with the scale of the inhomogeneities in a medium and the effect is related to the smallness of the longitudinal momentum transferred to the medium by an emitting relativistic particle. Consequently, radiation of frequency ω is generated over a distance of the order of $l_c \approx (c\gamma^2/\omega)[1 + (\omega_p^2\gamma^2/\omega^2)]$ (γ is the Lorentz factor of a particle and ω_p is the plasma frequency), which may be of macroscopic magnitude. If the medium contains density inhomogeneities of the same or smaller scale and the positions of electrons and nuclei in these inhomogeneities are correlated, an additional contribution to bremsstrahlung of relativistic particles appears in the form of coherent radiation.

The ordered distribution of particles in a medium is not the only factor that can be responsible for the emission of radiation by a fast particle in matter. The coherence effect may be manifested also in an amorphous medium when a particle is traveling at a superluminal velocity. This gives rise to the familiar effect of emission of the Vavilov–Cherenkov radiation. The factor responsible for the coherence in the case of the Vavilov–Cherenkov radiation is the incident particle itself: an electromagnetic field of a moving particle can be represented as a superposition of waves traveling at a velocity c/n (n is the refractive index of the medium). In a direction at an angle of θ relative to the particle velocity $\mathbf{v}$, a sequence of waves which is emitted is characterized by the same phases if v, n, and θ are related by

$$\frac{c}{n} = v\cos\theta. \tag{1}$$

The phase-locked waves propagate along the direction θ and they represent the Vavilov–Cherenkov radiation.

We shall consider these processes from the microscopic point of view in order to understand better the physical reason for the coherence of the bremsstrahlung, of the Vavilov–Cherenkov radiation, and of the radiation emitted by a medium under the influence of the field of the incident particle (this last process is called in Ref. 3 the transition scattering or the transition bremsstrahlung, whereas in Ref. 4 it is referred to as the polarization bremsstrahlung). The Feynman diagrams of these three processes are shown in Figs. 1–3.

In the bremsstrahlung case a fast particle exchanges a virtual photon of momentum $\mathbf{q}$ with a medium and emits a photon of momentum $\mathbf{k}$ (Fig. 1). In a disordered medium this gives rise to the familiar noncoherent Bethe–Heitler radiation. However, if a medium contains inhomogeneities

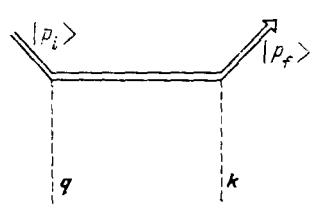

FIG. 1.

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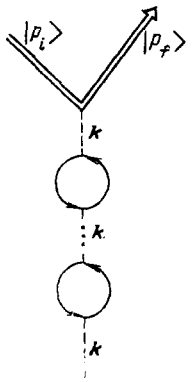

FIG. 2.

with a scale of l, then if $q_{\parallel} \approx l_c^{-1} \sim 2\pi l^{-1}$, the square of the Fourier component of the electromagnetic field of a virtual photon includes a contribution proportional to the square of the number of particles (see Sec. 2). This means that the particles in the medium interact in an ordered manner (via their fields) with the motion of a fast particle and this is the reason for the coherent bremsstrahlung. In the course of emission of radiation the particle drops from a state $|p_i\rangle$ to a state $|p_f\rangle$, which is different from $|p_i\rangle$. A transition of the medium to a different quantum state is not essential. Moreover, we can regard the medium as an object without any dynamic degrees of freedom creating simply an external field which acts on a particle.

In the case of the polarization bremsstrahlung a particle again exchanges a virtual photon with a momentum $\mathbf{q}$ with a medium, but in this case a photon is emitted by electrons in a medium (Fig. 3). Therefore, an allowance for the degrees of freedom of the medium is essential although the quantum state of the medium may not change: we can have either a process which is elastic in respect of the medium $|n_f\rangle = |n_i\rangle$, or there may be inelastic processes $|n_f\rangle \neq |n_i\rangle$. Coherent effects associated with the polarization bremsstrahlung can be of two kinds. In the frequency interval $\omega \lesssim c/R$, where R is the screening radius (equal to the radius of an atom or the Debye radius in a plasma) all the electrons in an atom or in a Debye sphere emit coherently. However, if the medium contains macroscopic inhomogeneities on a scale of l, then one additional coherent effect appears when $|\mathbf{q} - \mathbf{k}| \sim 2\pi l^{-1}$. This will be discussed in Sec. 4. In both cases (ordinary and polarization bremsstrahlung) the medium receives or gives up a nonzero momentum.

