On the possible generation of cosmic rays in plasma pinches

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The possibility of generating galactic cosmic rays in cosmic plasma cylindrical pinches is discussed for the case where there is an increase in constriction perturbations from which streams of plasma are squeezed. It is shown that when the constrictions break away, an energy distribution function for accelerated particles is formed in the stream with the form $dN/dE = CE^{-\nu}$, where $\nu = 1 + \sqrt{3} = 2.732$. This actually coincides with the average value observed in the 10^{10}–10^{15} eV energy range of $\nu_0 = 2.74$ (see Ref. 9), which makes this hypothesis very plausible. At higher energies the observed exponent of the spectrum is approximately equal to $\nu_0 = 3.1$, and here, apparently, another "electrodynamic" acceleration mechanism is operating, which also is accompanied by a break in current in the pinch. The model of the "plasma universe" proposed by Alfvén predicts the presence of a number of pinches in space.

1. INTRODUCTION

Particles with energies up to 10^{20} eV are observed in cosmic rays. Particles with energies less than 10^{10} eV are generated in the Sun, and their spectrum varies sharply during solar flares. Particles with energies greater than 10^{10} eV are apparently of galactic origin. Their spectrum is stable, although small variations in the 10^{9}–10^{11} eV energy range have recently been observed which apparently indicate some sort of explosive processes in the galaxy. In the 10^{10}–10^{15} eV energy range the spectrum of galactic cosmic rays has a power law of the form $dN/dE = CE^{-\nu}$, where C is a constant and the exponent can be considered to be equal to $\nu_0 = 2.74$ with great accuracy (see Ref. 9). However, existing generation theories[1,2] do not yield such a spectrum unambiguously, and require additional hypotheses. At energies of the order of $2\cdot 10^{15}$ eV a "break" is observed in the spectrum, and at 10^{15}–10^{17} eV the exponent of the spectrum is approximately equal to $\nu_0 = 3.1$; at 10^{17}–10^{19} eV we again have $\nu_0 = 2.7$ (see Fig. 1, which we have drawn from Ref. 35). We note that the electron component of cosmic rays also has a power law spectrum with an exponent $\nu_e = 2.7$ (see Ref. 35).

The hypothesis is made in the article that in cosmic plasma nebulae or in the plasma atmosphere of stars like pulsars or black holes, electrical discharges may be generated, like "noiseless cosmic lightning," in which cylindrical plasma pinches are formed which are similar to laboratory pinches. Accelerated particles are observed in the latter,[3] and for this reason one can assume that they are produced in the same way, that is, due to constriction type instabilities, in galactic cosmic rays as well.

The presence of various current plasma pinches in space is predicted in the "plasma universe" model proposed by Alfven.[5] In this model, an important role is ascribed to these very currents on cosmic scales (up to a current of the order of 10^{19} A). Various aspects of this model were discussed at the International Working Group on Plasma Cosmology. The meeting was held on February 20–22, 1989 at La Jolla, California. A special issue of the journal Plasma Science was devoted to this meeting (see Ref. 36).

The fruitfulness of this model is manifested, for example, in the spiral structure of magnetic fields observed in the galactic plane of several galaxies. The magnetic fields evidently should be due to spiral currents in the plane. It is also reasonable to assume that these currents are extended by currents which are already not directly visible. These currents diverge from the galaxy along the axis of rotation and form a spheroidal closed current loop (see Fig. 3 below).

It is assumed that near the poles of such a structure electric double layers are sporadically formed in the axial pinches,[29] and it is in these layers that ions and electrons are accelerated. These layers also yield two plasma clouds in the shape of butterfly wings with strong synchrotron radiation, as occurs in the radio galaxy Cygnus A. It is proposed in Ref. 29 (see also Ref. 38) that in such pinches it is possible for not only double layers, but also pinch constrictions to be generated near stars. Below it is shown that in plasma streams squeezed from the constrictions in a relativistic skinned pinch, a spectrum $\sim E^{-\nu}$ is generated with $\nu = 1 + \sqrt{3} \simeq 2.732$, which is very close to the observed $\nu_0 = 2.74$. Moreover, it is *unambiguous*, which increases the plausibility of this hypothesis. The spectrum after the "break" is apparently formed by some other inductive acceleration mechanism, which should also arise in the process of the break in current, but occur after it.

2. A USEFUL ANALOGY—BREAKING A STREAM OF WATER INTO DROPS

It is well known that a stream of water flowing smoothly from a tap is broken into drops by the force of surface tension, as is clear in the accompanying figure (Fig. 2). In order to eliminate an increase in random perturbations, in experiments[6] the stream is subjected to the external effect of a specific wavelength of sound from a loudspeaker, which also determines the length of the period of priming perturbations, which then increase. In the long wavelength approximation $\lambda \gg a$, where $a(x,t)$ is the radius of the stream, one can assume that a surface tension with the coefficient σ_0 creates a pressure in the stream, $p = \sigma_0/a$, so that in a constriction with a small radius a the pressure increases and water is squeezed from the constriction. Pressure is the cause of instability here.

This picture is fully analogous to the one which we will want to calculate later for a plasma z pinch with a longitudinal current, and here it is important to note that if the compulsory external periodic action is absent, random "spontaneous" local perturbations will increase which initially do not interact with each other. A break in the stream is likely to

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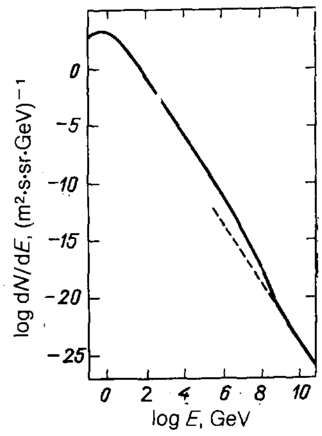

FIG. 1. Spectrum of galactic cosmic rays.

occur first at the location of the largest local priming constriction, which should be considered the most typical form of perturbations in the absence of external actions.

When water is squeezed into drops from the constrictions, the water particles are initially at rest (in a system of coordinates moving with the stream, one can formally assume that they are not moving at time $t = -\infty$), and then acquire a specific longitudinal velocity v, so one can calculate a velocity or energy distribution function of particles. Such an energy distribution is called an "energy spectrum." An analogous spectrum can be found for plasma particles squeezed from the constrictions of a plasma pinch, which we think may be a source of cosmic rays.

3. BASIC INFORMATION ABOUT COSMIC RAYS[7-9]

Near Earth, cosmic rays, which were discovered by Hess in 1912, consist of 90% protons, 7% α particles, and 1% heavier nuclei. Their integral spectrum has the form $I \sim E^{-\mu}$, where $\mu = 1.7$ at $E = 10^{10}$–10^{15} eV, $\mu = 2.2$ at $E = 10^{15}$–10^{17} eV, and $\mu = 1.7$ again at $E = 10^{17}$–10^{20} eV (see Fig. 1). It is assumed that particles with the highest observed energies, $E = 10^{19}$–10^{20} eV, are of extragalactic origin. Compared with the average abundance of elements in the universe, cosmic rays have relatively more light nuclei (Li, Be, B) and more heavy nuclei (with $Z \sim 20$). It is assumed that the enrichment of cosmic rays with heavy nuclei is the result of more efficient acceleration of these nuclei in the source, and that the higher content of Li, Be, and B is associated with the splitting of heavy nuclei when they collide with the nuclei of atoms in the interstellar medium.

Cosmic rays interact relatively weakly with matter in our galaxy, which includes the central core, the thin disk, and halo, and has a total mass $M_G = 10^{11} M_\odot = 2 \cdot 10^{44}$ g in a volume $V_G = 10^{68}$ cm^3. Most of the matter is in the form of stars and clouds of interstellar gas and dust concentrated in the galactic disk, which has a size of 0.5 kpc$\times$30 kpc (1 pc = 3 light years).

The total energy of cosmic rays in the galaxy is equal to $E_{cr} = 10^{68}$ eV $= 10^{56}$ erg, and coincides with good accuracy with the total energy of the chaotic magnetic field of the galaxy, which has an average strength of $H_G = 3 \cdot 10^{-6}$ G. The correlation $E_{cr} = V_G H_G^2/8\pi$ indicates an equilibrium between the magnetic field and the movement of cosmic rays.

