Radiation emitted by optically coupled lasers

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Optically coupled lasers are a striking example of a class of self-organized nonlinear dynamic systems which are currently attracting much interest. From a practical standpoint, the coupling of lasers makes it possible to increase relatively simply both the output power and brightness of a light source. The present review gives an account of the current status of the theory and experimental results obtained for two or more optically coupled lasers. The stress is on the structure of collective modes and their stability. Frequency and phase locking of laser arrays is considered. Optical coupling systems tested experimentally are discussed and the agreement between the experiment and theory is considered. A description is given of the operation of systems in which lasers are detuned by an amount exceeding the width of a locking band. Methods for increasing the brightness of the output radiation of systems with a synthesized aperture are also discussed.

INTRODUCTION

Progress in the theory of nonlinear dynamic systems with large numbers of degrees of freedom is currently attracting lively interest of specialists working on very different topics.[1,2] These systems include, for example, chains of coupled nonlinear oscillators, spin and atomic chains, coupled Josephson junctions, etc. Optically coupled lasers belong to the same class of systems and they are now being investigated in many laser laboratories. The principle of modular construction of laser systems is of practical importance because it provides opportunities for generating high-power high-quality radiation. In particular, the development of a chemical laser for optical data processing systems has followed this approach.[3] The modular structure of a laser system makes it possible to increase the output power quite simply while retaining the high quality of radiation typical of a single module.

The configuration of a system of modules and the number of modules N are usually determined by the type of laser and the technology used in their manufacture. The following examples can be quoted: linear and two-dimensional arrays of semiconductor lasers ($N \geqslant 10$),[4] arrays of tubular or waveguide CO_2 lasers ($N \geqslant 50$),[5,6] arrays of fiber-optic Nd^{3+} lasers ($N = 1300$),[7] arrays of Nd lasers ($N = 7$),[8] and an array of CO_2 lasers with unstable resonators ($N = 6$).[9]

When laser modules generate noncoherent (not phase-locked) radiation, the maximum density of the power which can be delivered to a target is N times greater than the power density obtainable from a single module. If it is compared with the power density obtainable from a single laser with an aperture $N^{1/2}$ times greater, it is found that the module array provides a higher brightness if the divergence of the radiation from a single laser exceeds $N^{1/2}\theta_{\rm diffr}$, where $\theta_{\rm diffr}$ is the diffraction limit of the divergence angle (for an aperture of $N^{1/2}d$ size).

When the radiation emitted by all the modules in an array is coherent, the maximum poower density which can be delivered to a target is N^2 times higher than for a single module. Therefore, a module array is superior to a single laser with a divergence exceeding $\theta_{\rm diffr}$ if the radiation from all the modules is phase-locked (the solution of this problem still leaves us with the task of aiming fields from N modules in such a way that they reach the same target).

We can distinguish three methods of frequency and phase locking of a laser array. One of them is based on the methods of adaptive optics[10] and involves locking by comparing a signal from each of the lasers with a standard (reference) signal. The standard can be a laser containing a different active medium and emitting at a different frequency. The locking is then provided by a system which mixes the frequencies.

The second method is based on utilization of a signal from a standard laser which is injected into all the lasers in an array.[11,12] This method includes also the case when a signal is injected into an array of amplifiers. The method of injection of a standard signal into a laser array presumes that all the resonator frequencies of the controlled lasers coincide to a high degree of accuracy with the frequency of emission by the standard.

Finally, the third locking method—which is the subject of the present review—is based on introduction of an optical coupling between lasers in an array.[6,13] When the properties of the lasers in an array differ little, they can generate a combined coherent radiation beam. As in the case of a conventional resonator, there are certain normal distributions of the fields which apply to the interiors of all the lasers in an array characterized by a high Q factor. Such normal distributions can be called supermodes or collective modes (in the case of two resonators it is usual to employ also the term "composite resonator modes"). The spectral density of such modes is governed by the total volume accessible to the lasing process. This volume is $NV_{\rm M} + V_{\rm c} \gg V_{\rm M}$, where $V_{\rm M}$ is the volume of a single module and $V_{\rm c}$ is the volume accessible to radiation in the communication channels. It would seem that in the case of such a high density of states the separation of a single mode needed to ensure complete coherence of the laser radiation is a serious problem. However, a complex configuration of an amplifying medium in a number of lasers produces a characteristic spatial filter which acts on the overall field and this results in selection of relatively few modes on the basis of the losses experienced by them. If the spectrum and structure of collective modes are

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known, one must consider the problem of their stability and competition between them. Obviously, important information on this topic can be obtained by considering the simplest system of two optically coupled lasers. This is why the present review will begin with an analysis of all the properties of a system of two lasers and this will be followed by a review of the experimental and theoretical results obtained for an array of N lasers and consideration, in the final section, of the quality of the radiation generated by a laser array as judged by the far-field pattern.

The development of special phase screens controlled by an external signal and designed for a coherent array of light sources makes it possible to control the overall radiation field practically at any rate and in accordance with any law. When such sources become available, it will be possible to extend significantly the range of applications of laser radiation.

1. TWO COUPLED LASERS

1.1. Principal equations (point model)

Two coupled lasers is the simplest example of optical coupling in an array. An investigation of the properties of this system is of major importance for the understanding of the properties of a large number of optically coupled lasers, which will be considered in Sec. 2.

To the best of our knowledge, the problem of operation of two optically coupled lasers was first considered theoretically in Ref. 14. The authors discussed the optical coupling provided by diffraction-induced exchange of radiation between two adjacent resonators containing plane mirrors. The coupling coefficient was found in Ref. 14 by numerical calculations of the change in the eigenvalues of the parameters of modes in empty resonators as a function of the distance between them. In an analysis of the stability of coupled operation of two lasers it was suggested that they can be described by a model which generalizes the system of equations for a single laser to the case of many lasers. If there are two lasers, the equations become

$$\begin{aligned} &\dot{E}_1 + i(\omega-\omega_1)E_1 + \frac{\omega_1}{2Q_1}E_1 + \omega M E_2 = -2\pi i\omega P_1, \\ &\dot{E}_2 + i(\omega-\omega_2)E_2 + \frac{\omega_2}{2Q_2}E_2 + \omega M E_1 = -2\pi i\omega P_2, \\ &\dot{P}_k + \frac{P_k}{T_2} + i(\omega-\omega_0)P_k = i\frac{|\mu|^2}{h}N_k E_k, \\ &\dot{N}_k + \frac{1}{T_1}(N_k - N_{0k}) = -\frac{i}{h}(E_k P_k^* - E_k^* P_k), \end{aligned} \tag{1.1}$$

where $k = 1$ or 2 refer to the characteristics of the first and second lasers, respectively; $E_{1,2}$ is the field intensity in a laser; $Q_{1,2}$ is the Q factor of a single laser; ω_0 is the frequency of the lasing transition in the active medium; ω_1 and ω_2 are the eigenfrequencies of the resonator assumed to be close to ω_0; P_k is the polarization of the active medium; T_1 and T_2 are the longitudinal and transverse relaxation times of the active medium; μ is the matrix element of a dipole transition; N_k is the density of the active particles; N_{0k} is the steady-state density of the active particles, in the absence of lasing. The parameter M represents the optical coupling between the lasers.

An analysis of the stability of steady-state phase locking was made using simply the real values of the coupling parameter, although the system (1.1) can be used to investigate also the general case. It should be pointed out that the results of the numerical calculations reported in Ref. 14 had been interpreted incorrectly. In reality, the quantity M in the system of equations (1.1) is complex, i.e., $M\sim f(d)\exp(i\psi(d))$, where $\psi(d)$ is a function steeper than $f(d)$ and d is the distance between the edges of the resonator mirrors.

The ability to control the emission spectrum and the problem of stability of operation of a system of two optically coupled resonators, one of which is passive and the other is filled with an active medium, was considered somewhat later.[15,16] The dynamic equations for the coupling by semitransparent mirrors were derived and the normal oscillation modes were identified for two coupled Fabry–Perot resonators under conditions such that their modes had similar frequencies.

The paper on this topic cited most frequently (in spite of the fact that it was published much later than Refs. 14–16) is that of Spencer and Lamb[17] where a consistent derivation (similar to that given in Ref. 15) is provided of the dynamic equations describing the amplitudes of the fields of two lasers coupled by a semitransparent mirror with a reflection coefficient close to unity. An attempt has been made to provide a more generally valid derivation of the dynamic equations describing coupled lasers.[18–25] A system of equations was obtained in Refs. 19 and 20 by expanding the fields in terms of the normal modes of the separate resonators. Calculation of the coupling coefficient requires the knowledge of the projections of the collective modes on the modes of the individual resonators. Expressions were obtained in the cited papers for the coupling coefficients in the case when two resonators are linked by a nonresonant interferometer with losses. The dynamic equations for the amplitudes of the fields of coupled resonators with a constant transverse structure of the modes in the resonators are derived in Ref. 21. The method used to obtain the dynamic equations[21] is familiar from the theory of nonlinear waves (see, for example, Ref. 27).

Following the same procedure we can obtain the equations describing the evolution of the fields of two lasers. These equations are simplest in the case of low values of the transmission coefficients of the mirrors:

$$\begin{aligned} &\frac{dE_1}{dt} = \frac{c}{2L_1}\left[r_0 r_{11}\exp(i\varphi_1 + g_1 l_1) - 1\right]E_1 + \frac{c}{2L_1}M_{21}E_2, \\ &\frac{dE_2}{dt} = \frac{c}{2L_2}\left[r_3 r_{22}\exp(i\varphi_2 + g_2 l_2) - 1\right]E_2 + \frac{c}{2L_2}M_{12}E_1, \end{aligned} \tag{1.2}$$

where the coefficients M_{12} and M_{21} can be expressed in terms of the reflection and transmission coefficients of the mirrors $M_{12} = t_{12}/r_{22}$ and $M_{21} = t_{21}/r_{11}$. In the plane-wave approximation when the coupling is provided by an interferometer with mirror reflection and transmission coefficients r_1, r_2, d_1,

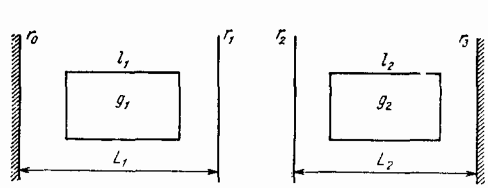

FIG. 1. Lasers coupled by semitransparent plane mirrors (schematic representation).

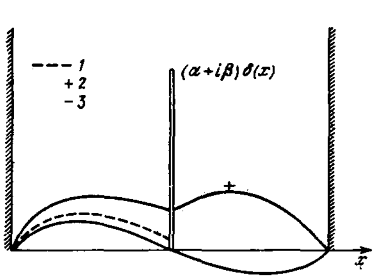

FIG. 2. Coupling of lasers by a δ-function barrier: 1) field distribution in one resonator in the absence of coupling; 2) field distribution in a cophasal mode; 3) field distribution in an antiphasal mode.

and d_2 (Fig. 1), the expressions for r_{11}, r_{22}, t_{12}, and t_{21} are

$$r_{11} = r_1 + r_2 d_1^2 \exp(2i\psi)[1 - r_1 r_2 \exp(2i\psi)]^{-1},$$
$$r_{22} = r_2 + r_1 d_2^2 \exp(2i\psi)[1 - r_1 r_2 \exp(2i\psi)]^{-1},$$
$$t_{12} = t_{21} = d_1 d_2 \exp(i\psi)[1 - r_1 r_2 \exp(2i\psi)]^{-1}.$$

Here, $\varphi_{1,2} = 2(\omega/c)L_{1,2}$, and $\psi = (\omega/c)l_c$ are the phase shifts in the resonators and in the coupling channel; $l_{1,2}$ are the lengths of the active media; $g_{1,2}$ are the gains of the active media. When two lasers are coupled using a Fabry–Perot interferometer, it is found from the approximation of plane waves in the absence of dissipation in the communication channel that the product $M_{12}M_{21}$ is a real negative quantity. If the reflection coefficients r_1 and r_2 of the coupling interferometer are equal, then $M_{12} = M_{21}$ and this quantity is purely imaginary. The system of equations (1.2) then reduces to corresponding equations obtained in Ref. 17. Identical results are obtained also by considering the modes in two resonators separated by a barrier $U(x) = (\alpha + i\beta)\delta(x)$ (Fig. 2). If there is no dissipation of the field in the barrier, i.e., if $\beta = 0$, then the coefficients representing the coupling between the lasers are purely imaginary $M_{12} = M_{21} = i\alpha c/2\omega$. In the case of absorption ($\beta > 0$) or amplification ($\beta < 0$) of a field across a barrier, the coupling coefficient has both real and imaginary parts:

$$M_{12} = M_{21} = \frac{i\alpha - \beta}{\alpha^2 + \beta^2}\frac{c}{2\omega}.$$

As pointed out in Ref. 21, all the investigations which deal with individual modes of separate resonators (we shall call them local modes) suffer from a shortcoming originating from the assumption that the overall (composite) field can be represented by a linear combination of the local mode fields. If the coupling between the resonators is strong, the errors due to this shortcoming may be important.

An approach which makes it possible to avoid this problem is proposed in Ref. 22. The starting point is a calculation of the field in a resonator in the presence of a signal injected from outside. This approach makes it possible to allow explicitly for the coupling out of the injected signal transformed by one trip through the resonator and it results in a renormalization of the coupling coefficients.

It should be pointed out however that all the theoretical derivations of the dynamic equations for local modes and calculations of the coupling coefficient are based on an implicit assumption that the deformation of the mode fields due to the nonlinearity of the active medium does not affect the coupling coefficient. Obviously, this hypothesis may not be obeyed, particularly in those cases when the transverse structure of the field is inhomogeneous (examples are the diffraction coupling,[14] the coupling via an aperture in a mirror,[28] and the coupling due to diffraction by a spatial filter).[29]

An alternative approach to the derivation of the dynamic equations is based on collective modes.[23] It is necessary to know how to calculate the characteristics of such modes, which is difficult even in the simplest case of two resonators coupled by a semitransparent mirror. Obviously, this problem of derivation of equations for the amplitudes of collective modes does not differ in any fundamental way from the general problem of finding the field in a conventional resonator with an active medium (see, for example, Ref. 30). The main cause of an instability of emission of a collective mode (on condition that in the absence of the active medium there is one lowest-loss mode) is an inhomogeneous "burnup" of the active medium when the fields of the collective modes have a variety of spatial distributions.

The magnitude and phase of the coupling coefficient become much more complex if we allow for the fact that the transverse structure of the fields in the resonators is inhomogeneous and this is also true of the case when the reflection coefficients of the coupling mirrors depend on the transverse coordinate.

1.2. Steady-state conditions in the point model

We shall now consider the normal modes in two coupled resonators making the approximations of the same kind as in the derivation of the system of equations (1.2). In the absence of any coupling between the mirrors the eigenfrequencies of the modes and the gains of the active media are given by the expressions

$$|r_0 r_{11}| \exp[\mathrm{Re}(g_1 l_1)] = 1,$$
$$|r_3 r_{22}| \exp[\mathrm{Re}(g_2 l_2)] = 1,$$
$$\exp[i\varphi_1 + i\,\mathrm{Im}(g_1 l_1) + i\arg(r_0 r_{11})] = 1,$$
$$\exp[i\varphi_2 + i\,\mathrm{Im}(g_2 l_2) + i\arg(r_3 r_{22})] = 1.$$

The simplest case is realized for identical parameters of the active media (when the pumping rates and lengths are equal) and almost equal resonator lengths ($L_1 \approx L_2 \approx L$). We shall first consider the modes closest to the center of the line profile. If the resonator modes are detuned only slightly, so that $|\Delta| \ll 1$, the system of equations (1.2) can be rewritten in the form[21,25]

$$\frac{2L}{c}\frac{dE_1}{dt} = \left[(g_1 l - k) + i\frac{\Delta}{2}\right]E_1 + ME_2,$$
$$\frac{2L}{c}\frac{dE_2}{dt} = \left[(g_2 l - k) - i\frac{\Delta}{2}\right]E_2 + ME_1, \quad (1.3)$$

where $k = \ln(1/|r_0 r_{11}|)$, $M_{12} = M_{21} = M$,

$$\Delta = \frac{2\pi}{\lambda}\left[(L_1 - L_2) - n\frac{\lambda}{2}\right], \quad \left|(L_1 - L_2) - n\frac{\lambda}{2}\right| \ll \frac{\lambda}{2},$$

and n is an integer.