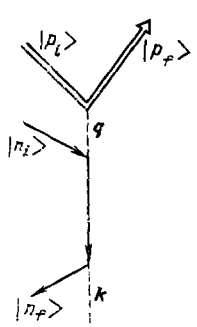

FIG. 3.

In the case of the Vavilov–Cherenkov radiation the interaction of a fast particle with a medium differs from the cases discussed above. A virtual photon emitted by the fast particle is scattered by the medium through zero angles, the state of the medium is not altered, and no momentum is transferred to the medium. The process of elastic scattering of a virtual photon is represented in Fig. 2 by a polarization operator. Repeated scattering of a virtual photon alters its dispersion law ("dressing" of a photon in a medium). For a set of values of $\mathbf{k}$ satisfying Eq. (1), such a "dressed" virtual photon reaches a material surface and is converted into a real photon giving rise to the Vavilov–Cherenkov radiation. Since no momentum is transferred to the medium, we cannot interpret the Vavilov–Cherenkov radiation as emitted by the particles of the medium excited by the incident particle. This was emphasized in the fundamental paper of Tamm and Frank.[5] The intensity of the Vavilov–Cherenkov radiation is governed solely by the refractive index and no other details of the structure of the medium traversed affect this radiation. Moreover, this radiation could appear also in vacuum if superluminal velocity of a charge particle would have been possible in vacuum.

We shall not discuss any further the coherent properties of the Vavilov–Cherenkov radiation, but concentrate on the coherent effects in the emission of radiation by an incident relativistic particle or by particles in a medium excited by the former and due to the presence of macroscopic inhomogeneities (see also Ref. 6). In some cases the coherent bremsstrahlung should give rise to significant observable effects which have not been mentioned in the literature; in other cases it reduces to the familiar effects, such as the inverse Compton effect representing the scattering of Langmuir waves into electromagnetic waves and the transition radiation. In the latter case we are dealing with the coherence in the polarization bremsstrahlung, which has been investigated intensively in recent years.[4] These effects can be considered not only macroscopically, but also as a result of coherent addition of the radiation fields associated with individual microparticles (atoms or electrons in a medium). This approach not only has methodological advantages, but also makes it possible to identify the conditions under which the coherent effects in the radiation are important.

2. THEORY OF THE BREMSSTRAHLUNG IN MEDIA WITH DENSITY INHOMOGENEITIES

The spectral and angular distribution of the density of the energy $\mathscr{E}_{\mathbf{n},\omega}$ emitted as radiation by a particle with a charge e during the whole of its motion can be represented by

$$\frac{d^2\mathscr{E}_{\mathbf{n},\omega}}{d\omega\, d\Omega} = \frac{\omega}{(2\pi)^2}\left|\mathbf{k}\int_{-\infty}^{+\infty} e^{-i\omega t}\mathbf{j}_{fi}(\mathbf{k};t)\,dt\right|^2, \quad \mathbf{n} = \frac{\mathbf{k}}{k}, \qquad (2)$$

where ω and $\mathbf{k}$ are the frequency and the wave vector of the emitted electromagnetic waves related to the dispersion law of the medium; Ω is the solid angle within which the radiation is emitted; $\mathbf{j}_{fi}(\mathbf{k}, t)$ is a matrix element of the current representing a transition between the initial i and final f states:

$$\mathbf{j}_{fi}(\mathbf{k}, t) = \langle f|\,\hat{\mathbf{j}}(\mathbf{r};t)\,e^{-i\mathbf{k}\mathbf{r}}\,|i\rangle. \qquad (3)$$

The energy of the interaction of a relativistic particle with a

medium is much less than its kinetic energy. We shall calculate the wave function of the incident particle using perturbation theory. In the zeroth order approximation, when the wave function represents a plane wave, Eq. (2) yields the Vavilov–Cherenkov radiation intensity if the condition (1) is satisfied. We shall consider the range of high frequencies when $n(\omega) < 1$. Allowing for the influence of the particles of the medium on the motion of the incident particle, we find that in the first order of perturbation theory Eq. (2) yields the familiar expression for the spectral density of the bremsstrahlung[1]:

$$d\mathscr{E}_\omega = \frac{e^4}{4\mathscr{E}^2 V (4\pi)^2} \frac{dq_0 d\mathbf{q} d\mathbf{k}}{(2\pi)^6} \frac{|\zeta(\mathbf{q}, q_0)|^2}{\varkappa_1^2 \varkappa_2^2} \Big[-2\varkappa_1\varkappa_2 \frac{q^2}{m^2} \Big(\frac{4\mathscr{E}^2}{m^2} + \varkappa_1 + \varkappa_2 - \frac{q^2}{m^2} - 2 \Big) + (\varkappa_1^2 + \varkappa_2^2) \Big(\varkappa_1\varkappa_2 + \frac{2q^2}{m^2} \Big) - \frac{8\mathscr{E}^2}{m^2} (\varkappa_1 + \varkappa_2)^2 \Big] \times \delta(\omega - q_0 - (\mathbf{k} - \mathbf{q})\mathbf{v}), \quad q = (q_0, \mathbf{q}), \quad p = (\mathscr{E}, \mathbf{p}), \quad (4)$$

