Demonstration of the law of electromagnetic induction

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At the university level demonstrations of the phenomenon of electromagnetic induction usually involve a solenoid connected to a galvanometer that registers the electromotive force (emf) generated by changes in the magnetic flux through the solenoid.[1] In turn, changes in the magnetic flux through the solenoid are accomplished either by deforming or moving the solenoid in an external magnetic field, or by time variation of the magnetic flux. All these techniques, as well as the demonstration in which the changes in the magnetic flux through a solenoid are effected via a second solenoid connected to a source of alternating current,[2] illustrate only the existence of inductive emf. These demonstrations shed no light on the time evolution of the electromagnetic induction process or its dependence on the electrical parameters of the circuit. In other words, while these demonstrations do prove the existence of inductive emf, they actually do not demonstrate the law of electromagnetic induction.

In this report we propose that the phenomenon of electromagnetic induction can be illustrated better by employing a special signal, consisting of an harmonically time-varying alternating voltage that contains narrow rectangular pulses[3] separated by π radians in the vector diagram. This signal is delivered to one of two neighboring solenoids and the time evolution of the electrical signals in both solenoids can be monitored on a two-beam oscilloscope.

The law of electromagnetic induction states that the electromagnetic induction emf $\mathscr{E}_i$ in a solenoid is equal in magnitude and opposite in sign to the rate of change in the magnetic flux Φ_m through the solenoid:

$$\mathscr{E}_i = -\frac{d\Phi_m}{dt}. \tag{1}$$

This law can be demonstrated by employing the apparatus shown schematically in Fig. 1. The setup consists of a harmonic voltage generator *1*, a special signal generator *2*, a solenoid *3* in series with a resistor *4*, another solenoid *5* in series with a resistor *6*, and a two-beam oscilloscope *7*. The function of resistor *4* is to produce a current of the required form, while resistor *6* prevents parasitic high-frequency oscillations. The voltage drops measured over the two resistors are fed into the first Y_I and second Y_{II} vertical channel inputs of the two-beam oscilloscope.

Solenoids *3* and *5* should be of the same length l and have approximately equal turn diameters. They are to be held motionless, with solenoid *5* fitting on-axis inside solenoid *3*.

Suppose that a current I flows through solenoid *3*. The magnetic flux Φ generated by this current is proportional to I; in vacuum we have

$$\Phi = L_1 I = \frac{\mu_0 N_1^2 S I}{l}, \tag{2}$$

where L_1 is the inductance of the first solenoid; N_1 is the number of winding turns in the first solenoid; S is the turn area; μ_0 is the permeability.

If the inductive impedance of the first solenoid is small compared to the resistance of resistor *4*, the current I is related to the voltage U applied to the first solenoid by Ohm's law

$$I = \frac{U}{R_1}. \tag{3}$$

Substituting (3) into (2) we obtain

$$\Phi = \frac{\mu_0 N_1^2 S U}{l R_1}, \tag{4}$$

where U can be taken as the voltage drop over resistor *4*.

Consequently, if the alternating voltage U is fed into the first vertical input Y_I of the two-beam oscilloscope, the vertical deviation of the first beam will be proportional to the magnetic flux Φ through the first solenoid *3*.

The magnetic flux through a single turn of the second solenoid *5* will be $\Phi = BS = \mu_0 N_1 S I / l$. The total magnetic flux through the second solenoid containing N_2 turns is then

$$\Phi_m = \Phi_1 N_2 = \frac{\mu_0 N_1 N_2 S I}{l}. \tag{5}$$

If $N_1 = N_2$, it follows from (2), (4), and (5) that

$$\Phi_m = \Phi = \frac{\mu_0 N_1^2 S U}{l R_1}. \tag{6}$$

The electromagnetic induction emf generated in the second solenoid is

$$\mathscr{E}_i = -\frac{d\Phi_m}{dt} = -\frac{\mu_0 N_1^2 S\, dU}{l R_1 dt} = -\frac{d\Phi}{dt}. \tag{7}$$

Consequently, if the emf $\mathscr{E}_i$ is fed into the second vertical input Y_{II} of the two-beam oscilloscope, then, as long as the resistor *6* is sufficiently small, the vertical beam deviation will be proportional to the time derivative $d\Phi/dt$ of the magnetic flux. The rate of change of the magnetic flux, in turn, will be proportional to the first time derivative of the applied voltage U.

If the voltage at resistor *4* consists of a time-varying sinusoid that contains rectangular pulses, then the time derivative signal will consist of a cosine function with deriva-

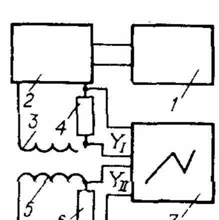

FIG. 1. Diagram of the instruments.