The solutions of the system (1.3) obtained for the fixed value of the gain $g_1 = g_2 = k/l$ can be found in the form $E \propto \exp(\gamma i)$. If $\Delta = 0$, the amplitudes of the fields in the two lasers are the same and the normal collective modes represent cophasal $[(E_1, E_2)_+ = E_0(1,1)]$ and antiphasal $[(E_1, E_2)_- = E_0(1, -1)]$ superpositions of the fields with eigenvalues $\gamma_\pm = \pm Mc/2L$. If the mismatch of the resonator lengths is finite, then the amplitudes of the fields in the lasers are different. If we ignore the difference between the gains of the active media ($g_1 = g_2$), we find that

$$(E_1, E_2)_\pm = E_0\left[1, \pm\left(1-\frac{\Delta^2}{4M^2}\right)^{1/2} - i\frac{\Delta}{2M}\right],$$

$$\gamma_\pm = \pm\frac{c}{2L}\left(M^2-\frac{\Delta^2}{4}\right)^{1/2}.$$

If the mismatch is considerable so that $|\Delta/2| \gg |M|$, the collective modes in the coupled lasers are identical with the modes of the individual lasers, i.e., in the case of one mode the field energy is concentrated in the first laser, and in the case of the other mode it is concentrated in the second laser.

If the coupling coefficient is imaginary, then $\gamma_\pm$ is an imaginary quantity. In this case the coupling between the lasers simply splits the frequencies. If the coupling coefficient has a real part, then $\gamma_\pm$ also has both imaginary and real parts.

The difference between the real parts of $\gamma_\pm$ is an indication of a difference between the Q factors of the cophasal and antiphasal modes. Figure 2 shows two collective modes for resonators coupled by a delta-function potential. In the presence of absorption by the potential barrier the antiphasal mode has a higher Q factor. If amplification of the field occurs at the potential barrier, then the cophasal collective mode has a higher Q factor. This is due to the fact that the relative intensity of the field at the barrier is different for the cophasal and antiphasal modes and, consequently, the losses experienced by these modes are different. In real laser systems the fields in the resonator cannot be regarded as plane waves. We shall give below the results of investigations of lasers with an inhomogeneous distribution of the field in the specific case of two unstable resonators.

1.3. Collective modes of coupled unstable resonators

We shall consider resonators with a cylindrical geometry of the mirrors shown schematically in Fig. 3. In numerical investigations we shall assume that the mirrors are finite, but we shall carry out an analytic investigation in the approximation of infinite mirrors.

The integral equations for the fields E_1 and E_2 obtained in the plane of the coupling mirror for identical radii of curvature of the mirrors and a small mismatch between the resonator lengths ($\Delta L \ll L_1$) can be represented in the following conditional form (see Refs. 31 and 32):

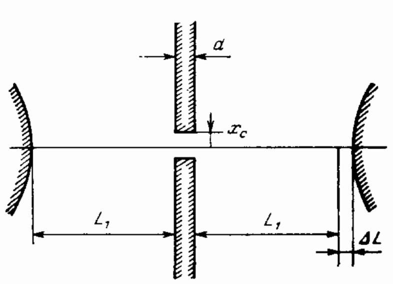

FIG. 3. Optical coupling of two lasers with unstable resonators (schematic representation.

$$\gamma\begin{pmatrix}E_1(x)\\ E_L(x)\end{pmatrix} = \left(\frac{ik}{4\pi Lq}\right)^{1/2}\begin{pmatrix}r(x) & d(x)\exp(2ik\Delta L)\\ d(x) & r(x)\exp(2ik\Delta L)\end{pmatrix}\int\limits_{-\infty}^{\infty} H(x,y)\begin{pmatrix}E_1(x)\\ E_2(y)\end{pmatrix}dy, \tag{1.4}$$

where γ is an eigenvalue of the integral equation. The modulus of γ determines the Q factor, whereas the phase of γ governs the collective mode frequency. Here, $q = 1 + (L_1/R)$, where R is the radius of curvature of the mirrors; $r(x)$ and $d(x)$ are the reflection and transmission coefficients (expressed in terms of the field amplitude) of the coupling mirror, which are generally functions of the complex variable;

$$H(x,y) = \exp\left\{i\frac{k}{2L_1}\left[\left(1-\frac{1}{2q}\right)(x^2+y^2)-\frac{xy}{q}\right]\right\}.$$

It is not possible to obtain the general solution of the problem described by the system (1.4). A complete analysis can be carried out if the coupling mirror is semitransparent and the transmission and reflection coefficients are independent of the transverse coordinate x, and also in the case when the optical coupling of the resonators occurs via a small aperture.

In the case of the semitransparent mirror coupling a collective mode in a resonator can be represented by a superposition of two identical normal modes of an empty unstable resonator ($E_1 \sim E_2$). The system (1.4) yields a dispersion equation describing the spectrum of the eigenvalues of γ:

$$\gamma_\pm(n) = rm^{-n-1/2}\left\{\frac{1+\exp(2ik\Delta L)}{2} \pm\left[\left(\frac{1+\exp(2ik\Delta L)}{2}\right)^2+\frac{d^2}{r^2}\exp(2ik\Delta L)\right]^{1/2}\right\}, \tag{1.5}$$

where m is the magnification of the unstable resonator; $n = 0, 1, 2, \ldots$. The Q factor of the collective modes is governed by the moduli of the eigenvalues:

$$|\gamma_\pm(n)|^2 = m^{-1-2n}\left\{|r|^2\frac{1+\cos\varphi}{2} + \left[|r|^4\left(\frac{1-\cos\varphi}{2}\right)^2+|d|^4-|dr|^2\cos\psi(1-\cos\varphi)\right]^{1/2} \pm\left(|r|^2\cos\frac{\varphi}{2}\left[\left|\frac{d}{r}\right|^2\exp(2i\psi)-\sin^2\frac{\varphi}{2}\right]^{1/2}+\text{c.c.}\right)\right\}, \tag{1.6}$$

where $\varphi = 2k\Delta L$ and $\psi = \arg(d/r)$.

The cumbersome expression in Eq. (1.6) can be simplified considerably in some special cases. If the difference (mismatch) ΔL between the resonator lengths is a multiple of an integral number of the radiation wavelengths $\varphi = 2\pi N$ ($N = 0, 1, \ldots$), then $|\gamma_\pm(n)| = m^{-1-2n}(|r|^2 + |d|^2 \pm 2|dr|\cos\psi)$. If $\varphi = 2\pi(N + 1/2)$, the Q factors of the two modes are identical: $|\gamma_\pm(n)|^2 = (|r|^4 + |d|^4 - 2|dr|^2\cos 2\psi)^{1/2}$. It should be pointed out that the phase ψ is found independently by solving the problem of reflection by a semitransparent mirror. In the absence of dissipation or amplification in the coupling channel, we find that $\psi = 5/2$ and the Q factors of the collective modes are the same.

We shall now consider the coupling of unstable resonators via a small aperture in a shared mirror. If the size of this aperture is sufficiently small {i.e., if x_0 is much less than the size of the equivalent Fresnel zone $[(k/2L)\ (m^2-1)/4qm]^{-1/2}$}, we can use perturbation theory.[33] The expression for the eigenvalues of the parameters of the modes with lower losses is then analogous to that given by Eq. (1.5):

$$\gamma_{1,2}=m^{-1/2}\left\{(1-\beta)\frac{1+e^{i\varphi}}{2}\pm\left[(1-\beta)^2\left(\frac{1-e^{i\varphi}}{2}\right)^2+\beta^2e^{2i\psi-i\varphi}\right]^{1/2}\right\},\quad(1.7)$$

where

$$\beta=x_0\left(\frac{k}{\pi L}\frac{m^2-1}{qm}\right)^{1/2}.$$

The Q factors of two collective modes in infinite resonators, governed by the modulus of $\gamma_{1,2}$ are identical when the mismatch between the resonators is $\varphi=\pi(2N+1)$ or if the phase shift in the coupling channel is $\psi=\pi(N\pm 1/2)$.

The physical reasons for the degeneracy of the collective modes in respect of the losses are as follows. If $\varphi=\pm\pi$, the frequencies of the longitudinal modes in one resonator are located exactly at midpoints of the intervals between the frequencies of the longitudinal modes in the second resonator. In view of this symmetry, the transverse distributions of the fields on the coupling mirror differ on transition from one collective mode to another by the fact that the field in the first resonator is replaced by the field in the second resonator and vice versa.

We can see how the Q factors of the collective modes depend on the phase ψ in the coupling channel if we consider the simplest case of equal resonator lengths. The collective modes can in this case be represented by even and odd superpositions of fields with the same transverse structure. Therefore, the eigenvalue for the problem of two identical resonators coupled through an aperture is identical with the solution for one resonator with a phase projection shift ψ (for an even superposition of the fields) or $-\pi+\psi$ (for an odd superposition of the fields) when this phase projection replaces a coupling aperture (Fig. 4). If the reflection from the edges of the resonator mirrors can be ignored, the losses associated with the scattering of the field by a phase projection are periodic (with a period 2π) and even functions of the phase shift. Therefore, the Q factors of the collective modes are identical if $\psi=\pi/2$.

In real situations the fields are always reflected by the resonator mirror edges. Then, in addition to a wave diverging from the resonator axis, we can expect a converging wave and the eigenvalues for the normal modes of coupled resonators may be greatly modified.

A numerical investigation of the mode structures of coupled unstable resonators was reported in Ref. 33. It was based on integration of a system of parabolic equations for a pair of counterpropagating waves in each of the resonators. The coupling between the fields was allowed for by specifying the appropriate boundary conditions at the coupling mirror. The transformations of the fields in one trip between the resonator mirrors were found by the traditional spectral method[34] or by the difference scheme method (see, for example, Ref. 35) employing an algorithm for the fast Fourier transform. The eigenvalues for some of the collective modes with the highest Q factor were obtained in Ref. 33 by the Prony method (see Ref. 36).

We shall now give the results of calculations carried out for specific resonator parameters and we shall later show which of them are of general validity and which apply to specific dimensions. In these calculations the resonator parameters were selected to be similar to those used in experiments reported in Ref. 28 where the configuration was similar to that shown in Fig. 3. The radii of curvature of the convex mirrors were 10 m and the dimensions of these mirrors were 0.5 cm. The distance between the convex and coupling mirrors was 1 m. The plane coupling mirror had an aperture 0.23 cm in radius. All the calculations were carried out for CO_2 laser radiation with the wavelength $\gamma=10^{-3}$ cm.

The moduli of the eigenvalues for two pairs of collective modes in resonators of identical length ($\varphi=0$) are plotted in Fig. 5 as a function of the phase shift ψ in the coupling channel. The resultant dependence is periodic, with a period 2π, and the Q factors of the even and odd collective modes reproduce one another after a shift of π. This behavior is in agreement with an analytic investigation carried out assuming that resonators are formed by infinite mirrors. However, there is a considerable difference: the Q factors of the even and odd modes differ for $\psi=\pi/2$. This is due to the influence of the waves reflected by the mirror edges, which are responsible for the difference between the losses in the case of a mode of a resonator (Fig. 4) with a phase projection observed on reversal of the sign of the phase shift.

The dependences of the moduli of the eigenvalues of the parameters of two collective modes on the mismatch φ between the resonator lengths are plotted in Fig. 6. These dependences are periodic and even functions of φ. In the interval $(-\pi,0)$ the phase shift in the coupling channel is $\psi=0$, whereas in the interval $(9,\pi)$ it is $\psi=0.6\pi$. It follows from the analytic expression (1.6) that the Q factors of the collective modes become identical if $\varphi=\pm\pi$ irrespective of the

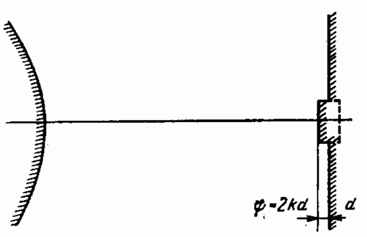

FIG. 4. Equivalent system used in calculation of the parameters of normal modes of two lasers coupled by an aperture in a shared mirror.

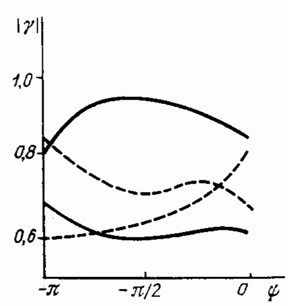

FIG. 5. Dependences of the moduli of the eigenvalues of the mode parameters on the phase of the coupling coefficient ψ in the case of identical lengths of unstable resonators.

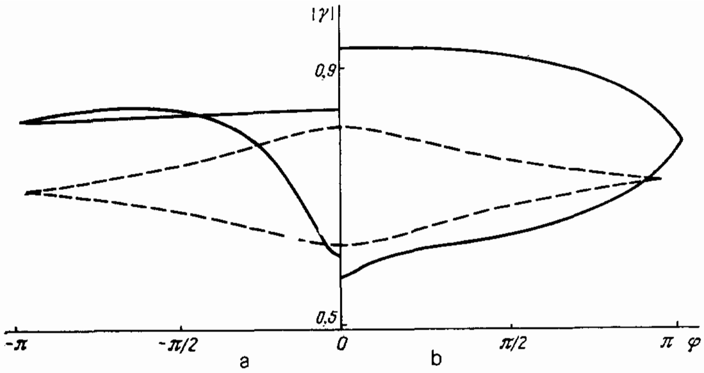

FIG. 6. Dependences of the moduli of the eigenvalues of the mode parameters of two optically coupled unstable resonators on the mismatch of their lengths: a) $\psi = 0$; b) $\psi = 0.6\pi$.

phase shift ψ. For comparison, Fig. 6 includes (dashed curve) an analytic dependence of $|\gamma|$ on φ for two collective modes, derived using Eq. (1.7) and assuming that $\psi = 0.6\pi$.

The phase of the eigenvalue determines the collective mode frequency. If the width of the gain profile of the active medium is comparable with the intermode spacing, then the difference between the collective mode frequencies can result in their discrimination in accordance with the Q factors. Figure 7 shows the dependences of the phases of the eigenvalues of the mode parameters on the mismatch between the resonator lengths φ in the case when $\psi = 0.6\pi$. The dashed curve represents the analytic dependence of the phase of the eigenvalue obtained from Eq. (1.7).

The field distributions on the mirror surfaces depend on the mismatch between the resonator lengths and on the phase shift in the coupling channel. If the resonator lengths are the same, the distributions of the fields are the same in both resonators. If the phase mismatch is $\varphi = \pm \pi$, the distributions of the fields over the resonator cross sections are very different. In this case the collective modes differ from one another only by replacement of the field distribution in one resonator with the field distribution in the other resonator.

It follows from the numerical calculations reported in Ref. 33 that in the case of symmetric unstable resonators coupled optically through an aperture in a shared mirror the properties of the collective modes are in many respects similar to the results of an analytic investigation, namely, the Q factors of the adjacent collective modes are identical if $\varphi = \pm \pi$ and the field distributions in them are identical apart from transposition.

The main deviation from the analytic results is due to a considerable influence of the waves reflected by the mirror edges on the structure of the fields in the resonators and on the Q factors of the modes. The transverse structure of the normal modes is governed by all the resonator parameters, by the dimensions of the coupling aperture, and by the phase shift in the coupling channel. Calculations of the eigenvalues of the parameters of the modes in empty coupled resonators make it possible to find the conditions under which single-mode emission can be achieved near the lasing threshold.

One of the main problems in the study of coupled lasers is the determination of the parameters of the resonators and of the active medium needed to achieve stable single-mode lasing. The results of investigations of this topic can be found in Refs. 14, 17, 24–26, 37–39.