where $\mathscr{E}$, $\mathbf{p}$, $\mathbf{v}$, and m are the energy, momentum, velocity, and mass of a relativistic particle (electron); $\varkappa_1 = 2k(p + q - k)/m^2$; $\varkappa_2 = -2kp/m^2$; $\mathbf{q}$ and q_0 are the momentum and energy transferred in a collision; V is the normalized volume [it is assumed in Eq. (4) that $\hbar = 1$ and $c = 1$]; $\zeta(\mathbf{q}, q_0)$ is the time component of the four-potential of the electromagnetic field created by all the particles in a plasma when they are in real motion. In the case of ordinary media and a nonrelativistic plasma the emission of radiation depends only on the scalar potential $\zeta(\mathbf{q}, q_0)$ and is independent of the vector potential $\mathbf{A}(\mathbf{q}, q_0)$. Summing the contributions of all the electrons and ions in a plasma, we find that $|\zeta(\mathbf{q}, q_0)|^2$ is described by

$$|\zeta|^2 = \frac{(4\pi e)^2}{q^2} \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dt' \exp[-iq_0(t - t')] \times \Big\langle \sum_{a=1}^{ZN} \sum_{b=1}^{ZN} \exp[i\mathbf{q}(\mathbf{r}_a(t) - \mathbf{r}_b(t'))] + Z^2 \sum_{A=1}^{N} \sum_{B=1}^{N} \exp[i\mathbf{q}(\mathbf{R}_A(t) - \mathbf{R}_B(t'))] - Z \sum_{a=1}^{ZN} \sum_{B=1}^{N} \exp[i\mathbf{q}(\mathbf{r}_a(t) - \mathbf{R}_B(t'))] - Z \sum_{A=1}^{N} \sum_{b=1}^{ZN} \exp[-i\mathbf{q}(\mathbf{R}_A(t) - \mathbf{r}_b(t'))] \Big\rangle, \quad (5)$$

where e and Ze are the charges of an electron and an ion; $\mathbf{r}_{a,b}$ and $\mathbf{R}_{A,B}$ are the position vectors of an electron and an ion; N is the total number of ions in the system.

Similar summing for media consisting of neutral atoms gives the following expression:

$$|\zeta|^2 = \frac{(4\pi e)^2 Z^2}{(q^2 + R^{-2})^2} \int dt \int dt' \exp[-iq_0(t - t')] \times \Big\langle \sum_{A=1}^{N} \sum_{B=1}^{N} \exp[-i\mathbf{q}(\mathbf{R}_A(t) - \mathbf{R}_B(t'))] \Big\rangle, \quad (6)$$

where R is the radius of an atom, and $\mathbf{R}_A$ and $\mathbf{R}_B$ are the position vectors of the nuclei. It is interesting that Eq. (5) can be reduced to Eq. (6) if a plasma exhibits no oscillations that would disturb its quasineutrality (such as Langmuir waves), whereas magnetoacoustic and shock waves may exist. This corresponds to the hypothesis of screening of the Coulomb fields of the nuclei by the electron Debye jackets, where R in Eq. (6) represents the Debye screening radius.

If the positions of different particles in a medium are completely random, then averaging of the double sum in Eq. (6) (representing the structure factor of the medium) results in a nonzero contribution only from the terms with $A = B$ and the radiation is described by the familiar Bethe–Heitler expression for the noncoherent bremsstrahlung.

However, if waves propagate in a medium (for example, if they are acoustic waves), or if structures are formed or if there are random inhomogeneities of the density, the positions of the particles may be correlated at sufficiently long distances so that the double sum in Eq. (6) differs from N. The corresponding corrections to $|\xi|^2$ describe, via Eq. (4), the process of generation of the coherent bremsstrahlung which in some cases can be quite strong and dominant within certain frequency intervals.