Explosions of supernovae of type I (old stars near the galactic center) and II (massive young stars) are considered sources of cosmic rays. The kinetic energy of the dispersed shells of these stars is estimated to be $K_I = 10^{49}$ erg for type I and $K_{II} = 10^{52}$ erg for type II stars. Supernovae explode in the galaxy approximately once every 10–30 years $= 3 \cdot 10^8$–$10 \cdot 10^8$ s; thus the power of cosmic ray generation is approximately $W = 10^{40}$ erg/s for a type I star, and $W = 10^{43}$ erg/s for a type II star. Since the characteristic time for cosmic ray particles to leave the galaxy due to diffusion in inhomogeneous magnetic fields is approximately $T = 10^8$ years $= 3 \cdot 10^{15}$ s, the power of all sources of cosmic rays should be equal to $W_{cr} = E_{cr}/T = 10^{56}$ erg$/3 \cdot 10^{15}$ s $= 3 \cdot 10^{40}$ erg/s. As we see, the power of the explosions of supernovae is such (10^{41} erg/s) that they may insure the generation of cosmic rays. Thus, the role of stars like the Sun (mass $M_\odot = 2 \cdot 10^{33}$ g) is insignificant, since the Sun generates low-energy, nonrelativistic cosmic rays with a power of the order of $W_\odot = 10^{23}$ erg/s. Since the galaxy contains 10^{11} stars like the Sun, their total power, 10^{34} erg/s, is small compared to the contribution of supernovae.

The maximum energy of cosmic rays which may be produced in the shells of supernovae is determined by the ability of supernova's magnetic field to confine the particles. The magnetic field has a strength of the order of 10^{-3} G at a shell radius of 1 pc, so the maximum energy is evaluated using the formula $E_{max} = eHR_{sh} = 2 \cdot 10^6$ erg $= 10^{18}$–10^{19} eV. Analogous estimates for the Sun yield values of $H_\odot = 1$ G, $R_{sh} = 1$ million km $= 10^{11}$ cm, so that $E_\odot^{max} = 10^{13}$ eV. For a galaxy with a radius $R_G = 5 \cdot 10^{22}$ cm and an average magnetic field strength $H_G = 2 \cdot 10^{-6}$ G, we find a value $E_G^{max} = 10^{19}$ eV. Thus, it is assumed that protons with energies 10^{19}–10^{20} come from the metagalaxy.

Intense formation and evolution of stars occurred in the early stage of development of the galaxy ($T_1 = 10^9$ years), and the "relic" cosmic rays formed then should have left the galaxy due to diffusion according to the law $N = N_0 \exp(-T_G/T_d)$, where $T_G = 10^{10}$ years is the age of the galaxy, and $T_d = 10^8$ years is the diffusion time, so that $N = N_0 e^{-100}$. This estimate shows that observed cosmic rays are not "relic" cosmic rays, but "continuously renewed" particles.

Finally, let us examine the processes which determine the lifetime of cosmic rays in the galaxy. Direct collisions with stars, which have a concentration $n = 10^{11}$ stars$/10^{68}$ cm$^3 = 10^{-57}$ cm^{-3}, and a typical radius $R_\odot = 10^{11}$ cm, occur in collision time $T_{coll} = 10^{24}$ $s = 3 \cdot 10^{16}$ yr, which substantially exceeds the age of the galaxy, so this process does not occur. Collisions with interstellar gas, which consists of 90% hydrogen, has a total mass of the order of 10^9 $M_\odot = 10^{42}$ g, and an atomic concentration $n_{at} = 10^{-2}$

FIG. 2. Breaking a stream of water into drops (from Ref. 6 album).

cm^{-3}, occurs in time $T_{nuc} = 1/n_{at}\sigma_{nuc}$ s $= 10^{17}$ s $= 3\cdot 10^9$ years; here $\sigma_{nuc} = 3\cdot 10^{-26}$ cm^2 is the cross section of nuclear interactions with hydrogen. As we see, this time T_{nuc} is comparable with the age of the galaxy, but exceeds the time for departure due to diffusion in the inhomogeneities of the magnetic field. The collisions of cosmic rays with particles of cosmic dust may be evaluated if one considers the total dust mass to be about 0.01 the mass of the gas, 10^{40} g. The radius of dust particles is $r_p = 4\cdot 10^{-5}$ cm (ice, ammonia, methane), the density is 1 g/cm^3, and the mass is $m = 3\cdot 10^{-13}$ g. Thus, the total number of dust particles is $10^{40}/3\cdot 10^{-13} = 10^{53}$, and their concentration is $n_p = 10^{53}/10^{68} = 10^{-15}$ cm^{-3}. The average time of collisions of cosmic rays with dust particles is $T_p = (n_p \pi r_p^2 c)^{-1} = 3\cdot 10^5$ years; however, cosmic rays almost freely penetrate these small particles, and about 10^6 collisions with dust particles are needed for a cosmic ray proton to experience a nuclear interaction with the nuclei of the dust particle atoms. Thus, this requires 10^{11} years, which exceeds the age of the galaxy, and so this process is immaterial as well.

As they twist in the magnetic field, the cosmic ray protons may lose energy into synchrotron radiation; however, the time for the energy to decay by a factor of two is about $2\cdot 10^{12}$ years, so this process can be ignored as well.

The main loss process occurs when cosmic rays leave due to diffusion in the inhomogeneities of the magnetic field. The characteristic scale of inhomogeneities is $L = 3\cdot 100$ pc $= 10^{19} - 10^{20}$ cm, and the coefficient of diffusion in the inhomogeneities is $D = cL = 10^{30}$ cm^2/s, so departure per unit length of the galaxy $R_G = 4\cdot 10^{22}$ cm occurs in time $T = R_G^2/D = 10^{15}$ s $= 10^8$ years, which defines the lifetime of cosmic rays.

4. EXISTING THEORIES FOR THE GENERATION OF COSMIC RAYS

The surprising regularity of the galactic cosmic ray spectrum makes it possible to assume the presence of some specific mechanism for the generation of these particles; however, no one of the existing theories *unambiguously* yields a spectrum with $\nu = 2.7$. Let us describe these theories.

I. Betatron acceleration of particles should occur in the shells of exploding stars due to the turbulent generation of a magnetic field in the plasma. The adiabatic invariant $J = p^2/B$ should be conserved, so an increase in the field B is accompanied by an increase in particle momentum p.

II. The Fermi mechanism assumes that cosmic rays collide with moving clouds of plasma which carry with them a "frozen-in magnetic field." An individual cloud with such a "framework" may be seen as a unique macroparticle, which elastically reflects charged cosmic ray particles with its magnetic field. Then, according to the laws of chaos in this set of macro- and micro-particles, as in a gas made up of various molecules, one should see a tendency toward the establishment of an equal distribution of the energies of all members of the ensemble participating in elastic collisions, so that the microparticle strives to acquire an energy equal to the energy of the cloud, which generates cosmic rays with superhigh energies.

III. The mechanism of quasilinear diffusion over momenta is based on the equation (see Refs. 10–12)

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p}\left(\hat{D}\frac{\partial}{\partial p}f\right); \qquad (4.1)$$

here $f(p)$ is the momentum distribution function of particles, and D is the tensor of the diffusion of particles in momentum space due to the collisions of particles with plasma waves of various types, and is proportional to the intensity of these waves. The waves themselves, for example, Langmuir, Alfven, or magnetosonic waves, are stimulated by some cosmic explosions, or, let us say, by the rotation of magnetic stars, such as pulsars. If $\hat{D} > 0$, even initially cold particles with zero momentum $p = 0$ will spread diffusively over the momentum space so that, over time, the particles may acquire large momenta and energies.

IV. The turbulent pulsations of cosmic plasma[37] may also lead to the diffusion over momenta described by Eq. 4.1, since they may play the same role that "regular" plasma waves played in the previous case.

V. In recent years, a hypothesis on the generation of cosmic rays by shock waves propagating in clouds of cosmic plasma has become popular. The shock waves are produced by some stellar explosions. Here, a single passage through the front of the shock wave, even a very intense one, does not greatly increase the energy of the particles. Thus, the authors of Ref. 2 (see also the references cited therein) added to the kinetics equation a diffusion term with diffusion of the particles in the usual coordinate space, assuming that this diffusion occurs in islands of local inhomogeneity of the magnetic field. It is as if these islands were imbedded in the main plasma, and some portion of the particles which are chaotically scattered on them find themselves again in front of the shock wave; then the wave again overtakes these particles. When such migrations are repeated multiple times, some small portion of the particles may acquire large energies.