1.4. Stability of single-mode lasing (point model)

An analysis of the mode structure and determination of the eigenvalues of the fields in empty coupled resonators makes it possible to select only approximately the range of parameters for realization of single-mode lasing. The main reason for the difference between the Q factors of the collective modes in empty resonators is the different spatial distribution of the fields, which results—in the presence of an

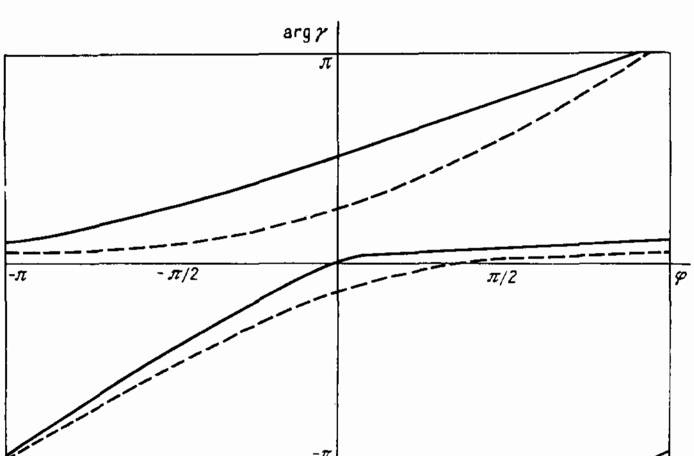

FIG. 7. Dependences of the phases of the eigenvalues of the mode parameters on the mismatch between the resonator lengths in the case when $\psi = 0.6\pi$. The continuous curves represent numerical calculations and the dashed curves are analytic dependences.

inhomogeneous distribution of the absorption or scattering—in selection of the mode losses. The presence of an active medium in a laser resonator can alter the field distributions and the conditions of stability of single-mode lasing.

As pointed out already, the first analysis of the stability of single-mode emission from coupled lasers was given in Ref. 14 for real values of the coupling coefficient. The results of this analysis were formulated by the authors of Ref. 14 by specifying three physical conditions for cophasal (phase-matched) lasing. The first condition, $M<0$ [see Eq. (1.1)], is that the lasing optimal from the energy point of view is stable. The second condition is $|\varphi| < 2|M|$ and it governs the maximum possible detuning of the eigenvalues of the laser frequencies which does not destroy cophasal (phase-matched) operation. The third condition requires self-excitation of the coupled lasers.

A numerical analysis of single-mode emission provided in Ref. 17 applies to the case of weak coupling of two lasers via a semitransparent mirror when the coupling coefficient is a purely imaginary quantity. Analytic expressions for the dependence of the stable locking band on the parameters of the active medium and on the coupling coefficient were obtained in Ref. 17 in a transcendental form, which is quite difficult to analyze. The necessary conditions for the existence of steady-state single-mode emission were obtained in Refs. 24–26 for a homogeneously broadened line profile of the active medium in the approximation of a small difference between the intensities of the fields in the resonators. Determination of the locking band (realization of single-mode emission) was reduced in Refs. 24–26 to finding the range of the parameters outside which steady-state solutions are no longer obtained. It should also be pointed out that an analysis given in Refs. 25 and 26 allows for linear interresonator absorption (amplification) of the fields under the conditions of phase-locked multifrequency emission from two lasers.[26] In the treatment given in Refs. 24–26 the gain of the active medium was assumed to be a function of the intensity at a given moment. The finite response time of the medium to a change in the intensity leads to a memory effect in the field dependence of the gain. An allowance for the dynamics of the gain can expand the instability region so that the reported investigations[24–26] set the upper limit to the range of existence of single-mode operation. On the other hand, the development of an instability does not always result in emission of the second collective mode and, consequently, the conditions of stability of the steady-state solution set the lower limit of the existence of single-mode operation (sufficient conditions).

The stability of steady-state single-mode lasing considered in the approximation of a small modulus of the coupling coefficient, allowing for the finite relaxation time of the active medium, was investigated earlier.[37,39] In the case of real values of the coupling coefficient when $|\varphi| < 2|M|$, either the cophasal ($M<0$) or antiphasal ($M>0$) lasing is stable. If the coupling coefficient is imaginary, the range of existence of single-mode lasing[11] characterized by $|\varphi| < (2g_0/g_{\rm th})(|M|^2/(g_0-g_{\rm th})l)$ (g_0 is the small-signal gain, $g_{\rm th}$ is the threshold value of the gain, and l is the length of the active medium) is not always identical with the stability zone governed by the system of the inequalities[39]

$$\frac{1}{\tau} > \frac{c}{2L}\frac{4|M|^2}{(g_0-g_{\rm th})l}|\sin\chi|,$$

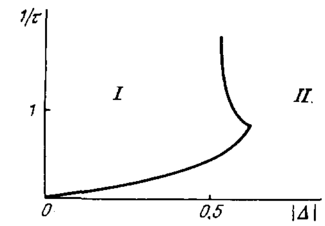

FIG. 8. Boundary between the regions of stability of single-mode lasing: I) stable region; II) unstable region.

$$\left(\frac{2L}{c}\right)^2\frac{1}{\tau^2}\left\{1-4|M|^2\left[\frac{g_0\sin\chi}{g_{\rm th}(g_0-g_{\rm th})l}\right]^2\right\} > \left[\frac{4|M|\sin\chi}{(g_0-g_{\rm th})l}\right]^2\left[\cos^2\chi-\left(\frac{g_0\sin\chi}{g_{\rm th}}\right)^2\frac{2L}{c\tau}\frac{1}{(g_0-g_{\rm th})l}\right], \qquad (1.8)$$

where τ is the relaxation time of the population inversion in the active medium and

$$\chi = -\sin^{-1}\left[\frac{\varphi}{2|M|^2}\frac{(g_0-g_{\rm th})l}{g_0/g_{\rm th}}\right].$$

A diagram demonstrating the stability of single-mode lasing in terms of the variables $1/\tau$ and φ is plotted in Fig. 8 on the assumption that $g_0 l = 1.4$, $g_{\rm th} l = 1$, $|M| = 0.3$, and $\psi = \pm\pi/2$. Near the lasing threshold $[(g_0-g_{\rm th})l < M]$ and in the limit of an instantaneous-response active medium ($\tau\to 0$) the condition of stability of single-mode lasing is the same as the condition for a real value of the coupling coefficient $|\varphi| < 2|M|$. As pointed out in Sec. 2, in the case of an imaginary coupling coefficient the Q factors of the cophasal and antiphasal modes are the same. It would then seem that saturation of the gain of the active medium should destabilize either of the modes. However, in the case of totally identical lasers ($\varphi = 0$) the distributions of the field intensities are completely identical for the cophasal and antiphasal modes. It is this circumstance that results in stabilization of both modes (attainability of one of the possible regimes is governed by the earlier stages of the growth of lasing). In fact, a small admixture of the second collective mode produces an asymmetric distribution of the intensity in the resonators and this distribution varies at a frequency $(c/L)M$. The finite relaxation time of the active medium results in damping of these oscillations and, consequently, it attenuates the second mode field. It should be pointed out that in this case the saturation of the active medium and the finite relaxation time do not create a multimode structure, in contrast to other lasers when spatial distributions of the modes are different, but it stabilizes single-frequency emission. Consequently, if $\varphi = 0$, there is a margin of the stability of single-mode emission associated with the finite relaxation time of the active medium. Single-mode lasing becomes neutrally stable in the limit $\tau\to\infty$.

The refractive index of semiconductor lasers depends on the gain of the active medium. The gain in turn is governed by the radiation intensity. A linear analysis of the stability of cophasal and antiphasal lasing of two semiconductor lasers can be found in Ref. 40. The range of stability of phase locking is found in terms of two variables which are the pump current and the coupling coefficient.

Unstable single-mode lasing may result also from the familiar[30] spatial and spectral "burning of a hole" in the gain by the single-mode radiation field. The conditions for the growth of adjacent longitudinal modes in coupled lasers are identical with the corresponding conditions for one laser.[30] The problem of the influence of saturation of the active medium on the stability of single-mode lasing is more complex is the transverse structure of the fields is inhomogeneous. Such a situation occurs in lasers with optically coupled unstable resonators.

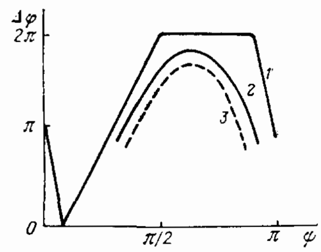

FIG. 9. Boundaries between the regions of stability of phase locking of two lasers obtained for different values of the small-signal gain (in terms of the variables φ and ψ): 1) empty resonator; 2) $g_0 = 1.5\, g_{th}$; 3) $g_0 = 1.5\, g_{th}$. The coupling aperture is displaced relative to the optic axis.

1.5. Numerical analysis of the stability of single-mode emission from lasers with unstable resonators

A numerical analysis of the stability of single-mode emission reported in Refs. 37 and 38 was made for an optical system similar to that shown in Fig. 3. The resonator fields were represented by a combination of counterpropagating waves with amplitudes satisfying a system of parabolic equations

$$\pm 2ik\frac{\partial E^{\pm}_{1,2}}{\partial z} + \frac{\partial^2 E^{\pm}_{1,2}}{\partial x^2} = ikg_{1,2}E^{\pm}_{1,2}; \qquad (1.9)$$

here $E^{+}_{1,2}$ are the amplitudes of the waves propagating along the z axis and $E^{-}_{1,2}$ are the amplitudes of the counterpropagating waves (i.e., the waves traveling in the opposite direction). The indices 1 and 2 identify the resonator. We shall model the active medium assuming that it is characterized by a homogeneously broadened spectral gain band with the following dependence of the gain on the intensities $|E^{\pm}|^2$ (fields in the resonators are normalized to the saturation intensity in the medium):

$$g_{1,2} = g^0_{1,2}\,(1 + |E^{+}_{1,2}|^2 + |E^{-}_{1,2}|^2)^{-1}, \qquad (1.10)$$

where $g^0_{1,2}$ is the small-signal gain. Coupling between the fields occurs on the mirrors and is described by relevant boundary conditions.[33] A new numerical method for an analysis of the stability of single-mode emission from coupled lasers was proposed in Ref. 37. It involves alternate application of the steady-state method[34] and of the Prony method.[36] The zeroth order approximation was the field distribution of a specific collective mode in empty resonators and the amplitude of this mode was normalized to a value estimated using the Rigrod formula (see Refs. 41 and 42). This was followed by several iterations using the steady-state method. The number of iterations was selected to be at least equal to the number of radiation trips through the resonator needed to establish the diffraction structure of the field in the resonator.[43] Next, the "frozen" distribution of the gain, obtained in the last stage of iteration by the steady-state method, was used to calculate the parameters of modes in coupled resonators by the Prony method. Projection of the distribution of the field on a given mode was used as a new initial condition in the steady-state method. If subsequent iterations did not result in convergence with the required precision, the distribution of the gain was "frozen" once more and the Prony method was used to calculate the parameters and select the required mode, etc.

This technique made it possible to calculate the parameters of lasing even in the case of modes with relatively low values of the Q factor. A physical analog of the method used in Ref. 37 is a resonator containing dispersive elements. Separation of a single mode in a dispersive resonator can be achieved by discrimination of the losses in the dispersive element.

Calculation of the distributions of the field and gain of the active medium was followed by a study of the stability of a given single-mode lasing regime. The linear problem of determination of the eigenvalues of the fundamental modes was solved numerically for a constant distribution of the gain. Obviously, the modulus of the eigenvalue for a mode involved in lasing is unity. When the moduli of the eigenvalues for all the remaining modes are less than unity, the single-mode lasing regime is stable in the case of constant parameters. Otherwise, it is obvious that a given single-mode regime is unstable. The parameters of mismatch between the resonators, corresponding to the point at which the modulus of the eigenvalue of one of the modes becomes equal to unity, determine the boundary of the region of stable phase locking of the lasers.

Figure 9 shows, in terms of the coordinates of the phase mismatch between the resonators and the phase shift in the coupling channel, the boundaries of the region of stable phase locking of two lasers for different values of the small-signal gain. When the excess above the threshold is slight, the phase-locking region is the same as in the case of an empty resonator. An increase in the small-signal gain increases the inhomogeneity of the distribution of the gain and reduces the range of stable phase locking. This range becomes narrower also as a result of a slight shift of the coupling aperture relative to the resonator axes (dashed curve in Fig. 9).

The numerical method developed in Ref. 37 makes it possible to identify the conditions when there is no steady-state lasing. A correct description of lasing can then be obtained by solving numerically a transient (time-dependent) problem. Characteristics of transient lasing of optically coupled lasers can be investigated using the simpler point model in the case when the transverse structure of the fields is unimportant and the coefficient representing coupling of the lasers is independent of the parameters of the active medium.

1.6. Dynamics of emission of radiation from two optically coupled lasers

The possibility of a fairly complex behavior of a dynamic system is supported indirectly by the existence of multistable regimes of a system. The conditions for the appearance of bistability in a system of coupled resonators were considered in Ref. 44 and it was shown that a fold catastrophe[45] can occur in a system of equations describing slowly varying field amplitudes. The analysis reported in Ref. 44 was carried out using equations derived in Refs. 15 and 16 and similar to those employed in Refs. 14 and 17 under the conditions when the active medium is in one of the resonators and the coupling is provided by a semitransparent mirror. The dynamics of lasing of two coupled active resonators has been investigated in Ref. 39.

The evolution of fields in two coupled resonators in the range of parameters where steady-state single-mode lasing is unstable can be described using the point model for the amplitudes of the fields and the gains of the active media (expressed using the amplitude and phase variables):

$$\begin{aligned} \dot A_1 &= \frac{1}{2}(g_1 - g_l)A_1 + MA_2\cos(\psi-\chi),\\ \dot A_2 &= \frac{1}{2}(g_2 - g_l)A_2 + MA_1\cos(\psi+\chi),\\ \dot\chi &= -\varphi + M\left[\sin(\psi-\chi)\frac{A_2}{A_1} - \sin(\psi+\chi)\frac{A_1}{A_2}\right],\\ \dot g_{1,2} &= \frac{g_0 - g_{1,2}}{\tau} - A_{1,2}^2 g_{1,2}; \end{aligned} \tag{1.11}$$

here, $A_{1,2}$ are the amplitudes of the fields in the lasers; $\chi = \chi_1 - \chi_2$ is the difference between the field phases; g_0 is the unsaturated gain per one trip through the resonator; $g_{1,2}$ and g_l are the values of the gain and the losses for the first and second lasers, respectively. The dimensionless quantity $\varphi = (\Delta\omega 2L/c)$ represents the detuning of the eigenfrequencies of the resonators and the optical coupling coefficient is characterized by the amplitude M and phase ψ of the coupling. All the quantities with the dimensions of time such as τ (relaxation time) and A^{-2} (stimulated transition time) are made dimensionless when divided by the time which light takes to cross one of the resonators $2L/c$.

The various dynamic regimes are realized depending on many parameters of the problems such as M, ψ, φ, τ, g_0, and g_l. The fullest results on the stability of steady-state conditions are given above for the values of the phase of the coupling coefficient $\psi = 0\ \pi$, and $\psi = \pm\pi/2$. We shall quote here the results of an investigation of the dynamics of lasing for these values of the phase of the coupling coefficient in accordance with Ref. 39.

If the coupling coefficient is a real quantity ($\psi = 0, \pi$) there are no steady-state solutions for $|\varphi| \leqslant 2M$. If $\psi = 0$, then a cophasal distribution of the fields in the lasers is stable, whereas for $\psi = \pi$ the antiphasal distribution is stable. When the mismatch between the resonators exceeds $2M$, lasing becomes time-dependent. The maximum degree of transformation of the energy of the active medium into radiation occurs when the amplitudes of the fields have the same time dependences. The system of equations (1.11) then simplifies greatly and becomes

$$\begin{aligned} \dot A &= \frac{1}{2}(g - g_l) \pm MA\cos\chi,\\ \dot g &= \frac{g_0 - g}{\tau} - A^2 g,\\ \dot\chi &= -\varphi \mp 2M\sin\chi. \end{aligned} \tag{1.12}$$

The upper sign corresponds to the coupling coefficient $\psi = 0$, and the lower to $\psi = \pi$. An analysis of the solution of the last of the equations in the system (1.12) shows that if $|\varphi| > 2M$ then $\cos\chi(t)$ is a periodic function of time with a period $(2\pi/(\varphi^2 - 4M^2)^{1/2}$. Therefore, a study of the dynamics of the radiation intensity can be reduced to the problem of emission from one laser with periodic modulation of the Q factor. Similar problems have been investigated in detail both experimentally and numerically.[46–50] These investigations have demonstrated that near a resonance between the modulation period of the Q factor and the period of normal damped oscillations in the laser system an increase in the modulation amplitude complicates greatly the dynamics of the intensity of laser radiation and may give rise to a chaotic behavior of the system. Similar dynamics of the radiation intensity of two lasers with a real coupling coefficient was investigated numerically in Ref. 39.