If density inhomogeneities are present in a medium, the average in Eq. (6) should be carried out with the aid of appropriate correlation functions separating in the sum the autocorrelation times ($A = B$) from the pair correlations ($A \neq B$):

$$\Big\langle \sum_{A=1}^{N} \sum_{B=1}^{N} \exp[i\mathbf{q}(\mathbf{R}_A(t) - \mathbf{R}_B(t'))] \Big\rangle = \frac{N}{V} \int f(\mathbf{p}) w_a(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2, \tau) \times \exp[i\mathbf{q}(\mathbf{r}_1 - \mathbf{r}_2)] d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{p}_1 d\mathbf{p}_2 + \frac{N(N-1)}{V^2} \int f^{(2)}(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2, \tau) \times \exp[i\mathbf{q}(\mathbf{r}_1 - \mathbf{r}_2)] d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{p}_1 d\mathbf{p}_2, \quad (7)$$

where $f(\mathbf{p})$ is the distribution function of the momenta of the particles in the medium, normalized to unity; w_a is the autocorrelation function describing the probability of a change in the coordinate and momentum of a single particle from the values $\mathbf{r}_1$ and $\mathbf{p}_1$ to $\mathbf{r}_2$ and $\mathbf{p}_2$ in a time τ; $f^{(2)}$ is a two-particle distribution function:

$$f^{(2)} = f(\mathbf{p}_1) f(\mathbf{p}_2) + g(\mathbf{r}_1 - \mathbf{r}_2, \mathbf{p}_1, \mathbf{p}_2, \tau), \quad (8)$$

which contains a two-particle correlation function g and V is the volume of the system. Equation (7) allows for the fact that in the case of a medium which is statistically homogeneous in space and time the functions w_a and g depend only on the difference between the times and coordinates.

Calculation of $f(\mathbf{p})$ and w_a makes it possible to find, with the aid of Eq. (7), the corrections to the usual bremsstrahlung associated with the motion of particles in the medium. However, this motion has little effect on the radiation emitted by an ultrarelativistic particle and we shall ignore it. The term $f(\mathbf{p}_1)f(\mathbf{p}_2)$ in Eq. (8) makes no contribution to the radiation, because its Fourier transform contains the delta function $\delta(\mathbf{q})$, representing the emission of a wave of infinite wavelength. Subject to these comments, we can transform Eq. (7) to

$$\Big\langle \sum_{A=1}^{N} \sum_{B=1}^{N} \exp[i\mathbf{q}(\mathbf{R}_A(t) - \mathbf{R}_B(t'))] \Big\rangle = N + \frac{N(N-1)}{V} G(\mathbf{q}, \tau), \quad (9)$$

where

$$G(\mathbf{q},\tau)=\frac{|\Delta N(\tau)|_{\mathbf{q}}^2}{N^2}$$
$$=\int g(\mathbf{r}_1-\mathbf{r}_2,\mathbf{p}_1,\mathbf{p}_2,\tau)\exp[i\mathbf{q}(\mathbf{r}_1-\mathbf{r}_2)]\,d(\mathbf{r}_1-\mathbf{r}_2)\,d\mathbf{p}_1\,d\mathbf{p}_2 \tag{10}$$

is the spectrum of inhomogeneities of the concentration of atoms, etc. in a medium; in Eq. (10) we can ignore the dependence on time τ if the velocity of propagation of inhomogeneities in the medium is low compared with the velocity of light.

Substituting Eq. (9) into Eq. (6) and then Eq. (6) into Eq. (4), eliminating the δ function, and integrating with respect to angles as shown in Ref. 1, we obtain (with c and $\hbar$ present explicitly):

$$I_\omega=I_\omega^{\mathrm{BH}}+\frac{4e^6Z^2\omega}{\mathscr{E}^2cV^2}\int_0^{mc/\hbar}q_\perp\,dq_\perp\int_{q_{\min}}^{mc/\hbar}dq_\parallel\,\frac{|\Delta N|_{\mathbf{q}}^2}{(q^2+R^{-2})^2}$$
$$\times\frac{q_\perp^2}{q_\parallel^2}\left(1+\frac{2q_{\min}}{q_\parallel}+\frac{2q_{\min}^2}{q_\parallel^2}\right), \tag{11}$$

where $\mathscr{E}$ is the energy of a relativistic particle and

$$q_{\min}(\omega)=\frac{\omega}{2c\gamma^2}\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right)=l_c^{-1} \tag{12}$$

is the reciprocal of the coherence length.