Let us examine this process in more detail. In a system of coordinates where the front of the shock wave is at rest in the $x = 0$ plane, we write the distribution function in the form $f = f_0 + f_1 \cos\theta$, where the second term takes anisotropy into account. Then for the isotropic part one can obtain the equation

$$v\frac{\partial f_0}{\partial x} + \langle \dot{p}\rangle \frac{\partial f_0}{\partial p} = D\frac{\partial^2 f_0}{\partial x^2}, \qquad (4.2)$$

which should be solved in two regions, ahead of the front and behind the front. Then one should join both solutions at the front; here $v = v_{1,2}$ is the velocity of the medium, D is the aforementioned coefficient of the diffusion over the coordinates, and finally, $\langle \dot{p}\rangle$ is the effective force accelerating a particle at the front. One can show that it is equal to

$$\langle \dot{p}\rangle = \frac{1}{3}p(v_1 - v_2)\,\delta(x), \qquad (4.3)$$

so that the solution of Eq. (4.2) can be sought in the form of separated variables $f_0 = \alpha(x)\beta(p)$. Since $\langle \dot{p}\rangle \sim p$, for β we get a power law spectrum $\beta \sim p^{-\nu}$ in the region behind the front. The exponent of the spectrum is found to be $\nu = 3\sigma/(\sigma - 1)$, where $\sigma = \rho_2/\rho_1 = v_1/v_2$ is the degree of compression in the shock wave defined by the Rankine–Hugoniot adiabatic curve

$$\sigma = (\gamma + 1)\,\mathrm{Ma}_1^2\,[(\gamma - 1)\,\mathrm{Ma}_1^2 + 2]^{-1}; \tag{4.4}$$

here $\gamma = c_P/c_V$ is the exponent of the common adaibatic curve, and $\mathrm{Ma}_1 = v_1/c_1$ is the Mach number ahead of the front. For the most intense wave we have $\mathrm{Ma}_1 \gg 1$, and then $\sigma = (\gamma + 1)/(\gamma - 1)$. For a monatomic gas plasma this leads to $\gamma = 5/3$, $\sigma = 4$, and $\nu = 4$ for the spectrum behind the front. Thus, it is assumed that before the front the function is monochromatic and has the form $\beta \sim \delta(p - p_0)$. The fact that the power law spectrum with exponent ν behind the front is independent of the value of the diffusion coefficient D and is sufficiently universal makes this mechanism of "regular acceleration in shock waves" very attractive in explaining the generation of galactic cosmic rays and their spectra. Yet this mechanism does not *unambiguously* yield an exponent $\nu = 2.7$, which is observed, at least, in the energy range 10^{10}–10^{15} eV. Thus, let us examine other mechanisms, continuing the list.

5. FORMATION OF DOUBLE LAYERS IN PINCHES IN COSMIC PLASMA

In addition to the shock wave hypothesis, in recent years the possibility has been discussed of the acceleration of cosmic ray particles in "double layers," which arise in current-carrying plasma (see, in particular, the survey of Ref. 29).

The double layer theory is very close to the theory of the Langmuir diode, in which an ion current $j_i = en_i v_i$ and an electron current $j_e = -en_e v_e$ flow such that for an electrostatic potential $\varphi(x)$ we have the Poisson equation

$$\varphi''_{xx} = 4\pi\left(\frac{j_e}{|v_e|} - \frac{j_i}{v_i}\right) = \frac{4\pi}{c}\left\{j_e\frac{1+\xi_e}{[\xi_e(2+\xi_e)]^{1/2}} - j_i\frac{1+\xi_i}{[\xi_i(2+\xi_i)]^{1/2}}\right\}. \tag{5.1}$$

Here the movement of particles is considered relativistic; thus the laws of conservation of energy are used, $\varepsilon = mc^2\gamma + e\varphi = \text{const}$, where $\gamma = (1-\beta^2)^{-1/2}$. It is assumed that ions with zero velocity emerge from the left electrode-anode at $x = 0$, which has a potential φ_0, and arrive at the cathode at $x = d$ with a zero potential, so the following notation is introduced into Eq. (5.1)

$$\xi_i = \frac{e(\varphi_0 - \varphi)}{m_i c^2}, \quad \xi_e = \frac{e\varphi}{m_e c^2}\,(e > 0). \tag{5.2}$$

Multiplying Eq. (5.1) by φ'_x and integrating, we obtain the law of conservation

$$e^{-1}(j_i p_i + j_e p_e) - \frac{E^2}{8\pi} = C_* = \text{const}, \quad p = mv\gamma, \tag{5.3}$$

where $E = -\varphi'_x$ is the electric field, and p_α are the momenta of the particles. This law indicates the requirement of the homogeneity of the total pressure of particles and the field in the space interval $0 < x < d$. If we further require that the field E go to zero at both ends for $x = 0, d$, we find the ratio of the currents

$$\frac{j_e}{j_i} = \left(\frac{2m_i c^2 + e\varphi_0}{2m_e c^2 + e\varphi_0}\right)^{1/2}. \tag{5.4}$$

In the nonrelativistic case $e\varphi_0 \ll 2m_e c^2 = 10^6$ eV we have $j_e/j_i = (m_i/m_e)^{1/2} \gg 1$, while at the ultrarelativistic limit $e\varphi \gg 2m_p c^2 = 2\cdot 10^9$ eV the ratio of currents is approximately equal to $j_e/j_i = 1 + (m_i c^2/e\varphi_0)$. Taking into account the boundary conditions from Eq. (5.3) we have

$$\frac{E^2}{8\pi} = \frac{c}{e}\,[j_i m_i\,[\xi_i(2+\xi_i)]^{1/2} - j_e m_e\{[\xi_e^0(2+\xi_e^0)]^{1/2} - [\xi_e(2+\xi_e)]^{1/2}\}], \tag{5.5}$$

where $\xi_e^0 = e\varphi_0/m_e c^2$. For ultrarelativistic electrons, when $\xi_e^0 \gg \xi_e \gg 1$, we get

$$\frac{E^2}{8\pi} = \frac{m_i c}{e}\,\{j_i\,[\xi_i(2+\xi_i)]^{1/2} - j_e\xi_i\}, \tag{5.6}$$

and if ions are also ultrarelativistic, $\xi_i \gg 1$, we have

$$\frac{E^2}{8\pi} = \frac{\varphi m_i j_i c}{e\varphi_0}, \quad \varphi'^2_x = 4A\varphi, \quad A = \frac{2\pi c m_i j_i}{e\varphi_0}, \tag{5.7}$$

and then integration leads to the result

$$\varphi(x) = (\varphi_0^{1/2} - xA^{1/2})^2, \quad \varphi_0 = \left[\pi d^2 (j_e + j_i)\frac{m_i c}{e}\right]^{1/2}. \tag{5.8}$$

It is important to stress that in laboratory diodes, current arises as a result of an applied voltage φ_0; however, in cosmic conditions it is assumed that the cause and effect switch positions, and current is held constant by an enormous external inductance. Equation (5.8) defines the difference in potentials which arises in the formation of a discharge in the form of a double layer in the current channel. In the review of Ref. 29 it is assumed for the estimate that the length of the interval d which arises is approximately equal to the radius of the pinch filament, which, consequently, carries the total current $I_0 = \pi d^2(j_i + j_e)$, and Eq. (5.8) assumes the form

$$\varphi_0 = (I_0 m_p c/e)^{1/2}, \quad E_{\max} = Ze\varphi_0 = Z(I_0 c e m_p)^{1/2}, \tag{5.9}$$

where $E_{\max}$ is the maximum energy of accelerated particles with a charge Z. For example, at a current of $3\cdot 10^{17}$ A, which is assumed in Ref. 29, for our galaxy we get $E_p^{\max} = 10^{14}$ eV, but since more energetic particles are found in galactic cosmic rays, it is noted in Ref. 29 that acceleration in double layers apparently can yield particles with energies of 10^{14}–10^{16} eV, while for larger energies some other mechanisms are necessary.

As shown below, higher energy particles may be acquired in an alternating induction field, which arises in an interruption of current I_0. The estimates here lead to a maximum energy of the order of $E_{\max} = I_0 e/c$, which at a current of $3\cdot 10^{17}$ yields $E = 10^{19}$ eV, which exceeds by five orders of magnitude the estimate from the double layer theory. The question arises, however, of where such pinches with large currents can be formed in space.

In this regard the "plasma universe" model with a multiplicity of currents is attractive. This model was proposed by Alfven in Ref. 5. Figure 3, which we have drawn from Ref. 5, shows a diagram of currents, which are denoted by arrows. The diagram makes it possible to explain the picture of observed isophots of two plasma clouds O with strong synchrotron radiation by electrons in the radio galaxy Cygnus A. These electrons are presumably accelerated in double layers (DL). The galaxy itself (G) acts as a unipolar inductor at the midpoint between the radio sources.

In the author's opinion, the most convincing evidence for the existence of currents in space was presented in Ref. 30

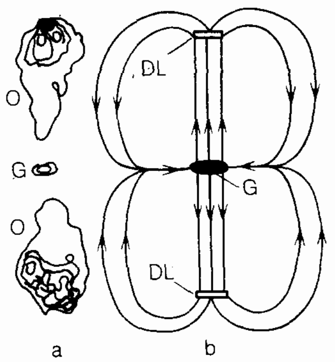

FIG. 3. a. Isophots of radio galaxy Cygnus A; b. Diagram of currents.[5]

using the same radio galaxy, Cygnus A, with currents up to 10^{19} A, as an example. Reference 31 lists possible cases with Birkeland currents from 10^5 to 10^{19} A with current filament sizes of 10^2 to 10^{21} m in space. The attractiveness of the "plasma universe" model for the problem of the generation of galactic cosmic rays is strengthened by the fact that accelerated particles are also observed in laboratory pinches that lead to the generation of neutrons in deuterium.[17] As a direct analogy one can point out lightning up to 1000 km long observed on Jupiter by the Voyager probe. There is no doubt that greater currents and pinches arise in solar protuberances and flares. It is assumed in Ref. 29 that currents $I = 10^{10}$–10^{12} A flow in solar flares, and double layers arise in them with potentials of up to $5 \cdot 10^{10}$ V. Particles are accelerated in these layers.