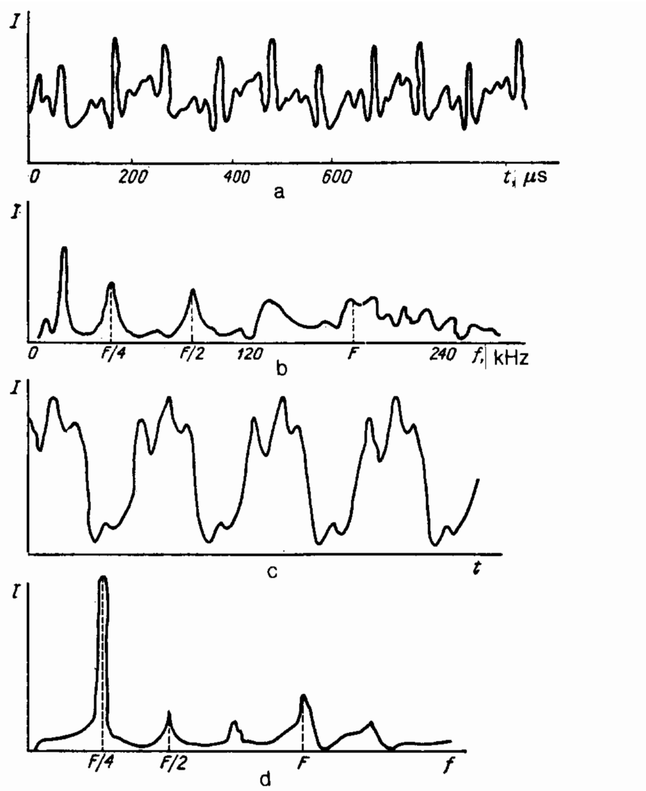

FIG. 10. Time dependences of the radiation intensity (*a*—experimental curve; *c*—calculated dependence) and the spectrum obtained on quadrupling of the period (*b*—experimental results, *d*—calculated curve).[39]

The dynamics of the radiation intensity was studied numerically in Ref. 37 assuming parameters for which the steady-state solutions are unstable or do not exist and this was done on the basis that the coupling coefficient is imaginary ($\psi = \pm\pi/2$). Effects typical of nonlinear dynamic systems were detected.[46] Figure 10 shows, by way of example, the time and spectral dependences of the radiation intensity for one of the lasers, corresponding to quadrupling of the period. The main period of the oscillations is governed by the time taken to transfer energy from one laser to the other. If the coupling coefficient is imaginary, it depends on the phase mismatch of the resonators and on the amplitude of the coupling coefficient:

$$T = \frac{2\pi}{(\varphi^2 - 4M^2)^{1/2}}\ .$$

Calculated phase diagrams corresponding to doubling and quadrupling of the period are shown in Fig. 11. In these calculations the frequencies of energy transfer between the lasers were selected to be close to the frequency of relaxa-

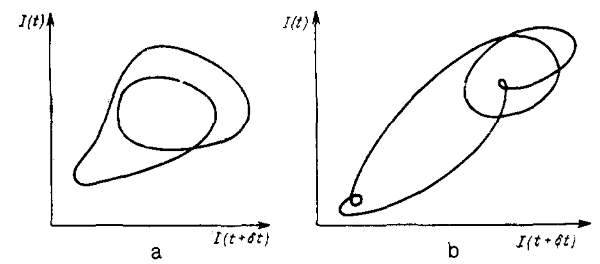

FIG. 11. Phase diagram of the period doubling (a) and quadrupling (b) regimes.

tional oscillations of the intensity in a laser. Variation of these parameters revealed also regimes corresponding to tripling of the period as well as more complex behavior corresponding to a transition of the system to a chaotic behavior.

1.7. Experimental investigations of phase locking of two lasers

One of the very first experimental investigations of phase locking of two CO_2 laser beams by optical coupling was reported in Ref. 28. The apparatus was similar to that shown in Fig. 3 and it included not only two semisymmetric unstable resonators, but also a system for stepwise motion of one of the convex mirrors and an optical system for recording the interference pattern formed by the two laser beams in the far-field zone. The coupling between the lasers was provided by a semitransparent mirror or a shared mirror with an aperture located on the optic axis. Measurements were made of the range where stable phase locking of the lasers was possible; this was done as a function of the detuning of the resonator frequencies and of the fraction of the radiation used to couple the lasers. The results were in qualitative agreement with the theoretical predictions in Ref. 17.

The feasibility of phase locking of fields generated by two pulsed ring CO_2 lasers was demonstrated experimentally by Holswade *et al.*[51] The field phase-locking effect was deduced from the visibility of the interference pattern formed by two laser beams. The fringe visibility was represented by a parameter $(I_{max} - I_{min})/(I_{max} + I_{min})$, which varied from one pulse to another between 0.3 and 0.9. No measures were taken to stabilize the optical lengths of the resonators or the mirror positions and this was clearly the reason why the results were not reproducible. Holswade *et al.*[51] planned to carry out more rigorous investigations of the range of stability of phase locking and of the quality of laser beams.

Detailed experimental studies of the optical coupling of two cw HF laser beams were reported by an American team in Refs. 52–54, describing the feasibility of locking of laser

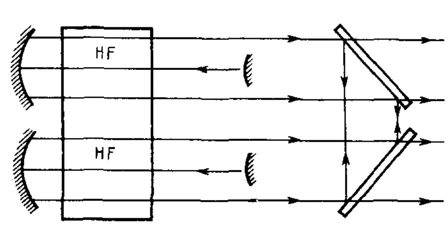

FIG. 12. Equivalent representation of the optical coupling between two HF lasers.[52–54]

fields under multi-frequency emission conditions. They used apparatus shown in a simplified manner in Fig. 12. Two identical confocal unstable resonators with magnifications $m = 1$ and 4 and equivalent Fresnel numbers 4 and 1 were coupled optically by a semitransparent ZnSe plate. The fraction of the energy injected from one laser to the other was 20%. The resonator lengths and the optical paths in the coupling channel were 1.5 m long. The active medium was excited HF formed in a supersonic diffusion generator.[55] The maximum small-signal gain of the active medium reached $g_0l = 1.8$. Simultaneous lasing as a result of nine transitions in HF was observed. The output beams were directed to two mirrors representing a synthesized aperture of the system. The efficiency of phase locking was studied when one or several transitions in HF were active and an investigation was made of the mode composition of the radiation and of the distribution in the far-field zone. A study of the phase locking efficiency was made by recording the interference pattern in the near-field zone in two ways: using a thermal visualization system or employing a radiation detector placed behind a screen with a small aperture, which received radiation reflected by a rotating mirror. The quality of the locking of the laser fields was deduced from the visibility of fringes in this interference pattern. The most important (from the practical point of view) was a high value of the visibility of the pattern (98% in the case of lasing as a result of one transition and 94% in the case of multifrequency lasing) and a low sensitivity of the fringe visibility to the mismatch of the resonator lengths. These results were evidence of almost complete phase locking of the two laser fields. Measurements of the difference frequencies in the spectrum of the longitudinal modes demonstrated that in the case of a single uncoupled laser the intermode spacing was governed by the resonator length $c/2L$, but when two lasers were optically coupled, the spectral components representing mode beats with a frequency interval governed by the total optical length of the system (i.e., by the sum of the two resonator lengths plus the length of the coupling channel) were predominant. It was also demonstrated in Refs. 52–54 that the intensity peak in the far-field zone became narrower by a factor of 2.5 in the case of emission as a result of a single transition in HF and by a factor of 2 in the case of multifrequency lasing. The results of these measurements were extremely sensitive to the difference between the optical paths of the output beams and the setting of the synthesized aperture mirror had to be accurate to within $\lambda/10$. A comparison of systems for phase locking of laser beams based on optical coupling and that based on the use of a master oscillator with several amplifiers was made in Ref. 54. There was no significant difference between the angular distributions of the fields in these two cases, but the optical paths of the laser beams had to be exactly equal.

Antyukhov *et al.*[38] were the first to investigate the influence of a phase shift (advance) in the coupling channel on the width of the stable phase-locking band of two CO_2 lasers with unstable resonators. The boundary conditions for a mismatch of the resonator lengths were a periodic function of the phase in the coupling channel and the period was π (in these experiments a phase shift by this amount corresponded to a change in the optical path by $\lambda/2$ in the coupling channel) and laser locking was found to be most effective when the optical path of the coupling channel was a multiple of λ /

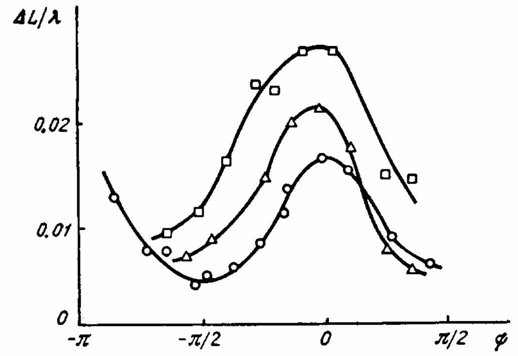

FIG. 13. Dependences of the band of stable phase locking $\Delta L/\lambda$ on the phase shift in the coupling channel (the symbols represent experimental results obtained using different coupling apertures).[38]

2. The fraction of the radiation energy used to couple the lasers was varied in the experiments described in Ref. 38 within the range 10^{-2}–10^{-3}, depending on the size of the coupling aperture. The maximum mismatch between the resonator lengths which still did not result in loss of coherence was $\Delta L = \lambda/20$. The dependence of the stable phase-locking band $\Delta L/\lambda$ on the phase shift ψ in the coupling channel is plotted in Fig. 13. The periodic dependence of the maximum mismatch of the resonator lengths on ψ was in full agreement with the results of a theoretical analysis given in Ref. 33.

The dynamics of the fields of two quasi-cw CO_2 lasers with mutual injection of the radiation was investigated experientally for the first time by Bondarenko *et al.*[39] They used the apparatus shown schematically in Fig. 14. Two waveguide CO_2 lasers *1* and *2* with the resonators formed by plane mirrors *4* and *5* were excited by an ac (10-kHz) discharge. The optical coupling between them was provided by an external mirror *3* and a matching lens *6*, and the strength of this coupling (representing the fraction of the energy used in the process) was $\sim 10^{-3}$–10^{-4}; it was varied by means of calibrated absorbers. The radiation intensity was recorded using a photodetector and a digital oscilloscope, which yielded the results that were analyzed with a microcomputer. The frequency of the beats of the combined intensity in the absence of optical coupling was used to monitor the difference between the normal laser frequencies. Different dynamic regimes were observed depending on the coupling coefficient, on the difference between the laser frequencies, and on the active-medium gain. When the frequency of exchange of energy between the lasers approached the natural frequency of relaxation oscillations, dynamic doubling regimes were observed, the period of the oscillations was quadrupled, and the radiation intensity exhibited a chaotic behavior. Figure 10 shows, by way of example, the time dependence of the intensity and the spectrum of the signal obtained under period quadrupling conditions. The spectrum included also components corresponding to the doubled frequency of the pump current of the active medium amounting to 20 kHz. The frequencies of the relaxational oscillations in the laser system were determined in advance and they could be varied within the range 100–200 kHz by altering the resonator losses. A satisfactory agreement between the experimental and calculated results was achieved employing the point model of coupled lasers.

1.8. Nonlinear optical coupling of two lasers

In addition to the linear methods for coupling lasers, investigations have been made employing nonlinear optical components.[56–59,61,62,64] In experimental studies[56–59,61,64] the coupling between two lasers was established by stimulated four-wave scattering. Photorefractive crystals were used as four-wave mirrors for coupling diode[56,58] and argon ion[57] lasers with relatively low output powers. A $BaTiO_3$ crystal used in these experiments was characterized by a high nonlinearity constant and a low relaxation frequency of ~ 1 Hz.

The optical systems used in extracavity coupling of lasers are shown in Fig. 15. Phase conjugation of the wavefront radiation reflected from a four-wave mirror was recorded in some of the experiments[57] by placing wavefront-distorting transparencies in the path of a beam. Studies had also been made of the spectral characteristics of coupled lasers. It was found that the coupling based on the four-wave interaction made it possible to compensate phase distortions of the wavefront, and the nonlinear interaction in a crystal (scattering back of the radiation from one laser to another by moving diffraction gratings in a crystal) pulled the frequencies of both lasers. Complete identity of the spectra was observed when one of the lasers was operated in the subthreshold regime in the absence of the four-wave mirror (Fig. 15b). It was also proposed to use such optical systems for phase locking of laser arrays.[57] Experiments of this kind were reported in Ref. 58, where in the presence of nonlinear coupling it was found that the transverse mode structure of the fields from arrays of diode lasers exhibited considerable modification and it was found from the intensity distributions in the far-field zone that cophasal and antiphasal field distributions were obtained. Experiments involving an addi-

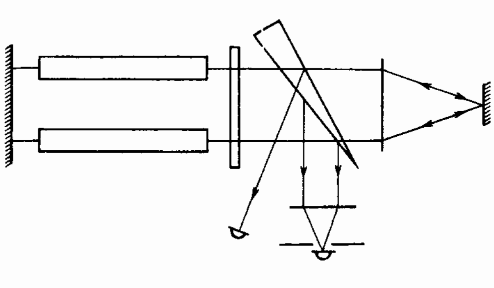

FIG. 14. Experimental setup used to investigate the dynamics of radiation emitted from two optically coupled CO_2 lasers.

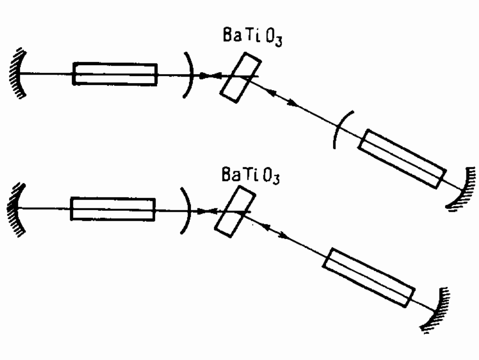

FIG. 15. Optical systems for extracavity coupling of lasers utilizing the four-wave interaction between optical beams.

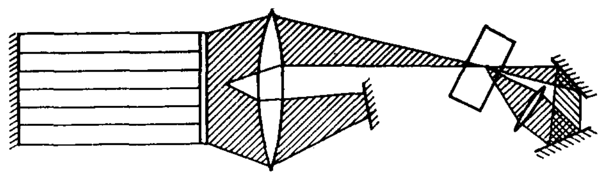

FIG. 16. Coupling of lasers with self-pumped phase-conjugating mirrors.

tional self-pumped phase-conjugating mirror (Fig. 16) were reported in Ref. 58. The reflection coefficient of this mirror was 10% and it altered the mode structure of the fields generated by a laser array. A detailed analysis of the properties of the resonators with phase conjugating mirrors was given in Ref. 63.

Phase locking of waveguide CO_2 lasers employing an extracavity four-wave mirror was described in Ref. 59. The apparatus used (Fig. 17) consisted of two CO_2 lasers with an output power of ~ 5 W emitting pulses up to 50 ms duration.

A matching lens was used to collect the radiation inside a cell formed by a plane-parallel KCl plate and a plane-parallel metal mirror, oriented normally to one of the laser beams. Toluene was used as a nonlinear liquid because it exhibited a weak dispersion of the absorption coefficient and a fairly high thermal nonlinearity constant.[60] A wedge made of KCl directed part of the radiation to a recording system consisting of a photodetector and a slit used to scan the region of overlap of the beams at the lens focus. This slit scanning took 0.5 ms and the time delay relative to the beginning of a lasing pulse could be varied. Two optical systems were used in the experiments reported in Ref. 59. In one of them a "seed" thermal grating was formed by beam splitters, whereas these splitters were absent from the second system so that such a grating did not form. In both cases it was found that the two lasers operated coherently after 2 ms measured from the beginning of lasing. The distributions recorded in the far-field zone corresponded to cophasal and antiphasal operation of the two lasers.