If we allow for the excitation of an electron in an atom the form of $q_{\min}(\omega)$ changes and this alters the frequency dependence of the component of the radiation associated with inelastic processes. Usually these processes can be allowed for if the factor Z^2 in the Bethe–Heitler expression is replaced with $Z(Z+A)$, where $A\approx 1$. In fact, A is not a constant but depends on the radiation frequency. We shall consider the simplest case of a medium consisting of hydrogen atoms. Then, the minimum value of the transferred momentum found allowing for the excitation of an atom to the first level is

$$q_{\min}(\omega)=\frac{\Delta E^{01}}{c\hbar}+\frac{\omega}{2c\gamma^2}\left(1+\frac{\omega_p^2\gamma^2}{\omega^2}\right), \tag{13}$$

where ΔE^{01} is the energy of the excitation to the first level. Substituting Eq. (13) into the radiation intensity (11), we find A in the form

$$A(\omega)=\frac{1}{1+\{2\Delta E^{01}\gamma^2/\hbar\omega\,[1+(\omega_p^2\gamma^2/\omega^2)]\}}. \tag{14}$$

Therefore, inelastic processes contribute in the frequency range $\omega>2\Delta E^{01}\gamma^2/\hbar$. It should be noted that in the case of inelastic processes the screening form factor is somewhat different from $(1+R^{-2}q^{-2})^2$ (it has other power exponents—see Ref. 7). This modifies somewhat the argument of the logarithm in the Bethe–Heitler expression and alters the radiation intensity by a few percent.

In the case of a many-electron atom the processes of excitation of the individual electrons make an additive contribution to the radiation intensity and the spectrum has the form shown in Fig. 4, where ΔE_{0f} represents the excitation energies of different levels. We shall consider only the elastic component of the radiation intensity, since the coherent effects appear at frequencies lower than $2\gamma^2\Delta E_{0f}/\hbar$.

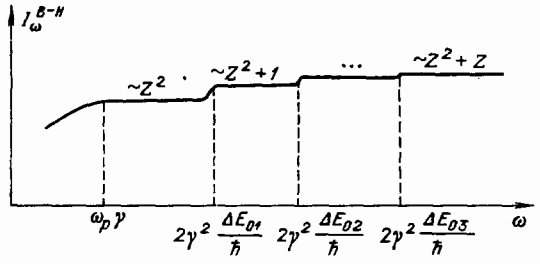

FIG. 4.

If the distribution of inhomogeneities in a substance is on the average isotropic, the corresponding spectrum

$$|\Delta n|_{\mathbf{q}}^2=\frac{|\Delta N|_{\mathbf{q}}^2}{V^2} \tag{15}$$

depends only on the modulus of the vector $\mathbf{q}$. Replacing the variables $q_\perp$ and $q_\parallel$ with q and θ and integrating with respect to the angle, we obtain

$$I_\omega=I_\omega^{\mathrm{BH}}+\frac{16}{3}\frac{e^2Z^2r_0^2}{1+(\omega_p^2\gamma^2/\omega^2)}\int_{q_{\min}}^{mc/\hbar}\frac{|\Delta n|_q^2q^2\,dq^2}{(q^2+R^{-2})^2}, \tag{16}$$

where r_0 is the classical radius of an electron.

A comparison of the noncoherent and coherent parts of Eq. (16) shows that the latter is always small compared with the former, since it contains a small factor $(R/l_c)^4$. Formally, this circumstance is due to the fact that Eq. (11) is dominated by large values $q\sim q_\perp\sim R^{-1}$, whereas we have $q_\parallel\ll q_\perp$. The correlation function $|\Delta n|_q^2$, occurring in Eq. (16) differs from zero over much longer distances $l\sim l_c\gg R$, and at $l\sim R$ practically vanishes.

It follows from the above analysis that the coherent bremsstrahlung may be of significant intensity only when the spectrum of inhomogeneities is strongly anisotropic. This intensity reaches its highest value if the longitudinal correlation length is of the order of, or greater than, the coherence length l_c and the transverse correlation length is of the order of the screening radius R. This is precisely the case in single crystals,[1] so that the coherent part of the radiation is high along certain directions. However, averaging of the radiation intensity over the angles of orientation of a crystal relative to the velocity of a particle has the effect that the coherent part of the radiation is small compared with the noncoherent Bethe–Heitler radiation. This is a familiar result,[1] which is in full agreement with the above conclusion on the negligibly low intensity of the coherent bremsstrahlung in a medium with isotropic macroscopic inhomogeneities.