Stronger pinches may be formed in plasma clouds of galactic size. Figure 4, which is drawn from Ref. 32, shows the center of our galaxy. The picture was obtained on a radio telescope at a wavelength of 20 cm from a region 300 light years in size. The multiple filaments here have a diameter of the order of 0.3 pc with a length of 10–50 pc. Similar filaments in other galaxies are presented in Ref. 33.

The astronomical objects called jets may be considered externally similar to pinches. Their radio emission is apparently due to beams of cold electrons accelerated in double layers.[34] The largest jets are observed in radio galaxies, and as a rule, consist (see Ref. 13) of two thin, long streams of plasma ejected symmetrically from a central source. Figure 5 shows on different scales the structure of the jet isophots of radio galaxy NGC 6251, which has a length of the order of 1 Mpc (the Andromeda nebula is about that far from us). The radio emission from the jets is of synchrotron nature[14] and is polarized. This polarization may be used to determine the direction of the magnetic field, which may be directed along and across the stream. The stream is not homogeneous; one can distinguish individual "spots" and "knots" in it. As a rule, in the knots the magnetic field is perpendicular to the direction of the stream, so knots are very similar to constrictions in laboratory pinches. Jets are also observed near some stars. The binary star SS 433, which is either a pulsar or a black hole, yields a jet which moves at a speed of 80,000 km/s. Two curved jets have also been observed near the center of our galaxy.

In our opinion, however, the simplest and most frequently arising process for the formation of cylindrical plasma pinches in space may be the process of the breaking up of flat pinches into a set of cylindrical ones as a result of a tearing instability. Under laboratory conditions, flat pinches, which are also called "neutral current layers" were studied in Refs. 15 and 16, where one can see the process of breaking up into a number of current filaments in fast photographs. An individual filament is in essence an individual cylindrical Z pinch. In cosmic conditions the flat pinches should arise any time there is a collision between two plasma condensations containing "frozen in" magnetic fields in op-

FIG. 4. Structure of plasma filaments near the center of our galaxy, obtained with a radio telescope at a wavelength of 20 cm.

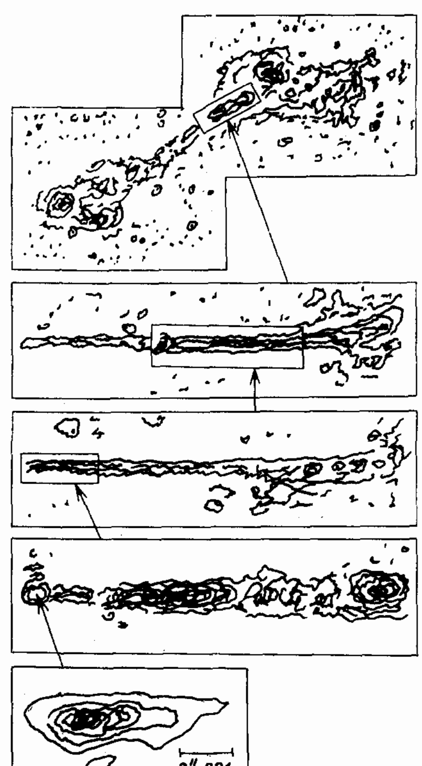

FIG. 5. Jet in radio galaxy NGG 6251 (Ref. 13).

posing directions, or at least in directions which do not coincide. At the interface a "neutral current layer" is formed. One can easily be convinced of the instability of this layer if one replaces it with a set of infinitely thin conductors with mass m_1 per unit length and current I_1 in each. The equation of motion of the n^{th} current

$$m_1\ddot{x}_n = I_1 c^{-1} B,\ B = B_y(x = x_n)$$

$$= 2I_1 c^{-1} \sum_{k=-\infty}^{\infty} (x_k - x_n)^{-1} \qquad (5.10)$$

in the case of long-wave perturbations of the system one can be replaced by two equations of hydrodyanmic type. For this we assume that at equilibrium the distance between neighboring conductors is a_0. Further, let us introduce the function $n(x,t)$, the number of conductors per unit length along the x axis. If we now introduce the dimensionless "effective density" $\rho = na_0$, then from Eq. (5.10) we find

$$\frac{\partial \rho}{\partial t} + \frac{\partial v\rho}{\partial x} = 0,\ \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = g\hat{H}\rho, \qquad (5.11)$$

$$\hat{H}\rho(x) = \frac{1}{\pi} \fint \frac{\rho(x')\,dx'}{x' - x},$$

where $g = 2\pi I_1^2/m_1 c^2 a_0 = \text{const}$ is a parameter with the dimension of acceleration, and H is the Hilbert integral operator in which the integral is taken in the sense of the principal value. In the linear approximation we assume $\rho = 1 + \rho_1$. For $v, \rho_1 \ll 1$, we get

$$\frac{\partial \rho_1}{\partial t} = -\frac{\partial v}{\partial x},\ \frac{\partial v}{\partial t} = g\hat{H}\rho_1,\quad v = C\exp(\gamma t)\sin(kx),$$

$$\gamma = (kg)^{1/2}, \qquad (5.12)$$

which indicates the instability of the current layer, which is striving to break up into a set of cylindrical pinches (Fig. 6). The instability of a cylindrical pinch in relation to the constrictions was first examined in Refs. 4 and 17, where it was shown that the instantaneous interruption of a thin current I generates a cylindrically diverging wave with a field $E = 2Ic^{-1}(c^2t^2 - r^2)^{-1/2}$, which is what accelerates particles. This solution for an infinitely thin wire is in essence a Green's function for the solution of the problem of the interruption of any distributed current. If in the xy plane a noninstantaneously interrupted current is distributed with a density $j(t, x, y)$, the field generated by the interruption will be equal to

$$E(t, x, y) = \int dE = \iint j(t', x', y')\,G\,dx'\,dy',$$

$$G = \frac{2}{c}[c^2(t - t')^2 - (x - x')^2 - (y - y')^2]^{-1/2}, \qquad (5.13)$$

where G is a Green's function. From here, as is shown in Ref. 18, one can find the Bulanov–Syrovatskiĭ solution[19] for the case where, in an infinitely thin flat pinch, a uniformly expanding region of "disconnected" currents is formed. This case imitates a single interruption of a current layer with ends which are moving away from the "X point" along an axis along which, it is assumed, the particles should be accelerated, since there is no magnetic field on the "X lines" and acceleration is not hindered. However, in experiments[15] a flat pinch broke up immediately into many current filaments. Under these conditions the acceleration should occur not along "X lines," but along "O lines," as happens in simply a single z pinch.

Ref. 20 proposed the original model for the formation of a z pinch from arcs of magnetic force lines rising above the Sun's photosphere which close again at some altitude and yield a separate loop with an axis in the form of a z pinch.

6. THE FIELD CREATED BY THE INTERRUPTION OF A CURRENT SURROUNDED BY PLASMA

The model of a "vacuum diode" described above should be corrected by taking into account the plasma surrounding the pinch. Below we will show that this problem is reduced to an equation of the form $\Delta\psi = C\psi$, which can be called a "screening equation."

It is curious that this equation is encountered in several plasma problems. For example, for $C = D^{-2}$, where D is the Debye radius, it describes the Debye screening of charge. If we have a point charge q, then the solution will be $\varphi = (q/r)\exp(-r/D)$. If we place an infinitely thin wire with charge q_1 per unit length in a plasma, then the potential it creates will be equal to

$$\varphi = 2q_1 K_0(x),\ x = \frac{r}{D},\ K_0(x \ll 1) = \ln\frac{2}{\gamma x},$$

$$K_0(x \gg 1) = \left(\frac{\pi}{2x}\right)^{1/2} e^{-x}, \qquad (6.1)$$

where $\gamma = 1.78$, and K_0 is a modified Bessel function. The screening of a weak alternating current is also described by an equation of the form $\Delta A = CA$, where A is the vector potential of the transverse wave, and $C = \delta^{-2}$, $\delta = c/\omega_0$ is the length of the "vacuum skin layer." In the two-dimensional case, its solution, $A \sim \exp(-x/\delta)$ describes the penetration of a low frequency radio wave into a flat layer of ionospheric plasma with subsequent reflection, which makes radio communication on Earth possible. If there is in the plasma an infinitely thin wire with a weak alternating current $I_1(t) \sim \cos\Omega t$, alternating with frequency Ω which is small in comparison with the plasma frequency $\omega_0 = (4\pi ne^2/m)^{-1/2}$, then we have the solution

$$A = a(t)K_0(x),\ x = \frac{r}{\delta},\ a(t) = 2I_1(t)c^{-1}. \qquad (6.2)$$

The factor $a(t)$ is defined here from the requirement that at small $r \to 0$ the potential (6.2) should yield a magnetic field $B = 2I_1(t)/cr$. Then one can find the induced electric field near a wire with a weak alternating current

$$E = E_z = -c^{-1}\frac{\partial A}{\partial t} = -2\dot{I}_1(t)c^{-2}K_0(x). \qquad (6.3)$$

Let us now examine a problem which we find more interesting, assuming that the pinch may be similar to an infi-

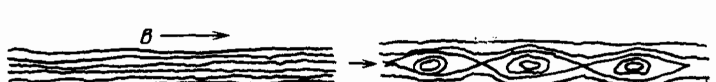

FIG. 6. Diagram of the formation of z pinches from a current layer.

nitely thin wire with a current of the form $I = I_0 + I_1(t)$, where I_0 is the constant component, and $I_1(t)$ is a small variable component. It can be assumed that in the gradual contraction of the pinch, the "main magnetic field" $B_0 = 2I_0/cr$ will penetrate into the "peripheral plasma" surrounding the pinch. For generality we will assume that its density is $n_* = C_* r^{-s}$ and attempt to define in it the induced electric field of the wave generated by the variable component of the current $I_1(t)$. This problem can be called "the problem of the screening of a current by a *magnetized* plasma."