Strong scattering by the thermal grating was observed after 2–3 ms from the beginning of lasing and the scattering coefficient averaged over one pulse (and representing the energy injected from one laser into the other) amounted to 3–4%. The resonator lengths had to be matched to within 10^{-4} cm in order to ensure stable locking of the fields.

An investigation of the operation of two neodymium lasers with resonators coupled by a dynamic hologram was described in Ref. 61. The optical system used was similar to that employed in earlier studies.[57,59] A nonlinear medium was a liquid with a thermal nonlinearity (isopropyl alcohol or acetone) in a cell 6-mm thick. The laser energy density of the beams in the cell did not exceed 40 J/cm^2.

It was found experimentally that simultaneous lasing resulted in matching of the fields in the two lasers, namely the time dependences of the intensities of the radiation during the advanced stage of lasing were identical and the transverse distributions of the fields were in agreement, as confirmed by the distributions in the near- and far-field zones. A resonator with one mirror in the form of a nonlinear element was nonselective in respect of the transverse structure of the laser fields. Introduction of additional optical inhomogeneities (lenses and a burner flame) did not alter the nature of lasing. Analytic investigations[61] of stable operation of such a system showed that an increase in the active-medium gain resulted in relaxation of the excitation conditions (because initially an injected external signal was needed for lasing). The ranges of the parameters corresponding to these regimes were determined. It was also found that a system of resonators coupled by a dynamic hologram could be bistable.

The use of optical systems with nonlinear elements inside the lasers made it possible to achieve not only mutual injection of the fields, but a nonlinear self-matching of the optical lengths of the resonators. The feasibility of phase locking of lasers with the aid of an intracavity nonlinear element was investigated by Likhanskiĭ *et al.*[62] allowing for the four-wave interaction. A system for coupling two lasers[62] shown in Fig. 18 was identical with the systems including a self-pumped phase-conjugating mirror.[63] The nonlinear solutions of the problem were found in Ref. 62 assuming an instantaneous-response nonlinearity and a slight difference between the normal resonator frequencies. In contrast to the case of linear coupling of the lasers the number of steady-state regimes was greater when the four-wave intracavity phase-locking method was used. For example, when the difference between the resonator frequencies was zero, the difference between the phases of the fields could have the values $0, \pi, \pm\pi/2, \pm\pi/3$, and $\pm 2\pi/3$. In the first four regimes the intensities were the same in both lasers, but otherwise the intensities differed. An analysis showed that solutions characterized by $\Delta\omega = 0$ and by a difference between the phases of the field amounting to $0, \pi$ should be stable against small perturbations. Single-frequency phase locking should occur when the phase mismatch of the resonator lengths is compensated by the phase advance in the nonlinear medium: $|\Delta\omega|2L/c < 2klI, \chi = knc/2\pi\omega$ is the nonlinear polarizability coefficient of the nonlinear medium and l is the size of the interaction region in the nonlinear medium. Locking of the frequencies of the two TEA-CO_2 lasers as a result of the intracavity four-wave interaction of light beams was achieved by Baranov *et al.*[64] They used a nonlinear medium in the form of gaseous SF_6 placed inside a cell 25-cm long. Radiation pulses consisted of a strong "spike" of ~ 1 mW power and ~ 200 ns duration followed by a gently sloping

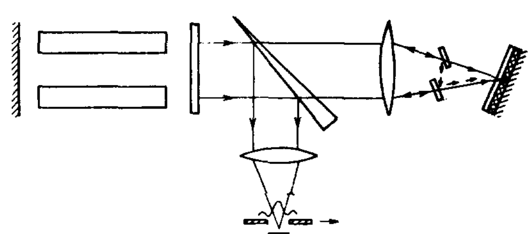

FIG. 17. Intracavity coupling of two CO_2 lasers in a nonlinear cell.

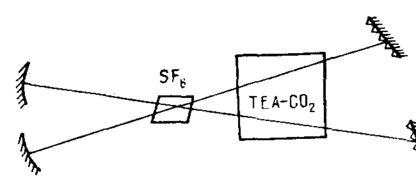

FIG. 18. Coupling of two CO_2 lasers with an intracavity nonlinear cell containing SF_6.

"tail" of $\sim 1.5\ \mu s$ duration, carrying half the pulse energy. The energy of the fundamental mode was 0.5 J. These experiments were carried out using different rotational–vibrational transitions in CO_2. Locking of the lasers was deduced from the visibility of the interference pattern formed by the two laser beams. In the absence of SF_6 the lasers operated independently. When the SF_6 pressure was ~ 2 Torr, the laser frequencies were locked in some of the pulses. An increase in the pressure above 3 Torr resulted in stable locking. The locking mechanism was as follows: When the frequencies of the fields in the SF_6 cell were identical, the refractive index and absorption gratings were formed and these resulted in the scattering of light from one resonator to the other. When the frequencies of the laser fields were unequal, such refractive index and absorption gratings did not form in SF_6 because of the inertia of the response of this medium. Locked operation was characterized by an additional (compared with that discussed in Ref. 62) stability margin, which was due to saturation of the absorption coefficient of SF_6.

Locking of the fields of two cw dye lasers by the intracavity four-wave interaction of optical beams was demonstrated in Ref. 65. Sodium vapor at 198 °C was used as a nonlinear element. The frequency of the resonant transition in Na ($\lambda = 5980$ Å) was close to the laser radiation frequency. The light beams interacted in a system similar to that shown in Fig. 18. The beams met at an angle not exceeding 0.5°. When the radiation intensities were 7 and 3.5 kW/cm^2, stable locking of the fields of two lasers was observed for a period of 1 min.

Many others[57–59,62,64] suggested using these systems for locking the fields of laser arrays.

2. OPTICAL COUPLING OF MANY LASERS

An analysis of the coupling of two lasers given in Sec. 1 shows that the main condition for stable phase locking is a small difference between the normal frequencies of the resonators compared with the strength of the coupling. Obviously, the same condition must apply also when locking of a large number of lasers is considered.

It is technologically convenient to fabricate laser arrays with periodically distributed components. It is then possible to couple both the adjacent components and all the lasers together. The nearest-neighbor coupling is used quite frequently in periodic arrays of semiconductor and waveguide lasers if the radiation from each waveguide can penetrate the adjacent waveguide.[66] Optical coupling can also be provided by external mirrors[6,67] that deflect a small fraction of the radiation to the adjacent lasers. Possible optical laser-coupling systems are shown in Fig. 19.

2.1. Simplest models. Collective modes

A simple model of coupled lasers is a medium with a periodic distribution of the refractive index and of the gain, placed between resonator mirrors. The propagation of radiation is then described by a system of parabolic equations for two counterpropagating waves:

$$\pm 2ik_z \frac{\partial E_\pm}{\partial z} + \Delta_\perp E_\pm + \left(\frac{\varepsilon(x,y)\,\omega^2}{c^2} - k_z^2\right) E_\pm = 0, \qquad (2.1)$$

where $E_\pm$ are the amplitudes of the fields of the counterpropagating waves and $\Delta_\perp = \partial^2/\partial x^2 + \partial^2/\partial y^2$. The fields should be matched in the plane of the resonator mirrors by suitable boundary conditions imposed on Eq. (2.1). Equation (2.1) is an analog of the secular Schrödinger equation describing the motion of a quantum particle in a periodic potential. If the dependence $\varepsilon(x,y)$ is in the form of periodic rectangular regions, then the problem can be factorized for each of the directions x and y. The solution can then be reduced to the familiar case of the Kronig–Penney potential,[68] and hence the spectrum of the eigenvalues (normal frequencies) and of the normal modes (field distributions) can be obtained. The spectrum has then a band structure. If we assume (as is usually done for periodic semiconductor laser arrays[69]) that the absorption and amplification regions have a periodic structure, it is found that collective modes are amplified to a different extent during a trip through a resonator. The strongest enhancement is experienced by the field distributions for which the functional

$$\mathrm{Im} \int \varepsilon(x,y)\,|E(x,y)|^2\,dx\,dy \left(\int |E(x,y)|^2\,dx\,dy\right)^{-1} \qquad (2.2)$$

reaches its minimum value. The condition $\mathrm{Im}\,\varepsilon > 0$ in Eq. (2.2) corresponds to an absorbing medium, whereas $\mathrm{Im}\,\varepsilon < 0$

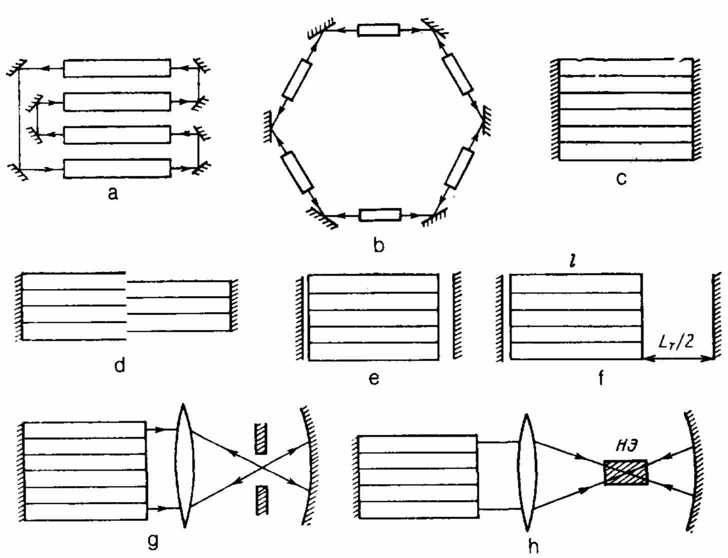

FIG. 19. Possible methods of optical coupling of laser arrays.

represents an amplifying medium. Usually the amplification regions are inside a waveguide and the absorption of radiation occurs partly in the regions separating the individual lasers. Obviously, a minimum of the function (2.2) is realized for an antiphasal distribution of the field corresponding to an alternation of the sign of the fields in neighboring waveguides, i.e., it corresponds to the upper edge of a zone. Then, the field amplitude falls to zero in an absorbing region between the waveguides, whereas it reaches its maximum value in the amplifying region inside the waveguide. It follows that in the case of a perfect rectangular periodic array of lasers coupled across waveguide walls it would be easy to ensure generation of an antiphasal collective mode.

Calculations of the generation of collective modes in semiconductor laser arrays are reported in Refs. 70–72. Usually the field distribution in the far-field zone is of the two-lobe type with an angular separation between the lobes (maxima) governed by the array period.

A self-consistent model for numerical calculation of the structure of the fields of an array of diode waveguide lasers with external signal injection was proposed in Ref. 73. The dependences of the intensity in the far-field zone on the entry angle and on the wavelength of the injected radiation were obtained. For certain parameters of the external signal the distribution in the far-field zone was of the single-lobe nature with an angular width governed by the total aperture of the array.

The problem of phase locking of lasers is closely related to the mode structures and emission from lasers utilizing retroreflector mirrors in the form of periodic sets of corner-cube reflectors or prisms.[74–76] Lasing may occur between any pair of reflectors. Two possible paths of rays in a resonator with a mirror composed of corner-cube reflectors are shown in Fig. 20. A system of this kind can be regarded as a set of separate lasers coupled by the diffraction of radiation when it passes between the mirrors. The distribution of the fields in a laser with a retroreflector mirror was investigated theoretically by Bel'dyugin.[77]

A model of a set of coupled lasers described by Eq. (2.1) can be used to represent directly semiconductor lasers with an internal coupling by tunnel diffraction.

A more general model, which represents a natural generalization of the model of two coupled resonators [Eq. (1.2)] is described by

$$\frac{2L}{c}\,\frac{\partial E_n}{\partial t} = E_n\left[(g_n - g_{\mathrm{th}})\,l + i\cdot\frac{2L}{c}\,\Delta\omega_n\right] + \sum_m M_{nm}E_m, \tag{2.3}$$

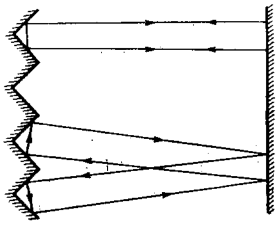

FIG. 20. Paths of rays in a resonator with a mirror composed of corner-cube reflectors.

where $2L/c$ is the time needed for one round trip in the resonator; g_{th} is the threshold gain; g_n is the gain of the nth channel; $\Delta\omega_n$ is the detuning of the normal frequency of a single laser from the laser transition frequency; M_{nm} is the matrix of the coefficients representing the coupling between the channels. Difficulties encountered in the determination of the coupling coefficients of real systems discussed in Sec. 1 remain also in the case of laser arrays. In particular, Eq. (2.3) is written down on the assumption that each laser in an array emits only one mode. It is also assumed that E_n changes little in the time $2L/c$.

We shall first consider the spectrum and structure of collective modes in the case of an ideal array of lasers on the assumption that $g_n = g_{\mathrm{th}}$ and $\Delta\omega_n = 0$. The problem can be reduced mathematically to finding the eigenvalues and the eigenvectors of the following system of linear equations:

$$\gamma^{(j)}E_n^{(j)} = \sum_m M_{nm}E_m^{(j)}, \tag{2.4}$$

where j is the mode number. Obviously, the number of the collective modes is equal to the number of lasers N. The spectrum of the modes is governed largely by the structure of the coupling matrix M_{nm}, which in turn depends on the geometry of the laser array and on the coupling method. Figure 19 shows a number of schemes which can be used for the optical coupling of laser arrays that have been discussed in the literature. In practically all the cases the coupling matrix can be regarded as depending only on the difference between the indices: $M_{nm} = M(n-m)$, where n and m are either simple (in the case of one-dimensional arrays) or vector (in the case of two-dimensional arrays) indices. They are known as the Toeplitz matrices[78] and their properties have been investigated in detail. In particular, in the case of large numbers of lasers when $N\to\infty$ the eigenvalue spectrum is easily found:

$$\gamma(q) = M(q) = \sum e^{iqn}M(n), \tag{2.5}$$

if q for large but finite values of N assumes discrete values separated by steps of the order of $\delta q \approx 2\pi/N^{1/2}$ for a two-dimensional array ($\delta q = 2\pi/N$ for a one-dimensional array).

We can easily see that the collective modes are described by $E_n(q) = c_q e^{iqn}$ if $\gamma(q)$ is a nondegenerate spectrum. If $M(n)$ is an even function (for example, if it represents symmetric coupling of the two nearest neighbors), then

$$E_n(q) = c_{q1}e^{iqn} + c_{q2}e^{-iqn}.$$

We must mention one further common property of the solutions of the systems (2.4) with a difference matrix. If the second moment of the matrix $M(n)$ does exist, i.e., if we have $\Sigma n^2M(n) < \infty$ when $N\to\infty$, then the collective modes with the eigenvalues at the edge of the band ($j \ll N$, $N-j \ll N$) are smooth functions of the variable, so that we can now adopt the Fokker–Planck approximation in Eq. (2.4). We then have

$$\sum_m M(n-m)E_m \to M_0E_n + U\frac{dE_n}{dn} + D\frac{d^2E_n}{dn^2},$$

$$M_0 = \sum_n M(n),\quad U = \sum_n nM(n),\quad D = \frac{1}{2}\sum_n n^2M(n).$$

In the Fokker–Planck approximation the problem of finding collective modes for a laser array reduces to the classical eigenvalue problem for a medium with diffusion–convective transport. We note that both the "velocity" U and the "diffusion coefficient" may be complex variables. The problem of the boundary conditions is solved in this formulation if N is large (and the results are then accurate to within terms of the order of $1/N$): at the edges of a laser array the field should be assumed to be zero if there is no additional coupling.

We shall use these general properties to discuss several simple fairly common cases.

In the case of the scheme labeled a in Fig. 19 with a unidirectional coupling we have $(M_{n,m} = M\delta_{n,m-1})$

$$\gamma(j) = M\exp\frac{2\pi i\,(j-1)}{N},\quad E_n^{(j)} = \exp\frac{2\pi i\,(j-1)\,n}{N}. \qquad (2.6)$$

Since $E_n(t) \propto \exp(\gamma_n t)E_n$, it follows that the mode with the highest Q factor has the largest Re γ. [The value of $\gamma(j)$ is measured from any point, because in this discussion it is assumed arbitrarily that $M(0) = 0$. When the real value of $M(0)$ is substituted, the condition Re $\gamma \leqslant 0$ is obviously satisfied, provided there is no amplifying medium.]