In the case of a one-dimensional distribution of inhomogeneities (such as a pile of plates with a relativistic particle moving at right-angles to them) described by

$$|\Delta n|_{\mathbf{q}}^2=\langle\Delta n^2\rangle\delta(\mathbf{q}_\perp)K(q_\parallel) \tag{17}$$

the coherent bremsstrahlung vanishes identically because of the presence of a factor $\delta(\mathbf{q}_\perp)$ (although there exists coherent radiation emitted by electrons in the medium and it is known as the transition radiation). However, if we consider a pile of plates oriented at an angle to the velocity of a particle, then the coherent bremsstrahlung becomes possible. In

the simplest case of a sinusoidal modulation of the density

$$n = n_0 + \Delta n \sin(\mathbf{k}_0\mathbf{r}), \tag{18}$$

the coherent radiation intensity is

$$I_\omega^{\rm coh} = \frac{2Z^2e^6\omega(\Delta n)^2}{\pi\mathscr{E}^2c}\,{\rm tg}^2\alpha\frac{R^4}{(1+k_0^2R^2\sin^2\alpha)^2} \times\left(1+\frac{2}{k_0l_c\cos\alpha}+\frac{2}{k_0^2l_c^2\cos^2\alpha}\right)\theta(k_0\cos\alpha - l_c^{-1}), \tag{19}$$

where

$$k_0 = |\mathbf{k}_0|,$$

α is the angle between $\mathbf{k}_0$ and $\mathbf{v}$, and θ is the Heaviside step function. The highest radiation intensity is along an angle described by $\cos\alpha \approx 1/k_0l_c \ll 1$, which corresponds to an almost grazing incidence of a particle in a pile of plates (it is assumed that $k_0^{-1} \ll l_c$). An estimate of the intensity gives

$$I_\omega^{\rm coh} \approx I_\omega^{\rm BH}\left(\frac{\Delta n}{n}\right)^2(nR^2l_c)(k_0^2R^2), \tag{20}$$

and the ratio of $I_\omega^{\rm coh}$ to the intensity I_ω^p of the transition radiation generated in a pile of plates in the case of oblique incidence is

$$\frac{I_\omega^{\rm coh}}{I_\omega^p} \approx \frac{R^4k_0^4l_c\omega}{[1+(\omega_p^2\gamma^2/\omega^2)]\,c}. \tag{21}$$

It follows from Eq. (21) that in this situation the coherent bremsstrahlung can be the main mechanism at frequencies exceeding $\omega_p\gamma$ if the condition $k_0R > \gamma^{-1/2}$ is satisfied.

The coherent bremsstrahlung is also possible if the investigated medium is confined in the transverse direction to a sufficiently small size d. In this case the factor $\delta(\mathbf{q}_\perp)$ is replaced with a different function $\Psi(\mathbf{q}_\perp)$ which has a maximum in the region $q_\perp \sim d^{-1}$. This situation may occur in the case of practical importance when an electron or a beam of electrons propagates in a plasma channel. The characteristic lengths in the longitudinal and transverse directions are then very different. If we assume that the spectrum of inhomogeneities along the channel is described by a power law $|\Delta n|_q^2 \sim q_\parallel^{-\nu}$, the spectrum of inhomogeneities is

$$|\Delta n|_{\mathbf{q}}^2 = \frac{1}{2}(\nu-1)\langle\Delta n^2\rangle\frac{k_0^{\nu-1}}{q_\parallel^\nu}\frac{J_1^2(q_\perp d)}{q_\perp^2}\left(1-\frac{n_{\rm out}}{n_{\rm ch}}\right)^2, \tag{22}$$

where $L_0 = 2\pi/k_0$ is the main scale of the turbulence; $n_{\rm out}$ and $n_{\rm ch}$ are the concentrations of particles outside and inside the channel; $\langle\Delta n^2\rangle$ is the average of the square of inhomogeneities on the scale of $l \approx L_0$. We should also mention that instead of a density jump there may be a jump in the screening radius and the coherence effect is then retained. Substituting $|\Delta n|_q^2$ in Eq. (11) and integrating with respect to $dq_\perp$ and $dq_\parallel$, we obtain the following expression for the bremsstrahlung spectrum in such a channel:

$$I_\omega^{\rm coh} = \frac{(\nu-1)(5\nu^2+19\nu+16)}{(\nu+1)(\nu+2)(\nu+3)}\frac{e^6Z^2\omega}{\mathscr{E}^2c}\frac{\langle\Delta n^2\rangle k_0^{\nu-1}R^3}{q_{\min}^{\nu+1}(\omega)\,d} \times\left(1-\frac{n_{\rm out}}{n_{\rm ch}}\right)^2 \tag{23}$$

The ratio of this intensity to the intensity of the Bethe–Heitler radiation is

$$\frac{I_\omega^{\rm coh}}{I_\omega^{\rm BH}} \approx \frac{\langle\Delta n^2\rangle}{n^2}\left(\frac{l_c}{L_0}\right)^{\nu-1}\frac{R}{d}(nR^2l_c). \tag{24}$$