To solve it we use the known dispersion law

$$N^2 = \xi - \eta^2\xi^{-1},\ N = \frac{ck}{\omega},\ \xi = 1 - \sum_{e,i}\frac{\omega_0^2}{\omega^2-\omega_B^2},$$
$$\eta = -\sum_{e,i}\frac{\omega_0^2\omega_B}{(\omega^2-\omega_B^2)\omega} \tag{6.4}$$

for a wave propagating in a plasma transverse to the magnetic field. Here ξ and η are components of the tensor of the permittivity of the plasma, which consists of ions with mass m_i and electrons with mass m_e. At the limit $m_i \to \infty$, $m_e \to 0$ when the contribution of ions can be ignored, and electrons experience only an electric drift with velocity $v = cE/B_0$, the values of ξ and η are found to be $\xi = 1$, $\eta = \omega_{0e}^2/\omega\omega_{Be}$, so the dispersion law [Eq. (6.4)] yields the relation $\omega^2 = c^2k^2 + (\omega_{0e}^2/\omega_{Be})^2$, which is very similar to the typical relation for a transverse wave $\omega^2 = c^2k^2 + \omega_0^{2e}$ in a nonmagnetized plasma. In the latter case, it is known that for $\omega \ll ck$ one obtains the relation $k^2 = (\omega_{0e}/c)^2$, which yields the equation $\Delta A = A\delta^{-2}$, which was examined earlier. Now for a magnetized plasma, for $\omega \ll ck$ we obtain $-k^2 = (\omega_{0e}^2/c\omega_{Be})^2$, which is equivalent to the equation

$$\Delta A = \frac{\partial}{r\,\partial r}\left(r\frac{\partial A}{\partial r}\right) = \left(\frac{4\pi ne}{B_0}\right)^2 A = \left(\frac{r}{R}\right)^{2-2s}R^{-2}A,$$
$$R = \left(\frac{I_0}{2\pi C_* ec}\right)^{1/(2-s)}. \tag{6.5}$$

Here we have substituted the density of the peripheral plasma $n = C_* r^{-s}$ and the field $B_0 = 2I_0/cr$. If we now introduce $\lambda = r/R$ and the new argument $x = \lambda^{2-s}/(2-s)$, we obtain the equation and its solution

$$\frac{\partial}{x\,\partial x}\left(x\frac{\partial A}{\partial x}\right) = A,\ A = a(t)\,K_0(x),\ a(t) = \frac{2}{2-s}I_1(t)\,c^{-1}. \tag{6.6}$$

Here the factor $a(t)$ again, as in the solution of Eq. (6.2), was defined so that at the limit $r \to 0$ the potential [Eq. (6.6)] would yield the magnetic field $B_1 = -\partial A/\partial r = 2I_1(t)/cr$, due to the correction $I_1(t)$ to the current. Finally, from Eq. (6.6) we find the electric field

$$E = E_z(t,\ r) = -\frac{\partial A}{c\,\partial t} = -\frac{2}{2-s}c^{-2}\dot{I}_1(t)\,K_0(x), \tag{6.7}$$

which, as is shown below, can effectively accelerate ions.

7. ELECTRODYNAMIC MECHANISM OF THE ACCELERATION OF PARTICLES IN A PINCH[21]

The field found above, Eq. (6.7), accelerates ions in accordance with the equation $\dot{p} = eE$. To integrate it, we note that in the derivation of Eq. (6.7) we assumed the correction $I_1(t)$ was small. Hereinafter, however, we will assume that current is actually completely interrupted in the pinch, so that during the time of the interruption the correction I_1 changes from $I_1(0) = 0$ to $I_1^{\max} = -I_0$, which is not small. Ignoring the inaccuracy of this calculation, one can assume that ions acquire suddenly the momentum

$$p = p_0 K_0(x),\quad p_0 = \frac{2eI_0}{(2-s)\,c^2}, \tag{7.1}$$

which is thus defined by the coordinate $\langle\langle r\rangle\rangle$, that is, the position of the ion before acceleration. This circumstance makes it possible to find the momentum distribution function of the ions.

For this we note that a ring layer dr in thickness and $L = dz$ in height contains a number of particles equal to

$$dN = n\cdot 2\pi r\,dr\,dz = 2\pi LC_* r^{1-s}dr = \frac{LI_0}{ce}dx = F(p)\,dp, \tag{7.2}$$

where x is the argument of the modified Bessel function, so that, inverting Eq. (7.1) we find the distribution function

$$F(p) = \frac{dN}{dp} = N_0\frac{dx}{dp} = \frac{N_0}{p}e^{-p/p_0},\ N_0 = \frac{LI_0}{ce}, \tag{7.3}$$

which is also suitable in the relativistic case. In a nonrelativistic approximation, Eq. (7.3) can be conveniently rewritten in the form of a distribution over the energy $E = p^2/2M$:

$$F(E) = \frac{dN}{dE} = \frac{N_0}{2E}\exp\left[-\left(\frac{E}{E_0}\right)^{1/2}\right],\quad E_0 = \frac{2e^2I_0^2}{Mc^4(2-s)^2}. \tag{7.4}$$

It is curious that, in essence, these formulas do not contain the density of the "peripheral" plasma $n = C_* r^{-s}$, although the derivation of the field, Eq. (6.7), is based on the assumption of its existence. What is more important, however, is that the nonrelativistic expression, Eq. (7.4), provides a good description of the spectrum of particles accelerated in laboratory pinches (Fig. 7). Thus, one would expect that the relativistic formula, Eq. (7.3), is also applicable to cosmic rays if they are generated in a similar manner. However, the parameter p_0 in Eq. (7.3) contains the length of the constriction L and the interrupted current I_0, which are known in each laboratory experiment with pinches, but are unknown for cosmic conditions. This makes the application of Eq. (7.3) to galactic cosmic rays more difficult. Below, however, we will examine yet another "pinch mechanism" for the acceleration of particles.

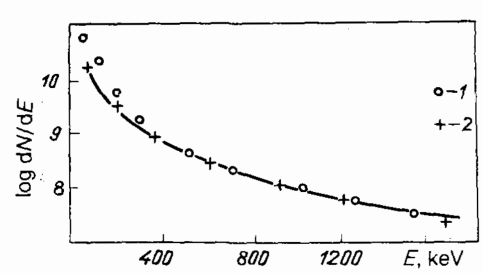

FIG. 7. Spectrum of deuterons accelerated in a laboratory pinch,[28] $I = 0.48$ MA.