In the case of a one-dimensional laser array with the nearest-neighbor coupling (see also Ref. 79)

$$\gamma(j) = 2M\cos\frac{2\pi\,(j-1)}{N},\quad E_n^{(j)} = \sin\frac{2\pi nj}{N}, \qquad (2.7)$$

where $M_{n,m} = M(\delta_{n,m} + \delta_{n,m+1})$. We can easily show that the solutions represented by Eq. (2.7) can be obtained, accurate to within terms $\sim 1/N$, using the Fokker–Planck approximation provided $j \ll N$, $N - j \ll N$.

We shall consider one further limiting case: all lasers are coupled pairwise with one another and have the same coupling coefficient.[79] We can then use Eq. (2.5) to show that there is only one eigenvalue $\gamma(0) = NM$, whereas the others obey $\gamma(j) \equiv 0$. The mode which is then generated is cophasal and its intensity is the same for all the lasers.

An example of a more complex coupling will be discussed in the next section (Sec. 2.2).

In the case of a laser array the problem of coherent emission and the problem of the influence of the scatter of normal frequencies of the individual resonators and of the nonlinearity of the medium on this stability is very difficult and cannot be solved in general terms. Some results, which give at least a qualitative idea of the requirements in respect of the parameters of an array can be obtained for the simplest models. For example, in the case of the scheme shown in Fig. 19a where a sequential coupling of lasers is assumed and a ring is closed by the $N \to 1$ coupling, we can readily obtain a nonlinear dispersion equation [if we assume that $E_j \sim \exp(i\Omega t)$]

$$\prod_{j=1}^{N}\left[(g_j - g_{\text{th}})\,l + i\Delta\omega_j\cdot\frac{2L}{c}\right] = \prod_{j=1}^{N}(-M_j), \qquad (2.8)$$

where M_j is the $(j-1)\to j$ coupling coefficient. The threshold gain, the resonator lengths, and the lengths of the active media are assumed to be the same for all the lasers. In the case of given values of the gain we find from this equation a general output frequency Ω, since $\Delta\omega_j = \omega_j - \Omega$. Substituting the above frequency into the system (2.3), we obtain the following equations:

$$E_j = \frac{-M_jE_{j-1}}{(g_j - g_{\text{th}})\,l + i\,(2L\Delta\omega_j/c)}, \qquad (2.9)$$

from which we can calculate consecutively all the values of E_j by specifying—for example—E_1. Then, in the simplest case, we have

$$g_j = \frac{g_{j0}}{1 + (|E_j|^2/E_s^2)},$$

where E_s is the amplitude of the saturation field. In particular, if $|E_j| = |E_{j-1}|$, we obtain the equations for g_j (on the assumption that g_{j0} and M_j are independent of j):

$$(g_j - g_{\text{th}})^2\,l^2 + \frac{4L^2}{c^2}(\Delta\omega_j)^2 = |M|^2, \qquad (2.10)$$

from which it follows that a stationary solution exists only if $(2L/c)|\Delta\omega_j| \geqslant |M|$. If one of the lasers in a chain is not phase-matched, then it emits generally multifrequency radiation. Therefore, coherence of the field of the whole array breaks down if the condition (2.10) is not satisfied by just one laser. If for some reason the detuning of the normal frequencies of the resonators (or the coupling coefficients) do not vary greatly for adjacent lasers, while the overall change may still be large as a result of accumulation, then it is convenient to employ for the analysis, the Fokker–Planck approximation mentioned above.

An instructive example which can be used to investigate conveniently the influence of the scatter of the normal frequencies of the resonators, is provided by a model of coupling "of all the lasers to all others". As pointed out already, in this case one collective mode of an ideal array is preferred (it has a high Q factor when Re $M > 0$). It is clear from Eq. (2.3) that the field injected into each laser is the same throughout $(\Sigma_{m=1}^N M_{nm}E_m = M\Sigma_m E_m \equiv ME_\Sigma)$. The steady-state solution of the system (2.3) is then obtained directly:

$$E_n = \frac{-ME_\Sigma}{(g_n - g_{\text{th}})\,l + i\,(\omega_n - \Omega)\,(2L/c)}. \qquad (2.11)$$

Here Ω is the general frequency for all the lasers.

Summing Eq. (2.11) over n, we obtain the coherent lasing condition

$$\sum_n \frac{M}{(g_n - g_{\text{th}})\,l + i\,(\omega_n - \Omega)\,(2L/c)} = -1, \qquad (2.12)$$

The gain for a single laser is then given by

$$g_n = \frac{g_0}{1 + |E_n|^2/E_s^2},$$

where E_s is the amplitude of the saturation field; the small-signal gain is assumed (for simplicity) to be the same for all the lasers, and the lengths l and L are also assumed to be the same. In the case of a large number of lasers the sum in Eq. (2.12) can be rewritten in terms of an integral by introducing the probability density of a specific value of the normal frequency of a laser $f(\omega)$, so that $\int f(\omega)d\omega = 1$:

$$NM\int\frac{f(\omega)\,d\omega}{(g(\omega) - g_{\text{th}})\,l + i\,(\omega - \Omega)\,(2L/c)} = -1. \qquad (2.13)$$

We shall give the solution of the problem formulated above by assuming that the probability density has the simplest form

$$f(\omega) = \frac{1}{2\Delta},\quad |\omega| \leqslant \Delta.$$
$$= 0,\quad |\omega| > \Delta.$$

(It should be stressed that all the frequencies are measured from the center of a gain profile and that the width of this profile is assumed to be large compared with the intermode spacing.) If $\Delta \ll NM$ (it is assumed also that M is purely real), then

$$g(\omega) = g_{st}(1+\alpha\omega^2),$$
$$I(\omega) = I_{st}(1-\beta\omega^2),$$

where

$$g_{st} = g_n - \frac{MN}{l}$$
$$\times \left[1 - \frac{1}{3}\left(\frac{\Delta}{MN}\right)^2 \frac{NM + (g_n l - MN)\left(1 - \frac{g_n l - MN}{g_0 l}\right)}{NM + 2(g_n l - MN)\left(1 - \frac{g_n l - MN}{g_0 l}\right)}\right],$$
$$I_{st} = I_s\left(\frac{g_0}{g_{st}} - 1\right),$$
$$\beta = \frac{1}{MN}\,\frac{1}{MN + 2g_{st}l\,[1-(g_{st}/g_0)]}, \quad \alpha = \beta\left(1 - \frac{g_{st}}{g_0}\right).$$

It follows from the meaning of the above expressions that g_{st} is the threshold gain for the collective lasing at the frequency $\Omega = 0$. In the absence of a scatter of the normal frequencies, we have $g_{st} = g_{th} - (MN/l)$, whereas the scatter increases the threshold. In the case of independent lasing the threshold is higher. Direct calculations show that if $\Delta < MN$, coherent lasing is stable in the sense that none of the coupled lasers can emit independently for just any value of the ratio g_0/g_{st}.

We must draw attention to the fact that if the normal frequencies exhibit a scatter, it follows from Eq. (2.11) that the phases of the fields in the individual lasers remain constant in time, but they vary in accordance with the resonator mismatch.

Obviously, the coupling of all lasers to all others is an ideal situation in the case of real and positive values of M and this situation can only be approached. Such a configuration is least affected by the scatter of the normal frequencies of the resonators. However, this configuration is very difficult to construct if the number of lasers is large. Some approximation to the properties of an array of this kind can be provided by a coupling between lasers due to the diffraction of radiation by a stop with a diameter a at the focus of a lens if $a \approx \lambda F/D$, where λ is the wavelength of the radiation, F is the focal length, and D is the overall diameter of the array. A similar effect is produced by a plane mirror located at a distance $z \approx d^2/\lambda$ from the laser array (d is the array period). It should be pointed out that these two coupling methods suffer from considerable losses and from an increase in the lasing threshold.

A plane mirror located at some distance from the aperture of a laser array is a relatively simple (from the technological point of view) optical coupling element. The radiation reflected by this mirror establishes an optical coupling between the components of the array because of diffraction spreading. However, at first sight the efficiency of such coupling seems to be fairly low. The proportion of the radiation transferred from one laser to another can be increased by moving the mirror away from the aperture. Then, because of diffraction, the radiation is "smeared out" over the complete aperture and the losses in the coupling channel increase. It is found that in the case of a periodic laser array such a simple analysis is invalid because it ignores reproduction of the periodic structure of the monochromatic field at a specific distance. If the coupling mirror is located at half the distance of the reproduction of the periodic structure, the images of the channel ends (if they emit in a cophasal manner) are projected back exactly on the same ends. On the other hand, since the image of each spot includes a contribution from many channels, the coupling between the individual lasers is fairly strong. If the radiation emitted by the individual channels is not cophasal, the diffraction image breaks down and the losses increase strongly. This effect thus results in selection of the regime characterized by phase locking. In view of the high selectivity and the relatively simple technology of this coupling method, we shall consider it in greater detail.

2.2. Phase locking because of the Talbot effect

The problem of reproduction of a periodic structure of the field at certain distances discovered by a French investigator Talbot over 150 years ago,[80] and first explained by Rayleigh,[81] has been "rediscovered" many times, but it has remained known only to a narrow circle of specialists, because until recently it had no practical application. The Talbot effect has quite recently been used in measurements of the phase inhomogeneities of the wavefront[82] in studies of the mode structures reached in plane–plane resonators with a periodic distribution of the reflection coefficient and with selectors in the form of periodic structures.[83,84]

We shall give the simplest derivation of the expression for the reproduction distance in a periodic structure by considering the example of a one-dimensional grating with the period a. We shall assume that the field distribution at some plane $z = 0$ (where z is the direction of propagation of the radiation) has the following form in the transverse direction x:

$$E(x, z=0) = \sum_{n=-\infty}^{+\infty} f(x - na) \equiv \sum_{m=-\infty}^{+\infty} \alpha_m \exp\left(\frac{2\pi i}{a} mx\right). \tag{2.14}$$

In the paraxial approximation, bearing in mind the monochromaticity of the signal, we find that the field distribution at a distance $z = L$ becomes

$$E(x, z=L) = \exp\left(i\frac{\omega_0}{c}L\right) \sum_{m=-\infty}^{\infty} \alpha_m$$
$$\times \exp\left(\frac{2\pi i}{a} mx\right) \exp\left[-i\frac{c}{2\omega_0}\left(\frac{2\pi m}{a}\right)^2 L\right]. \tag{2.15}$$

The field distribution, accurate up to a common phase factor, is identical with the initial distribution described by Eq. (2.14) if $L = (2a^2/\lambda)P$ (P is an integer). This is known as the Talbot distance for a one-dimensional periodic structure.

The distribution of radiation generated by an array of lasers with the Talbot coupling can be analyzed if we divide the process of establishment of this distribution into two main stages. The first is a double passage along the waveguides containing the active medium and the second is the passage to the coupling mirror and back again. We shall assume that the distribution of the radiation in a waveguide is characterized by a certain set of normal transverse modes and a discrete spectrum of the normal frequencies. We shall

TABLE I.

Type of lattice	Translation vectors	Reciprocal lattice vectors	Number of modes n depending on distance to mirror z_M
Triangular	$a_1 = (1,\ 0)\,a$ $a_2 = (1/2,\ \sqrt{3}/2)\,a$	$b_1 = \frac{2}{a/\sqrt{3}}(\sqrt{3}/2,\ -1/2)$ $b_2 = \frac{2}{a/\sqrt{3}}(0,\ 1)$	Если $z_3 = mz_T/2$ (m — целое), то $n = 3m^2$
Rectangular with integral period ratio P	$a_1 = (Pa,\ 0)$ $a_2 = (0,\ a)$	$b_1 = (1/Pa,\ 0)$ $b_2 = (0,\ 1/a)$	if $z_M = mz_T/4$, then $n = m^2P^2$

assume that one transverse mode is selected and that it can be described by the field distribution $f(r)$ at the end of a channel. Without limiting the generality of the treatment, we can select the function $f(r)$ to be real and normalized in such a way that

$$\int_S f^2(r)\,dr = 1,$$

where S is the area of the channel end. The field in a plane coinciding with the channel ends is represented in the form

$$E(\rho) = \sum_{m,n} C(R_{m,n})\, f(\rho - R_{mn}), \tag{2.16}$$

where R_{mn} is the coordinate of the center of the channel with the number (m,n), i.e., $R_{mn} = ma_1 + na_2$; a_1 and a_2 are the vectors representing the translation of the lattice of ends of the channels; ρ is the coordinate in the investigated plane; $C(R_{mn})$ is the amplitude of the envelope of the field. The vectors a_1 and a_2 representing different types of lattices are listed in Table I. After traveling a distance z the field becomes

$$E(z,\rho) = \frac{iK_0}{2\pi z}\exp(iK_0 z)\int E(\rho')\exp\left[\frac{iK_0}{2z}(\rho-\rho')^2\right]d\rho'. \tag{2.17}$$

If we assume that at a distance $z/2$ from the plane of the ends of the channels there is a plane mirror with a sufficiently large aperture and that the projection of Eq. (2.17) onto Eq. (2.16) is reproduced apart from a constant factor, the problem of finding the field in the resonator can be reduced to the problem of eigenvalues of a system of linear equations:

$$\gamma' C(R) = A\sum_R M(R,R')\,C(R'), \tag{2.18}$$

where

$$M(R,R') = \frac{iK_0}{2\pi z}\int d\rho\, d\rho' f(\rho - R)\, f(\rho' - R') \times \exp\left[\frac{iK_0}{2z}(\rho-\rho')^2\right]; \tag{2.19}$$

γ' is the eigenvalues whose modulus determines the losses and the phase governs the normal mode frequency; A is a constant representing the total phase shift (advance) and the change in the amplitude in the waveguides. We shall use $\gamma = \gamma'/A$.

When the distance from the nontransmitting mirror to the waveguide ends is $z/2$, which is half the Talbot distance, we can obtain the limiting solution of the system (2.18) for $N\to\infty$, which corresponds to complete reproduction of a constant-phase distribution of the field between the components; $C(R) = \text{const}$. The expressions for z_t in the case of gratings of different types are given in Ref. 85. We then find that $|\gamma| = 1$.

This can be demonstrated more simply by rewriting Eq. (2.17) for the Fourier transforms of the field:

$$E(z,q) = E(0,q)\exp\left(-i\frac{zq^2}{2k_0}\right)\exp(ik_0 z), \tag{2.20}$$

and bearing mind that $E(\rho)$ is a periodic function so that q is a multiple of $2\pi b$, where b is the reciprocal lattice vector. Then, $b = mb_1 + nb_2$ (m and n are integers and the vectors b_1 and b_2 are listed in Table I).

Since $2\pi^2 z_T b^2/k_0$ for all values of b is a multiple of 2π, the phase factor in Eq. (2.20) is the same for all the Fourier components, which implies the self-reproduction effect.

In addition to the normal modes corresponding to a constant-phase distribution of the field, periodic arrays can exhibit also other self-reproduced field distributions. In the case of an infinite array of lasers the eigenvalues of the system (2.18) are easily found from

$$\gamma = M(q) = \sum_R M(R)\exp(-iqR), \tag{2.21}$$

since the matrix $M(R,R')$ is a function of the difference; $M(R,R') = M(R - R')$. The eigenvectors $C(R) = \exp(iqR)$ represent a discrete analog of plane waves. After calculation of $M(q)$, allowing for the finite and normalized nature of the function $f(r)$, we can readily demonstrate that $M(q=0) = 1$ (in the case of a cophasal mode) and $|M(q)| = 1$ also when $q = q_1 = k_0 a_1/z_T$ and $q_2 = k_0 a_2/z_T$.

In the case of a triangular lattice the Brillouin zone has the form shown in Fig. 21, including the vectors q_1 and q_2. The phases of the three adjacent lasers encountered in the

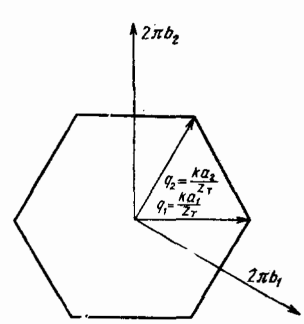

FIG. 21. Brillouin zone of a triangular array (lattice).

clockwise direction are shifted relative to one another by an amount $2\pi/3$ in the case of a self-reproducing mode with $q=q_1$, whereas in the case of a mode with $q=q_2$ they are shifted by $-2\pi/3$. The eigenvalues for these models are the same: $M(q_1)=M(q_2)=\exp(-2\pi i/3)$, which implies degeneracy of the normal frequencies of these modes differing by $c/6L_0$ from the fundamental-mode ($q=0$) frequency (L_0 is the distance between the resonator mirrors).