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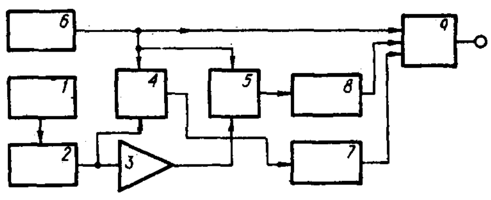

FIG. 2. Block diagram of the generator of signals of a special shape.

tives of the rectangular pulses that are shifted by $\pi/2$ with respect to the pulses themselves.

The apparatus for generating the special voltage signal operates as follows. Short rectangular pulses are superposed on the output of the harmonic signal generator *1* (see Fig. 1). The positions of these pulses correspond to the points in time when the absolute value of the sinusoidal signal exceeds a trigger level. The special signal required in the experiment–a sinusoidal voltage containing rectangular pulses–is generated by employing the signal generating circuit illustrated in Fig. 2. The $\pm$ 10 V voltage source *1* is fed into a potentiometer *2* that sets a base voltage U, $0 \leqslant U \leqslant 10$ V, and a voltage inverter *3* that sets a voltage $-U$. The inputs of the comparators *4* and *5* are fed with the signal of the sinusoidal voltage generator *6* (in Fig. 1 this generator was labeled with the index *1*) together with the signals $+U$ and $-U$ respectively (Fig. 3, *1*). When the sinusoidal signal falls below $-U$ the output of comparator 4 switches positive. Likewise, when the sinusoidal signal goes above $+U$ the output of comparator *5* switches positive (see Fig. 3, *2*). The monovibrators *7* and *8* generate short rectangular pulses corresponding to the rising edges of the comparator signals. Therefore, the leading edges of the monovibrator signals correspond to the points when the sinusoidal signal goes above $+U$ and below $-U$ (see Fig. 3, *3*). The monovibrator signals and the original sinusoidal signal are fed into a summing amplifier *9*. The resulting output signal is shown in Fig. 3, *4*. This signal is sent to solenoid *3* (see Fig. 1). The voltage drop measured over resistor *4*, which sets the current

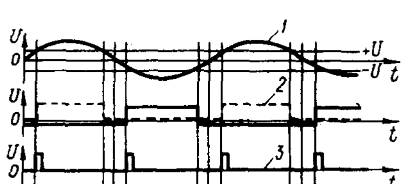

FIG. 3. Time diagrams of formation of a signal of a special shape.

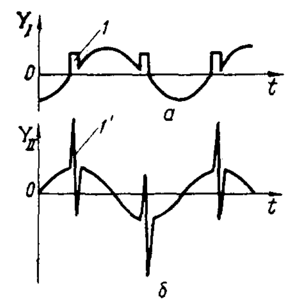

FIG. 4. Oscillograms.

through solenoid *3*, is fed into the Y_I input of the oscilloscope. The trace of this voltage is shown in Fig. 4, *1*. The time dependence of the flux Φ in solenoid *3* is the same as the time dependence of the voltage drop in resistor *4*, since both are proportional to the current. According to the law of electromagnetic induction an electromagnetic force $\mathscr{E}_i = -\mathrm{d}\Phi/\mathrm{d}t$ should appear in the other solenoid *5*, i.e., the induction emf is the first time derivative of the magnetic flux. The time-varying voltage drop measured over the resistor *6* and fed into the Y_{II} input of the oscilloscope (Fig. 4, *2*) also represents the first time derivative of the signal (see Fig. 4, *1*), since we observe that marker *1* is shifted by $\pi/2$ in the signal (see Fig. 4, *2*) and moves to *1'*, i.e., the electromagnetic induction emf is the first time derivative of the magnetic flux. Furthermore, as the marker *1* moves from left to right through the maximum in Y_I in Fig. 4, *1*, corresponding to $\mathrm{d}\Phi > 0$ and $\mathrm{d}\Phi < 0$, marker *1'* in Fig. 4, *2* shifts according to the sign of the time derivative $\mathrm{d}\Phi/\mathrm{d}t > 0$ and $\mathrm{d}\Phi/\mathrm{d}t < 0$, demonstrating Faraday's law of electromagnetic induction.

In our particular set-up we employed a toroid of 60 mm inner diameter and 100 mm outer diameter. The primary and secondary windings each contained 100 turns of 0.8 mm diameter wire. The resistance of resistor *4* was 1.2 kΩ, while that of resistor *6* was 0.2 kΩ (see Fig. 1). The special signal circuit was fed a sinusoidal voltage of 1 V amplitude at a frequency of 1180 Hz from a G3-33 generator. The oscilloscope traces were taken on an S1-93 two-beam oscilloscope.

Thus a comparison of the traces on a two-beam oscilloscope can be employed to demonstrate the law of electromagnetic induction.

[1]T. I. Trofimova, *A Course of Physics* (in Russian), Vysshaya shkola, M., 1985.
[2]Jay Orear, Physics, Macmillan, NY, 1979. [Russ. Transl., 2 Vols., Mir, M., 1981].
[3]V. A. Trofimov and Yu. V. Rublev, Usp. Fiz. Nauk **94**, 743 (1968) [Sov. Phys. Usp. **11**, 276 (1968–69)].

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