8. GAS-DYNAMIC MECHANISM FOR THE ACCELERATION OF PARTICLES IN A PINCH[21]

In addition to acceleration in a field of Eq. (6.7), acceleration is also possible directly in the process of squeezing the plasma from the constrictions. In the simplest nonrelativistic variant, this process can be calculated if one assumes that the pinch is completely skinned and that the plasma inside the pinch is described by the gas-dynamic equations

$$\frac{\partial \rho}{\partial t}+\operatorname{div} \rho \mathbf{v}=0, \frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v} \nabla) \mathbf{v}=-\rho^{-1} \nabla p, \quad p=p_0\left(\frac{\rho}{\rho_0}\right)^s . \tag{8.1}$$

For comparison it is also useful to return to the problem which was described earlier, the stream of incompressible liquid broken into drops due to the force of surface tension σ_0. This problem is described by the system of equations

$$\operatorname{div} \mathbf{v}=0, \quad \frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v} \nabla) \mathbf{v}=-\rho_0^{-1} \nabla p, \tag{8.2}$$

simpler than Eq. (8.1). However, both systems are difficult to solve in three dimensions, and so we will replace them with approximate one-dimensional versions, using in both cases the known "approximation of a narrow stream or channel," where all values are considered constant across the cross section $S=\pi a^2$, where $a=a(t,x)$ is the radius of the channel. In this approximation the conservation of mass equation for an incompressible stream takes the form $S_t'+(Sv)_x'=0$, and for a compressible pinch $(S\rho)_t'+(S\rho v)_x'=0$. If in both cases we introduce a dimensionless "linear density" $\rho_*=\rho S/\rho_0 S_0$ per unit length, then both equations are written in the form

$$\rho_{*t}'+(\rho_* v)_x'=0. \tag{8.3}$$

In the narrow stream or channel approximation, only long-wave perturbations with $\lambda \gg a$ are examined, so in the equation of motion it is sufficient to consider only one x component, the longitudinal one. Then the pressure of surface tension in the stream of liquid will be equal to $p=\sigma_0/a=p_0(a_0/a)=p_0\rho_*^{-1/2}$, and the equation of motion takes the form

$$\frac{\partial}{\partial t} v+v \frac{\partial}{\partial x} v=-2 c_0^2 \frac{\partial \rho_*^{-1/2}}{\partial x}, \quad c_0^2=\frac{p_0}{2 \rho_0} . \tag{8.4}$$

In the case of a skinned pinch, we assume that a constant current I_0 flows in it, creating at the boundary $r=a(t,x)$ a magnetic field $B=2I_0/ca$, whose pressure $B^2/8\pi=p_0(a_0/a)^2$ balances the plasma pressure $p=p_0(\rho/\rho_0)^s$, so that $(a_0/a)^2=(\rho/\rho_0)^s$. It is easy to verify that when one takes this relation into account the equation of motion for the pinch acquires the form

$$\frac{\partial v}{\partial t}+v \frac{\partial v}{\partial x}=-c_0^2 \frac{\partial}{\partial x} \rho_*^{-1}, \quad c_0^2=\frac{s p_0}{(s-1) \rho_0} . \tag{8.5}$$

Thus, both problems are described by one system in the form

$$\rho_{*t}'+(\rho_* v)_x'=0, \quad v_t'+v v_x'=m c_0^2\left(\rho_*^{1/m}\right)_x' \tag{8.6}$$

and differ only by the value of the "azimuthal number," $m=-2$ for a stream and $m=-1$ for a pinch, and by the values of the constant parameter c_0. In Refs. 22–24 it is shown that this "quasi-Chaplygin" system [Eq. (8.6)] describes about 50 different "quasi-gas" unstable media in the long-wave approximation, and, what is most important, permits a complete solution.

To solve the system in Eq. (8.6) we introduce new dimensionless functions $r=\rho_*^{1/2m}$ and $z=v/2mc_0$, which can be conveniently regarded as cylindrical coordinates in some three-dimensional "phase" space r, φ, z. For these we find the equations

$$c_0^{-1} r_t'+r z_x'+2 m z r_x'=0, \quad c_0^{-1} z_t'-r r_x'+2 m z z_x'=0. \tag{8.7}$$

If we then introduce inverse functions $tc_0=T(r,z)$, $x=X(r,z)$ and a fictitious "potential" $\psi=r^m T\cos(m\varphi)$, we get

$$\begin{gathered}X_r'=r T_z'+2 m z T_r', \quad X_z'=-r T_r'+2 m z T_z', \\ T_{zz}''+T_{rr}''+\frac{2 m+1}{r} T_r'=0\end{gathered} \tag{8.8}$$

and the Laplace equation for the potential

$$\Delta \psi(r, \varphi, z)=\frac{\partial}{r \partial r}\left(r \frac{\partial \psi}{\partial r}\right)+\frac{1}{r^2} \frac{\partial^2 \psi}{\partial \varphi^2}+\frac{\partial^2 \psi}{\partial z^2}=0, \tag{8.9}$$

which for clarity can be regarded as an electrostatics equation. This equation may have many different solutions which describe perturbations; however, the most interesting are the special "spontaneous" solutions which describe perturbations which disappear at the opposite time limit $t\to-\infty$, when it is assumed that the system was in an unperturbed state. The absence of perturbations means $\rho_*=1$, $v=0$, which in three-dimensional phase space r, φ, z corresponds to a circumference of unit radius $r=1$, $z=0$ on which fictitious "charges" ρ_{eff} should be placed. These charges generate a potential

$$\psi(r, \varphi, z)=\int\left|\mathbf{R}'-\mathbf{R}\right|^{-1} \rho_{\text{eff}}(\mathbf{R}')\, dV', \tag{8.10}$$

defined by a Poisson integral. This potential may be expanded into a multipole series. Then one can show that the first "Coulomb" term of the expansion describes perturbations which are periodic over the length of the system, in our case, over the length x of the pinch. It is important to stress that for such perturbations one needs periodic "primers" which are introduced into the system by some external factor. If such a factor is absent, then the Coulomb term should be considered to be equal to zero. Then the main role will be played by the next term in the expansion, the "dipole" term, which describes not a periodic perturbation but a local perturbation. A set of such local perturbations should be considered the most typical picture of the development of instability in the indicated "quasi-Chaplygin media." It is clear that the break in the medium first occurs at the location of the largest local perturbation, which can be examined separately. For a pinch, the simplest local solution is

$$\begin{gathered}T=L_0\left(1-\frac{1+r^2+z^2}{\varkappa}\right)\left(1-r^2-z^2+\varkappa\right)^{1/2}, \\ \varkappa=\left[\left(1+r^2+z^2\right)^2-4 r^2\right]^{1/2}.\end{gathered} \tag{8.11}$$

This spontaneous solution describes the hump located between two constrictions in the pinch. At the limit $t\to-\infty$ the perturbation is absent, then rises. At time $t=0$ the constrictions break, and the hump acquires the form of a flat "pancake" in which the particles squeezed from the constrictions are concentrated. At this moment the momentum distribution function of the particles can be found if one assumes that in length dx there is a number of particles

$$dN = n\pi a^2 dx = N_1 \rho_* dx = F(p)\, dp, \quad N_1 = \pi a_*^2 n_0. \quad (8.12)$$

Using the formulas presented above, we find the function

$$F(p) = \left(\frac{dN}{dp}\right)_{t=0} = \frac{N_1}{p_0}\left[(T_r'' + T_z')\frac{1}{rT_r'}\right]_{r=0} = \frac{2N_0}{\pi p_0}(1+z^2)^{-2}, \quad (8.13)$$

where $N_0 = 2\pi\sqrt{2}L_0 N_1$ is the total number of particles drawn into the "pancake." For $z = v/2c_0 \gg 1$, we find the asymptotic spectrum $dN/dE \sim E^{-2.5}$, which, like the "electrodynamic" formula, Eq. (7.4), also provides a very good description of particles accelerated in laboratory pinches. This is shown in Fig. 7. Qualitatively close spectra are also obtained in numerical calculations.[25] It is curious that the dependence $\sim E^{-2.5}$ is also close to the spectrum of galactic cosmic rays, which are, however, ultrarelativistic in the energy region in which we are interested. Therefore below we examine a relativistic pinch.

9. PERTURBATIONS IN A RELATIVISTIC "SKINNED" PINCH[26,27]

We will describe the interior of a pinch with relativistic gas-dynamics equations without a magnetic field. Written in covariant form, these equations have the form

$$\frac{\partial (nu^i)}{\partial x^i} = 0, \quad wu^k \frac{\partial u_i}{\partial x^k} = \frac{\partial p}{\partial x^i} - u_i u^k \frac{\partial p}{\partial x^k}. \quad (9.1)$$

Here in four-dimensional space $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, a 4-velocity is introduced with the components

$$u^i = (\gamma, \gamma\bar{\beta}), \quad u_i = (\gamma, -\gamma\bar{\beta}),$$
$$\bar{\beta} = \frac{\bar{v}}{c}, \quad \gamma = (1-\beta^2)^{-1/2}, \quad (9.2)$$

while the quantities n, w, p are scalar, and denote the number density of particles, the enthalpy density, and the pressure, which are given in their "proper" system of coordinates moving with the liquid. Assuming that the plasma is nonrelativistic in this system, we use the nonrelativistic relations for the plasma, $w = nM_0c^2$, $p = p_0(n/n_0)^s$, $s = c_p/c_V$. Again, as in the nonrelativistic case, Eq. (8.1), we replace the three-dimensional equations, Eq. (9.1), with one-dimensional ones in the narrow stream or channel approximation.

Again it is convenient to introduce a dimensionless linear density per unit length $\rho_* = nS/n_0S_0$; we then obtain the equation

$$(\gamma\rho_*')_t' + c(u\rho_*)_z' = 0,$$
$$u_t' + \frac{c}{\gamma}uu_z' = -\varepsilon\left(c\gamma\frac{\partial}{\partial z} + u\frac{\partial}{\partial t}\right)\rho_*^{-1}, \quad (9.3)$$

where $\varepsilon = sp_0/(s-1)n_0M_0c^2$ is constant. We again assume that the pressure p is equal to the magnetic pressure $B^2/8\pi$, which is created by the current I_0 which is constant over time and which flows only along the surface of the pinch. The current is maintained by the external inductance of the portion of the pinch where there is no local constriction.