In the case of a rectangular lattice the infinite system of equations (2.18) becomes factorized. In addition to a cophasal mode, there are then also modes with $q_1=ka_1/z_T$ and $q_2=ka_2/z_T$, corresponding to an antiphasal (alternating-sign) distribution of the fields in the channel. In contrast to a triangular lattice, where the self-reproduction of the fields appears at the minimum z_T, the minimum distance for a square lattice is $z_T/4$. One mode with the phase modulation $(0,\pi)$, i.e., $C(n,m)=\exp[i\pi(n+m)]$ is then reproduced.

We shall consider the mode selection method later; we shall now study the influence of the finite nature of an array and the mirror alignment precision.

When the number of lasers is reduced, the diffraction image deteriorates and this should increase the losses. The size of the array at which the lasers become effectively coupled is governed by the angle of the divergence of the radiation from one channel $\theta_{\text{diffr}}\sim\lambda/\delta$, where δ is the internal radius of the channel. Then, $\sim a^2/\delta^2$ are strongly coupled. If the size of an array exceeds a^2/δ, there should be little change in the stucture of the fundamental modes. We shall seek the solution of the system (2.18) in the form

$$C(R)=\psi(R)\exp(iq_0R),\quad q_0=0,\ q_1,\ q_2, \tag{2.22}$$

where q_0 corresponds to one of the self-reproducing modes and $\psi(R)$ is a function with a characteristic size equal to the aperture of the array. Under these assumptions the system (2.18) can be reduced to an equation for an envelope of the field amplitudes:

$$(\gamma-\gamma_0)\psi=\gamma_0 D\Delta_\perp\psi, \tag{2.23}$$

where

$$\gamma_0=\exp\left(-i\frac{z_T q_0^2}{2k_0}\right),$$

$$D=\left(\frac{z_T}{2k_0}\right)^2\int(\nabla f(\rho))^2\,d\rho-\frac{iz_T}{2k_0}. \tag{2.24}$$

The complex "diffusion coefficient" can be estimated from $\sim Aa^2(a/\delta)^2-iBa^2$, where A and B are constants of the order of unity, depending on the nature of the periodic array and on the field distribution over the waveguide cross section. In the limit of a small gap $a\gg\delta$, Eq. (2.23) reduces to the familiar boundary-value diffusion problem.

Equation (2.23) must be supplemented by the boundary conditions in order to complete formulation of the problem. Near the edge of the region occupied by the lasers the amplitude should decrease because of the uncompensated loss of radiation by diffraction. The law describing the fall of $\psi(R)$ in the direction of the edge can be found by solving the system (2.18) by the Wiener–Hopf method for a semiinfinite laser array. The solution of this problem can be found in Ref. 86, from which it follows that the boundary condition for Eq. (2.23) requires that $\psi(R)$ should vanish at an "extrapolated" distance l, which is of the order of $l\approx a^2/\delta$.

We shall now assume that lasers occupy a strip defined by $-L/2\leqslant x\leqslant L/2$. Then, the solution of Eq. (2.23) for the lowest mode subject to the conditions $\psi[\pm(L/2+l)]=0$ [or by the "impedance" conditions[85] $\partial\psi/\partial x\pm(\psi/l)=0$ at $x=\pm L/2$ which are equivalent in the leading order in l/L] is

$$\psi_1(x)=\cos\frac{\pi x}{L+2l}. \tag{2.25}$$

We then have

$$\frac{\gamma}{\gamma_0}=1-\frac{\pi^2 D}{(L+2l)^2}.$$

In general, the length l is a complex quantity[86] but since $|l|\ll L$, it follows that in calculation of the losses, i.e., of the quantity $|\gamma|$ it is sufficient to use the expression

$$|\gamma|\approx1-\frac{\pi^2}{L^2}\operatorname{Re}D. \tag{2.26}$$

We have allowed here also for the fact that $a^2/L\delta$, $a/L\ll1$.

If an array occupies a circle of radius L, the lowest mode is described by a Bessel function

$$\psi_1(R)=J_0\left(\frac{|R|}{L+l}\mu_0\right), \tag{2.27}$$

where

$$\frac{\gamma}{\gamma_0}=1-\frac{\mu_0^2 D}{(L+l)^2},$$

whereas μ_0 is the first zero of the function J_0. The losses experienced by the collective mode increase if the distance from the ends of the channels differs from $z_T/2$. For a small deviation from this distance $|\delta z|\ll z_T$, the expression for the eigenvalue can be obtained in the form

$$\frac{\gamma}{\gamma_0}=\int|f(q)|^2\exp\left(-i\frac{\delta zq^2}{2\varkappa_0}\right)\frac{dq}{(2\pi)^2}, \tag{2.28}$$

where $f(q)$ is the Fourier transform of the function $f(\rho)$. An order-of-magnitude estimate gives

$$\delta|\gamma|\approx-\left(\frac{\delta z}{z_T}\right)^2\left(\frac{a^2}{2\pi\delta^2}\right)^2.$$

Hence, it follows that if the ratio δ/a is reduced, the losses associated with the longitudinal misalignment of the mirror positions increase strongly.

In the case of a small angular misalignment $\theta\ll\delta/z_T$ a change in the Q factor of the modes can be deduced from perturbation theory, as was done in Ref. 85. The order of magnitude of these changes is given by the expression

$$\delta|\gamma|\approx\frac{k_0^2\theta^2L^4}{D}\sim\left(\frac{\theta}{\theta_{\text{diffr}}}\right)^2\left(\frac{L}{a}\right)^4,$$

which demonstrates that the angular alignment becomes critical as the size of an array increases.

We have considered so far only the field distributions in the absence of saturation of the active medium. Obviously, such saturation flattens the field of the fundamental mode in the central part of the laser array. If an active medium is present in the waveguides, the system (2.18) yields the following equation for the field envelope

$$\left[\frac{\gamma}{\gamma_0}\exp(-gL_k)-1\right]\psi=D\Delta_\perp\psi, \tag{2.29}$$

where the eigenvalue can naturally be presented in the form $\gamma=|\gamma|e^{i\nu}=\exp(g_nL_k+i\nu)$, where g and g_n are the gain in general and the threshold gain of the active medium in particular; L_k is the length of the channel containing the active medium; ν is a phase factor governed by the resonator parameters. In general, the gain is a falling function of $|\psi|^2$. It is easiest to determine the field distribution near the threshold when $(g-g_n)L_k<1$ and $g=g_0(1-|\psi|^2/|\psi_s|^2)$; here, g_0 is the unsaturated gain and $|\psi_s|^2$ is the saturation value of the intensity. We shall use the approximate boundary conditions $\psi(\pm L_\perp/2)=0$ for a strip of width $L_\perp$. Under the assumptions made above, Eq. (2.19) for the fundamental mode reduces to the equation for $|\psi|$:

$$\frac{d^2|\psi|}{dx^2}+\beta\left(\frac{g_0-g_n-g_0|\psi|^2}{|\psi_s|^2}\right)|\psi|=0, \qquad (2.30)$$

where $\beta=L_k\,\mathrm{Re}\,D/|D|^2$. This equation has been investigated thoroughly and its solution is expressed in terms of elliptic Jacobi functions (see Ref. 87). The lasing threshold is given by

$$g_{\mathrm{th}}=g_n+\frac{\pi^2}{\beta L_\perp^2}.$$

When the threshold $g_n+(\pi^2/\beta L_\perp{}^2)$ is exceeded slightly, so that

$$g_0-g_n-\frac{\pi^2}{\beta L_\perp^2}<\frac{\pi^2}{\beta L_\perp^2},$$

the field distribution is described by (2.25). If the size of the array $L_\perp$ is sufficiently large so that $g_0-g_n\gg\pi^2/\beta L_\perp{}^2$, the amplitude distribution is given by

$$|\psi|=|\psi_s|\left(\frac{g_0-g_n}{g_0}\right)^{1/2}\frac{\mathrm{ch}\left(\varkappa\frac{L_\perp}{2}\right)-\mathrm{ch}\,(\varkappa x)}{\mathrm{ch}\left(\varkappa\frac{L_\perp}{2}\right)+\mathrm{ch}\,(\varkappa x)},$$

$$\varkappa=[2\beta(g_0-g_n)]^{1/2}.$$

The total power is then governed by the excess of the gain above the threshold and can be calculated using the Rigrod formula (see Refs. 41 and 42).

A system proposed in Ref. 101 is close to the use of the Talbot effect. In this system (Fig. 19g) a plane mirror located some distance from a laser array is replaced by a reflecting telescope with a magnification $M=-1$ which transfers the images of the ends of the elements back to the initial positions of these ends. This makes it possible to avoid practically completely the diffraction losses in the coupling arm. A spatial filter can be used for the coupling and then the scattering of the radiation emitted by the individual lasers results in the coupling. In the language of collective modes the role of a spatial filter is to select specific *a priori* chosen coherent superpositions of the laser fields. Efficient selection naturally requires that the structure of the spatial filter be matched to the structure of one of the fields of the different collective modes and that the distributions of the fields of the different collective modes should differ at the position of the spatial filter. These conditions are satisfied, for example, if the filter is in the form of a stop at the focus of a telescope[29] transmitting the radiation of a cophasal collective mode, ensuring the minimum size of the focal spot. However, if the ends of the elements form a periodic lattice, then the diffraction images of this lattice which appear at certain points because of the Talbot effect are slightly reduced. If a spatial filter is located at these places in the form of a periodic lattice of shadows, we can select the required collective mode. The problem of finding the normal modes in such a system also reduces (see Ref. 101) to the solution of the system of the (24) type, where M_{nm} is the real difference frequency. Theoretical (analytic) and numerical estimates of the spectrum of losses experienced by the normal collective modes and the permissible scatter of the normal frequencies emitted by individual lasers are given in Ref. 101.

The refractive index of active laser media may depend on the radiation intensity. The problem of the influence of this dependence on the phase locking of lasers was investigated recently.[88] It was found that in the case of a sufficiently strong nonlinearity of the refractive index the laser frequencies become equalized and phase-locked operation is possible. In the case of a one-dimensional array of N lasers with the nearest-neighbor coupling the phase-locked operation is obtained when

$$\left|\frac{\partial n}{\partial I}kl\,\mathrm{Im}\,M\right|>\left|\frac{\partial g}{\partial I}\frac{2L}{c}\langle\Delta\omega^2\rangle^{1/2}N^{3/2}\right|.$$

The conditions of stability of such cophasal operation are the inequalities $\mathrm{Re}M>0$ and $(\partial n/\partial I)\mathrm{Im}M<0$. The physical meaning of these quantities is as follows. If $\mathrm{Re}M>0$, then the radiation losses in the coupling channel are minimal for the cophasal operation. Changes in the phases of the fields emitted by the lasers should alter the amplitude of a signal in the coupling channel, resulting in compensating changes in the refractive index. Therefore, there must be a nonzero imaginary part of the coupling coefficient and the sign of this part is governed by the sign of $\partial n/\partial I$: $\partial n/\partial I$ $\mathrm{Im}M<0$. Therefore, "self-matching" of the optical lengths of the resonators takes place and cophasal lasing is possible.

2.3. Experiments on phase locking of many lasers

The coupling between semiconductor lasers by tunnel diffraction between individual components of an array had been investigated experimentally.[66,69] The difficulty of ensuring single-mode lasing in the case of such coupling is that the parameters of the waveguides are different and they depend on the pump current. The normal frequencies of the individual lasers are different and a weak optical coupling fails to establish stable phase locking.

Fields with a constant phase on the exit aperture were generated in a study described in Ref. 89 by employing an optical Y coupling between two periodic semiconductor laser arrays. This coupling is shown schematically in Fig. 22. In two active regions A and B the radiation propagates along separate periodically distributed waveguides, whereas in a region C the field of each waveguide in an array is coupled to the fields of two waveguides in another array. The lowest losses in the matching of the fields of these two laser arrays are observed when the phases are equal.

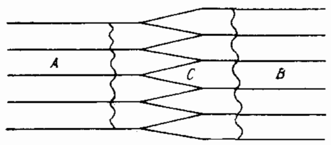

FIG. 22. Y coupling of two semiconductor laser arrays.

It was reported in Ref. 90 that one of the collective modes was generated by an array of CO_2 waveguide lasers because of the special nature of the construction of the waveguides performing selection of the mode losses. An array of waveguides with an active medium consisted of two parts (Fig. 19d), so that the metal walls of one half of the waveguide separated the section of each waveguide in the other half into two equal parts. In this system the minimum losses were experienced by the distribution of the field scattered least at the contact between the two parts of the waveguides. This occurs if the field distributions of the normal modes are identical in these two parts of the waveguides. Obviously, the common features of these waveguides are collective modes, which vanish on metal walls of both arrays. In the case of this selection method the optical coupling of the adjacent waveguide fields can reach a value of the order of unity. The resultant field distribution is a collective mode with a period identical with the transverse size of one waveguide and the field distribution is antisymmetric relative to the center of the rectangular cross section of each waveguide. The intensity of the radiation in the far-field zone then exhibits two lobes.

Considerable narrowing of the angular distribution of the radiation emitted by two-dimensional laser arrays was first achieved in the experiment reported in Ref. 91. The brightness of the radiation of a pulse multicomponent semiconductor cadmium sulfide laser pumped by an electron beam was achieved[91] using a system with an unstable resonator. The authors of Ref. 91 were able to reduce the angular divergence of the radiation to 30′ (in the absence of optical coupling between the components of the array the divergence was $\sim 10°$) when the output power was 200 kW and this was achieved at the expense of a small reduction in the efficiency of the system.

Phase-locked operation of a large number (~ 60) waveguide CO_2 lasers assembled to form a two-dimensional array, with a cross section in the form of a triangular periodic lattice, was reported in Ref. 6: It was achieved using a conventional mirror separated by a short distance from the aperture of the array. A single laser in the investigation reported in Ref. 6 emitted quasicontinuously and was a glass tube 1.1 m long with outer and inner diameters of 8.5 and 5.5 mm, respectively; the active medium inside the tube was excited by an ac discharge. The average power of the cw radiation obtained from each tube was ~ 10 W. The distance from the coupling mirror to the exit aperture of the array was selected to be ~ 1 m, so as to ensure exchange of radiation between the adjacent tubes as a result of diffraction spreading, but the change in the losses (due to escape of the radiation outside the aperture of each laser) was slight. Phase-locked operation of all the lasers was achieved in these experiments, but it was unstable: the increase in the brightness of the radiation in the far-field zone was a factor of ~ 10 compared with independent lasing.

Phase locking of an array of lasers based on the self-reproduction of the periodic structure of the field was first reported in Ref. 92. A detailed investigation of the properties of the coherent radiation emitted by a two-dimensional array of CO_2 lasers was described in Ref. 93. The parameters of a multibeam CO_2 laser were given in Ref. 94. In this case the exit mirror of the resonator was a germanium plane-parallel plate with a reflection coefficient of 0.5. A plane metal mirror located at half the Talbot reproduction distance z_T from the tube ends ($z_T \approx 10$ m at $\lambda = 10.6\ \mu$m) was replaced in Ref. 93 by a telescope consisting of a focusing lens and a convex copper mirror with its center coinciding with the focus of the lens. Such an optical system was equivalent to a plane mirror[95] separated by $z_T/2$ from the tube ends on condition that $L_l = z_T/2 - f[(f/R) - 1]$, where L_l is the distance from the ends to the lens. The use of a telescope not only reduced the dimensions of the whole system, but also made it less critically sensitive to the precision of alignment of the optical components. The distributions of the radiation in the near- and far-field zones were obtained by scanning the laser beam with a rotating mirror and using stopped-down photodetectors.