This ratio may be much greater than unity. In particular, if $l_c(\omega_p\gamma) \approx L_0$, $d/R \approx 1$, $n \sim 10^{12}$–10^{16} cm^{-3}, $\gamma = 10^4$, $R \sim 10^{-3}$–10^{-5} cm, we find that at the frequency $\omega_{\max} = \omega_p\gamma$ this ratio is

$$\frac{I_{\omega_p\gamma}^{\rm coh}}{I_{\omega_p\gamma}^{\rm BH}} \sim 5\cdot(10^6-10^8)\frac{\langle\Delta n\rangle^2}{n^2}, \tag{25}$$

where $\omega_{\max} \sim 10^{15}$–$10^{17}$ s^{-1} and the required channel width is $d \approx 10^{-3}$–10^{-5} cm. We can easily see that this transverse channel becomes greater in plasmas of lower density. In experiments involving the passage of high-current beams through a plasma such coherent radiation has not been observed because the dimensions of the beam and the channel have been much greater than R (Ref. 8).

Anisotropic quasi-one-dimensional distributions can in some cases be observed in condensed media. This applies in particular to materials composed of linear polymer molecules oriented parallel to one another.

3. INVERSE COMPTON SCATTERING OF LANGMUIR WAVES IN A PLASMA

We shall consider the coherent bremsstrahlung of a relativistic electron traveling in a plasma where Langmuir waves are excited and the spectrum of these waves can be arbitrary. In calculation of the intensity of the radiation in this case we shall use the initial expression (5) which contains four double sums. Since Langmuir waves are associated with oscillations of just the electron component of a plasma, two-particle correlations related to these waves are present only in the electron sum

$$\left\langle\sum_{a\neq b}^{ZN}\sum^{ZN}\exp[i\mathbf{q}(\mathbf{r}_a(t)-\mathbf{r}_b(t'))]\right\rangle = \frac{ZN(ZN-1)}{V}g_{\mathbf{q}}(\tau) = \frac{ZN(ZN-1)}{V}\frac{|\Delta n|_{\mathbf{q}}^2}{n^2}e^{i\omega_p\tau}. \tag{26}$$

On the other hand, it follows from the Maxwell equation div $\mathbf{E} = 4\pi\rho$ (in the case of isotropically distributed wave vectors) that

$$|\Delta n|_{\mathbf{q}}^2 = \left(\frac{q}{4\pi e}\right)^2|\mathbf{E}^l|_{\mathbf{q}}^2, \tag{27}$$

where $|\mathbf{E}^l|_q^2$ is the Fourier spectrum of the Langmuir wave field. Substituting Eqs. (26) and (27) into Eq. (5), we obtain

$$|\zeta|_{\mathbf{q},q_0}^2 = \frac{|\mathbf{E}^l|_{\mathbf{q}}^2}{q^2}\delta(q_0-\omega_p). \tag{28}$$

We can readily see that Eq. (28) is the potential of the field of Langmuir waves and the radiation emitted by an electron moving in such a potential represents, by definition, the inverse Compton scattering of Langmuir waves into electromagnetic waves.[3] It is interesting to note that this result is obtained here by coherent addition of the bremsstrahlung emitted by a relativistic electron as it collides with individual thermal electrons oscillating in a Langmuir wave.

4. TRANSITION RADIATION AS THE COHERENT POLARIZATION BREMSSTRAHLUNG[9]

The Bethe–Heitler expression is known to describe only part of the total bremsstrahlung of particles in a medium. The other part represents the noncoherent radiation emitted by the particles of the medium excited by the relativistic particle and known as the transition bremsstrahlung[3] or the polarization bremsstrahlung.[4]

It is shown above that under certain conditions we can expect not only the ordinary bremsstrahlung in a periodic or a randomly inhomogeneous medium, but also the coherent bremsstrahlung. Here, we shall discuss the corresponding coherent effect in the case of the polarization bremsstrahlung. It is found that the coherent polarization bremsstrahlung is identical with the transition radiation of a relativistic particle in a spatially homogeneous medium.

We shall assume that a relativistic particle is traveling uniformly along a straight line in a spatially periodic medium with the concentration of atoms varying periodically in space:

$$n(z) = n_0 + \frac{\Delta n}{2} \sin k_0 z, \quad k_0 R \ll 1, \tag{29}$$

where z is the coordinate along the particle velocity. The polarization bremsstrahlung is described by the expression[4]

$$I^p_{\omega,n} = \frac{4vr_0^2 e^2 Z^2}{(2\pi)^2 cV} \int d^2\mathbf{k}'_\perp \frac{k'^2_\perp + (\omega^2/v^2\gamma^4) - [\mathbf{nk}' - \mathbf{nv}(\omega/c^2)]^2}{\{k'^2_\perp + (\omega^2/v^2)[(1/\gamma^2) + (\omega_p^2/\omega^2)]\}^2} \times \frac{\left\langle \sum_{A=1}^{N} \sum_{B=1}^{N} \exp[i(\mathbf{k}-\mathbf{k}')(\mathbf{R}_A - \mathbf{R}_B)] \right\rangle}{[1 + (\mathbf{k}-\mathbf{k}')^2 R^2]^2},$$