In the nonrelativistic approximation from Eq. (9.3) we obtain the equations

$$\rho_{*t}' + (v\rho_*)_z' = 0, \quad v_t' + vv_z' = -c_0^2(\rho_*^{-1})_z',$$
$$c_0^2 = \frac{sp_0}{(s-1)\rho_0}, \quad (9.4)$$

which coincide with Eq. (8.5) examined earlier.

To solve the relativistic system, Eq. (9.3), we introduce new functions $x(t,z)$ and $y(t,z)$, assuming that $\rho_* = \varepsilon/x$, $u = \operatorname{sh} y$. Then we introduce the inverse functions $ct = T(x, y)$ and $z = Z(x,y)$, for which we obtain

$$T_y' + Z_x' = (T_x' + Z_y')\operatorname{th} y, \quad Z_y' - xT_x' = (T_y' - xZ_x')\operatorname{th} y. \quad (9.5)$$

If we now write the introduced inverse functions in the form

$$T = (\psi \operatorname{sh} y - \varphi \operatorname{ch} y)xe^{-x}, \quad Z = (\psi\operatorname{ch} y - \varphi\operatorname{sh} y)xe^{-x}, \quad (9.6)$$

then for $\varphi(x,y)$ and $\psi(x,y)$ we obtain two equations

$$\varphi_y' = \psi_x' + \frac{1}{x}\psi, \quad \psi_y' = x(\varphi - \varphi_x'), \quad (9.7)$$

the consistency condition for which can be conveniently written in operator form

$$\Delta^*\varphi = 0, \quad \Delta^*\varphi = \varphi_{yy}'' - 2\varphi + \hat{L}\varphi, \quad \hat{L}\varphi = x\varphi_{xx}'' + (2-x)\varphi_x'. \quad (9.8)$$

Here the eigenfunctions of the operator $\hat{L}$ are orthonormalized Laguerre polynomials $\lambda_n = L_n^1(x)$ with the superscript 1, which are defined by the relations

$$\hat{L}\lambda_n = -n\lambda_n, \quad \int_0^\infty \lambda_m\lambda_n w\, dx = \delta_{mn}, \quad w = xe^{-x},$$
$$\lambda_n = \frac{w^{-1}d^n}{n!(n+i)^{1/2}dx^n}(wx^n), \quad (9.9)$$

and since the set $\lambda_n(x)$ is complete, one can expand any function in terms of it. The general solution has the form

$$\varphi(x,y) = \sum_{n=0}^\infty \lambda_n(x) f_n(y), \quad \frac{d^2 f_n}{dy^2} = (n+2)f_n,$$
$$f_n = C_n \exp[-|y|(n+2)^{1/2}], \quad (9.10)$$

where the coefficients C_n should be obtained from definite boundary conditions, as is done below.

10. EXPANSION OF SPONTANEOUS PERTURBATIONS IN TERMS OF "MULTIPOLES"

To determine the coefficients C_n in Eq. (9.10), we note that the state of development of constrictions in the pinch under natural cosmic conditions should be preceded by a state of relatively calm constriction of the plasma and the formation of the pinch itself. Only after this will the constrictions in the pinch increase, and it is from these constrictions that plasma is intensely squeezed out in the form of jets.

In our model, we cannot examine this preliminary state, but imitating it we introduce the requirement that there should be no perturbations at the opposite time limit $t \to -\infty$. The absence of perturbations means that $\rho_* = 1$,

$v=0$. In our variables this corresponds to the "point" $x=\varepsilon$, $y=0$, at which the first formula, Eq. (9.6) acquires the form $T=-\varphi\varepsilon e^{-\varepsilon}$, so that the limit $t\to-\infty$ means that at this point the "potential" has the characteristic $\varphi\to+\infty$. In other words, the potential should be generated by point "charges" located at this point; thus, it is more correct to write Eq. (9.8) in the form of a Poisson equation

$$\Delta^*\varphi=-4\pi\rho,\ \rho(x,y)$$

$$=\int_0^\infty dx'\int_{-\infty}^{+\infty}dy'\rho(x',y')\,\delta(x'-x)\,\delta(y'-y), \tag{10.1}$$

and seek its solution in the form of an integral

$$\varphi(x,y)=\int_0^\infty dx'\int_{-\infty}^{+\infty}dy'\rho(x',y')\,G(x,y;\ x',y'). \tag{10.2}$$

Then, using the formulas

$$\delta(x'-x)=w(x')\sum_{n=0}^{\infty}\lambda_n(x)\,\lambda_n(x'),\quad \delta(y'-y)$$

$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[ik(y'-y)\right]dk, \tag{10.3}$$

it is easy to find the Green's function in the form of a sum

$$G=2\pi w(x')\sum_{n=0}^{\infty}(n+2)^{-1/2}\lambda_n(x)\lambda_n(x')$$
$$\times\exp[-|y-y'|(n+2)^{1/2}], \tag{10.4}$$

and the integral, Eq. (10.2), yields a general solution to Eq. (10.1) for a random distribution of "charges" ρ. Hereinafter, however, we will take into account that our charges are located only at the point $x=\varepsilon, y=0$. Thus, it is expedient to switch to new variables x_1, y_1, taking $x'=\varepsilon+x_1$, $y'=-y_1$. Introducing the notation

$$\rho_{\text{eff}}=2\pi w\rho,\quad S(\varepsilon,y)=\sum_{n=0}^{\infty}\frac{\lambda_n(x)}{(n+2)^{1/2}}\lambda_n(\varepsilon)\,Y^{(n+2)^{1/2}},$$

$$Y=e^{-|y|}, \tag{10.5}$$

we rewrite the general solution of Eq. (10.2) in a more suitable form

$$\varphi(x,y)=\iint\rho_{\text{eff}}(x_1,y_1)\,S(\varepsilon+x_1,\ y+y_1)\,dx_1dy_1. \tag{10.6}$$

Since the quantities x_1, and y_1 are considered to be small, we expand the last sum in a Taylor series

$$S(\varepsilon+x_1,\ y+y_1)$$
$$=S(x;\ \varepsilon,y)+\left(x_1\frac{\partial S}{\partial\varepsilon}+y_1\frac{\partial S}{\partial y}\right)+O(x_1^2,\ y_1^2)+\dots \tag{10.7}$$

Thus, we obtain an expansion of the "potential" φ in a series in terms of "multipoles," where the first term is equal to

$$\varphi^0=QS(x;\ \varepsilon,y),\quad Q=\iint\rho_{\text{eff}}(x_1,y_1)\,dx_1dy_1 \tag{10.8}$$

and will be nonzero only when the "total charge of the system" Q is nonzero. However, as in the nonrelativistic case, Eq. (8.10), one can show that solutions with $Q\neq0$ describe perturbations which are periodic over the length of the pinch, and these perturbations require corresponding periodic "primers" which, as we assume, can not arise under natural cosmic conditions. Thus, we will consider "Coulomb" solutions with $Q\neq0$ to be *unrealizable.*

The following terms of the expansion describe not periodic, but only local perturbations. The simplest of them will be a "dipole solution" in the form

$$\varphi^1=D\frac{\partial S}{\partial\varepsilon}=D\sum_{n=0}^{\infty}\frac{\lambda_n(x)}{(n+2)^{1/2}}\,Y^{(n+2)^{1/2}}\frac{\partial}{\partial\varepsilon}\lambda_n(\varepsilon),$$
$$D=\iint x_1\rho_{\text{eff}}\,dx_1dy_1. \tag{10.9}$$

Since $\lambda_0=1$ and $\partial\lambda_0/\partial\varepsilon=0$, the sum here begins with the term $Y^{\sqrt{3}}$, which, as is shown below, is what yields the "cosmic spectrum."

11. THE SPECTRUM OF ACCELERATED PARTICLES

To determine the spectrum we note that in length dz the number of particles is equal to $dN=\pi a^2n\gamma dz=F(u)du$. Writing $N_0=\pi a_0^2n_0$, we find the desired function from the formulas presented above

$$F(u)=\left(\frac{dN}{du}\right)_t=N_0\frac{\varepsilon}{\gamma}e^{-x}\left[(\varphi-\psi_y')^2+x(\psi-\varphi_y')^2\right]\left[\varphi-\psi_y'\right.$$
$$\left.+x(\psi-\varphi_y')\,\text{th}\,y\right]^{-1}, \tag{11.1}$$

which is what yields a spectrum of accelerated particles at a random moment in time. Hereinafter, however, we will assume that the typical local perturbation has the form of a disk located between two growing constrictions, which are broken at time $t=0$. At the same moment in time the disk is flattened into a "pancake" which contains the particles squeezed from the constrictions.