The dependence of the output radiation power on the distance from the plane mirror z is plotted in Fig. 23. An increase of z from 0 to 3.5 m reduced the power monotonically, which was related to an increase in the intracavity diffraction losses. When z was increased from $z_T/2 \approx 5$ m, which corresponded to $L_l \approx 20$ cm in the presence of a telescope, the radiation power increased again. Variation of z near $z_T/2$ was accompanied also by modification of the spectral composition of the laser radiation. Lasing occurred as a result of vibrational–rotational transitions in the CO_2 molecules selected so that the output radiation wavelength corrsponded best to the value of z needed to ensure reproduction of the periodic structure.

An experimental proof of coherent operation of all the lasers was provided by the appearance of interference structures in the far-field zones: These structures corresponded to the three fundamental supermodes in the case of a triangular array. Coherent operation of a laser array also modified the intensity distribution not only in the far- but also in the near-field zones. In the case of independent lasing ($z = 0$) the laser powers were additive. Under phase-locked lasing conditions ($z = z_T/2$) the diffraction losses suffered by the lasers at the periphery were higher than the losses experienced by the lasers located in the central part of the array and the power distribution became inhomogeneous. An analysis of the main physical factors (finite number of components, angular misalignment of the optical components, differences between the normal frequencies of the individual lasers) resulting in additional losses in laser arrays with the Talbot coupling were analyzed in Ref. 93. It was reported that an increase in the brightness for a phase-locked array was a factor of 10 compared with independent lasing of the components of the array.

Locking of the phases of the fields in a one-dimensional array of diode lasers by the self-reproduction effect was

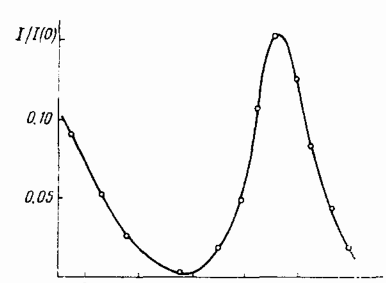

FIG. 23. Dependence of the output power of an array of CO_2 lasers on the distance to the coupling mirror.

achieved experimentally and described in Ref. 96. A periodic array of microlenses and a coupling mirror, located at a distance $z_T/2$, made it possible to achieve cophasal operation of seven AlGaAs lasers. The main peak (lobe) in the far-field zone carried 82% of the output radiation power. The need to use microlenses in the study reported in Ref. 96 was due to considerable inhomogeneities of the refractive index over a cross section in individual lasers. Hence, the radiation emitted by a single laser represented a diverging wave with a small radius of curvature when it reached the exit aperture. Microlenses transformed the laser field into radiation with a plane wavefront.

A new method for phase locking of a set of CO_2 lasers, described briefly in the preceding section, was proposed and implemented experimentally in Ref. 101. Coherent operation of a laser array was ensured by employing a spatial filter located above the planes of self-reproduction or reproduction of the periodic laser array.[101] This method ensured reliable selection of various collective modes with a high degree of coherence and low power losses. The total power obtained from such an array (one-dimensional or square in the plane of the ends) under coherent conditions represented up to 0.6 of the power attainable in the case of independent operation of the individual lasers, whereas in the Talbot coupling case (see Ref. 92) the output power reached only 0.2 of the power obtainable as a result of independent operation of lasers.

2.4. Methods for increasing the brightness of a source with a synthesized aperture

Optical coupling does not always result in selection of one collective mode. For example, when the Talbot coupling is employed, there may be several collective modes differing slightly in respect of the losses. Moreover, if the margin of stability against multimode operation is insufficient, frequency detuning of the individual resonators may take place and the active medium may become saturated by the inhomogeneous radiation field. The simplest mode selection method involves the use of angular and spatial filters. This selection method has been used successfully in the case of laser arrays and for individual lasers with retroreflector mirrors.[29] In the case of angular selection by means of a circular stop located at the lens focus the minimal losses are exhibited by a cophasal mode.

Reproduction of the periodic structure of the field is used to phase-lock a laser array, and the role of the spatial filter is performed by the array aperture. However, the Talbot coupling does not always result in selection of a specific selective mode. The use of periodic spatial filters located in the reproduction planes[84] can result in further mode selection.

By way of example, we shall consider a one-dimensional array of lasers with a period a and a coupling mirror located at half the replication distance $L_T/2$. Two collective modes may then be generated: cophasal and antiphasal. In the case of a cophasal mode with a field distribution across the aperture $E(x, z=0) = \Sigma f(x - na)$, the distribution in the coupling mirror plane is $E(x, z = L_T/2) = \Sigma f(x - n/2 - na)$, i.e., the image is shifted by $a/2$ relative to the original image. In the case of antiphasal mode with the distribution $E(x, z=0) = \Sigma(-1)^n f(x - na)$ across the aperture, the image in the $x = L_T/2$ plane is completely identical with the original. If a periodic absorbing screen is placed in the mirror plane and if the effective reflection coefficient of the mirror is then a periodic function $R(x) = \Sigma\varphi((x - a/2) - na)$ with maxima at the points $x = (a/2) + na$, a cophasal mode is selected. For a screen which establishes the reflection coefficient of the mirror $R(x) = \Sigma\varphi(x - na)$, an antiphasal mode has a higher Q factor. We can similarly select modes emitted by two-dimensional laser arrays.

A cophasal mode can be separated by injection of an external signal with a plane wavefront.[71,73,91] This topic has hardly been investigated for laser arrays. Obviously, the required power of an external signal in the case of generation of a cophasal mode can be estimated exactly as in the case of a single laser.[30]

Certain optical coupling methods[66,90] ensure the emission of one antiphasal mode from periodic arrays and this is based on selection in accordance with the Q factor. A phase screen which shifts the phases of the fields of adjacent neighbors by π can produce a field distribution similar to that in the case of a cophasal mode.

The radiation emitted by periodic laser arrays usually does not fill the whole area of the exit aperture. Therefore, even in the case of full phase locking of the components, the radiation energy in the far-field zone is distributed between different orders of the diffraction structure. We shall find the characteristic parameters of the radiation in the far-field zone by considering the example of a one-dimensional array of phase-locked lasers. Therefore, the field distribution in a phase-locked laser system is of the form

$$E(x) = \sum_n T(n) f(x - na),$$

where a is the period of the array; $T(n)$ is the envelope of a cophasal mode; $f(y)$ is the field distribution at the exit from one laser. The distribution in the far-field zone is described by the Fourier transform $E(x)$:

$$E(q) \sim \int E(x) \exp(iqx)\, dx$$
$$= \sum_n T(n) \exp(iqna) f(q) = f(q) \sum_m T\left(q - \frac{2\pi m}{a}\right).$$

It therefore follows that the distribution of the radiation in the far-field zone consists of a number of peaks $t(q)$ of width $\sim 2\pi/Na$, governed by the linear size Na of the array and separated in the Fourier representation by distances $2\pi/a$ from one another. The number of such peaks is governed by the Fourier transform of the field distribution obtained for one laser $f(q)$. The size is $f(q) \sim 2\pi/\delta$, where δ is the size of the emitting region of one laser. The bulk of the energy in the far-field zone is carried by the $\sim a/\delta$ diffraction peaks.

In the case of a two-dimensional array the energy carried by the central peak (lobe) is proportional to the ratio of the area occupied by the radiation S_n to the total aperture area S_a. Expansion of phase-locked beams to fill the aperture completely makes it possible to reduce the fraction of the energy scattered into side diffraction orders and thus increase the axial brightness.

An increase in the fraction of the aperture occupied by radiation can be achieved using a profiled mirror. The cross section of such a mirror for a one-dimensional periodic array of phase-locked laser separated by a certain distance is shown in Fig. 24. The difference between the optical paths

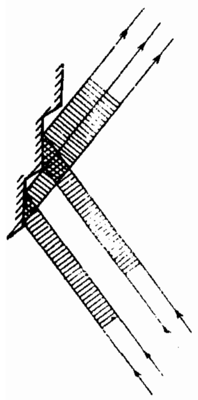

FIG. 24. Mirror increasing the degree of field occupancy of the aperture of a one-dimensional periodic array of phase-locked lasers.

for two neighboring beams should be a multiple of an integral number of the radiation wavelengths and efficient operation requires a high precision of alignment of such a mirror (because the difference between the optical lengths is a function of the angle).

An excellent suggestion for expansion of the field of a phase-locked laser array to fill the complete aperture is put forward in Ref. 97. The main idea in Ref. 97 is as follows. If a lens is located in the path of a periodic structure of an electromagnetic field, which can have one of two values, then the field distribution in the focal plane is a Fourier transform of the original periodic structure. The central peak (lobe) represents the zeroth Fourier component. It is found that if the phase of the zeroth Fourier component of the field is shifted by a certain amount and the lens transformation is applied again, the output radiation may be constant over the aperture and the phase may have a periodic distribution which assumes one of two values. A plane wave may then be generated using a periodic phase screen. A system of this kind is shown schematically in Fig. 25. We shall now provide an analytic theory of this method based on Ref. 97.

Let the field distribution over the aperture of a periodic laser array be

$$A(r)=A_0+A_1(r),$$

where A_0 is the degree of occupancy of the aperture by the field defined as $A_0=S_n/S_a$; $A(r)$ is the complement of the distribution needed to obtain the original structure; $A_1(r)=1-A_0$ applies in the regions occupied by the field; $A_1(r)=-A_0$ applies in the regions which are not so occupied. It should be noted that $\int_S A_1(r)d^2r=0$ and $A_1(r)$ makes no contribution to the central peak in the far-field zone. A phase shift by an amount θ in the central part of the

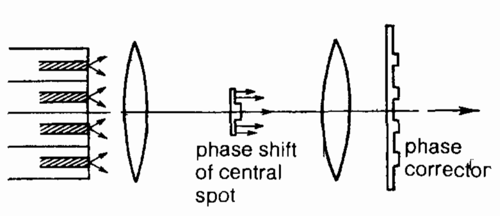

FIG. 25. Optical systems used in Refs. 97–99 to increase the brightness of the output radiation obtained from a periodic array of phase-locked lasers.

field produces the following distribution in the far-field zone at the exit from a telescopic system:

$$E_1(r)=A_0\frac{\exp(i\theta)}{m}+A_2(r).$$

The periodic structure of $A_2(r)$ has the same symmetry as $A_1(r)$, but it may be shifted relative to the initial structure, because a telescopic transformation alters the field distribution in accordance with the law $A_2(r)=A_1/m(-r/m)$, where m is the telescope magnification.

The field distribution amplitude is constant if the phase shift obeys

$$\theta=\cos^{-1}\frac{2A_0-1}{2A_0},$$

which is possible for $S_n\geqslant S_a/4$. The output radiation then has a periodic phase structure with an amplitude

$$\Delta\psi=\tan^{-1}\frac{A_0-1}{(4A_0-1)^{1/2}},$$

spatially identical with the distribution $A_1(r)$. If a phase screen is placed in the path of the field, a plane wave is generated.

Experiments reported in Ref. 98 involved formation of a coherent speckle field from He–Ne laser radiation transmitted by a mask characterized by $S_n/S_a=1/4$. The energy in the central peak was then 92% of the total energy transmitted by the mask. The results of a numerical investigation[98] also showed that this method for suppressing the side diffraction orders was effective in the case of a real field distribution in each laser. Experiments on an increase in the energy in the central peak for a set of ten GaAlAs diode lasers with Y coupling were reported in Ref. 99.

The radiation obtained from a diode laser array was 50 mW. The main diffraction peak carried 51% of the energy. After expansion of the laser beams to fill the full aperture the power carried by the central peak was 45 mW (90%).

A method for increasing the axial brightness of the radiation, based on the property of reproduction of the images of the periodic field structure at certain distances, was used in Ref. 100. The periodic structure of the field

$$E(x,z=0)=\sum_n f(x-na)=\sum_m C_m\exp\left(\frac{2\pi i}{a}mx\right)$$

is transformed at a distance $z=L_T/4$, in accordance with Eq. (2.15), into

$$E(x,z=L_T/4)=\sum_k C_{2k}\left(\frac{2\pi i}{a}\cdot 2kx\right)+i\sum_k C_{2k+1}\exp\left[\frac{2\pi i}{a}(2k+1)x\right].$$

The first term describes a cophasal structure of the field with a period $a/2$ and the second an antiphasal structure with a period a. The cophasal and antiphasal components are characterized by the same relative phase $\pi/2$. Figure 26 shows the distributions of the fields over the entry aperture and in the reproduction plane $z=L_T/4$. The number of images in the case of a one-dimensional periodic structure doubles at $z=L_T/4$. If the area occupied by the radiation is relatively small ($S_n\ll S_a$), the characteristic scale of the change in the Fourier component C_m is much greater than unity. In this approximation we have $|C_m|-|C_{m+1}|\ll|C_{m+1}|$ and the amplitude of the combined field at $z=L_T/4$ is $\sim\sqrt{2}$ times less than the field amplitude at the entry aperture, whereas

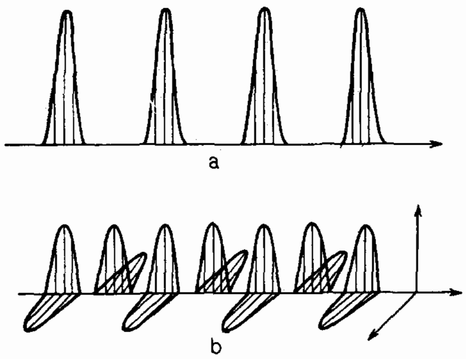

FIG. 26. Field distributions: a) over the entry aperture; b) in the reproduction plane at $L_T/4$.

the phases of the fields of the adjacent images differ by $\pi/2$. If a phase screen with a period $a/2$ and an amplitude $\pi/2$ is located in the reproduction plane, a periodic cophasal field distribution with the degree of occupancy of the aperture approximately twice the original one is obtained.

A similar result can be achieved by reproduction of the images of speckles by some other method, ensuring that the necessary phase relationships are obeyed.

We can also mention conceptually simplest, but difficult to realize in practice, methods for increasing each of the images of a periodic structure of speckles using a matched periodic array of telescopes.

Recent papers[102–104] reported the construction of an experimental system in which self-reproduction of the periodic structure of the field was used to couple one-dimensional and two-dimensional laser arrays. The brightness of the radiation obtained from an array was increased additionally[104] by employing a phase corrector located in one of the planes of reproduction of the diffraction image.

CONCLUSIONS

The reported investigations demonstrate that optical coupling is a promising method of achieving phase locking of lasers and thus increasing the brightness of the radiation emitted by multimodule systems. The structure of collective modes has been investigated thoroughly and the main factors influencing its stability have been established. Several methods for effective optical coupling and selection of collective modes have been proposed and implemented. Methods are available for increasing the axial brightness of a source with a periodic structure of the field.

There are however many topics of general physical importance and of practical interest which have not yet been investigated. They include, for example, the dynamics of coupled oscillators (lasers). As pointed out in Sec. 1.6, a system of two optically coupled lasers exhibits a complex dynamic behavior of the output radiation intensity. Numerical and experimental investigations of the behavior of axial brightness of the radiation under various dynamic conditions and its correlation with such parameters as the fractal dimensionality, Kolmogorov entropy, etc. are interesting. In the case of a large number of lasers one could expect the dynamics of stimulated emission and the behavior of the brightness of the output fields to become more complex, depending on the actual conditions. In some cases (when the coupling coefficient is imaginary) a system of coupled lasers is analogous to a system of coupled nonlinear oscillators.[1,2] Therefore, in addition to phase locking of many lasers, we can expect a complex chaotic behavior which can be described by statistical methods.[105]

Since the gain and the refractive index of laser media depend on the radiation intensity, an optically coupled system of lasers may exhibit a variety of nonlinear phenomena known also for other optical systems such as bistability, hysteresis,[106,107] formation of transverse periodic structures, and motion of switching waves.[107–110] In the process of organization of optical coupling between components involving rotation of the field about the optic axis we can expect the appearance of transient radiation in structures with an angular symmetry, similar to those found in the case of nonlinear interferometers with a feedback loop.[111]

The problem of the influence of statistical scatter of the parameters of individual lasers on the efficiency of phase locking of the fields by various optical coupling methods and on the structure of the resultant fields is of practical importance but has not yet been studied.

Another urgent task is to develop controlled phase transparencies because they would provide means for controlling the radiation, in accordance with a selected law, emitted by a system of phase-locked lasers.

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