$$\mathbf{n} = \frac{\mathbf{k}}{k}, \quad k'_z = \frac{\omega}{v}, \tag{30}$$

in which the structure factor should be calculated allowing for the spatial modulation of the density of the medium [Eq. (16)]. The velocity of the relativistic particle is assumed to be $v \approx c$. We then find, by analogy with Eqs. (7) and (9), that

$$\left\langle \sum_{A=1}^{N} \sum_{B=1}^{N} \exp[i(\mathbf{k}-\mathbf{k}')(\mathbf{R}_A - \mathbf{R}_B)] \right\rangle = N + \frac{N(N-1)}{V^2} \times \int \Delta f(z_1)\Delta f(z_2) \exp[i(\mathbf{k}-\mathbf{k}')(\mathbf{r}_1 - \mathbf{r}_2)]\, d\mathbf{r}_1\, d\mathbf{r}_2$$

$$= N + \frac{N(N-1)}{V} \delta(\mathbf{k}_\perp - \mathbf{k}'_\perp) \left(\frac{\Delta n}{n}\right)^2 \delta(k_z - k'_z - k_0), \tag{31}$$

where $\Delta f(z) = \Delta n \sin(k_0 z)/2n$.

After substitution of Eq. (31) into Eq. (30), we find that integration in Eq. (30) carried out using the delta function of $d^2\mathbf{k}'_\perp$ and the angles of the vectors $\mathbf{n}$ is a trivial procedure. The spectrum of the coherent polarization bremsstrahlung is

$$I^{p\,\mathrm{coh}}_\omega = \frac{e^2\omega_p^4}{16(2k_0c)^2} \left(\frac{\Delta n}{n}\right)^2 \frac{1}{c\omega} \left(\frac{2k_0c}{\omega} - \frac{1}{\gamma^2} - \frac{\omega_p^2}{\omega^2}\right), \tag{32}$$

where

$$k_0 c \leqslant \omega \leqslant 2k_0 c\gamma^2.$$

The above expression is easily demonstrated[3] to be identical with the expression for the intensity of the transition radiation in a medium with a periodic variation of the concentration of atoms given by Eq. (29). This is also true in the case when a medium contains random inhomogeneities of the density, including an isotropic distribution over the wave vectors.[10] This means that the transition scattering[3] of the field of a relativistic particle by macroscopic inhomogeneities of the density of a medium represents the coherent sum of the corresponding scattering events involving individual particles of the medium subject to an allowance for their spatially inhomogeneous distribution. Since the electrons in the medium excited consecutively by the propagating relativistic particle emit coherently, the coherent (i.e., transition) radiation has a narrow angular distribution along the particle velocity in contrast to the noncoherent polarization bremsstrahlung with the almost isotropic angular distribution.

In this case the situation is analogous to that discussed in the preceding section which deals with the inverse Compton scattering of Langmuir waves in a plasma. In both cases the electrons in the medium play an important part in the emission of the radiation, but the ions do not. Therefore, in one case the macroscopic electric field of oscillating electrons is uncompensated and in the other this is true of the electron polarization current.

However, in the case of the bremsstrahlung of an electron in a medium with inhomogeneities that do not disturb its quasineutrality, a significant radiation can be observed only under certain special conditions described more fully in Sec. 2.

The discussion in the present section, based on the summation of microscopic fields of the individual particles in the medium, allows us to solve a wider range of problems than the traditional macroscopic approach. In particular, our method can be applied also to microscopic inhomogeneities when the parameter $k_0 R$ is not small.

5. CONCLUSIONS

The analysis given above shows that in general coherent bremsstrahlung can form if the spectrum of fluctuations of the density of particles in a medium is sufficiently anisotropic, as in the case of a single crystal, a narrow plasma channel, and certain other systems. The long-range order over distances of the order of the coherence length of the radiation is sufficient.

Experimental investigations of the coherent bremsstrahlung in these special cases is extremely desirable, because it will extend our knowledge of the radiation emitted by relativistic particles propagating in a plasma or in solids.

We also analyzed the close relationship between the various radiation emission mechanisms. We found that, in particular, the coherent bremsstrahlung in a plasma with excited Langmuir waves is identical with inverse Compton scattering of these waves into electromagnetic waves, whereas coherent polarization bremsstrahlung is identical with transition radiation (or scattering) by the same inhomogeneities.

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