In this pancake $\rho_*\to\infty$, $x\to0$. Since at small $\langle\!\langle x\rangle\!\rangle$ one can obtain the expansion of potentials from the formulas in Eq. (9.7)

$$\varphi=\varphi_0(y)+x\varphi_1(y)+x^2\varphi_2(y)+\dots,$$

$$\psi=\frac{x}{2}\varphi_0'(y)+\frac{x^2}{3}\varphi_1'(y)+\dots \tag{11.2}$$

in the limit $x\to0$, which corresponds to the moment of the complete break in the constrictions, the distribution function is equal to $F_0=N_0\varepsilon\varphi_0(y)/\gamma$. In particular, for the solution of Eq. (10.9) we find

$$\varphi_0(y)=D\sum_{k=1}^{\infty}\left(\frac{k+1}{k+2}\right)^{1/2}Y^{(k+2)^{1/2}}\frac{d\lambda_k(\varepsilon)}{d\varepsilon},$$
$$F_0(y\gg1)=CE^{-(1+\sqrt{3})}. \tag{11.3}$$

The last asymptotic property holds true for all "dipole" local perturbations, and coincides with the galactic cosmic ray spectrum observed for the energy range $E = 10^{10}$–10^{15} eV, which, in our opinion, makes the examined "pinch mechanism" very plausible. It is reasonable to assume that this squeezing mechanism may accelerate ions to the maximum energy at which the Larmor radius of the ion is equal to the radius of the pinch, which in the ultrarelativistic case corresponds to the energy $E^{\max}_{(\mathrm{eV})} = 60 I_{\mathrm{A}}$. For example, an energy of 10^{15} eV is reached at a current of $2\cdot 10^{13}$ A.

At higher energies, $E = 10^{15}$–10^{17} eV, the exponent of the observed spectrum is approximately equal to $\nu_0 = 3.1$, and here, it appears that another acceleration mechanism is in effect. In the framework of our model we can assume that the next term, that is, the third "quadrupole" term of the expansion, Eq. (10.7), is "switched on." This should yield an exponent $\nu = 1 + \sqrt{4} = 3$, which is close to the observed value. The second possibility consists of the assumption that the most energetic galactic cosmic rays are generated by an inductive "electrodynamic" mechanism which yields a spectrum of the form of Eq. (7.3), in which the parameter p_0 which depends on the current I_0 plays a role. We do not know, however, of any statistics on currents in space, which complicates the procedure of averaging over currents and making comparisons with the observed spectrum. Finally, the third possibility consists of the assumption that in the 10^{18}–10^{20} eV energy range we observe particles of extragalactic origin with a mechanism of the type of Eq. (11.3), which again yields an exponent $\nu = 1 + \sqrt{3}$ in pinches assumed in Fig. 3. Then the 10^{15}–10^{17} eV energy range can be seen simply as a transition region from galactic to extragalactic particles (see Fig. 1).

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[1] V. L. Ginzburg and S. I. Syrovatskiĭ, *The Origin of Cosmic Rays*, Pergamon, Oxford, 1964 [Russ. original, Izd-vo AN SSSR, M, 1963.]

[2] E. G. Berezhko *et al.*, *The Generation of Cosmic Rays By Shock Waves* (in Russian), edited by A. I. Kuz'min, Nauka, Novosibirsk, 1988.

[3] N. V. Filippov, Zh. Eksp. Teor. Fiz. **76**, 1547 (1979) [Sov. Phys. JETP **49**, 785 (1979).

[4] B. A. Trubnikov, *The Physics of Plasma and the Problem of Controlled Thermonuclear Reactions, Volume 1* (in Russian), Izd-vo AN SSSR, M, 1958.

[5] H. Alfven, *Cosmic Plasma*, Reidel, Dordrecht, 1981 [Russ. transl., Mir, M., 1983.

[6] M. Van Dyke, *An Album of Fluid Motion, Parabolic Press*, Stanford, 1982, [Russ. transl.,] Mir, M., 1987.

[7] V. L. Ginzburg, Usp. Fiz. Nauk **124**, 307 (1978) [Sov. Phys. Usp. **21**, 155 (1978)]; Usp. Fiz. Nauk **155**, 185 (1988) [Sov. Phys. Usp. **31**, 491 (1988)].

[8] L. I. Dorman, *Experimental and Theoretical Foundations of the Astrophysics of Cosmic Rays* (in Russian), Nauka, M., 1975.

[9] V. S. Murzin, *Introduction to the Physics of Cosmic Rays* (in Russian), Izdatel'stvo MGU, M., 1988.

[10] Yu. A. Romanov and G. F. Fillippov, Zh. Eksp. Teor. Fiz. **40**, 123 (1961) [Sov. Phys. JETP **13**, 87 (1961)].

[11] A. A. Vedenov, E. P. Velikhov, and R. Z. Sagdeev, Yad. sintez. **3**, 465 (1962).

[12] W. E. Drummond and D. Pines, Nucl. Fusion Suppl. 3, 1049 (1962).

[13] A. A. Suchkov, *Galaxies Familiar and Enigmatic* (in Russian), Nauka, M., 1988.

[14] B. A. Trubnikov, Dokl. Akad. Nauk SSSR **118**, 913 (1958).

[15] V. M. Fadeev, I. F. Fvartskhava, and N. N. Komarov, Yad. sintez. **5**, 202 (1965).

[16] S. I. Syrovatskiĭ, A. G. Frank, and A. Z. Khodzhaev, Zh. Tekh. Fiz. **43**, 912 (1973) [Sov. Phys. Tech. Phys. **18**, 580 (1973)].

[17] B. A. Trubnikov, *The Physics of Plasma and the Problem of Controlled Thermonuclear Reactions, Volume 4* (in Russian), Izd-vo AN SSSR, M., 1958.

[18] B. A. Trubnikov, Fiz. Plazmy **12**, 468 (1986). [Sov. J. Plasma Phys. **12**, 271 (1986)]

[19] S. V. Bulanov and S. I. Syrovatskiĭ, Trudy FIAN SSSR **74**, 88 (1974) [Proc. (Tr.) P. N. Lebedev Phys. Inst. Acad. Sci. USSR **74** (1974)].

[20] G. A. Rozanova, *Numerical Modeling of Plasma in the Final Stage of Pinch Compression* (in Russian), Author's abstract to the dissertation for Candidacy in Physical and Mathematical Sciences, IAÉ, M., 1989.

[21] B. A. Troubnikov, V. P. Vlasov, and S. K. Zhdanov, *Proc. 1989 Intern. Conf. on Plasma Phys., Vol. 1*, New Delhi, 1989.

[22] S. K. Zhdanov and B. A. Trubnikov, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 178 (1986) [JETP Lett. **43**, 226 (1986)].

[23] B. A. Trubnikov and S. K. Zhdanov, Phys. Rep. **155**, 137 (1987).

[24] B. A. Trubnikov and S. K. Zhdanov, *Proc. of the Intern. Workshop on Plasma Physics and Turbulence, Vol. 2*, Kiev, 1987.

[25] S. I. Ananin and V. V. Vikhrev, Preprint IAÉ-3299/6, (In Russian), M., 1980.

[26] V. P. Vlasov, S. K. Zhdanov, and B. A. Trubnikov, Preprint IAÉ-4828/6, (In Russian) M., 1989.

[27] V. P. Vlasov, S. K. Zhdanov, and B. A. Trubnikov, Pis'ma Zh. Eksp. Teor. Fiz. **49**, 581 (1989) [JETP Lett. **49**, 667 (1989)].

[28] K. Hirano *et al.*, *Proc. of the 11th Intern. Conf. on Plasma Physics and Controlled Nuclear Fusion Research* IAEA-CN-47/D-IV-5-1, Kyoto, 1986.

[29] M. A. Raadu, Phys. Rep. **178**, 25 (1989).

[30] A. L. Peratt, IEEE Trans. Plasma Sci. **PS-18**, 26 (1990).

[31] A. L. Peratt, IEEE Trans. Plasma Sci. **PS-14**, 639 (1986).

[32] F. Yusef-Zadeh, M. Morris, and D. Chance, Nature (London) 310, 557 (1984).

[33] R. Beck, IEEE Trans. Plasma Sci. **PS-18**, 33 (1990).

[34] G. Benford, Mon. Not. R. Astron. Soc. **183**, 29 (1978).

[35] K. Graĭzen, *The Physics of Cosmic X-Rays, Gamma-Rays and High Energy Particles* [Russ. Transl., Mir, M., 1975.]

[36] IEEE Trans. Plasma Sci. **PS-18**, 1, 1990.

[37] V. N. Fedorenko, On the Cosmic-Ray Generated Magnetohydrodynamic Turbulence, Preprints 1441 and 1442, FTI AN SSSR, L, 1990.

[38] B. A. Troubnikov, *Proc. of Workshop on Plasma Astrophysics*, ESA SP-311, Telavi, Georgia, USSR, 1990.

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